33

Electromagnetic Waves

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP33.1** **Conceptualize** Reread the material on solar sailing in Section 33.5 to make sure you understand the concept completely.

Categorize We will model the spacecraft as a *particle in equilibrium* to find the minimum area of solar sail needed.

Analyze (a) Set up a force equation for the spacecraft from the particle in equilibrium model:

$$\sum F = 0 \quad \rightarrow \quad F_{\text{radiation}} - F_{\text{gravity}} = 0 \qquad (1)$$

For the force due to the radiation from the Sun, use Equation 33.37 for the radiation pressure:

$$F_{\text{radiation}} = PA = \frac{2S}{c}A \qquad (2)$$

The magnitude of the Poynting vector is the intensity of the light from the Sun:

$$F_{\text{radiation}} = \frac{2I}{c}A = \frac{2P_{\text{Sun}}}{4\pi r^2 c}A = \frac{P_{\text{Sun}}}{2\pi r^2 c}A \qquad (3)$$

where r is the distance of the sail from the Sun. Now, the gravitational force on the spacecraft is given by

$$F_{\text{gravity}} = mg_{\text{Sun}} = mG\frac{M_{\text{Sun}}}{r^2} \qquad (4)$$

where we have used a solar version of Equation 13.6 for the acceleration due to gravity from the Sun. Substitute Equations (3) and (4) into Equation (1):

$$\frac{P_{\text{Sun}}}{2\pi r^2 c}A - mG\frac{M_{\text{Sun}}}{r^2} = 0 \quad \rightarrow \quad A = 2\pi cG\frac{mM_{\text{Sun}}}{P_{\text{Sun}}} \qquad (5)$$

Substitute numerical values:

$$A = 2\pi\left(3.00 \times 10^{8}\text{ m/ s}\right)\left(6.674 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{ kg}^2\right)\frac{\left(15\,000\text{ kg}\right)\left(1.99 \times 10^{30}\text{ kg}\right)}{3.85 \times 10^{26}\text{ W}}$$

$$= 9.75 \times 10^{6}\text{ m}^2 = \boxed{9.75\text{ km}^2}$$

(b) What about your supervisor's question about requiring less area as you move farther away from the Sun? Notice that the distance to the Sun does not appear in Equation (5). $\boxed{\text{The intensity of solar radiation and the gravitational force both fall off as } 1/r^2\text{, so we need the same area of sail at all distances from the Sun.}}$

Finalize Notice that the sail is very large. Despite this size, solar sailing is receiving serious consideration. The sail of the *IKAROS* spacecraft spins while is use and has an area of 196 m^2.]

Answers: (a) 9.75 km^2 (b) Solar radiation and gravity both fall off as $1/r^2$, so the same area of sail is needed at all distances from the Sun

***TP 33.2** **Conceptualize** The surface temperature of the planet will be determined by a simple energy balance equation: light from the star coming in and infrared radiation from the planet surface going out. The gravity at the surface will depend on the mass and radius of the planet.

Categorize The planet is modeled as a *nonisolated system in steady state* for *energy*.

Analyze Write the appropriate reduction of Equation 8.2 for the planet:

$$0 = T_{\text{ER}} \qquad (1)$$

The only remaining term in the equation consists of two transfers of energy by electromagnetic radiation:

$$0 = T_{\text{ER}}(\text{in}) + T_{ER}(\text{out}) \quad \rightarrow \quad T_{\text{ER}}(\text{in}) = -T_{ER}(\text{out}) \qquad (2)$$

If we differentiate Equation (2) with respect to time, we have

$$P_{\text{ER}}(\text{in}) = -P_{ER}(\text{out}) \qquad (3)$$

The input radiation is from the star. The power absorbed by the planet will be 70.0% (because 30.0% is reflected away) of that reaching the planet. Because the star radiates in all directions, only a small portion of the output power of the star will reach the planet. That portion is the fraction with the cross sectional (circular) area of the planet of radius R in the numerator divided by the surface area through which all the radiation passes: that of a sphere whose radius is the orbital radius$a$ of the planet:

$$P_{\text{ER}}(\text{in}) = (0.700)eL_{\text{star}}\left(\frac{\pi R^2}{4\pi a^2}\right) = (0.175)e\frac{L_{\text{star}}R^2}{a^2} \qquad (4)$$

where e is the emissivity of the surface, which is the same as the absorptivity. (See Section 19.6.) The output power from the surface of the planet is given by Stefan's law, Equation 19.21:

$$P_{ER}(\text{out}) = -\sigma AeT^4 \qquad (5)$$

where the minus sign indicates that the energy is leaving the system by this radiation and A is the surface area of the planet. Substitute for the surface area:

$$P_{ER}(\text{out}) = -\sigma\left(4\pi R^2\right)eT^4 = -4\pi\sigma R^2 eT^4 \qquad (6)$$

Substitute Equations (4) and (6) into Equation (3) and solve for the surface temperature of the planet:

$$(0.175)e\frac{L_{\text{star}}R^2}{a^2} = -\left(-4\pi\sigma R^2 eT^4\right) \quad \rightarrow \quad T = \sqrt[4]{\frac{(0.043\,75)L_{\text{star}}}{\sigma\pi a^2}} \qquad (7)$$

So, for example, for GJ 436b, find the surface temperature by substituting numerical values:

$$T = \sqrt[4]{\frac{(0.043\,75)(0.025)(3.85\times10^{26}\ \mathrm{W})}{(5.67\times10^{-8}\ \mathrm{W/\,m^2\cdot K^4})\pi\left[(0.028\,9\ \mathrm{AU})(1.50\times10^{11}\ \mathrm{m/\,AU})\right]^2}}$$
$$= 596\ \mathrm{K}$$

Use this same procedure for all the planets in the table and use Equation 13.6 to find the surface gravity. The results are shown in the last two columns of the following table:

Planet	Mass M (M_{Earth})	Radius R (R_{Earth})	Orbital Radius a (AU)	Luminosity L_{star} of Star (P_{Sun})	Surface Temperature (K)	Surface Gravity (m/s²)
GJ 436b	22.3	4.17	0.0289	0.025	594	12.6
GJ 674b	12.7	12.4	0.039	0.016	459	0.811
Gliese 581c	5.40	1.50	0.073	0.013	318	23.6
HAT-P-11b	26.2	4.63	0.053	0.26	790	12.0
GJ 3470b	13.9	3.14	0.035 6	0.029	557	13.8
Kepler-42b	2.86	0.768	0.011 6	0.002 4	523	47.6
Kepler-42c	1.91	0.713	0.006 0	0.002 4	728	36.9
Kepler-42d	0.955	0.209	0.015 4	0.002 4	454	215
Kepler-138b	21.3	0.571	0.074 6	0.060	461	641
HD 219134b	3.82	1.57	0.038 5	0.28	944	15.2
Kepler-452b	2.86	1.50	1.05	1.2	260	12.5

There are only two candidate planets for evolution like ours based on temperature: Gliese 581c (318 K) and Kepler-452b (260 K). However, Gliese 581c has a surface gravity of 23.6 m/s², which is 2.4 times that on

Earth. Imagine the stress on your spine and other bones in this type of gravity. On the other hand, Kepler-452b has a surface gravity of 12.5 m/s^2, only 28% larger than ours. $\boxed{\text{Therefore, there is only one exoplanet, Kepler-452b, that meets both criteria for the evolution of life like ours.}}$

Finalize A calculation of the surface temperature of Earth (without regard for the atmosphere) results in a value of 255 K. Therefore, Kepler-452b is very similar to Earth based on these criteria. Of course, we have no idea of the composition of the atmosphere of this exoplanet, so we can't count on life like ours having evolved there!]

Answer: only one: Kepler-452b

***TP 33.3** **Conceptualize** In both cases, the astronaut is causing something to move in a direction away from the ISS: either the laser, or light from the laser. As a result, she will move in the opposite direction, back toward the station.

Categorize When the laser is thrown, the system of the laser and the astronaut is modeled as an *isolated system* for *momentum*. Then the astronaut is modeled as a *particle under a constant velocity*. With the photon rocket, the system of the laser, the astronaut and the emitted light is modeled as an *isolated system* for *momentum*. Then the astronaut–laser combination is modeled as a *particle under a net force* and a *particle under constant acceleration*.

Analyze Let us look first at throwing the laser. All velocities and momenta are along what we define as the x axis. Conservation of momentum for an isolated system gives us

$$p_i = p_f \quad \rightarrow \quad (m_a + m_l)(0) = m_a v_a + m_l v_l \quad \rightarrow \quad v_a = -\frac{m_l}{m_a} v_l \tag{1}$$

Then use the particle under constant velocity model to find the time at which the astronaut arrives at the station if the time $t = 0$ is just as the laser is thrown,

$$x_f = x_i + v_a t \quad \rightarrow \quad t = \frac{x_f - x_i}{v_a} = \frac{0 - x_i}{\left(-\frac{m_l}{m_a} v_l\right)} = \frac{m_a x_i}{m_l v_l} \tag{2}$$

Substitute numerical values:

$$t = \frac{(147\text{ kg})(10.0\text{ m})}{(103\text{ kg})(0.200\text{ m/ s})} = 71.4\text{ s} = \boxed{1.19\text{ min}}$$

Now consider using the laser as a photon rocket. Use the particle under a net force model to find the acceleration of the astronaut as the laser is operating:

$$F = (m_a + m_l)a \quad \rightarrow \quad a = \frac{F}{m_a + m_l} = \frac{1}{m_a + m_l}\frac{dp_a}{dt} \tag{3}$$

where p_a is the momentum of the astronaut and the laser. Applying the isolated system model for momentum to the system of the laser, the astronaut and the emitted light, the rate of change of momentum of the astronaut and laser must be the same in magnitude as that of the emitted radiation, but opposite in direction. Equation 33.34 gives the momentum transported to a surface due to complete absorption. The same equation represents the momentum transferred away from a light source with complete emission. Therefore, using Equation (3) and Equation 33.34,

$$a = \frac{1}{m_a + m_l}\frac{dp_a}{dt} = \frac{1}{m_a + m_l}\left(-\frac{dp_{\text{rad}}}{dt}\right)$$
$$= -\frac{1}{m_a + m_l}\frac{d}{dt}\left(\frac{T_{\text{ER}}}{c}\right) = -\frac{1}{(m_a + m_l)c}\frac{dT_{\text{ER}}}{dt} = -\frac{P}{(m_a + m_l)c} \qquad (4)$$

where P is the power of the radiation emitted by the laser. Equation (4) shows that the acceleration of the astronaut and laser is constant, so we apply the particle under constant acceleration model. The position of the astronaut is given by Equation 2.16:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}at^2 \qquad (5)$$

Enter the information that the astronaut begins from rest at a nonzero value x_i and ends up at $x_f = 0$, and solve for the time at which the astronaut arrives at this position:

$$0 = x_i + 0 + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{-2x_i}{a}} \qquad (6)$$

Substitute the acceleration from Equation (4):

$$t = \sqrt{\frac{-2x_i}{\left(-\frac{P}{(m_a + m_l)c}\right)}} = \sqrt{\frac{2x_i(m_a + m_l)c}{P}} \qquad (7)$$

Substitute numerical values:

$$t = \sqrt{\frac{2(10.0\text{ m})(250\text{ kg})(3.00 \times 10^8\text{ m/s})}{9.50 \times 10^4\text{ W}}} = 3.97 \times 10^3\text{ s} = \boxed{1.10\text{ h}}$$

Using the laser as a photon rocket requires a time interval longer than that of the remaining oxygen, so the astronaut would not survive.

Finalize The only way to save the astronaut is by throwing the laser, in which case, she arrives at the station in a little over a minute. You would have an excellent argument that saving a life is more important than saving a piece of hardware.

Answer: Throwing the laser causes her to return to the ISS in 1.19 min. Using the laser would have required 1.10 h to return to the ISS, longer than the remaining oxygen would last.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 33.1 Displacement Current and the Generalized Form of Ampère's Law

P33.1 For the capacitor,

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$$

(a) $$\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \frac{0.200\ \text{A}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left[\pi\left(10.0\times10^{-2}\ \text{m}\right)\right]}$$

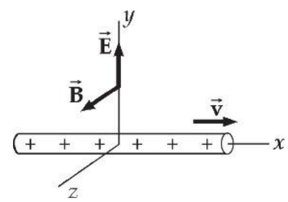
$$= \boxed{7.19\times10^{11}\ \text{V/m}\cdot\text{s}}$$

(b) $$\oint B\cdot ds = \epsilon_0\mu_0\frac{d\Phi_E}{dt}: \qquad 2\pi rB = \cancel{\epsilon_0}\mu_0\frac{d}{dt}\left[\frac{Q}{\cancel{\epsilon_0}A}\cdot\pi r^2\right]$$

$$B = \frac{\mu_0 Ir}{2A} = \frac{\mu_0(0.200\ \text{A})\left(5.00\times10^{-2}\ \text{m}\right)}{2\left[\pi\left(10.0\times10^{-2}\ \text{m}\right)^2\right]} = \boxed{2.00\times10^{-7}\ \text{T}}$$

Section 33.2 Maxwell's Equations and Hertz's Discoveries

P33.2 (a) The very long rod creates the same electric field that it would if stationary. We apply Gauss's law to a cylinder, concentric with the rod, of radius r = 20.0 cm and length ℓ:



ANS. FIG. P33.2

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$E(2\pi r\ell)\cos 0° = \frac{\lambda\ell}{\epsilon_0}$$

$$\vec{\mathbf{E}} = \frac{\lambda}{2\pi\epsilon_0 r} \text{ radially outward}$$

$$= \frac{35.0\times 10^{-9}\ \text{C/m}}{2\pi\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.200\ \text{m})}\hat{\mathbf{j}}$$

$$= \boxed{3.15\times 10^3\,\hat{\mathbf{j}}\ \text{N/C}}$$

(b) The charge in motion constitutes a current of

$$(35.0 \times 10^{-9}\ \text{C/m}) \times (15.0 \times 10^6\ \text{m/s}) = 0.525\ \text{A}$$

This current creates a magnetic field.

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$$

$$= \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(0.525\ \text{A})}{2\pi(0.200\ \text{m})}\hat{\mathbf{k}} = \boxed{5.25\hat{\mathbf{k}}\times 10^{-7}\ \text{T}}$$

(c) The Lorentz force on the electron is $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$.

$$\vec{\mathbf{F}} = \left(-1.60\times10^{-19}\text{ C}\right)\left(3.15\times10^{3}\,\hat{\mathbf{j}}\text{ N/C}\right) + \left(-1.60\times10^{-19}\text{ C}\right)\left(240\times10^{6}\,\hat{\mathbf{i}}\text{ m/s}\right) \times\left(5.25\times10^{-7}\,\hat{\mathbf{k}}\text{ T}\right)$$

$$\vec{\mathbf{F}} = 5.04\times10^{-16}\left(-\hat{\mathbf{j}}\right)\text{ N} + 2.02\times10^{-17}\left(+\hat{\mathbf{j}}\right)\text{ N} = \boxed{4.83\left(-\hat{\mathbf{j}}\right)\times10^{-16}\text{ N}}$$

P33.3 The net force on the proton is the Lorentz force, as described by

$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \quad \text{so that} \quad \vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right]$$

Taking the cross product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$,

$$\vec{\mathbf{v}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\hat{\mathbf{j}} + 200(0.300)\hat{\mathbf{k}}$$

Then, $$\vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right] = \left(\frac{1.60\times10^{-19}}{1.67\times10^{-27}}\right)\left[50.0\,\hat{\mathbf{j}} - 80.0\,\hat{\mathbf{j}} + 60.0\,\hat{\mathbf{k}}\right]\text{ m/s}^2$$

$$= \boxed{\left(-2.87\times10^{9}\hat{\mathbf{j}} + 5.75\times10^{9}\hat{\mathbf{k}}\right)\text{ m/s}^2}$$

Section 33.3 Plane Electromagnetic Waves

P33.4 From Equation 33.14,

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{27.33\times10^{6}\text{ Hz}} = \boxed{11.0\text{ m}}$$

P33.5 (a) Since the light from this star travels at 3.00×10^8 m/s, the last bit of light will hit the Earth in

$$t = \frac{d}{c} = \frac{6.44\times10^{18}\ \text{m}}{2.998\times10^{8}\ \text{m/s}} = 2.15\times10^{10}\ \text{s} = \boxed{681\ \text{years}}$$

(b) From Table C.4 (in Appendix C of the textbook), the average Earth-Sun distance is $d = 1.496 \times 10^{11}$ m, giving the transit time as

$$t = \frac{d}{c} = \left(\frac{1.496\times10^{11}\ \text{m}}{2.998\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{8.32\ \text{min}}$$

(c) Also from Table C.4, the average Earth-Moon distance is $d = 3.84 \times 10^{8}$ m, giving the time for the round trip as

$$t = \frac{2d}{c} = \frac{2\left(3.84\times10^{8}\ \text{m}\right)}{2.998\times10^{8}\ \text{m/s}} = \boxed{2.56\ \text{s}}$$

P33.6 Time to reach object

$$= \frac{1}{2}(\text{total time of flight}) = \frac{1}{2}\left(4.00\times10^{-4}\ \text{s}\right) = 2.00\times10^{-4}\ \text{s}$$

Thus,

$$d = vt = \left(3.00\times10^{8}\ \text{m/s}\right)\left(2.00\times10^{-4}\ \text{s}\right) = 6.00\times10^{4}\ \text{m} = \boxed{60.0\ \text{km}}$$

P33.7 From Equation 33.21,

$$v = \frac{1}{\sqrt{\kappa\mu_0\epsilon_0}} = \frac{1}{\sqrt{1.78}}c = 0.750c = \boxed{2.25\times10^{8}\ \text{m/s}}$$

***P33.8** **Conceptualize** Imagine the television signal from the episode traveling out into space and being intercepted by the alien civilization. The intensity of the signal will be very small.

Categorize We categorize this problem as a simple substitution problem.

Imagine that the signal traveled to the alien civilization, which processed it immediately, prepared an answer, and transmitted the answer. In this case, the travel time for the electromagnetic radiation to and back from the alien civilization is 2019 – 1952 = 67 yr. Taking half that value means that the signal took 33.5 years to reach the alien civilization. Therefore, they are located 33.5 light-years away from us.

However, we cannot say that this answer is exact. Suppose that the alien civilization faced bureaucratic and political obstacles regarding the reception of the signal, and some time interval was required for their government to decide to process the signal, prepare an answer, and transmit the answer. Then the actual travel time for the electromagnetic radiation would be less than 67 years; we need to subtract the time the aliens took to decide to answer. Therefore, the distance of $\boxed{\text{33.5 light-years is a } \textit{maximum} \text{ distance away}}$; the aliens could be closer!]

Answer: Answer will increase with time after this book is published; for the publication year of 2019, the alien civilization is at most 33.5 light-years away

P33.9 Since the separation of the burn marks is $d_{\text{A to A}} = 6\text{ cm} \pm 5\% = \dfrac{\lambda}{2}$, then $\lambda = 12\text{ cm } \pm 5\%$ and

$$\begin{aligned} v = \lambda f &= (0.12\text{ m } \pm 5\%)\left(2.45 \times 10^9\text{ s}^{-1}\right) \\ &= \boxed{2.9 \times 10^8\text{ m/s } \pm 5\%} \end{aligned}$$

P33.10 $E = E_{\max}\cos(kx - \omega t)$

$$\frac{\partial E}{\partial x} = -E_{\max}\sin(kx - \omega t)(k) \quad \rightarrow \quad \frac{\partial^2 E}{\partial x^2} = -E_{\max}\cos(kx - \omega t)\left(k^2\right)$$
$$\frac{\partial E}{\partial t} = -E_{\max}\sin(kx - \omega t)(-\omega) \quad \rightarrow \quad \frac{\partial^2 E}{\partial t^2} = -E_{\max}\cos(kx - \omega t)(-\omega)^2$$

We must show: $\dfrac{\partial^2 E}{\partial x^2} = \mu_0 \in_0 \dfrac{\partial^2 E}{\partial t^2}$

That is, $-\left(k^2\right)E_{\max}\cos\left(kx-\omega t\right) = -\mu_0 \in_0 (-\omega)^2 E_{\max}\cos\left(kx-\omega t\right)$.

But this is true, because $\dfrac{k^2}{\omega^2} = \left(\dfrac{1}{f\lambda}\right)^2 = \dfrac{1}{c^2} = \mu_0 \in_0$.

The proof for the wave of the magnetic field follows precisely the same steps.

P33.11 The amplitudes of the electric and magnetic fields are in the correct ratio so that $E_{\max}/B_{\max} = c$. The ratio of ω to k, however, must also equal the speed of light:

$$\frac{\omega}{k} = \frac{3.00 \times 10^{15}\ \text{s}^{-1}}{9.00 \times 10^{6}\ \text{m}^{-1}} = 3.33 \times 10^{8}\ \text{m/s}$$

This value is higher than the speed of light in a vacuum, so the wave as described is impossible.

Section 33.4 Energy Carried by Electromagnetic Waves

P33.12 From Equation 16.34, we recall that the intensity I a distance r from a point or spherical source is inversely proportional to the square of the distance: $I = P/4\pi r^2$. At the Earth, $r_1 = 1.496 \times 10^{11}$ m, the intensity is $I_1 = I_E$, then at distance r_2, the intensity $I_2 = 3I_E$. Then,

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

and

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = \left(1.496\times10^{11}\ \text{m}\right)\sqrt{\frac{1}{3}} = \boxed{8.64\times10^{10}\ \text{m}}$$

P33.13 In time interval Δt, sunlight travels distance $\Delta x = c\Delta t$. The intensity of the sunlight passing into a volume $\Delta V = A\Delta x$ in time Δt is

$$S = I = \frac{U}{A\Delta t} = \frac{U}{A\,\Delta x/c} = \frac{Uc}{V} = uc$$

$$\frac{\text{Energy}}{\text{Unit Volume}} = u = \frac{I}{c} = \frac{1\,000\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = \boxed{3.33\ \mu\text{J/m}^3}$$

33.14 (a) $\dfrac{P}{\text{area}} = \dfrac{\text{energy}}{\Delta t\cdot\text{area}} = \dfrac{600\times10^3\ \text{Wh}}{(30\ \text{d})(13.0\ \text{m})(9.50\ \text{m})}\left(\dfrac{1\ \text{d}}{24\ \text{h}}\right) = \boxed{6.75\ \text{W/m}^2}$

(b) The car uses gasoline at the rate of $(55\ \text{mi/h})\left(\dfrac{\text{gal}}{25\ \text{mi}}\right)$. Its rate of energy conversion is

$$P = 44.0\times10^6\ \text{J/kg}\left(\frac{2.54\ \text{kg}}{1\ \text{gal}}\right)(55\ \text{mi/h})\left(\frac{\text{gal}}{25\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)$$
$$= 6.83\times10^4\ \text{W}$$

Its power-per-footprint-area is

$$\frac{P}{\text{area}} = \frac{6.83\times10^4\ \text{W}}{(2.10\ \text{m})(4.90\ \text{m})} = \boxed{6.64\times10^3\ \text{W/m}^2}$$

(c) A powerful automobile that is running on sunlight would have to carry on its roof a solar panel huge compared with the size of the car.

(d) Agriculture and forestry for food and fuels, space heating of large and small buildings, water heating, and heating for drying and many other processes are current and potential applications of solar energy.

P33.15 (a) $B_{max} = \dfrac{E_{max}}{c}$: $B_{max} = \dfrac{7.00\times10^5 \text{ N/C}}{3.00\times10^8 \text{ m/s}} = \boxed{2.33 \text{ mT}}$

(b) $I = \dfrac{E_{max}^2}{2\mu_0 c}$:

$$I = \frac{\left(7.00\times10^5 \text{ V/m}\right)^2}{2\left(4\pi\times10^{-7} \text{ T}\cdot\text{m/A}\right)\left(3.00\times10^8 \text{ m/s}\right)} = 6.50\times10^8 \text{ W/m}^2$$
$$= \boxed{650 \text{ MW/m}^2}$$

(c) $I = \dfrac{P}{A}$: $P = IA = \left(6.50\times10^8 \text{ W/m}^2\right)\left[\dfrac{\pi}{4}\left(1.00\times10^{-3} \text{ m}\right)^2\right] = \boxed{511 \text{ W}}$

P33.16 The energy put into the water in each container by electromagnetic radiation can be written as $\Delta E = eP\Delta t = eIA\Delta t$, where e is the percentage absorption efficiency. This energy has the same effect as heat in raising the temperature of the water:

$$eIA\Delta t = mc\Delta T = \rho Vc\Delta T$$
$$\Delta T = \frac{eI\ell^2\Delta t}{\rho\ell^3 c} = \frac{eI\Delta t}{\rho\ell c}$$

where ℓ is the edge dimension of the container and c the specific heat of water. For the small container,

$$\Delta T = \frac{0.700\left(25.0\times10^3 \text{ W/m}^2\right)(480 \text{ s})}{\left(10^3 \text{ kg/m}^3\right)(0.060\ 0 \text{ m})\left(4\,186 \text{ J/kg}\cdot°\text{C}\right)} = \boxed{33.4°\text{C}}$$

For the larger,

$$\Delta T = \frac{0.910\left(25.0\times10^3 \text{ W/m}^2\right)(480 \text{ s})}{\left(10^3 \text{ kg/m}^3\right)(0.120 \text{ m})\left(4\,186 \text{ J/°C}\right)} = \boxed{21.7°\text{C}}$$

***P33.17** **Conceptualize**You may have seen solar energy facilities with solar panels. Or you may have seen solar panels on rooftops, as well as on free-standing streetlights, emergency telephones, or other electrical devices. These solar panels take in energy that would otherwise warm up the surface of the Earth and convert some of it to useful electricity.

CategorizeThis problem involves the discussion of intensity and power of light in Section 33.4.

AnalyzeThe relationship between power and intensity was first introduced in our discussion of sound as Equation 16.38, which we will use for sunlight:

$$I = \frac{P_{\text{avg}}}{A} \qquad (1)$$

Solve Equation (1) for the area:

$$A = \frac{P_{\text{avg}}}{I} \qquad (2)$$

The term P_{avg} is the average power of sunlight required by the community to supply its power needs. However, because the solar panels are not 100% efficient, but rather have an efficiency e, the required power of sunlight is larger than the actual power needs of the community. Therefore,

$$A = \frac{\left(\frac{P_{\text{community}}}{e}\right)}{I} = \frac{P_{\text{community}}}{Ie} \qquad (3)$$

Also, the solar intensity of 1 000 W/m^2 is not available 24 hours a day. If we assume that this intensity is available for 8 hours a day, then the average intensity over a 24-hour period is one-third of the solar intensity during the day:

$$A = \frac{P_{\text{community}}}{\left(\frac{1}{3}I\right)e} = \frac{3P_{\text{community}}}{Ie} \qquad (4)$$

Substitute numerical values:

$$A = \frac{3\left(1.00 \square 10^{6}\ \text{W}\right)}{\left(1\,000\ \text{W/ m}^{2}\right)\left(0.300\right)} = \boxed{1.00 \square 10^{4}\ \text{m}^{2}}$$

FinalizeThis area is equivalent to about 2.5 acres. However, more land will be needed for at least four reasons: (1) The solar panels should not lie horizontally, but should be angled toward the south (in the northern hemisphere) to be oriented as perpendicularly to the Sun's rays as possible. This would require rows of angled panels separated from each other by some distance so as not to cast shadows on other panels. (2) The Sun's rays will not be perpendicular to the panel surface for 8 hours a day. (3) Additional land would be needed for service buildings, technical equipment, and any roadways needed to access the cells for maintenance and repairs. (4) Some land would be needed around the perimeter of the cells to provide security. Your advice to the council will depend on how much area is available in the community.]

Answer: $\sim 1 \times 10^4\ \text{m}^2$

P33.18 (a) The intensity of the broadcast waves is

$$I = \frac{B_{\text{max}}^2 c}{2\mu_0} = \frac{P}{4\pi r^2}$$

solving,

$$B_{\max} = \sqrt{\left(\frac{P}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\left(\frac{P}{2\pi r^2}\right)\left(\frac{\mu_0}{c}\right)}$$

$$= \sqrt{\frac{(10.0\times 10^3\ \text{W})(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(5.00\times 10^3\ \text{m})^2(3.00\times 10^8\ \text{m/s})}} = \boxed{5.16\times 10^{-10}\ \text{T}}$$

(b) Since the magnetic field of the Earth is approximately 5×10^{-5} T, the Earth's field is some 100 000 times stronger.

P33.19 Power $= SA = \dfrac{E_{\max}^2}{2\mu_0 c}(4\pi r^2)$

Solving for r,

$$r = \sqrt{\frac{P\mu_0 c}{2\pi E_{\max}^2}} = \sqrt{\frac{(100\ \text{W})(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(3.00\times 10^8\ \text{m/s})}{2\pi(15.0\ \text{V/m})^2}}$$

$$= \boxed{5.16\ \text{m}}$$

P33.20 (a) $E_{\text{rms}} = cB_{\text{rms}} = (3.00\times 10^8\ \text{m/s})(1.80\times 10^{-6}\ \text{T}) = \boxed{540\ \text{V/m}}$

(b) From Equation 33.32,

$$u_{\text{avg}} = \frac{(B_{\max})^2}{2\mu_0} = \frac{(B_{\text{rms}})^2}{\mu_0} = \frac{(1.80\times 10^{-6}\ \text{T})^2}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = \boxed{2.58\ \mu\text{J/m}^3}$$

(c) $S_{\text{avg}} = cu_{\text{avg}} = (3.00\times 10^8\ \text{m/s})(2.58\times 10^{-6}\ \text{J/m}^3) = \boxed{773\ \text{W/m}^2}$

Section 33.5 Momentum and Radiation Pressure

P33.21 The intensity of the beam is $I = \dfrac{P_{\text{power}}}{\pi r^2}$, where $r = 1.00 \times 10^{-3}$ m. By Equation 33.36, the radiation pressure on the mirror is

$$P = \frac{2S}{c} = \frac{2I}{c} = \frac{2P_{\text{power}}}{\pi r^2 c}$$
$$= \frac{2\left(25.0\times 10^{-3}\ \text{W}\right)}{\pi\left(1.00\times 10^{-3}\ \text{m}\right)^2\left(3.00\times 10^{8}\ \text{m/s}\right)} = \boxed{5.31\times 10^{-5}\ \text{N/m}^2}$$

P33.22 (a) The light pressure on the absorbing Earth is $P = \dfrac{S}{c} = \dfrac{I}{c}$.

The force is

$$F = PA = \frac{I}{c}\left(\pi R^2\right) = \frac{(1\ 370\ \text{W/m}^2)\pi(6.37\times 10^{6}\ \text{m})^2}{3.00\times 10^{8}\ \text{m/s}}$$
$$= \boxed{5.82\times 10^{8}\ \text{N}}$$

away from the Sun.

(b) The attractive gravitational force exerted on Earth by the Sun is

$$F_g = \frac{GM_S M_M}{r_M^2}$$
$$= \frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times 10^{30}\ \text{kg}\right)\left(5.98\times 10^{24}\ \text{kg}\right)}{\left(1.496\times 10^{11}\ \text{m}\right)^2}$$
$$= 3.55\times 10^{22}\ \text{N}$$

which is $\boxed{6.10\times 10^{13}\ \text{times stronger}}$ compared to the repulsive force in part (a).

P33.23 (a) $I = \dfrac{P}{\pi r^2} = \dfrac{E_{\max}^2}{2\mu_0 c}$, and $r = 1.00 \times 10^{-3}$ m:

$$E_{\max} = \sqrt{\frac{2\mu_0 cP}{\pi r^2}}$$
$$= \sqrt{\frac{2\left[4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right]\left(3.00\times 10^{8}\ \text{m/s}\right)\left(15.0\times 10^{-3}\ \text{W}\right)}{\pi\left(1.00\times 10^{-3}\ \text{m}\right)^2}}$$
$$= 1.90\times 10^{8}\ \text{J} = \boxed{1.90\ \text{kN/C}}$$

(b) The beam carries power P. The amount of energy ΔE in the length of a beam of length ℓ is the amount of power that passes a point in time interval $\Delta t = \ell/c$:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\ell/c}$$

or $$\Delta E = \frac{P\ell}{c} = \frac{15.0\times10^{-3}\ \text{W}}{3.00\times10^{8}\ \text{m/s}}(1.00\ \text{m}) = \boxed{50.0\ \text{pJ}}.$$

(c) From Equation 33.34 and our result in part (b), the momentum and energy carried a light beam are related by

$$p = \frac{T_{\text{ER}}}{c} = \frac{\Delta E}{c} = \frac{50.0\times10^{-12}\ \text{J}}{3.00\times10^{8}\ \text{m/s}} = \boxed{1.67\times10^{-19}\ \text{kg}\cdot\text{m/s}}$$

P33.24 (a) $$I = \frac{P}{\pi r^2} = \frac{E_{\text{max}}^2}{2\mu_0 c} \rightarrow E_{\text{max}} = \boxed{\sqrt{\frac{2\mu_0 cP}{\pi r^2}}}$$

(b) The beam carries power P. The amount of energy ΔE in the length of a beam of length ℓ is the amount of power that passes a point in time interval $\Delta t = \ell/c$:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\ell/c} \rightarrow \Delta E = \boxed{\frac{P\ell}{c}}$$

(c) From Equation 33.34 and our result in part (b), the momentum and energy carried a light beam are related by

$$p = \frac{T_{\text{ER}}}{c} = \frac{\Delta E}{c} = \boxed{\frac{P\ell}{c^2}}$$

P33.25 (a) The magnitude of the momentum transferred to the assumed totally reflecting surface in time interval Δt is (from Equation 33.36)

$$\Delta p = \frac{2T_{ER}}{c} = \frac{2SA\Delta t}{c}$$

Then the momentum transfer is

$$\Delta\vec{\mathbf{p}} = \frac{2\vec{\mathbf{S}}A\Delta t}{c} = \frac{2(6.00\ \hat{\mathbf{i}}\ \text{W/m}^2)(40.0\times10^{-4}\ \text{m}^2)(1.00\ \text{s})}{3.00\times10^8\ \text{m/s}}$$

$$\Delta\vec{\mathbf{p}} = \boxed{1.60\times10^{-10}\ \hat{\mathbf{i}}\ \text{kg}\cdot\text{m/s each second}}$$

(b) The force is

$$\vec{\mathbf{F}} = PA\,\hat{\mathbf{i}} = \frac{2SA}{c}\hat{\mathbf{i}} = \frac{2\left(6.00\ \text{W/m}^2\right)\left(40.0\times10^{-4}\ \text{m}^2\right)(1.00\ \text{s})}{3.00\times10^8\ \text{m/s}}$$
$$= \boxed{1.60\times10^{-10}\ \hat{\mathbf{i}}\ \text{N}}$$

(c) The answers are the same. Force is the time rate of momentum transfer.

P33.26 (a) If P_S is the total power radiated by the Sun, and r_E and r_M are the radii of the orbits of the planets Earth and Mars, then the intensities of the solar radiation at these planets are:

$$I_E = \frac{P_S}{4\pi r_E^2} \quad \text{and} \quad I_M = \frac{P_S}{4\pi r_M^2}$$

Thus,

$$I_M = I_E\left(\frac{r_E}{r_M}\right)^2 = \left(1\,370\ \text{W/m}^2\right)\left(\frac{1.496\times10^{11}\ \text{m}}{2.28\times10^{11}\ \text{m}}\right)^2$$
$$= \boxed{590\ \text{W/m}^2}$$

(b) Mars intercepts the power falling on its circular face:

$$P_M = I_M\left(\pi R_M^2\right) = \left(590\ \text{W/m}^2\right)\left[\pi\left(3.37\times10^6\ \text{m}\right)^2\right]$$
$$= \boxed{2.10\times10^{16}\ \text{W}}$$

(c) If Mars behaves as a perfect absorber, it feels pressure

$$P = \frac{S_M}{c} = \frac{I_M}{c},$$

so the light-pressure force is

$$F_L = PA = \frac{I_M}{c}\left(\pi R_M^2\right) = \frac{P_M}{c} = \frac{2.10\times 10^{16}\ \text{W}}{3.00\times 10^{8}\ \text{m/s}} = \boxed{7.01\times 10^{7}\ \text{N}}$$

(d) Using our results from above, we have $F_L = I_M \dfrac{\pi R_M^2}{c}$ and

$I_M = I_E \dfrac{r_E^2}{r_M^2}$, so the light-pressure force on Mars is $F_L = I_E \dfrac{r_E^2}{r_M^2}\dfrac{\pi R_M^2}{c}$.

The attractive gravitational force exerted on Mars by the Sun is

$F_g = \dfrac{GM_S M_M}{r_M^2}$. Their ratio is

$$\frac{F_g}{F_L} = \frac{GM_S M_M}{\cancel{r_M^2}} \cdot \frac{1}{I_E}\frac{\cancel{r_M^2}}{r_E^2}\frac{c}{\pi R_M^2} = \left(\frac{cGM_S}{\pi I_E r_E^2}\right)\frac{M_M}{R_M^2}$$

Suppressing units,

$$\frac{F_g}{F_L} = \left[\frac{\left(3.00\times 10^{8}\right)\left(6.67\times 10^{-11}\right)\left(1.991\times 10^{30}\right)}{\pi\left(1\,370\right)\left(1.496\times 10^{11}\right)^2}\right]\left(\frac{M_M}{R_M^2}\right)$$

$$\frac{F_g}{F_L} = \left(414\ \text{m}^2/\text{kg}\right)\frac{M_M}{R_M^2} = \left(414\ \text{m}^2/\text{kg}\right)\frac{\left(6.42\times 10^{23}\ \text{kg}\right)}{\left(3.37\times 10^{6}\ \text{m}\right)^2}$$

$$= 2.34\times 10^{13}$$

The attractive gravitational force exerted on Mars by the Sun is $\boxed{\sim 10^{13}\ \text{times stronger}}$ than the repulsive light-pressure force of part (c).

(e) The expression for the ratio of the gravitational force to the light-pressure force for Earth is similar to that used in part (d) for Mars (replace M with E):

$$\frac{F_g}{F_L} = \left(414 \text{ m}^2/\text{kg}\right)\frac{M_E}{R_E^2} = \left(414 \text{ m}^2/\text{kg}\right)\frac{\left(5.98\times 10^{24} \text{ kg}\right)}{\left(6.37\times 10^{6} \text{ m}\right)^2}$$
$$= 6.10\times 10^{13}$$

$\boxed{\text{The values are similar for both planets because both the forces follow inverse-square laws. The force ratios are not identical for the two planets because of their different radii and masses.}}$

Section 33.6 Production of Electromagnetic Waves by an Antenna

P33.27 (a) The wavelength of an ELF wave of frequency 75.0 Hz is

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^8 \text{ m/s}}{75.0 \text{ Hz}} = 4.00\times 10^6 \text{ m}$$

The length of a quarter-wavelength antenna would be

$$L = 1.00\times 10^6 \text{ m} = \boxed{1.00\times 10^3 \text{ km}}$$

or $$L = \left(1\,000 \text{ km}\right)\left(\frac{0.621 \text{ mi}}{1.00 \text{ km}}\right) = \boxed{621 \text{ mi}}$$

(b) $\boxed{\text{While the project may be theoretically possible, it is not very practical.}}$

P33.28 (a) The magnetic field $\vec{\mathbf{B}} = \frac{1}{2}\mu_0 J_{\max}\cos(kx - \omega t)\hat{\mathbf{k}}$ applies for $x > 0$, since it describes a wave moving in the $\hat{\mathbf{i}}$ direction. The electric field direction must satisfy $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ as $\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{k}}$ so the direction of the electric field is $\hat{\mathbf{j}}$ when the cosine is positive. For its magnitude we have $E = cB$, so altogether we have

$$\boxed{\vec{\mathbf{E}} = \frac{1}{2}\mu_0 c J_{\max}\cos(kx-\omega t)\hat{\mathbf{j}}}.$$

(b) $$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\frac{1}{4}\mu_0^2 c J_{\max}^2\cos^2(kx-\omega t)\hat{\mathbf{i}}$$

$$\boxed{\vec{\mathbf{S}} = \frac{1}{4}\mu_0 c J_{\max}^2\cos^2(kx-\omega t)\hat{\mathbf{i}}}$$

(c) The intensity is the magnitude of the Poynting vector averaged over one or more cycles. The average of the cosine-squared function is $\frac{1}{2}$, so $\boxed{I = \frac{1}{8}\mu_0 c J_{\max}^2}$.

(d) $$J_{\max} = \sqrt{\frac{8I}{\mu_0 c}} = \sqrt{\frac{8(570\text{ W/m}^2)}{4\pi\times10^{-7}(\text{Tm/A})3\times10^8\text{ m/s}}} = \boxed{3.48\text{ A/m}}$$

P33.29 For the proton, Newton's second law gives

$$\sum F = ma: \qquad qvB\sin 90.0° = \frac{mv^2}{R}.$$

The period and frequency of the proton's circular motion are therefore:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB} = \frac{2\pi(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(0.350\text{ T})} = 1.87\times10^{-7}\text{ s}$$

and $$f = 5.34\times10^6\text{ Hz}.$$

The charge will radiate at this same frequency, with

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{5.34\times10^{6}\ \text{Hz}} = \boxed{56.2\ \text{m}}$$

P33.30 For the proton, $\sum F = ma$ yields

$$qvB\sin 90.0^\circ = \frac{mv^2}{R} \rightarrow v = \frac{qBR}{m}$$

The period of the proton's circular motion is therefore:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

The frequency of the proton's motion is $f = \dfrac{1}{T}$

The charge will radiate electromagnetic waves at this frequency, with

$$\lambda = \frac{c}{f} = cT = \boxed{\frac{2\pi mc}{qB}}$$

Section 33.7 The Spectrum of Electromagnetic Waves

P33.31 (a) $f = \dfrac{c}{\lambda} = \dfrac{3\times10^{8}\ \text{m/s}}{1.7\ \text{m}}\ \boxed{\sim 10^{8}\ \text{Hz}}\ \boxed{\text{radio wave}}$

(b) 1 000 pages, 500 sheets, is about 3 cm thick so one sheet is about 6×10^{-5} m thick.

$$f = \frac{3.00\times10^{8}\ \text{m/s}}{6\times10^{-5}\ \text{m}}\ \boxed{\sim 10^{13}\ \text{Hz}}\ \boxed{\text{infrared}}$$

P33.32 The time interval for the radio signal to travel 100 km is:

$$\Delta t_r = \frac{100\times10^{3}\ \text{m}}{3.00\times10^{8}\ \text{m/s}} = 3.33\times10^{-4}\ \text{s}$$

The sound wave travels 3.00 m across the room in:

$$\Delta t_s = \frac{3.00\text{ m}}{343\text{ m/s}} = 8.75\times10^{-3}\text{ s}$$

Therefore, $\boxed{\text{listeners 100 km away}}$ will receive the news before the people in the newsroom by a total time difference of

$$\Delta t = 8.75\times10^{-3}\text{ s} - 3.33\times10^{-4}\text{ s} = 8.41\times10^{-3}\text{ s}$$

Additional Problems

P33.33 (a) From $P = SA$, we have

$$P = \left(1\,370\text{ W/m}^2\right)\left[4\pi\left(1.496\times10^{11}\text{ m}\right)^2\right] = \boxed{3.85\times10^{26}\text{ W}}$$

(b) $S = \dfrac{E_{\text{max}}^2}{2\mu_0 c}$ so

$$\begin{aligned} E_{\text{max}} &= \sqrt{2\mu_0 cS} \\ &= \sqrt{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(3.00\times10^8\text{ m/s}\right)\left(1\,370\text{ W/m}^2\right)} \\ &= \boxed{1.02\text{ kV/m}} \end{aligned}$$

(c) $S = \dfrac{cB_{\text{max}}^2}{2\mu_0}$ so

$$\begin{aligned} B_{\text{max}} &= \sqrt{\frac{2\mu_0 S}{c}} = \sqrt{\frac{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1\,370\text{ W/m}^2\right)}{3.00\times10^8\text{ m/s}}} \\ &= \boxed{3.39\ \mu\text{T}} \end{aligned}$$

P33.34 From the electromagnetic spectrum chart and accompanying text discussion, the following identifications are made:

Frequency, f	Wavelength, $\lambda = \frac{c}{f}$	Classification
2 Hz = 2×10^0 Hz	150 Mm	Radio
2 KHz = 2×10^3 Hz	150 km	Radio
2 MHz = 2×10^6 Hz	150 m	Radio
2 GHz = 2×10^9 Hz	15 cm	Microwave
2 THz = 2×10^{12} Hz	150 μm	Infrared
2 PHz = 2×10^{15} Hz	150 nm	Ultraviolet
2 EHz = 2×10^{18} Hz	150 pm	X-ray
2 ZHz = 2×10^{21} Hz	150 fm	Gamma ray
2 YHz = 2×10^{24} Hz	150 am	Gamma ray
Wavelength, λ	Frequency, $f = \frac{c}{\lambda}$	Classification
2 km = 2×10^3 m	1.5×10^5 Hz	Radio
2 m = 2×10^0 m	1.5×10^8 Hz	Radio
2 mm = 2×10^{-3} m	1.5×10^{11} Hz	Microwave
2 μm = 2×10^{-6} m	1.5×10^{14} Hz	Infrared
2 nm = 2×10^{-9} m	1.5×10^{17} Hz	Ultraviolet/X-ray
2 pm = 2×10^{-12} m	1.5×10^{20} Hz	X-ray/Gamma ray

2 fm = 2×10^{-15} m	1.5×10^{23} Hz	Gamma ray
2 am = 2×10^{-18} m	1.5×10^{26} Hz	Gamma ray

P33.35 The wavelength is found from

$$f\lambda = c \;\rightarrow\; \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.45 \times 10^{14} \text{ Hz}} = \boxed{5.50 \times 10^{-7} \text{ m}}$$

P33.36 The angular frequency of the wave is

$$\omega = 2\pi f = 2\pi\left(3.00 \times 10^9 \text{ s}^{-1}\right) = 1.88 \times 10^{10} \text{ s}^{-1}$$

and the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = 2\pi\left(\frac{3.00 \times 10^9 \text{ s}^{-1}}{3.00 \times 10^8 \text{ m/s}}\right) = 20.0\pi \text{ m}^{-1} = 62.8 \text{ m}^{-1}$$

Also,

$$B_{\max} = \frac{E}{c} = \frac{300 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.00 \;\mu\text{T}$$

Then,

$$\boxed{E = 300\cos\left(62.8x - 1.88 \times 10^{10} t\right)}$$

$$\boxed{B = 1.00\cos\left(62.8x - 1.88 \times 10^{10} t\right)}$$

where E is in volts per meter (V/m), B is in microtesla (μT), x is in meters, and t is in seconds.

***P33.37** **Conceptualize** The standing waves set up between the metal plates are analogous to the standing waves set up on a string fixed at both ends in Chapter 17.

Categorize We model the radio waves as waves under boundary conditions.

Analyze Because t here is no smaller separation distance between the plates at which standing waves can be set up, the situation described must correspond to the fundamental mode of oscillation. For this mode, the distance between the plates is half a wavelength. Therefore,

$$\lambda = 2L \qquad (1)$$

Now, use Equation 16.12 to find the frequency of the wave and substitute Equation (1):

$$f = \frac{c}{\lambda} = \frac{c}{2L} \qquad (2)$$

Substitute numerical values:

$$f = \frac{3.00 \times 10^8 \text{ m/ s}}{2(2.00 \text{ m})} = 7.50 \times 10^7 \text{ Hz} = \boxed{75.0 \text{ MHz}}$$

Finalize Notice in Figure 33.13 that this radio wave is close to the lower end of the "TV, FM" frequency range.

Answer: 75.0 MHz

P33.38 (a) The power incident on the mirror is:

$$P_I = IA = (1\ 370 \text{ W/m}^2)\left[\pi(100 \text{ m})^2\right] = 4.30 \times 10^7 \text{ W}.$$

The power reflected through the atmosphere is

$$P_R = 0.746(4.30 \times 10^7 \text{ W}) = \boxed{3.21 \times 10^7 \text{ W}}$$

(b) $$S = \frac{P_R}{A} = \frac{3.21 \times 10^7 \text{ W}}{\pi(4.00 \times 10^3 \text{ m})^2} = \boxed{0.639 \text{ W/m}^2}$$

(c) Noon sunshine in St. Petersburg produces this power-per-area on a horizontal surface:

$$\frac{P_N}{A} = 0.746\left(1\ 370\ \text{W/m}^2\right)\sin 7.00° = 125\ \text{W/m}^2$$

The radiation intensity received from the mirror is

$$\left(\frac{0.639\ \text{W/m}^2}{125\ \text{W/m}^2}\right)100\% = \boxed{0.513\%}$$ of that from the noon Sun in January.

P33.39 Suppose you cover a 1.7 m × 0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the effective target area you fill in the Sun's light is

$$A = (1.7\ \text{m})(0.3\ \text{m})\cos 30° = 0.4\ \text{m}^2$$

Now $I = \frac{P}{A} = \frac{\Delta E}{A\Delta t}$, so

$$\Delta E = IA\Delta t = (0.5)\left[(0.6)\left(1\,370\ \text{W/m}^2\right)\right]\left(0.4\ \text{m}^2\right)(3\,600\ \text{s})$$

$$\boxed{\sim 10^6\ \text{J}}$$

P33.40 Of the intensity $S = 1\ 370\ \text{W/m}^2$, the 38.0% that is reflected exerts a pressure

$$P_1 = \frac{2S_r}{c} = \frac{2(0.380)S}{c}$$

The absorbed light exerts pressure

$$P_2 = \frac{S_a}{c} = \frac{0.620S}{c}$$

Altogether the pressure at the subsolar point on Earth is

(a) $$P_{\text{total}} = P_1 + P_2 = \frac{1.38S}{c} = \frac{1.38\left(1370\ \text{W/m}^2\right)}{3.00\times 10^8\ \text{m/s}} = \boxed{6.30\times 10^{-6}\ \text{Pa}}$$

(b) Compared to normal atmospheric pressure,

$$\frac{P_a}{P_{\text{total}}} = \frac{1.01\times 10^5\ \text{N/m}^2}{6.30\times 10^{-6}\ \text{N/m}^2}$$
$$= \boxed{1.60\times 10^{10}\ \text{times smaller than atmospheric pressure}}$$

P33.41 The gravitational force exerted by the Sun on the particle is given by

$$F_{\text{grav}} = \frac{GM_S m}{R^2} = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

where M_S = mass of Sun, r = radius of particle, and R = distance from Sun to particle. The force exerted by solar radiation on the particle is given by $F_{\text{rad}} = PA$, and since the particle absorbs all the radiation, by Equation 33.35, we have

$$F_{\text{rad}} = PA = \frac{S}{c}\pi r^2$$

When the particle is in equilibrium, the gravitational force toward the Sun is balanced by the force of radiation away from the Sun, $F_{\text{rad}} = F_{\text{grav}}$, so

$$\frac{S}{c}\pi r^2 = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

Solving for r, the radius of the particle, then gives

$$r = \frac{3SR^2}{4cGM_S\rho}$$

Suppressing units,

$$r = \frac{3(214)\left(3.75\times10^{11}\right)^2}{4\left(3.00\times10^{8}\right)\left(6.67\times10^{-11}\right)\left(1.991\times10^{30}\right)(1\ 500)}$$

$$= 3.78\times10^{-7}\ \text{m} = \boxed{378\ \text{nm}}$$

P33.42 The gravitational force exerted by the Sun on the particle is given by

$$F_{\text{grav}} = \frac{GM_S m}{R^2} = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

where M_S = mass of Sun, r = radius of particle, and R = distance from Sun to particle. The force exerted by solar radiation on the particle is given by $F_{\text{rad}} = PA$, and since the particle absorbs all the radiation, by Equation 33.35, we have

$$F_{\text{rad}} = PA = \frac{S}{c}\pi r^2$$

When the particle is in equilibrium, the gravitational force toward the Sun is balanced by the force of radiation away from the Sun, $F_{\text{rad}} = F_{\text{grav}}$, so

$$\frac{S}{c}\pi r^2 = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

Solving for r, the radius of the particle, then gives

$$r = \boxed{\frac{3SR^2}{4cGM_S\rho}}$$

P33.43 The mirror intercepts power

$$P = I_1A_1 = \left(1.00\times10^3\ \text{W/m}^2\right)\left[\pi(0.500\ \text{m})^2\right] = 785\ \text{W}.$$

(a) In the image, $I_2 = \dfrac{P}{A_2}$, so

$$I_2 = \frac{785\text{ W}}{\pi(0.020\,0\text{ m})^2} = \boxed{625\text{ kW/m}^2}$$

(b) $I_2 = \frac{E_{\max}^2}{2\mu_0 c}$, so

$$\begin{aligned} E_{\max} &= \sqrt{2\mu_0 c I_2} \\ &= \sqrt{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(3.00\times10^8\text{ m/s})(6.25\times10^5\text{ W/m}^2)} \\ &= \boxed{21.7\text{ kN/C}} \end{aligned}$$

(c) $B_{\max} = \frac{E_{\max}}{c} = \boxed{72.4\ \mu\text{T}}$

(d) We obtain the time interval from

$$0.400(P\Delta t) = mc\Delta T$$

solving,

$$\begin{aligned} \Delta t &= \frac{mc\Delta T}{0.400P} = \frac{(1.00\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 20.0^\circ\text{C})}{0.400(785\text{ W})} \\ &= 1.07\times10^3\text{ s} = \boxed{17.8\text{ min}} \end{aligned}$$

P33.44 (a) In $\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} = \frac{\Phi}{4\pi r^2}\hat{\mathbf{r}} = \frac{487\text{ N}\cdot\text{m}^2/\text{C}}{4\pi r^2}\hat{\mathbf{r}}$,

$$\boxed{\vec{\mathbf{E}} = \frac{38.8}{r^2}\hat{\mathbf{r}} \text{ where } \vec{\mathbf{E}} \text{ is in volts per meter and } r \text{ is in meters.}}$$

(b) The radiated intensity is

$$I = \frac{P}{4\pi r^2} = \frac{E_{\max}^2}{2\mu_0 c}$$

solving,

$$E_{\max} = \sqrt{\frac{2\mu_0 cP}{4\pi r^2}} = \frac{1}{r}\sqrt{\frac{\mu_0 cP}{2\pi}}$$
$$= \frac{1}{r}\sqrt{\frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(3.00 \times 10^{8}\ \mathrm{m/s})(25.0\ \mathrm{W})}{2\pi}}$$

$$\boxed{E_{\max} = \frac{38.7}{r} \text{ where } E \text{ is in volts per meter and } r \text{ is in meters.}}$$

(c) For $E_{\max} = \dfrac{38.7}{r} = 3.00 \times 10^{6} \rightarrow r = 1.29 \times 10^{-5} = 12.9 \times 10^{-6}$, so r is $\boxed{12.9\ \mu\text{m, but the expression in part (b) does not apply if this point is inside the source.}}$

(d) From part (c), we see that $\boxed{\text{in the radiated wave, the field amplitude is inversely proportional to distance. As the distance doubles, the amplitude is cut in half.}}$

(e) $\boxed{\text{In the static case, the field is inversely proportional to the square of the distance. As the distance doubles, the field is reduced by a factor of 4.}}$

P33.45 (a) At steady state, $P_{\text{in}} = P_{\text{out}}$ and the power radiated out is $P_{\text{out}} = e\sigma A T^4$. Thus,

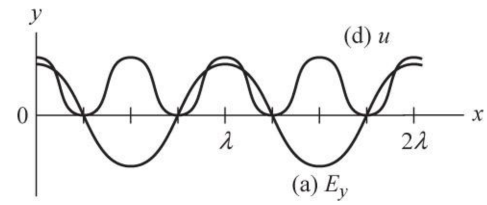
$$T = \left[\frac{P_{\text{out}}}{e\sigma A}\right]^{1/4} = \left[\frac{900\ \mathrm{W/m^2}}{0.700(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})}\right]^{1/4}$$
$$= \boxed{388\ \mathrm{K}} = 115^\circ\mathrm{C}$$

(b) The box of horizontal area A presents projected area A sin 50.0° perpendicular to the sunlight. Then by the same reasoning,

$$0.900(1\,000\ \mathrm{W/m^2})A\sin 50.0^\circ$$
$$= 0.700(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})AT^4$$

or $$T = \left[\frac{(900\ \text{W/m}^2)\sin 50.0°}{0.700(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{363\ \text{K}} = 90.0°\text{C}$$

P33.46 (a) See ANS. FIG. P33.46



ANS. FIG. P33.46

(b) $$u_E = \frac{1}{2}\epsilon_0 E^2 = \boxed{\frac{1}{2}\epsilon_0 E_{\text{max}}^2\cos^2(kx)}$$

(c) $$u_B = \frac{1}{2\mu_0}B^2 = \boxed{\frac{1}{2\mu_0}B_{\text{max}}^2\cos^2(kx)}$$

(d) Note that

$$u_B = \frac{1}{2\mu_0}\frac{E_{\text{max}}^2}{c^2}\cos^2(kx) = \frac{1}{2\mu_0}\frac{E_{\text{max}}^2}{(1/\mu_0\epsilon_0)}\cos^2(kx)$$
$$= \frac{1}{2}\epsilon_0 E_{\text{max}}^2\cos^2(kx) = u_E$$

Therefore, $u = u_E + u_B = \boxed{\epsilon_0 E_{\text{max}}^2\cos^2(kx)}$.

(e) $$E_\lambda = \int_0^\lambda uA\,dx$$

$$E_\lambda = \int_0^\lambda \epsilon_0 E_{\text{max}}^2\cos^2(kx)A\,dx = \int_0^\lambda \epsilon_0 E_{\text{max}}^2 A\left[\frac{1}{2}+\frac{1}{2}\cos(2kx)\right]A\,dx$$
$$= \frac{1}{2}\epsilon_0 E_{\text{max}}^2 A\,x\Big|_0^\lambda + \frac{\epsilon_0 E_{\text{max}}^2}{4k}A\sin(2kx)\Big|_0^\lambda$$
$$= \frac{1}{2}\epsilon_0 E_{\text{max}}^2 A\lambda + \frac{\epsilon_0 E_{\text{max}}^2}{4k}A[\sin(4\pi)-\sin(0)]$$
$$= \boxed{\frac{1}{2}\epsilon_0 E_{\text{max}}^2\lambda A}$$

(f) $$P = \frac{E_\lambda}{T} = \frac{1}{2}\frac{\epsilon_0 E_{\max}^2 \lambda A}{(1/f)} = \frac{1}{2}\epsilon_0 E_{\max}^2 (\lambda f) A = \boxed{\frac{1}{2}\epsilon_0 c E_{\max}^2 A}$$

(g) $$I = \frac{P}{A} = \frac{\frac{1}{2}\epsilon_0 c E_{\max}^2 A}{A} = \boxed{\frac{1}{2}\epsilon_0 c E_{\max}^2}$$

(h) From part (g), we have

$$\frac{1}{2}\epsilon_0 c E_{\max}^2 = \frac{\mu_0}{\mu_0}\frac{\epsilon_0 c E_{\max}^2}{2} = (\mu_0 \epsilon_0)\frac{c E_{\max}^2}{2\mu_0} = \frac{1}{c^2}\frac{c E_{\max}^2}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c}$$

$$\boxed{\text{The result in part (g) agrees with } I = \frac{E_{\max}^2}{2\mu_0 c} \text{ in Equation 33.27.}}$$

***P33.47** **Conceptualize** Imagine exposing the system to sunlight. The right-hand plate in Figure P33.47 is black, and will therefore absorb all the light incident upon it. The left-hand plate is perfectly reflecting, so it experiences twice as much force as the right-hand plate. As a result, the system will rotate clockwise when viewed from above. Because the radiation pressure is small in magnitude, we assume that the rotation during the time interval during which the plates have light incident upon them is sufficiently small that we can treat the plates as being perpendicular to the sunlight throughout the process.

CategorizeThe system is analyzed using the *rigid object under a net torque model*.

AnalyzeWe can write the force on one plate using Equation 9.3:

$$F = \frac{dp}{dt} \qquad (1)$$

In Equation (1) substitute for the radiation momentum p from Equation 33.34:

$$F_{\text{black}} = \frac{d}{dt}\left(\frac{T_{\text{ER}}}{c}\right) = \frac{1}{c}\left(\frac{dT_{\text{ER}}}{dt}\right) = \frac{1}{c}\left(Power\right)_{\text{avg}} \qquad (2)$$

where we have identified this result as the force on the black plate. Now use Equation 16.38 to express the power in terms of the intensity of the light striking the plates:

$$F_{\text{black}} = \frac{1}{c} I_s A_p \qquad (3)$$

whereA_p is the area on one plate. Based on the discussion in Section 33.5, the force on the reflecting plate must be twice as great:

$$F_{\text{reflecting}} = \frac{2}{c} I_s A_p \qquad (4)$$

Now, from the definition of torque, find the net torque on the system:

$$\sum \tau = \left(\frac{1}{c} I_s A_p\right)\left(\frac{\ell}{2}\right) - \left(\frac{2}{c} I_s A_p\right)\left(\frac{\ell}{2}\right) = -\frac{1}{2c} I_s A_p \ell \qquad (5)$$

where we have defined the direction of the torque by looking downward on the system from above. Now apply the rigid object under a net torque model:

$$\square\ \tau = I\alpha \qquad (6)$$

Substitute Equation (5) for the left side, $\Delta\omega/\Delta t$ for α on the right side, and evaluate the moment of inertia of the system as the sum of that of a spherical shell, a rod around the middle, and two particles representing the plates:

$$-\frac{1}{2c}I_s A_p \ell = \left[\tfrac{2}{3}mR^2 + \tfrac{1}{12}m_r\ell^2 + 2m_p\left(\frac{\ell}{2}\right)^2\right]\frac{\Delta\omega}{\Delta t}$$

$$= \left[\tfrac{2}{3}mR^2 + \tfrac{1}{12}m_r\ell^2 + 2m_p\left(\frac{\ell}{2}\right)^2\right]\frac{\left(\omega_f - 0\right)}{\Delta t}$$

$$= \left[\tfrac{2}{3}mR^2 + \tfrac{1}{12}m_r\ell^2 + 2m_p\left(\frac{\ell}{2}\right)^2\right]\frac{\omega_f}{\Delta t} \qquad (7)$$

Solve Equation (7) for ω_f:

$$\omega_f = -\frac{I_s \pi r_p{}^2 \ell}{\left[\tfrac{4}{3}mR^2 + \left(\tfrac{1}{6}m_r + m_p\right)\ell^2\right]c}\Delta t$$

Substitute numerical values:

$$\Delta\omega = -\frac{\left(1\,000\text{ W/ m}^2\right)\pi\left(0.020\,0\ m\right)^2\left(1.00\text{ m}\right)}{\left[\begin{array}{l}\tfrac{4}{3}\left(0.500\text{ kg}\right)\left(0.150\text{ m}\right)^2 \\ \qquad +\left(\dfrac{0.050\,0\text{ kg}}{6} + 0.010\,0\text{ kg}\right)\left(1.00\text{ m}\right)^2\end{array}\right]\left(3.00\times10^8\text{ m/ s}\right)}\left(120\text{ s}\right)$$

$$= \boxed{-1.51\times10^{-5}\text{ rad/ s}}$$

Finalize Because the system began from rest, this final result is the angular velocity with which the system is turning after the light is removed. The negative sign tells us that the system is rotating clockwise when viewed from above. This is a very small angular velocity, consistent with the fact that radiation pressure on small objects is tiny. On a spacecraft, we can extend the length of the arms, make the plate radius larger, and increase the time interval during which sunlight strikes the plates. In addition, we can replace the black plate with a small object of the same mass, or a plate turned edge-on to the Sun. This would maintain the mass balance of the system, but because the small object would absorb negligible radiation, it would not supply a counter-torque, so that the net torque on the system would be doubled.]

Answer: -1.51×10^{-5} rad/s

P33.48 (a) On the right side of the equation,

$$\frac{\mathrm{C}^2\left(\mathrm{m/s^2}\right)^2}{\left(\mathrm{C^2/N\cdot m^2}\right)\left(\mathrm{m/s}\right)^3} = \frac{\mathrm{N\cdot m^2\cdot C^2\cdot m^2\cdot s^3}}{\mathrm{C^2\cdot s^4\cdot m^3}} = \frac{\mathrm{N\cdot m}}{\mathrm{s}} = \frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}$$

(b) $F = ma = qE$, or

$$a = \frac{qE}{m} = \frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(100\ \mathrm{N/C}\right)}{9.11\times10^{-31}\ \mathrm{kg}} = \boxed{1.76\times10^{13}\ \mathrm{m/s^2}}$$

(c) The radiated power is then:

$$P = \frac{q^2a^2}{6\pi\in_0 c^3} = \frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)^2\left(1.76\times10^{13}\ \mathrm{m/s^2}\right)^2}{6\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)^3}$$
$$= \boxed{1.75\times10^{-27}\ \mathrm{W}}$$

(d) $F = ma_c = m\left(\dfrac{v^2}{r}\right) = qvB$,

so $v = \dfrac{qBr}{m}$

The proton accelerates at

$$a = \frac{v^2}{r} = \frac{q^2B^2r}{m^2} = \frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)^2\left(0.350\ \mathrm{T}\right)^2\left(0.500\ \mathrm{m}\right)}{\left(1.67\times10^{-27}\ \mathrm{kg}\right)^2}$$
$$= 5.62\times10^{14}\ \mathrm{m/s^2}$$

The proton then radiates

$$P = \frac{q^2 a^2}{6\pi \in_0 c^3} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)^2\left(5.62\times10^{14}\ \text{m/s}^2\right)^2}{6\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.00\times10^{8}\ \text{m/s}\right)^3}$$
$$= \boxed{1.80\times10^{-24}\ \text{W}}$$

P33.49 (a) A hemisphere is half a sphere:

$$m = \rho V = \rho\left[\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\right] = 5.50 + 4(0.800)\ \text{kg} = 8.70\ \text{kg}$$

$$r = \left(\frac{6m}{\rho 4\pi}\right)^{1/3} = \left(\frac{6(8.7\ \text{kg})}{\left(990\ \text{kg/m}^3\right)4\pi}\right)^{1/3} = \boxed{0.161\ \text{m}}$$

(b) $A = \frac{1}{2}4\pi r^2 = 2\pi(0.161\ \text{m})^2 = \boxed{0.163\ \text{m}^2}$

(c) $P = e\sigma AT^4$ and $T = 31.0 + 273.0 = 304$ K:

$$P = 0.970\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.163\ \text{m}^2\right)(304\ \text{K})^4$$
$$= \boxed{76.8\ \text{W}}$$

(d) $I = \frac{P}{A} = e\sigma T^4$

$$I = 0.970\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(304\ \text{K})^4$$
$$= \boxed{470\ \text{W/m}^2}$$

(e) $I = \frac{E_{\text{max}}^2}{2\mu_0 c}$

$$E_{\text{max}} = \left(2\mu_0 cI\right)^{1/2}$$
$$= \left[2\left(4\pi\times10^{-7}\ \text{Tm/A}\right)\left(3.00\times10^{8}\ \text{m/s}\right)\left(470\ \text{W/m}^2\right)\right]^{1/2}$$
$$= \boxed{595\ \text{N/C}}$$

(f) $E_{\text{max}} = cB_{\text{max}} \rightarrow B_{\text{max}} = \frac{595\ \text{N/C}}{3\times10^{8}\ \text{m/s}} = \boxed{1.98\ \mu\text{T}}$

(g) Each kitten has radius $r_k = \left(\frac{6m}{\rho 4\pi}\right)^{1/3} = \left[\frac{6(0.800)}{990 \times 4\pi}\right]^{1/3} = 0.072\,8\text{ m}$

and radiating area $2\pi(0.072\,8\text{ m})^2 = 0.033\,3\text{ m}^2$. The mother cat

has area $2\pi\left[\frac{6(5.50)}{990 \times 4\pi}\right]^{2/3} = 0.120\text{ m}^2$. The total glowing area is

$0.120\text{ m}^2 + 4(0.033\,3\text{ m}^2) = 0.254\text{ m}^2$ and has power output

$P = IA = (470\text{ W/m}^2)(0.254\text{ m}^2) = \boxed{119\text{ W}}$.

Challenge Problems

P33.50 We can approximate the magnetic field as uniform over the area of the loop while it oscillates in time as $B = B_{\max}\cos\omega t$. The induced voltage is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\theta) = -A\frac{d}{dt}\left(B_{\max}\cos\omega t\cos\theta\right)$$

or $\mathcal{E} = AB_{\max}\,\omega(\sin\omega t\cos\theta)$

(a) Since the angular frequency is $\omega = 2\pi f$, and the area of the loop is πr^2, the amplitude of this emf is

$$\boxed{\mathcal{E}_{\max} = 2\pi^2 r^2 f B_{\max}\cos\theta}$$

where θ is the angle between the magnetic field and the normal to the loop.

(b) If $\vec{\mathbf{E}}$ is vertical, $\vec{\mathbf{B}}$ is horizontal, so $\boxed{\text{the plane of the loop should be vertical}}$ and $\boxed{\text{the plane should contain the line of sight of the transmitter}}$.

P33.51 We are given $f = 90.0$ MHz and $E_{\max} = 200\text{ mV/m} = 2.00 \times 10^{-3}\text{ V/m}$

(a) The wavelength of the wave is $\lambda = \dfrac{c}{f} = \boxed{3.33 \text{ m}}$

(b) Its period is $T = \dfrac{1}{f} = 1.11 \times 10^{-8} \text{ s} = \boxed{11.1 \text{ ns}}$

(c) We obtain the maximum value of the magnetic field from

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 6.67 \times 10^{-12} \text{ T} = \boxed{6.67 \text{ pT}}$$

(d) $$\boxed{\vec{\mathbf{E}} = \left(2.00 \times 10^{-3}\right)\cos 2\pi\left(\frac{x}{3.33} - 90.0 \times 10^{6} t\right)\hat{\mathbf{j}}}$$

$$\boxed{\vec{\mathbf{B}} = \left(6.67 \times 10^{-12}\right)\cos 2\pi\left(\frac{x}{3.33} - 90.0 \times 10^{6} t\right)\hat{\mathbf{k}}}$$

where $\vec{\mathbf{E}}$ is in V/m, $\vec{\mathbf{B}}$ in tesla, x in meters, and t in seconds.

(e) $$I = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{\left(2.00 \times 10^{-3} \text{ V/m}\right)^2}{2\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)}$$
$$= \boxed{5.31 \times 10^{-9} \text{ W/m}^2}$$

(f) From Equation 33.33, $I = cu_{\text{avg}}$ so $u_{\text{avg}} = \dfrac{I}{c} = \boxed{1.77 \times 10^{-17} \text{ J/m}^3}$

(g) From Equation 33.37, the pressure is

$$P = \frac{2I}{c} = \frac{(2)\left(5.31 \times 10^{-9} \text{ W/m}^2\right)}{3.00 \times 10^{8} \text{ m/s}} = \boxed{3.54 \times 10^{-17} \text{ Pa}}$$

ANSWERS TO QUICK-QUIZZES

1. (i) (b) **(ii)** (c)

2. (c)

3. (c)

4. (b)

5. (a)

6. (c)

7. (a)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P33.2 (a) $3.15 \times 10^3 \hat{\mathbf{j}}$ N/C; (b) $5.25\hat{\mathbf{k}} \times 10^{-7}$ T; (c) $4.83(-\hat{\mathbf{j}}) \times 10^{-16}$ N

P33.4 11.0m

P33.6 60.0km

P33.8 Answer will increase with time after this book is published; for the publication year of 2019, the alien civilization is at most 33.5 light-years away

P33.10 See P33.10 for full explanation.

P33.12 8.64×10^{10} m

P33.14 (a) 6.75 W/m^2; (b) 6.64×10^3 W/m^2; (c) A powerful automobile running on sunlight would have to carry on its roof a solar panel that is huge compared to the size of the car; (d) Agriculture and forestry for food and fuels, space heating of large and small buildings, water heating, and heating for drying and many other processes are current and potential applications of solar energy.

P33.16 33.4°C，21.7°C

P33.18 (a) 5.16×10^{-10} T; (b) Since the magnetic field of the Earth is approximately 5×10^{-5} T, the Earth's field is some 100 000 times stronger.

P33.20 (a) 540 V/m; (b) 2.58 μJ/m^3; (c) 773 W/m^2

P33.22 (a) 5.82×10^8 N; (b) 6.10×10^{13} times stronger

P33.24 (a) $\sqrt{\dfrac{2\mu_0 cP}{\pi r^2}}$; (b) $\dfrac{P\ell}{c}$; (c) $\dfrac{P\ell}{c^2}$

P33.26 (a) 590 W/m^2; (b) 2.10×10^{16} W; (c) 7.01×10^7 N; (d) ~10^{13} times stronger; (e) The values are similar for both planets because both the forces follow inverse-square laws. The force ratios are not identical for the two planets because of their different radii and masses.

P33.28 (a) $\dfrac{1}{2}\mu_0 cJ_{\max}\cos(kx-\omega t)\hat{\mathbf{j}}$; (b) $\dfrac{1}{4}\mu_0 cJ_{\max}^2\cos^2(kx-\omega t)\hat{\mathbf{i}}$; (c) $\dfrac{1}{8}\mu_0 cJ_{\max}^2$; (d) 3.48 A/m

P33.30 $\dfrac{2\pi mc}{qB}$

P33.32 8.41×10^{-3} s

P33.34 See table in P33.34 for full description.

P33.36 $E = 300\cos\left(62.8x - 1.88\times10^{10}t\right)$ and $B = 1.00\cos\left(62.8x - 1.88\times10^{10}t\right)$

P33.38 (a) 3.21×10^7 W; (b) 0.639 W/m^2; (c) 0.513%

P33.40 (a) 6.30×10^{-6} Pa; (b) 1.60×10^{10} times smaller than atmospheric pressure

P33.42 $\dfrac{3SR^2}{4cGM_S\rho}$

P33.44 (a) $\vec{\mathbf{E}} = \dfrac{38.8}{r^2}\hat{\mathbf{r}}$, where $\vec{\mathbf{E}}$ is in volts per meter and r is in meters;

(b) $E_{\max} = \dfrac{38.7}{r}$ where E is in volts per meter and r is in meters;

(c) 12.9 μm, but the expression in part (b) does not apply if this point is inside the source; (d) From part (c), we see that in the radiated wave, the field amplitude is inversely proportional to distance. As the distance doubles, the amplitude is cut in half; (e) In the static case, the field is inversely proportional to the square of distance. As the distance doubles, the field is reduced by a factor of 4.

P33.46 (a) See Fig 33.46 (b) $\dfrac{1}{2}\epsilon_0 E_{\max}^2 \cos^2(kx)$; (c) $\dfrac{1}{2\mu_0} B_{\max}^2 \cos^2(kx)$;

(d) $\epsilon_0 E_{\max}^2 \cos^2(kx)$; (e) $\dfrac{1}{2}\epsilon_0 E_{\max}^2 \lambda A$;(f) $\dfrac{1}{2}\epsilon_0 cE_{\max}^2 A$;

(g) $\dfrac{1}{2}\epsilon_0 cE_{\max}^2$;(h) $\dfrac{E_{\max}^2}{2\mu_0 c}$

P33.48 (a) W (b) 1.76×10^{13} m/s^2; (c) 1.75×10^{-27} W;

(d) 1.80×10^{-24} W

P33.50 (a) $\mathcal{E}_{\max} = 2\pi^2 r^2 f B_{\max}\cos\theta$ (b) If $\vec{\mathbf{E}}$ is vertical, $\vec{\mathbf{B}}$ is horizontal, so the plane of the loop should be vertical and the plane should contain the line of sight of the transmitter.