16

Wave Motion

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP16.1** **Conceptualize** Figure 16.4 shows a typical wind-driven wave on the surface of the ocean. A tsunami would be similar to this diagram, except that the wavelength would be huge compared to normal ocean waves.

Categorize The waves on the ocean, whether wind-driven or tsunami, are described by the *traveling wave* model.

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Analyze (a) Let us first use the traveling wave model to calculate the frequency of the wave:

$$v = \lambda f \quad \rightarrow \quad f = \frac{v}{\lambda} = \frac{800\ \text{km/h}}{200\ \text{km}} = 4\ \text{h}^{-1}$$

Inverting this result, the period of the tsunami is 0.25 h, or 15 minutes. Therefore, on the ship in the ocean, the passage of the tsunami would be represented by your ship moving up and down by the crest-to-trough vertical distance of 2.0 m in 15 minutes. $\boxed{\text{This is essentially undetectable}}$, especially in the presence of the motion of the ship due to the normal wind-driven waves.

(b) Based on our frequency calculation of part (a), if the water begins to recede at a given time, it will begin coming back in about half the period, or $\boxed{7.5 \text{ minutes}}$, so you have about that much time to warn people to move off the beach and head far inland to high ground.

(c) Set up a ratio of the power of the wave in the open ocean to the power in the shallow water:

$$\frac{P_{\text{shallow}}}{P_{\text{deep}}} \sim \frac{\omega_{\text{shallow}}^2 A_{\text{shallow}}^2 v_{\text{shallow}}}{\omega_{\text{deep}}^2 A_{\text{deep}}^2 v_{\text{deep}}} \qquad (1)$$

Introduce two facts into this equation: (1) our assumed conservation of energy means that $P_{\text{shallow}} = P_{\text{deep}}$, and (2) the frequency of a wave does not change when it enters a new medium, so $\omega_{\text{shallow}} = \omega_{\text{deep}}$. Therefore,

$$\frac{P_{\text{shallow}}}{P_{\text{deep}}} = 1 = \frac{A_{\text{shallow}}^2 v_{\text{shallow}}}{A_{\text{deep}}^2 v_{\text{deep}}} \qquad (2)$$

Solve for the amplitude of the wave in the shallow water:

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$$A_{\text{shallow}} = A_{\text{deep}}\sqrt{\frac{v_{\text{deep}}}{v_{\text{shallow}}}} \qquad (3)$$

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Substitute numerical values:

$$A_{\text{shallow}} = 1.0 \text{ m}\sqrt{\frac{800 \text{ km/ h}}{75 \text{ km/ h}}} = \boxed{3.3 \text{ m}}$$

(d) If half the energy is reflected, then only half the power is transmitted into the shallow water, so Equation (2) is modified to be

$$\frac{P_{\text{shallow}}}{P_{\text{deep}}} = \frac{1}{2} = \frac{A_{\text{shallow}}^2 v_{\text{shallow}}}{A_{\text{deep}}^2 v_{\text{deep}}} \qquad (4)$$

Solve for the amplitude of the wave in the shallow water:

$$A_{\text{shallow}} = A_{\text{deep}}\sqrt{\frac{v_{\text{deep}}}{2v_{\text{shallow}}}} \qquad (5)$$

Substitute numerical values:

$$A_{\text{shallow}} = 1.0 \text{ m}\sqrt{\frac{800 \text{ km/ h}}{2(75 \text{ km/ h})}} = \boxed{2.3 \text{ m}}$$

Finalize The amplitude in part (d) is still very dangerous. Keep in mind that the water in a tsunami arriving on shore will not just reach heights above sea level equal to the amplitude. The tremendous amount of momentum in the water will cause it to rise up the slope of a beach and well into any buildings and houses in a developed area located there. The moving water can cause great damage both as it enters the area and also as it recedes back into the ocean.

Answers: (a) Wave is undetectable. (b) 7.5 min (c) 3.3 m (d) 2.3 m

***TP16.2** **Conceptualize** Imagine the sound waves spreading out from the speaker as suggested by Figure 16.21. Any sound waves aimed toward the ground are absorbed, so we don't need to worry about those.

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Categorize We categorize the sound waves as spherical and use the discussion in Section 16.8.

Analyze (a) Use Equation 16.40 and determine the intensity from 16.41 to find the power output of the speaker:

$$(Power)_{\text{avg}} = 4\pi r^2 I = 4\pi r^2 I_0 10^{(\beta/10)} \qquad (1)$$

Substitute numerical values:

$$(Power)_{\text{avg}} = 4\pi(100\text{ m})^2(1\times10^{-12}\text{ W/m}^2)10^{(83/10)} = \boxed{25.1\text{ W}}$$

(b) Substitute new numerical values into Equation (1):

$$(Power)_{\text{avg}} = 4\pi(100\text{ m})^2(1\times10^{-12}\text{ W/m}^2)10^{(70.8/10)} = \boxed{1.51\text{ W}}$$

(c) The power found in part (b) is clearly less than the desired 25 W and much less than the 150 W figure stated by the salesman. Hopefully, however, after doing some research, you learned the following: loudspeakers are rated in electronics stores by the *input* electrical power. That is the figure given by the salesman. That number is far different, however, from the *output* sound power. Loudspeakers have an efficiency of about 1%. Therefore, for the 150 W of input electrical power, about 1.50 W of sound power comes out, agreeing with the result in part (b). The rest of the input energy goes into warming up the loudspeaker and the air surrounding it.

Finalize The salesman was just doing his job. It's the entrepreneur's responsibility to understand the difference between input electrical power and output sound power for the loudspeakers.

Answers: (a) 25.1 W (b) 1.51 W (c) Answers will vary.

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***TP16.3** **Conceptualize** Imagine the earthquake occurring, with waves spreading out in all directions along the surface of the Earth. In particular, think about the waves that will arrive at Sacramento, San Francisco, and Los Angeles.

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Categorize While we might want to use the traveling wave model, we have no information about wavelengths or frequencies. What we have is a leading edge of the wave traveling through the Earth at a fixed speed. Therefore, we can model the leading edge of the wave as a *particle under constant velocity*.

Analyze From Equation 2.7, the time at which the leading edge a wave arrives at the seismic station, if the earthquake occurred at $t = 0$, is given by

$$t = \frac{x_f - x_i}{v} = \frac{d}{v}$$

where d is the distance between the earthquake epicenter and the seismic station. For the P and S waves, we don't know the actual value of t because we don't know when the earthquake occurred. But we do know *two* values of t relative to an unknown reference time: one for the P waves and one for the S waves. So let's subtract the arrival times:

$$t_S - t_P = \frac{d}{v_S} - \frac{d}{v_P}$$

Substituting the wave speeds, we have

$$t_S - t_P = \frac{d}{4.00 \text{ km/ s}} - \frac{d}{8.00 \text{ km/ s}} = (0.125 \text{ s/ km})d$$
$$\rightarrow \quad d = (8.00 \text{ km/ s})(t_S - t_P)$$

Using this equation, we can subtract to find the S–P time difference and then find the distance from the epicenter to each of the seismic stations:

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Seismic Station	Arrival Time of P Wave (h:min:s)	Arrival Time of S Wave (h:min:s)	$t_S - t_P$ (s)	d (km)
Sacramento	3:21:34.4 PM	3:22:04.8 PM	30.4	243
San Francisco	3:21:35.9 PM	3:22:07.8 PM	31.9	255
Los Angeles	3:21:52.1 PM	3:22:40.2 PM	48.1	385

Find a California map on the Internet that contains a scale. Using the scale, determine the radii of the three circles that you would draw centered on Sacramento, San Francisco, and Los Angeles representing the points from which the earthquake could have originated. Such a drawing appears below:

There is only one point belonging to all three circles, showing that the earthquake epicenter is near the city of Fresno, California.

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(b) Because the primary wave travels at exactly twice the speed of the S wave, the S–P time difference is exactly the same as the time interval between the occurrence of the earthquake and the arrival of the leading edge of the P wave. Therefore, in the table, subtract the S–P time difference from the arrival time of the P wave:

Seismic Station	**Arrival Time of P Wave (h:min:s)**	**Arrival Time of S Wave (h:min:s)**	$t_S - t_P$ **(s)**	**Time of earthquake (h:min:s)**
Sacramento	3:21:34.4 PM	3:22:04.8 PM	30.4	3:21:04.0
San Francisco	3:21:35.9 PM	3:22:07.8 PM	31.9	3:21:04.0
Los Angeles	3:21:52.1 PM	3:22:40.2 PM	48.1	3:21:04.0

All three sets of data show that the earthquake occurred at $\boxed{\text{3:21:04.0 PM}}$.

Finalize Notice that the two circles centered on Sacramento and San Francisco would isolate the earthquake to two possible locations: Fresno and a point to the northwest of Ukiah. We need the third circle to identify which of these two locations is the correct one.

Answers: (a) Fresno (b) 3:21:04.0

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 16.1 Propagation of a Disturbance

P16.1 The distance the waves have traveled is $d = (7.80 \text{ km/s})t = (4.50 \text{ km/s})(t + 17.3 \text{ s})$, where t is the travel time for the faster wave.

Then, $$(7.80 - 4.50)(\text{km/s})t = (4.50 \text{ km/s})(17.3 \text{ s})$$

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or $$t = \frac{(4.50 \text{ km/s})(17.3 \text{ s})}{(7.80 - 4.50) \text{ km/s}} = 23.6 \text{ s}$$

and the distance is $d = (7.80 \text{ km/s})(23.6 \text{ s}) = \boxed{184 \text{ km}}$

P16.2 (a) The $\boxed{\text{longitudinal P wave}}$ travels a shorter distance and is moving faster, so it will arrive at point B first.

(b) The P wave that travels through the Earth must travel a distance of $2R\sin 30.0° = 2(6.37 \times 10^6 \text{ m})\sin 30.0° = 6.37 \times 10^6 \text{ m}$ at a speed of 7 800 m/s.

Therefore, it takes $\Delta t_P = \dfrac{6.37 \times 10^6 \text{ m}}{7\,800 \text{ m/s}} \simeq 817 \text{ s}.$

The Rayleigh wave that travels along the Earth's surface must travel a distance of

$$s = R\theta = R\left(\frac{\pi}{3} \text{ rad}\right) = 6.67 \times 10^6 \text{ m}$$

at a speed of 4 500 m/s.

Therefore, it takes $\Delta t_S = \dfrac{6.67 \times 10^6 \text{ m}}{4\,500 \text{ m/s}} \simeq 1\,482 \text{ s}.$

The time difference is $\Delta T = \Delta t_S - \Delta t_P = \boxed{666 \text{ s}} = 11.1 \text{ min}.$

***P16.3** **Conceptualize** Notice that the speed of sound in the pipe is much higher than the speed of sound in air. Therefore, you will hear the sound through the pipe first, followed later by the sound through the air. The time interval between the sounds will be short, which is why you will need your smartphone to measure them.

Categorize The sound pulses through the air and the pipe can each be modeled as a *particle under constant velocity.*

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Analyze (a) Use Equation 2.7 to find the time at which each pulse arrives at the end of the pipe, where the initial position of the pulse at the struck end is $x_i = 0$ and the final position of the pulse is $x_f = L$, the length of the pipe:

$$x_f = x_i + vt \quad \rightarrow \quad L = 0 + vt \quad \rightarrow \quad t = \frac{L}{v} \qquad (1)$$

Find the time interval Δt between the arrivals of the pulses for the two sound waves:

$$\Delta t = t_{\text{air}} - t_{\text{copper}} = \frac{L}{v_{\text{air}}} - \frac{L}{v_{\text{copper}}} = L\left(\frac{1}{v_{\text{air}}} - \frac{1}{v_{\text{copper}}}\right)$$

$$\rightarrow \quad L = \frac{\Delta t}{\left(\dfrac{1}{v_{\text{air}}} - \dfrac{1}{v_{\text{copper}}}\right)} \qquad (2)$$

Substitute numerical values:

$$L = \frac{\Delta t}{\left(\dfrac{1}{343\text{ m/ s}} - \dfrac{1}{3\,560\text{ m/ s}}\right)} \quad \rightarrow \quad \boxed{L = (380\text{ m/ s})\Delta t} \qquad (3)$$

(b) Substitute the numerical value for the time interval into Equation (3):

$$L = (380\text{ m/ s})(127 \times 10^{-3}\text{ s}) = \boxed{48.2\text{ m}}$$

(c) Assuming that the speeds of sound are correct, the 1.0% error in the time interval will translate to a 1.0% error in the length measurement:

$$\Delta L = (0.010)L = (0.010)(48.2\text{ m}) = 0.48\text{ m} = \boxed{48\text{ cm}}$$

Finalize Your supervisor may or may not be happy with the half-meter uncertainty expressed in part (b). On the other hand, he may be willing to accept that uncertainty in exchange for the much faster

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measurement time for your technique. Your technique may also have other uncertainties. For example, the speed of sound will depend on temperature, so you would need to take that into account in your measurement.

Answers: (a) $L = (380 \text{ m/ s})\Delta t$ (b) 48.2 m (c) 48 cm

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***P16.4** **Conceptualize** You are likely to have seen a wave at a stadium; they are common all over the world. If not, videos are available online. While it is called a "wave," it is actually a single pulse.

Categorize For lack of any information to the contrary, we will model the wave as a *particle under constant speed*.

Analyze The speed of the wave, in seats per second, is

$$v = \frac{\#\ \text{seats}}{\Delta t} = \frac{974\ \text{seats}}{47.4\ \text{s}} = 20.5\ \text{seats/ s}$$

When the wave arrives at a certain seat, the person in that seat begins to rise and then sit down. During the time interval of standing and sitting, the wave has progressed at its speed across more seats. By the time you sit down, the leading edge of the wave is arriving at a distance from you of

$$x_f = x_i + vt = 0 + (20.5\ \text{seats/ s})(0.95\ \text{s}) = 19.5\ \text{seats}$$

Therefore, $\boxed{\text{19 or 20 people}}$ are up out their seats and not paying attention to buying concessions.

Finalize This result applies to only one row. We can't simply multiply by the number of rows, because different rows will have different numbers of seats, but we can argue that at least several hundred people in the stadium are up out of the seats at any time during a wave, cutting onto concession sales.

Answer: 19 or 20 people

Section 16.2 Analysis Model: Traveling Wave

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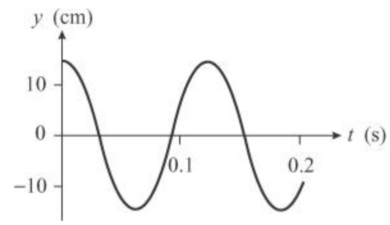
P16.5 The speed of waves along this wire is

$$v = f\lambda = (4.00 \text{ Hz})(60.0 \text{ cm}) = 240 \text{ cm/s} = \boxed{2.40 \text{ m/s}}$$

P16.6 (a) ANS. FIG. P16.6 shows the y vs. t plot of the given wave.



ANS. FIG. P16.6

(b) The time from one peak to the next one is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3 \text{ s}^{-1}} = \boxed{0.125 \text{ s}}$$

(c) $\boxed{\text{This agrees with the period found in the example in the text.}}$

P16.7 At time t, the motion at point A, where $x = 0$, is

$$y_A = (1.50 \text{ cm})\cos(-50.3t)$$

At point B, the motion is

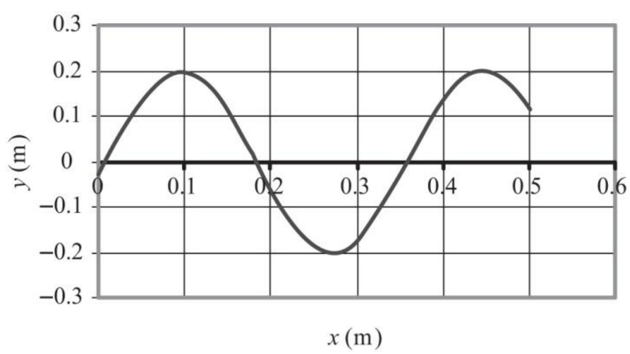
$$y_B = (15.0 \text{ cm})\cos(15.7x_B - 50.3t) = (15.0 \text{ cm})\cos\left(-50.3t \pm \frac{\pi}{3}\right)$$

which implies

$$15.7x_B = \left(15.7 \text{ m}^{-1}\right)x_B = \pm\frac{\pi}{3}$$

or $$x_B = -0.066\ 7 \text{ m} = \boxed{\pm 6.67 \text{ cm}}$$

P16.8 (a) ANS. FIG. P16.8 (a) shows a sketch of the wave at $t = 0$.



ANS FIG. P16.8 (a)

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(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.350 \text{ m}} = \boxed{18.0 \text{ rad/m}}$

(c) $T = \frac{1}{f} = \frac{1}{12.0/\text{s}} = \boxed{0.083\ 3 \text{ s}}$

(d) $\omega = 2\pi f = 2\pi 12.0/\text{s} = \boxed{75.4 \text{ rad/s}}$

(e) $|v| = f\lambda = (12.0/\text{s})(0.350 \text{ m}) = \boxed{4.20 \text{ m/s}}$

(f) $y = A \sin(kx + \omega t + \phi)$ specializes to

$$\boxed{y = (0.200 \text{ m}) \sin(18.0x/\text{m} + 75.4t/\text{s} + \phi)}$$

(g) At $x = 0$, $t = 0$ we require

$$-3.00 \times 10^{-2} \text{ m} = (0.200 \text{ m}) \sin(+\phi)$$
$$\phi = -8.63° = -0.151 \text{ rad}$$

so

$$\boxed{y(x,\ t) = 0.200 \sin(18.0x + 75.4t - 0.151), \text{ where } x \text{ and } y \text{ are in meters and } t \text{ is in seconds.}}$$

P16.9 Using the traveling wave model, we can put constants with the right values into $y = A \sin(kx + \omega t + \phi)$ to have the mathematical representation of the wave. We have the same (positive) signs for both kx and ωt so that a point of constant phase will be at a decreasing value of x as t increases—that is, so that the wave will move to the left.

The amplitude is $A = y_{max} = 8.00 \text{ cm} = 0.080\ 0 \text{ m}$

The wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.800 \text{ m}} = 2.50\pi \text{ m}^{-1}$

The angular frequency is $\omega = 2\pi f = 2\pi(3.00 \text{ s}^{-1}) = 6.00\pi \text{ rad/s}$

(a) In $y = A \sin(kx + \omega t + \phi)$, choosing $\phi = 0$ will make it true that $y(0, 0) = 0$. Then the wave function becomes upon substitution of the constant values for this wave

$$\boxed{y = (0.080\ 0) \sin(2.50\pi x + 6.00\pi t)}$$

(b) In general, $y = (0.080\ 0)\sin(2.50\pi x + 6.00\pi t + \phi)$

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If $y(x, 0) = 0$ at $x = 0.100$ m, we require

$$0 = (0.080\ 0)\sin(2.50\pi + \phi)$$

so we must have the phase constant be $\phi = -0.250\pi$ rad.

Therefore, the wave function for all values of x and t is

$\boxed{y = 0.080\ 0 \sin(2.50\pi x + 6.00\pi t - 0.250\pi)}$, where x and y are in meters and t is in seconds.

Section 16.3 The Speed of Waves on Strings

P16.10 If the tension in the wire is T, the tensile stress is

$$\text{stress} = \frac{T}{A} \qquad \text{so} \qquad T = A(\text{stress})$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{m/L}} = \sqrt{\frac{\text{Stress}}{m/AL}} = \sqrt{\frac{\text{Stress}}{m/\text{Volume}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70 \times 10^8 \text{ Pa}}{7860 \text{ kg/m}^3}} = \boxed{185 \text{ m/s}}$$

P16.11 The two wave speeds can be written as

$$v_1 = \sqrt{T_1/\mu} \qquad \text{and} \qquad v_2 = \sqrt{T_2/\mu}$$

Since μ is constant, $\mu = \frac{T_2}{v_2^2} = \frac{T_1}{v_1^2}$, and

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{30.0 \text{ m/s}}{20.0 \text{ m/s}}\right)^2 (6.00 \text{ N}) = \boxed{13.5 \text{ N}}$$

P16.12 The tension in the string is $T = mg$, where g is the acceleration of gravity on the Moon, about one-sixth that of Earth. From the data

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given, what is the acceleration of gravity on the Moon?

The wave speed is

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{Mg}{m/L}}=\sqrt{\frac{MgL}{m}}=\frac{L}{t}\rightarrow\frac{MgL}{m}=\frac{L^2}{t^2}\rightarrow g=\frac{mL}{Mt^2}$$

$$g=\frac{mL}{Mt^2}=\frac{\left(4.00\times10^{-3}\text{ kg}\right)\left(1.60\text{ m}\right)}{\left(3.00\text{ kg}\right)\left(26.1\times10^{-3}\text{ s}\right)^2}=3.13\text{ m/s}^2$$

The calculated gravitational acceleration of the Moon is almost twice that of the accepted value.

P16.13 (a) The tension in the string is

$$F=mg=\left(3.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)=29.4\text{ N}$$

Then, from $v=\sqrt{\frac{F}{\mu}}$, the mass per unit length is

$$\mu=\frac{F}{v^2}=\frac{29.4\text{ N}}{\left(24.0\text{ m/s}\right)^2}=\boxed{0.0510\text{ kg/m}}$$

(b) When m = 2.00 kg, the tension is

$$F=mg=\left(2.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)=19.6\text{ N}$$

and the speed of transverse waves in the string is

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{19.6\text{ N}}{0.0510\text{ kg/m}}}=\boxed{19.6\text{ m/s}}$$

Section 16.4 Rate of Energy Transfer by Sinusoidal Waves on Strings

P16.14 We use $v=\sqrt{\frac{T}{\mu}}$ to solve for the tension:

$$T=\mu v^2=\rho Av^2=\rho\pi r^2v^2$$

$$T=\left(8920\text{ kg/m}^3\right)\left(\pi\right)\left(7.50\times10^{-4}\text{ m}\right)^2\left(200\text{ m/s}\right)^2$$

$$T=\boxed{631\text{ N}}$$

P16.15 We are given T = constant; we use the equation for the speed of a wave

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on a string, $v = \sqrt{\dfrac{T}{\mu}}$, and the power supplied to a vibrating string, $P = \dfrac{1}{2}\mu\omega^2 A^2 v$.

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(a) If L is doubled, μ is still the same, so v remains constant: therefore P is constant: $\boxed{1}$.

(b) If A is doubled and ω is halved, $P \propto \omega^2 A^2$ remains constant: $\boxed{1}$.

(c) If λ and A are doubled, the product $\omega^2 A^2 \propto \dfrac{A^2}{\lambda^2}$ remains constant, so $\boxed{1}$.

(d) If L and λ are halved, μ is still the same, and $\omega^2 \propto \dfrac{1}{\lambda^2}$ is quadrupled, so P $\boxed{\text{is increased by a factor of 4}}$.

P16.16 (a) $\boxed{\text{As for a string wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy and the frequency stays constant. As the speed drops the amplitude must increase.}}$

(b) We write $P = FvA^2$, where F is some constant. With no absorption of energy,

$$Fv_{\text{granite}}A^2_{\text{granite}} = Fv_{\text{mudfill}}A^2_{\text{mudfill}}$$

$$\frac{A_{\text{mudfill}}}{A_{\text{granite}}} = \sqrt{\frac{v_{\text{granite}}}{v_{\text{mudfill}}}} = \sqrt{\frac{v_{\text{granite}}}{v_{\text{granite}}/25.0}} = \sqrt{\frac{25.0v_{\text{granite}}}{v_{\text{granite}}}} = 5.00$$

$\boxed{\text{The amplitude increases by 5.00 times.}}$

P16.17 We are given $\mu = 30.0\ \text{g/m} = 30.0 \times 10^{-3}\ \text{kg/m}$, with

$$\lambda = 1.50\ \text{m}$$

$$f = 50.0\ \text{Hz:} \qquad \omega = 2\pi f = 314\ \text{s}^{-1}$$

$$2A = 0.150\ \text{m:} \quad A = 7.50 \times 10^{-2}\ \text{m}$$

(a) From $y = A \sin\left(\dfrac{2\pi}{\lambda}x - \omega t\right)$, $\boxed{y = (0.075)\sin(4.19x - 314t)}$

(b) $$P = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(30.0\times10^{-3}\right)(314)^2\left(7.50\times10^{-2}\right)^2\left(\frac{314}{4.19}\right)\ \text{W}$$

$\boxed{P = 625\ \text{W}}$

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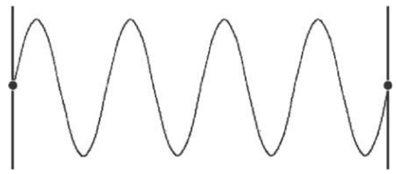

ANS. FIG. P16.17

P16.18 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source. It is spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{P}{2\pi r}$$

is proportional to amplitude squared, so amplitude is proportional to

$$\sqrt{\frac{P}{2\pi r}}$$

P16.19 Originally,

$$P_0 = \frac{1}{2}\mu\omega^2 A^2 v$$

$$P_0 = \frac{1}{2}\mu\omega^2 A^2 \sqrt{\frac{T}{\mu}}$$

$$P_0 = \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$$

The doubled string will have doubled mass per length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times:

$$P = \frac{1}{2}\omega^2 A^2 \sqrt{T(2\mu)} = \sqrt{2}\left(\frac{1}{2}\omega^2 A^2 \sqrt{T\mu}\right) = \boxed{\sqrt{2}P_0}$$

Section 16.5 The Linear Wave Equation

P16.20 The important thing to remember with partial derivatives is that **you treat all variables as constants, except the single variable of interest**. Keeping this in mind, we must apply two standard rules of differentiation to the function $y = \ln[b(x - vt)]$:

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$$\frac{\partial}{\partial x}\left[\ln f(x)\right]=\frac{1}{f(x)}\frac{\partial\left[f(x)\right]}{\partial x} \qquad \textbf{[1]}$$

$$\frac{\partial}{\partial x}\left[\frac{1}{f(x)}\right]=\frac{\partial}{\partial x}\left[f(x)\right]^{-1}=(-1)\left[f(x)\right]^{-2}\frac{\partial\left[f(x)\right]}{\partial x}$$

$$=-\frac{1}{\left[f(x)\right]^{2}}\frac{\partial\left[f(x)\right]}{\partial x} \qquad [2]$$

Applying [1],

$$\frac{\partial y}{\partial x}=\left(\frac{1}{b(x-vt)}\right)\frac{\partial(bx-bvt)}{\partial x}=\left(\frac{1}{b(x-vt)}\right)(b)=\frac{1}{x-vt}$$

Applying [2],

$$\frac{\partial^2 y}{\partial x^2}=-\frac{1}{(x-vt)^2}$$

In a similar way,

$$\frac{\partial y}{\partial t}=\frac{-v}{x-vt} \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2}=\frac{v^2}{(x-vt)^2}$$

From the second-order partial derivatives, we see that it is true that

$$\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

so the proposed function **is** one solution to the wave equation.

P16.21 The linear wave equation is $\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

If $y=e^{b(x-vt)}$

Then $\frac{\partial y}{\partial t}=-bve^{b(x-vt)}$ and $\frac{\partial y}{\partial x}=be^{b(x-vt)}$

$\frac{\partial^2 y}{\partial t^2}=b^2v^2e^{b(x-vt)}$ and $\frac{\partial^2 y}{\partial x^2}=b^2e^{b(x-vt)}$

Therefore, $\frac{\partial^2 y}{\partial t^2}=v^2\frac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution.

P16.22 (a) From $y=x^2+v^2t^2$,

evaluate $\frac{\partial y}{\partial x}=2x$ and $\frac{\partial^2 y}{\partial x^2}=2$

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Also, $\dfrac{\partial y}{\partial t} = v^2 2t$ and $\dfrac{\partial^2 y}{\partial t^2} = 2v^2$

Does $\dfrac{\partial^2 y}{\partial t^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$?

By substitution, we must test $2 = \dfrac{1}{v^2}\left(2v^2\right)$ and this is true, so the wave function does satisfy the wave equation.

(b) Note

$$\frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2 = \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2$$
$$= x^2 + v^2t^2$$

as required. So

$$\boxed{f(x+vt) = \frac{1}{2}(x+vt)^2} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}$$

(c) $y = \sin x \cos vt$ makes

$$\frac{\partial y}{\partial x} = \cos x \cos vt \qquad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$$

$$\frac{\partial y}{\partial t} = -v \sin x \sin vt \qquad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$$

Then $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$ becomes $-\sin x \cos vt = \dfrac{-1}{v^2} v^2 \sin x \cos vt$ which is true, as required.

Note $\sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt$

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$$\boxed{f(x+vt) = \frac{1}{2}\sin(x+vt)} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}\sin(x-vt)}$$

Section 16.6 Sound Waves

P16.23 (a) $A = \boxed{2.00\ \mu\text{m}}$

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(b) $\lambda = \dfrac{2\pi}{15.7} = 0.400\text{ m} = \boxed{40.0\text{ cm}}$

(c) $v = \dfrac{\omega}{k} = \dfrac{858}{15.7} = \boxed{54.6\text{ m/s}}$

(d) $s = 2.00\cos\left[(15.7)(0.050\,0) - (858)\left(3.00\times10^{-3}\right)\right] = \boxed{-0.433\ \mu\text{m}}$

(e) $v_{\max} = A\omega = (2.00\ \mu\text{m})\left(858\text{ s}^{-1}\right) = \boxed{1.72\text{ mm/s}}$

Section 16.7 Speed of Sound Waves

P16.24 The speed of longitudinal waves in a fluid is $v = \sqrt{B/\rho}$. Considering the Earth's crust to consist of a very viscous fluid, our estimate of the average bulk modulus of the material in Earth's crust is

$$B = \rho v^2 = \left(2\,500\,\text{kg/m}^3\right)\left(7\times10^3\text{ m/s}\right)^2 = \boxed{1\times10^{11}\text{ Pa}}$$

P16.25 We use $\Delta P_{\max} = \rho v\omega s_{\max} = \rho v\left(\dfrac{2\pi v}{\lambda}\right)s_{\max}$:

$$\lambda_{\min} = \frac{2\pi\rho v^2 s_{\max}}{\Delta P_{\max}} = \frac{2\pi\left(1.20\text{ kg/m}^3\right)(343\text{ m/s})^2\left(5.50\times10^{-6}\text{ m}\right)}{0.840\text{ Pa}} = \boxed{5.81\text{ m}}$$

P16.26 (a) The speed gradually changes from

$$v = (331\text{ m/s})\left(1+\frac{27.0°\text{C}}{273°\text{C}}\right)^{1/2} = 347\text{ m/s}$$

to

$$v = (331\text{ m/s})\left(1+\frac{0°\text{C}}{273°\text{C}}\right)^{1/2} = 331\text{ m/s}$$

a 4.6% decrease. The cooler air at the same pressure is more dense.

(b) The frequency is unchanged because every wave crest in the hot air becomes one crest without delay in the cold air.

(c) The wavelength decreases by 4.6%, from

$$v/f = (347\text{ m/s})/(4000/\text{s}) = 86.7\text{ mm}$$

to

$$v/f = (331\text{ m/s})/(4000/\text{s}) = 82.8\text{ mm}$$

The crests are more crowded together when they move more slowly.

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***P16.27** **Conceptualize** The sound from the clap travels to the cliff and back, so the distance traveled by the sound is twice the distance to the cliff.

Categorize The pulse of sound from the clap will be modeled as a *particle under constant speed*.

Analyze Several sites are available online for calculating the distance between two coordinates. A typical value for a distance between the coordinates given in the problem is 78.8 m.

From this value, calculate the speed of sound:

$$v = \frac{2d}{\Delta t} = \frac{2(78.8 \text{ m})}{0.47 \text{ s}} = \boxed{335 \text{ m/ s}}$$

Finalize This is surprisingly close, differing from the speed of sound at 20° by a little over 2%. Of course, we have no information about the temperature of the air on the day you perform the experiment, so we don't really know the actual value of the speed of sound.

Answer: 335 m/s

P16.28 It is easiest to solve part (b) first:

(b) The distance the sound travels to the plane is

$$d_s = \sqrt{h^2 + \left(\frac{h}{2}\right)^2} = \frac{h\sqrt{5}}{2}$$

The sound travels this distance in 2.00 s, so

$$d_s = \frac{h\sqrt{5}}{2} = (343 \text{ m/s})(2.00 \text{ s}) = 686 \text{ m}$$

giving the altitude of the plane as $h = \frac{2(686 \text{ m})}{\sqrt{5}} = \boxed{614 \text{ m}}$

(a) The distance the plane has traveled in 2.00 s is

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$$v(2.00\ \text{s}) = \frac{h}{2} = 307\ \text{m}$$

Thus, the speed of the plane is:

$$v = \frac{307\ \text{m}}{2.00\ \text{s}} = \boxed{153\ \text{m/s}}$$

P16.29 (a) At 9 000 m, $\Delta T = (9\ 000\ \text{m})\left(\frac{-1.00°\text{C}}{150\ \text{m}}\right) = -60.0°\text{C}$, so $T = -30.0°\text{C}$.

Using the chain rule,

$$\frac{dv}{dt} = \frac{dv}{dT_C}\frac{dT_C}{dx}\frac{dx}{dt} = v\frac{dv}{dT_C}\frac{dT_C}{dx} = v(0.607)\left(\frac{1}{150}\right) = \frac{v}{247}$$

so $dt = (247\ \text{s})\dfrac{dv}{v}$. Integrating,

$$\int_0^t dt = (247\ \text{s})\int_{v_i}^{v_f}\frac{dv}{v}$$

$$t = (247\ \text{s})\ln\left(\frac{v_f}{v_i}\right) = (247\ \text{s})\ln\left[\frac{331.5 + 0.607(30.0)}{331.5 + 0.607(-30.0)}\right]$$

which gives $t = \boxed{27.2\ \text{s}}$ for sound to reach the ground.

(b) $$t = \frac{h}{v} = \frac{9\ 000\ \text{m}}{331.5\ \text{m/s} + 0.607(30.0°\text{C})} = \boxed{25.7\ \text{s}}$$

$\boxed{\text{The time interval in (a) is longer.}}$

P16.30 Since $\cos^2\theta + \sin^2\theta = 1$, $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ (each sign applying half the time),

$$\Delta P = \Delta P_{\max}\ \sin\,(kx - \omega t) = \pm\rho v\omega s_{\max}\sqrt{1-\cos^2\,(kx-\omega t)}$$

Therefore,

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$$\Delta P = \pm\,\rho v\omega\,\sqrt{s_{\max}^2 - s_{\max}^2\cos^2(kx-\omega t)} = \pm\,\rho v\omega\,\sqrt{s_{\max}^2 - s^2}$$

Section 16.8 Intensity of Sound Waves

P16.31 We use $I = \frac{1}{2}\rho\omega^2 s_{\max}^2 v.$

(a) At f = 2 500 Hz, the frequency is increased by a factor of 2.50, so the intensity (at constant $s_{\max}$) increases by $(2.50)^2 = 6.25$.

Therefore, $6.25(0.600) = \boxed{3.75\ \text{W/m}^2}$

(b) The changes cancel each other: frequency $f \rightarrow f' = f/2$, and displacement amplitude $s_{\max} \rightarrow s'_{\max} = 2s_{\max}$

original intensity: $I = \frac{1}{2}\rho\omega^2 s_{\max}^2 v = 0.600\ \text{W/m}^2$

new intensity: $I' = \frac{1}{2}\rho\omega'^2 s'^2_{\max} v = \frac{1}{2}\rho\left(\frac{\omega}{2}\right)^2 (2s_{\max})^2 v = \frac{1}{2}\rho\omega^2 s_{\max}^2 v$
$= \boxed{600\ \text{W/m}^2}$

P16.32 The original intensity is $I_1 = \frac{1}{2}\rho\omega^2 s_{\max}^2 v = 2\pi^2\rho v f^2 s_{\max}^2$

(a) If the frequency is increased to f' while a constant displacement amplitude is maintained, the new intensity is

$$I_2 = 2\pi^2\rho v (f')^2\, s_{\max}^2 \text{ so } \frac{I_2}{I_1} = \frac{2\pi^2\rho v(f')s_{\max}^2}{2\pi^2\rho v f^2 s_{\max}^2} = \left(\frac{f'}{f}\right)^2$$

or $\boxed{I_2 = \left(\frac{f'}{f}\right)^2 I_1}$

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(b) If the frequency is reduced to $f' = \frac{f}{2}$ while the displacement amplitude is doubled, the new intensity is

$$I_2 = 2\pi^2 \rho v \left(\frac{f}{2}\right)^2 (2s_{\max})^2 = 2\pi^2 \rho v f^2 s_{\max}^2 = I_1$$

or the $\boxed{\text{intensity is unchanged}}$.

P16.33 (a) From the sound level equation,

$$120 \text{ dB} = (10 \text{ dB})\log\left[\frac{I}{10^{-12} \text{ W/m}^2}\right]$$

$$I = 1.00 \text{ W/m}^2 = \frac{P}{4\pi r^2}$$

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{6.00 \text{ W}}{4\pi(1.00 \text{ W/m}^2)}} = \boxed{0.691 \text{ m}}$$

We have assumed the speaker is an isotropic point source.

(b) Again from the sound level equation,

$$0 \text{ dB} = (10 \text{ dB})\log\left(\frac{I}{10^{-12} \text{ W/m}^2}\right)$$

$$I = 1.00 \times 10^{-12} \text{ W/m}^2$$

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{6.00 \text{ W}}{4\pi(1.00 \times 10^{-12} \text{ W/m}^2)}} = \boxed{691 \text{ km}}$$

We have assumed a uniform medium that absorbs no energy.

P16.34 (a) The energy transferred by sound from the explosion is

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$$
\begin{aligned}
T_{MW} &= P\Delta t = 4\pi r^2 I\Delta t \\
&= 4\pi(100\text{ m})^2\left(7.00\times10^{-2}\text{ W/m}^2\right)(0.200\text{ s}) \\
&= \boxed{1.76\text{ kJ}}
\end{aligned}
$$

(b) $\beta = (10\text{ dB})\log\left(\dfrac{7.00\times10^{-2}}{1.00\times10^{-12}}\right) = \boxed{108\text{ dB}}$

***P16.35** **Conceptualize** As the audience members climb up the ladder steps, they are moving closer to the loudspeakers. As a result, the intensity of the sound they hear increases, as does the sound level.

Categorize We are told that the sound is emitted uniformly in all directions, so we can use the discussion in Section 16.8 to help us with this problem.

Analyze Based on Equation 16.40, we can relate the intensity measured near the speakers to that located below the speakers a distance *r*:

$$
\frac{I_r}{I_{\text{near}}} = \frac{r_{\text{near}}^2}{r^2} \quad\rightarrow\quad r = r_{\text{near}}\sqrt{\frac{I_{\text{near}}}{I_r}} \qquad (1)
$$

Rearrange Equation 16.41 so that it expresses the intensity in terms of the sound level:

$$
\beta = 10\log\left(\frac{I}{I_0}\right) \quad\rightarrow\quad I = I_0\left(10^{\beta/10}\right) \qquad (2)
$$

Use Equation (2) to substitute for the two intensities in Equation (1):

$$
r = r_{\text{near}}\sqrt{\frac{I_0\left(10^{\beta_{\text{near}}/10}\right)}{I_0\left(10^{\beta_r/10}\right)}} = r_{\text{near}}\sqrt{10^{(\beta_{\text{near}}-\beta_r)/10}} \qquad (3)
$$

Substitute numerical values, to find the distance from the speaker at which the sound level is at the threshold of pain, 120 dB:

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$$r = (0.200 \text{ m})\sqrt{10^{(150-120)/10}} = 6.32 \text{ m}$$

This is the distance of the audience member from the speaker at which he hears the sound at the threshold of pain. We need to subtract this from the height of the speaker to find the height above the ground at which to mount the barriers:

$$h_{\text{barrier}} = h_{\text{column}} - r = 10.6 \text{ m} - 6.32 \text{ m} = \boxed{4.28 \text{ m}}$$

Finalize Even hearing sound at sustained levels of 120 dB can be damaging to your hearing. Many people who attend rock concerts (as well as the performers) suffer hearing loss due to these activities.

Answer: 4.28m

P16.36 We assume that both lawn mowers are equally loud and approximately the same distance away. We found in Example 16.8 that a sound of twice the intensity results in an increase in sound level of 3 dB. We also see from the What If? section of that example that a doubling of loudness requires a 10-dB increase in sound level. Therefore, the sound of two lawn mowers will not be twice the loudness, but only a little louder than one!

P16.37 We begin with $\beta_2 = (10 \text{ dB})\log\left(\frac{I_2}{I_0}\right)$ and $\beta_1 = (10 \text{ dB})\log\left(\frac{I_1}{I_0}\right)$, so

$$\beta_2 - \beta_1 = (10 \text{ dB})\log\left(\frac{I_2}{I_1}\right)$$

Also, $I_2 = \frac{P}{4\pi r_2^2}$ and $I_1 = \frac{P}{4\pi r_1^2}$, giving $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

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Then, $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{r_1}{r_2}\right)^2 = \boxed{20\log\left(\frac{r_1}{r_2}\right)}$

Section 16.9 The Doppler Effect

P16.38 (a) Equation 16.46, $f' = f\left(\frac{v + v_o}{v - v_s}\right)$, applies to an observer on $\boxed{\text{B}}$ because B is receiving sound from source A.

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(b) The sign of v_s should be $\boxed{\text{positive}}$ because the source is moving toward the observer, resulting in an increase in frequency.

(c) The sign of v_o should be $\boxed{\text{negative}}$ because the observer is moving away from the source, resulting in a decrease in frequency.

(d) The speed of sound should be that of the medium of seawater, $\boxed{1\ 533\ \text{m/s}}$.

(e) $$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right) = \left(5.27\times10^3\ \text{Hz}\right)\left[\frac{(1\ 533\ \text{m/s})+(-3.00\ \text{m/s})}{(1\ 533\ \text{m/s})-(+11.0\ \text{m/s})}\right]$$
$$= \boxed{5.30\times10^3\ \text{Hz}}$$

P16.39 The *half angle* of the shock wave cone is given by $\sin\theta = \dfrac{v_{\text{light}}}{v_S}$.

$$v_S = \frac{v_{\text{light}}}{\sin\theta} = \frac{2.25\times10^8\ \text{m/s}}{\sin(53.0°)} = \boxed{2.82\times10^8\ \text{m/s}}$$

P16.40 The apparent frequency drops because of the Doppler effect. Using a T subscript for the situation when the athlete moves *toward* the horn, and A for movement away from the horn, we have,

$$\frac{f'_A}{f'_T} = \frac{\left(\dfrac{v+v_{OA}}{v-v_S}\right)f}{\left(\dfrac{v+v_{OT}}{v-v_S}\right)f} = \frac{v+v_{OA}}{v+v_{OT}} = \frac{v+(-v_O)}{v+(+v_O)} = \frac{v-v_O}{v+v_O}$$

where v_0 is the constant speed of the athlete. Setting this ratio equal to 5/6, we have

$$\frac{5}{6} = \frac{v-v_O}{v+v_O} \quad\rightarrow\quad 5v+5v_O = 6v-6v_O \quad\rightarrow\quad 11v_O = v$$

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Solving for the speed of the athlete,

$$v_O = \frac{v}{11} = \frac{343 \text{ m/s}}{11} = 31.2 \text{ m/s}$$

$\boxed{\text{This is much faster than a human athlete can run.}}$

P16.41 (a) The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$
$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0 \text{ N/m}}{5.00 \text{ kg}}}(0.500 \text{ m}) = 1.00 \text{ m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\max} = f\left(\frac{v}{v - v_{\max}}\right) = 440 \text{ Hz}\left(\frac{343}{343 - 1.00}\right) = \boxed{441 \text{ Hz}}$$

to

(b) $$f'_{\min} = f\left(\frac{v}{v + v_{\max}}\right) = 440 \text{ Hz}\left(\frac{343}{343 + 1.00}\right) = \boxed{439 \text{ Hz}}$$

(c) $$\beta = (10 \text{ dB})\log\left(\frac{I}{I_0}\right) = (10 \text{ dB})\log\left(\frac{P/4\pi r^2}{I_0}\right)$$

The maximum intensity level $\beta_{\max} = 60.0$ dB occurs at $r = r_{\min} = 1.00$ m. The minimum intensity level occurs when the speaker is farthest from the listener, i.e., when $r = r_{\max} = r_{\min} + 2A = 2.00$ m.

Thus, $$\beta_{\max} - \beta_{\min} = (10 \text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\min}^2}\right) - (10 \text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\max}^2}\right)$$

or

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$$\beta_{\max} - \beta_{\min} = (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\min}^2}\frac{4\pi I_0 r_{\max}^2}{P}\right)$$

$$= (10\text{ dB})\log\left(\frac{r_{\max}^2}{r_{\min}^2}\right) = (20\text{ dB})\log\left(\frac{r_{\max}}{r_{\min}}\right)$$

This gives:

$$60.0\text{ dB} - \beta_{\min} = (20\text{ dB})\log(2.00) = 6.02\text{ dB}$$

or $\beta_{\min} = \boxed{54.0\text{ dB}}$

P16.42 The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A$$

The frequencies heard by the stationary observer range from

(a) $f'_{\max} = \boxed{\dfrac{vf}{v - A\sqrt{\dfrac{k}{m}}}}$ to (b) $f'_{\max} = \boxed{\dfrac{vf}{v + A\sqrt{\dfrac{k}{m}}}}$

where v is the speed of sound.

(c) $\beta = (10\text{ dB})\log\left(\dfrac{I}{I_0}\right) = (10\text{ dB})\log\left(\dfrac{P/4\pi r^2}{I_0}\right)$

The maximum intensity level $\beta_{\max} = \beta$ occurs at $r = r_{\min} = d$. The minimum intensity level occurs when the speaker is farthest from the listener, i.e., when $r = r_{\max} = r_{\min} + 2A = d + 2A$.

Thus,

$$\beta_{\max} - \beta_{\min} = (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\min}^2}\right) - (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\max}^2}\right)$$

or

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$$\beta_{\text{max}} - \beta_{\text{min}} = (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\text{min}}^2}\frac{4\pi I_0 r_{\text{max}}^2}{P}\right)$$

$$= (10\text{ dB})\log\left(\frac{r_{\text{max}}^2}{r_{\text{min}}^2}\right) = (20\text{ dB})\log\left(\frac{r_{\text{max}}}{r_{\text{min}}}\right)$$

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This gives:

$$\beta - \beta_{\min} = (20 \text{ dB})\log\left(\frac{d+2A}{d}\right)$$

or $\beta_{\min} = \boxed{\beta - (20 \text{ db})\log\left(1+\frac{2A}{d}\right)}$

Additional Problems

P16.43 The speed of the wave on the rope is $v = \sqrt{\frac{T}{\mu}}$ and in this case $T = mg$; therefore, $m = \frac{\mu v^2}{g}$.

Now $v = f\lambda$ implies $v = \frac{\omega}{k}$ so that

$$m = \frac{\mu}{g}\left(\frac{\omega}{k}\right)^2 = \frac{0.250 \text{ kg/m}}{9.80 \text{ m/s}^2}\left[\frac{18\pi \text{ s}^{-1}}{0.750\pi \text{ m}^{-1}}\right]^2 = \boxed{14.7 \text{ kg}}$$

P16.44 Assume a typical distance between adjacent people ~ 1 m.

Then the wave speed is $v = \frac{\Delta x}{\Delta t} \sim \frac{1 \text{ m}}{0.1 \text{ s}} \sim 10 \text{ m/s}$.

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10 \text{ m/s}} = 63 \text{ s} \boxed{\sim 1 \text{ min}}$$

P16.45 At normal body temperature of $T = 37.0°\text{C}$, the speed of sound in air is

$$v = (331 \text{ m/s})\sqrt{1+\frac{T_C}{273}} = (331 \text{ m/s})\sqrt{1+\frac{37.0}{273}} = 353 \text{ m/s}$$

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and the wavelength of sound having a frequency of $f = 20\ 000$ Hz is

$$\lambda = \frac{v}{f} = \frac{353 \text{ m/s}}{20\ 000 \text{ Hz}} = 1.76 \times 10^{-2} \text{ m} = \boxed{1.76 \text{ cm}}$$

Thus, the diameter of the eardrum is $\boxed{1.76 \text{ cm.}}$

P16.46 (a) $\boxed{\text{The energy a wave crest carries is constant in the absence of absorption. Then the rate at which energy passes a stationary point, which is the power of the wave, is constant.}}$

(b) $\boxed{\text{The power is proportional to the square of the amplitude and to the wave speed. The speed decreases as the wave moves into shallower water near shore, so the amplitude must increase.}}$

(c) For the wave described, with a single direction of energy transport, the power is the same at the deep-water location ① and at the place ② with depth 9.00 m. Because power is proportional to the square of the amplitude and the wave speed, to express the constant power we write,

$$A_1^2 v_1 = A_2^2 v_2 = A_2^2 \sqrt{g d_2}$$

$$(1.80 \text{ m})^2 (200 \text{ m/s}) = A_2^2 \sqrt{(9.80 \text{ m/s}^2)(9.00 \text{ m})}$$

$$= A_2^2 (9.39 \text{ m/s})$$

$$A_2 = 1.80 \text{ m} \left(\frac{200 \text{ m/s}}{9.39 \text{ m/s}} \right)^{1/2}$$

$$= \boxed{8.31 \text{ m}}$$

(d) $\boxed{\text{As the water depth goes to zero, our model would predict zero speed and infinite amplitude. In fact the amplitude must be finite as the wave comes ashore. As the speed decreases the wavelength also decreases. When it becomes comparable to the water depth, or smaller, our formula } \sqrt{gd} \text{ for wave speed no longer applies.}}$

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P16.47 (a) From $y = (0.150 \text{ m}) \sin(0.800x - 50.0t) = A \sin(kx - \omega t)$

we compute

$$\partial y / \partial t = (0.150 \text{ m})(-50.0 \text{ s}^{-1}) \cos(0.800x - 50.0t)$$

and $a = \partial^2 y / \partial t^2 = -(0.150 \text{ m})(-50.0 \text{ s}^{-1})^2 \sin(0.800x - 50.0t)$

Then $a_{\max} = (0.150 \text{ m})(50.0 \text{ s}^{-1})^2 = \boxed{375 \text{ m/s}^2}$

(b) For the 1.00-cm segment with maximum force acting on it,

$$\sum F = ma = \left(\frac{12.0 \times 10^{-3} \text{ kg}}{100 \text{ cm}} \right)(1.00 \text{ cm})\left(375 \text{ m/s}^2\right) = \boxed{0.045\ 0 \text{ N}}$$

(c) To find the tension in the string, we first compute the wave speed

$$v = \lambda f = \frac{\omega}{k} = \frac{50.0 \text{ s}^{-1}}{0.800 \text{ m}^{-1}} = 62.5 \text{ m/s}$$

then,

$$v = \sqrt{\frac{T}{\mu}} \text{ gives } T = \mu v^2 = \left(\frac{12.0 \times 10^{-3} \text{ kg}}{1.00 \text{ m}} \right)(62.5 \text{ m/s})^2 = \boxed{46.9 \text{ N}}$$

$\boxed{\text{The maximum transverse force is very small compared to the tension, more than a thousand times smaller.}}$

P16.48 Imagine a short transverse pulse traveling from the bottom to the top of the rope. When the pulse is at position x above the lower end of the rope, the wave speed of the pulse is given by $v = \sqrt{\frac{T}{\mu}}$, where $T = \mu x g$ is the tension required to support the weight of the rope below position x.

Therefore, $v = \sqrt{gx}$.

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But $v = \dfrac{dx}{dt}$, so that $dt = \dfrac{dx}{\sqrt{gx}}$

and $t = \displaystyle\int_0^L \frac{dx}{\sqrt{gx}} = \frac{1}{\sqrt{g}} \left.\frac{\sqrt{x}}{\frac{1}{2}}\right|_0^L \approx \boxed{2\sqrt{\frac{L}{g}}}$

P16.49 (a) $\mu = \dfrac{dm}{dL} = \rho A \dfrac{dx}{dx} = \rho A$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{\left[\rho\left(ax+b\right)\right]}} = \sqrt{\frac{T}{\left[\rho\left(10^{-3}x+10^{-2}\right)\text{cm}^2\right]}}$$

With all SI units,

$\boxed{v = \sqrt{\dfrac{T}{\left[\rho\left(1.00\times10^{-5}x+1.00\times10^{-6}\right)\right]}}}$ where x is in meter, T is in newtons, and v is in meters per second.

(b) $v(0) = \sqrt{\dfrac{24.0}{\left[(2700)\left(0+10^{-6}\right)\right]}} = \boxed{94.3\text{ m/s}}$

$$v(10.0\ m) = \sqrt{\frac{24.0}{\left[(2700)\left(10^{-4}+10^{-6}\right)\right]}} = \boxed{9.38\text{ m/s}}$$

P16.50 $v = \left(\dfrac{4\ 450\times10^3\text{ m}}{5.88\text{ h}}\right)\left(\dfrac{1\text{ h}}{3\ 600\text{ s}}\right) = 210\text{ m/s}$

$$d_{\text{avg}} = \frac{v^2}{g} = \frac{(210\text{ m/s})^2}{9.80\text{ m/s}^2} = 4\ 500\text{ m}$$

The given speed corresponds to an ocean depth that is greater than the average ocean depth, about 4 280 m.

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P16.51 (a) $P(x) = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\mu\omega^2 A_0^2 e^{-2bx}\left(\frac{\omega}{k}\right) = \boxed{\frac{\mu\omega^3}{2k}A_0^2 e^{-2bx}}$

(b) $P(0) = \boxed{\frac{\mu\omega^3}{2k}A_0^2}$

(c) $\frac{P(x)}{P(0)} = \boxed{e^{-2bx}}$

P16.52 (a) We have $f' = \frac{fv}{v-u}$ and $f'' = \frac{fv}{v-(-u)}$. We then have

$$f' - f'' = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right)$$

$$\Delta f = \frac{fv(v+u-v+u)}{v^2-u^2} = \frac{2uvf}{v^2\left(1-\frac{u^2}{v^2}\right)} = \boxed{\frac{2\left(\frac{u}{v}\right)}{1-\frac{u^2}{v^2}}f}$$

(b) $130 \text{ km/h} = 36.1 \text{ m/s}$

$$\Delta f = \frac{2(36.1 \text{ m/s})(400 \text{ Hz})}{(340 \text{ m/s})\left[1-\frac{(36.1 \text{ m/s})^2}{(340 \text{ m/s})^2}\right]} = \boxed{85.9 \text{ Hz}}$$

P16.53 The gliders stick together and move with final speed given by momentum conservation for the two-glider system:

$$m_1v_1 + m_2v_2 = m_1v_1 + 0 = (m_1+m_2)v$$

$$v = \frac{m_1v_1}{m_1+m_2} = \frac{(0.150 \text{ kg})(2.30 \text{ m/s})}{0.150 \text{ kg} + 0.200 \text{ kg}} = 0.986 \text{ m/s}$$

The missing mechanical energy is

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$$\begin{aligned}\Delta K &= \frac{1}{2}m_1v_1^2 - \frac{1}{2}(m_1+m_2)v^2 \\ &= \frac{1}{2}(0.150\text{ kg})(2.30\text{ m/s})^2 - \frac{1}{2}(0.350\text{ kg})(0.986\text{ m/s})^2 \\ &= 0.227\text{ J}\end{aligned}$$

We imagine one-half of 227 mJ going into internal energy and half into sound radiated isotropically in 7.00 ms. Its intensity 0.800 m away is

$$I = \frac{E}{At} = \frac{\frac{1}{2}(0.227\text{ J})}{4\pi(0.800\text{ m})^2(7.00\times10^{-3}\text{ s})} = 2.01\text{ W/m}^2$$

Its intensity level is

$$\beta = (10\text{ dB})\log\left(\frac{2.01\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = 123\text{ dB}$$

$\boxed{\text{It is unreasonable, implying a sound level of 123 dB. Nearly all of the decrease in mechanical energy becomes internal energy in the latch.}}$

P16.54 (a) $\boxed{\text{The wave moves outward equally in all directions. (We can tell it is outward because of the negative sign in } 1.36r - 2\ 030t.)}$

(b) $\boxed{\text{Its amplitude is inversely proportional to its distance from the center. Its intensity is proportional to the square of the amplitude, so the intensity follows the inverse-square law, with no absorption of energy by the medium.}}$

(c) $\boxed{\text{Its speed is constant at } v = f\lambda = \omega/k = (2030/\text{s})/(1.36/\text{m}) = 1.49\text{ km/s. By comparison to the table in the chapter, it can be moving through water at 25°C, and we assume that it is.}}$

(d) $\boxed{\text{Its frequency is constant at } (2030/\text{s})/2\pi = 323\text{ Hz.}}$

(e) $\boxed{\text{Its wavelength is constant at } 2\pi/k = 2\pi/(1.36/\text{m}) = 4.62\text{ m.}}$

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(f) Its pressure amplitude is (25.0 Pa/r). Its intensity at this distance is

$$I = \frac{\Delta P_{\text{max}}^2}{2\rho v} = \frac{\left[\left(25\ \text{N/m}^2\right)/r\right]^2}{2(1000\ \text{kg/m}^3)(1490\ \text{m/s})} = \frac{209\ \mu\text{W/m}^2}{r^2}$$

so the power of the source and the net power of the wave at all distances is

$$P = I4\pi r^2 = \left(\frac{2.09\times 10^{-4}\,\text{W/m}^2}{r^2}\right)4\pi r^2 = 2.63\ \text{mW}$$

(g) Its intensity follows the inverse-square law; at $r = 1$ m, the intensity is 209 μW/m^2.

P16.55 For the longitudinal wave $\quad v_L = \left(\frac{Y}{\rho}\right)^{1/2}$

For the transverse wave $\quad v_T = \left(\frac{T}{\mu}\right)^{1/2}$

If we require $\frac{v_L}{v_T} = 8.00$, we have $T = \frac{\mu Y}{64.0\rho}$ where $\mu = \frac{m}{L}$ and

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 L}$$

This gives

$$T = \frac{\pi r^2 Y}{64.0} = \frac{\pi\left(2.00\times 10^{-3}\ \text{m}\right)^2\left(6.80\times 10^{10}\ \text{N/m}^2\right)}{64.0}$$
$$= \boxed{1.34\times 10^4\ \text{N}}$$

P16.56 (a) Model your loud, sharp sound impulse as a single narrow peak in a graph of air pressure versus time. It is a noise with no frequency, wavelength, or period. It radiates away from you in all directions and some of it is incident on each one of the solid vertical risers of the bleachers.

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$\boxed{\text{The repeated reflections from the steps create a repetition frequency so that the ear/brain combination assigns a pitch to the sound heard by the listener.}}$

Suppose that, at the ambient temperature, sound moves at 343 m/s; and suppose that the horizontal width of each row of seats is 60 cm. Then there is a time delay of

$$\frac{0.60\text{ m}}{343\text{ m/s}} = 0.001\ 7\text{ s}$$

between your sound impulse reaching each riser and the next. Whatever its material, each will reflect much of the sound that reaches it. The reflected wave sounds very different from the sharp pop you made.

(b) If there are twenty rows of seats, you hear from the bleachers a tone with twenty crests, each separated from the next in time by

$$\frac{2(0.60\text{ m})}{343\text{ m/s}} = 0.003\ 5\text{ s}$$

This is the extra time for it to cross the width of one seat twice, once as an incident pulse and once again after its reflection. Thus, you hear a sound of definite pitch, with a period of about 0.0035 s, and frequency,

$$\frac{1}{0.003\ 5\text{ s}} = 290\text{ Hz}\ \boxed{\sim \text{a few hundred Hz}}$$

(c) Wavelength

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{290\text{ s}^{-1}} = 1.2\text{ m} \sim \boxed{1\text{ m}}$$

(d) and duration

$$20(0.003\ 5\text{ s}) \sim \boxed{0.1\text{ s}}$$

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Challenge Problems

P16.57 (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x) = mx + b$.

To have $\mu(0) = \mu_0$ we require $b = \mu_0$. Then $\mu(L) = \mu_L = mL + \mu_0$

so $m = \dfrac{\mu_L - \mu_0}{L}$.

Then $\boxed{\mu(x) = \dfrac{(\mu_L - \mu_0)x}{L} + \mu_0}$.

(b) Imagine the crest of a short transverse pulse traveling from one end of the string to the other. Consider the pulse to be at position x. From $v = \dfrac{dx}{dt}$, the time interval required to move from x to $x + dx$ is $\dfrac{dx}{v}$. The time interval required to move from 0 to L is

$$\Delta t = \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{T/\mu}} = \frac{1}{\sqrt{T}} \int_0^L \sqrt{\mu(x)}\, dx$$

$$\Delta t = \frac{1}{\sqrt{T}} \int_0^L \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{1/2} \left(\frac{\mu_L - \mu_0}{L} \right) dx \left(\frac{L}{\mu_L - \mu_0} \right)$$

$$\Delta t = \frac{1}{\sqrt{T}} \left(\frac{L}{\mu_L - \mu_0} \right) \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{3/2} \frac{1}{\left(\frac{3}{2}\right)} \Bigg|_0^L$$

$$\boxed{\Delta t = \frac{2L}{3\sqrt{T}(\mu_L - \mu_0)} \left(\mu_L^{3/2} - \mu_0^{3/2} \right)}$$

P16.58 Refer to Problem 60. At distance x from the bottom, the tension is $T = \left(\dfrac{mxg}{L} \right) + Mg$, so the wave speed is:

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$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \sqrt{xg + \left(\frac{MgL}{m}\right)} = \frac{dx}{dt} \rightarrow dt = \frac{dx}{\sqrt{xg + \left(\frac{MgL}{m}\right)}}$$

(a) Then

$$t = \int_0^t dt = \int_0^L \left[xg + \left(\frac{MgL}{m}\right)\right]^{-1/2} dx$$

gives $$t = \frac{1}{g}\frac{\left[xg + \left(MgL/m\right)\right]^{1/2}}{\frac{1}{2}}\Bigg|_{x=0}^{x=L}$$

$$t = \frac{2}{g}\left[\left(Lg + \frac{MgL}{m}\right)^{1/2} - \left(\frac{MgL}{m}\right)^{1/2}\right]$$

$$\boxed{t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m} - \sqrt{M}\right)}$$

(b) When $M = 0$,

$$t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m} - 0}{\sqrt{m}}\right) = \boxed{2\sqrt{\frac{L}{g}}}$$

(c) As $m \rightarrow 0$ we expand

$$\sqrt{M+m} = \sqrt{M}\left(1 + \frac{m}{M}\right)^{1/2} = \sqrt{M}\left(1 + \frac{1}{2}\frac{m}{M} - \frac{1}{8}\frac{m^2}{M^2} + \cdots\right)$$

to obtain $$t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M} + \frac{1}{2}\left(\frac{m}{\sqrt{M}}\right) - \frac{1}{8}\left(m^2/M^{3/2}\right) + \cdots - \sqrt{M}\right)$$

$$t \approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

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where we neglect terms $\frac{1}{8}\left(\frac{m^2}{M^{3/2}}\right)$ and higher because terms with m^2 and higher powers are very small.

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P16.59 Figure 17.10 shows that each wavefront that passes the observer is spherical. Let T represent the period of the source vibration, and T_{MW} be the energy put into each wavefront during one vibration. Then $(Power)_{\text{avg}} = \dfrac{T_{\text{MW}}}{T}$. At the moment when the observer is at distance r in front of the source, he is receiving a spherical wavefront of radius $R_w = v\Delta t$, where Δt is the time interval since this energy was radiated. Since the wavefront was radiated, the source has moved forward distance $d_s = v_s\Delta t$, so the total distance the wavefront has traveled is

$$R_w = r + d_s \quad \rightarrow \quad v\Delta t = r + v_s\Delta t$$

therefore,

$$\Delta t = \frac{r}{v - v_S}$$

The surface area of the sphere is $4\pi R_w^2 = 4\pi(v\Delta t)^2 = \dfrac{4\pi v^2 r^2}{(v - v_S)^2}$. The energy per unit area emitted during one cycle and carried by one spherical wavefront is uniform with the value

$$I = \frac{T_{\text{MW}}}{A} = \frac{(Power)_{\text{avg}}\, T(v - v_S)^2}{4\pi v^2 r^2}$$

The energy carried by the wavefront passes the observer in the time interval $T' = 1/f'$, where f' is the Doppler-shifted frequency

$$f' = f\left(\frac{v}{v - v_S}\right) = \frac{v}{T(v - v_S)}$$

so the observer receives a wave with intensity

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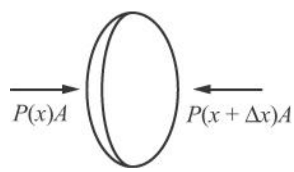
$$I=\left(\frac{T_{\mathrm{MW}}}{A}\right)\frac{1}{T'}=\left(\frac{T_{\mathrm{MW}}}{A}\right)f'=\left(\frac{(Power)_{\mathrm{avg}}T(v-v_S)^2}{4\pi v^2r^2}\right)\left(\frac{v}{T(v-v_S)}\right)$$

$$I=\boxed{\frac{(Power)_{\mathrm{avg}}}{4\pi r^2}\left(\frac{v-v_S}{v}\right)}$$

P16.60 (a) ANS. FIG. P16.60 shows a force diagram of an element of gas indicating the forces exerted on the left and right surfaces due to the pressure of the gas on either side of the element.

(b) Let $P(x)$ represent absolute pressure as a function of x. The net force to the right on the chunk of air is $+P(x)A-P(x+\Delta x)A$. Atmospheric pressure subtracts out, leaving

$$\left[-\Delta P(x+\Delta x)+\Delta P(x)\right]A=-\frac{\partial\Delta P}{\partial x}\Delta xA$$



ANS. FIG. P16.60

The mass of the air is $\Delta m=\rho\Delta V=\rho A\Delta x$ and its acceleration is $\frac{\partial^2 s}{\partial t^2}$. So Newton's second law becomes

$$-\frac{\partial\Delta P}{\partial x}\Delta xA=\rho A\Delta x\frac{\partial^2 s}{\partial t^2}$$

(c) From the result above, we have

$$-\frac{\partial\Delta P}{\partial x}\Delta xA=\rho A\Delta x\frac{\partial^2 s}{\partial t^2}\quad\rightarrow\quad-\frac{\partial\Delta P}{\partial x}=\rho\frac{\partial^2 s}{\partial t^2}$$

Substituting $\Delta P=-(B\partial s/\partial x)$ (Eq. 16.30), we have

$$-\frac{\partial}{\partial x}\left(-B\frac{\partial s}{\partial x}\right)=\rho\frac{\partial^2 s}{\partial t^2}\quad\rightarrow\quad\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}=\frac{\partial^2 s}{\partial t^2}$$

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(d) Into this wave equation we substitute a trial solution $s(x,\ t) = s_{\max}\cos(kx - \omega t)$. We find

$$\frac{\partial s}{\partial x} = -ks_{\max}\sin(kx - \omega t)$$
$$\frac{\partial^2 s}{\partial x^2} = -k^2 s_{\max}\cos(kx - \omega t)$$
$$\frac{\partial s}{\partial t} = +\omega s_{\max}\sin(kx - \omega t)$$
$$\frac{\partial^2 s}{\partial t^2} = -\omega^2 s_{\max}\cos(kx - \omega t)$$

$\dfrac{B}{\rho}\dfrac{\partial^2 s}{\partial x^2} = \dfrac{\partial^2 s}{\partial t^2}$ becomes

$$-\frac{B}{\rho}k^2 s_{\max}\cos(kx - \omega t) = -\omega^2 s_{\max}\cos(kx - \omega t)$$

This is true provided that $\dfrac{B}{\rho}k^2 = \omega^2 \to \dfrac{\omega}{k} = \sqrt{\dfrac{B}{\rho}}$, that is, provided it propagates with speed $v = \sqrt{\dfrac{B}{\rho}}$.

ANSWERS TO QUICK-QUIZZES

1. (i) (b) (ii) (a)

2. (i) (c) (ii) (b) (iii) (d)

3. (c)

4. (f) and (h)

5. (d)

6. (c)

7. (b)

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8. (b)

9. (e)

10. (e)

11. (b)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P16.2 (a) longitudinal P wave; (b) 666 s

P16.4 19 or 20 people

P16.6 (a) See ANS FIG P16.6; (b) 0.125 s; (c) This agrees with the period found in the example in the text.

P16.8 (a) See ANS FIG P16.8 (a); (b) 18.0 rad/m; (c) 0.083 3 s; (d) 75.4 rad/s; (e) 4.20 m/s; (f) $y = (0.200\ \text{m})\sin\left(18.0x/\text{m} + 75.4t/s + \phi\right)$; (g) $y(x, t) =$ 0.200 sin ($18.0x + 75.4t - 0.151$), where x and y are in meters and t is in seconds.

P16.10 185 m/s

P16.12 The calculated gravitational acceleration of the Moon is almost twice that of the accepted value.

P16.14 631N

P16.16 (a) As for a string wave, the rate of energy transfer is proportional to the square of the amplitude to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy, and the frequency stays constant. As the speed drops, the amplitude must increase; (b) The amplitude increases by 5.00 times

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P16.18 See P16.18 for the full explanation.

P16.20 The proposed function **is** one solution to the wave equation

P16.22 (a) See P16. 22 (a) for full explanation; (b) $f(x+vt)=\frac{1}{2}(x+vt)^2$ and $g(x-vt)=\frac{1}{2}(x-vt)^2$; (c) $f(x+vt)=\frac{1}{2}\sin(x+vt)$ and $g(x-vt)=\frac{1}{2}\sin(x-vt)$

P16.24 1×10^{11} Pa

P16.26 (a) The speed gradually changes from v = (331 m/s)(1 + 27°C/273°C)$^{1/2}$ = 347 m/s to (331 m/s) (1 + 0/273°C)$^{1/2}$ = 331 m/s, a 4.6% decrease. The cooler air at the same pressure is more dense; (b) The frequency is unchanged because every wave crest in the hot air becomes one crest without delay in the cold air; (c) The wavelength decreases by 4.6%, from v/f = (347 m/s) (4 000/s) = 86.7 mm to (331 m/s)(4 000/s) = 82.8 mm. The crests are more crowded together when they move more slowly.

P16.28 (a) 153 m/s; (b) 614 m

P16.30 See P16.30 for complete solution.

P16.32 (a) $I_2=\left(\frac{f'}{f}\right)^2 I_1$; (b) intensity is unchanged

P16.34 (a) 1.76 kJ; (b) 108 dB

P16.36 We assume that both lawn mowers are equally loud and approximately the same distance away. We found in Example 16.8 that a sound of twice the intensity results in an increase in sound level of 3

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dB. We also see from the What If? section of that example that a doubling of loudness requires a 10-dB increase in sound level. Therefore, the sound of two lawn mowers will not be twice the loudness, but only a little louder than one!

P16.38 (a) B; (b) positive; (c) negative; (d) 1 533 m/s; (e) 5.30×10^3 Hz

P16.40 This is much faster than a human athlete can run.

P16.42 (a) $\dfrac{vf}{v - A\sqrt{\dfrac{k}{m}}}$; (b) $\dfrac{vf}{v + A\sqrt{\dfrac{k}{m}}}$; (c) $\beta - (20 \text{ dB})\log\left(1 + \dfrac{2A}{d}\right)$

P16.44 ~1 min

P16.46 (a) The energy a wave crest carries is constant in the absence of absorption. Then the rate at which energy passes a stationary point, which is the power of the wave, is constant; (b) The power is proportional to the square of the amplitude and to the wave speed. The speed decreases as the wave moves into shallower water near shore, so the amplitude must increase; (c) 8.31 m; (d) As the water depth goes to zero, our model would predict zero speed and infinite amplitude. In fact, the amplitude must be finite as the wave comes ashore. As the speed decreases, the wavelength also decreases. When it becomes comparable to the water depth, or smaller, our formula $\sqrt{gd}$ for wave speed no longer applies.

P16.48 $2\sqrt{\dfrac{L}{g}}$

P16.50 The given speed corresponds to an ocean depth that is greater than the average ocean depth, about 4 280 m.

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P16.52 (a) $\dfrac{2\frac{u}{v}}{1-\frac{u^2}{v^2}}f$; (b) 85.9 Hz

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P16.54 (a) The wave moves outward equally in all directions; (b) Its amplitude is inversely proportional to its distance from the center. Its intensity is proportional to the square of the amplitude, so the intensity follows the inverse-square law, with no absorption of energy by the medium; (c) Its speed is constant $v = f\lambda = \omega/k = (2\ 030/\text{s})(1.36/\text{m}) = 1.49\,\text{km/s}$. By comparison to the table, it can be moving through water at 25° C, and we assume it is; (d) Its frequency is constant at $(2\ 030/\text{s})/2\pi = 323\ \text{Hz}$; (e) Its wavelength is constant at $2\pi/\text{k} = 2\pi/(1.36/\text{m}) = 4.62\ \text{m}$;

(f) $P = I4\pi r^2 = \left(\dfrac{2.09\times10^{-4}\ \text{W/m}^2}{r^2}\right)4\pi r^2 = 2.63\ \text{mW}$; (g) Its intensity follows the inverse-square law; at $r = 1$ m, the intensity is $209\ \mu\text{W/m}^2$

P16.56 (a) The repeated reflections from the steps create a repetition frequency so that the ear/brain combination assigns a pitch to the sound heard by the listener; (b) ~ a few hundred Hz; (c) ~ 1 m; (d) ~ 0.1 s

P16.58 (a) $t = 2\sqrt{\dfrac{L}{g}}\left(\sqrt{M+m} - \sqrt{M}\right)$; (b) $2\sqrt{\dfrac{L}{g}}$; (c) $\sqrt{\dfrac{mL}{Mg}}$

P16.60 (a) See ANS. FIG P16.60; (b) See P16.60(b) for full explanation; (c) See P16.60 (c) for full explanation; (d) See P16.60 (d) for full explanation.

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