5

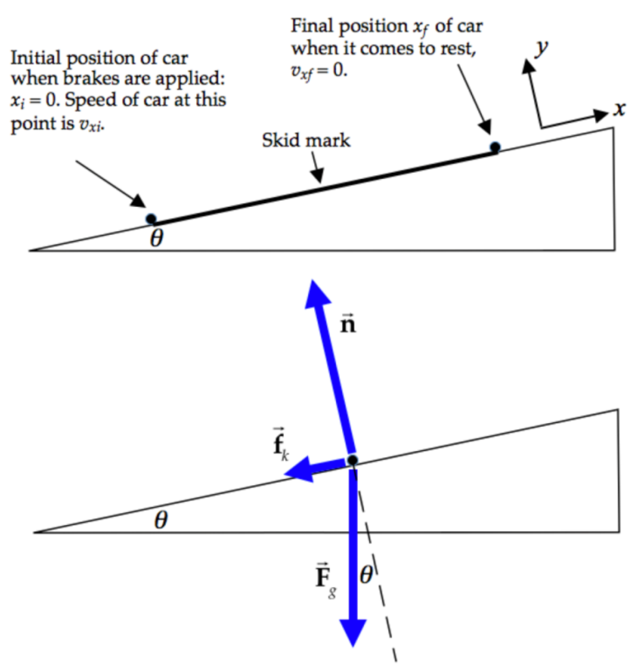
The Laws of Motion

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP5.1 Conceptualize** Imagine the situation occurring: The car is moving up the hill and suddenly the brakes are slammed on, locking the wheels. The car slows down due to two effects: the kinetic friction force on the wheels, and a component of the gravitational force acting down the hill. Because the wheels are locked, the car leaves a nice, long evidential trail of its motion on the street: the skid mark. We will assume that we can distinguish the skid marks from the front tires from those of the rear, so we do not need to know the distance between the front and rear wheels. Either set of skid marks is 17.0 m long. Let's make a drawing of the situation:



The upper drawing shows the kinematic information, while the lower drawing is a free-body diagram for the car during its motion.

Categorize The car is subject to forces and exhibits a change in its motion, so we use the *particle under a net force* model for the x direction, parallel to the incline, and the *particle in equilibrium* model perpendicular to the incline. We also know that the forces on the car are constant, resulting in a constant acceleration, so we will use the *particle under constant acceleration* model.

Analyze (a) From the particle under a net force and particle in equilibrium models, write equations for Newton's second law in the x and y directions, that is, parallel and perpendicular to the plane:

$$x: \quad -f_k - F_g \sin\theta = ma \qquad (1)$$
$$y: \quad n - F_g \cos\theta = 0 \qquad (2)$$

In Equation (1), m is the combined mass of the driver and the car. We can also use our standard model for kinetic friction forces to relate the friction force on the car and the normal force on it:

$$f_k = \mu_k n \qquad (3)$$

Combine Equations (1) through (3), eliminating f_k and n, leading to

$$-F_g\left(\sin\theta + \mu_k \cos\theta\right) = ma \qquad (4)$$

Now, the gravitational force is $F_g = mg$, where, again, m is the combined mass of the driver and the car. Making this substitution, Equation (4) becomes

$$a = -g\left(\sin\theta + \mu_k \cos\theta\right) \qquad (5)$$

This is the constant acceleration of the car. Now, from the particle under constant acceleration model, choose the equation relating velocity and position, Equation 2.17:

$$v_{xf}^2 = v_{xi}^2 + 2a\left(x_f - x_i\right)$$

Incorporate the initial position and the final velocity, and solve for the initial speed of the car:

$$0 = v_{xi}^2 + 2a\left(x_f - 0\right) \quad \rightarrow \quad v_{xi} = \sqrt{-2ax_f} \qquad (6)$$

Substitute the acceleration from Equation (5) into Equation (6):

$$v_{xi} = \sqrt{-2\left[-g\left(\sin\theta + \mu_k \cos\theta\right)\right]x_f} = \sqrt{2gx_f\left(\sin\theta + \mu_k \cos\theta\right)} \qquad (7)$$

Substitute numerical values:

$$\begin{aligned} v_{xi} &= \sqrt{2\left(9.80 \text{ m/ s}^2\right)\left(17.0 \text{ m}\right)\left[\sin 17.5\square + \left(0.580\right)\cos 17.5\square\right]} \\ &= 16.9 \text{ m/ s}\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right)\left(\frac{1 \text{ mi}}{1\,609 \text{ m}}\right) = 37.7 \text{ mi/ h} \end{aligned}$$

(b) This initial speed of the car exceeds the posted speed limit by over 50%, so your group should *not* agree to offer testimony for the defense in this case.

Finalize Notice what data we did *not* need for a solution: the mass of the car, the mass of the driver, and the coefficient of static friction.

Answer: (a) no (b) The initial speed of the car exceeds the posted speed limit.

***TP5.2** **Conceptualize** Imagine throwing the egg so that it follows a trajectory like that in Figure 4.8. When the egg lands in your hand, it will be brought to rest over a short distance if you hold your hand stationary. You may be tempted to think that it is brought to rest over zero distance, but two things happen to provide some distance. One, your hand is soft, so the flesh of your hand will compress a bit when the egg hits it. More importantly, experience shows us that the egg breaks in this situation, so the egg will be brought to rest over a distance comparable to its diameter as the leading edge stops and parts of the egg behind the leading edge continue to move. When you move your hands backward while catching the egg, the stopping distance is increased significantly.

Categorize The analysis involves a combination of a trajectory problem for the thrown egg and a *particle under a net force* as the egg is brought to rest. Without more detailed knowledge, and since we are making an estimate, let's assume that the acceleration of the egg is constant as it is brought to rest by the hand, so we can use the *particle under constant acceleration* model.

Analyze In a typical egg-throwing contest, there is no fixed separation between the thrower and the catcher, because the game is often played by starting with the participants close together, and then moving apart as successful transfers are made, until the egg breaks. Let us choose a separation distance of 10 m as a possibility. Assume the egg is thrown at 45° and returns to the same height from which it is thrown. We can then use Equation 4.20 to find the speed with which the egg is thrown, which is the same as the speed with which it lands in the hand:

$$R = \frac{v_i^2 \sin 2\theta_i}{g} \quad \rightarrow \quad v_i = v_f = \sqrt{\frac{gR}{\sin 2\theta_i}} \qquad (1)$$

where R = 10 m is the range of the egg toss. Now, let's focus on the egg. A medium egg has a mass of about 50 g = 0.050 kg. A typical diameter might be 40 mm = 0.040 m. When the hand is held stiffly, the egg will be brought to rest over a distance similar to its diameter. When the hand moves backward while catching the egg, the egg will be brought to rest over a longer distance, lets say, 1.0 m.

From the particle under constant acceleration model, let's find the acceleration of the egg as it is brought to rest. Assume that the direction of this acceleration is opposite that of the final velocity vector of the egg at the end of its flight, which we will call the s direction. Then, using Equation 2.17,

$$v_{sf}^2 = v_{si}^2 + 2a_s\left(s_f - s_i\right) \quad \rightarrow \quad a_s = \frac{v_{sf}^2 - v_{si}^2}{2\left(s_f - s_i\right)} = \frac{-v_{si}^2}{2d} \qquad (2)$$

where d is the distance over which the egg is brought to rest (v_{sf} = 0). The initial speed v_{si} in Equation (2) for the acceleration portion of the motion is the same as the final speed v_f for the trajectory found above in Equation (1). From the particle under a net force model, the force exerted on the egg while it is brought to rest is

$$F_s = ma_s \quad (3)$$

Combining Equations (1), (2), and (3), we have the force on the egg in terms of estimated values:

$$F_s = -m\left[\frac{gR}{2d\sin 2\theta_i}\right]$$

Substituting numerical values for the egg caught in the stiffly held hand,

$$F_s = -(0.050 \text{ kg})\left[\frac{(9.80 \text{ m/ s}^2)(10 \text{ m})}{2(0.040 \text{ m})\sin 90°}\right] = -61 \text{ N}$$

For the egg caught in the hand that moves backward while catching it,

$$F_s = -(0.050 \text{ kg})\left[\frac{(9.80 \text{ m/ s}^2)(10 \text{ m})}{2(1.0 \text{ m})\sin 90°}\right] = -2.5 \text{ N}$$

Finalize Therefore, with these estimates and assumptions, the magnitude of the force on the egg is reduced by a factor of 25 by moving the hands backward while catching the egg.

Answer: Answers will vary, based on estimates.

***TP5.3** *Answers:* (a) The coefficient of static friction is given by the equation in Example 5.11, $\mu_s = \tan\theta_c$, where θ_c is the critical angle. Answers will vary, depending on the coin and the condition of the book cover. (b) Ideally, the coefficient should not depend on the number of coins.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 5.1 The Concept of Force

P5.1 Using the reference axes shown in Figure P5.4, we see that

$$\sum F_x = T\cos 14.0° - T\cos 14.0° = 0$$

and

$$\sum F_y = -T\sin 14.0° - T\sin 14.0° = -2T\sin 14.0°$$

Thus, the magnitude of the resultant force exerted on the tooth by the wire brace is

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{0 + (-2T\sin 14.0°)^2} = 2T\sin 14.0°$$

or

$$R = 2(18.0\text{ N})\sin 14.0° = \boxed{8.71\text{ N}}$$

P5.2 (a) Force exerted by spring on hand, to the left; force exerted by spring on wall, to the right.

(b) Force exerted by wagon on handle, downward to the left. Force exerted by wagon on planet, upward. Force exerted by wagon on ground, downward.

(c) Force exerted by football on player, downward to the right. Force exerted by football on planet, upward.

(d) $\boxed{\text{Force exerted by small-mass object on large-mass object, to the left.}}$

(e) $\boxed{\text{Force exerted by negative charge on positive charge, to the left.}}$

(f) $\boxed{\text{Force exerted by iron on magnet, to the left.}}$

Section 5.4 Newton's Second Law

P5.3 We use Newton's second law to find the force as a vector and then the Pythagorean theorem to find its magnitude. The givens are $m = 3.00$ kg and $\vec{\mathbf{a}} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$.

(a) The total vector force is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (3.00\ \text{kg})(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\ \text{m/s}^2 = \boxed{(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}})\ \text{N}}$$

(b) Its magnitude is

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\left(F_x\right)^2 + \left(F_y\right)^2} = \sqrt{(6.00\ \text{N})^2 + (15.0\ \text{N})^2} = \boxed{16.2\ \text{N}}$$

P5.4 (a) Let the x axis be in the original direction of the molecule's motion. Then, from $v_f = v_i + at$, we have

$$a = \frac{v_f - v_i}{t} = \frac{-670\ \text{m/s} - 670\ \text{m/s}}{3.00 \times 10^{-13}\ \text{s}} = \boxed{-4.47 \times 10^{15}\ \text{m/s}^2}$$

(b) For the molecule, $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Its weight is negligible.

$$\vec{\mathbf{F}}_{\text{wall on molecule}} = \left(4.68\times10^{-26}\ \text{kg}\right)\left(-4.47\times10^{15}\ \text{m/s}^2\right)$$
$$= -2.09\times10^{-10}\ \text{N}$$
$$\vec{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09\times10^{-10}\ \text{N}}$$

P5.5 (a) We start from the sum of the two forces:

$$\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(-6.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) + \left(-3.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)$$
$$= \left(-9.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{N}$$

The acceleration is then:

$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{\left(-9.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{N}}{2.00\ \text{kg}}$$
$$= \left(-4.50\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

and the velocity is found from

$$\vec{\mathbf{v}}_f = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \vec{\mathbf{a}}t$$

$$\vec{\mathbf{v}}_f = \left[\left(-4.50\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\ \text{m/s}^2\right](10.0\ \text{s})$$
$$= \boxed{\left(-45.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\ \text{m/s}}$$

(b) The direction of motion makes angle θ with the x direction.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{15.0\ \text{m/s}}{45.0\ \text{m/s}}\right)$$

$$\theta = -18.4^\circ + 180^\circ = \boxed{162^\circ \text{ from the } +x \text{ axis}}$$

(c) Displacement:

$$x\text{-displacement} = x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$= \frac{1}{2}\left(-4.50\ \text{m/s}^2\right)(10.0\ \text{s})^2 = -225\ \text{m}$$

$$y\text{-displacement} = y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$$

$$= \frac{1}{2}\left(+1.50\ \text{m/s}^2\right)(10.0\ \text{s})^2 = +75.0\ \text{m}$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(-225\hat{\mathbf{i}} + 75.0\hat{\mathbf{j}}\right)\ \text{m}}$$

(d) Position: $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$

$$\vec{\mathbf{r}}_f = \left(-2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}\right) + \left(-225\hat{\mathbf{i}} + 75.0\hat{\mathbf{j}}\right) = \boxed{\left(-227\hat{\mathbf{i}} + 79.0\hat{\mathbf{j}}\right)\ \text{m}}$$

P5.6 Since the two forces are perpendicular to each other, their resultant is

$$F_R = \sqrt{(180\ \text{N})^2 + (390\ \text{N})^2} = 430\ \text{N}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{390\ \text{N}}{180\ \text{N}}\right) = 65.2°\ \text{N of E}$$

From Newton's second law,

$$a = \frac{F_R}{m} = \frac{430\ \text{N}}{270\ \text{kg}} = 1.59\ \text{m/s}^2$$

or

$$\vec{\mathbf{a}} = \boxed{1.59\ \text{m/s}^2\ \text{at}\ 65.2°\ \text{N of E}}$$

P5.7 $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ reads

$$\left(-2.00\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{i}}-3.00\hat{\mathbf{j}}-45.0\hat{\mathbf{i}}\right)\text{ N}=m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of $\vec{\mathbf{a}}$:

$$\left(-42.0\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)\text{ N}=m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

$$\sum\vec{\mathbf{F}}=\sqrt{(42.0)^2+(1.00)^2}\text{ N at }\tan^{-1}\left(\frac{1.00}{42.0}\right)\text{ below the }-x\text{ axis}$$

$$\sum\vec{\mathbf{F}}=42.0\text{ N at }181°=m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) Therefore $\boxed{\hat{\mathbf{a}}\text{ is at }181°}$ counter-clockwise from the x axis

(b) $m=\dfrac{42.0\text{ N}}{3.75\text{ m/s}^2}=\boxed{11.2\text{ kg}}$

(c) $v=|\vec{\mathbf{v}}|=0+|\vec{\mathbf{a}}|t=(3.75\text{ m/s}^2)(10.00\text{ s})=\boxed{37.5\text{ m/s}}$

(d) $\vec{\mathbf{v}}=\vec{\mathbf{v}}_i+|\vec{\mathbf{a}}|\,t=0+\dfrac{\vec{\mathbf{F}}}{m}t$

$$\vec{\mathbf{v}}=\frac{\left(-42.0\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)\text{ N}}{11.2\text{ kg}}(10.0\text{ s})=\left(-37.5\hat{\mathbf{i}}-0.893\hat{\mathbf{j}}\right)\text{ m/s}$$

So, $\vec{\mathbf{v}}_f=\boxed{\left(-37.5\hat{\mathbf{i}}-0.893\hat{\mathbf{j}}\right)\text{ m/s}}$

P5.8 $v=v_i-kx$ implies the acceleration is given by

$$a=\frac{dv}{dt}=0-k\frac{dx}{dt}=-kv$$

Then the total force is

$$\sum F = ma = m(-kv)$$

The resistive force is opposite to the velocity:

$$\boxed{\sum \vec{\mathbf{F}} = -km\vec{\mathbf{v}}}$$

P5.9 We find the mass of the baseball from its weight: $w = mg$, so $m = w/g$ = 2.21 N/9.80 m/s^2 = 0.226 kg.

(a) We use $x_f = x_i + \frac{1}{2}(v_i + v_f)t$ and $x_f - x_i = \Delta x$, with $v_i = 0$, v_f = 18.0 m/s, and $\Delta t = t = 170 \text{ ms} = 0.170 \text{ s}$:

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$$
$$\Delta x = \frac{1}{2}(0 + 18.0 \text{ m/s})(0.170 \text{ s}) = \boxed{1.53 \text{ m}}$$

(b) We solve for acceleration using $v_{xf} = v_{xi} + a_x t$, which gives

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

where a is in m/s^2, v is in m/s, and t in s. Substituting gives

$$a_x = \frac{18.0 \text{ m/s} - 0}{0.170 \text{ s}} = 106 \text{ m/s}^2$$

Call $\vec{\mathbf{F}}_1$ = force of pitcher on ball, and $\vec{\mathbf{F}}_2$ = force of Earth on ball (weight). We know that

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \text{m}\vec{\mathbf{a}}$$

Writing this equation in terms of its components gives

$$\sum F_x = F_{1x} + F_{2x} = ma_x \qquad \sum F_y = F_{1y} + F_{2y} = ma_y$$

$$\sum F_x = F_{1x} + 0 = ma_x \qquad \sum F_y = F_{1y} - 2.21 \text{ N} = 0$$

Solving,

$$F_{1x} = (0.226 \text{ kg})(106 \text{ m/s}^2) = 23.9 \text{ N} \text{ and } F_{1y} = 2.21 \text{ N}$$

Then,

$$\begin{aligned} F_1 &= \sqrt{(F_{1x})^2 + (F_{1y})^2} \\ &= \sqrt{(23.9 \text{ N})^2 + (2.21 \text{ N})^2} = 24.0 \text{ N} \end{aligned}$$

and $\theta = \tan^{-1}\left(\dfrac{2.21 \text{ N}}{23.9 \text{ N}}\right) = 5.29°$

$\boxed{\text{The pitcher exerts a force of 24.0 N forward at 5.29° above the horizontal.}}$

Section 5.5 The Gravitational Force and Weight

P5.10 (a) Use $\Delta x = \dfrac{1}{2}(v_i + v_f)\Delta t$, where $v_i = 0$, $v_f = v$, and $\Delta t = t$:

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t = \boxed{\frac{1}{2}vt}$$

(b) Use $v_{xf} = v_{xi} + a_x t$:

$$v_{xf} = v_{xi} + a_x t \rightarrow a_x = \frac{v_{xf} - v_{xi}}{t} \rightarrow a_x = \frac{v - 0}{t} = \frac{v}{t}$$

Call $\vec{\mathbf{F}}_1$ = force of pitcher on ball, and $\vec{\mathbf{F}}_2 = -F_g = -mg =$ gravitational force on ball. We know that

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \mathrm{m}\vec{\mathbf{a}}$$

writing this equation in terms of its components gives

$$\sum F_x = F_{1x} + F_{2x} = ma_x \qquad \sum F_y = F_{1y} + F_{2y} = ma_y$$

$$\sum F_x = F_{1x} + 0 = ma_x \qquad \sum F_y = F_{1y} - mg = 0$$

Solving and substituting from above,

$$F_{1x} = mv/t \qquad F_{1y} = mg$$

then the magnitude of F_1 is

$$F_1 = \sqrt{\left(F_{1x}\right)^2 + \left(F_{1y}\right)^2}$$
$$= \sqrt{(mv/t)^2 + (mg)^2} = \boxed{m\sqrt{(v/t)^2 + g^2}}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{mg}{mv/t}\right) = \boxed{\tan^{-1}\left(\frac{gt}{v}\right)}$$

P5.11 Since this is a linear acceleration problem, we can use Newton's second law to find the force as long as the electron does not approach relativistic speeds (as long as its speed is much less than 3×10^8 m/s), which is certainly the case for this problem. We know the initial and final velocities, and the distance involved, so from these we can find the acceleration needed to determine the force.

(a) From $v_f^2 = v_i^2 + 2ax$ and $\sum F = ma$, we can solve for the acceleration and then the force: $a = \dfrac{v_f^2 - v_i^2}{2x}$

Substituting to eliminate a, $\sum F = \dfrac{m\left(v_f^2 - v_i^2\right)}{2x}$

Substituting the given information,

$$\sum F = \frac{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left[\left(7.00 \times 10^{5}\ \text{m/s}\right)^2 - \left(3.00 \times 10^{5}\ \text{m/s}\right)^2\right]}{2(0.050\,0\ \text{m})}$$

$$\sum F = \boxed{3.64 \times 10^{-18}\ \text{N}}$$

(b) The Earth exerts on the electron the force called weight,

$$F_g = mg = (9.11 \times 10^{-31}\,\text{kg})(9.80\ \text{m/s}^2) = 8.93 \times 10^{-30}\,\text{N}$$

The accelerating force is

$$\boxed{4.08 \times 10^{11}\ \text{times the weight of the electron.}}$$

P5.12 We are given $F_g = mg = 900\ \text{N}$, from which we can find the man's mass,

$$m = \frac{900\ \text{N}}{9.80\ \text{m/s}^2} = 91.8\ \text{kg}$$

Then, his weight on Jupiter is given by

$$\left(F_g\right)_{\text{on Jupiter}} = 91.8\ \text{kg}\left(25.9\ \text{m/s}^2\right) = \boxed{2.38\ \text{kN}}$$

P5.13 (a) You and the Earth exert equal forces on each other: $m_y g = M_E a_E$. If your mass is 70.0 kg,

$$a_E = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)}{5.98 \times 10^{24}\ \text{kg}} = \boxed{\sim 10^{-22}\ \text{m/s}^2} \qquad \textbf{[1]}$$

(b) You and the planet move for equal time intervals Δt according to $\Delta x = \frac{1}{2}a(\Delta t)^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2\Delta x_y}{a_y}} = \sqrt{\frac{2\Delta x_E}{a_E}}$$

$$\Delta x_E = \frac{a_E}{a_y}\Delta x_y$$

We substitute for $\frac{a_E}{a_y}$ from [1] to obtain

$$\Delta x_E = \frac{70.0\ \text{kg}(0.500\ \text{m})}{5.98 \times 10^{24}\ \text{kg}}$$

$$\boxed{\Delta x_E \sim 10^{-23}\ \text{m}}$$

Section 5.6 Newton's Third Law

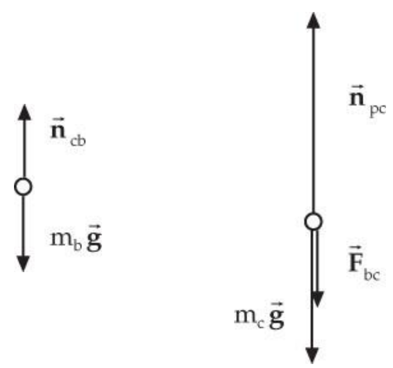
P5.14 The free-body diagrams are shown in ANS. FIG. P5.14 below.

(a)

$\vec{\mathbf{n}}_{cb}$ = normal force of cushion on brick
$m_b\vec{\mathbf{g}}$ = gravitational force on brick

(b)

$\vec{\mathbf{n}}_{pc}$ = normal force of pavement on cushion
$m_b\vec{\mathbf{g}}$ = gravitational force on cushion
$\vec{\mathbf{F}}_{bc}$ = force of brick on cushion



brick cushion

(a) (b)

ANS. FIG.P5.14

(c)

force: normal force of cushion on brick ($\vec{\mathbf{n}}_{cb}$) $\rightarrow$ reaction force: force of brick on cushion ($\vec{\mathbf{F}}_{bc}$)
force: gravitational force of Earth on brick ($m_b\vec{\mathbf{g}}$) $\rightarrow$ reaction force: gravitational force of brick on Earth
force: normal force of pavement on cushion ($\vec{\mathbf{n}}_{pc}$) $\rightarrow$ reaction force: force of cushion on pavement
force: gravitational force of Earth on cushion ($m_c\vec{\mathbf{g}}$) $\rightarrow$ reaction force: gravitational force of cushion on Earth

Section 5.7 Analysis Models Using Newton's Second Law

P5.15 As the worker through the pole exerts on the lake bottom a force of 240 N downward at 35° behind the vertical, the lake bottom through the pole exerts a force of 240 N upward at 35° ahead of the vertical. With the x axis horizontally forward, the pole force on the boat is

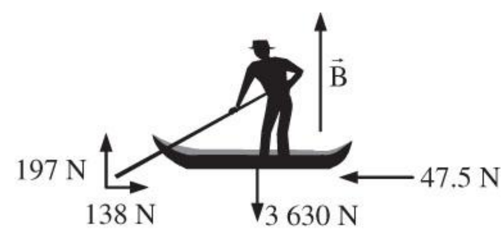
$$\left(240\cos 35°\hat{\mathbf{j}} + 240\sin 35°\hat{\mathbf{i}}\right)\text{ N} = \left(138\hat{\mathbf{i}} + 197\hat{\mathbf{j}}\right)\text{ N}$$

The gravitational force of the whole Earth on boat and worker is F_g = mg = 370 kg (9.8 m/s^2) = 3 630 N down. The acceleration of the boat is purely horizontal, so

$$\sum F_y = ma_y \text{ gives } +B + 197\text{ N} - 3\,630\text{ N} = 0$$

(a) The buoyant force is $B = \boxed{3.43\times 10^3\text{ N}}$.

(b) The acceleration is given by



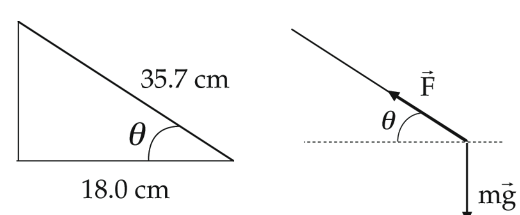
$$\sum F_x = ma_x: \quad +138\text{ N} - 47.5\text{ N} = (370\text{ kg})a$$

ANS. FIG. P5.15

$$a = \frac{90.2\text{ N}}{370\text{ kg}} = 0.244\text{ m/s}^2$$

According to the constant-acceleration model,

ANS. FIG. P5.16(a)



P5.16 (a) The left-hand diagram in ANS. FIG. P5.16(a) shows the geometry of the situation and lets us find the angle of the string with the horizontal:

$$\cos\theta = 28/35.7 = 0.784$$

or $\theta = 38.3°$

The right-hand diagram in ANS. FIG. P5.16(a) is the free-body diagram. The weight of the bolt is

$$w = mg = (0.065 \text{ kg})(9.80 \text{ m/s}^2) = 0.637 \text{ N}$$

(b) To find the tension in the string, we apply Newton's second law in the x and y directions:

$$\sum F_x = ma_x: -T\cos 38.3° + F_{\text{magnetic}} = 0 \qquad \textbf{[1]}$$

$$\sum F_y = ma_y: +T\sin 38.3° - 0.637 \text{ N} = 0 \qquad \textbf{[2]}$$

from equation [2],

$$T = \frac{0.637 \text{ N}}{\sin 38.3°} = \boxed{1.03 \text{ N}}$$

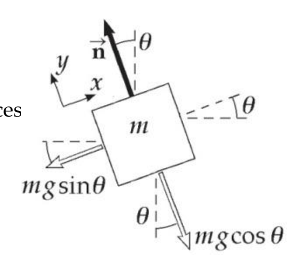
(c) Now, from equation [1],

$$F_{\text{magnetic}} = T\cos 38.3° = (1.03 \text{ N})\cos 38.3° = \boxed{0.805 \text{ N to the right}}$$

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ &= 0.857 \text{ m/s} + (0.244 \text{ m/s}^2)(0.450 \text{ s}) \\ &= 0.967 \text{ m/s} \end{aligned}$$

$$\vec{\mathbf{v}}_f = \boxed{0.967\hat{\mathbf{i}} \text{ m/s}}$$

P5.17 (a) ANS. FIG. P5.17(a) shows the forces on the



ANS. FIG. P5.17(a)

object. The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x axis is chosen to be parallel to the plane, then the free-body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive x direction), we have

$$\sum F_y = n - mg\cos\theta = 0\text{:}\ \ n = mg\cos\theta$$

$$\sum F_x = -mg\sin\theta = ma\text{:}\ \ a = -g\sin\theta$$

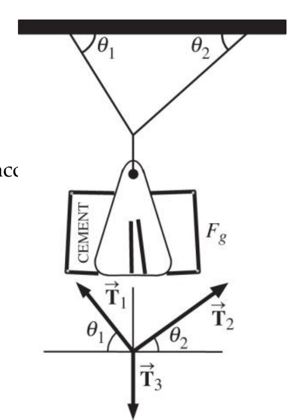
(b) When $\theta = 15.0°$,

$$a = \boxed{-2.54\ \text{m/s}^2}$$

(c) Starting from rest,

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2a\Delta x$$
$$\left|v_f\right| = \sqrt{2|a|\Delta x} = \sqrt{2\left|-2.54\ \text{m/s}^2\right|(2.00\ \text{m})} = \boxed{3.19\ \text{m/s}}$$

P5.18 From equilibrium of the sack:



$T_3 = F_g$ **[1]**

From $\sum F_y = 0$ for the knot:

$T_1 \sin\theta_1 + T_2 \sin\theta_2 = F_g$ **[2]**

From $\sum F_x = 0$ for the knot:

$T_1 \cos\theta_1 = T_2 \cos\theta_2$ **[3]**

ANS. FIG. P5.18

Eliminate T_2 by using $T_2 = T_1 \cos\theta_1 / \cos\theta_2$

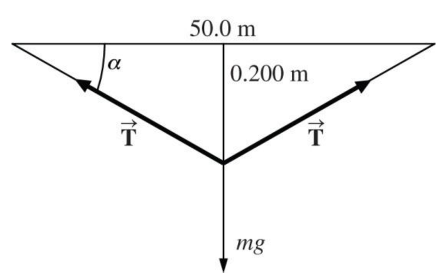
and solve for T_1:

$$T = \frac{F_g cos\theta_2}{sin\theta_1 cos\theta_2 + cos\theta_1 sin\theta_2}$$

$$T = \frac{F_g cos\theta_2}{\sin(\theta_1 + \theta_2)}$$

P5.19 We use Newton's second law with the forces in the x and y directions in equilibrium.

(a) At the point where the bird is perched, the wire's midpoint, the forces acting on the wire are the tension forces and the force of gravity acting on the bird. These forces are shown in ANS. FIG. P5.19 below.



ANS. FIG. P5.19

(b) The mass of the bird is m = 1.00 kg, so the force of gravity on the bird, its weight, is mg = (1.00 kg)(9.80 m/s^2) = 9.80 N. To calculate the angle α in the free-body diagram, we note that the base of the triangle is 25.0 m, so that

$$\tan\alpha = \frac{0.200 \text{ m}}{25.0 \text{ m}} \quad \rightarrow \quad \alpha = 0.458°$$

Each of the tension forces has x and y components given by

$$T_x = T\cos\alpha \quad \text{and} \quad T_y = T\sin\alpha$$

The x components of the two tension forces cancel out. In the y direction,

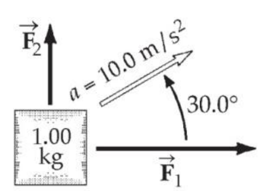
$$\sum F_y = 2T\sin\alpha - mg = 0$$

which gives

$$T = \frac{mg}{2\sin\alpha} = \frac{9.80 \text{ N}}{2\sin 0.458°} = \boxed{613 \text{ N}}$$

P5.20 Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North. The acceleration is

$$\begin{aligned}\vec{\mathbf{a}} &= [(10.0 \cos 30.0°)\hat{\mathbf{i}} + (10.0 \sin 30.0°)\hat{\mathbf{j}}] \text{ m/s}^2 \\ &= (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) \text{ m/s}^2\end{aligned}$$



ANS. FIG. P5.20

From Newton's second law,

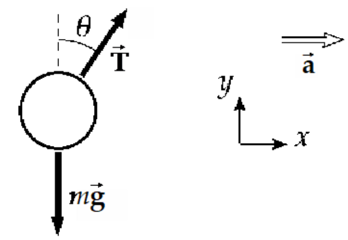
$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (1.00\text{ kg})(8.66\hat{\mathbf{i}}\text{ m/s}^2 + 5.00\hat{\mathbf{j}}\text{ m/s}^2)$$
$$= (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{ N}$$

and $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

So the force we want is

$$\vec{\mathbf{F}}_1 = \sum \vec{\mathbf{F}} - \vec{\mathbf{F}}_2 = (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{j}})\text{ N}$$
$$= 8.66\hat{\mathbf{i}}\text{ N} = \boxed{8.66\text{ N east}}$$

P5.21 (a) Assume the car and mass accelerate horizontally. We consider the forces on the suspended object.



ANS. FIG. P5.21

$$\sum F_y = ma_y:\ +T\cos\theta - mg = 0$$

$$\sum F_x = ma_x:\ +T\sin\theta = ma$$

Substitute $T = \dfrac{mg}{\cos\theta}$ from the first equation into the second,

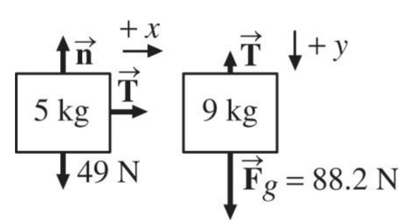
$$\frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = ma$$

$$\boxed{a = g\tan\theta}$$

(b) $a = (9.80\text{ m/s}^2)\tan 23.0° = \boxed{4.16\text{ m/s}^2}$

P5.22 (a) The forces on the objects are shown in ANS. FIG. P5.22.

(b) and (c) First, consider m_1, the block moving



ANS. FIG. P5.22

along the horizontal. The only force in the direction of movement is T. Thus,

$$\sum F_x = ma$$

or $\quad T = (5.00\ \text{kg})a$ **[1]**

Next consider m_2, the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

We have $\sum F_y = ma$:

$$88.2\ \text{N} - T = (9.00\ \text{kg})a \qquad \textbf{[2]}$$

Note that both blocks must have the same magnitude of acceleration. Equations [1] and [2] can be added to give 88.2 N = (14.0 kg)a. Then

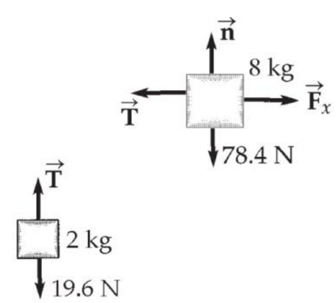
$$\boxed{a = 6.30\ \text{m/s}^2 \text{ and } T = 31.5\ \text{N}}$$

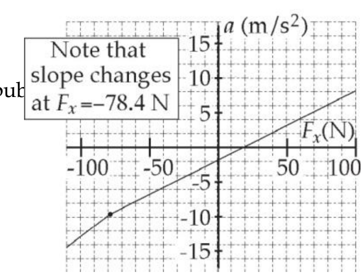
P5.23 Forces acting on m_1 = 2.00-kg block:

$$T - m_1 g = m_1 a \qquad \textbf{[1]}$$

Forces acting on m_2 = 8.00-kg block:

$$F_x - T = m_2 a \qquad \textbf{[2]}$$





(a) Eliminate T and solve for a:

$$a = \frac{F_x - m_1 g}{m_1 + m_2}$$

$$\boxed{a > 0 \text{ for } F_x > m_1 g = 19.6\text{ N}}$$

(b) Eliminate a and solve for T:

ANS. FIG. P5.23

$$T = \frac{m_1}{m_1 + m_2}\left(F_x + m_2 g\right)$$

$$\boxed{T = 0 \text{ for } F_x \le -m_2 g = -78.4\text{ N}}$$

Note that if $F_x < -m_2 g$, the cord is loose, so mass m_2 is in free fall and mass m_1 accelerates under the action of F_x only.

(c) See ANS. FIG. P5.23.

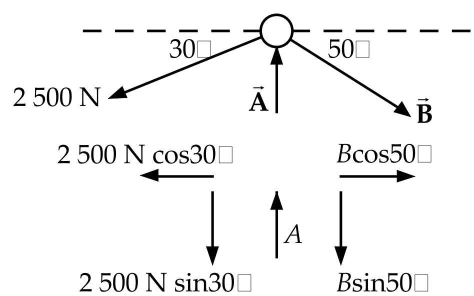
F_x, N	–100	–78.4	–50.0	0	50.0	100
a_x, m/s^2	–12.5	–9.80	–6.96	–1.96	3.04	8.04

P5.24 We assume the vertical bar is in compression, pushing up on the pin with force A, and the tilted bar is in tension, exerting force B on the pin at –50.0°.

$\sum F_x = 0$:

$$-2\,500\text{ N}\cos 30° + B\cos 50° = 0$$

$$\boxed{B = 3.37 \times 10^3\text{ N}}$$



$\sum F_y = 0$:

$$-2\ 500\ \text{N}\sin 30° + A - 3.37\times 10^3\ \text{N}\ \sin 50° = 0$$

ANS. FIG. P5.24

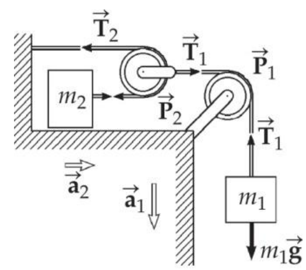
$$\boxed{A = 3.83\times 10^3\ \text{N}}$$

Positive answers confirm that

$$\boxed{B \text{ is in tension and } A \text{ is in compression.}}$$

P5.25 (a) Pulley P_2 has acceleration a_1.

Since m_2 moves *twice* the distance P_2 moves in the same time, m_2 has twice the acceleration of P_2, i.e., $\boxed{a_2 = 2a_1}$.



ANS. FIG. P5.25

(b) From the figure, and using

$$\sum F = ma: \quad m_1 g - T_1 = m_1 a_1 \qquad \textbf{[1]}$$
$$T_2 = m_2 a_2 = 2m_2 a_1 \qquad \textbf{[2]}$$
$$T_1 - 2T_2 = 0 \qquad \textbf{[3]}$$

Equation [1] becomes $m_1 g - 2T_2 = m_1 a_1$. This equation combined with equation [2] yields

$$\frac{T_2}{m_2}\left(2m_2 + \frac{m_2}{2}\right) = m_1 g$$

$$\boxed{T_2 = \frac{m_1 m_2}{2m_2 + \frac{1}{2}m_1} g} \quad \text{and} \quad \boxed{T_2 = \frac{m_1 m_2}{m_2 + \frac{1}{4}m_1} g}$$

(c) From the values of T_2 and T_1, we find that

$$a_2 = \frac{T_2}{m_2} = \boxed{\frac{m_1 g}{2m_2 + \frac{1}{2}m_1}} \quad \text{and} \quad a_1 = \frac{1}{2}a_2 = \boxed{\frac{m_1 g}{4m_2 + m_1}}$$

Section 5.8 Forces of Friction

P5.26 Find the acceleration of the car, which is the same as the acceleration of the book because the book does not slide.

For the car: $v_i = 72.0$ km/h = 20.0 m/s, $v_f = 0$, $\Delta x = (x_f - x_i) = 30.0$ m. Using $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the acceleration of the car:

$$a = -6.67 \text{ m/s}^2$$

Now, find the maximum acceleration that friction can provide. Because the book does not slide, static friction provides the force that slows down the book. We have the coefficient of static friction, $\mu_s = 0.550$, and we know $f_s \le \mu_s n$. The book is on a horizontal seat, so friction acts in the horizontal direction, and the vertical normal force that the seat exerts on the book is equal in magnitude to the force of gravity on the book: $n = F_g = mg$. For maximum acceleration, the static friction force will be a maximum, so $f_s = \mu_s n = \mu_s mg$. Applying Newton's second law, we find the acceleration that friction can provide for the book:

$$\sum F_x = ma_x:$$

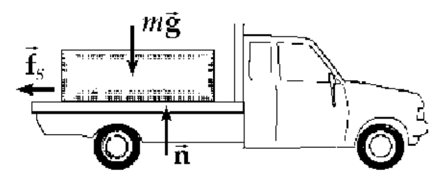
$$-f_s = ma$$

$$-\mu_s mg = ma$$

which gives $a = -\mu_s g = -(0.550)(9.80 \text{ m/s}^2) = -5.39 \text{ m/s}^2$, which is too small for the stated conditions.

$\boxed{\text{The situation is impossible because maximum static friction cannot provide the acceleration necessary to keep the book stationary on the seat.}}$

P5.27 If the load is on the point of sliding forward on the bed of the slowing truck, static friction acts backward on the load with its maximum value, to give it the same acceleration as the truck:



ANS. FIG.P5.26

$$\Sigma F_x = ma_x: \quad -f_s = m_{\text{load}} a_x$$

$$\Sigma F_y = ma_y: \quad n - m_{\text{load}} g = 0$$

Solving for the normal force and substituting into the x equation gives:

$$-\mu_s m_{\text{load}} g = m_{\text{load}} a_x \quad \text{or} \quad a_x = -\mu_s g$$

We can then use

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Which becomes

$$0 = v_{xi}^2 + 2(-\mu_s g)\left(x_f - 0\right)$$

(a) $x_f = \dfrac{v_{xi}^2}{2\mu_s g} = \dfrac{(12.0\ \text{m/s})^2}{2(0.500)(9.80\ \text{m/s}^2)} = \boxed{14.7\ \text{m}}$

(b) From the expression $x_f = \dfrac{v_{xi}^2}{2\mu_s g}$,

$\boxed{\text{neither mass affects the answer}}$

P5.28 We assume that all the weight is on the rear wheels of the car.

(a) We find the record time from

$$F = ma: \quad \mu_s mg = ma \quad \text{or} \quad a = \mu_s g$$

But

$$\Delta x = \frac{at^2}{2} = \frac{\mu_s g t^2}{2}$$

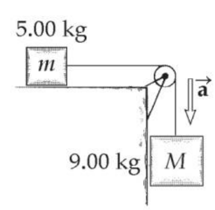
so $\mu_s = \dfrac{2\Delta x}{gt^2}$

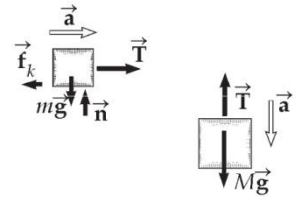
$$\mu_s = \frac{2(0.250\ \text{mi})(1\,609\ \text{m/mi})}{(9.80\ \text{m/s}^2)(4.43\ \text{s})^2} = \boxed{4.18}$$

(b) $\boxed{\text{Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.}}$

P5.29 Newton's second law for the 5.00-kg mass gives

$T - f_k = (5.00\ \text{kg})a$





Similarly, for the 9.00-kg mass,

$$(9.00\text{ kg})g - T = (9.00\text{ kg})a$$

Adding these two equations gives:

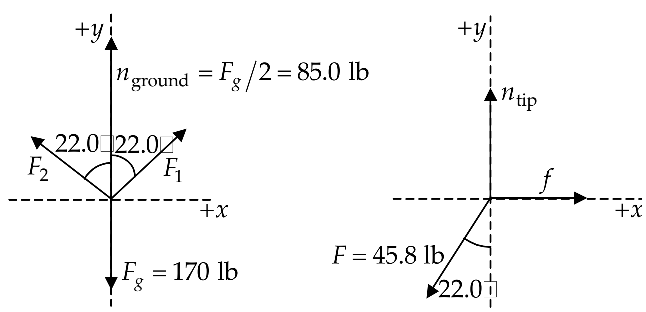
$$\begin{aligned}&(9.00\text{ kg})(9.80\text{ m/s}^2)\\ &\quad -0.200(5.00\text{ kg})(9.80\text{ m/s}^2)\\ &\qquad = (14.0\text{ kg})a\end{aligned}$$

ANS. FIG.P5.29

Which yields $a = 5.60\text{ m/s}^2$. Plugging this into the first equation above gives

$$T = (5.00\text{ kg})(5.60\text{ m/s}^2) + 0.200(5.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{37.8\text{ N}}$$

P5.30 The free-body diagrams for this problem are shown in ANS. FIG. P5.62.



Free-Body Diagram   Free-Body Diagram

ANS.FIG. P5.30

From the free-body diagram for the person,

$$\sum F_x = F_1\sin(22.0°) - F_2\sin(22.0°) = 0$$

which gives $F_1 = F_2 = F$. Then, $\sum F_y = 2F\cos 22.0° + 85.0 \text{ lbs} - 170 \text{ lbs} = 0$ yields $F = 45.8$ lb.

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8 \text{ lb})\sin 22.0° = 0$$

or $f = 17.2$ lb.

$$\sum F_y = n_{\text{tip}} - (45.8 \text{ lb})\cos 22.0° = 0$$

which gives $n_{\text{tip}} = 42.5$ lb.

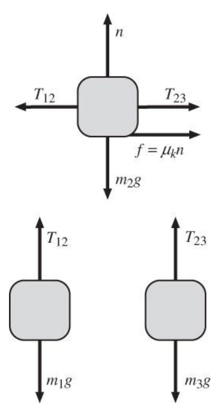
For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so $f = (f_s)_{\max} = \mu_s n_{\text{tip}}$ and

$$\mu_s = \frac{f}{n_{\text{tip}}} = \frac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.404}$$

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8 \text{ lb}}$$

P5.31 (a) The free-body diagrams for each object appear on the right.



ANS. FIG. P5.64(a)

(b) Let a represent the positive magnitude of the acceleration $-a\hat{\mathbf{j}}$ of m_1, of the acceleration $-a\hat{\mathbf{i}}$ of m_2, and of the acceleration $+a\hat{\mathbf{j}}$ of m_3. Call T_{12} the tension in the left cord and T_{23} the tension in the cord on the right.

For m_1, $\sum F_y = ma_y$:

$$+T_{12} - m_1 g = -m_1 a$$

For m_2, $\sum F_x = ma_x$:

$$-T_{12} + \mu_k n + T_{23} = -m_2 a$$

and $\sum F_y = ma_y$, giving $n - m_2 g = 0$.

For m_3, $\sum F_y = ma_y$, giving $T_{23} - m_3 g = +m_3 a$.

We have three simultaneous equations:

$$\begin{aligned} -T_{12} + 39.2\text{ N} &= (4.00\text{ kg})a \\ +T_{12} - 0.350(9.80\text{ N}) - T_{23} &= (1.00\text{ kg})a \\ +T_{23} - 19.6\text{ N} &= (2.00\text{ kg})a \end{aligned}$$

Add them up (this cancels out the tensions):

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\ \text{m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}$$

(c) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\ \text{m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\ \text{m/s}^2)$

$$\boxed{T_{23} = 24.2\text{ N}}$$

(d) If the tabletop were smooth, friction disappears ($\mu_k = 0$), and so the acceleration would become larger. For a larger acceleration, according to the equations above, the tensions change:

$$T_{12} = m_1 g - m_1 a \rightarrow \boxed{T_{12}\text{ decreases}}$$

$$T_{23} = m_3 g + m_3 a \rightarrow \boxed{T_{23} \text{ increases}}$$

***P5.32** Maximum static friction provides the force that produces maximum acceleration, resulting in a minimum time interval to accelerate through Δx = 3.00 m. We know that the maximum force of static friction is $f_s = \mu_s n$. If the shoe is on a horizontal surface, friction acts in the horizontal direction. Assuming that the vertical normal force is maximal, equal in magnitude to the force of gravity on the person, we have $n = F_g = mg$; therefore, the maximum static friction force is

$$f_s = \mu_s n = \mu_s mg$$

Applying Newton's second law:

$$\sum F_x = ma_x:$$

$$f_s = ma$$

$$\mu_s mg = ma \quad \rightarrow \quad a = \mu_s g$$

We find the time interval $\Delta t = t$ to accelerate from rest through Δx= 3.00 m using $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$\Delta x = \frac{1}{2}a_x(\Delta t)^2 \rightarrow \Delta t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2\Delta x}{\mu_s g}}$$

(a) For $\mu_s = 0.500$, $\Delta t = \boxed{1.11 \text{ s}}$

(b) For $\mu_s = 0.800$, $\Delta t = \boxed{0.875 \text{ s}}$

***P5.33 Conceptualize** Imagine beginning from rest and suddenly running across the floor. If the coefficient of static friction is too low, your feet will slip and you will not be able to move away rapidly enough in an emergency. You want your feet to stay planted on the floor in order to give your body as large an acceleration as possible.

Categorize The problem statement suggests assuming that you be modeled as a *particle under constant acceleration*. You are also a *particle under a net force,* where the force of static friction between your feet and the floor is causing your acceleration.

Analyze Modeling your body of mass m as a particle under a net force, we have, in the horizontal direction,

$$F_x = ma_x \quad \rightarrow \quad f_{s,\max} = ma_x$$

where we have recognized that the only force on your body in the horizontal direction is the force of static friction and we have assumed an impending motion situation where your foot is just about to slip. In this case, we can relate the friction force to the normal force on your body:

$$f_{s,\max} = ma_x \quad \rightarrow \quad \mu_s n = ma_x$$

Because your body is not accelerating in the vertical direction, it is a particle in equilibrium for this direction and the normal force is equal to your weight:

$$\mu_s n = ma_x \quad \rightarrow \quad \mu_s mg = ma_x \quad \rightarrow \quad \mu_s = \frac{a_x}{g} \qquad (1)$$

Now, let’s model your body as a particle under constant acceleration in order to determine the acceleration a_x. From Equation 2.16,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}a_x t^2 \quad \rightarrow \quad a_x = \frac{2x_f}{t^2} \qquad (2)$$

Substituting Equation (2) into Equation (1), we have

$$\mu_s = \frac{2x_f}{gt^2}$$

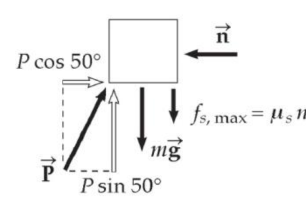
Substituting numerical values,

$$\mu_s = \frac{2(4.23\ \text{m})}{\left(9.80\ \text{m/s}^2\right)(1.20\ \text{s})^2} 0.599$$

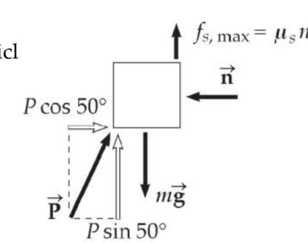
Your shoes do qualify because the coefficient of static friction is larger than 0.5.

Finalize Though we have made numerous simplifications in modeling the interactions of a shoe sole with the tile floor during a run, the fact remains that the forward acceleration you experience is due entirely to the force of static friction between the soles of your shoes and the floor.

Answer: Yes, the coefficient of static friction is $\mu_s \approx 0.6$, which is greater than regulated minimum value of 0.5.



P5.34 (a) To find the maximum possible value of P,



imagine impending upward motion as case 1. Setting $\sum F_x = 0$:

$$P\cos 50.0° - n = 0$$

with $f_{s,\,\max} = \mu_s n$:

$$\begin{aligned} f_{s,\,\max} &= \mu_s P\cos 50.0° \\ &= 0.250(0.643)P = 0.161P \end{aligned}$$

Setting $\sum F_y = 0$:

ANS. FIG. P5.34

$$\begin{aligned} &P\sin 50.0° - 0.161P \\ &\quad -(3.00\text{ kg})(9.80\text{ m/s}^2) = 0 \\ &P_{\max} = \boxed{48.6\text{ N}} \end{aligned}$$

To find the minimum possible value of P, consider impending downward motion. As in case 1,

$$f_{s,\,\max} = 0.161P$$

Setting $\sum F_y = 0$:

$$P\sin 50.0° + 0.161P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$

$$P_{\min} = \boxed{31.7\text{ N}}$$

(b) $\boxed{\text{If } P > 48.6\text{ N, the block slides up the wall. If } P < 31.7\text{ N, the block slides down the wall.}}$

(c) We repeat the calculation as in part (a) with the new angle.

Consider impending upward motion as case 1. Setting

$$\begin{aligned} &\sum F_x = 0: & &P\cos 13° - n = 0 \\ &f_{s,\,\max} = \mu_s n: & &f_{s,\,\max} = \mu_s P\cos 13° \\ & & &\quad = 0.250(0.974)P = 0.244P \end{aligned}$$

Setting

$$\sum F_y = 0: \quad P\sin 13° - 0.244P - (3.00 \text{ kg})(9.80 \text{ m/s}^2) = 0$$
$$P_{\text{max}} = -1\ 580 \text{ N}$$

The push cannot really be negative. However large or small it is, it cannot produce upward motion. To find the minimum possible value of P, consider impending downward motion. As in case 1,

$$f_{s,\ \text{max}} = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° + 0.244P - (3.00 \text{ kg})(9.80 \text{ m/s}^2) = 0$$
$$P_{\text{min}} = \boxed{62.7 \text{ N}}$$

$P \geq 62.7$ N. The block cannot slide up the wall. If $P < 62.7$ N, the block slides down the wall.

P5.35 The motion of the salmon as it breaks the surface of the water and eventually leaves must be modeled in two steps. The first is over a distance of 0.750 m, until half of the salmon is above the surface, while a constant force, P, is applied upward. In this motion, the initial velocity of the salmon as it nears the surface is 3.58 m/s and ends with the salmon having a velocity, $v_{½}$, when it is half out of the water. This is then the initial velocity for the second motion, where gravity is a second force to be considered acting on the fish. This motion is again over a distance of 0.750 m, and results with the salmon having a final velocity of 6.26 m/s.

The vertical motion equations, in each case, would be

$$a_{1y} = \frac{v_{1yf}^2 - v_{1yi}^2}{2\ \Delta y} = \frac{v_{1/2}^2 - \left(3.58\ \text{m/s}\right)^2}{2\ (0.750\ \text{m})} = \frac{v_{1/2}^2 - \left(12.8\ \text{m}^2/\text{s}^2\right)}{1.50\ \text{m}}$$

and

$$a_{2y} = \frac{v_{2yf}^2 - v_{2yi}^2}{2\ \Delta y} = \frac{\left(6.26\ \text{m/s}\right)^2 - v_{1/2}^2}{2\ (0.750\ \text{m})} = \frac{\left(39.2\ \text{m}^2/\text{s}^2\right) - v_{1/2}^2}{1.50\ \text{m}}$$

Solving for the square of the velocity in each case and equating the expressions, we find

$$v_{1/2}^2 = (1.50\ \text{m})a_{1y} + \left(12.8\ \text{m}^2/\text{s}^2\right)$$

$$v_{1/2}^2 = \left(39.2\ \text{m}^2/\text{s}^2\right) - (1.50\ \text{m})a_{2y}$$

$$(1.50\ \text{m})a_{1y} + \left(12.8\ \text{m}^2/\text{s}^2\right) = \left(39.2\ \text{m}^2/\text{s}^2\right) - (1.50\ \text{m})a_{2y}$$
$$a_{1y} = \left(17.6\ \text{m/s}^2\right) - a_{2y}$$

In the first motion, the relationship between the net acceleration and the net force can be written as

$$\sum F_y = P = ma_{1y}$$
$$P = \left(61.0\ \text{kg}\right)a_{1y}$$

Substituting from above,

$$P = \left(61.0\ \text{kg}\right)\left[\left(17.6\ \text{m/s}^2\right) - a_{2y}\right]$$
$$P = 1\ 070\ \text{N} - \left(61.0\ \text{kg}\right)a_{2y}$$

In the second motion, the relationship between the net acceleration and the net force can be written as

$$\sum F_y = P - mg = ma_{2y}$$
$$P = mg + ma_{2y} = (61.0\text{ kg})(9.80\text{ m/s}^2) + (61.0\text{ kg})a_{2y}$$
$$P = 598\text{ N} + (61.0\text{ kg})a_{2y}$$

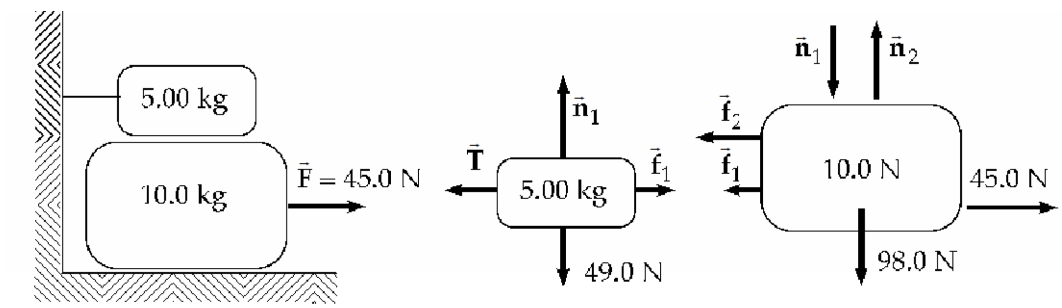
Equating these two equations for, *P*,

$$1\ 070\text{ N} - (61.0\text{ kg})a_{2y} = 598\text{ N} + (61.0\text{ kg})a_{2y}$$
$$-(122.0\text{ kg})a_{2y} = -472\text{ N}$$
$$a_{2y} = 3.87\text{ m/s}^2$$

Plugging into either of the above,

$$P = 598\text{ N} + (61.0\text{ kg})(3.87\text{ m/s}^2)$$
$$P = \boxed{834\text{ N}}$$

P5.36 (a) The free-body diagrams are shown in the figure below.



ANS. FIG. P5.36

f_1 and n_1 appear in both diagrams as action-reaction pairs.

(b) For the 5.00-kg mass, Newton's second law in the *y* direction gives:

$$n_1 = m_1 g = (5.00\text{ kg})(9.80\text{ m/s}^2) = 49.0\text{ N}$$

In the *x* direction,

$$f_1 - T = 0$$

$$T = f_1 = \mu mg = 0.200(5.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{9.80 \text{ N}}$$

For the 10.0-kg mass, Newton's second law in the x direction gives:

$$45.0 \text{ N} - f_1 - f_2 = (10.0 \text{ kg})a$$

In the y direction,

$$n_2 - n_1 - 98.0 \text{ N} = 0$$

$$f_2 = \mu n_2 = \mu(n_1 + 98.0 \text{ N}) = 0.20(49.0 \text{ N} + 98.0 \text{ N}) = 29.4 \text{ N}$$

$$45.0 \text{ N} - 9.80 \text{ N} - 29.4 \text{ N} = (10.0 \text{ kg})a$$

$$a = \boxed{0.580 \text{ m/s}^2}$$

Additional Problems

P5.37 (a) Choose the black glider plus magnet as the system.

$$\sum F_x = ma_x \rightarrow +0.823 \text{ N} = (0.24 \text{ kg})a$$

$$a = \boxed{3.43 \text{ m/s}^2 \text{ toward the scrap iron}}$$

(b) The force of attraction the magnet exerts on the scrap iron is the same as in (a):

$$a_{\text{black}} = \boxed{3.43 \text{ m/s}^2 \text{ toward the scrap iron}}$$

By Newton's third law, the force the black glider exerts on the magnet is equal and opposite to the force exerted on the scrap iron:

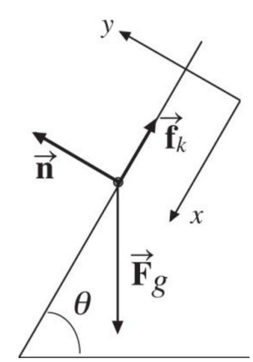
$$\sum F_x = ma_x \rightarrow -0.823 \text{ N} = -(0.12 \text{ kg})\ a$$

$$a = \boxed{-6.86 \text{ m/s}^2 \text{ toward the magnet}}$$

P5.38 Find the acceleration of the block according to the kinematic equations. The book travels through a displacement of 1.00 m in a time interval of 0.483 s. Use the equation $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$, where $\Delta x = x_f - x_i = 1.00$ m, $\Delta t = t = 0.483$ s, and $v_i = 0$:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow a = \frac{2\Delta x}{t^2} = 8.57 \text{ m/s}^2$$

Now, find the acceleration of the block caused by the forces. See the free-body diagram below. We take the positive y axis is perpendicular to the incline; the positive x axis is parallel and down the incline.



ANS. FIG. P5.38

$\sum F_y = ma_y$:

$$n - mg \cos\theta = 0 \rightarrow n = mg \cos\theta$$

$\sum F_x = ma_x$:

$$mg \sin\theta - f_k = ma$$

where $\quad f_k = \mu_k n = \mu_k mg \cos\theta$

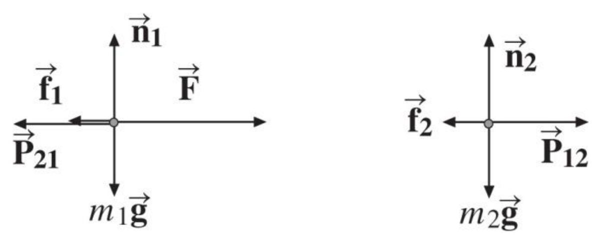
Substituting the express for kinetic friction into the x-component equation gives

$$mg \sin \theta - \mu_k mg \cos \theta = ma \rightarrow a = g(\sin \theta - \mu_k \cos \theta)$$

For $\mu_k = 0.300$, and $\theta = 60.0°$, $a = 7.02 \text{ m/s}^2$.

$\boxed{\text{The situation is impossible because these forces on the book cannot produce the acceleration described.}}$

P5.39 (a) The free-body diagrams of the two blocks shown in ANS. FIG. P5.39(a):



ANS. FIG. P5.39(a)

Vertical forces sum to zero because the blocks move on a horizontal surface; therefore, $a_y = 0$ for each block.

$\sum F_{1y} = m_1 a_y$: | $\sum F_{2y} = m_2 a_y$:

$-m_1 g + n_1 = 0 \rightarrow n_1 = m_1 g$ | $-m_2 g + n_2 = 0 \rightarrow n_2 = m_2 g$

Kinetic friction is: | Kinetic friction is:

$f_1 = \mu_1 n_1 = \mu_1 m_1 g$ | $f_2 = \mu_2 n_2 = \mu_2 m_2 g$

(b) The net force on the system of the blocks would be equal to the magnitude of the force, F, minus the friction force on each block. The blocks will have the same acceleration.

(c) The net force on the mass, m_1, would be equal to the force, F, minus the friction force on m_1 and the force P_{21}, as identified in the free-body diagram.

(d) The net force on the mass, m_2, would be equal to the force, P_{12}, minus the friction force on m_2, as identified in the free-body diagram.

(e) The blocks are pushed to the right by force $\vec{\mathbf{F}}$, so kinetic friction $\vec{\mathbf{f}}$ acts on each block to the left. Each block has the same horizontal acceleration, $a_x = a$. Each block exerts an equal and opposite force on the other, so those forces have the same magnitude: $P_{12} = P_{21} = P$.

$\sum F_{1x} = m_1 a_x$: $\qquad$ $\sum F_{2x} = m_2 a_x$:

$F - P - f_1 = m_1 a$ $\qquad$ $P - f_2 = m_2 a$

$\boxed{F - P - \mu_1 m_1 g = m_1 a}$ $\qquad$ $\boxed{P - \mu_2 m_2 g = m_2 a}$

(f) Adding the above two equations of x components, we find

$$F - P - \mu_1 m_1 g + P - \mu_2 m_2 g = m_1 a + m_2 a$$
$$F - \mu_1 m_1 g - \mu_2 m_2 g = (m_1 + m_2)a \rightarrow$$
$$\boxed{a = \frac{F - \mu_1 m_1 g - \mu_2 m_2 g}{m_1 + m_2}}$$

(g) From the x component equation for block 2, we have

$$P - \mu_2 m_2 g = m_2 a \rightarrow P = \mu_2 m_2 g + m_2 a$$

$$\boxed{P = \left(\frac{m_2}{m_1 + m_2}\right)\left[F + (\mu_2 - \mu_1) m_1 g\right]}$$

We see that when the coefficients of friction are equal, $\mu_1 = \mu_2$, the magnitude P is independent of friction.

P5.40 (a) Let x represent the position of the glider along the air track. Then $z^2 = x^2 + h_0^2$, $x = \left(z^2 - h_0^2\right)^{1/2}$, and $v_x = \dfrac{dx}{dt} = \dfrac{1}{2}\left(z^2 - h_0^2\right)^{-1/2}(2z)\dfrac{dz}{dt}$.

Now $\dfrac{dz}{dt}$ is the rate at which the string passes over the pulley, so it is equal to v_y of the counterweight.

$$v_x = z\left(z^2 - h_0^2\right)^{-1/2} v_y = u v_y$$

(b) $a_x = \dfrac{dv_x}{dt} = \dfrac{d}{dt} u v_y = u\dfrac{dv_y}{dt} + v_y \dfrac{du}{dt}$

At release from rest, $v_y = 0$ and $a_x = u a_y$.

(c) $\sin 30.0° = \dfrac{80.0\text{ cm}}{z}$, $z = 1.60\text{ m}$,

$$u = \left(z^2 - h_0^2\right)^{-1/2} z = \left[(1.6\text{ m})^2 - (0.8\text{ m})^2\right]^{-1/2}(1.6\text{ m}) = 1.15\text{ m}$$

For the counterweight, $\sum F_y = m a_y$:

$$T - (0.5\text{ kg})\left(9.80\text{ m/s}^2\right) = -(0.5\text{ kg})a_y$$

$$a_y = \left(-2\text{ kg}^{-1}\right)T + \left(9.80\text{ m/s}^2\right)$$

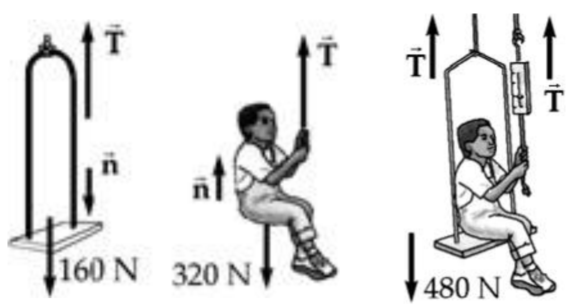
For the glider, $\sum F_x = m a_x$:

$$T\cos 30° = (1.00\text{ kg})\ a_x = (1.15\text{ kg})a_y$$
$$= (1.15\text{ kg})\left[(-2\text{ kg}^{-1})T + 9.80\text{ m/s}^2\right]$$
$$= -2.31T + 11.3\text{ N}$$

$$3.18T = 11.3\text{ N}$$
$$T = \boxed{3.56\text{ N}}$$

P5.41 (a) ANS. FIG. P5.41(a) shows the free-body diagrams for this problem.

Note that the same-size force n acts up on Nick and down on chair, and cancels out in the diagram. The same-size force T = 250 N acts up on Nick and up on chair, and appears twice in the diagram.



ANS. FIG. P5.41(a)

(b) First consider Nick and the chair together as the system. Note that **two** ropes support the system, and $T = 250$ N in each rope.

Applying $\sum F = ma$, $\qquad 2T - (160\text{ N} + 320\text{ N}) = ma$

where $\qquad m = \dfrac{480\text{ N}}{9.80\text{ m/s}^2} = 49.0\text{ kg}$

Solving for a gives $$a = \frac{(500 - 480)\ \text{N}}{49.0\ \text{kg}} = \boxed{0.408\ \text{m/s}^2}$$

(c) On Nick, we apply

$\sum F = ma$: $$n + T - 320\ \text{N} = ma$$

where $$m = \frac{320\ \text{N}}{9.80\ \text{m/s}^2} = 32.7\ \text{kg}$$

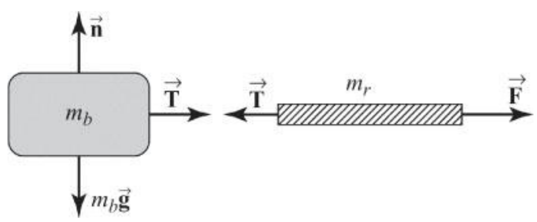
The normal force is the one remaining unknown:

$$n = ma + 320\,\text{N} - T$$

Substituting, $$n = (32.7\ \text{kg})\left(0.408\ \text{m/s}^2\right) + 320\ \text{N} - 250\ \text{N}$$

gives $$n = \boxed{83.3\ \text{N}}$$

P5.42 (a) free-body diagrams of block and rope are shown in ANS. FIG. P5.42(a):



ANS. FIG. P5.42(a)

(b) Applying Newton's second law to the rope yields

$$\sum F_x = ma_x \quad \Rightarrow \quad F - T = m_r a \quad \text{or} \quad T = F - m_r a \qquad \textbf{[1]}$$

Then, applying Newton's second law to the block, we find

$$\sum F_x = ma_x \quad \Rightarrow T = m_b a \quad \text{or} \quad F - m_r a = m_b a$$

which gives

$$\boxed{a = \frac{F}{m_b + m_r}}$$

(c) Substituting the acceleration found above back into equation [1] gives the tension at the left end of the rope as

$$T = F - m_r a = F - m_r\left(\frac{F}{m_b + m_r}\right) = F\left(\frac{m_b + \cancel{m_r} - \cancel{m_r}}{m_b + m_r}\right)$$

or $$\boxed{T = \left(\frac{m_b}{m_b + m_r}\right)F}$$

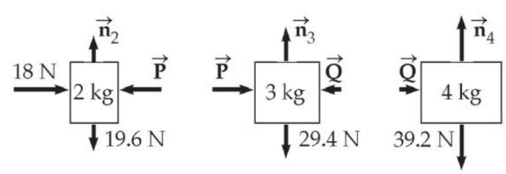
(d) From the result of (c) above, we see that as m_r approaches zero, T approaches F. Thus,

$\boxed{\text{the tension in a cord of negligible mass is constant along its length.}}$

P5.43 (a) See free-body diagrams in ANS. FIG. P5.43.

(b) We write $\sum F_x = ma_x$ for each object.

$$18\text{ N} - P = (2\text{ kg})a$$
$$P - Q = (3\text{ kg})a$$
$$Q = (4\text{ kg})a$$



ANS. FIG. P5.43

Adding gives

$$18\text{ N} = (9\text{ kg})a \rightarrow a = \boxed{2.00\text{ m/s}^2}$$

(c) The resultant force on any object is $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$: All have the same acceleration:

$$\sum \vec{\mathbf{F}} = (4\text{ kg})(2\text{ m/s}^2) = \boxed{8.00\text{ N on the 4-kg object}}$$

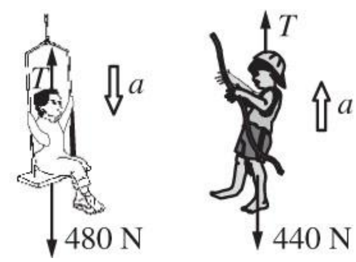
$$\sum \vec{\mathbf{F}} = (3\text{ kg})(2\text{ m/s}^2) = \boxed{6.00\text{ N on the 3-kg object}}$$

$$\sum \vec{\mathbf{F}} = (2\text{ kg})(2\text{ m/s}^2) = \boxed{2.00\text{ N on the 2-kg object}}$$

(d) From above, $P = 18\text{ N} - (2\text{ kg})a \to \boxed{P = 14.0\text{ N}}$, and $Q = (4\text{ kg})a \to \boxed{Q = 8.00\text{ N}}$.

(e) Introducing the heavy block reduces the acceleration because the mass of the system (plasterboard-heavy block-you) is greater. The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects.

P5.44 See ANS. FIG. P5.44 showing the free-body diagrams. The rope has tension T.



ANS. FIG. P5.44

(a) As soon as Nick passes the rope to the other child,

Nick and the seat, with total weight 480 N, will accelerate down and the child, with smaller weight 440 N, will accelerate up.

On Nick and the seat,

$$\sum F_y = +480\text{ N} - T = \frac{480\text{ N}}{9.80\text{ m/s}^2}a$$

On the child,

$$\sum F_y = +T - 440\ \text{N} = \frac{440\ \text{N}}{9.80\ \text{m/s}^2}a$$

Adding,

$$+\,480\ \text{N} - T + T - 440\ \text{N} = (49.0\ \text{kg} + 44.9\ \text{kg})a$$

$$a = \frac{40\ \text{N}}{93.9\ \text{kg}} = \boxed{0.426\ \text{m/s}^2 = a}$$

The rope tension is $T = 440\ \text{N} + (44.9\ \text{kg})(0.426\ \text{m/s}^2) = 459\ \text{N}$.

(b) The rope must support Nick and the seat, so the rope tension is 480 N.

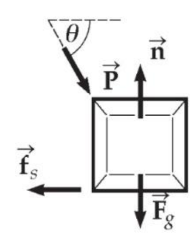
In problem 81, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.

The tension in the chain supporting the pulley is 480 N + 480 N = 960 N, so the chain may break first.

P5.45 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be n and the friction force, f_s.

Resolving vertically: $\sum F_y = ma_y$ gives

$$n = F_g + P\sin\theta$$



ANS. FIG. P5.45

Horizontally, $\sum F_x = ma_x$ gives

$$P\cos\theta = f$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P\left(\cos\theta - \mu_s\sin\theta\right) \le \mu_s F_g$$

Divide by cos θ:

$$P\left(1 - \mu_s\tan\theta\right) \le \mu_s F_g\sec\theta$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g \sec\theta}{1 - \mu_s\tan\theta}}$$

(b) To set the crate into motion, the x component ($P\cos\theta$) must overcome friction $f_s = \mu_s n$:

$$P\cos\theta \ge \mu_s n = \mu_s\,(F_g + P\sin\theta)$$

$$P\left(\cos\theta - \mu_s\sin\theta\right) \ge \mu_s\,F_g$$

For this condition to be satisfied, it must be true that

$$\left(\cos\theta - \mu_s\sin\theta\right) > 0 \rightarrow \mu_s\tan\theta < 1 \rightarrow \tan\theta < \frac{1}{\mu_s}$$

If this condition is not met, no value of P can move the crate.

P5.46 (a) When block 2 moves down 1 cm, block 1 moves 2 cm forward, so block 1 always has twice the speed of block 2, and $\boxed{a_1 = 2a_2}$ relates the magnitudes of the accelerations.

(b) Let T represent the uniform tension in the cord.

For block 1 as object,

$$\sum F_x = m_1 a_1: \quad T = m_1 a_1 = m_1(2a_2)$$
$$T = 2m_1 a_2 \qquad [1]$$

For block 2 as object,

$$\sum F_y = m_2 a_2: \quad T + T - m_2 g = m_2(-a_2)$$
$$2T - m_2 g = -m_2 a_2 \qquad [2]$$

To solve simultaneously we substitute equation [1] into equation [2]:

$$2(2m_1 a_2) - m_2 g = -m_2 a_2 \rightarrow 4m_1 a_2 + m_2 a_2 = m_2 g$$
$$a_2 = \frac{m_2 g}{4m_1 + m_2}$$

for $m_2 = 1.30$ kg: $\boxed{a_2 = 12.7 \text{ N } (1.30 \text{ kg} + 4\ m_1)^{-1} \text{ down}}$

(c) If m_1 is very much less than 1.30 kg, a_2 approaches

12.7 N/1.30 kg = $\boxed{9.80 \text{ m/s}^2 \text{ down}}$

(d) If m_1 approaches infinity, $\boxed{a_2 \text{ approaches zero}}$.

(e) From equation (2) above, $2T = m_2 g + m_2 a_2 = 12.74 \text{ N} + 0$,

$\boxed{T = 6.37 \text{ N}}$

(f) Yes. As m_1 approaches zero, block 2 is essentially in free fall. As m_2 becomes negligible compared to m_1, m_2 has very little weight, so the system is nearly in equilibrium.

***P5.47 Conceptualize** Make sure you are clear on the physical setup of the problem. The ship is moving toward the reef with an initial velocity when the rudder jams. There is a force toward the reef from the wind and a force away from the reef due to the propellers. The question is this: Under the action of these forces, can the ship be reversed before it strikes the reef?

Categorize The ship is modeled as a *particle under a net force*. Because the forces are all constant, it is also modeled as a *particle under constant acceleration.*

Analyze From Equation 2.17 in the particle under constant acceleration model, solve for the final position of the boat:

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right) \quad \rightarrow \quad x_f = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x} \quad (1)$$

Let's imagine that the reef is not there and simply find out how far the boat would move before coming to rest. If this distance is less than 900 m, the captain could have stopped before hitting the reef. If it is greater, the captain could not stop in time. We define the initial position of the boat when the rudder jams as $x_i = 0$, and we set $v_{xf} = 0$ to find out how far the boat moves before stopping. Therefore, Equation (1) becomes

$$x_f = -\frac{v_{xi}^2}{2a_x} \quad (2)$$

Now, use Newton's second law in the particle under a net force model to substitute for the acceleration in Equation (2):

$$x_f = -\frac{v_{xi}^2}{2\left(\frac{\sum F_x}{m}\right)} = -\frac{mv_{xi}^2}{2\left(F_{\text{wind}} - F_{\text{propellers}}\right)}$$

Substitute numerical values:

$$x_f = -\frac{\left(5.50\times10^7\ \text{kg}\right)\left(2.50\ \text{m/ s}\right)^2}{2\left(9.00\times10^3\ \text{N} - 1.25\times10^5\ \text{N}\right)} = 1.48\times10^3\ \text{m}$$

Under these conditions, the ship would require almost 1.5 km to come to rest if the reef were not there. Because the reef is only 900 m away, it was inevitable that the ship was going to hit the reef. There is nothing that the captain could have done to avoid the accident.

Finalize The actions of the captain did slow the ship so that the ship did not hit the reef as strongly, resulting in less damage to the cargo and fewer injuries to the crew. By putting the ship's engines in reverse, the captain slowed down the ship so that it did not hit the reef as strongly. From Newton's second law, the constant acceleration of the ship before it struck the reef was

$$\begin{aligned} a &= \frac{\sum F_x}{m} = \frac{F_{\text{wind}} - F_{\text{propellers}}}{m} = \frac{9.00\times10^3\ \text{N} - 1.25\times10^5\ \text{N}}{5.50\times10^7\ \text{kg}} \\ &= -2.11\times10^{-3}\ \text{m/s}^2 \end{aligned}$$

At this rate of deceleration, the ship's final velocity as it struck the reef, computed from Equation 2.17, was

$$\begin{aligned} v_{xf} &= \sqrt{v_{xi}^2 + 2a\left(x_f - x_i\right)} \\ &= \sqrt{\left(2.50\ \text{m/s}\right)^2 + 2\left(-2.11\times10^{-3}\ \text{m/s}^2\right)\left(900\ \text{m}\right)} \\ &= 1.57\ \text{m/s} \end{aligned}$$

Had the captain not reversed the engines, the ship would have increased its speed because of the wind force and struck the reef at a much higher speed than 2.50 m/s.

Answer: Ship requires 1.5 km to come to rest.

P5.48 (a) The net force on the cushion is in a fixed direction, downward and forward making angle $\theta = \tan^{-1}(F/mg)$ with the vertical.

Because the cushion starts from rest, the direction of its line of motion will be the same as that of the net force.

We show the path is a straight line another way. In terms of a standard coordinate system, the x and y coordinates of the cushion are

$$y = h - \frac{1}{2}gt^2$$

$$x = \frac{1}{2}(F/m)t^2 \rightarrow t^2 = (2m/F)x$$

Substitution of t^2 into the equation for y gives

$$y = h - (mg/f)x$$

which is an equation for a straight line.

(b) Because the cushion starts from rest, it will move in the direction of the net force which is the direction of its acceleration; therefore, it will move with increasing speed and its velocity changes in magnitude.

(c) Since the line of motion is in the direction of the net force, they both make the same angle with the vertical. Refer to Figure P5.91 in the textbook: in terms of a right triangle with angle θ, height h, and base x,

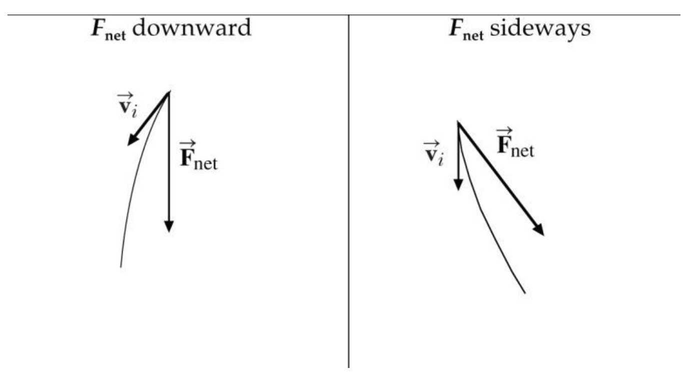
$$\tan\theta = x/h = F/mg \to x = hF/mg$$

$$x = \frac{(8.00 \text{ m})(2.40 \text{ N})}{(1.20 \text{ kg})(9.80 \text{ m/s}^2)}$$

and the cushion will land a distance

$x = \boxed{\text{1.63 m from the base of the building}}$.

(d) The cushion will move along a tilted parabola. If the cushion were experiencing a constant net force directed vertically downward (as is normal with gravity), and if its initial velocity were down and somewhat to the left, the trajectory would have the shape of a parabola that we would expect for projectile motion. Because the constant net force is "sideways"—at an angle θ counterclockwise from the vertical—the cushion would travel a similar trajectory as described above, but rotated counterclockwise by the angle θ so that the initial velocity is directed downward. See the figures.



ANS. FIG. P5.48(d)

P5.49 We will use $\sum F = ma$ on each object, so we draw force diagrams for the $M + m_1 + m_2$ system, and also for blocks m_1 and m_2. Remembering that normal forces are always perpendicular to the contacting surface, and always **push** on a body, draw n_1 and n_2 as shown. Note that m_1 is in contact with the cart, and therefore feels a normal force exerted by the cart. Remembering that ropes always **pull** on bodies toward the center of the rope, draw the tension force $\vec{\mathbf{T}}$. Finally, draw the gravitational force on each block, which always points downwards.

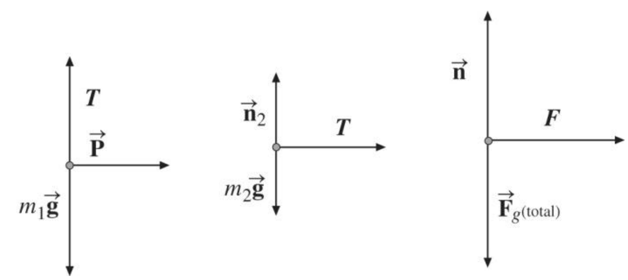
Applying $\sum F = ma$,

For m_1: $\quad T - m_1 g = 0$

For m_2: $\quad T = m_2 a$

Eliminating T,

$$a = \frac{m_1 g}{m_2}$$



ANS. FIG. P5.49

For all three blocks:

$$\boxed{F = (M + m_1 + m_2)\frac{m_1 g}{m_2}}$$

P5.50 (a) $\sum F_y = ma_y$:

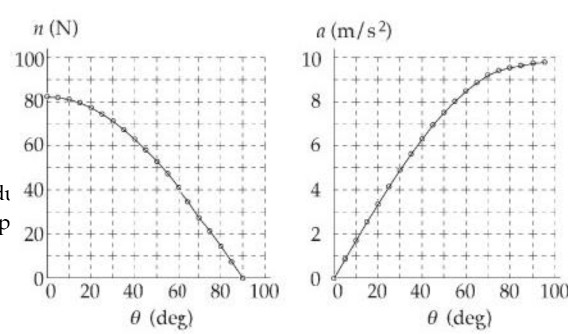
$$n - mg\cos\theta = 0$$

or $n = (8.40\text{ kg})(9.80\text{ m/s}^2)\cos\theta$

$$\boxed{n = (82.3\text{ N})\cos\theta}$$

(b) $\sum F_x = ma_x$:

$$mg\sin\theta = ma$$



or

$$a = g\sin\theta$$

$$\boxed{a = \left(9.80\ \text{m/s}^2\right)\sin\theta}$$

ANS. FIG. P5.50

(c)

θ, deg	n, N	a, m/s^2
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30

45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80

(d) At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

Challenge Problems

P5.51 (a) The cord makes angle θ with the horizontal where

$$\theta = \tan^{-1}\left(\frac{0.100 \text{ m}}{0.400 \text{ m}}\right) = 14.0°$$

Applying Newton's second law in the y direction gives

$$\sum F_y = ma_y:$$

$$T\sin\theta - mg + n = 0$$

$$(+10\text{ N})\sin 14.0° - (2.20\text{ kg})(9.80\text{ m/s}^2) + n = 0$$

which gives n = 19.1 N. Applying Newton's second law in the x direction then gives

$$\sum F_x = ma_x:$$

$$T\cos\theta - f_k = ma$$

$$T\cos\theta - \mu_k n = ma$$

$$(+10\text{ N})\cos 14.0° - 0.400(19.1\text{ N}) = (2.20\text{ kg})\, a$$

which gives

$$a = \boxed{0.931\text{ m/s}^2}$$

(b) When x is large we have $n = 21.6$ N, $f_k = 8.62$ N, and $a = (10\text{ N} - 8.62\text{ N})/2.2\text{ kg} = 0.625\text{ m/s}^2$.

As x decreases, the acceleration increases gradually, passes through a maximum, and then drops more rapidly, becoming negative. At x = 0 it reaches the value $a = [0 - 0.4(21.6\text{ N} - 10\text{ N})]/2.2\text{ kg} = -2.10\text{ m/s}^2$.

(c) We carry through the same calculations as in part (a) for a variable angle, for which $\cos\theta = x[x^2 + (0.100\text{ m})^2]^{-1/2}$ and $\sin\theta = (0.100\text{ m})[x^2 + (0.100\text{ m})^2]^{-1/2}$. We find

$$a = \left(\frac{1}{2.20\text{ kg}}\right)(10\text{ N})x\left[x^2 + 0.100^2\right]^{-1/2}$$
$$- 0.400\left(21.6\text{ N} - (10\text{ N})(0.100)\left[x^2 + 0.100^2\right]^{-1/2}\right)$$
$$a = 4.55x\left[x^2 + 0.100^2\right]^{-1/2} - 3.92 + 0.182\left[x^2 + 0.100^2\right]^{-1/2}$$

Now to maximize a we take its derivative with respect to x and set it equal to zero:

$$\frac{da}{dx} = 4.55\left(x^2 + 0.100^2\right)^{-1/2} + 4.55x\left(-\frac{1}{2}\right)2x\left(x^2 + 0.100^2\right)^{-3/2}$$
$$+ 0.182\left(-\frac{1}{2}\right)2x\left(x^2 + 0.100^2\right)^{-3/2} = 0$$

Solving,

$$4.55\left(x^2 + 0.1^2\right) - 4.55x^2 - 0.182x = 0$$

or $x = \boxed{0.250\text{ m}}$

At this point, suppressing units,

$$a = (4.55)(0.250)\left[0.250^2 + 0.100^2\right]^{-1/2} - 3.92$$
$$+ 0.182\left[0.250^2 + 0.100^2\right]^{-1/2}$$
$$= \boxed{0.976\text{ m/s}^2}$$

(d) We solve, suppressing units,

$$0 = 4.55x\left[x^2 + 0.100^2\right]^{-1/2} - 3.92 + 0.182\left[x^2 + 0.100^2\right]^{-1/2}$$
$$3.92\left[x^2 + 0.100^2\right]^{1/2} = 4.55x + 0.182$$
$$15.4\left[x^2 + 0.100^2\right] = 20.7x^2 + 1.65x + 0.033\ 1$$

which gives the quadratic equation

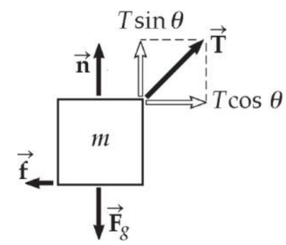
$$5.29x^2 + 1.65x - 0.121 = 0$$

Only the positive root is directly meaningful, so

$$x = \boxed{0.0610\text{ m}}$$

P5.52 The force diagram is shown on the right. With motion impending,

$$n + T\sin\theta - mg = 0$$
$$f = \mu_s\left(mg - T\sin\theta\right)$$



ANS. FIG. P5.52

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so $$T = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$:

$$\frac{d}{d\theta}\left(\cos\theta + \mu_s\sin\theta\right) = 0 = -\sin\theta + \mu_s\cos\theta$$

Therefore, the angle where tension T is a minimum is

$$\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}(0.350) = 19.3°$$

What is the tension at this angle? From above,

$$T = \frac{0.350(1.30\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{\cos 19.3° + 0.350\sin 19.3°} = 4.21\text{ N}$$

$$\boxed{\text{The situation is impossible because at the angle of minimum tension, the tension exceeds 4.00 N.}}$$

P5.53 We apply Newton's second law to each of the three masses, reading the forces from ANS. FIG. P5.53:

$$m_2(a-A)=T \Rightarrow \qquad a=\frac{T}{m_2}+A \qquad \textbf{[1]}$$

$$MA=R_x=T \Rightarrow \qquad A=\frac{T}{M} \qquad \textbf{[2]}$$

$$m_1a=m_1g-T \Rightarrow \qquad T=m_1(g-a) \qquad \textbf{[3]}$$

(a) Substitute the value for a from [1] into [3] and solve for T:

$$T=m_1\left[g-\left(\frac{T}{m_2}+A\right)\right]$$

Substitute for A from [2]:

$$T=m_1\left[g-\left(\frac{T}{m_2}+\frac{T}{M}\right)\right]\Rightarrow T=\boxed{m_2g\left[\frac{m_1M}{m_2M+m_1(m_2+M)}\right]}$$

(b) Solve [3] for a and substitute value of T:

$$\begin{aligned}a&=g-\frac{T}{m_1}=g-m_2g\left[\frac{M}{m_2M+m_1(m_2+M)}\right]\\&=g\left[1-\frac{m_2M}{m_2M+m_1(m_2+M)}\right]\\&=\boxed{\left[\frac{gm_1(m_2+M)}{m_2M+m_1(m_2+M)}\right]}\end{aligned}$$

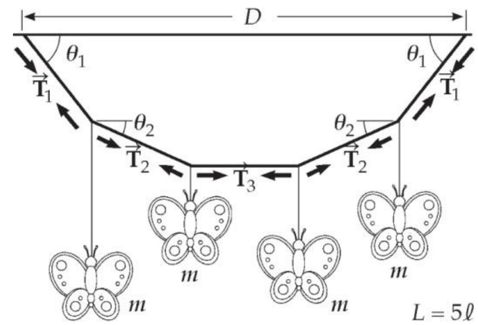
(c) From [2], $A=\dfrac{T}{M}$. Substitute the value of T:

$$A=\frac{T}{M}=\boxed{\left[\frac{m_1m_2g}{m_2M+m_1(m_2+M)}\right]}$$

(d) The acceleration of m_1 is given by

$$a - A = \boxed{\left[\frac{m_1 M g}{m_2 M + m_1 (m_2 + M)}\right]}$$

P5.54 (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (the other half is the same, by symmetry).



ANS. FIG. P5.54

$T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$ **[1]**

$T_1 \sin\theta_1 - T_2 \sin\theta_2 - mg = 0$ **[2]**

$T_2 \cos\theta_2 - T_3 = 0$ **[3]**

$T_2 \sin\theta_2 - mg = 0$ **[4]**

Substituting [4] into [2] for $T_2 \sin\theta_2$,

$$T_1 \sin\theta_1 - mg - mg = 0$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}$$

Substitute [3] into [1] for $T_2 \cos\theta_2$:

$$T_3 - T_1 \cos\theta_1 = 0, T_3 = T_1 \cos\theta_1$$

Substitute value of T_1:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}$$

From equation [4],

$$T_2 = \frac{mg}{\sin\theta_2}$$

(b) Divide [4] by [3]:

$$\frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \frac{mg}{T_3}$$

Substitute value of T_3:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg}, \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}$$

Then we can finish answering part (a):

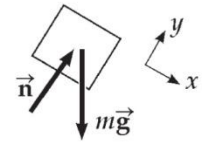
$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}$$

(c) D is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$\boxed{\begin{aligned}&D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell\\ &D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\}\end{aligned}}$$

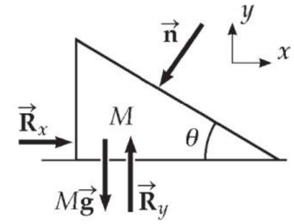
P5.55 Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$



Let $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$ represent the force of table on incline. We have

$$\sum F_x = ma_x: \quad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta$$



ANS. FIG. P5.102

$$\boxed{\vec{\mathbf{R}} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

ANSWERS TO QUICK-QUIZZES

1. (d)

2. (a)

3. (d)

4. (b)

5. **(i)** (c) **(ii)** (a)

6. (b)

7. (b) Pulling up on the rope decreases the normal force, which, in turn, decreases the force of kinetic friction.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P5.2 (a) force exerted by spring on hand, to the left; force exerted by spring on wall, to the right (b) force exerted by wagon on handle, downward to the left; force exerted by wagon on planet, upward; force exerted by wagon on ground, downward (c) force exerted by football on player, downward to the right; force exerted by football on planet, upward (d) force exerted by small-mass object on large-mass object, to the left (e) force exerted by negative charge on positive charge, to the left (f) force exerted by iron on magnet, to the left

P5.4 (a) -4.47×10^{15} m/s^2; (b) $+2.09 \times 10^{-10}$ N

P5.6 1.59 m/s^2 at 65.2° N of E

P5.8 $\sum \vec{\mathbf{F}} = -km\vec{\mathbf{v}}$

P5.10 (a) $\frac{1}{2}vt$; (b) magnitude: $m\sqrt{(v/t)^2 + g^2}$, direction: $\tan^{-1}\left(\frac{gt}{v}\right)$

P5.12 2.38 kN

P5.14 (a–c) See free-body diagrams and corresponding forces in P5.14.

P5.16 (a) See ANS. FIG. P5.16; (b) 1.03 N; (c) 0.805 N to the right

P5.18 See P5.18 for complete derivation.

P5.20 8.66 N east

P5.22 (a) See ANS FIG P5.22; (b) 3.57 m/s^2; (c) 26.7 N; (d) 7.14 m/s

P5.24 $B = 3.37 \times 10^3$ N, $A = 3.83 \times 10^3$ N, B is in tension and A is in compression.

P5.26 The situation is impossible because maximum static friction cannot provide the acceleration necessary to keep the book stationary on the seat.

P5.28 (a) 4.18; (b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

P5.30 (a) 0.404; (b) 45.8 lb

P5.32 (a) See ANS. FIG. P5.32; (b) 2.31 m/s^2, down for m_1, left for m_2, and up for m_3; (c) T_{12} = 30.0 N and T_{23} = 24.2 N; (d) T_{12} decreases and T_{23} increases

P5.34 (a) 48.6 N, 31. 7 N; (b) If P > 48.6 N, the block slides up the wall. If P < 31.7 N, the block slides down the wall; (c) 62.7 N, P > 62.7 N, the block cannot slide up the wall. If P < 62.7 N, the block slides down the wall

P5.36 (a) See P5.36 for complete solution; (b) 9.80 N, 0.580 m/s^2

P5.38 The situation is impossible because these forces on the book cannot produce the acceleration described.

P5.40 (a) and (b) See P5.40 for complete derivation; (c) 3.56 N

P5.42 (a) See ANS. FIG. P5.42(a); (b) $a = \dfrac{F}{m_b + m_r}$; (c) $T = \left(\dfrac{m_b}{m_b + m_r}\right)F$; (d) the tension in a cord of negligible mass is constant along its length

P5.44 (a) Nick and the seat, with total weight 480 N, will accelerate down and the child, with smaller weight 440 N, will accelerate up; (b) In P5.44, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.

P5.46 (a) $a_1 = 2a_2$; (b) $a_2 = 12.7\text{ N }(1.30\text{ kg} + 4m_1)^{-1}$ down; (c) 9.80 m/s^2 down; (d) a_2 approaches zero; (e) $T = 6.37$ N; (f) yes

P5.48 (a) The net force on the cushion is in a fixed direction, downward and forward making angle tan21 (F/mg) with the vertical. Starting from rest, it will move along this line with (b) increasing speed. Its velocity changes in magnitude. (c) 1.63 m (d) It will move along a parabola. The axis of the parabola is parallel to the line described in part (a). If the cushion is thrown in a direction above this line, its path will be concave downward, making its velocity become more and more nearly parallel to the line over time. If the cushion is thrown down more steeply, its path will be concave upward, again making its velocity turn toward the fixed direction of its acceleration.

P5.50 (a) $n = (8.23\text{ N})\cos\theta$; (b) $a = (9.80\text{ m/s}^2)\sin\theta$; (c) See ANS. FIG P5.50; (d) At 0°, the normal force is the full weight, and the acceleration is zero. At 90° the mass is in free fall next to the vertical incline.

P5.52 The situation is impossible because at the angle of minimum tension, the tension exceeds 4.00 N

P5.54 (a) $T_1 = \dfrac{2mg}{\sin\theta_1}, \dfrac{2mg}{\tan\theta_1} = T_3;$

(b) $\theta_2 = \tan^{-1}\left(\dfrac{\tan\theta_1}{2}\right) T_2 = -\dfrac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]};$

(c) See P5.54 for complete explanation