32

Alternating-Current Circuits

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK – PAIR – SHARE AND ACTIVITIES

***TP32.1** **Conceptualize** Be sure you are clear on the resonant behavior of an *RLC* circuit as discussed in Section 32.7. Equation 32.38 shows the rms current in the circuit, which, of course, maximizes at the same frequency for all three elements. The voltage across each element, however, depends on both the current and the reactance, so it maximizes at a different frequency for each element.

Categorize The circuit in question is an *RLC* circuit, so we can use the material we studied in Sections 32.5 and 32.7.

Analyze (a) Take the ratio of the magnitude of the output voltage across the inductor, using Equation 32.15, to the magnitude of the voltage across the capacitor, using Equation 32.23:

$$\frac{|\Delta v_L|}{|\Delta v_C|} = \frac{I_{\max} X_L}{I_{\max} X_C} = \frac{X_L}{X_C} = \frac{\omega L}{\left(\dfrac{1}{\omega C}\right)} = \omega^2 LC$$

The question asks about the relationship between the voltages at resonance, so substitute the frequency at resonance from Equation 32.39:

$$\frac{|\Delta v_L|}{|\Delta v_C|} = \omega_0{}^2 LC = \left(\frac{1}{\sqrt{LC}}\right)^2 LC = 1$$

(b) Take the ratio of the magnitude of the output voltage across the resistor, using Equation 32.5, to the magnitude of the voltage across the inductor, using Equation 32.15:

$$\frac{|\Delta v_R|}{|\Delta v_L|} = \frac{I_{\max} R}{I_{\max} X_L} = \frac{R}{\omega_0 L} = \frac{R}{L}\sqrt{LC} = R\sqrt{\frac{C}{L}} \qquad (1)$$

This ratio is either larger than 1 or smaller than 1, depending on the particular values of *R*, *L*, and *C*:

$$R > \sqrt{\frac{L}{C}} \quad \rightarrow \quad \frac{|\Delta v_R|}{|\Delta v_L|} > 1 \qquad (2)$$

$$R < \sqrt{\frac{L}{C}} \quad \rightarrow \quad \frac{|\Delta v_R|}{|\Delta v_L|} < 1 \qquad (3)$$

Because the output voltage across the capacitor has the same magnitude as that across the inductor at resonance, the same conditions (2) and (3) hold if Δv_L is replaced with Δv_C. Finally, all three output voltages have the same magnitude at resonance when

$$R = \sqrt{\frac{L}{C}} \qquad (4)$$

(c) From Equation 32.38, the current in the circuit at any frequency is

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} \qquad (5)$$

From Equation 32.15, the magnitude of the voltage across the inductor is

$$\left|\Delta v_L\right|_{\max} = I_{\max} X_L = \frac{\Delta V_{\max} X_L}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{\Delta V_{\max}\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad (6)$$

We can simplify this expression a little by multiplying both numerator and denominator by ωC:

$$\left|\Delta v_L\right|_{\max} = \frac{\Delta V_{\max}\omega^2 LC}{\sqrt{R^2C^2\omega^2 + \left(\omega^2 LC - 1\right)^2}} \qquad (7)$$

Differentiate Equation (7) and set the result equal to zero to find the frequency at which the magnitude of the voltage across the inductor maximizes:

$$\frac{d\left|\Delta v_L\right|_{\max}}{d\omega}=\frac{d}{d\omega}\left[\frac{\Delta V_{\max}\omega^2 LC}{\sqrt{R^2C^2\omega^2+\left(\omega^2 LC-1\right)^2}}\right]=0$$

$$\rightarrow \quad \omega_{L,\max}=\sqrt{\frac{\frac{2}{C^2}}{2\frac{L}{C}-R^2}} \qquad (8)$$

From Equation 32.23, the magnitude of the voltage across the capacitor is

$$\left|\Delta v_C\right|_{\max}=I_{\max}X_C=\frac{\Delta V_{\max}X_C}{\sqrt{R^2+\left(X_L-X_C\right)^2}}=\frac{\Delta V_{\max}\left(\frac{1}{\omega C}\right)}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} \qquad (9)$$

We can simplify this expression a little by multiplying both numerator and denominator by ωC:

$$\left|\Delta v_C\right|_{\max}=\frac{\Delta V_{\max}}{\sqrt{R^2C^2\omega^2+\left(\omega^2 LC-1\right)^2}} \qquad (10)$$

Differentiate Equation (10) and set the result equal to zero to find the frequency at which the magnitude of the voltage across the capacitor maximizes:

$$\frac{d\left|\Delta v_C\right|_{\max}}{d\omega}=\frac{d}{d\omega}\left[\frac{\Delta V_{\max}}{\sqrt{R^2C^2\omega^2+\left(\omega^2 LC-1\right)^2}}\right]=0$$

$$\rightarrow \quad \omega_{C,\max}=\sqrt{\frac{\frac{2L}{C}-R^2}{2L^2}} \qquad (11)$$

Now divide Equation (8) by Equation (11):

$$\frac{\omega_{L,\max}}{\omega_{C,\max}} = \frac{\sqrt{\dfrac{\dfrac{2}{C^2}}{2\dfrac{L}{C}-R^2}}}{\sqrt{\dfrac{\dfrac{2L}{C}-R^2}{2L^2}}} = \sqrt{\frac{4\dfrac{L^2}{C^2}}{\left(\dfrac{2L}{C}-R^2\right)^2}} = \sqrt{\frac{4L^2}{\left(2L-R^2C\right)^2}} \qquad (12)$$

Substitute the resistance from Equation (4) in part (b) into Equation (12):

$$\frac{\omega_{L,\max}}{\omega_{C,\max}} = \sqrt{\frac{4L^2}{\left[2L-\left(\sqrt{\dfrac{L}{C}}\right)^2 C\right]^2}} = \sqrt{\frac{4L^2}{\left[2L-\left(\sqrt{\dfrac{L}{C}}\right)^2 C\right]^2}} = 2$$

Finalize Perhaps it is satisfying to see somewhat complicated expressions reducing to just 2 at the end of part (c)!]

Answer: (b) $R = \sqrt{\dfrac{L}{C}}$

***TP32.2** (a) At high frequencies, the capacitor acts as if it is almost a continuous wire, because the reactance in Equation 32.21 is very low, so the voltage across it is small. Therefore, the voltage across the resistor will be equal to the input voltage. (b) At low frequencies, the capacitor acts as if it is almost a break in the wire, because the reactance in Equation 32.21 is high, so the voltage across it is equal to the input voltage.

Possibility (i) is called a *high-pass filter*. The output voltage is low for low frequencies and rises to the input voltage as the frequency becomes very high. Figure ANS TP32.2a shows the behavior of the ratio of the output voltage to the input voltage as a function of frequency.

[Fig. ANS TP32.2a = New (see submitted file "9781337553278_Serway_PSE10e_Ch32_Figure_ANS_TP32.2a.jpg")]

Possibility (ii) is called a *low-pass filter*. The output voltage is equal to the input voltage for very low frequencies and approaches zero as the frequency becomes very high. Figure ANS TP32.2b shows the behavior of the ratio of the output voltage to the input voltage as a function of frequency.

[Fig. ANS TP32.2b = New (see submitted file "9781337553278_Serway_PSE10e_Ch32_Figure_ANS_TP32.2b.jpg")]

An *RLC* circuit acts as a crude *band-pass filter,* with large output voltage near the resonance frequency. Band-pass filters with a more "flat" response can be designed with combinations of *RLC* circuits.

Answers: (a) (i) (b) (ii)

***TP32.3** **Conceptualize** Remind yourself of the details of connecting elements in parallel to the voltage source. Since all elements are connected directly to the source, they all must have the same voltages across them.

Categorize We didn't study the parallel *RLC* circuit in the chapter, but we can apply the material studied in Sections 32.5 and 32.7 to this new circuit.

Analyze (a) The current in each element must be equal to the voltage across the element divided by its impedance. As with any parallel circuit, the current from the voltage source must equal the sum of the currents in the individual elements. Because of the phasor relationship shown in Figure TP32.3b, we must have

$$I_{\max} = \left[I_R^2 + \left(I_C - I_L\right)^2\right]^{1/2} = \left[\left(\frac{\Delta V_{\max}}{R}\right)^2 + \left(\frac{\Delta V_{\max}}{\frac{1}{\omega C}} - \frac{\Delta V_{\max}}{\omega L}\right)^2\right]^{1/2}$$

$$= \Delta V_{\max}\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2} \qquad (1)$$

Using Equations 32.6 and 32.8, we find

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{\Delta V_{\max}}{\sqrt{2}}\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2} = \Delta V_{\text{rms}}\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}$$

(b) From the phasor diagram, we see that

$$\tan\phi = \frac{I_C - I_L}{I_R} = \frac{\frac{\Delta V_{\text{rms}}}{X_C} - \frac{\Delta V_{\text{rms}}}{X_L}}{\frac{\Delta V_{\text{rms}}}{R}} = R\left(\frac{1}{X_C} - \frac{1}{X_L}\right) = R\left(\omega C - \frac{1}{\omega L}\right)$$

(c) At resonance $X_C = X_L$, so Equation (1) becomes

$$I_{\text{rms}} = \Delta V_{\text{rms}}\left[\frac{1}{R^2} + (0)^2\right]^{1/2} = \frac{\Delta V_{\text{rms}}}{R}$$

This is the minimum value because any nonzero value of $X_C - X_L$ increases the value of I_{rms}.

(d) In general, the rms current in a circuit is related to the rms voltage and impedance of the circuit according to an rms version of Equation 32.29:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} \qquad (2)$$

Comparing Equations (1) and (2), we see that

$$Z = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{-1/2} \qquad (3)$$

Finalize Notice that some of the results for the parallel *RLC* circuit look somewhat similar to those for the series circuit, but there are some clear differences. Equation (3) can be written as

$$Z = \left[\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L} \right)^2 \right]^{-1/2}$$

Compare this expression for the impedance for the parallel *RLC* circuit to that for the series *RLC* circuit in Equation 32.28. Another major difference is that the parallel circuit is an *antiresonator,* as seen in part (c): the current reaches a *minimum* at resonance.

Answers: (d) $Z = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{-1/2}$

***TP32.4** **Conceptualize** Notice that the current is not varying sinusoidally, as were the currents in the discussions in this chapter. Notice also that we are given the *current,* and we want to find the *voltage* in part (b).

Categorize The circuit is a simple one formed from a time-varying voltage source and an inductor. Note that the source voltage is periodic but not sinusoidal.

Analyze (a) The expressions for $i(t)$ in each of the two segments of every second are recognized as linear functions. Therefore, we graph them as lines with the slopes and intercepts given in the expression:

[Fig. ANS TP32.4a (see submitted file "9781337553278_Serway_PSE10e_Ch32_Figure_ANS_TP32.4a.jpg")]

(b) From Equation 31.1, we have the voltage across the inductor:

$$\Delta v_L = -L\frac{di}{dt} \qquad (1)$$

Therefore, the voltage across the inductor is proportional to the time derivative of the current. Because the current varies linearly with time, the derivative of the current is just the constant slope of the current graph. Therefore, for each second,

$$\Delta v_L(t) = \begin{cases} -\left(3.5\times10^{-3}\text{ H}\right)\left(6.00\times10^{-3}\text{ A}\right) & 0 < t < 0.600\text{ s} \\ -\left(3.5\times10^{-3}\text{ H}\right)\left(-9.00\times10^{-3}\text{ A}\right) & 0.600\text{ s} < t < 1.00\text{ s} \end{cases}$$

Carrying out the multiplications gives us

$$\Delta v_L(t) = \begin{cases} -2.10\times10^{-5}\text{ V} = -21.0\ \mu V & 0 < t < 0.600\text{ s} \\ \ \ 3.15\times10^{-5}\text{ V} = \ \ 31.5\ \mu V & 0.600\text{ s} < t < 1.00\text{ s} \end{cases}$$

Graphing this function gives us

[Fig. ANS TP32.4b (see submitted file "9781337553278_Serway_PSE10e_Ch32_Figure_ANS_TP32.4b.jpg")]

Finalize This turned out to be a relatively simple exercise in applying the equation for the voltage across an inductor to a nonsinusoidal situation.

Answers: (a) **[see submitted file "9781337553278_Serway_PSE10e_Ch32_Figure_ANS_TP32.4a.jpg"]**

(b) **[see submitted file "9781337553278_Serway_PSE10e_Ch32_Figure_ANS_TP32.4b.jpg"**

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 32.2 Resistors in an AC Circuit

P32.1 The rms voltage is

$$\Delta V_{\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$$

(a) $P = \frac{(\Delta V_{\text{rms}})^2}{R} \quad \rightarrow \quad R = \frac{(120\text{ V})^2}{75.0\text{ W}} = \boxed{193\ \Omega}$

(b) $R = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$

P32.2 (a) We compute the peak voltage from the rms voltage:

$$\Delta V_{R,\max} = \sqrt{2}\left(\Delta V_{R,\text{rms}}\right) = \sqrt{2}\left(120\text{ V}\right) = \boxed{170\text{ V}}$$

(b) From the definition of power,

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \frac{\Delta V_{\text{rms}}^2}{R}$$

Solving for the resistance,

$$R = \frac{\Delta V_{\text{rms}}^2}{P_{\text{avg}}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = \boxed{2.40 \times 10^2\ \Omega}$$

(c) Because $P_{\text{avg}} = \frac{(\Delta V_{\text{rms}})^2}{R} \quad \rightarrow \quad R = \frac{(\Delta V_{\text{rms}})^2}{P_{\text{avg}}}$, a 100-W bulb has less resistance than a 60.0-W bulb.

P32.3 The current as a function of time is $i = \frac{\Delta v}{R} = \left(\frac{\Delta V_{\max}}{R}\right)\sin\omega t$. Given the value of t, we want to identify a point with

$$0.600\frac{\Delta V_{\max}}{R}=\frac{\Delta V_{\max}}{R}\sin(\omega t)$$

or $\omega t = \sin^{-1} 0.600$

To find the lowest frequency we choose the smallest angle satisfying this relation:

$$(0.007\ 00\ \text{s})\omega = \sin^{-1}(0.600) = 0.644\ \text{rad}$$

Thus, $\omega = 91.9\ \text{rad/s} = 2\pi f$ so $\boxed{f = 14.6\ \text{Hz}}$

P32.4 All lamps are connected in parallel with the voltage source, so $\Delta V_{\text{rms}} = 120\ \text{V}$ for each lamp. Also, the current is $I_{\text{rms}} = P_{\text{avg}}/\Delta V_{\text{rms}}$ and the resistance is $R = \Delta V_{\text{rms}}/I_{\text{rms}}$.

(a) For the 150-W bulbs,

$$I_{\text{rms}} = \frac{150\ \text{W}}{120\ \text{V}} = 1.25\ \text{A}$$

For the 100-W bulb,

$$I_{\text{rms}} = \frac{100\ \text{W}}{120\ \text{V}} = 0.833\ \text{A}$$

$\boxed{\text{The rms current in each 150-W bulb is 1.25 A. The rms current in the 100-W bulb is 0.833 A.}}$

(b) The resistance in bulbs 1 and 2 is

$$R_1 = R_2 = \frac{120\ \text{V}}{1.25\ \text{A}} = \boxed{96.0\ \Omega}$$

and the resistance in bulb 3 is

$$R_3 = \frac{120\ \text{V}}{0.833\ \text{A}} = \boxed{144\ \Omega}$$

(c) The bulbs are in parallel, so

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{96.0\ \Omega} + \frac{1}{96.0\ \Omega} + \frac{1}{144\ \Omega}$$

$$R_{eq} = \boxed{36.0\ \Omega}$$

P32.5 (a) From Equation 32.5, $\Delta v_R = \Delta V_{max} \sin \omega t$. To find the angular frequency, we write

$$\Delta v_R = 0.250(\Delta V_{max})$$

so $\sin \omega t = 0.250$

or $\omega t = \sin^{-1}(0.250)$

The smallest angle for which this is true is $\omega t = 0.253$ rad. Thus, if $t = 0.010\,0$ s,

$$\omega = \frac{0.253\ \text{rad}}{0.010\,0\ \text{s}} = \boxed{25.3\ \text{rad/s}}$$

(b) The second time when $\Delta v_R = 0.250(\Delta V_{max})$, $\omega t = \sin^{-1}(0.250)$ again. For this occurrence, $\omega t = \pi - 0.253\ \text{rad} = 2.89\ \text{rad}$ (to understand why this is true, recall the identity $\sin(\pi - \theta) = \sin\theta$ from trigonometry). Thus,

$$t = \frac{2.89\ \text{rad}}{25.3\ \text{rad/s}} = \boxed{0.114\ \text{s}}$$

Section 32.3 Inductors in an AC Circuit

P32.6 (a) $X_L = \omega L = \dfrac{\Delta V_{max}}{I_{max}}$

$$L = \frac{\Delta V_{\max}}{\omega I_{\max}} = \frac{100\text{ V}}{2\pi(50.0\text{ Hz})(7.50\text{ A})} = \boxed{0.0424\text{ H}}$$

(b) From $I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{\Delta V_{\max}}{\omega L}$, we see that is current inversely proportional to angular frequency:

$$\frac{I_{\max}}{I'_{\max}} = \frac{\omega'}{\omega}$$

$$\omega' = \omega\frac{I_{\max}}{I'_{\max}} = [2\pi(50.0\text{ Hz})]\frac{7.50\text{ A}}{2.50\text{ A}} = \boxed{942\text{ rad/s}}$$

P32.7 The inductive reactance is

$$X_L = \omega L = (65.0\ \pi\text{s}^{-1})(70.0 \times 10^{-3}\text{ V}\cdot\text{s/A}) = 14.3\ \Omega$$

The amplitude of the current is

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{80.0\text{ V}}{14.3\ \Omega} = 5.60\text{ A}$$

Equation 32.10 lets us evaluate the current:

$$i = -I_{\max}\cos\omega t = -(5.60\text{ A})\cos\left[(65.0\pi\text{ s}^{-1})(0.015\ 5\text{ s})\right]$$
$$= -(5.60\text{ A})\cos(3.17\text{ rad}) = \boxed{+5.60\text{ A}}$$

P32.8 In the inductor, because $U_B = \frac{1}{2}Li_L^2 = 0$ when $t = 0$, $i_L = I_{\max}\sin(\omega t)$.

Then,

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{\Delta V_{\text{rms}}}{\omega L} = \frac{120\text{ V}}{\left[2\pi(60.0)\text{ s}^{-1}\right](0.020\,0\text{ H})} = 15.9\text{ A}$$

and $I_{\max} = \sqrt{2}I_{\text{rms}} = \sqrt{2}(15.9\text{ A}) = 22.5\text{ A}$

the current in the inductor is

$$i_L = I_{\max} \sin \omega t = (22.5 \text{ A}) \sin\left[2\pi(60.0) \text{ s}^{-1} \cdot \left(\frac{1}{180} \text{ s}\right)\right]$$
$$= (22.5 \text{ A}) \sin 120° = 19.5 \text{ A}$$

and the energy stored is

$$U_B = \frac{1}{2} L i_L^2 = \frac{1}{2}(0.020\,0 \text{ H})(19.5 \text{ A})^2 = \boxed{3.80 \text{ J}}$$

P32.9 (a) $X_L = 2\pi f L = 2\pi(80.0\text{ Hz})(25.0\times10^{-3}\text{ H}) = \boxed{12.6\ \Omega}$

(b) $I_{\text{rms}} = \dfrac{\Delta V_{L,\text{rms}}}{X_L} = \dfrac{78.0\text{ V}}{X_L} = \boxed{6.21\text{ A}}$

(c) $I_{\max} = \sqrt{2}\,I_{\text{rms}} = \sqrt{2}(6.21\text{ A}) = \boxed{8.78\text{ A}}$

P32.10 The flux Φ_B through each turn of the inductor is related to the inductance by

$$L = \frac{N\Phi_B}{i}$$

Then, for an N-turn inductor,

$$N\Phi_{B,\max} = LI_{\max} = \frac{X_L}{\omega}\frac{(\Delta V_{L,\max})}{X_L} = \frac{(\Delta V_{L,\max})}{\omega}$$

$$N\Phi_{B,\max} = \frac{\sqrt{2}(\Delta V_{L,\text{rms}})}{2\pi f} = \frac{120\text{ V}}{\sqrt{2}\pi(60.0\text{ s}^{-1})} = \boxed{0.450\text{ Wb}}$$

Section 32.4 Capacitors in an AC Circuit

P32.11 Current leads voltage by 90° in a capacitor, and because charge is proportional to voltage, current leads charge by 90°. If $\Delta v_C = \Delta V_{\max}\sin\omega t$, then $q = C(\Delta V_{\max})\sin\omega t$ so that the stored energy is $U_C = \dfrac{q^2}{2C} = 0$ when $t = 0$. Therefore, the current is given by

$$i_C = I_{\max}\sin(\omega t + 90°) = \frac{\Delta V_{\max}}{X_C}\sin(\omega t + 90°)$$

The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0/\text{s})(1.00\times10^{-3}\ \text{C/V})} = 2.65\ \Omega$$

and the current at $t = \frac{1}{180}$ s is

$$\begin{aligned} i_C &= \frac{\Delta V_{\max}}{X_C}\sin(\omega t+\phi) \\ &= \frac{\sqrt{2}(120\ \text{V})}{2.65\ \Omega}\sin\left[2\pi(60.0\ \text{s}^{-1})\cdot\left(\frac{1}{180}\ \text{s}\right)+\frac{\pi}{2}\right] \\ &= -32.0\ \text{A} \end{aligned}$$

The magnitude of the current is $\boxed{32.0\ \text{A}}$.

P32.12 (a) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0\ \text{Hz})(12.0\times10^{-6}\ \text{F})} = \boxed{221\ \Omega}$

(b) $I_{\text{rms}} = \dfrac{\Delta V_{C,\text{rms}}}{X_C} = \dfrac{36.0\ \text{V}}{221\ \Omega} = \boxed{0.163\ \text{A}}$

(c) $I_{\max} = \sqrt{2}\,I_{\text{rms}} = \boxed{0.230\ \text{A}}$

(d) $\boxed{\text{No.}}$ Current leads voltage, and thus charge, by 90° in a capacitor. The current reaches its maximum value one-quarter cycle before the voltage reaches its maximum value. From the definition of capacitance, the capacitor reaches its maximum charge when the voltage across it is also a maximum. Consequently, the maximum charge and the maximum current do not occur at the same time.

P32.13 The maximum current in the capacitor is given by

$$I_{\max} = \sqrt{2}I_{\text{rms}} = \frac{\sqrt{2}(\Delta V_{\text{rms}})}{X_C} = \sqrt{2}(\Delta V_{\text{rms}})2\pi f C$$

(a) For the North American electrical outlet,

$$I_{\max} = \sqrt{2}(120\text{ V})2\pi(60.0/\text{s})(2.20\times 10^{-6}\text{ C/V}) = \boxed{141\text{ mA}}$$

(b) For the European electrical outlet,

$$I_{\max} = \sqrt{2}(240\text{ V})2\pi(50.0/\text{s})(2.20\times 10^{-6}\text{ F}) = \boxed{235\text{ mA}}$$

P32.14 The maximum charge is given by

$$Q_{\max} = C(\Delta V_{\max}) = C\left[\sqrt{2}(\Delta V_{\text{rms}})\right] = \boxed{\sqrt{2}C(\Delta V_{\text{rms}})}$$

Section 32.5 The *RLC* Series Circuit

P32.15 The resistance of the circuit is $R = 300\ \Omega$. The inductive reactance of the circuit is

$$X_L = \omega L = 2\pi\left(\frac{500}{\pi}\text{ s}^{-1}\right)(0.200\text{ H}) = 200\ \Omega$$

The capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C} = \left[2\pi\left(\frac{500}{\pi}\text{ s}^{-1}\right)(11.0\times 10^{-6}\text{ F})\right]^{-1} = 90.9\ \Omega$$

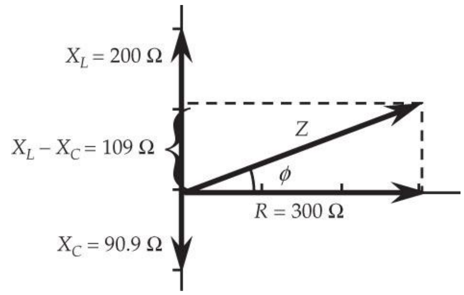
The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\ \Omega)^2 + (200\ \Omega - 90.0\ \Omega)^2} = 319\ \Omega$$

and

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{200\ \Omega - 90.9\ \Omega}{300\ \Omega}\right) = 20.0°$$

The phasor diagram is shown in ANS. FIG. P32.15.



ANS. FIG. P32.15

P32.16 We first determine the reactances of the circuit. The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)(65.0\times10^{-6}\ \text{F})} = 49.0\ \Omega$$

the inductive reactance is,

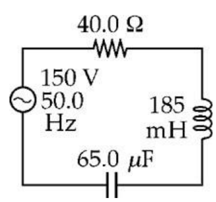
$$X_L = \omega L = 2\pi(50.0)(185\times10^{-3}\ \text{H}) = 58.1\ \Omega$$

and the impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(40.0\ \Omega)^2 + (58.1\ \Omega - 49.0\ \Omega)^2}$$
$$= 41.0\ \Omega$$

The current in the circuit is then

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{150\ \text{V}}{41.0\ \Omega} = 3.66\ \text{A}$$



ANS. FIG. P32.16

(a) The maximum voltage between points *a* and *b* is the potential drop across the resistor:

$$\Delta V_R = I_{\text{max}}R = (3.66\ \text{A})(40.0\ \Omega) = \boxed{146\ \text{V}}$$

(b) The maximum voltage between points *b* and *c* is the potential drop across the coil:

$$\Delta V_L = I_{\text{max}}X_L = (3.66\ \text{A})(58.1\ \Omega) = 212.5\ \text{V} = \boxed{212\ \text{V}}$$

(c) The maximum voltage between points *c* and *d* is the potential drop across the capacitor:

$$\Delta V_C = I_{\max} X_C = (3.66\ \text{A})(49.0\ \Omega) = 179.1\ \text{V} = \boxed{179\ \text{V}}$$

(d) The potential drop between points *b* and *d* is

$$\Delta V_L - \Delta V_C = 212.5\ \text{V} - 179.1\ \text{V} = \boxed{33.4\ \text{V}}$$

***P32.17** **Conceptualize** Model the existing factory as the circuit shown in Figure 32.13, with the capacitor replaced by a simple wire. Unmodified Figure 32.13 represents the circuit for the factory with the proposed capacitor installed.

Categorize The factory is categorized as an AC circuit, so we can use the techniques we learned in this chapter.

Analyze (a) The power factor is given in Equation 32.35 as

$$\cos\phi = \frac{R}{Z} \qquad (1)$$

Incorporate Equation 32.28 for the impedance of the circuit:

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad (2)$$

For the existing factory with no capacitive load, $X_C = 0$, and

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}} \qquad (3)$$

Incorporate Equation 32.13 for the inductive reactance:

$$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{R}{\sqrt{R^2 + (2\pi f L)^2}} \qquad (4)$$

Substitute numerical values for your demonstration circuit:

$$\cos\phi = \frac{20.0\ \Omega}{\sqrt{(20.0\ \Omega)^2 + \left[2\pi(60.0\ \text{Hz})(25.0\times10^{-3}\ \text{H})\right]^2}} = \boxed{0.905}$$

(b) Now, to make the power factor equal to 1.00, we need to have the impedance equal the resistance, which we can do by demanding that $X_C = X_L$:

$$\omega L = \frac{1}{\omega C} \quad \rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} \tag{5}$$

Substitute numerical values for your demonstration circuit:

$$C = \frac{1}{\left[2\pi(60.0\ \text{Hz})\right]^2 (25.0\times10^{-3}\ \text{H})} = 2.81\times10^{-4}\ \text{F} = \boxed{281\ \mu\text{F}}$$

(c) Finally, to convince the owners of the increased power to the factory, show them the ratio of the power with the capacitor to that without. Assuming the supply voltage stays the same, use Equation 32.34:

$$\frac{P_{\text{new}}}{P_{\text{old}}} = \frac{I_{\text{rms,new}}\Delta V_{\text{rms}}\cos\phi_{\text{new}}}{I_{\text{rms,old}}\Delta V_{\text{rms}}\cos\phi_{\text{old}}} = \frac{I_{\text{rms,new}}\cos\phi_{\text{new}}}{I_{\text{rms,old}}\cos\phi_{\text{old}}} \tag{6}$$

Use an rms version of Equation 32.29 for the currents:

$$\frac{P_{\text{new}}}{P_{\text{old}}} = \frac{\left(\dfrac{\Delta V_{\text{rms}}}{Z_{\text{new}}}\right)\cos\phi_{\text{new}}}{\left(\dfrac{\Delta V_{\text{rms}}}{Z_{\text{old}}}\right)\cos\phi_{\text{old}}} = \frac{Z_{\text{old}}\cos\phi_{\text{new}}}{Z_{\text{new}}\cos\phi_{\text{old}}} \tag{7}$$

Use Equation (1) to substitute for the power factors:

$$\frac{P_{\text{new}}}{P_{\text{old}}} = \frac{Z_{\text{old}}\left(\dfrac{R}{Z_{\text{new}}}\right)}{Z_{\text{new}}\left(\dfrac{R}{Z_{\text{old}}}\right)} = \left(\frac{Z_{\text{old}}}{Z_{\text{new}}}\right)^2 \tag{8}$$

Substitute for the impedances:

$$\frac{P_{\text{new}}}{P_{\text{old}}} = \frac{R^2 + (2\pi fL)^2}{R^2} = 1 + \left(\frac{2\pi fL}{R}\right)^2 \tag{9}$$

Substitute numerical values:

$$\frac{P_{\text{new}}}{P_{\text{old}}} = 1 + \left[\frac{2\pi(60.0\text{ Hz})(25.0\times10^{-3}\text{ H})}{20.0\ \Omega}\right]^2 = 1.22$$

Notice that the simple addition of the capacitance results in a $\boxed{22\%}$ increase in power delivered to the demonstration circuit.

Finalize Keep in mind that this is a demonstration circuit, and scaling up to a factory-sized circuit may entail many practical considerations that are not present in a small circuit.

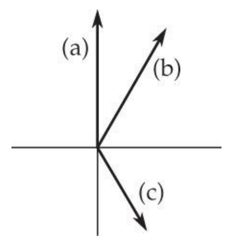
Answers: (a) 0.905 (b) 281 μF (c) 22%

P32.18 The Phasors for the three cases are shown in ANS. FIG. P32.18.

(a) 25.0 sin ωt at $\omega t = 90.0°$

(b) 30.0 sin ωt at $\omega t = 60.0°$

(c) 18.0 sin ωt at $\omega t = 300°$



ANS. FIG. P32.18

P32.19 The reactance of the inductor is

$$X_L = 2\pi f L = 2\pi\left(60.0\ \text{s}^{-1}\right)(0.460\ \text{H}) = 173\ \Omega$$

The reactance of the capacitor is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(60.0\ \text{s}^{-1}\right)\left(21.0 \times 10^{-6}\ \text{F}\right)} = 126\ \Omega$$

(a) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{173\ \Omega - 126\ \Omega}{150\ \Omega}\right) = \boxed{17.4°}$

(b) Since $X_L > X_C$, ϕ is positive, so $\boxed{\text{voltage leads the current}}$. This means that the power-supply or total voltage goes through each maximum, zero-crossing, and minimum earlier in time than the current does.

P32.20 (a) The capacitive reactance of the circuit is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \text{Hz})\left(30.0 \times 10^{-6}\ \text{F}\right)} = \boxed{88.4\ \Omega}$$

(b) The impedance of the circuit is

$$Z = \sqrt{R^2 + \left(0 - X_C\right)^2} = \sqrt{R^2 + X_C^2} = \sqrt{(60.0\ \Omega)^2 + (88.4\ \Omega)^2}$$
$$= \boxed{107\ \Omega}$$

(c) $I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{1.20 \times 10^2\ \text{V}}{107\ \Omega} = \boxed{1.12\ \text{A}}$

(d) The phase angle in this RC circuit is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 88.4\ \Omega}{60.0\ \Omega}\right) = -55.8°$$

Since $\phi < 0$, $\boxed{\text{the voltage lags behind the current by } 55.8°}$.

(e) Adding an inductor will change the impedance, and hence the current in the circuit. The current could be larger or smaller, depending on the inductance added. The largest current would result when the inductive reactance equals the capacitive reactance, the impedance has its minimum value, equal to 60.0 Ω, and the current in the circuit is

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{\Delta V_{\text{max}}}{R} = \frac{1.20 \times 10^2\ \text{V}}{60.0\ \Omega} = 2.00\ \text{A}$$

Section 32.6 Power in an AC Circuit

P32.21 From the definition of impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, we have

$$(X_L - X_C) = \sqrt{Z^2 - R^2}$$

Substituting numerical values,

$$(X_L - X_C) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

The phase angle of the circuit is then

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1°$$

The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\ \text{V}}{75.0\ \Omega} = 2.80\ \text{A}$$

Therefore, the power delivered to the circuit is

$$P = (\Delta V_{\text{rms}}) I_{\text{rms}} \cos\phi = (210 \text{ V})(2.80 \text{ A})\cos(53.1°) = \boxed{353 \text{ W}}$$

P32.22 The power factor for a series *RLC* circuit is given by

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The circuit in this problem has no capacitance, so the power factor becomes

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^{\,2}}}$$

In order for the power factor to be equal to 1.00, we would have to have $X_L = 0$, which would require either L or f to be zero. Because this is not the case, the situation is impossible.

P32.23 The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{160\ \text{V}}{80.0\ \Omega} = 2.00\ \text{A}$$

and the average power delivered to the circuit is

$$P_{\text{avg}} = I_{\text{rms}}^2 R = (2.00\ \text{A})^2(22.0\ \Omega) = \boxed{88.0\ \text{W}}$$

P32.24 Given $v = \Delta V_{\max}\sin(\omega t) = (90.0\ \text{V})\sin(350t)$, observe that $\Delta V_{\max} = 90.0\ \text{V}$ and $\omega = 350$ rad/s. Also, the net reactance is

$$X_L - X_C = 2\pi fL - \frac{1}{2\pi fC} = \omega L - \frac{1}{\omega C}$$

(a) To find the impedance, we first compute

$$\begin{aligned} X_L - X_C &= \omega L - \frac{1}{\omega C} \\ &= (350\ \text{rad/s})(0.200\ \text{H}) - \frac{1}{(350\ \text{rad/s})(25.0\times10^{-6}\ \text{F})} \\ &= -44.3\ \Omega \end{aligned}$$

so the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (-44.3\ \Omega)^2} = \boxed{66.8\ \Omega}$$

(b) The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\max}/\sqrt{2}}{Z} = \frac{90.0\ \text{V}}{\sqrt{2}(66.8\ \Omega)} = \boxed{0.953\ \text{A}}$$

(c) The phase difference between the applied voltage and the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{-44.3\ \Omega}{50.0\ \Omega}\right) = -41.5°$$

so the average power delivered to the circuit is

$$P_{\text{avg}} = I_{\text{rms}}\Delta V_{\text{rms}}\cos\phi = I_{\text{rms}}\left(\frac{\Delta V_{\max}}{\sqrt{2}}\right)\cos\phi$$
$$= (0.953\ \text{A})\left(\frac{90.0\ \text{V}}{\sqrt{2}}\right)\cos(-41.5°) = \boxed{45.4\ \text{W}}$$

Section 32.7 Resonance in a Series RLC Circuit

P32.25 (a) The resonance frequency of a *RLC* circuit is $f_0 = 1/2\pi\sqrt{LC}$. Thus, the inductance is

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2(9.00\times10^9\ \text{Hz})^2(2.00\times10^{-12}\ \text{F})}$$
$$= 1.56\times10^{-10}\ \text{H} = \boxed{156\ \text{pH}}$$

(b) At resonance,

$$X_L = X_C = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi\left(9.00\times10^{9}\ \text{Hz}\right)\left(2.00\times10^{-12}\ \text{F}\right)}$$
$$= \boxed{8.84\ \Omega}$$

P32.26 We are given $L = 0.020\,0$ H, $C = 100 \times 10^{-9}$ F, $R = 20.0\ \Omega$, and $\Delta V_{\max} = 100$ V.

(a) The resonant frequency for a series *RLC* circuit is

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \boxed{3.56\text{ kHz}}$$

(b) At resonance,

$$I_{\max} = \frac{\Delta V_{\max}}{R} = \boxed{5.00\text{ A}}$$

(c) From Equation 32.43,

$$Q = \frac{\omega_0 L}{R} = \boxed{22.4}$$

(d) At resonance, the amplitude of the voltage across the inductor is

$$\Delta V_{L,\,\max} = X_L I_{\max} = \omega_0 L I_{\max} = \boxed{2.24\text{ kV}}$$

***P32.27** **Conceptualize** We can connect the capacitors in series or parallel. We need to determine which type of connection we need. It doesn't matter how you connect the resistors, because the resonance frequency doesn't depend on the resistance. The resistance will, however, determine the maximum current in the circuit at resonance as well as the Q factor.

Categorize We categorize the problem as a simple substitution problem.

The resonance frequency is given by Equation 32.39:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

From this equation, we see that, to minimize the resonance frequency, you want to maximize the capacitance. Therefore, you should connect the capacitors in a parallel combination and attach that combination in series with the resistors and the inductor. Capacitors in parallel add, so the resonance frequency will be

$$\omega_0 = \frac{1}{\sqrt{L(2C)}}$$

where C is the capacitance of each individual capacitor that you have. Therefore, substituting numerical values,

$$\omega_0 = \frac{1}{\sqrt{2(5.00 \times 10^{-3}\ \text{H})(5.00 \times 10^{-9}\ \text{F})}} = \boxed{1.41 \times 10^5\ \text{rad/ s}}$$

To maximize the current, the resistors should be connected in parallel to provide the lowest possible equivalent resistance.

Answer: 1.41×10^5 rad/s

P32.28 The resonance frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$, then

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$$

and $$X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)}$$

so the rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$$

The power delivered to the circuit is

$$P = (I_{\text{rms}})^2 R = \left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}$$

and the energy delivered in one period is $E = P\Delta t$:

$$E = P\Delta t = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right)$$

$$= \frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C + 9L}$$

Substituting numerical values,

$$E = \frac{4\pi(50.0\text{ V})^2(10.0\ \Omega)(100\times10^{-6}\text{ F})\left[(10.0\times10^{-3}\text{ H})(100\times10^{-6}\text{ F})\right]^{1/2}}{4(10.0\ \Omega)^2(100\times10^{-6}\text{ F}) + 9(10.0\times10^{-3}\text{ H})}$$

$$= \boxed{242\text{ mJ}}$$

P32.29 The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$, then

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$$

and $$X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)}$$

so the rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$$

The power delivered to the circuit is

$$P = (I_{\text{rms}})^2 R = \left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}$$

and the energy delivered in one period is $E = P\Delta t$:

$$E = P\Delta t = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right)$$

$$= \boxed{\frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C + 9L}}$$

Section 32.8 The Transformer and Power Transmission

P32.30 The rms primary voltage is

$$\Delta V_{1,\text{rms}} = \frac{170\ \text{V}}{\sqrt{2}} = 120\ \text{V}$$

The rms voltage across the bigger coil is

$$\Delta V_{2,\text{rms}} = \left(\frac{N_2}{N_1}\right)\Delta V_{1,\text{rms}} = \left(\frac{2\,000}{350}\right)(120\ \text{V}) = \boxed{687\ \text{V}}$$

P32.31 The capacitive reactance of this “circuit” is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \text{Hz})(20.0\times10^{-12}\ \text{F})} = 1.33\times10^{8}\ \Omega$$

and the impedance is

$$Z = \sqrt{\left(50.0\times10^{3}\ \Omega\right)^{2} + \left(1.33\times10^{8}\ \Omega\right)^{2}} \approx 1.33\times10^{8}\ \Omega$$

The rms current is then

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{5\,000\ \text{V}}{1.33\times10^{8}\ \Omega} = 3.77\times10^{-5}\ \text{A}$$

and the rms voltage across the person's body is

$$\left(\Delta V_{\text{rms}}\right)_{\text{body}} = I_{\text{rms}}R_{\text{body}} = \left(3.77\times10^{-5}\ \text{A}\right)\left(50.0\times10^{3}\ \Omega\right)$$
$$= \boxed{1.88\ \text{V}}$$

P32.32 (a) The total resistance of the transmission line is

$$R = \left(4.50\times10^{-4}\ \Omega/\text{m}\right)\left(6.44\times10^{5}\ \text{m}\right) = 290\ \Omega$$

and the rms current in the line is

$$I_{\text{rms}} = \frac{P}{\Delta V_{\text{rms}}} = \frac{5.00\times10^{6}\ \text{W}}{5.00\times10^{5}\ \text{V}} = 10.0\ \text{A}$$

The power loss during transmission is

$$P_{\text{loss}} = I_{\text{rms}}^{2}R = (10.0\ \text{A})^{2}(290\ \Omega) = \boxed{29.0\ \text{kW}}$$

(b) The fraction of input power lost is

$$\frac{P_{\text{loss}}}{P} = \frac{2.90\times10^{4}\ \text{W}}{5.00\times10^{6}\ \text{W}} = \boxed{5.80\times10^{-3}}$$

(c) $\boxed{\text{It is impossible to transmit so much power at such low voltage.}}$

Maximum power transfer occurs when load resistance equals the line resistance of 290 Ω, and is $\dfrac{\left(4.50\times10^{3}\ \text{V}\right)^{2}}{2\cdot2(290\ \Omega)} = 17.5\ \text{kW}$, far below the required 5 000 kW.

Additional Problems

P32.33 From Equation 32.39, the resonance frequency for this circuit is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi\sqrt{\left(2.80 \times 10^{-6}\ \text{H}\right)\left(0.910 \times 10^{-12}\ \text{F}\right)}} = 99.7\ \text{MHz}$$

This frequency is not in the range of North American AM frequencies, which can be found from Internet research to be 520 kHz – 1 610 kHz. The frequency above is appropriate for an North American FM radio station.

P32.34 (a) The impedance is given by

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

From which we obtain

$$X_C = X_L \pm \sqrt{Z^2 - R^2}$$
$$X_C = X_L + \sqrt{Z^2 - R^2}$$
$$= 700\ \Omega + \sqrt{(760\ \Omega)^2 - (400\ \Omega)^2} = 1\ 346\ \Omega = 1.35\ \text{k}\Omega$$

or

$$X_C = X_L - \sqrt{Z^2 - R^2}$$
$$= 700\ \Omega - \sqrt{(760\ \Omega)^2 - (400\ \Omega)^2} = 53.8\ \Omega$$

$$\boxed{X_C \text{ could be } 53.8\ \Omega \text{ or it could be } 1.35\ \text{k}\Omega.}$$

(b) The power delivered to the circuit is given by

$$P = \left(I_{\text{rms}}\right)^2 R = \frac{\left(\Delta V_{\text{rms}}\right)^2}{Z^2} R$$

If the power is decreased as the frequency is raised, then the impedance is increased, so the inductive reactance is greater than the capacitive reactance, and the circuit must be above resonance:

$$X_L > X_C \quad \rightarrow \quad \omega L > 1/\omega C \quad \rightarrow \quad \omega > 1\big/\sqrt{LC} \quad \rightarrow \quad \omega > \omega_0$$

Therefore, the inductive reactance 700 Ω and the

$\boxed{\text{capacitive reactance is } 53.8\ \Omega.}$

(c) Now,

$$X_C = X_L \pm \sqrt{Z^2 - R^2} = 700 \pm \sqrt{(760)^2 - (200)^2} = 700 \pm 733$$

Here $X_C = 700 - 733 = -33\ \Omega$ is impossible, but

$X_C = 700 + 733 = 1433 = 1.43\ \text{k}\Omega$ is possible.

$\boxed{X_C \text{ must be } 1.43\ \text{k}\Omega.}$

P32.35 Consider a two-wire transmission line: each wire has resistance R, the total power transmitted is P, and the current in the wires is $I_{\text{rms}} = \dfrac{P}{\Delta V_{\text{rms}}}$. The power loss is 1.00% of the transmitted power P.

Therefore,

$$P_{\text{loss}} = I_{\text{rms}}^2 R_{\text{line}} = \frac{P}{100}$$

$$P_{\text{loss}} = \left(\frac{P}{\Delta V_{\text{rms}}}\right)^2 (2R) = \frac{P}{100}$$

Solving for the resistance gives

$$R = \frac{(\Delta V_{\text{rms}})^2}{200P}$$

The resistance of one wire is

$$R = \frac{\rho_{\text{Cu}}\ell}{A} = \frac{(\Delta V_{\text{rms}})^2}{200P}$$

Solving for the area gives

$$A = \frac{\pi d^2}{4} = \frac{200\rho_{\text{Cu}}P\ell}{(\Delta V_{\text{rms}})^2}$$

and the diameter is

$$\begin{aligned} d &= \sqrt{\frac{800\rho_{\text{Cu}}P\ell}{\pi(\Delta V_{\text{rms}})^2}} \\ &= \sqrt{\frac{800(1.7\times10^{-8}\ \Omega\cdot\text{m})\ (20\ 000\ \text{W})(18\ 000\ \text{m})}{\pi\left(1.50\times10^{3}\ \text{V}\right)^2}} \\ &= 0.026\ \text{m} = \boxed{2.6\ \text{cm}} \end{aligned}$$

P32.36 Consider a two-wire transmission line: each wire has resistance R, the total power transmitted is P, and the current in the wires is $I_{\text{rms}} = \dfrac{P}{\Delta V_{\text{rms}}}$. The fractional power loss is f of the transmitted power P. Therefore,

$$P_{\text{loss}} = I_{\text{rms}}^2 R_{\text{line}} = fP$$

$$P_{\text{loss}} = \left(\frac{P}{\Delta V_{\text{rms}}}\right)^2 (2R) = fP \quad \rightarrow \quad R = \frac{f(\Delta V_{\text{rms}})^2}{2P}$$

The resistance of one wire is

$$R = \frac{\rho_{\text{Cu}}\ell}{A} = \frac{(\Delta V_{\text{rms}})^2}{200P}$$

Solving for the area gives

$$A = \frac{\pi d^2}{4} = \frac{200\rho_{\text{Cu}}P\ell}{(\Delta V_{\text{rms}})^2}$$

and the diameter is

$$d = \boxed{\sqrt{\frac{8\rho_{\text{Cu}}\,\ell P}{\pi f\left(\Delta V_{\text{rms}}\right)^2}}}$$

P32.37 The turns ratio is the factor of change in voltage:

$$\frac{N_1}{N_2} = \frac{\Delta V_1}{\Delta V_2}$$

with $Z_1 = \dfrac{\Delta V_1}{I_1}$ and $Z_2 = \dfrac{\Delta V_2}{I_2}$

we have $\dfrac{N_1}{N_2} = \dfrac{Z_1 I_1}{Z_2 I_2}$

(a) Since $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$, we find

$$\frac{N_1}{N_2} = \frac{Z_1 N_2}{Z_2 N_1} \quad \text{so} \quad \frac{N_1^2}{N_2^2} = \frac{Z_1}{Z_2} \quad \text{and} \quad \frac{N_1}{N_2} = \boxed{\sqrt{\frac{Z_1}{Z_2}}}$$

(b) $\dfrac{N_1}{N_2} = \sqrt{\dfrac{8\,000\ \Omega}{8.00\ \Omega}} = \boxed{31.6}$

P32.38 The equation for $\Delta v(t)$ during the first period (using $y = mx + b$) is:

$$\Delta v = \frac{2\left(\Delta V_{\text{max}}\right)t}{T} - \Delta V_{\text{max}} = \Delta V_{\text{max}}\left[\frac{2t}{T} - 1\right]$$

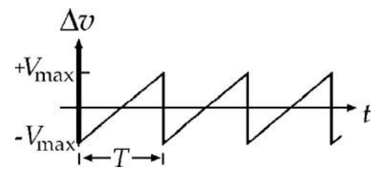
Therefore,

$$\left[(\Delta v)^2\right]_{\text{avg}} = \frac{1}{T}\int_0^T \left[\Delta v(t)\right]^2 dt = \frac{\left(\Delta V_{\text{max}}\right)^2}{T}\int_0^T \left[\frac{2}{T}t - 1\right]^2 dt$$

$$\left[(\Delta v)^2\right]_{\text{avg}} = \frac{(\Delta V_{\max})^2}{T}\left(\frac{T}{2}\right)\frac{\left[2t/T-1\right]^3}{3}\Bigg|_{t=0}^{t=T}$$

$$= \frac{(\Delta V_{\max})^2}{6}\left[(+1)^3-(-1)^3\right] = \frac{(\Delta V_{\max})^2}{3}$$

Then, $$\Delta V_{\text{rms}} = \sqrt{\left[(\Delta v)^2\right]_{\text{avg}}} = \sqrt{\frac{(\Delta V_{\max})^2}{3}} = \boxed{\frac{\Delta V_{\max}}{\sqrt{3}}}$$



ANS. FIG. P32.38

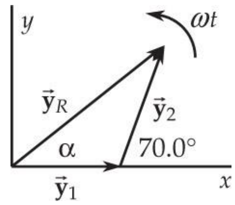
P32.39 (a) We can use $\sin A + \sin B = 2\sin\left(\frac{A}{2}+\frac{B}{2}\right)\cos\left(\frac{A}{2}-\frac{B}{2}\right)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0\text{ cm})\sin(4.50t + 35.0°)\cos 35.0°$$
$$E_1 + E_2 = (19.7\text{ cm})\sin(4.50t + 35.0°)$$

Thus, the total wave has amplitude $\boxed{19.7\text{ cm}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) Refer to ANS. FIG. P32.39 (b). In units of cm, the resultant phasor is

$$\vec{\mathbf{y}}_R = \vec{\mathbf{y}}_1 + \vec{\mathbf{y}}_2 = \left(12.0\hat{\mathbf{i}}\right) + \left(12.0\cos(70.0°)\hat{\mathbf{i}} + 12.0\sin(70.0)\hat{\mathbf{j}}\right)$$
$$= 16.1\hat{\mathbf{i}} + 11.3\hat{\mathbf{j}}$$
$$\vec{\mathbf{y}}_R = \sqrt{(16.1)^2 + (11.3)^2}\ \text{at}\ \tan^{-1}\left(\frac{11.3}{16.1}\right) = \boxed{19.7\text{ cm at }35.0°}$$



ANS. FIG. P32.39 (b)

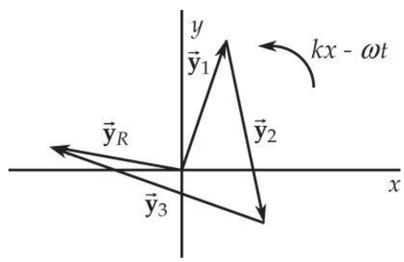
(c) $\boxed{\text{The answers are identical.}}$

(d) Refer to ANS. FIG. P32.39 (d). Adding the three waves yields

$$\begin{aligned}\vec{\mathbf{y}}_R &= 12.0\cos(70.0°)\hat{\mathbf{i}} + 12.0\sin(70.0°)\hat{\mathbf{j}} \\ &\quad + 15.5\cos(-80.0°)\hat{\mathbf{i}} + 15.5\sin(-80.0°)\hat{\mathbf{j}} \\ &\quad + 17.0\cos(160°)\hat{\mathbf{i}} + 17.0\sin(160°)\hat{\mathbf{j}} \\ \vec{\mathbf{y}}_R &= -9.18\hat{\mathbf{i}} + 1.83\hat{\mathbf{j}} = \boxed{9.36\text{ cm at }169°}\end{aligned}$$

The wave function of the total wave is

$$y_R = (9.36\text{ cm})\sin(15x - 4.5t + 169°)$$



ANS. FIG. P32.39 (d)

P32.40 (a) $\boxed{\text{Higher. At the resonance frequency, } X_L = X_C\text{. As the frequency increases, } X_L \text{ goes up and } X_C \text{ goes down.}}$

(b) $\boxed{\text{It is possible. We have three independent equations in the three unknowns } L, C, \text{ and the certain } f.}$

(c) The equations are $\omega_0^2 = \dfrac{1}{LC} = 2\ 000\text{ s}^{-1}$, $X_C = \dfrac{1}{\omega C} = 8.00\ \Omega$, and $X_L = \omega L = 12.0\ \Omega$. From the inductive reactance,

$$X_L = \omega L \qquad \rightarrow \omega = \frac{X_L}{L}$$

then from the capacitive reactance,

$$X_C \omega = \frac{1}{\omega C} \frac{X_L}{L}$$

solving for the angular frequency gives

$$\omega^2 = \frac{X_L}{X_C} \frac{1}{LC} = \frac{X_L}{X_C} \omega_0^2 = \left(\frac{12.0\ \Omega}{8.00\ \Omega}\right)\left(2\ 000\ \text{s}^{-1}\right)^2$$

from which we obtain

$$\omega = 2\ 450\ \text{s}^{-1}$$

Then,

$$L = \frac{X_L}{\omega} = \frac{12.0\ \Omega}{2\ 450\ \text{s}^{-1}} = 4.90 \times 10^{-3}\ \text{H} = \boxed{4.90\ \text{mH}}$$

$$C = \frac{1}{\omega X_C} = \frac{1}{\left(2\ 450\ \text{s}^{-1}\right)(8.00\ \Omega)} = 5.10 \times 10^{-5}\ \text{F} = \boxed{51.0\ \mu\text{F}}$$

P32.41 (a) The lowest-frequency standing-wave pattern is N-A-N. The distance between the clamps we represent as $d = d_{\text{NN}} = \dfrac{\lambda}{2}$. The speed of transverse waves on the string is $v = f\lambda = \sqrt{\dfrac{T}{\mu}} = f2d$.

The magnetic force on the wire oscillates at 60 Hz, so the wire will oscillate in resonance at 60 Hz. From the speed of transverse waves,

$$v = f\lambda = \sqrt{\frac{T}{\mu}} = f2d$$

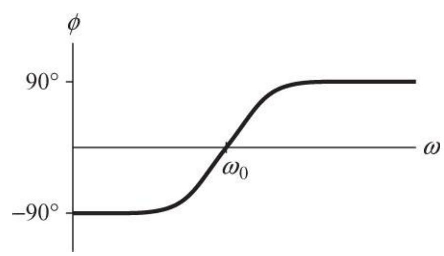
we obtain the period as

$$T = 4\mu f^2 d^2 = 4\left(19.0\times10^{-3}\text{ kg/m}\right)(60.0\text{ Hz})^2 d^2$$
$$= \left(274\text{ kg/m}\cdot\text{s}^2\right)d^2$$

$\boxed{\text{Tension } T \text{ and separation } d \text{ must be related by } T = 274d^2 \text{ where } T \text{ is in newtons and } d \text{ is in meters.}}$

(b) $\boxed{\text{One possibility is } T = 10.9\text{ N and } d = 0.200\text{ m.}}$ Any values of T and d related according to this expression will work. We did not need to use the value of the current and magnetic field.

P32.42 (a) See the graph in ANS. FIG. P32.42(a).



ANS. FIG. P32.42 (a)

(b) $\phi = \tan^{-1}\left(\dfrac{\omega L - 1/\omega C}{R}\right)$ changes from –90° for $\omega = 0$ to 0 at the resonance frequency to +90° as ω goes to infinity.

The slope of the graph is $d\phi/d\omega$:

$$\frac{d\phi}{d\omega} = \frac{1}{1+\left(\dfrac{\omega L - 1/\omega C}{R}\right)^2}\frac{1}{R}\left(L - \frac{1}{C}(-1)\frac{1}{\omega^2}\right)$$
$$= \boxed{\frac{R}{R^2 + (\omega L - 1/\omega C)^2}\left(L + \frac{1}{\omega^2 C}\right)}$$

(c) At resonance we have $\omega_0^2 = 1/LC$; substituting, we find the slope at the resonance point is

$$\left.\frac{d\phi}{d\omega}\right|_{\omega_0} = \frac{1}{R+0^2}\left(L+\frac{LC}{C}\right) = \frac{2L}{R} = \frac{2Q}{\omega_0}$$

where $Q = \omega_0 L/R$.

P32.43 (a) The inductive reactance of the circuit is

$$X_L = 2\pi f L = 2\pi(50.0\text{ Hz})(0.250\text{ H}) = \boxed{78.5\ \Omega}$$

(b) The capacitive reactance of the circuit is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50.0\text{ Hz})(2.00\times10^{-6}\text{ F})}$$
$$= 1.59\times10^3\ \Omega = \boxed{1.59\text{ k}\Omega}$$

(c) The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(150\ \Omega)^2 + (78.5\ \Omega - 1590\ \Omega)^2}$$
$$= 1.52\times10^3\ \Omega = \boxed{1.52\text{ k}\Omega}$$

(d) The maximum current is

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{2.10\times10^2\text{ V}}{1.52\times10^3\ \Omega} = 0.138\text{ A} = \boxed{138\text{ mA}}$$

(e) $$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{78.5\ \Omega - 1590\ \Omega}{150\ \Omega}\right) = \boxed{-84.3^\circ}$$

(f) $$\cos\phi = \cos\left[\tan^{-1}\left(\frac{78.5\ \Omega - 1\,590\ \Omega}{150\ \Omega}\right)\right] = \boxed{0.098\,7}$$

(g) The power input into the circuit is

$$P = I_{\text{rms}}\Delta V_{\text{rms}}\cos\phi = \frac{(\Delta V_{\text{rms}})^2}{Z}\cos\phi = \frac{\left(\Delta V_{\max}/\sqrt{2}\right)^2}{Z}\cos\phi$$
$$= \frac{(\Delta V_{\max})^2}{2Z}\cos\phi$$
$$P = \frac{(210\text{ V})^2}{2(1.52\times10^3\ \Omega)}(0.098\,7) = \boxed{1.43\text{ W}}$$

P32.44 (a) With both switches closed, the current goes only through the generator and resistor.

$$\boxed{i = \frac{\Delta V_{\max}}{R}\cos\omega t}$$

(b) $$\boxed{P = \frac{1}{2}\frac{(\Delta V_{\max})^2}{R}}$$

(c) $$\boxed{i = \frac{\Delta V_{\max}}{\sqrt{R^2+\omega^2 L^2}}\cos\left[\omega t + \tan^{-1}\left(\frac{\omega L}{R}\right)\right]}$$

(d) For $0 = \phi = \tan^{-1}\left(\dfrac{\omega_0 L - (1/\omega_0 C)}{R}\right)$,

We require $\omega_0 L = \dfrac{1}{\omega_0 C}$, so $\boxed{C = \dfrac{1}{\omega_0^2 L}}$

(e) The frequency is the resonance frequency: $Z = \boxed{R}$

For parts (f) and (g), the circuit is at resonance, so $Z = R$ and $X_C = X_L = \omega_0 L$.

(f) To find the maximum energy stored in the capacitor, we start with

$$U = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}C(IX_C)^2$$

When $I = I_{\max}$,

$$U_{\max} = \frac{1}{2}CI_{\max}^2 X_C^2 = \frac{1}{2}C\left(\frac{\Delta V_{\max}}{R}\right)^2(\omega_0 L)^2 = \boxed{\frac{(\Delta V_{\max})^2 L}{2R^2}}$$

(g) $$U_{\max} = \frac{1}{2}LI_{\max}^2 = \boxed{\frac{1}{2}L\frac{(\Delta V_{\max})^2}{R^2}}$$

(h) Now $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$, so

$$\phi = \tan^{-1}\left(\frac{\omega L - (1/\omega C)}{R}\right) = \tan^{-1}\left(\frac{2\sqrt{L/C} - (1/2)\sqrt{L/C}}{R}\right)$$

$$= \boxed{\tan^{-1}\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}$$

(i) Now $\omega L = \frac{1}{2}\frac{1}{\omega C}$, so $\omega = \boxed{\frac{1}{\sqrt{2LC}}} = \frac{\omega_0}{\sqrt{2}}$

***P32.45** **Conceptualize** The RC circuit shown here acts as a *low-pass filter* or a *high-pass filter*. At high frequencies, the capacitor has a low impedance, so most of the output voltage is across the resistor. Conversely, at low frequencies, the capacitor acts almost as an open circuit, so the output voltage is high across the capacitor.

Categorize This is a simple AC circuit, to which we apply the material from this chapter.

Analyze Based on the discussion in the Conceptualize step, we see that we attach the woofer to (a) the $\boxed{\text{capacitor}}$, and the tweeter to (b) the $\boxed{\text{resistor}}$.

(c) Set up the ratio of the magnitude of the output voltage across the resistor to the magnitude of the input voltage:

$$\frac{|\Delta v_R|}{|\Delta v_{\text{in}}|} = \frac{I_{\max}R}{I_{\max}Z} = \boxed{\frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}}$$

where we have used Equation 32.28 for the impedance, with $L = 0$.

(d) Follow the same procedure as we did in part (c), but for the capacitor output:

$$\frac{|\Delta v_C|}{|\Delta v_{\text{in}}|} = \frac{I_{\max} X_C}{I_{\max} Z} = \frac{\dfrac{1}{\omega C}}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}} = \boxed{\frac{1}{\sqrt{R^2 C^2 \omega^2 + 1}}}$$

Finalize Notice the behavior of the expressions in parts (c) and (d) as the frequencies vary. In part (c), as the frequency *increases,* the ratio of the voltages *increases*; the tweeter should be connected to this output. In part (d), as the frequency *decreases*, the ratio of the voltages *increases*; the woofer should be connected to this output.

Answers: (a) capacitor (b) resistor (c) $\dfrac{R}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$ (d) $\dfrac{1}{\sqrt{R^2 C^2 \omega^2 + 1}}$

P32.46 (a) We are given $X_L = X_C = 1\,884\ \Omega$ when $f = 2\,000$ Hz. The impedance is then

$$L = \frac{X_L}{2\pi f} = \frac{1\,884\ \Omega}{4\,000\pi\ \text{rad/s}} = 0.150\ \text{H}$$

and the capacitance is

$$C = \frac{1}{(2\pi f)X_C} = \frac{1}{(4\,000\pi\ \text{rad/s})(1\,884\ \Omega)}$$
$$= 42.2\ \text{nF}$$

therefore,

$$X_L = 2\pi f(0.150\ \text{H})$$

$$X_C = \frac{1}{(2\pi f)(4.22 \times 10^{-8} \text{ F})}$$

and

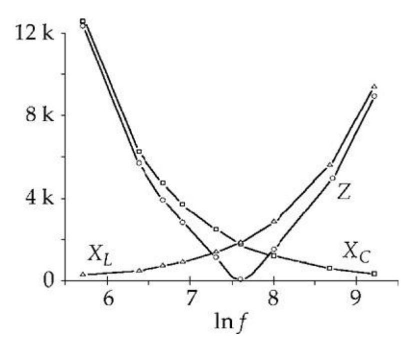
$$Z = \sqrt{(40.0\ \Omega)^2 + (X_L - X_C)^2}$$

TABLE P32.46 lists the inductive reactance, the capacitive reactance, and the impedance for the frequencies listed in the problem statement.

f (Hz)	X_L (Ω)	X_C (Ω)	Z (Ω)
300	283	12 600	12 300
600	565	6 280	5 720
800	754	4 710	3 960
1 000	942	3 770	2 830
1 500	1 410	2 510	1 100
2 000	1 880	1 880	40
3 000	2 830	1 260	1 570
4 000	3 770	942	2 830
6 000	5 650	628	5 020
10 000	9 420	377	9 040

TABLE P32.46

(b) ANS. FIG. P32.46 (b) shows a graph of X_L, X_C, and Z as a function of the frequency f.



ANS. FIG. P32.46 (b)

***P32.47** **Conceptualize** The circuit in Figure P32.47a is the *RLC* circuit that we analyzed in Section 31.6 and Figure 31.15, with the addition of switches in parallel with each of the three circuit elements. The circuit in Figure P32.47b is the *RLC* circuit driven by an AC source in Figure 32.13, again with the addition of the switches in parallel.

Categorize Based on the description in the Conceptualize step, we categorize the circuit as an *RLC* circuit, powered by a battery in Figure P32.47a and by an AC source in Figure P32.47b.

Analyze Consider Figure P32.47a first, with the battery as the energy source for the circuit. If switch S_L is closed and S_C and S_R are open, there is a short circuit across the inductor, so the inductor plays no role in the circuit. Therefore, when switch S is thrown to the position *b*, the capacitor discharges through the resistor; the circuit is an *RC* circuit, and the time constant is given by Equation 27.20:

$$\tau_1 = RC \quad \rightarrow \quad C = \frac{\tau_1}{R} \qquad (1)$$

If switch S_C is closed and S_L and S_R are open, there is a short circuit across the capacitor, so the capacitor plays no role in the circuit. Therefore, when switch S is thrown to the position *b*, the circuit is an *RL* circuit, and the time constant is given by Equation 31.8:

$$\tau_2 = \frac{L}{R} \quad \rightarrow \quad L = \tau_2 R \qquad (2)$$

Now, let's move to Figure P32.47b. With switch S at position *b*, the battery is no longer part of the circuit. Switches S_C, S_L, and S_R are all open, so we can ignore them. What we have is an *RLC* circuit driven by an AC source, just like in Figure 32.13. From Equation 32.39, the resonance frequency of the circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (3)$$

Substitute Equations (1) and (2) into Equation (3):

$$\omega_0 = \frac{1}{\sqrt{(\tau_2 R)\left(\frac{\tau_1}{R}\right)}} = \frac{1}{\sqrt{\tau_1 \tau_2}} \tag{4}$$

Substitute numerical values:

$$\omega_0 = \frac{1}{\sqrt{(0.200\text{ ms})(0.050\,0\text{ ms})}} = \boxed{1.00 \square 10^4 \text{ rad/ s}}$$

Finalize Look at how many quantities we did *not* know in this problem: ε, r, L, R, and C! We also did not know the maximum voltage ΔV_{max} of the AC source. Yet we were still able to find the answer. Notice the importance of solving problems algebraically before substituting numerical values. If you solve problems by substituting numbers at every step, you would have been stuck at Equation (1), because you don't know R. But, by carrying the calculation through algebraically, we see that we only need the time constants in the final expression; we *don't need R*, or any of the other quantities.

Answer: 1.00×10^4 rad/s

P32.48 (a) At resonance,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00 \times 10^{-3}\text{ H})(1.00 \times 10^{-9}\text{ F})}} = 1.00 \times 10^6 \text{ rad/s}$$

At that point,

$$Z = R = 1.00\ \Omega \quad \text{and} \quad I = \frac{1.00\text{ V}}{1.00\ \Omega} = 1.00\text{ A}$$

The power is

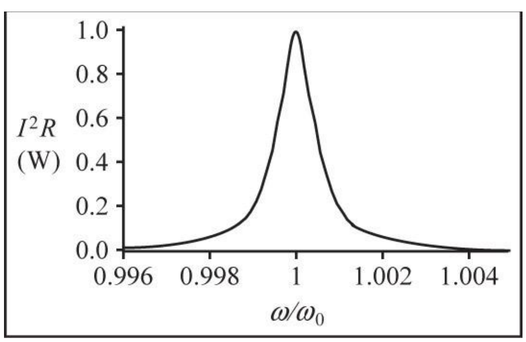
$$P = I^2R = (1.00\text{ A})^2(1.00\ \Omega) = 1.00\text{ W}$$

We compute the power at some other angular frequencies, listed in TABLE P32.48.

$\dfrac{\omega}{\omega_0}$	$\omega L\ (\Omega)$	$\dfrac{1}{\omega C}\ (\Omega)$	$Z\ (\Omega)$	$P = I_{\text{rms}}^{\ 2}\ R\ (W)$
0.9990	999.0	1001.0	2.24	0.19984
0.9991	999.1	1000.9	2.06	0.23569
0.9993	999.3	1000.7	1.72	0.33768
0.9995	999.5	1000.5	1.41	0.49987
0.9997	999.7	1000.3	1.17	0.73524
0.9999	999.9	1000.1	1.02	0.96153
1.0000	1000	1000.0	1.00	1.00000
1.0001	1000.1	999.9	1.02	0.96154
1.0003	1000.3	999.7	1.17	0.73535
1.0005	1000.5	999.5	1.41	0.50012
1.0007	1000.7	999.3	1.72	0.33799
1.0009	1000.9	999.1	2.06	0.23601
1.0010	1001	999.0	2.24	0.20016

TABLE P32.48

ANS. FIG. P32.48 shows a graph of the results tabulated above.



ANS. FIG. P32.48

(b) The angular frequencies giving half the maximum power are

$$0.999\,5 \times 10^{6}\ \text{rad/s} \quad \text{and} \quad 1.000\,5 \times 10^{6}\ \text{rad/s}$$

so the full width at half the maximum is

$$\Delta\omega = (1.000\,5 - 0.999\,5) \times 10^{6}\ \text{rad/s}$$

$$\Delta\omega = 1.00 \times 10^{3}\ \text{rad/s}$$

Since $\Delta\omega = 2\pi\,\Delta f$, $\quad \Delta f = 159\ \text{Hz}$

and for comparison,

$$\frac{R}{2\pi L} = \frac{1.00\ \Omega}{2\pi\left(1.00 \times 10^{-3}\,\text{H}\right)} = 159\ \text{Hz}$$

The two quantities agree.

Challenge Problems

P32.49 We start with

$$\frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \frac{IR}{IZ} = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

$$= \frac{8.00\ \Omega}{\sqrt{(8.00\ \Omega)^2 + \left[2\pi\,fL - 1/2\pi\,f\,C\right]^2}}$$

Then, at 200 Hz,

$$\left(\frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}\right)^2 = \left(\frac{R}{Z}\right)^2 = \frac{1}{4} = \frac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + \left[400\pi\,L - 1/400\pi\,C\right]^2}$$

and at 4 000 Hz,

$$\left(\frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}\right)^2 = \left(\frac{R}{Z}\right)^2 = \frac{1}{4} = \frac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + \left[8000\pi\,L - 1/8000\pi\,C\right]^2}$$

At the low frequency, $X_L - X_C < 0$. This reduces to

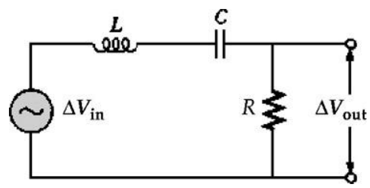
$$400\pi L - \frac{1}{400\pi C} = -13.9\ \Omega \qquad \textbf{[1]}$$

For the high frequency half-voltage point,

$$8\,000\pi L - \frac{1}{8\,000\pi C} = +13.9\ \Omega \qquad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives

(a) $L = \boxed{580\ \mu\text{H}}$



ANS. FIG. P32.49(a)

(b) $C = \boxed{54.6\ \mu\text{F}}$

(c) When $X_L = X_C$, $\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \left(\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}\right)_{\text{max}} = \boxed{1.00}$

(d) $X_L = X_C$ requires

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5.80\times 10^{-4}\ \text{H})(5.46\times 10^{-5}\ \text{F})}} = \boxed{894\ \text{Hz}}$$

(e) $\boxed{\text{At 200 Hz}}$, $\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_C > X_L$, so the phasor diagram is as shown in ANS. FIG. P32.49 (e).

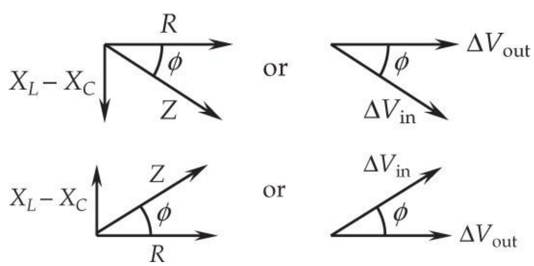
$$\boxed{\phi =} -\cos^{-1}\left(\frac{R}{Z}\right) = -\cos^{-1}\left(\frac{1}{2}\right) = \boxed{-60.0^\circ}$$

so $\boxed{(\Delta v_{out} \text{ leads } \Delta v_{in})}$.

$\boxed{\text{At } f_0}$, $X_L = X_C$ so

$$\boxed{\phi = 0 \ (\Delta v_{out} \text{ is in phase with } \Delta v_{in})}.$$

$\boxed{\text{At 4 000 Hz}}$, $\dfrac{\Delta v_{out}}{\Delta v_{in}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_L - X_C > 0$.



ANS. FIG. P32.49 (e)

Thus, $\boxed{\phi =} \cos^{-1}\left(\dfrac{1}{2}\right) = \boxed{+60.0°}$

or $\boxed{\Delta v_{out} \text{ lags } \Delta v_{in}}$.

(f) At 200 Hz and at 4 kHz,

$$P = \frac{\left(\Delta v_{out,\,rms}\right)^2}{R} = \frac{\left[(1/2)\Delta v_{in,\,rms}\right]^2}{R} = \frac{(1/4)\left(\Delta v_{in,\,max}/\sqrt{2}\right)^2}{R}$$
$$= \frac{(1/2)(10.0\text{ V})^2}{4(8.00\ \Omega)} = \boxed{1.56\text{ W}}$$

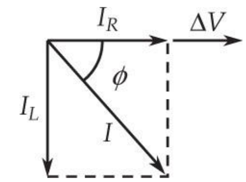
At f_0,

$$P = \frac{\left(\Delta v_{out,\,rms}\right)^2}{R} = \frac{\left(\Delta v_{in,\,rms}\right)^2}{R} = \frac{\left(\Delta v_{in,\,max}/\sqrt{2}\right)^2}{R} = \frac{(1/2)(10.0\text{ V})^2}{(8.00\ \Omega)}$$
$$= \boxed{6.25\text{ W}}$$

(g) We take

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi(894\ \text{Hz})(5.80\times10^{-4}\ \text{H})}{8.00\ \Omega} = \boxed{0.408}$$

P32.50 (a) $I_{R,\ \text{rms}} = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{100\ \text{V}}{80.0\ \Omega} = \boxed{1.25\ \text{A}}$



ANS. FIG. P32.50

(b) The total current will $\boxed{\text{lag}}$ the applied voltage as seen in the phasor diagram shown in ANS. FIG. P32.50.

$$I_{L,\ \text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{100\ \text{V}}{2\pi(60.0\ \text{s}^{-1})(0.200\ \text{H})} = 1.33\ \text{A}$$

Thus, the phase angle is:

$$\phi = \tan^{-1}\left(\frac{I_{L,\ \text{rms}}}{I_{R,\ \text{rms}}}\right) = \tan^{-1}\left(\frac{1.33\ \text{A}}{1.25\ \text{A}}\right) = \boxed{46.7°}$$

P32.51 We have $P = I_{\text{rms}}^2 R = \left(\dfrac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \dfrac{(\Delta V_{\text{rms}})^2}{Z^2} R$, and

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Therefore,

$$Z^2 = \frac{(\Delta V_{\text{rms}})^2 R}{P} \quad \rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{(\Delta V_{\text{rms}})^2 R}{P}$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{(\Delta V_{\text{rms}})^2 R}{P} - R^2$$

which is a fourth order equation in ω. But this can be simplified to two equations:

$$\omega L - \frac{1}{\omega C} = \pm A \quad \rightarrow \quad \omega^2 LC \mp \omega CA - 1 = 0$$

where $A = \sqrt{\dfrac{(\Delta V_{\text{rms}})^2 R}{P} - R^2}$.

We will solve for ω when ΔV_{rms} = 100 V and P = 250 W. From Figure P32.16, we have R = 40.0 Ω, L = 185 mH = 0.185 H, and C = 65.0 μF = 65.0×10^{-6} F.

The quantity A is

$$\begin{aligned} A &= \sqrt{\frac{(\Delta V_{\text{rms}})^2 R}{P} - R^2} = \sqrt{\frac{(120\ \text{V})^2(40.0\ \Omega)}{250\ \text{W}} - (40.0\ \Omega)^2} \\ &= \sqrt{704}\,\Omega \end{aligned}$$

The angular frequency ω must be positive, so we solve for the positive roots. (In the following, we suppress all units.)

For $\omega^2 LC - \omega CA - 1 = 0$,

$$\begin{aligned} \omega &= \frac{-(-CA) + \sqrt{(-CA)^2 - 4LC(-1)}}{2LC} \\ &= \frac{A + \sqrt{A^2 + 4L/C}}{2L} \\ &= \frac{\sqrt{704} + \sqrt{704 + 4(0.185)/(65.0 \times 10^{-6})}}{2(0.185)} \\ &= 226\ \text{s}^{-1} = 2\pi f \quad \rightarrow \quad f = \boxed{58.7\ \text{Hz}} \end{aligned}$$

For $\omega^2 LC + \omega CA - 1 = 0$,

$$\begin{aligned} \omega &= \frac{-(CA) + \sqrt{(-CA)^2 - 4LC(-1)}}{2LC} \\ &= \frac{-A + \sqrt{A^2 + 4L/C}}{2L} \\ &= \frac{-\sqrt{704} + \sqrt{704 + 4(0.185)/(65.0 \times 10^{-6})}}{2(0.185)} \\ &= 225\ \text{s}^{-1} = 2\pi f \quad \rightarrow \quad f = \boxed{35.9\ \text{Hz}} \end{aligned}$$

There are two answers because $\boxed{\text{the circuit can be either above or below resonance.}}$

ANSWERS TO QUICK-QUIZZES

1. **(i)** (c) **(ii)** (b)

2. (b)

3. (a)

4. (b)

5. (a) $XL < XC$ (b) $XL = XC$ (c) $XL > XC$

6. (c)

7. (c)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P32.2 (a) 170 V; (b) $2.40 \times 10^2\ \Omega$;

(c) Because $P_{\text{avg}} = \dfrac{(\Delta V_{\text{rms}})^2}{R} \quad \rightarrow \quad R = \dfrac{(\Delta V_{\text{rms}})^2}{P_{\text{avg}}}$, a 100-W bulb has less resistance than a 60.0-W bulb.

P32.4 (a) The rms current in each 150-W bulb is 1.25 A. The rms current in the 100-W bulb is 0.833 A; (b) $R_1 = 96.0\ \Omega,\ R_2 = 96.0\ \Omega,$ and $R_3 = 144\ \Omega$; (c) 36.0 Ω

P32.6 (a) 0.0424 H; (b) 942 rad/s

P32.8 3.80 J

P32.10 0.450 Wb

P32.12 (a) 221 Ω; (b) 0.163 A; (c) 0.230 A; (d) no

P32.14 $\sqrt{2}C(\Delta V_{\text{rms}})$

P32.16 (a) 146 V ; (b) 212 V; (c) 179 V; (d) 32.4 V

P32.18 See P32.18 for explanation.

P32.20 (a) 88.4 Ω; (b) 107 Ω; (c) 1.12 A; (d) the voltage lags behind the current by 55.8°; (e) Adding an inductor will change the impedence, and hence the current in the circuit. The current could be larger or smaller, depending on the inductance added. The largest current would result when the inductive reactance equals the capacitive reactance, the impedance has its minimum value, equal to 60.0 Ω, and the current in the circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{\Delta V_{\max}}{R} = \frac{1.20 \times 10^2 \text{ V}}{60.0 \ \Omega} = 2.00 \text{ A}$$

P32.22 See P32.22 for full explanation

P32.24 (a) 66.8 Ω; (b) 0.953 A; (c) 45.4 W

P32.26 (a) 3.56 kHz; (b) 5.00 A; (c) 22.4; (d) 2.24 kV

P32.28 242 mJ

P32.30 687 V

P32.32 (a) 29.0 kW (b) 5.80×10^{-3}

(c) It is impossible to transmit so much power at such low voltage

P32.34 (a) $\boxed{X_C \text{ could be } 53.8\ \Omega \text{ or it could be } 1.35\ \text{k}\Omega.}$

(b) capacitive reactance is 53.8 Ω (c) X_C 쟬ust be 1.43 kΩ

P32.36 $\sqrt{\dfrac{8\rho_{\text{Cu}}\,\ell P}{\pi f\left(\Delta V_{\text{rms}}\right)^2}}$

P32.38 $\dfrac{\Delta V_{\max}}{\sqrt{3}}$

P32.40 (a) Higher. At the resonance frequency, $X_L = X_C$. As the frequency increases, X_L goes up and X_C goes down; (b) It is possible. We have three independent equations in the three unknowns L, C, and the certain f; (c) $L = 4.90$ mH and $C = 51.0\ \mu$F

P32.42 (a) See ANS. FIG. P32.42(a); (b) $\dfrac{R}{R^2 + (\omega L - 1/\omega C)^2}\left(L + \dfrac{1}{\omega^2 C}\right)$; (c) See P32.42 (c) for full explanation.

P32.44 (a) $i = \dfrac{\Delta V_{\max}}{R}\cos\omega t$; (b) $P = \dfrac{1}{2}\dfrac{(\Delta V_{\max})^2}{R}$;

(c) $i = \dfrac{\Delta V_{\max}}{\sqrt{R^2 + \omega^2 L^2}}\cos\left[\omega t + \tan^{-1}\left(\dfrac{\omega L}{R}\right)\right]$; (d) $C = \dfrac{1}{\omega_0^2 L}$; (e) R;

(f) $\dfrac{(\Delta V_{\max})^2 L}{2R^2}$; (g) $\dfrac{1}{2}L\dfrac{(\Delta V_{\max})^2}{R^2}$; (h) $\tan^{-1}\left(\dfrac{3}{2R}\sqrt{\dfrac{L}{C}}\right)$; (i) $\dfrac{1}{\sqrt{2LC}}$

P32.46 (a) See Table P32.46; (b) See ANS. FIG. P32.46 (b).

P32.48 (a) See ANS. FIG. P32.48; (b) See P32.48 for full explanation.

P32.50 (a) 1.25 A; (b) lag, 46.7°