Randomised Algorithms

Sometimes Randomness is faster

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Before diving deep into Randomized Algorithms...

What is a Randomized Algorithm?

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Real-Life Applications of Randomness

- How does **NASA** make decisions for Mars Rovers?

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These examples reveal the power of randomness!

Notation

Definition (Randomized Algorithms)

A randomized algorithm is an algorithm that incorporates randomness as part of its operation.

Deterministic Algorithms

Aspect	Deterministic Algorithm
Definition	Predefined set of rules, no randomness
Objective	Exact solutions with good worst-case behavior
Complexity	Often more complex with nuanced correctness proofs
Performance	Good in worst-case scenarios
Error Tolerance	Guaranteed correctness

Table: Characteristics of Deterministic Algorithms

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What is Min-Cut?

Goal

Divide graph $G = (V, E)$ into two parts. Minimize edges between parts.

Uses

- Break networks.
- Cluster data.
- Optimize circuits.

Naive Algorithm

Steps

- Randomly pick an edge.

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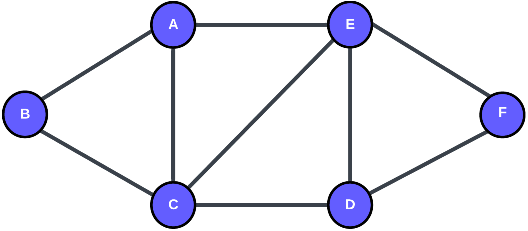
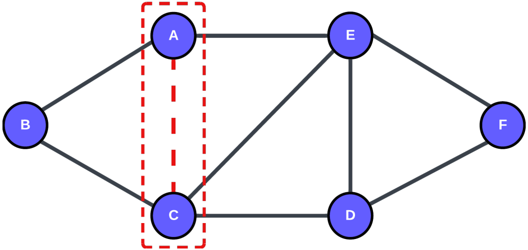
- Merge the two vertices connected by the chosen edge.
- Repeat the process until only two vertices remain.
- Perform this procedure for all possible edge combinations to find the minimum cut.

Simulation



Order of Selection: AC

Simulation



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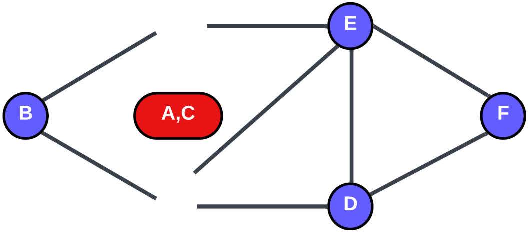

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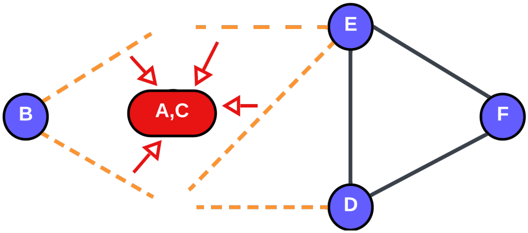

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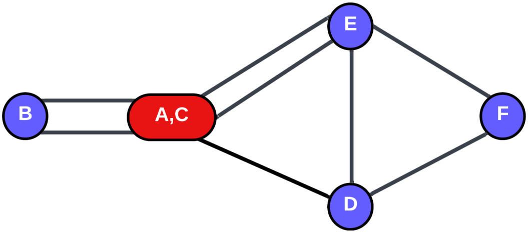

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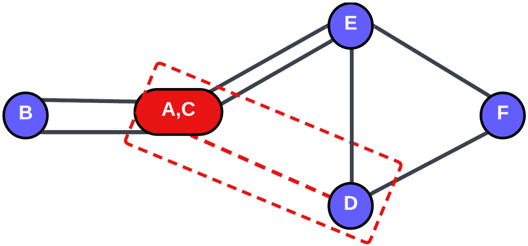
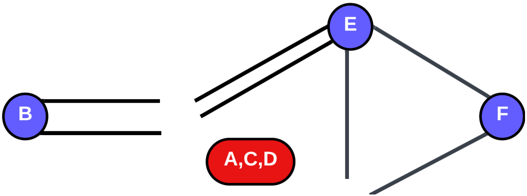


Order of Selection: AC,AC-D



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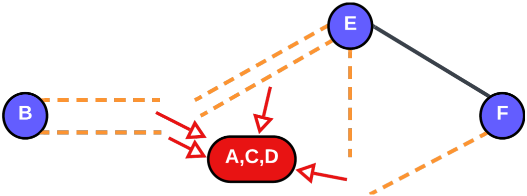

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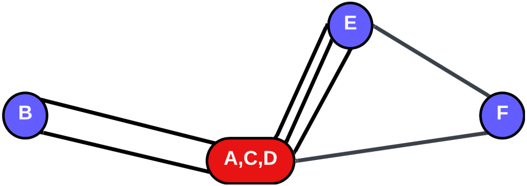

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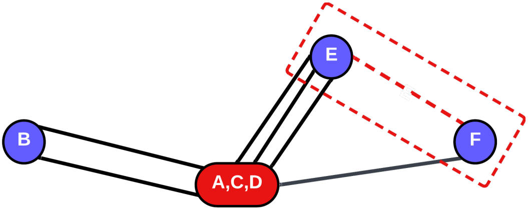
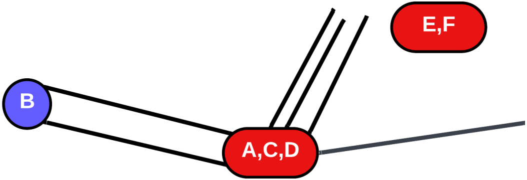


Order of Selection: AC,ACD,E-F

Simulation



Order of Selection: AC,ACD,EF

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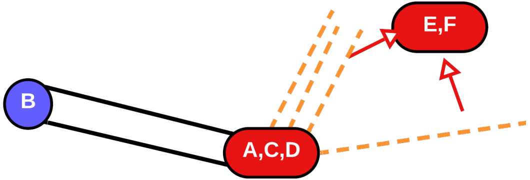

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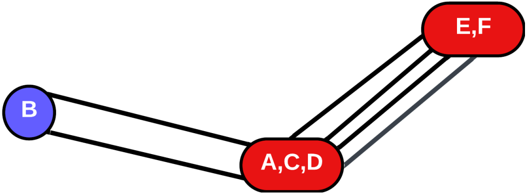

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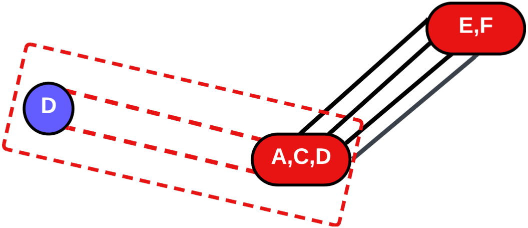
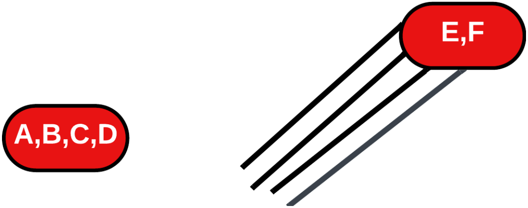


Order of Selection: AC,ACD,EF,ACD-B

Simulation

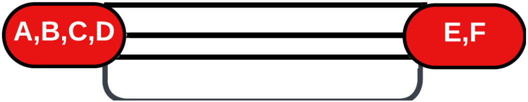


Order of Selection: AC,ACD,EF,ABCD

Simulation

Order of Selection: AC,ACD,EF,ABCD-EF

Simulation



Min-Cut: ABCD and EF

Naive Algorithm

Problems

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Problems

- Highly time-consuming.
- Requires exploring all possible combinations.
- Time complexity: $\mathcal{O}(n^2 \cdot 2^m)$, where n is the number of vertices and m is the number of edges.

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Next?

Let's improve it using **randomized cuts** for better efficiency.

Towards Randomized Min-Cut

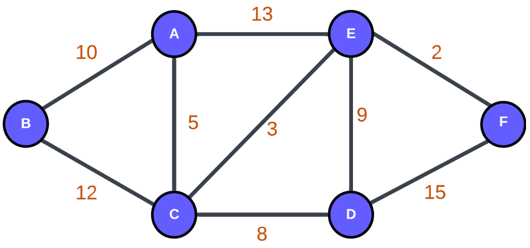
Key Idea: Randomized Algorithm

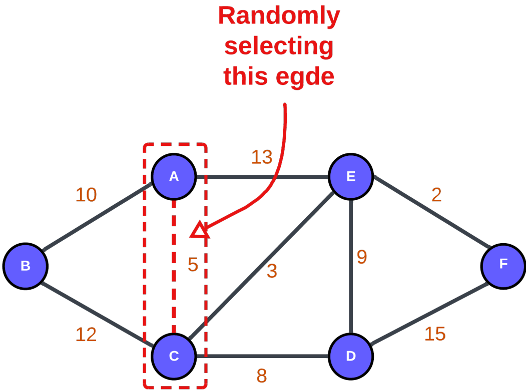
Randomness can help:

- Explore diverse contraction orders.
- Increase the chance of finding a true min-cut.

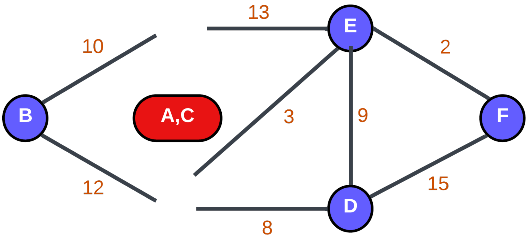
A Monte Carlo algorithm guarantees high success probability.

Simulation



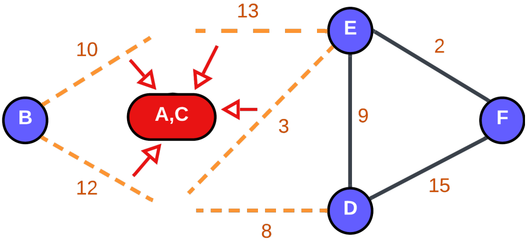


Selected Edge: AC

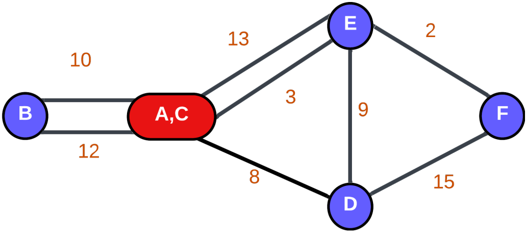


Merge A,C

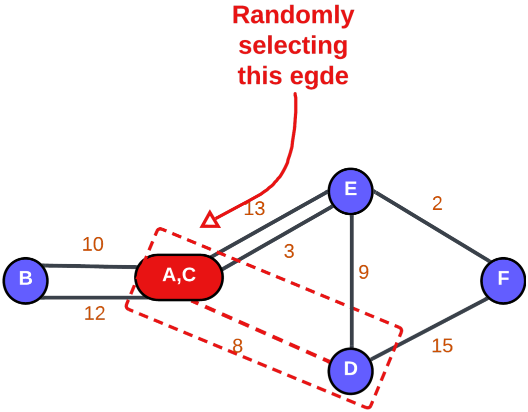
Simulation



Merge Adjacent edges of A,C

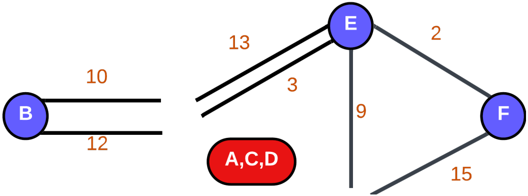


Merged Adjacent edges of A,C

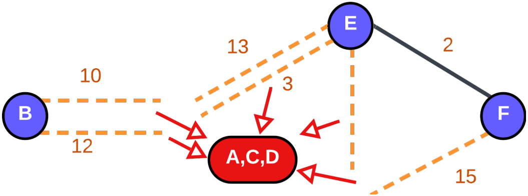


Select edge A,C,D

Simulation

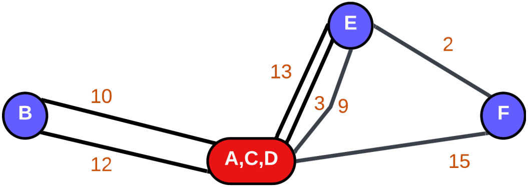


Merge the vertices A,C,D



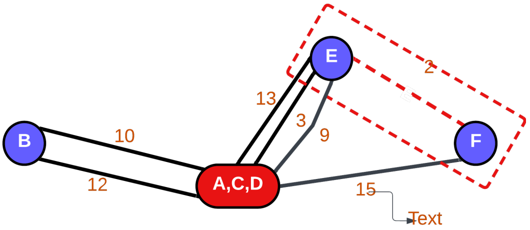
Merge the adjacent edges of A,C,D

Simulation



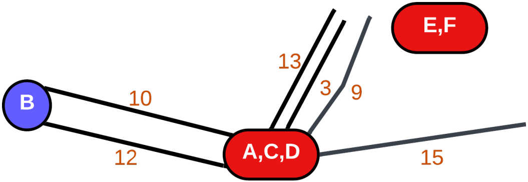
Merged the edges

Simulation



Select the edge E,F

Simulation



Merge the vertices E,F

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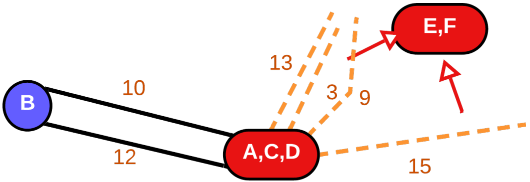
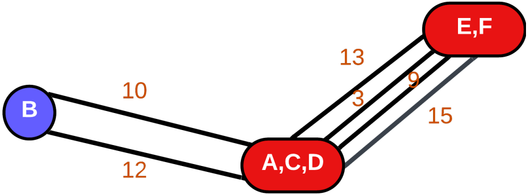


Merge the Adjacent edges



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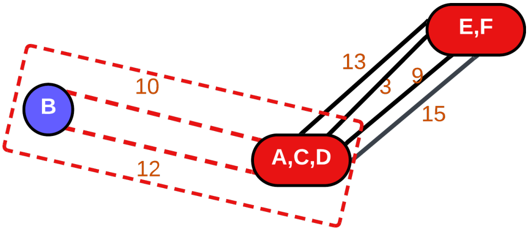
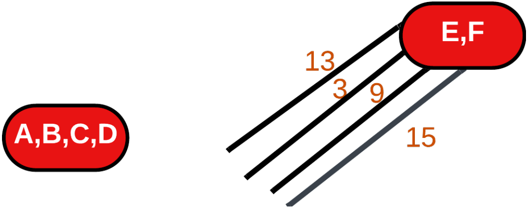
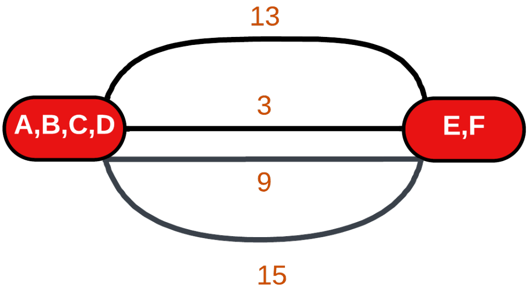


Select Edge B, ACD



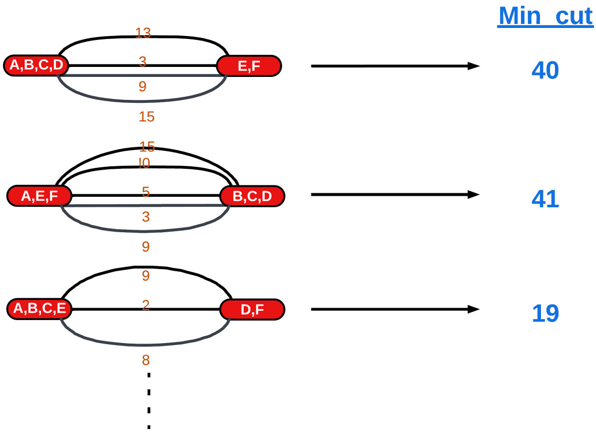
Merge the vertices B,ACD

Simulation



Only Two Vertices Remain;

Choose the Minimum



Time Complexity: Min-Cut Algorithm

- Efficient Algorithm for finding Minimum Cut.
- **Time Complexity:** $O(E \cdot \log(V)^2)$
  - E: **Edges** in the graph.
  - V: **Vertices** in the graph.

Summary

- Explored **Min-Cut Algorithm**.
- Key Aspects:

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- Key Aspects:
  - Divide graph into **disjoint subsets**.
  - Use **flow-based methods**.
- **Efficient:** $O(E \cdot \log(V)^2)$ for large, sparse graphs.
- Conclusion: Practical, impactful for graph problems.

Pivot Sort!!

Divide and Conquer:

- Recursive partitions
- Independent sorts

Time Complexity:

- Best/Average Case: $O(n \log n)$ (Balanced splits)
- Worst Case: $O(n^2)$ (Highly unbalanced splits)

Key Insight

Pivot choice impacts efficiency.
Poor Splits=?

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QuickSort Algorithm

Algorithm QuickSort (Divide and Conquer)

1: **Input:** Array A, indices $low, high$
2: **Output:** Sorted Array
3: **if** $low < high$ **then**
4: $pivot \leftarrow$ **Partition**$(A, low, high)$
5: QuickSort(A, low, pivot - 1)
6: QuickSort(A, pivot + 1, high)
7: **end if**

Algorithm Partition (Pivot Selection)

1: **Input:** Array A, indices $low, high$
2: **Output:** Pivot index
3: $pivot \leftarrow A[high]$
4: $i \leftarrow low - 1$
5: **for** $j = low$ **to** $high - 1$ **do**
6: **if** $A[j] \leq pivot$ **then**
7: $i \leftarrow i + 1$
8: Swap $A[i], A[j]$
9: **end if**
10: **end for**
11: Swap $A[i+1], A[high]$
12: **return** $i + 1$

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Example Simulation

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Pivot: ■ **i:** ■ **j:** ■ **Swap:** ■

i=-1	5	4	2	1	3

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i ← low − 1
for j = low to high − 1 do
    if A[j] ≤ pivot then
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        Swap(A[i], A[j])
    end if   i not getting updated
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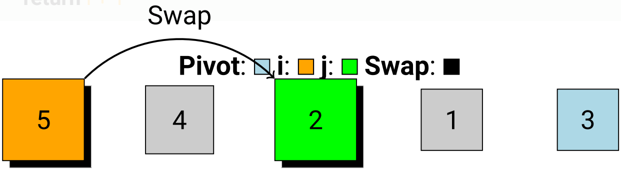

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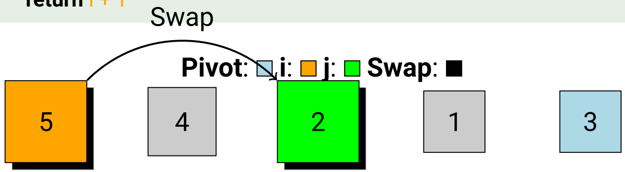

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Pivot: ▢ i: ▢ j: ▢ Swap: ■

2	4	5	1	3

Example Simulation

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pivot ← A[high]
i ← low − 1
for j = low to high − 1 do
    if A[j] ≤ pivot then
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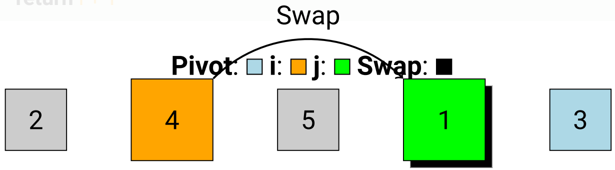

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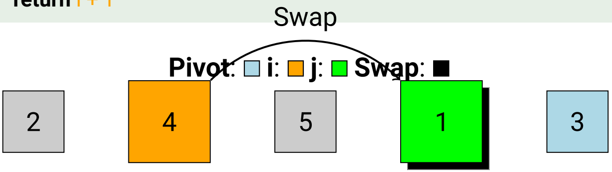

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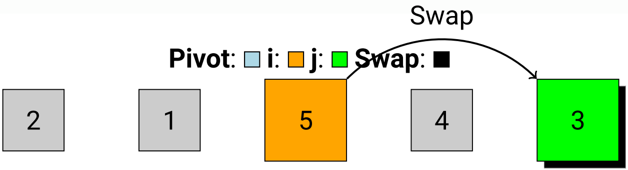

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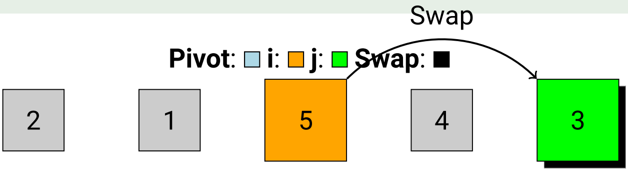

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New Pivot

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Continuing QuickSort

Step 1: Pick a pivot
Step 2: Partition subarrays
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Final sorted array: Coming next!

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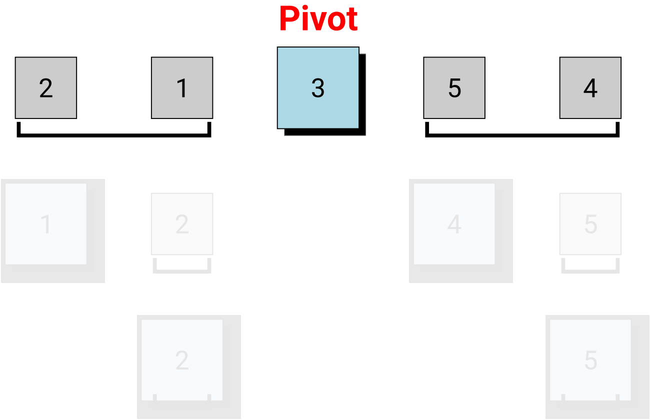

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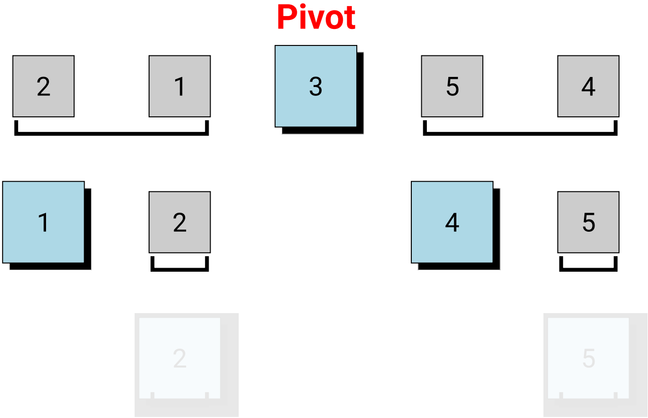

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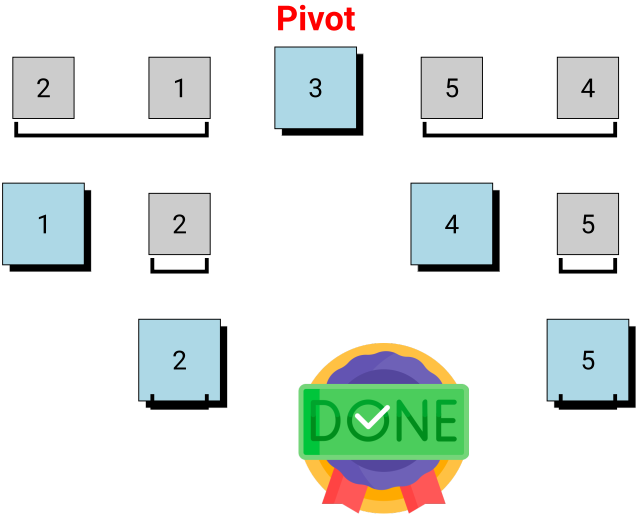
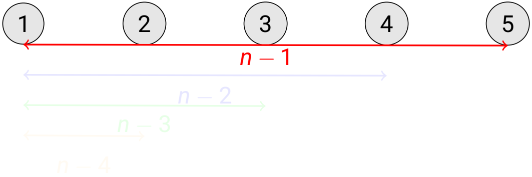


Worst Case Analysis



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Key Points:

- **Pivot Speciality**
- **When!!**
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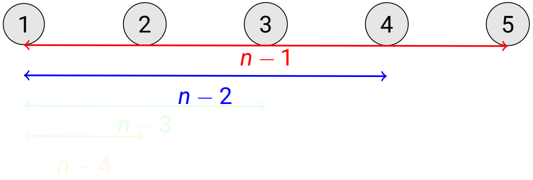

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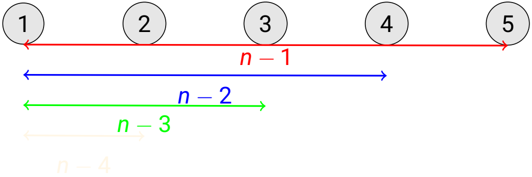

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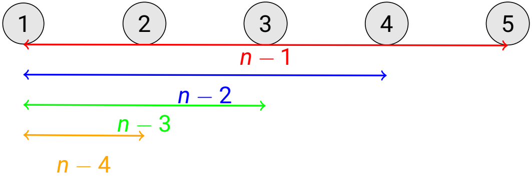

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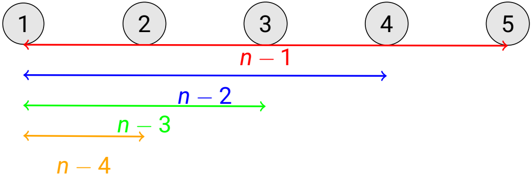

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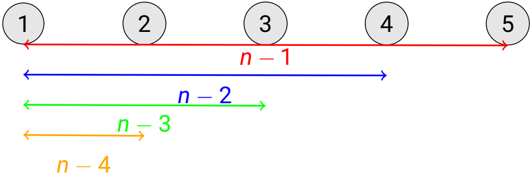

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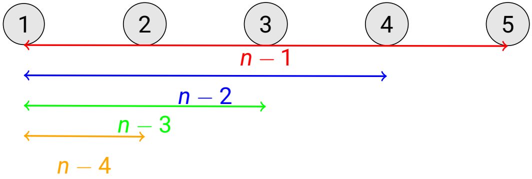

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Key Idea

Random Pivot selection

Avoid Worst-Case partitions

Objective

- **Balanced Partition** (on average)
- **No Sorted/Reverse** worst-case

How It Works

1. Pick **Random Pivot**
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Randomized QuickSort Algorithm

Algorithm QuickSort (Divide and Conquer)

Input: Array A, indices $low, high$
Output: Sorted Array
if $low < high$ **then**
    $pivot \leftarrow$ **Partition**$(A, low, high)$
    QuickSort(A, low, pivot - 1)
    QuickSort(A, pivot + 1, high)
end if

Algorithm Partition (Pivot Selection)

Input: Array A, indices $low, high$
Output: Pivot index

$$\boxed{pivot \leftarrow Random[low, high]}$$

$i \leftarrow low - 1$
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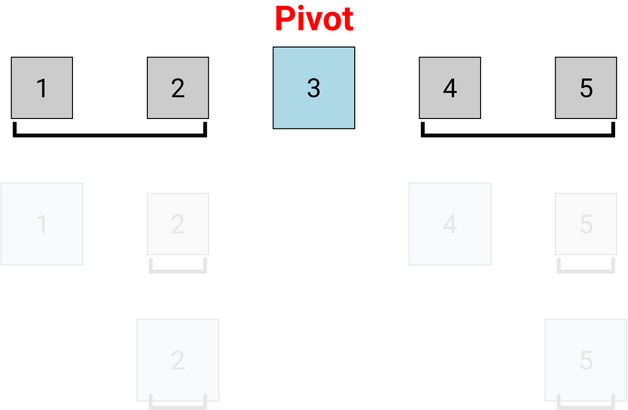
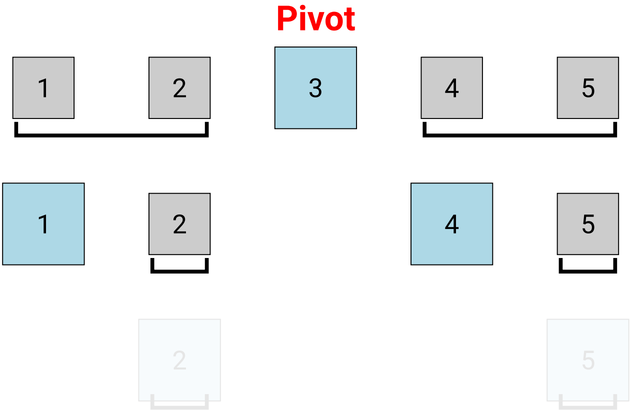
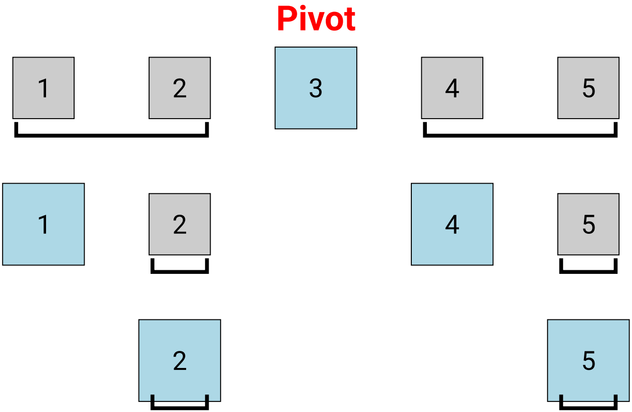
Worst Case Review




Pivot
1
2
3
4
5
1
2
4
5
2
5

Worst Case Review



Pivot Choice Order: 3 1 2 4 5

Performance Overview

- **Better Pivot Selection:**
  - More uniform distribution.
  - Reduces probability.
- **Average Case:**
  - Same.
  - Avoiding $O(n^2)$.

Comparison: Deterministic vs Randomized Q

Aspect	Deterministic Q	Randomized Q
Pivot Selection	Fixed (e.g., first/last)	Randomly chosen
Worst Case	$O(n^2)$	Low probability of $O(n^2)$
Average Case	$O(n \log n)$	$O(n \log n)$

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Trade-Off

Probabilistic outcomes.

- Do you think randomization is always beneficial?
- Are there cases where deterministic algorithms outperform randomized ones?

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Why Randomized Algorithms?

- ϟ Use randomness for faster problem-solving.

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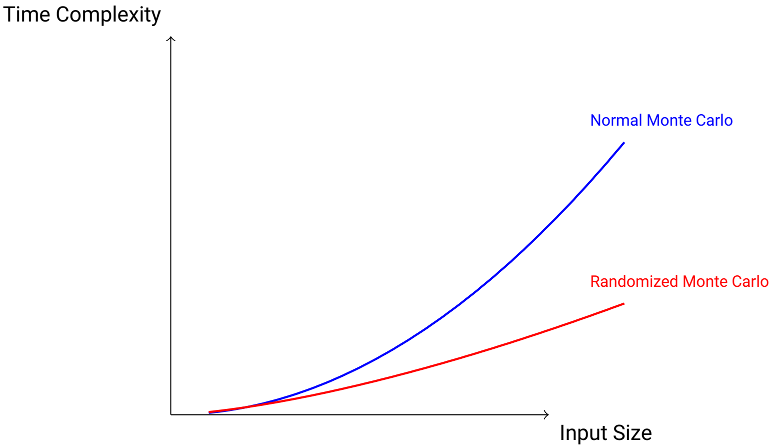
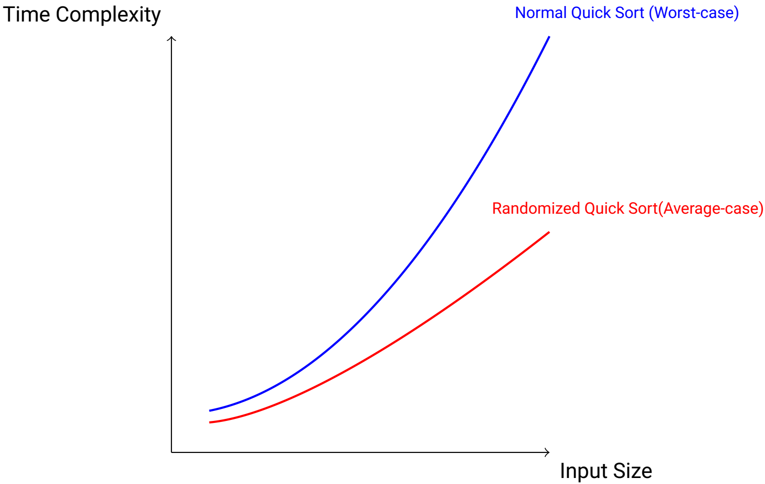
Comparison of time complexity of Monte Carlo Algorithm



Comparison of time complexity of Quick Sort



Fields Transformed by Randomization

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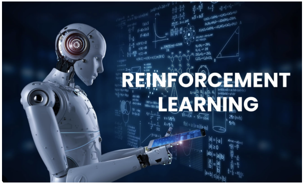

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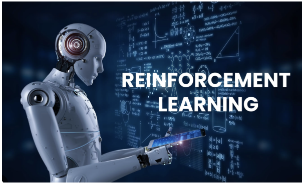

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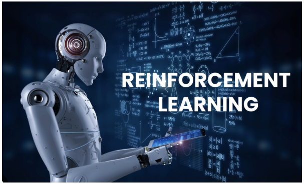

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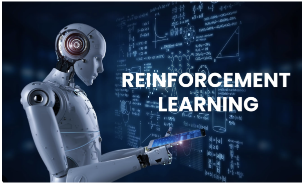
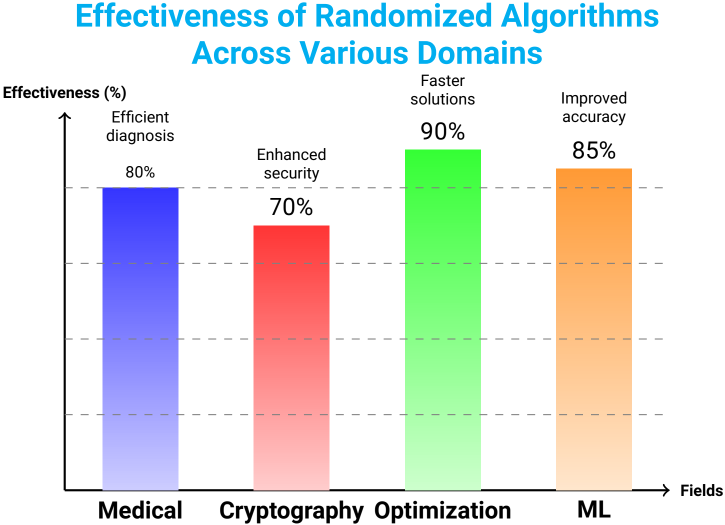


Effectiveness of Randomized Algorithms Across Various Domains

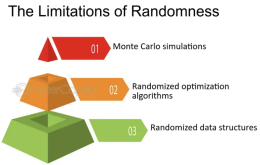


Advantages vs. Challenges

Advantages

- ✔ Avoid worst-case outcomes probabilistically.

Challenges



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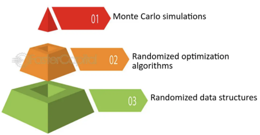
The Limitations of Randomness



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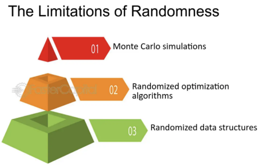

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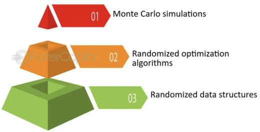

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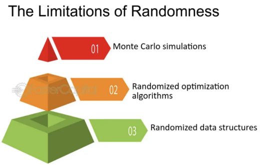

”In randomness, there is the seed of order.”

– Someone Inspirational

Thank You for Your Attention! Questions or

Comments?