9

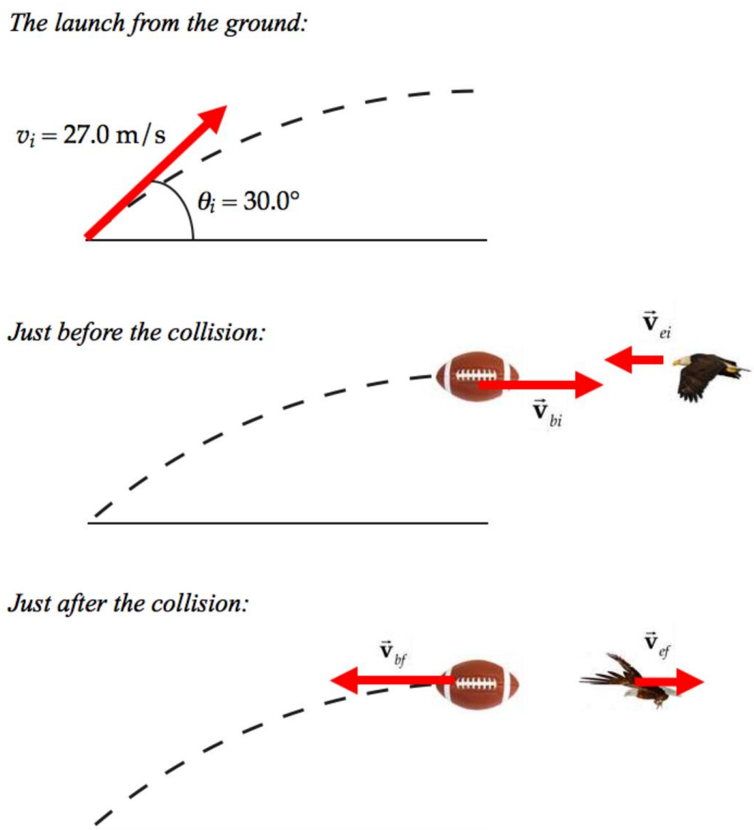
Linear Momentum and Collisions

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP9.1** **Conceptualize** Imagine the situation occurring: The ball rises in a parabolic trajectory. Just as it reaches its highest point, a horizontally flying eagle collides with it and reverses the direction of the velocity of the ball. The diagram below helps us to understand the situation.



Categorize This is a collision problem, so we use the *isolated system* model for *momentum*. We also see that we categorize the collision as elastic. One of the objects in the collision is a kicked football, so we also categorize the problem as one involving a projectile.

Analyze The point in the trajectory at which the ball rebounded from the eagle is the highest point. Therefore, the football is traveling horizontally at that instant. According to the problem statement, the ball also travels horizontally just after the collision. Therefore, the time interval required to fall back to the ground is the same as that for it to rise to its highest point. Furthermore, because the ball lands back at the point from which it was kicked, it travels the same horizontal distance as it did when rising. With the same horizontal distance and the same time interval, we see that the velocity vector of the ball must be simply reversed during the collision, but will maintain the same magnitude.

(a) We begin by writing Equations 9.16 and 9.20:

$$m_b v_{bi} + m_e v_{ei} = m_b v_{bf} + m_e v_{ef} \qquad (1)$$

$$v_{bi} - v_{ei} = -\left(v_{bf} - v_{ef}\right) \qquad (2)$$

where the ball is represented by mass m_b, initial velocity v_{bi}, and final velocity v_{bf}, and the eagle by m_e, v_{ei}, and v_{ef}. Multiply Equation (2) by m_e:

$$m_e v_{bi} - m_e v_{ei} = -m_e v_{bf} + m_e v_{ef} \qquad (3)$$

Subtract Equation (3) from Equation (1) to eliminate the final velocity of the eagle:

$$\left(m_b - m_e\right)v_{bi} + 2m_e v_{ei} = \left(m_b + m_e\right)v_{bf}$$
$$\rightarrow \quad v_{ei} = \frac{\left(m_b + m_e\right)v_{bf} - \left(m_b - m_e\right)v_{bi}}{2m_e} \qquad (4)$$

The velocity component v_{bi} of the ball just before the collision is the x component of the velocity of the ball after it is kicked, so $v_{bi} = v_i\cos\theta_i$, where v_i is the initial velocity magnitude of the kicked ball and θ_i is the launch angle. We have also argued that the velocity of the ball just after the collision has the same magnitude but opposite direction. Therefore, $v_{bf} = -v_i\cos\theta_i$. Making these substitutions into Equation (4) gives

$$v_{ei} = \frac{\left(m_b + m_e\right)\left(-v_i\cos\theta_i\right) - \left(m_b - m_e\right)\left(v_i\cos\theta_i\right)}{2m_e} = -\frac{m_b}{m_e}v_i\cos\theta_i \qquad (5)$$

Substituting numerical values gives

$$v_{ei} = -\frac{0.400\text{ kg}}{4.18\text{ kg}}\left(27.0\text{ m/ s}\right)\cos 30.0^\circ = -2.24\text{ m/ s}$$

(b) As the collision occurs, the eagle is too startled to begin flapping his wings to recover, so we can continue to ignore air resistance for a moment. Solve Equation (2) for the final velocity of the eagle:

$$v_{ef} = v_{bi} - v_{ei} + v_{bf} = v_i\cos\theta_i - v_{ei} + \left(-v_i\cos\theta_i\right) = -v_{ei} = 2.24\text{ m/ s}$$

(c) In the case of the inelastic collision, some kinetic energy will be transformed to other forms of energy. Therefore, to provide the necessary velocity of the ball to return to the point from which it was kicked, the eagle must fly faster than what we found in part (a).

Finalize Notice that the eagle's velocity is simply reversed, with the same magnitude, just as for the ball. This conclusion follows immediately from the conservation of kinetic energy in an elastic collision: because the ball's velocity simply reverses after the collision, its speed remains the same, and therefore its kinetic energy remains the same. Because the total kinetic energy is conserved in an elastic collision, it follows that the eagle's speed cannot change either, and this implies that the eagle's velocity simply reverses as well.

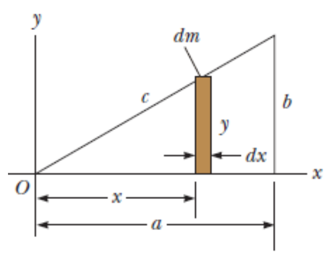
Answers: (a) –2.24 m/s (b) +2.24 m/s (c) higher

***TP9.2** **Conceptualize** If the cardboard triangle is to hang with its long leg parallel to the table, the string must be attached at a point directly above the center of gravity of the triangle, which is the same as its center of mass because it is in a uniform gravitational field.

Categorize As in the case of Example 9.11, we categorize this example as an analysis problem because it is necessary to identify infinitesimal mass elements of the triangle to perform the integration in Equation 9.32.

Analyze We assume the cardboard triangle has a total mass M. Because the triangle is a continuous distribution of mass, we must use the integral expression in Equation 9.32 to find the x coordinate of the center of mass.

We divide the triangle with legs a and b into narrow strips of width dx and height y as shown in Figure ANS P9.2, where y is the height of the hypotenuse of the triangle above the x axis for a given value of x. The mass of each strip is the product of the volume of the strip and the density of the cardboard from which the triangle is made: $dm =$ $yt\,dx$, where t is the thickness of the triangle. The density of the material is the total mass of the triangle divided by its total volume (area of the triangle times thickness).



ANS FIG P9.2

Evaluate dm:

$$dm = \rho yt\,dx = \left(\frac{M}{\frac{1}{2}abt}\right)yt\,dx = \frac{2My}{ab}\,dx \qquad (1)$$

Use Equation 9.32 to find the x coordinate of the center of mass:

$$x_{\mathrm{CM}} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^a x\frac{2My}{ab}\,dx = \frac{2}{ab}\int_0^a xy\,dx \qquad (2)$$

To proceed further and evaluate the integral, we must express y in terms of x. The line representing the hypotenuse of the triangle in Figure ANS P9.2 has a slope of b/a and passes through the origin, so the equation of this line is $y = (b/a)x$. Substitute for y in Equation (2):

$$x_{\mathrm{CM}} = \frac{2}{ab}\int_0^a x\left(\frac{b}{a}x\right)dx = \frac{2}{a^2}\int_0^a x^2\,dx = \frac{2}{a^2}\left[\frac{x^3}{3}\right]_0^a = \tfrac{2}{3}a$$

Therefore, the string must be attached to the triangle at a distance two-thirds of the length of the bottom edge from the left end.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 9.1 Linear Momentum

P9.1 (a) The momentum is $p = mv$, so $v = p/m$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$ so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

P9.2 We are given $m = 3.00$ kg and $\vec{\mathbf{v}} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right)$ m/s.

(a) The vector momentum is then

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (3.00 \text{ kg})\left[\left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) \text{ m/s}\right]$$
$$= \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right) \text{ kg} \cdot \text{m/s}$$

Thus, $\boxed{p_x = 9.00 \text{ kg} \cdot \text{m/s}}$ and $\boxed{p_y = -12.0 \text{ kg} \cdot \text{m/s}}$.

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00 \text{ kg} \cdot \text{m/s})^2 + (12.0 \text{ kg} \cdot \text{m/s})^2}$
$= \boxed{15.0 \text{ kg} \cdot \text{m/s}}$

at an angle of

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

P9.3 We apply the impulse-momentum theorem to find the average force the bat exerts on the baseball:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t \rightarrow \vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = m\left(\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t}\right)$$

Choosing the direction toward home plate as the positive x direction, we have $\vec{\mathbf{v}}_i = (45.0 \text{ m/s})\hat{\mathbf{i}}$, $\vec{\mathbf{v}}_f = (55.0 \text{ m/s})\hat{\mathbf{j}}$, and $\Delta t = 2.00$ ms:

$$\vec{\mathbf{F}}_{\text{on ball}} = m\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t} = (0.145 \text{ kg})\frac{(55.0 \text{ m/s})\hat{\mathbf{j}} - (45.0 \text{ m/s})\hat{\mathbf{i}}}{2.00 \times 10^{-3} \text{ s}}$$

$$\vec{\mathbf{F}}_{\text{on ball}} = \left(-3.26\hat{\mathbf{i}} + 3.99\hat{\mathbf{j}}\right) \text{ N}$$

By Newton's third law,

$$\vec{\mathbf{F}}_{\text{on bat}} = -\vec{\mathbf{F}}_{\text{on ball}} \quad \text{so} \quad \boxed{\vec{\mathbf{F}}_{\text{on bat}} = \left(+3.26\hat{\mathbf{i}} - 3.99\hat{\mathbf{j}}\right)\text{ N}}$$

Section 9.2 Analysis Model: Isolated System (Momentum)

P9.4 (a) Brother and sister exert equal-magnitude oppositely-directed forces on each other for the same time interval; therefore, the impulses acting on them are equal and opposite. Taking east as the positive direction, we have

impulse on boy: $I = F\Delta t = \Delta p = (65.0\text{ kg})(-2.90\text{ m/s}) = -189\text{ N}\cdot\text{s}$

impulse on girl: $I = -F\Delta t = -\Delta p = +189\text{ N}\cdot\text{s} = mv_f$

Her speed is then

$$v_f = \frac{I}{m} = \frac{189\text{ N}\cdot\text{s}}{40.0\text{ kg}} = 4.71\text{ m/s}$$

meaning $\boxed{\text{she moves at 4.71 m/s east}}$.

(b) original chemical potential energy in girl's body = total final kinetic energy

$$\begin{aligned} U_{\text{chemical}} &= \frac{1}{2}m_{\text{boy}}v_{\text{boy}}^2 + \frac{1}{2}m_{\text{girl}}v_{\text{girl}}^2 \\ &= \frac{1}{2}(65.0\text{ kg})(2.90\text{ m/s})^2 + \frac{1}{2}(40.0\text{ kg})(4.71\text{ m/s})^2 \\ &= \boxed{717\text{ J}} \end{aligned}$$

(c) $\boxed{\text{Yes.}}$ System momentum is conserved with the value zero.

(d) The forces on the two siblings are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated}}$.

(e) $\boxed{\begin{array}{l}\text{Even though there is motion afterward, the final momenta are}\\ \text{of equal magnitude in opposite directions so the final momentum}\\ \text{of the system is still zero.}\end{array}}$

P9.5 (a) For the system of two blocks $\Delta p = 0$, or $p_i = p_f$. Therefore,

$$0 = mv_m + (3m)(2.00\text{ m/s})$$

Solving gives $v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

(b)
$$\begin{aligned}\frac{1}{2}kx^2 &= \frac{1}{2}mv_M^2 + \frac{1}{2}(3m)v_{3M}^2\\ &= \frac{1}{2}(0.350\text{ kg})(-6.00\text{ m/s})^2 + \frac{3}{2}(0.350\text{ kg})(2.00\text{ m/s})^2\\ &= \boxed{8.40\text{ J}}\end{aligned}$$

(c) $\boxed{\text{The original energy is in the spring.}}$

(d) A force had to be exerted over a displacement to compress the spring, transferring energy into it by work.

$\boxed{\text{The cord exerts force, but over no displacement.}}$

(e) $\boxed{\text{System momentum is conserved with the value zero.}}$

(f) The forces on the two blocks are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated.}}$

(g) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

P9.6 I have mass 72.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a\left(x_f - x_i\right): \qquad 0 - v_i^2 = 2\left(-9.80\ \text{m/s}^2\right)\left(0.250\ \text{m}\right)$$

$$v_i = 2.20\ \text{m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the planet down and myself up:

$$0 = \left(5.98\times 10^{24}\ \text{kg}\right)\left(-v_e\right) + \left(85.0\ \text{kg}\right)\left(2.20\ \text{m/s}\right)$$

$$v_e \sim \boxed{10^{-23}\ \text{m/s}}$$

Section 9.3 Analysis Model: Nonisolated System (Momentum)

P9.7 (a) The mechanical energy of the isolated spring-mass system is conserved:

$$K_i + U_{si} = K_f + U_{sf}$$

$$0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{\frac{k}{m}}$$

(b) $I = \left|\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i\right| = mv_f - 0 = mx\sqrt{\dfrac{k}{m}} = x\sqrt{km}$

(c) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

P9.8 **Conceptualize** Imagine the crash occurring. While the seat belt brings the wife to a rapid stop, the only thing bringing the child to a stop is the force from the mother's arms. In your experience driving, you know that you are "thrown forward" relative to the car when it brakes hard. The acceleration in a violent crash has a much larger magnitude, so a very strong force is necessary to keep an occupant from sliding forward relative to the car during the crash.

Categorize We model the toddler as a *nonisolated system* for *momentum*.

Analyze Write Equation 9.13 for the toddler:

$$\Delta p_x = I_x \quad (1)$$

where the x direction is along the direction of velocity of the car in which the toddler is driving. Now use Equations 9.2 and 9.11 to substitute for both sides of Equation (1):

$$mv_{xf} - mv_{xi} = F_{x,\text{avg}}\Delta t \quad (2)$$

where m is the mass of the toddler, Δt is the time interval during which the toddler is brought to rest, and $F_{x,\text{avg}}$ is the force that your brother's wife must apply with her arms to bring the toddler to rest along with her. Solve Equation (2) for this force:

$$F_{x,\text{avg}} = m\frac{v_{xf} - v_{xi}}{\Delta t}$$

Substitute numerical values:

$$F_{x,\text{avg}} = (12\text{ kg})\frac{0 - 60\text{ mi/h}}{0.10\text{ s}}\left(\frac{1\,609\text{ m}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = -3.2 \times 10^{3}\text{ N}$$

where the minus sign indicates that your brother's wife must apply a force toward herself to keep the child from moving forward relative to her. The magnitude of this force is $\boxed{3.2 \times 10^3\text{ N}}$.

Finalize This is a tremendous amount of force, equivalent to over 700 pounds. Ask your brother if you think his wife could lift a 700-pound barbell with her arms.

Answer: 3.2×10^3 N

P9.9 (a) From the kinematic equations,

$$\Delta t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_f + v_i} = \frac{2(1.20\text{ m})}{0 + 25.0\text{ m/s}} = \boxed{9.60 \times 10^{-2}\text{ s}}$$

(b) We find the average force from the momentum-impulse theorem:

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(1\ 400\text{ kg})(25.0\text{ m/s} - 0)}{9.60 \times 10^{-2}\text{ s}} = \boxed{3.65 \times 10^5\text{ N}}$$

(c) Using the particle under constant acceleration model,

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{25.0\text{ m/s} - 0}{9.60 \times 10^{-2}\text{ s}} = (260\text{ m/s}^2)\left(\frac{1\,g}{9.80\text{ m/s}^2}\right) = \boxed{26.5g}$$

P9.10 (a) The impulse is in the x direction and equal to the area under the F-t graph:

$$I = \left(\frac{0+4\ \text{N}}{2}\right)(2\ \text{s}-0)+(4\ \text{N})(3\ \text{s}-2\ \text{s})+\left(\frac{4\ \text{N}+0}{2}\right)(5\ \text{s}-3\ \text{s})$$
$$= 12.0\ \text{N}\cdot\text{s}$$

$$\boxed{\vec{\mathbf{I}} = 12.0\ \text{N}\cdot\text{s}\ \hat{\mathbf{i}}}$$

(b) From the momentum-impulse theorem,

$$m\vec{\mathbf{v}}_i + \vec{\mathbf{F}}\Delta t = m\vec{\mathbf{v}}_f$$
$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = 0 + \frac{12.0\ \hat{\mathbf{i}}\ \text{N}\cdot\text{s}}{2.50\ \text{kg}} = \boxed{4.80\ \hat{\mathbf{i}}\ \text{m/s}}$$

(c) From the same equation,

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = -2.00\ \hat{\mathbf{i}}\ \text{m/s} + \frac{12.0\ \hat{\mathbf{i}}\ \text{N}\cdot\text{s}}{2.50\ \text{kg}} = \boxed{2.80\ \hat{\mathbf{i}}\ \text{m/s}}$$

(d) $$\vec{\mathbf{F}}_{\text{avg}}\Delta t = 12.0\hat{\mathbf{i}}\ \ \text{N}\cdot\text{s} = \vec{\mathbf{F}}_{\text{avg}}(5.00\ \text{s}) \quad \rightarrow \quad \vec{\mathbf{F}}_{\text{avg}} = \boxed{2.40\hat{\mathbf{i}}\ \text{N}}$$

P9.11 After 3.00 s of pouring, the bucket contains

(3.00 s)(0.250 L/s) = 0.750 liter

of water, with mass (0.750 L)(1 kg/1 L) = 0.750 kg, and feeling gravitational force (0.750 kg)(9.80 m/s^2) = 7.35 N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.750-kg bucket itself.

Water is entering the bucket with speed given by

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{impact}}^2$$

$$v_{\text{impact}} = \sqrt{2gy_{\text{top}}} = \sqrt{2\left(9.80\text{ m/s}^2\right)(2.60\text{ m})}$$
$$= 7.14\text{ m/s, downward}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{\text{impact}} + F_{\text{extra}}t = mv_f$$

The rate of change of momentum is the force itself:

$$\left(\frac{dm}{dt}\right)v_{\text{impact}} + F_{\text{extra}} = 0$$

which gives

$$F_{\text{extra}} = -\left(\frac{dm}{dt}\right)v_{\text{impact}} = -\left(0.250\text{ kg/s}\right)\left(-7.14\text{ m/s}\right) = 1.78\text{ N}$$

Altogether the scale must exert 7.35 N + 7.35 N + 1.78 N = $\boxed{16.5\text{ N}}$

Section 9.4 Collisions in One Dimension

P9.12 (a) Conservation of momentum gives

$$m_T v_{Tf} + m_C v_{Cf} = m_T v_{Ti} + m_C v_{Ci}$$

Solving for the final velocity of the truck gives

$$v_{Tf} = \frac{m_T v_{Ti} + m_C\left(v_{Ci} - v_{Cf}\right)}{m_T}$$

$$= \frac{(9\ 000\ \text{kg})(20.0\ \text{m/s}) + (1\ 200\ \text{kg})[(25.0 - 18.0)\ \text{m/s}]}{9\ 000\ \text{kg}}$$

$$v_{Tf} = \boxed{20.9\ \text{m/s East}}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta KE = KE_f - KE_i = \left[\frac{1}{2}m_C v_{Cf}^2 + \frac{1}{2}m_T v_{Tf}^2\right] - \left[\frac{1}{2}m_C v_{Ci}^2 + \frac{1}{2}m_T v_{Ti}^2\right]$$

$$= \frac{1}{2}\left[m_C\left(v_{Cf}^2 - v_{Ci}^2\right) + m_T\left(v_{Tf}^2 - v_{Ti}^2\right)\right]$$

$$= \frac{1}{2}\Big\{(1\ 200\ \text{kg})\left[(18.0\ \text{m/s})^2 - (25.0\ \text{m/s})^2\right]$$

$$+ (9\ 000\ \text{kg})\left[(20.9\ \text{m/s})^2 - (20.0\ \text{m/s})^2\right]\Big\}$$

$$\Delta KE = \boxed{-8.68 \times 10^3\ \text{J}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333 as the answer to part (a), the answer would be very different. We have kept extra digits in all intermediate answers until the problem is complete.

(c) The mechanical energy of the car-truck system has decreased. Most of the energy was transformed to internal energy with some being carried away by sound.

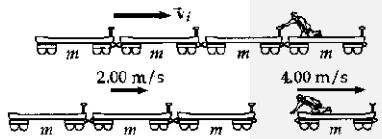
P9.13 (a) We write the law of conservation of momentum as

$$mv_{1i} + 3mv_{2i} = 4mv_f$$

or $$v_f = \frac{4.00 \text{ m/s} + 3(2.00 \text{ m/s})}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right]$$
$$= \frac{1}{2}\left(2.50\times10^4 \text{ kg}\right)[4(2.50 \text{ m/s})^2$$
$$- (4.00 \text{ m/s})^2 - 3(2.00 \text{ m/s})^2]$$
$$= \boxed{-3.75\times10^4 \text{ J}}$$

P9.14 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor. Conservation of momentum gives



ANS. FIG. P9.14

$$(4m)v_i = (3m)(2.00 \text{ m/s}) + m(4.00 \text{ m/s})$$
$$v_i = \frac{6.00 \text{ m/s} + 4.00 \text{ m/s}}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $$W_{\text{actor}} = K_f - K_i$$
$$= \frac{1}{2}\left[(3m)(2.00 \text{ m/s})^2 + m(4.00 \text{ m/s})^2\right] - \frac{1}{2}(4m)(2.50 \text{ m/s})^2$$

$$W_{\text{actor}} = \frac{\left(2.50\times10^4 \text{ kg}\right)}{2}(12.0 + 16.0 - 25.0)(\text{m/s})^2 = \boxed{37.5 \text{ kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.15 The collision is completely inelastic.

(a) Momentum is conserved by the collision:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$mv_1 + (2m)v_2 = mv_f + 2mv_f = 3mv_f$$
$$v_f = \frac{mv_1 + 2mv_2}{3m} \rightarrow \boxed{v_f = \frac{1}{3}(v_1 + 2v_2)}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta K = K_f - K_i = \frac{1}{2}(3m)v_f^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$
$$\Delta K = \frac{3m}{2}\left[\frac{1}{3}(v_1 + 2v_2)\right]^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$
$$\Delta K = \frac{3m}{2}\left(\frac{v_1^2}{9} + \frac{4v_1v_2}{9} + \frac{4v_2^2}{9}\right) - \frac{mv_1^2}{2} - mv_2^2$$
$$= m\left(\frac{v_1^2}{6} + \frac{2v_1v_2}{3} + \frac{2v_2^2}{3} - \frac{v_1^2}{2} - v_2^2\right)$$
$$\Delta K = m\left(\frac{v_1^2}{6} + \frac{4v_1v_2}{6} + \frac{4v_2^2}{6} - \frac{3v_1^2}{6} - \frac{6v_2^2}{6}\right)$$
$$= m\left(-\frac{2v_1^2}{6} + \frac{4v_1v_2}{6} - \frac{2v_2^2}{6}\right)$$
$$\Delta K = \boxed{-\frac{m}{3}\left(v_1^2 + v_2^2 - 2v_1v_2\right)}$$

P9.16 Let's first analyze the situation in which the wood block, of mass $m_w = 1.00$ kg, is held in a vise. The bullet of mass $m_b = 7.00$ g is initially moving with speed v_b and then comes to rest in the block due to the kinetic friction force f_k between the block and the bullet as the bullet deforms the wood fibers and moves them out of the way. The result is an increase in internal energy in the wood and the bullet. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left(0 - \frac{1}{2} m_b v_b^2\right) + f_k d = 0 \qquad \textbf{[1]}$$

where d = 8.00 cm is the depth of penetration of the bullet in the wood.

Now consider the second situation, where the block is sitting on a frictionless surface and the bullet is fired into it. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left[\frac{1}{2}\left(m_b + m_w\right) v_f^2 - \frac{1}{2} m_b v_b^2\right] + f_k d' = 0 \qquad \textbf{[2]}$$

where v_f is the speed with which the block and imbedded bullet slide across the table after the collision and d' is the depth of penetration of the bullet in this situation. Identify the wood and the bullet as an isolated system for momentum during the collision:

$$\Delta p = 0 \quad \rightarrow \quad p_i = p_f \quad \rightarrow \quad m_b v_b = \left(m_b + m_w\right) v_f \qquad \textbf{[3]}$$

Solving equation [3] for v_b, we obtain

$$v_b = \frac{(m_b + m_w)v_f}{m_b} \tag{4}$$

Solving equation [1] for $f_k d$ and substituting for v_b from equation [4] above:

$$f_k d = \frac{1}{2}m_b v_b^2 = \frac{1}{2}m_b\left[\frac{(m_b + m_w)v_f}{m_b}\right]^2 = \frac{1}{2}\frac{(m_b + m_w)^2}{m_b}v_f^2 \tag{5}$$

Solving equation [2] for $f_k d'$ and substituting for v_b from equation [4]:

$$\begin{aligned} f_k d' &= -\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b v_b^2\right] \\ &= -\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b\left[\frac{(m_b + m_w)v_f}{m_b}\right]^2\right] \\ f_k d' &= \frac{1}{2}\left[\frac{m_w}{m_b}(m_b + m_w)\right]v_f^2 \end{aligned}$$

[6]

Dividing equation [6] by [5] gives

$$\frac{f_k d'}{f_k d} = \frac{d'}{d} = \frac{\frac{1}{2}\left[\frac{m_w}{m_b}(m_b + m_w)\right]v_f^2}{\frac{1}{2}\left[\frac{(m_b + m_w)^2}{m_b}\right]v_f^2} = \frac{m_w}{m_b + m_w}$$

Solving for d' and substituting numerical values gives

$$d' = \left(\frac{m_w}{m_b + m_w}\right) d = \left[\frac{1.00\text{ kg}}{0.007\ 00\text{ kg} + 1.00\text{ kg}}\right](8.00\text{ cm}) = \boxed{7.94\text{ cm}}$$

P9.17 (a) The speed v of both balls just before the basketball reaches the ground may be found from $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

$$v = \sqrt{v_{yi}^2 + 2a_y\Delta y} = \sqrt{0 + 2(-g)(-h)} = \sqrt{2gh}$$
$$= \sqrt{2(9.80\text{ m/s}^2)(1.20\text{ m})} = \boxed{4.85\text{ m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two balls just before collision are

for the tennis ball (subscript t): $v_{ti} = -v$

and for the basketball (subscript b): $v_{bi} = +v$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$m_t v_{tf} + m_b v_{bf} = m_t v_{ti} + m_b v_{bi}$$

or $m_t v_{tf} + m_b v_{bf} = (m_b - m_t)v$ **[1]**

From the criteria for a perfectly elastic collision:

$$v_{ti} - v_{bi} = -(v_{tf} - v_{bf})$$

or $v_{bf} = v_{tf} + v_{ti} - v_{bi} = v_{tf} - 2v$ **[2]**

Substituting equation [2] into [1] gives

$$m_t v_{tf} + m_b\left(v_{tf} - 2v\right) = \left(m_b - m_t\right)v$$

or the upward speed of the tennis ball immediately after the collision is

$$v_{tf} = \left(\frac{3m_b - m_t}{m_t + m_b}\right)v = \left(\frac{3m_b - m_t}{m_t + m_b}\right)\sqrt{2gh}$$

The vertical displacement of the tennis ball during its rebound following the collision is given by $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

$$\Delta y = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{0 - v_{tf}^2}{2(-g)} = \left(\frac{1}{2\cancel{g}}\right)\left(\frac{3m_b - m_t}{m_t + m_b}\right)^2\left(2\cancel{g}h\right)$$
$$= \left(\frac{3m_b - m_t}{m_t + m_b}\right)^2 h$$

Substituting,

$$\Delta y = \left[\frac{3(590\text{ g}) - (57.0\text{ g})}{57.0\text{ g} + 590\text{ g}}\right]^2(1.20\text{ m}) = \boxed{8.41\text{ m}}$$

P9.18 (a) Using conservation of momentum, $\left(\sum \vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum \vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.00\text{ kg})(5.00\text{ m/s}) + (10.0\text{ kg})(3.00\text{ m/s})$$
$$+ (3.00\text{ kg})(-4.00\text{ m/s}) = \left[(4.00 + 10.0 + 3.00)\text{ kg}\right]v$$

Therefore, $v = +2.24$ m/s, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No.}}$ For example, if the 10.0-kg and 3.00-kg masses were to stick together first, they would move with a speed given by solving

$$(13.0 \text{ kg})v_1 = (10.0 \text{ kg})(3.00 \text{ m/s}) + (3.00 \text{ kg})(-4.00 \text{ m/s})$$

or $v_1 = +1.38 \text{ m/s}$

Then when this 13.0-kg combined mass collides with the 4.00-kg mass, we have

$$(17.0 \text{ kg})v = (13.0 \text{ kg})(1.38 \text{ m/s}) + (4.00 \text{ kg})(5.00 \text{ m/s})$$

and $v = +2.24$ m/s, just as in part (a).

$\boxed{\text{Coupling order makes no difference to the final velocity.}}$

Section 9.5 Collisions in Two Dimensions

P9.19 Conceptualize See Figure 9.13 for an approximation of the collision. The numbers in this problem will differ from those in the figure.

Categorize The two cars are modeled as an *isolated system* for *momentum* in the horizontal plane. If we look at the instants just before the collision and just after, any friction between the cars and the roadway will be negligible because of the very short time interval, and we can use the impulse approximation.

Analyze Write a conservation of momentum equation for the system in the east–west direction:

$$\Delta p_x = 0 \quad \rightarrow \quad p_{xf} = p_{xi} \quad \rightarrow \quad v_{xf\text{N}} + v_{xf\text{E}} = v_{xi\text{N}} + v_{xi\text{E}} \quad (1)$$

where we have recognized that both cars have the same mass and have used subscripts N and E to indicate the northbound (defendant's) car and the eastbound (plaintiff's) car, respectively.

Similarly, for the north–south direction,

$$\Delta p_y = 0 \quad \rightarrow \quad p_{yf} = p_{yi} \quad \rightarrow \quad v_{yf\mathrm{N}} + v_{yf\mathrm{E}} = v_{yi\mathrm{N}} + v_{yi\mathrm{E}} \qquad (2)$$

Substitute for the various velocity components:

East–west: $V_f \cos\theta + V_f \cos\theta = 0 + v_{xi\mathrm{E}} \quad \rightarrow \quad 2V_f \cos\theta = v_{xi\mathrm{E}} \qquad (3)$

North–south: $V_f \sin\theta + V_f \sin\theta = v_{yi\mathrm{N}} + 0 \quad \rightarrow \quad 2V_f \sin\theta = v_{yi\mathrm{N}} \qquad (4)$

In these equations, we do know the angle , but we do not know the final speed V_f of the combined wreckage. Divide Equation (4) by Equation (3) and solve for the initial speed of the defendant:

$$\frac{2V_f \sin\theta}{2V_f \cos\theta} = \frac{v_{yi\mathrm{N}}}{v_{xi\mathrm{E}}} \quad \rightarrow \quad v_{yi\mathrm{N}} = v_{xi\mathrm{E}} \tan\theta$$

Substitute numerical values:

$$v_{yi\mathrm{N}} = (13.0\ \mathrm{m/\ s})\tan 55.0\square = 18.6\ \mathrm{m/\ s}$$

Convert the result to mi/h:

$$v_{yi\mathrm{N}} = 18.6 \text{ m/s}\left(\frac{1 \text{ mi}}{1\,609 \text{ m}}\right)\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right) = \boxed{41.5 \text{ mi/h}}$$

The data from the accident scene show that the defendant was exceeding the speed limit of 35 mi/h.

Finalize Notice that we were able to solve this problem even without knowing the final speed of the combined wreckage. That's good news, because that quantity would have been virtually impossible to measure. We also did not have to know anything about the length of the skid marks. Although this measurement would have been easy to make, we don't need it!

Answer: The defendant was traveling at 41.5 mi/h.

P9.20 We write the conservation of momentum in the x direction, $p_{xf} = p_{xi}$, as

$$mv_\mathrm{O}\cos 37.0° + mv_\mathrm{Y}\cos 53.0° = m(5.00 \text{ m/s})$$

$$0.799v_\mathrm{O} + 0.602v_\mathrm{Y} = 5.00 \text{ m/s} \qquad \textbf{[1]}$$

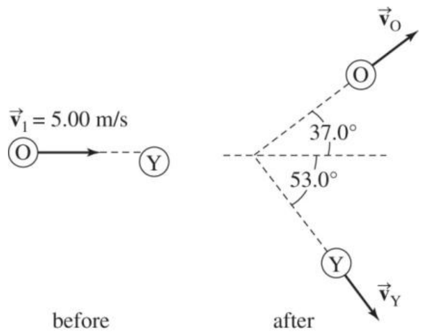
and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_\mathrm{O}\sin 37.0° - mv_\mathrm{Y}\sin 53.0° = 0$$

$$0.602v_\mathrm{O} = 0.799v_\mathrm{Y} \qquad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives,

$$\boxed{v_\mathrm{O} = 3.99 \text{ m/s}} \text{ and } \boxed{v_\mathrm{Y} = 3.01 \text{ m/s}}$$



ANS. FIG. P9.20

P9.21 ANS. FIG. P9.20 illustrates the collision. We write the conservation of momentum in the x direction, $p_{xf} = p_{xi}$, as

$$mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$$
$$v_O \cos\theta + v_Y \sin\theta = v_i \quad \textbf{[1]}$$

and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_O \sin\theta - mv_Y \cos(90.0° - \theta) = 0$$
$$v_O \sin\theta = v_Y \cos\theta \quad \textbf{[2]}$$

From equation [2],

$$v_O = v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \quad \textbf{[3]}$$

Substituting into equation [1],

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so

$$v_Y\left(\cos^2\theta+\sin^2\theta\right)=v_i\sin\theta, \text{ and } \boxed{v_Y=v_i\sin\theta}$$

Then, from equation [3], $\boxed{v_O=v_i\cos\theta}$.

We did not need to write down an equation expressing conservation of mechanical energy. In this situation, the requirement on perpendicular final velocities is equivalent to the condition of elasticity.

P9.22 (a) The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is perfectly inelastic.

(b) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback):

$$(90.0\text{ kg})(5.00\text{ m/s})+0=(185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta=2.43\text{ m/s} \qquad \textbf{[1]}$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent):

$$(95.0\text{ kg})(3.00\text{ m/s})+0=(185\text{ kg})V\sin\theta$$

which gives

$$V\sin\theta=1.54\text{ m/s} \qquad \textbf{[2]}$$

Divide equation [2] by [1]:

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which, $\boxed{\theta = 32.3°}$.

Then, either [1] or [2] gives $V = \boxed{2.88 \text{ m/s}}$.

(c) $K_i = \frac{1}{2}(90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55 \times 10^3 \text{ J}$

$$K_f = \frac{1}{2}(185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67 \times 10^2 \text{ J}$$

Thus, the kinetic energy lost is $\boxed{\text{786 J into internal energy}}$.

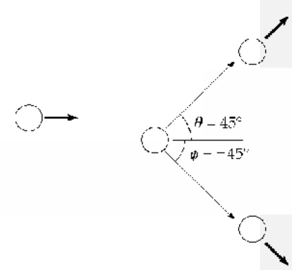
P9.23 (a) The vector expression for conservation of momentum,

$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$ gives $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$.

$$mv_i = mv\cos\theta + mv\cos\phi \qquad \textbf{[1]}$$

$$0 = mv\sin\theta + mv\sin\phi \qquad [2]$$

From [2], $\sin\theta = -\sin\phi$ so $\theta = -\phi$.



ANS. FIG. P9.23

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so

$$\boxed{v = \frac{v_i}{\sqrt{2}}}$$

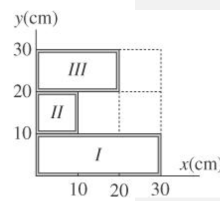
(b) Hence, [1] gives

$$v_i = \frac{2v_i \cos\theta}{\sqrt{2}}$$

with $\theta = \boxed{45.0°}$ and $\phi = \boxed{-45.0°}$.

Section 9.6 The Center of Mass

P9.24 We could analyze the object as nine squares, each represented by an equal-mass particle at its center. But we will have less writing to do if we think of the sheet as composed of three sections, and consider the mass of each section to be at the geometric center of that section. Define the mass per unit area to be σ, and number the rectangles as shown. We can then calculate the mass and identify the center of mass of each section.



ANS. FIG. P9.24

$m_I = (30.0 \text{ cm})(10.0 \text{ cm})\sigma$ with $CM_I = (15.0 \text{ cm}, 5.00 \text{ cm})$

$m_{II} = (10.0 \text{ cm})(20.0 \text{ cm})\sigma$ with $CM_{II} = (5.00 \text{ cm}, 20.0 \text{ cm})$

$m_{III} = (10.0 \text{ cm})(10.0 \text{ cm})\sigma$ with $CM_{III} = (15.0 \text{ cm}, 25.0 \text{ cm})$

The overall center of mass is at a point defined by the vector equation:

$$\vec{\mathbf{r}}_{\text{CM}} \equiv \left(\sum m_i \vec{\mathbf{r}}_i\right) / \sum m_i$$

Substituting the appropriate values, $\vec{\mathbf{r}}_{\text{CM}}$ is calculated to be:

$$\vec{\mathbf{r}}_{\text{CM}} = \left(\frac{1}{\sigma\left(300\text{ cm}^2 + 200\text{ cm}^2 + 100\text{ cm}^2\right)} \right)$$
$$\times \left\{ \sigma[(300)(15.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) + (200)(5.00\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}}) + (100)(15.0\hat{\mathbf{i}} + 25.0\hat{\mathbf{j}})]\text{ cm}^3 \right\}$$

Calculating,

$$\vec{\mathbf{r}}_{\text{CM}} = \frac{4\,500\hat{\mathbf{i}} + 1\,500\hat{\mathbf{j}} + 1\,000\hat{\mathbf{i}} + 4\,000\hat{\mathbf{j}} + 1\,500\hat{\mathbf{i}} + 2\,500\hat{\mathbf{j}}}{600}\text{ cm}$$

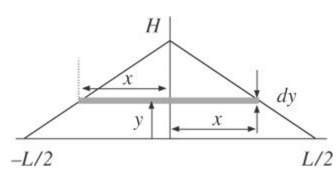
and evaluating, $\vec{\mathbf{r}}_{\text{CM}} = \boxed{(11.7\hat{\mathbf{i}} + 13.3\hat{\mathbf{j}})\text{ cm}}$

P9.25 The volume of the monument is that of a thick triangle of base $L = 64.8$ m, height $H = 15.7$ m, and width $W = 3.60$ m: $V = \frac{1}{2} LHW = 1.83 \times 10^3$ m^3. The monument has mass $M = \rho V = (3\,800\text{ kg/m}^3)V = 6.96 \times 10^6$ kg. The height of the center of mass (CM) is $y_{\text{CM}} = H/3$ (derived below). The amount of work done on the blocks is

$$U_g = Mgy_{\text{CM}}$$
$$= Mg\frac{H}{3} = \left(6.96 \times 10^6\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(\frac{15.7\text{ m}}{3}\right)$$
$$= \boxed{3.57 \times 10^8\text{ J}}$$

We derive $y_{\text{CM}} = H/3$ here:

We model the monument with the figure shown above. Consider the



ANS. FIG. P9.25

monument to be composed of slabs of infinitesimal thickness dy stacked on top of each other. A slab at height y has a infinitesimal volume element $dV = 2xWdy$, where W is the width of the monument and x is a function of height y.

The equation of the sloping side of the monument is

$$y = H - \frac{H}{L/2}x \to y = H - \frac{2H}{L}x \to y = H\left(1 - \frac{2}{L}x\right)$$

where x ranges from 0 to + $L/2$. Therefore,

$$x = \frac{L}{2}\left(1 - \frac{y}{H}\right)$$

where y ranges from 0 to H. The infinitesimal volume of a slab at height y is then

$$dV = 2xWdy = LW\left(1 - \frac{y}{H}\right)dy.$$

The mass contained in a volume element is $dm = \rho dV$.

Because of the symmetry of the monument, its CM lies above the origin of the coordinate axes at position y_{CM}:

$$y_{\text{CM}} = \frac{1}{M}\int_0^M y\,dm = \frac{1}{M}\int_0^V y\rho\,dV = \frac{1}{M}\int_0^H y\rho LW\left(1-\frac{y}{H}\right)dy$$

$$y_{\text{CM}} = \frac{\rho LW}{M}\int_0^H\left(y-\frac{y^2}{H}\right)dy = \frac{\rho LW}{M}\left(\frac{y^2}{2}-\frac{y^3}{3H}\right)\Bigg|_0^H$$

$$= \frac{\rho LW}{M}\left(\frac{H^2}{2}-\frac{H^3}{3H}\right)$$

$$y_{\text{CM}} = \frac{\rho LWH^2}{M}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{1}{6}\frac{\rho LWH^2}{\left(\frac{1}{2}\rho LWH\right)} = \left(\frac{2}{1}\right)\frac{H}{6}$$

$$y_{\text{CM}} = \frac{H}{3}$$

where we have used $M = \rho\left(\frac{1}{2}LHW\right)$.

P9.26 This object can be made by wrapping tape around a light, stiff, uniform rod.

(a) $M = \int_0^{0.300\text{ m}} \lambda\,dx = \int_0^{0.300\text{ m}} [50.0 + 20.0x]dx$

$$M = \left[50.0x + 10.0x^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

$$x_{\text{CM}} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda\,x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x + 20.0x^2\right]dx$$

(b) $$x_{\text{CM}} = \frac{1}{15.9\text{ g}}\left[25.0x^2 + \frac{20x^3}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

Section 9.7 Systems of Many Particles

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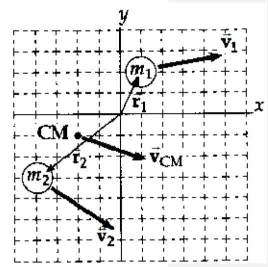
P9.27 (a) ANS. FIG. P9.27 shows the position vectors and velocities of the particles.



ANS. FIG. P9.27

(b) Using the definition of the position vector at the center of mass,

$$\vec{\mathbf{r}}_{CM} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2}$$

$$\vec{\mathbf{r}}_{CM} = \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(1.00\text{ m},\ 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m},\ -3.00\text{ m})]$$

$$\vec{\mathbf{r}}_{CM} = \boxed{\left(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ m}}$$

(c) The velocity of the center of mass is

$$\vec{\mathbf{v}}_{CM} = \frac{\vec{\mathbf{P}}}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

$$= \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(3.00\text{ m/s},\ 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s},\ -2.00\text{ m/s})]$$

$$\vec{\mathbf{v}}_{CM} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\vec{\mathbf{P}} = M\vec{\mathbf{v}}_{CM}$ or as $\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$. Either gives

$$\vec{\mathbf{P}} = \boxed{\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$$

P9.28 The vector position of the center of mass is (suppressing units)

$$\vec{r}_{CM} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} = \frac{3.5\left[\left(3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)t + 2\hat{\mathbf{j}}t^2\right] + 5.5\left[3\hat{\mathbf{i}} - 2\hat{\mathbf{i}}t^2 + 6\hat{\mathbf{j}}t\right]}{3.5 + 5.5}$$
$$= \left(1.83 + 1.17t - 1.22t^2\right)\hat{\mathbf{i}} + \left(-2.5t + 0.778t^2\right)\hat{\mathbf{j}}$$

(a) At $t = 2.50$ s,

$$\vec{\mathbf{r}}_{CM} = (1.83 + 1.17\cdot 2.5 - 1.22\cdot 6.25)\hat{\mathbf{i}} + (-2.5\cdot 2.5 + 0.778\cdot 6.25)\hat{\mathbf{j}}$$
$$= \boxed{\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right)\text{ cm}}$$

(b) The velocity of the center of mass is obtained by differentiating the expression for the vector position of the center of mass with respect to time:

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\vec{\mathbf{r}}_{\text{CM}}}{dt} = (1.17 - 2.44t)\hat{\mathbf{i}} + (-2.5 + 1.56t)\hat{\mathbf{j}}$$

At $t = 2.50$ s,

$$\vec{\mathbf{v}}_{\text{CM}} = (1.17 - 2.44\cdot 2.5)\hat{\mathbf{i}} + (-2.5 + 1.56\cdot 2.5)\hat{\mathbf{j}}$$
$$= (-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})\text{ cm/s}$$

Now, the total linear momentum is the total mass times the velocity of the center of mass.

$$\vec{\mathbf{p}} = (9.00\text{ g})(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})\text{ cm/s}$$
$$= \boxed{(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})\text{ g}\cdot\text{cm/s}}$$

(c) As was shown in part (b), $\boxed{\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)\text{cm/s}}$

(d)k Differentiating again, $\vec{\mathbf{a}}_{\text{CM}} = \dfrac{d\vec{\mathbf{v}}_{CM}}{dt} = (-2.44)\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}$

The center of mass acceleration is $\boxed{\left(-2.44\hat{\mathbf{i}}+1.56\hat{\mathbf{j}}\right)\text{cm/s}^2}$ at t = 2.50 s and at all times.

(e) The net force on the system is equal to the total mass times the acceleration of the center of mass:

$$\vec{\mathbf{F}}_{\text{net}}=\left(9.00\text{ g}\right)\left(-2.44\hat{\mathbf{i}}+1.56\hat{\mathbf{j}}\right)\text{ cm/s}^2=\boxed{\left(-220\hat{\mathbf{i}}+140\hat{\mathbf{j}}\right)\mu\text{N}}$$

P9.29 Conceptualize Imagine the drone hovering at rest above the house. It explodes into four pieces, which fly off in different directions. Your task is to determine if the drone simply exploded, or if it were struck by a meteorite.

Categorize If the drone were struck by a meteorite, we have a collision process. But we don't know whether or not this occurred, and, if it did, we have no information about the meteorite: its mass, speed, and direction of velocity are all unknown. The important piece of information in the problem is that the drone was *at rest*. Therefore, its center of mass had zero velocity and was located over the center of the house before the explosion/collision. If the drone simply exploded, the center of mass of the pieces must still be at the center of the house. If there were a collision, the center of mass of the drone would move during the collision and no longer be at the center of the house. Therefore, we categorize this problem as one involving the calculation of the center of mass of the four pieces.

Analyze From Equation 9.31, we calculate the center of mass of the four pieces of the drone, using the center of the house as the origin, the east–west direction as the x axis, and the north–south direction as the y axis:

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i \vec{\mathbf{r}}_i$$

$$= \frac{\begin{bmatrix} 80.0\left(-150\hat{\mathbf{i}}\right) + 120\left(75.0\hat{\mathbf{j}}\right) + 50.0\left(-90.0\sin 20.0°\hat{\mathbf{i}} - 90.0\cos 20.0°\hat{\mathbf{j}}\right) \\ +150\left(50.0\cos 20.0°\hat{\mathbf{i}} + 50.0\sin 20.0°\hat{\mathbf{j}}\right) \end{bmatrix} \mathrm{kg}\cdot\mathrm{m}}{400\ \mathrm{kg}}$$

$$= \left(-16.2\hat{\mathbf{i}} + 18.3\hat{\mathbf{j}}\right)\ \mathrm{m}$$

If the drone were at rest and experienced an explosion, its center of mass would be at the center of the house both before and after the explosion. Because the final center of mass of the pieces is not at the center of the house, the evidence suggests that $\boxed{\text{the drone was hit by a meteorite}}$.

Finalize The final center of mass is roughly northwest of the house, suggesting that that is the direction of travel of the meteorite that hit the drone.

Answer: The drone was struck by a meteorite.

Section 9.8 Deformable Systems

P9.30 (a) $\boxed{\text{Yes}}$ The only horizontal force on the vehicle is the frictional force exerted by the floor, so it gives the vehicle all of its final momentum, $(6.00\ \mathrm{kg})(3.00\hat{\mathbf{i}}\ \mathrm{m/s}) = \boxed{18.0\hat{\mathbf{i}}\ \mathrm{kg}\cdot\mathrm{m/s}}$.

(b) No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work.

(c) Yes, we could say that the final momentum of the cart came from the floor or from the Earth through the floor.

(d) No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount $KE = \left(\frac{1}{2}\right)(6.00\text{ kg})(3.00\text{ m/s})^2 = 27.0\text{ J}$.

(e) Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.

P9.31 (a) $\boxed{\text{Yes}}$ The floor exerts a force, larger than the person's weight over time as he is taking off.

(b) $\boxed{\text{No}}$ The work by the floor on the person is zero because the force exerted by the floor acts over zero distance.

(c) He leaves the floor with a speed given by $\frac{1}{2}mv^2 = mgy_f$, or

$$v = \sqrt{2gy_f} = \sqrt{2(9.80\text{ m/s}^2)(0.150\text{ m})} = 1.71\text{ m/s}$$

so his momentum immediately after he leaves the floor is

$$p = mv = (60.0\text{ kg})(1.71\text{ m/s up}) = \boxed{103\text{ kg}\cdot\text{m/s up}}$$

(d) Yes. You could say that it came from the planet, that gained momentum $103\text{ kg}\cdot\text{m/s}$ down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact.

(e) His kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(60.0\text{ kg})(1.71\text{ m/s})^2 = \boxed{88.2\text{ J}}$$

(f) **[** No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person. **]**

Section 9.9 Rocket Propulsion

P9.32 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s}) - 0}{1.00\text{ s}}$$
$$= \boxed{15.0\text{ N}}$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0-N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

P9.33 In $v = v_e \ln \dfrac{M_i}{M_f}$ we solve for M_i.

(a) $M_i = e^{v/v_e} M_f \quad \rightarrow \quad M_i = e^5 \left(3.00 \times 10^3 \text{ kg}\right) = 4.45 \times 10^5 \text{ kg}$

The mass of fuel and oxidizer is

$$\begin{aligned} \Delta M = M_i - M_f &= (445 - 3.00) \times 10^3 \text{kg} \\ &= \boxed{442 \text{ metric tons}} \end{aligned}$$

(b) $\Delta M = e^2 \left(3.00 \text{ metric tons}\right) - 3.00 \text{ metric tons} = \boxed{19.2 \text{ metric tons}}$

(c) This is much less than the suggested value of 442/2.5. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed, by counting again and again in the speed the body attains second after second during its burn.

P9.34 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\dfrac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \dfrac{k}{M_i} t\right)$

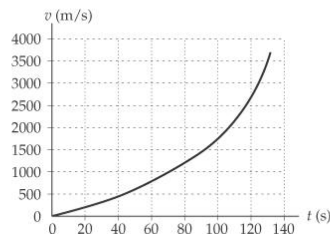
With the definition, $T_p \equiv \dfrac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s,

$$v = -(1\,500\text{ m/s})\ln\left(1 - \frac{t}{144\text{ s}}\right)$$

t (s)	v (m/s)
0	0
20	224
40	488
60	808
80	1 220
100	1 780
120	2 690
132	3 730



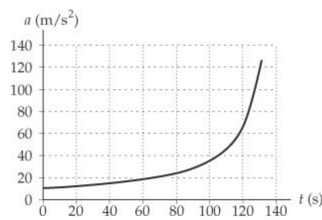
ANS. FIG. P9.34(b)

(c) $$a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]}{dt} = -v_e\left(\frac{1}{1 - \frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1 - \frac{t}{T_p}}\right),$$

or $\boxed{a(t) = \dfrac{v_e}{T_p - t}}$

(d) With, v_e = 1 500 m/s, and T_p = 144 s, $a = \dfrac{1\ 500\ \text{m/s}}{144\ \text{s} - t}$.

t (s)	a (m/s^2)
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125



ANS. FIG. P9.34(d)

(e) $$x(t) = 0 + \int_0^t v\,dt = \int_0^t \left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right] dt = v_e T_p \int_0^t \ln\left[1 - \frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

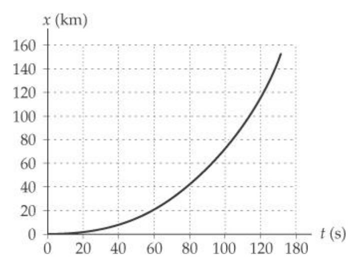
$$x(t) = v_e T_p \left[\left(1 - \frac{t}{T_p}\right) \ln\left(1 - \frac{t}{T_p}\right) - \left(1 - \frac{t}{T_p}\right) \right]_0^t$$

$$x(t) = \boxed{v_e \left(T_p - t\right) \ln\left(1 - \frac{t}{T_p}\right) + v_e t}$$

(f) With, v_e = 1.500 m/s = 1.50 km/s, and T_p = 144 s,

$$x = 1.50(144 - t)\ln\left(1 - \frac{t}{144}\right) + 1.50t$$

t (s)	x (m)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153



ANS. FIG. P9.34(f)

Additional Problems

P9.35 (a) The system is isolated because the skater is on frictionless ice — if it were otherwise, she would be able to move. Initially, the horizontal momentum of the system is zero, and this quantity is conserved; so when she throws the gloves in one direction, she will move in the opposite direction because the total momentum will remain zero. The system has total mass M. After the skater throws the gloves, the mass of the gloves, m, is moving with velocity $\vec{\mathbf{v}}_{\text{gloves}}$ and the mass of the skater less the gloves, $M - m$, is moving with velocity $\vec{\mathbf{v}}_{\text{girl}}$:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

$$0 = (M - m)\vec{\mathbf{v}}_{\text{girl}} + m\vec{\mathbf{v}}_{\text{gloves}} \rightarrow \vec{\mathbf{v}}_{\text{girl}} = \boxed{-\left(\frac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}}$$

The term $M - m$ is the total mass less the mass of the gloves.

(b) $\boxed{\text{As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her (Newton's third law) that causes her to accelerate from rest to reach the velocity } \vec{\mathbf{v}}_{\text{girl}}.}$

P9.36 (a) In the same symbols as in the text's Example, the original kinetic energy is

$$K_A = \frac{1}{2}m_1 v_{1A}^2$$

The example shows that the kinetic energy immediately after latching together is

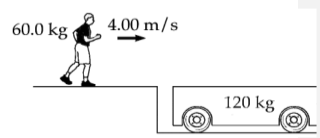
$$K_B = \frac{1}{2}\left(\frac{m_1 v_{1A}^2}{m_1 + m_2}\right)$$

so the fraction of kinetic energy remaining as kinetic energy is

$$K_B/K_A = \boxed{m_1/(m_1 + m_2)}$$

(b) Momentum is conserved in the collision so momentum after divided by momentum before is $\boxed{1.00}$.

P9.37 (a) Conservation of momentum for this totally inelastic collision gives



ANS. FIG. P9.37

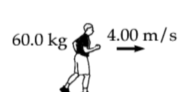
$$m_p v_i = (m_p + m_c) v_f$$

$$(60.0 \text{ kg})(4.00 \text{ m/s}) = (120 \text{ kg} + 60.0 \text{ kg}) v_f$$

$$\vec{\mathbf{v}}_f = \boxed{1.33\hat{\mathbf{i}} \text{ m/s}}$$

(b) To obtain the force of friction, we first consider Newton's second law in the y direction, $\sum F_y = 0$, which gives

$$n - (60.0 \text{ kg})(9.80 \text{ m/s}) = 0$$



or $n = 588$ N. The force of friction is then

$$f_k = \mu_k n = (0.400)(588\ \text{N}) = 235\ \text{N}$$

$$\vec{\mathbf{f}}_k = \boxed{-235\hat{\mathbf{i}}\ \text{N}}$$

(c) The change in the person's momentum equals the impulse, or

$$p_i + I = p_f$$

$$mv_i + Ft = mv_f$$

$$(60.0\ \text{kg})(4.00\ \text{m/s}) - (235\ \text{N})t = (60.0\ \text{kg})(1.33\ \text{m/s})$$
$$t = \boxed{0.680\ \text{s}}$$

(d) The change in momentum of the person is

$$m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i = (60.0\ \text{kg})(1.33 - 4.00)\hat{\mathbf{i}}\ \text{m/s} = \boxed{-160\hat{\mathbf{i}}\ \text{N}\cdot\text{s}}$$

The change in momentum of the cart is

$$(120\ \text{kg})(1.33\ \text{m/s}) - 0 = \boxed{+160\hat{\mathbf{i}}\ \text{N}\cdot\text{s}}$$

(e) $x_f - x_i = \dfrac{1}{2}(v_i + v_f)t = \dfrac{1}{2}[(4.00 + 1.33)\ \text{m/s}](0.680\ \text{s}) = \boxed{1.81\ \text{m}}$

(f) $x_f - x_i = \dfrac{1}{2}(v_i + v_f)t = \dfrac{1}{2}(0 + 1.33\ \text{m/s})(0.680\ \text{s}) = \boxed{0.454\ \text{m}}$

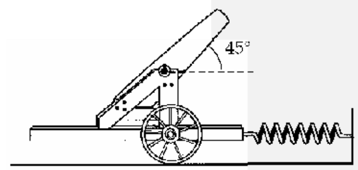
(g) $\dfrac{1}{2}mv_f^2 - \dfrac{1}{2}mv_i^2 = \dfrac{1}{2}(60.0\ \text{kg})(1.33\ \text{m/s})^2$
$$- \frac{1}{2}(60.0\ \text{kg})(4.00\ \text{m/s})^2 = \boxed{-427\ \text{J}}$$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(120\text{ kg})(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) The force exerted by the person on the cart must be equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about *why*. The distance moved by the cart is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, –320 J, becomes +320 J of additional internal energy in this perfectly inelastic collision.

(c) Energy is an entirely different thing from momentum. A comparison: When a photographer's single-use flashbulb flashes, a magnesium filament oxidizes. Chemical energy disappears. (Internal energy appears and light carries some energy away.) The measured mass of the flashbulb is the same before and after. It can be the same in spite of the 100% energy conversion, because energy and mass are totally different things in classical physics. In the ballistic pendulum, conversion of energy from mechanical into internal does not upset conservation of mass or conservation of momentum.

P9.38 (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing:



ANS. FIG. P9.38

$$p_{xf} = p_{xi}$$

$$m_{\text{shell}} v_{\text{shell}} \cos 45.0° + m_{\text{cannon}} v_{\text{recoil}} = 0$$

$$(200 \text{ kg})(125 \text{ m/s})\cos 45.0° + (5\,000 \text{ kg}) v_{\text{recoil}} = 0$$

or $v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$$

$$0 + 0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$$

$$x_{\max} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000 \text{ kg})(-3.54 \text{ m/s})^2}{2.00 \times 10^4 \text{ N/m}}} = \boxed{1.77 \text{ m}}$$

(c) $\left|F_{s,\max}\right| = kx_{\max}$

$$\left|F_{s,\max}\right| = (2.00 \times 10^4 \text{ N/m})(1.77 \text{ m}) = \boxed{3.54 \times 10^4 \text{ N}}$$

(d) $\boxed{\text{No.}}$ The spring exerts a force on the system during the firing. The force represents an impulse, so the momentum of the system is not conserved in the horizontal direction. Consider the vertical direction. There are two vertical forces on the system: the normal force from the ground and the gravitational force. During the firing, the normal force is larger than the gravitational force. Therefore, there is a net impulse on the system in the upward direction. The impulse accounts for the initial vertical momentum component of the projectile.

P9.39 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$
$$mv_i + M(0) = mv + Mv = (m + M)v$$
$$\rightarrow \qquad v_i = \frac{(m+M)}{m} v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block v and the height h the block climbs to:

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}(m+M)v^2 + 0 = (m+M)gh \rightarrow v = \sqrt{2gh}$$

Combining our results, we find

$$v_i = \frac{m+M}{m}\sqrt{2gh} = \left(\frac{1.255\ \text{kg}}{0.005\ 00\ \text{kg}}\right)\sqrt{2\left(9.80\ \text{m/s}^2\right)\left(0.220\ \text{m}\right)}$$
$$v_i = \boxed{521\ \text{m/s}}$$

P9.40 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$
$$mv_i + M(0) = mv + Mv = (m+M)v$$
$$\rightarrow \quad v_i = \frac{(m+M)}{m}v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block v and the height h the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m+M)v^2 + 0 = (m+M)gh \to v = \sqrt{2gh}$$

Combining our results, we find $\boxed{v_i = \frac{m+M}{m}\sqrt{2gh}}$.

P9.41 (a) When the spring is fully compressed, each cart moves with same velocity v. Apply conservation of momentum for the system of two gliders

$$p_i = p_f: \qquad m_1v_1 + m_2v_2 = (m_1 + m_2)v \quad \to \quad \boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

(b) Only conservative forces act; therefore, $\Delta E = 0$.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \left(\frac{1}{k(m_1+m_2)}\right)[(m_1+m_2)m_1v_1^2 + (m_1+m_2)m_2v_2^2$$
$$-(m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2]$$

$$x_m = \sqrt{\frac{m_1m_2(v_1^2 + v_2^2 - 2v_1v_2)}{k(m_1+m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}}$$

(c) $m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$

Conservation of momentum: $m_1(v_1 - v_{1f}) = m_2(v_{2f} - v_2)$ **[1]**

Conservation of energy: $$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$$

which simplifies to: $$m_1\left(v_1^2-v_{1f}^2\right)=m_2\left(v_{2f}^2-v_2^2\right)$$

Factoring gives

$$m_1\left(v_1-v_{1f}\right)\left(v_1+v_{1f}\right)=m_2\left(v_{2f}-v_2\right)\cdot\left(v_{2f}+v_2\right)$$

and with the use of the momentum equation (equation [1]),

this reduces to $$v_1+v_{1f}=v_{2f}+v_2$$

or $$v_{1f}=v_{2f}+v_2-v_1 \qquad \textbf{[2]}$$

Substituting equation [2] into equation [1] and simplifying yields

$$\boxed{v_{2f}=\frac{2m_1v_1+\left(m_2-m_1\right)v_2}{m_1+m_2}}$$

Upon substitution of this expression for into equation [2], one finds

$$\boxed{v_{1f}=\frac{\left(m_1-m_2\right)v_1+2m_2v_2}{m_1+m_2}}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

P9. 42 (a) $\boxed{\text{The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the } x \text{ direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the sleigh afterward, and the velocity of the seat relative to the ground.}}$

(b) We substitute v_{1f} = 8.00 m/s – v_{2f}:

$$(270\text{ kg})(7.50\text{ m/s}) = (15.0\text{ kg})(-8.00\text{ m/s} + v_{2f}) + (255\text{ kg})v_{2f}$$
$$2\ 025\text{ kg}\cdot\text{m/s} = -120\text{ kg}\cdot\text{m/s} + (270\text{ kg})v_{2f}$$

$$v_{2f} = \frac{2\ 145\text{ m/s}}{270} = 7.94\text{ m/s}$$
$$v_{1f} = 8.00\text{ m/s} - 7.94\text{ m/s} = 0.055\ 6\text{ m/s}$$

$\boxed{\text{The final velocity of the seat is } -0.055\,6\hat{\mathbf{i}}\text{ m/s. That of the sleigh is } 7.94\hat{\mathbf{i}}\text{ m/s.}}$

(c) You transform potential energy stored in your body into kinetic energy of the system:

$$\Delta K + \Delta U_{\text{body}} = 0$$
$$\Delta U_{\text{body}} = -\Delta K = K_i - K_f$$

$$\Delta U_{\text{body}} = \frac{1}{2}(270\text{ kg})(7.50\text{ m/s})^2 - \left[\frac{1}{2}(15.0\text{ kg})(0.0556\text{ m/s})^2 + \frac{1}{2}(255\text{ kg})(7.94\text{ m/s})^2\right]$$
$$\Delta U_{\text{body}} = 7\ 594\text{ J} - [0.023\ 1\text{ J} + 8\ 047\text{ J}]$$
$$\Delta U_{\text{body}} = \boxed{-453\text{ J}}$$

P9.43 (a) We can obtain the initial speed of the projectile by utilizing conservation of momentum:

$$m_1 v_{1A} + 0 = (m_1 + m_2) v_B$$

Solving for v_{1A} gives

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$
$$v_{1A} \cong \boxed{6.29\ \text{m/s}}$$

(b) We begin with the kinematic equations in the x and y direction:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$
$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

And simplify by plugging in $x_0 = y_0 = 0$, $v_{y0} = 0$, $v_{x0} = v_{1A}$, $a_x = 0$, and $a_y = g$:

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1At}$$

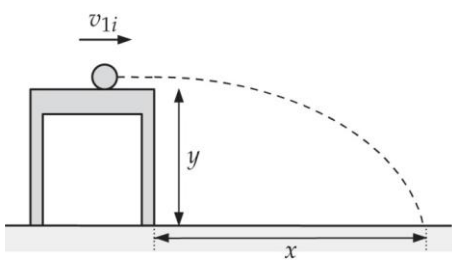
Combining them gives

$$v_{1A} = \frac{x}{\sqrt{2y/g}} = x\sqrt{\frac{g}{2y}}$$

Substituting the numerical values from the problem statement gives

$$v_{1A} = x\sqrt{\frac{g}{2y}} = (2.57\ \text{m})\sqrt{\frac{9.80\ \text{m/s}^2}{2(0.853\ \text{m})}} = \boxed{6.16\ \text{m/s}}$$

(c) $\boxed{\text{Most of the 2\% difference between the values for speed could be accounted for by air resistance.}}$



ANS. FIG. P9.43

P9.44 We hope the momentum of the equipment provides enough recoil so that the astronaut can reach the ship before he loses life support! But can he do it?

Relative to the spacecraft, the astronaut has a momentum p = (150 kg)(20 m/s) = 3 000 kg · m/s away from the spacecraft. He must throw enough equipment away so that his momentum is reduced to at least zero relative to the spacecraft, so the equipment must have momentum of at least 3 000 kg · m/s relative to the spacecraft. If he throws the equipment at 5.00 m/s relative to himself in a direction away from the spacecraft, the velocity of the equipment will be 25.0 m/s away from the spacecraft. How much mass travelling at 25.0 m/s is necessary to equate to a momentum of 3 000 kg · m/s?

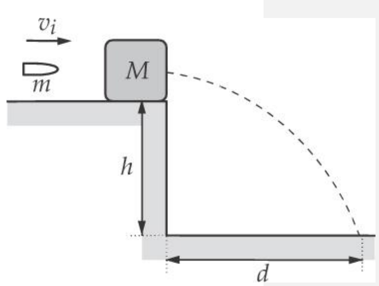
$$p = 3\ 000\ \text{kg}\cdot\text{m/s} = m(25.0\ \text{m/s})$$

which gives

$$m = \frac{3\ 000\ \text{kg}\cdot\text{m/s}}{25.0\ \text{m/s}} = 120\ \text{kg}$$

In order for his motion to reverse under these condition, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.

P9.45 Using conservation of momentum from just before to just after the impact of the bullet with the block:



ANS. FIG. P9.45

$$mv_i = (M + m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \text{ and } v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives

$$v_i = \left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}} = \left(\frac{250\ \text{g} + 8.00\ \text{g}}{8.00\ \text{g}}\right)\sqrt{\frac{(9.80\ \text{m/s}^2)(2.00\ \text{m})^2}{2(1.00\ \text{m})}}$$
$$= \boxed{143\ \text{m/s}}$$

P9.46 Refer to ANS. FIG. P9.45. Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M + m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \text{ and } v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

P9.47 (a) From conservation of momentum,

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$

$$\begin{aligned}(0.500\text{ kg})&\left(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} + 1.00\hat{\mathbf{k}}\right)\text{ m/s} \\ &+ (1.50\text{ kg})\left(-1.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m/s} \\ &= (0.500\text{ kg})\left(-1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 8.00\hat{\mathbf{k}}\right)\text{ m/s} \\ &+ (1.50\text{ kg})\vec{\mathbf{v}}_{2f}\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{v}}_{2f} &= \left(\frac{1}{1.50\text{ kg}}\right)\Big[\left(-0.500\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} \\ &\quad + \left(0.500\hat{\mathbf{i}} - 1.50\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\Big] \\ &= \boxed{0}\end{aligned}$$

The original kinetic energy is

$$\frac{1}{2}(0.500\text{ kg})\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2 + \frac{1}{2}(1.50\text{ kg})\left(1^2+2^2+3^2\right)\text{m}^2/\text{s}^2 = 14.0\text{ J}$$

The final kinetic energy is

$$\frac{1}{2}(0.500\text{ kg})\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0 = 18.5\text{ J}$$

different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} = (0.500\text{ kg})\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s} + (1.50\text{ kg})\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = \left(\frac{1}{1.50\text{ kg}}\right)\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s} + \left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}$$
$$= \boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{m/s}}$$

We see $\vec{\mathbf{v}}_{2f} = \vec{\mathbf{v}}_{1f}$ so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Again, from conservation of momentum,

$$\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} = (0.500\text{ kg})\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{m/s}+(1.50\text{ kg})\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = \left(\frac{1}{1.50\text{ kg}}\right)\left(-0.500\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}$$
$$+\left(0.500\hat{\mathbf{i}} - 1.50\hat{\mathbf{j}} - 0.500a\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}$$
$$= \boxed{(-2.67 - 0.333a)\hat{\mathbf{k}}\text{ m/s}}$$

Then, from conservation of energy:

$$14.0\text{ J} = \frac{1}{2}(0.500\text{ kg})\left(1^2 + 3^2 + a^2\right)\text{ m}^2/\text{s}^2$$
$$+\frac{1}{2}(1.50\text{ kg})(2.67 + 0.333a)^2\text{ m}^2/\text{s}^2$$
$$= 2.50\text{ J} + 0.250a^2 + 5.33\text{ J} + 1.33a + 0.0833a^2$$

This gives, suppressing units, a quadratic equation in a,

$$0 = 0.333a^2 + 1.33a - 6.167 = 0$$

which solves to give

$$a = \frac{-1.33 \pm \sqrt{1.33^2 - 4(0.333)(-6.167)}}{0.667}$$

With $\boxed{a = 2.74}$,

$$\vec{\mathbf{v}}_{2f} = \left(-2.67 - 0.333(2.74)\right)\hat{\mathbf{k}}\text{ m/s} = \boxed{-3.58\hat{\mathbf{k}}\text{ m/s}}$$

With $\boxed{a = -6.74}$,

$$\vec{\mathbf{v}}_{2f} = \left(-2.67 - 0.333(-6.74)\right)\hat{\mathbf{k}}\text{ m/s} = \boxed{-0.419\hat{\mathbf{k}}\text{ m/s}}$$

P9.48 (a) Proceeding step by step, we find the stone's speed just before collision, using energy conservation for the stone-Earth system:

$$m_a g y_i = \frac{1}{2} m_a v_i^2$$

which gives

$$v_i = \sqrt{2gh} = [2(9.80\ \text{m/s}^2)(1.80\ \text{m})]^{1/2} = 5.94\ \text{m/s}$$

Now for the elastic collision with the stationary cannonball, we use the specialized Equation 9.22 from the chapter text, with $m_1 = 80.0$ kg and $m_2 = m$:

$$\begin{aligned} v_{\text{cannonball}} = v_{2f} &= \frac{2m_1 v_{1i}}{m_1 + m_2} = \frac{2(80.0\ \text{kg})(5.94\ \text{m/s})}{80.0\ \text{kg} + m} \\ &= \frac{950\ \text{kg}\cdot\text{m/s}}{80.0\ \text{kg} + m} \end{aligned}$$

The time for the cannonball's fall into the ocean is given by

$$\Delta y = v_{yi} t + \frac{1}{2} a_y t^2 \to -36.0 = \frac{1}{2}(-9.80)t^2 \to t = 2.71\ \text{s}$$

so its horizontal range is

$$\begin{aligned} R = v_{2f} t &= (2.71\ \text{s})\left(\frac{950\ \text{kg}\cdot\text{m/s}}{80.0\ \text{kg} + m}\right) \\ &= \boxed{\frac{2.58\times 10^3\ \text{kg}\cdot\text{m}}{80.0\ \text{kg} + m}} \end{aligned}$$

(b) The maximum value for R occurs for $m \to 0$, and is

$$R = \frac{2.58\times 10^3\ \text{kg}\cdot\text{m}}{80.0\ \text{kg} + m} = \frac{2.58\times 10^3\ \text{kg}\cdot\text{m}}{80.0\ \text{kg} + 0} = \boxed{32.2\ \text{m}}$$

(c) As indicated in part (b), the maximum range corresponds to

$\boxed{m \to 0}$

(d) Yes, until the cannonball splashes down. No; the kinetic energy of the system is split between the stone and the cannonball after the collision and we don't know how it is split without using the conservation of momentum principle.

(e) The range is equal to the product of $v_{\text{cannonball}}$, the speed of the cannonball after the collision, and t, the time at which the cannonball reaches the ocean. But $v_{\text{cannonball}}$ is proportional to v_i, the speed of the stone just before striking the cannonball, which is, in turn, proportional to the square root of g. The time t at which the cannonball strikes the ocean is inversely proportional to the square root of g. Therefore, the product $R = (v_{\text{cannonball}})t$ is *independent* of g. At a location with weaker gravity, the stone would be moving more slowly before the collision, but the cannonball would follow the same trajectory, moving more slowly over a longer time interval.

P9.49 The force exerted by the spring on each block is in magnitude.

$$\left|F_s\right| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}$$

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring. From conservation of energy,

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$\frac{1}{2}(3.85\ \text{N/m})(0.080\ 0\ \text{m})^2 = \frac{1}{2}(0.250\ \text{kg})v_{1f}^2 + \frac{1}{2}(0.500\ \text{kg})v_{2f}^2 \qquad [1]$$

And from conservation of linear momentum,

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = (0.250\ \text{kg})v_{1f}(-\hat{\mathbf{i}}) + (0.500\ \text{kg})v_{2f}\hat{\mathbf{i}}$$

$$v_{1f} = 2v_{2f}$$

Substituting this into [1] gives

$$0.012\ 3\ \text{J} = \frac{1}{2}(0.250\ \text{kg})\left(2v_{2f}\right)^2 + \frac{1}{2}(0.500\ \text{kg})v_{2f}^2 = \frac{1}{2}(1.50\ \text{kg})v_{2f}^2$$

Solving,

$$v_{2f} = \left(\frac{0.012\ 3\ \text{J}}{0.750\ \text{kg}}\right)^{1/2} = 0.128\ \text{m/s} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 0.128\hat{\mathbf{i}}\ \text{m/s}}$$

$$v_{1f} = 2(0.128\ \text{m/s}) = 0.256\ \text{m/s} \qquad \boxed{\vec{\mathbf{v}}_{1f} = -0.256\hat{\mathbf{i}}\ \text{m/s}}$$

(b) For the lighter block,

$$\sum F_y = ma_y, \quad n - 0.250\ \text{kg}\left(9.80\ \text{m/s}^2\right) = 0, \quad n = 2.45\ \text{N},$$

$$f_k = \mu_k n = 0.1\left(2.45\ \text{N}\right) = 0.245\ \text{N}.$$

We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: f_k = 0.490 N. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compression becomes x_f given by

$$|F_s| = kx,\ 0.245 \text{ N} = (3.85 \text{ N·m})x_f,\ 0.063\ 6 \text{ m} = x_f$$

Now for the energy of the lighter block as it moves to this maximum-speed point, we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\ 3 \text{ J} - (0.245 \text{ N})(0.08 - 0.063\ 6 \text{ m}) = \frac{1}{2}(0.250 \text{ kg})v_f^2 + \frac{1}{2}(3.85 \text{ N/m})(0.063\ 6 \text{ m})^2$$

$$0.012\ 3 \text{ J} - 0.004\ 01 \text{ J} = \frac{1}{2}(0.250 \text{ kg})v_f^2 + 0.007\ 80 \text{ J}$$

$$\left(\frac{2(0.000\ 515 \text{ J})}{0.250 \text{ kg}}\right)^{1/2} = v_f = 0.064\ 2 \text{ m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter block, $\boxed{-0.064\ 2\hat{\mathbf{i}} \text{ m/s}}$.

(c) For the lighter block, f_k = 0.462(2.45 N) = 1.13 N. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P9.50 The orbital speed of the Earth is

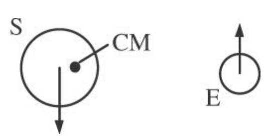
$$v_E = \frac{2\pi r}{T} = \frac{2\pi\left(1.496\times 10^{11}\text{ m}\right)}{3.156\times 10^{7}\text{ s}} = 2.98\times 10^{4}\text{ m/s}$$

In six months the Earth reverses its direction, to undergo momentum change

$$\begin{aligned} m_E\left|\Delta\vec{\mathbf{v}}_E\right| &= 2m_E v_E = 2\left(5.98\times 10^{24}\text{ kg}\right)\left(2.98\times 10^{4}\text{ m/s}\right) \\ &= 3.56\times 10^{29}\text{ kg}\cdot\text{m/s} \end{aligned}$$

Relative to the center of mass, the Sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S\left|\Delta\vec{\mathbf{v}}_S\right| = 3.56\times 10^{29}\text{ kg}\cdot\text{m/s}$

Then $\left|\Delta\vec{\mathbf{v}}_S\right| = \dfrac{3.56\times 10^{29}\text{ kg}\cdot\text{m/s}}{1.991\times 10^{30}\text{ kg}} = \boxed{0.179\text{ m/s}}$



ANS. FIG. P9.50

P9.51 (a) We have, from the impulse-momentum theorem, $\vec{\mathbf{p}}_i + \vec{\mathbf{F}}t = \vec{\mathbf{p}}_f$:

$$(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + \left(12.0\hat{\mathbf{i}}\text{ N}\right)(5.00\text{ s}) = (3.00\text{ kg})\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \boxed{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) The particle's acceleration is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t} = \frac{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}\right)\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(c) From Newton's second law,

$$\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{12.0\hat{\mathbf{i}}\text{ N}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(d) The vector displacement of the particle is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$= \left(7.00\text{ m/s}\,\hat{\mathbf{j}}\right)(5.00\text{ s}) + \frac{1}{2}\left(4.00\text{ m/s}^2\hat{\mathbf{i}}\right)(5.00\text{ s})^2$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}\right)\text{ m}}$$

(e) Now, from the work-kinetic energy theorem, the work done on the particle is

$$W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = \left(12.0\hat{\mathbf{i}}\text{ N}\right)\left(50.0\hat{\mathbf{i}}\text{ m} + 35.0\hat{\mathbf{j}}\text{ m}\right) = \boxed{600\text{ J}}$$

(f) The final kinetic energy of the particle is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\cdot\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m}^2/\text{s}^2$$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})\left(449\text{ m}^2/\text{s}^2\right) = \boxed{674\text{ J}}$$

(g) The final kinetic energy of the particle is

$$\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$$

(h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

Challenge Problems

P9.52 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\dfrac{dp}{dt} = \dfrac{d(mv)}{dt} = v\dfrac{dm}{dt} = (0.750\text{ m/s})(5.00\text{ kg/s}) = \boxed{3.75\text{ N}}$

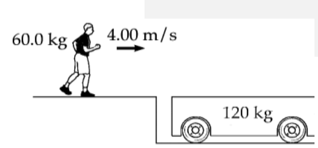
(b) The only horizontal force on the sand is belt friction, which causes the momentum of the sand to change: $F = \dfrac{dp}{dt} = \boxed{3.75\text{ N}}$ as above.

(c) The belt is in equilibrium:

$$\sum F_x = ma_x: \quad +F_{\text{ext}} - f = 0 \quad \text{and} \quad F_{\text{ext}} = \boxed{3.75\text{ N}}$$

(d) $W = F\Delta r\cos\theta = (3.75\text{ N})(0.750\text{ m})\cos 0° = \boxed{2.81\text{ J}}$

(e) $\dfrac{dK}{dt} = \dfrac{d\left(\frac{1}{2}mv^2\right)}{dt} = \dfrac{1}{2}v^2\dfrac{dm}{dt} = \dfrac{1}{2}(0.750\text{ m/s})^2(5.00\text{ kg/s}) = \boxed{1.41\text{ J/s}}$



(f) $\boxed{\text{One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.}}$

P9.53 The x component of momentum for the system of the two objects is

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$-mv_i + 3mv_i = 0 + 3mv_{2x}$$

The y component of momentum of the system is

$$0 + 0 = -mv_{1y} + 3mv_{2y}$$

By conservation of energy of the system,

$$+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$$

we have $v_{2x} = \dfrac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes

$$4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $$v_{2y} = \frac{\sqrt{2}v_i}{3}$$

(a) The object of mass m has final speed

$$v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$$

and the object of mass $3m$ moves at

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $$\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$$

P9.54 Depending on the length of the cord and the time interval Δt for which the force is applied, the sphere may have moved very little when the force is removed, or we may have x_1 and x_2 nearly equal, or the sphere may have swung back, or it may have swung back and forth several times. Our solution applies equally to all of these cases.

(a) The applied force is constant, so the center of mass of the glider-sphere system moves with constant acceleration. It starts, we define, from $x = 0$ and moves to $(x_1 + x_2)/2$. Let v_1 and v_2 represent the horizontal components of velocity of glider and sphere at the moment the force stops. Then the velocity of the center of mass is $v_{CM} = (v_1 + v_2)/2$, and because the acceleration is constant we have

$$\frac{x_1+x_2}{2}=\left(\frac{v_1+v_2}{2}\right)\left(\frac{\Delta t}{2}\right)$$

which gives

$$\Delta t = 2\left(\frac{x_1+x_2}{v_1+v_2}\right)$$

The impulse-momentum theorem for the glider-sphere system is

$$F\Delta t = mv_1 + mv_2$$

or

$$2F\left(\frac{x_1+x_2}{v_1+v_2}\right)=m\left(v_1+v_2\right)$$
$$2F\left(x_1+x_2\right)=m\left(v_1+v_2\right)^2$$

Dividing both sides by 4m and rearranging gives

$$\frac{2F\left(x_1+x_2\right)}{4m}=\frac{m\left(v_1+v_2\right)^2}{4m}$$
$$\frac{F\left(x_1+x_2\right)}{2m}=\frac{\left(v_1+v_2\right)^2}{4}=v_{\text{CM}}^2$$

or

$$v_{\text{CM}}=\boxed{\sqrt{\frac{F\left(x_1+x_2\right)}{2m}}}$$

(b) The applied force does work that becomes, after the force is removed, kinetic energy of the constant-velocity center-of-mass motion plus kinetic energy of the vibration of the glider and sphere relative to their center of mass. The applied force acts only on the glider, so the work-energy theorem for the pushing process is

$$Fx_1 = \frac{1}{2}(2m)v_{\text{CM}}^2 + E_{\text{vib}}$$

Substitution gives

$$Fx_1 = \frac{1}{2}(2m)\left[\frac{F(x_1 + x_2)}{2m}\right] + E_{\text{vib}} = \frac{1}{2}Fx_1 + \frac{1}{2}Fx_2 + E_{\text{vib}}$$

Then,

$$E_{\text{vib}} = \frac{1}{2}Fx_1 - \frac{1}{2}Fx_2$$

When the cord makes its largest angle with the vertical, the vibrational motion is turning around. No kinetic energy is associated with the vibration at this moment, but only gravitational energy:

$$mgL(1 - \cos\theta) = F(x_1 - x_2)/2$$

Solving gives

$$\boxed{\theta = \cos^{-1}[1 - F(x_1 - x_2)/2mgL]}$$

ANSWERS TO QUICK-QUIZZES

1. (d)

2. (b), (c), (a)

3. (i) (c), (e) (ii) (b), (d)

4. (a) All three are the same. (b) dashboard, seat belt, air bag

5. (a)

6. (b)

7. (b)

8. (i) (a) (ii) (b)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P9.2 (a) $p_x = 9.00 \text{ kg}\cdot\text{m/s}$, $p_y = -12.0 \text{ kg}\cdot\text{m/s}$; (b) 15.0 kg · m/s

P9.4 (a) 4.71 m/s East; (b) 717 J

P9.6 10^{-23} m/s

P9.8 (a) 3.22×10^3 N, 720 lb; (b) not valid; (c) These devices are essential for the safety of small children.

P9.10 (a) 12.0 i cap N . s (b) 4.80 I cap m/s (c) 2.80 I cap m/s (d) 2.40 i capN

P9.12 (a) 20.9 m/s East; (b) −8.68 × 103 J; (c) Most of the energy was transformed to internal energy with some being carried away by sound.

P9.14 (a) 2.50 m/s; (b) 37.5 kJ; (c) The event considered in this problem is the time reversal of the perfectly inelastic collision in Problem 9.15. The same momentum conservation equation describes both processes.

P9.16 7.94 cm

P9.18 (a) 2.24 m/s toward the right; (b) No. Coupling order makes no difference to the final velocity.

P9.20 $v_{\mathrm{O}} = 3.99\ \mathrm{m/s}$ and $v_{\mathrm{Y}} = 3.01\ \mathrm{m/s}$

P9.22 The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction; (b) $\theta = 32.3°$, 2.88 m/s; (c) 786 J into internal energy

P9.24 11.7 cm; 13.3 cm

P9.26 (a) 15.9 g (b) 0.153 m

P9.28 (a) $\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right)\mathrm{cm}$; (b) $\left(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}}\right)\mathrm{g \cdot cm/s}$;

(c) $\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)\mathrm{cm/s}$; (d) $\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\mathrm{cm/s^2}$;

(e) $\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\mu\mathrm{N}$

P9.30 (a) Yes. $18.0\hat{\mathbf{i}}\ \mathrm{kg \cdot m/s}$; (b) No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work; (c) Yes, we could say that the final momentum of the card came from the floor or from the Earth through the floor; (d) No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount 27.0 J; (e) Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.

P9.32 15.0 N in the direction of the initial velocity of the exiting water stream.

P9.34 (a) $-v_e \ln\left(1-\dfrac{t}{T_p}\right)$; (b) See ANS. FIG. P9.34(b); (c) $\dfrac{v_e}{T_p - t}$; (d) See ANS. FIG. P9.34(d); (e) $v_e\left(T_p - t\right)\ln\left(1-\dfrac{t}{T_p}\right) + v_e t$; (f) See ANS. FIG. P9.34(f)

P9.36 (a) $K_E/K_A = m_1/\left(m_1 + m_2\right)$; (b) 1.00; (c) See P9.68(c) for argument.

P9.38 (a) –3.54 m/s; (b) 1.77 m; (c) 3.54×10^4 N; (d) No

P9.40 (a) See P9.40(a) for description; (b) $v_i = \dfrac{m+M}{m}\sqrt{2gh}$

P9.42 (a) See P9.42 for complete statement; (b) The final velocity of the seat is $-0.055\,6\,\hat{\mathbf{i}}$ m/s. That of the sleigh is $7.94\,\hat{\mathbf{i}}$ m/s; (c) −453 J

P9.44 In order for his motion to reverse under these conditions, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.

P9.46 $\left(\dfrac{M+m}{m}\right)\sqrt{\dfrac{gd^2}{2h}}$

P9.48 (a) $2.58\times10^3\ \text{kg}\cdot\text{m}/(80\ \text{kg}+m)$; (b) 32.2 m; (c) $m\rightarrow0$; (d) See P9.48(d) for complete answer; (e) See P9.48(e) for complete answer.

P9.50 0.179 m/s

P9.52 (a) 3.75 N; (b) 3.75 N; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J/s; (f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.54 (a) $v_{CM}=\sqrt{\frac{F}{2m}(x_1+x_2)}$

(b) $\theta=\cos^{-1}\left[1-\frac{F}{2mgL}(x_1+x_2)\right]$