24

Electric Potential

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP24.1** **Conceptualize** Be sure you are clear on the two experiments that are performed and what can be measured. In the first experiment, the potential difference between the plates is adjusted until a drop is at rest. Then, the potential difference ΔV_{rest} is measured. We assume we can measure d, the distance between the plates, and that we know g and the density ρ_{oil} of the oil. We do not know q or r.

In the second experiment, the electric field is turned off and the oil drifts downward at constant speed. Because we can see a calibrated scale through the telescope and have access to a stopwatch, we can determine the terminal speed v_T of the drop. We also assume we know the viscosity and density of air.

Categorize In both experiments, the oil drop is modeled as a *particle in equilibrium.*

Analyze (a) For the oil drop at rest, write out the equilibrium equation for forces in the vertical direction from the particle in equilibrium model:

$$\sum F_y = 0 \quad \rightarrow \quad F_{e,y} + F_{g,y} = 0 \quad \rightarrow \quad F_{e,y} = -F_{g,y} \qquad (1)$$

Substitute for the force components in Equation (1) using Equations 22.8, 24.6, and 5.5:

$$(-q)(E_y) = -m(-g) \quad \rightarrow \quad (-q)\left(-\frac{\Delta V_{\text{rest}}}{d}\right) = (-\rho_{\text{oil}}V)(-g) = \rho_{\text{oil}}\left(\tfrac{4}{3}\pi r^3\right)g$$

$$\rightarrow \quad \Delta V_{\text{rest}} = \frac{4\pi\rho_{\text{oil}}gd}{3q}r^3 \qquad (2)$$

(b) For the oil drop moving downward at constant terminal speed, write out the equilibrium equation for force components in the vertical direction from the particle in equilibrium model:

$$\sum F_y = 0 \quad \rightarrow \quad R_y + B_y + F_{g,y} = 0 \quad \rightarrow \quad R_y + B_y = -F_{g,y} \qquad (3)$$

Substitute for the forces in Equation (3):

$$6\pi\eta r v_T + \rho_{\text{air}} g V_{\text{disp}} = -m(-g) \quad \rightarrow \quad 6\pi\eta r v_T + \rho_{\text{air}}\left(\tfrac{4}{3}\pi r^3\right)g = \rho_{\text{oil}}\left(\tfrac{4}{3}\pi r^3\right)g$$

$$\rightarrow \quad \tfrac{4}{3}\pi r^2 g(\rho_{\text{oil}} - \rho_{\text{air}}) = 6\pi\eta v_T \quad \rightarrow \quad r = 3\sqrt{\frac{\eta v_T}{2g(\rho_{\text{oil}} - \rho_{\text{air}})}} \qquad (4)$$

(c) Substitute Equation (4) for r into Equation (2) and solve for q:

$$\Delta V_{\text{rest}} = \frac{4\pi\rho_{\text{oil}}gd}{3q}\left[3\sqrt{\frac{\eta v_T}{2g(\rho_{\text{oil}} - \rho_{\text{air}})}}\right]^3 = 18\pi\frac{\rho_{\text{oil}}d}{q}\sqrt{\frac{\eta^3 v_T^3}{2g(\rho_{\text{oil}} - \rho_{\text{air}})^3}}$$

$$\rightarrow \quad q = \boxed{18\pi \frac{\rho_{\text{oil}} d}{\Delta V_{\text{rest}}} \sqrt{\frac{\eta^3 v_T^3}{2g\left(\rho_{\text{oil}} - \rho_{\text{air}}\right)^3}}} \qquad (5)$$

Finalize Equation (5) allows us to determine the charge on the oil drop by measuring the plate separation d, the voltage ΔV_{rest} at which the drop is stationary, and the terminal speed v_T of the drop when the electric field is turned off.

Often, the buoyant force is ignored because it is so much smaller than the other forces. We can see this in Equation (5) by realizing that the density of air (~1 kg/m^3) is much smaller than that of oil (~800 kg/m^3). If we ignore ρ_{air} compared to ρ_{oil} in Equation (5), the equation simplifies to

$$q = 18\pi \frac{\rho_{\text{oil}} d}{\Delta V_{\text{rest}}} \sqrt{\frac{\eta^3 v_T^3}{2g\left(\rho_{\text{oil}} - 0\right)^3}} = 18\pi \frac{d}{\Delta V_{\text{rest}}} \sqrt{\frac{\eta^3 v_T^3}{2g\rho_{\text{oil}}}}$$

Answer: (c) $18\pi \dfrac{\rho_{\text{oil}} d}{\Delta V_{\text{rest}}} \sqrt{\dfrac{\eta^3 v_T^3}{2g\left(\rho_{\text{oil}} - \rho_{\text{air}}\right)^3}}$

***TP24.2** **Conceptualize** Be sure that you understand clearly the relationship between electric potential and electric field as discussed in Section 24.4. Because the graph of potential vs. position consists of a series of straight lines, we expect the electric field in each region to be constant in magnitude.

Categorize This problem will use the relationship between electric potential and electric field as discussed in Section 24.4.

Analyze Use Equation 24.16 to evaluate the electric field in each of the five distinct regions in the graph:

$$0 < x < 1\text{ cm}:\ E_x = -\frac{dV}{dx} = -\frac{50\text{ V} - 0}{1\text{ cm}} = -50\text{ V/ cm} = -5.0 \times 10^3\text{ V/ m}$$

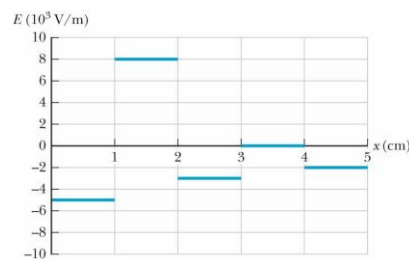
$$1\text{ cm} < x < 2\text{ cm}:\ E_x = -\frac{dV}{dx} = -\frac{-30\text{ V} - 50\text{ V}}{1\text{ cm}} = 80\text{ V/ cm} = 8.0 \times 10^3\text{ V/ m}$$

$$2\text{ cm} < x < 3\text{ cm}:\ E_x = -\frac{dV}{dx} = -\frac{0 - (-30\text{ V})}{1\text{ cm}} = -30\text{ V/ cm} = -3.0 \times 10^3\text{ V/ m}$$

$$3\text{ cm} < x < 4\text{ cm}:\ E_x = -\frac{dV}{dx} = -\frac{0 - 0}{1\text{ cm}} = 0\text{ V/ cm} = 0\text{ V/ m}$$

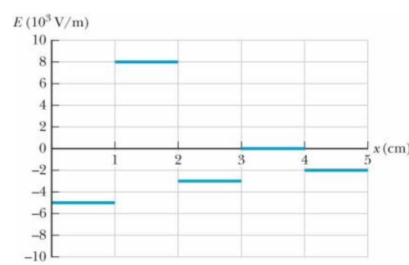
$$4\text{ cm} < x < 5\text{ cm}:\ E_x = -\frac{dV}{dx} = -\frac{20\text{ V} - 0}{1\text{ cm}} = -20\text{ V/ cm} = -2.0 \times 10^3\text{ V/ m}$$

Finally, using these values, make a graph of the *x* component of the electric field as a function of position:



Finalize The electric field graph comes out to be relatively simple because the lines in the potential graph were all straight. One could imagine setting up this system with a series of parallel plates held at different voltages.

Answer:



***TP24.3** **Conceptualize** Be sure you are clear on the two experiments that are performed and what can be measured. In the first experiment, the oil drifts downward at constant speed. Because we can see a calibrated scale through the

telescope and have access to a stopwatch, we can determine the terminal speed v_T of the drop. We also assume we know the viscosity and density of air.

In the second experiment, a potential difference is applied between the plates and adjusted until the same drop is at rest. Then, the potential difference ΔV_{rest} is measured. We assume we can measure d, the distance between the plates, and that we know g and the density ρ_{oil} of the oil. We do not know q or r.

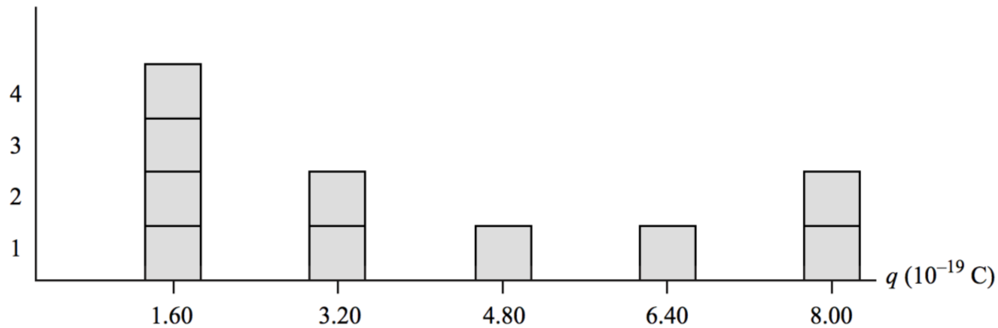
Categorize In both experiments, the oil drop is modeled as a *particle in equilibrium.* In the first experiment, the oil drop is moving at a constant downward velocity, so we can model it is a *particle under constant speed*.

Analyze (a) From the particle under constant speed model, we can find the terminal speed of each oil drop. We can use the equations provided in the problem to find the radius of each drop and the charge on it. These values appear in the expanded table below.

Drop #	**Δt to fall 1.00 mm (s)**	**ΔV_{rest} (V)**	**v_T (10^{-5} m/s)**	**r (μm)**	**q (10^{-19} C)**
1	40.0	8.95	2.50	0.502	4.79
2	31.8	9.44	3.14	0.563	6.40
3	22.7	12.55	4.41	0.667	7.98
4	40.3	26.65	2.48	0.500	1.59
5	64.1	13.32	1.56	0.397	1.59
6	38.3	14.25	2.61	0.513	3.21
7	31.6	7.65	3.16	0.565	7.97
8	49.2	19.62	2.03	0.453	1.60
9	112	5.70	0.893	0.300	1.60

10	27.3	23.7	3.66	0.608	3.21

Looking at the last column of values, we see that there are repeating values of the some of the charges. There are only five different values, allowing for some small uncertainty in the measurements, so let's make a histogram of the results for the charges on the oil drops:



We notice the following:

(1) The higher four values are integer multiples of the lowest value.

(2) There are no values of charges other than these integer multiples.

The histogram suggests that the charges on the oil drops are quantized. It's possible that the lowest value represents two fundamental charges and the next ones four, six, eight, and ten, but that's unlikely. Why would there always be an even number of elementary charges? It could also be 3, 6, 9, 12, and 15 charges. That's even more unlikely. The most likely interpretation is that the lowest value represents one elementary arge, and higher ones represent two, three, four, and five elementary charges. Therefore, we conclude that

$$e = \boxed{1.60 \times 10^{-19}\,\text{C}}$$

(b) From the expanded table, we see that indeed all ten drops are between 0.1 and 1 μm in radius, so the manufacturer's claim is $\boxed{\text{confirmed}}$ for these ten drops.

Finalize Millikan's experiment involved painstaking observations over several years. He received the 1923 Nobel Prize in Physics for his work, which was one of the earliest experiments to show the quantized nature of matter.

Answer: (a) 1.60×10^{-19} C (b) yes

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 24.1 Electric Potential and Potential Difference

P24.1 The potential difference is

$$\Delta V = V_f - V_i = -5.00 \text{ V} - 9.00 \text{ V} = -14.0 \text{ V}$$

and the total charge to be moved is

$$Q = -N_A e = -\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19} \text{ C}\right) = -9.63 \times 10^4 \text{ C}$$

Now, from $\Delta V = \dfrac{W}{Q}$, we obtain

$$W = Q\Delta V = (-9.63 \times 10^4 \text{ C})(-14.0 \text{ J/C}) = \boxed{1.35 \text{ MJ}}$$

P24.2 (a) The electron-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}m_e v_i^2 + (-e)V_i = 0 + (-e)V_f$$
$$e\left(V_f - V_i\right) = -\frac{1}{2}m_e v_i^2$$

The potential difference is then

$$\Delta V_e = -\frac{m_e v_i^2}{2e} = -\frac{(9.11\times10^{-31}\text{ kg})(2.85\times10^{7}\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})}$$
$$= -2.31\times10^{3}\text{ V} = \boxed{-2.31\text{ kV}}$$

(b) From (a), we see that the stopping potential is proportional to the kinetic energy of the particle.

> Because a proton is more massive than an electron, a proton traveling at the same speed as an electron has more initial kinetic energy and requires a greater magnitude stopping potential.

(c) The proton-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}m_p v_i^2 + eV_i = 0 + eV_f$$
$$e(V_f - V_i) = \frac{1}{2}m_p v_i^2$$

The potential difference is

$$\Delta V_p = \frac{m_p v_i^2}{2e}$$

Therefore, from (a),

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_p v_i^2/2e}{-m_e v_i^2/2e} \quad \rightarrow \quad \boxed{\Delta V_p/\Delta V_e = -m_p/m_e}$$

Section 24.2 Potential Difference in a Uniform Electric Field

P24.3 (a) From Equation 24.6,

$$E = \frac{|\Delta V|}{d} = \frac{600\text{ J/C}}{5.33\times10^{-3}\text{ m}} = \boxed{1.13\times10^{5}\text{ N/C}}$$

(b) The force on an electron is given by

$$F = |q|E = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(1.13 \times 10^{5}\ \text{N/C}\right) = \boxed{1.80 \times 10^{-14}\ \text{N}}$$

(c) Because the electron is repelled by the negative plate, the force used to move the electron must be applied in the direction of the electron's displacement. The work done to move the electron is

$$W = F \cdot s\cos\theta = \left(1.80\times10^{-14}\ \text{N}\right)\left[(5.33-2.00)\times10^{-3}\ \text{m}\right]\cos 0°$$
$$= \boxed{4.37\times10^{-17}\ \text{J}}$$

P24.4 Assume the opposite. Then at some point A on some equipotential surface the electric field has a nonzero component E_p in the plane of the surface. Let a test charge start from point A and move some distance on the surface in the direction of the field component. Then $\Delta V = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ is nonzero. The electric potential charges across the surface and it is not an equipotential surface. The contradiction shows that our assumption is false, that $E_p = 0$, and that the field is perpendicular to the equipotential surface.

P24.5 Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$.

(a) The rod-field system is isolated:

$$K_i + U_i = K_f + U_f$$
$$0 + 0 = \frac{1}{2}m_{\text{rod}}v^2 - qV$$
$$0 = \frac{1}{2}\mu Lv^2 - \lambda LEd$$
$$\frac{1}{2}\mu Lv^2 = \lambda LEd$$

Solving for the speed gives

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2\left(40.0\times10^{-6}\ \text{C/m}\right)(100\ \text{N/C})(2.00\ \text{m})}{(0.100\ \text{kg/m})}}$$
$$= \boxed{0.400\ \text{m/s}}$$

(b) $\boxed{\text{The same.}}$ Each bit of the rod feels a force of the same size as before.

P24.6 (a) The system consisting of the mass-spring-electric field is $\boxed{\text{isolated.}}$

(b) The system has both $\boxed{\text{electric potential energy and elastic potential energy}}$: U_e and U_{sp}.

(c) Taking the electric potential to be zero at the initial configuration, after the block has stretched the spring a distance x, the final electric potential is (from equation 24.3)

$$\Delta V = V = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}} = -Ex$$

By energy conservation within the system,

$$\left(K + U_{sp} + U_e\right)_i = \left(K + U_{sp} + U_e\right)_f$$
$$0 + 0 + 0 = 0 + \frac{1}{2}kx^2 + QV$$
$$0 = \frac{1}{2}kx^2 + Q(-Ex) \quad \rightarrow \quad x = \boxed{\frac{2QE}{k}}$$

(d) $\boxed{\text{Particle in equilibrium}}$

(e) $$\sum F = 0 \quad \rightarrow \quad -kx_0 + QE = 0 \quad \rightarrow \quad x_0 = \boxed{\frac{QE}{k}}$$

(f) The particle is no longer in equilibrium; therefore, the force equation becomes

$$\sum F = ma \quad \rightarrow \quad -kx + QE = m\frac{d^2x}{dt^2}$$
$$-k\left(x - \frac{QE}{k}\right) = m\frac{d^2x}{dt^2}$$

Defining $x' = x - x_0$, we have $\dfrac{d^2x'}{dt^2} = \dfrac{d^2\left(x - x_0\right)}{dt^2} = \dfrac{d^2x}{dt^2}$.

Substitute $x' = x - x_0$ into the force equation:

$$-k\left(x - \frac{QE}{k}\right) = m\frac{d^2x}{dt^2} \qquad \rightarrow \qquad -kx' = m\frac{d^2x'}{dt^2}$$

$$\rightarrow \qquad \boxed{\frac{d^2x'}{dt^2} = -\frac{kx'}{m}}$$

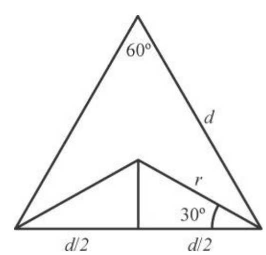
(g) The result of part (f) is the equation for simple harmonic motion $a_{x'} = -\omega^2 x'$ with

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \qquad \rightarrow \qquad T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$$

(h) The period does not depend on the electric field. The electric field just shifts the equilibrium point for the spring, just like a gravitational field does for an object hanging from a vertical spring.

Section 24.3 Electric Potential and Potential Energy Due to Point Charges

P24.7 By symmetry, a line from the center to each vertex forms a 30° angle with each side of the triangle. The figure shows the relationship between the length d of a side of the equilateral triangle and the distance r from a vertex to the center:



ANS. FIG. P24.7

$$r\cos 30.0° = d/2$$
$$\rightarrow r = d/(2\cos 30.0°)$$

The electric potential at the center is

$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$= k_e \left(\frac{Q}{d/(2\cos 30.0°)} + \frac{Q}{d/(2\cos 30.0°)} + \frac{2Q}{d/(2\cos 30.0°)} \right)$$

$$V = (4)\left(2\cos 30.0°\, k_e \frac{Q}{d} \right) = \boxed{6.93 k_e \frac{Q}{d}}$$

P24.8 (a) From Equation 24.12, the electric potential due to the two charges is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)$$
$$\times \left(\frac{5.00\times 10^{-9}\ \text{C}}{0.175\ \text{m}} + \frac{-3.00\times 10^{-9}\ \text{C}}{0.175\ \text{m}} \right) = \boxed{103\ \text{V}}$$

(b) The potential energy of the pair of charges is

$$U = \frac{k_e q_1 q_2}{r_{12}} = \left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)$$
$$\times \frac{\left(5.00\times 10^{-9}\ \text{C}\right)\left(-3.00\times 10^{-9}\ \text{C}\right)}{0.350\ \text{m}}$$
$$= \boxed{-3.85\times 10^{-7}\ \text{J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges by an infinite distance (that is, to bring them to a state of zero potential energy).

***P24.9** **Conceptualize** Figure 24.24 shows the physical setup for this attempt to reduce the outside electric field.

Categorize This problem can be argued from our understanding of Gauss's law and the relationship between the potential and the electric field of a sphere.

Analyze As discussed in part (A) of Example 23.6, the electric field of a charged sphere is the same as that of a point particle of the same charge located at the center of the sphere. If the electric *field* is the same as a point charge, then the electric *potential* will also be the same as that of a point charge, as given by Equation 24.11:

$$V_{\text{sphere}} = k_e \frac{Q}{r} \qquad (1)$$

For points outside the shell, where $r > R$, the potential due to the shell will also be the same as that of a point particle at the center:

$$V_{\text{shell}} = k_e \frac{Q_{\text{shell}}}{r} \qquad (2)$$

Therefore, the total potential outside the shell is that due to both sources:

$$V_{\text{total}} = V_{\text{sphere}} + V_{\text{shell}} = k_e \frac{Q}{r} + k_e \frac{Q_{\text{shell}}}{r} = \frac{k_e}{r}\left(Q + Q_{\text{shell}}\right) \qquad (3)$$

In order for the electric field outside the shell to be zero, the derivative of the potential must be zero (Eq. 24.16):

$$E_r = -\frac{dV}{dr} \quad \rightarrow \quad 0 = -\frac{d}{dr}\left[\frac{k_e}{r}\left(Q + Q_{\text{shell}}\right)\right] = \frac{k_e}{r^2}\left(Q + Q_{\text{shell}}\right)$$
$$\rightarrow \quad Q_{\text{shell}} = -Q$$

Therefore, from Equation (2), the electric potential of the shell where $r = R$, is

$$V_{\text{shell}} = k_e \frac{(-Q)}{R} = \boxed{-k_e \frac{Q}{R}}$$

Finalize This solution employs an understanding of Gauss's law. Compare this problem to Problem 23.30, where an insulating shell was placed around your experimental apparatus.

Answer: $-k_e \dfrac{Q}{R}$

***P24.10** **Conceptualize** Study Figure P24.10 carefully and make sure you understand the structure of the NaCl primitive cell.

Categorize The problem involves a single evaluation of the energy of a system in its final configuration, so no analysis models are needed.

Analyze (a) According to the discussion above Equation 24.14, the total electric potential energy is that associated with every pair of charges in the structure. Let's count the pairs:

Nearest-neighbor Na+–Cl–: 12 pairs

Body diagonal Na+–Cl–: 4 pairs

Face diagonal Na+–Na+: 6 pairs

Face diagonal Cl— Cl–: 6 pairs

Now, let's evaluate the total energy for each type of pair.

Nearest-neighbor Na^+–Cl^-:

$$12\text{ pairs} \square k_e \frac{(e)(-e)}{d} = -12k_e \frac{e^2}{d} \qquad (1)$$

Body diagonal Na^+–Cl^-:

$$4\text{ pairs} \square k_e \frac{(e)(-e)}{\left(\sqrt{d^2+d^2+d^2}\right)} = -4k_e \frac{e^2}{\sqrt{3}d} \qquad (2)$$

Face diagonal Na^+–Na^+:

$$6\text{ pairs} \square k_e \frac{(e)(e)}{\left(\sqrt{d^2+d^2}\right)} = +6k_e \frac{e^2}{\sqrt{2}d} \qquad (3)$$

Face diagonal Cl^-– Cl^-:

$$6\text{ pairs} \square k_e \frac{(-e)(-e)}{\left(\sqrt{d^2+d^2}\right)} = +6k_e \frac{e^2}{\sqrt{2}d} \qquad (4)$$

Adding all the pairs gives

$$U_e = -12k_e \frac{e^2}{d} - 4k_e \frac{e^2}{\sqrt{3}d} + 6k_e \frac{e^2}{\sqrt{2}d} + 6k_e \frac{e^2}{\sqrt{2}d}$$

$$= k_e \frac{e^2}{d}\left(-12 - \tfrac{4}{\sqrt{3}} + \tfrac{6}{\sqrt{2}} + \tfrac{6}{\sqrt{2}}\right) = \boxed{-5.82k_e \frac{e^2}{d}}$$

(b) The energy of the system is *negative*. Therefore, it is energetically favorable—that is, the energy of the system is reduced—when the crystal forms from the free ions.

Finalize In a macroscopic crystal with many primitive cells, calculations of energies become complicated because there are next-nearest neighbors, next-next-nearest neighbors, etc. Of course, the magnitudes of the energies of these interactions decrease because these higher-generation neighbors are farther apart than those in the primitive cell.

Answer: (a) $-5.82k_e \dfrac{e^2}{d}$ (b) See solution.

P24.11 (a) Each charge is a distance $\sqrt{a^2 + a^2}\big/2 = a\big/\sqrt{2}$ from the center.

$$V = k_e \sum_i \frac{q_i}{r_i} = 4k_e\left(\frac{Q}{a/\sqrt{2}}\right) = \boxed{4\sqrt{2}k_e \frac{Q}{a}}$$

(b) The potential at infinity is zero. The work done by an external agent is

$$W = q\Delta V = q\left(V_f - V_i\right) = q\left(4\sqrt{2}k_e \frac{Q}{a} - 0\right) = \boxed{4\sqrt{2}k_e \frac{qQ}{a}}$$

P24.12 (a) $V_A = k_e \sum_i \dfrac{q_i}{r_i} = k_e\left(\dfrac{Q}{d} + \dfrac{2Q}{d\sqrt{2}}\right) = k_e \dfrac{Q}{d}\left(1 + \sqrt{2}\right)$

$$V = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(\frac{5.00 \times 10^{-9} \text{ C}}{2.00 \times 10^{-2} \text{ m}}\right)\left(1 + \sqrt{2}\right) = \boxed{5.43 \text{ kV}}$$

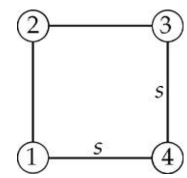
(b) $V_B = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{Q}{d\sqrt{2}} + \frac{2Q}{d} \right) = k_e \frac{Q}{d} \left(\frac{1}{\sqrt{2}} + 2 \right)$

$$V_B = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{5.00 \times 10^{-9} \text{ C}}{2.00 \times 10^{-2} \text{ m}} \right) \left(\frac{1}{\sqrt{2}} + 2 \right) = \boxed{6.08 \text{ kV}}$$

(c) $V_B - V_A = k_e \frac{Q}{d} \left(\frac{1}{\sqrt{2}} + 2 \right) - k_e \frac{Q}{d} \left(1 + \sqrt{2}\right) = k_e \frac{Q}{d} \left(\frac{1}{\sqrt{2}} + 1 - \sqrt{2} \right)$

$$V_B - V_A = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{5.00 \times 10^{-9} \text{ C}}{2.00 \times 10^{-2} \text{ m}} \right) \left(\frac{1}{\sqrt{2}} + 1 - \sqrt{2} \right)$$
$$= \boxed{658 \text{ V}}$$

P24.13 The work required equals the sum of the potential energies for all pairs of charges. No energy is involved in placing q_4 at a given position in empty space. When q_3 is brought from far away and placed close to q_4, the system potential energy can



ANS. FIG. P24.13

be expressed as q_3V_4, where V_4 is the potential at the position of q_3 established by charge q_4. When q_2 is brought into the system, it interacts with two other charges, so we have two additional terms q_2V_3 and q_2V_4 in the total potential energy. Finally, when we bring the fourth charge q_1 into the system, it interacts with three other charges, giving us three more energy terms. Thus, the complete expression for the energy is:

$$U = U_1 + U_2 + U_3 + U_4$$
$$U = 0 + U_{12} + \left(U_{13} + U_{23}\right) + \left(U_{14} + U_{24} + U_{34}\right)$$

$$U = 0 + \frac{k_eQ^2}{s} + \frac{k_eQ^2}{s} \left(\frac{1}{\sqrt{2}} + 1 \right) + \frac{k_eQ^2}{s} \left(1 + \frac{1}{\sqrt{2}} + 1 \right)$$
$$U = \frac{k_eQ^2}{s} \left(4 + \frac{2}{\sqrt{2}} \right) = \boxed{5.41 \frac{k_eQ^2}{s}}$$

We can visualize the term $\left(4+\frac{2}{\sqrt{2}}\right)$ as arising directly from the 4 side pairs and 2 face diagonal pairs.

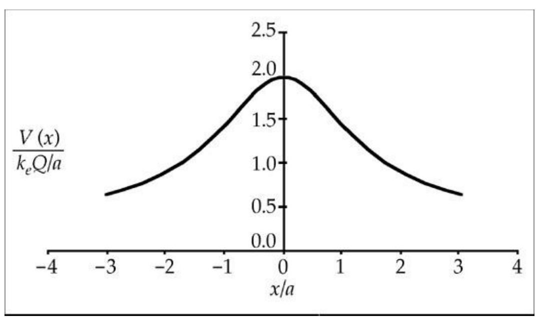
P24.14 (a) The potential due to the two charges along the x axis is

$$V(x)=\frac{k_eQ_1}{r_1}+\frac{k_eQ_2}{r_2}=\frac{k_e(+Q)}{\sqrt{x^2+a^2}}+\frac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$$

$$V(x)=\frac{2k_eQ}{\sqrt{x^2+a^2}}=\frac{k_eQ}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

$$\frac{V(x)}{(k_eQ/a)}=\boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$

ANS. FIG. P24.14 (a) shows the plot of this function for $|x/a|<3$.



ANS. FIG. P24.14 (a)

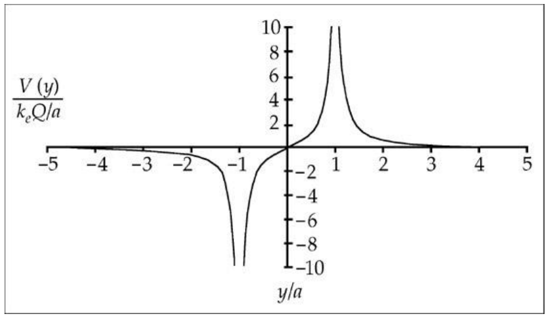
(b) The potential due to the two charges along the y axis is

$$V(y)=\frac{k_eQ_1}{r_1}+\frac{k_eQ_2}{r_2}=\frac{k_e(+Q)}{|y-a|}+\frac{k_e(-Q)}{|y+a|}$$

$$V(y)=\frac{k_eQ}{a}\left(\frac{1}{|y/a-1|}-\frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_eQ/a)}=\boxed{\left(\frac{1}{|y/a-1|}-\frac{1}{|y/a+1|}\right)}$$

ANS. FIG. P24.14(b) shows the plot of this function for $|y/a| < 4$.



ANS. FIG. P24.14 (b)

P24.15 (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$, located at a finite distance from the charges, at which this total potential is zero.

(b) $V = \dfrac{k_e q}{a} + \dfrac{k_e q}{a} = \boxed{\dfrac{2k_e q}{a}}$

P24.16 The original electrical potential energy is

$$U_e = qV = q\frac{k_e q}{d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are

$$F_{\text{spring}} + F_{\text{charge}} = -k(2d) + q\frac{k_e q}{(3d)^2} = 0$$

Then $\quad k = \dfrac{k_e q^2}{18d^3}$

In the final configuration the total potential energy is

$$\frac{1}{2}kx^2 + qV = \frac{1}{2}\frac{k_e q^2}{18d^3}(2d)^2 + q\frac{k_e q}{3d} = \frac{4}{9}\frac{k_e q^2}{d}$$

The missing energy must have become internal energy, as the system is isolated:

$$\Delta U + \Delta E_{\text{int}} = 0$$
$$\frac{4k_e q^2}{9d} - \frac{k_e q^2}{d} + \Delta E_{\text{int}} = 0$$

The increase in internal energy of the system is then

$$\Delta E_{\text{int}} = \boxed{\frac{5k_e q^2}{9d}}$$

P24.17 Consider the two spheres as a system.

(a) Conservation of momentum:

$$0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right) \quad \text{or} \quad v_2 = \frac{m_1 v_1}{m_2}$$

By conservation of energy,

$$0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$$

and $\dfrac{k_e q_1 q_2}{r_1 + r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}\dfrac{m_1^2 v_1^2}{m_2}$, which yields

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

suppressing units,

$$v_1 = \sqrt{\frac{2(0.700)(8.99\times 10^9)(2\times 10^{-6})(3\times 10^{-6})}{(0.100)(0.800)}\left(\frac{1}{8\times 10^{-3}} - \frac{1}{1.00}\right)}$$
$$= \boxed{10.8\ \text{m/s}}$$
$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.100\ \text{kg})(10.8\ \text{m/s})}{0.700\ \text{kg}} = \boxed{1.55\ \text{m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P24.18 Consider the two spheres as a system.

(a) Conservation of momentum:

$$0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right)$$

or $$v_2 = \frac{m_1 v_1}{m_2}.$$

By conservation of energy,

$$0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$$

and $$\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2 v_1^2}{m_2}.$$

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P24.19 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, 2 × 6 = 12 face diagonal pairs separated by $\sqrt{2}s$, and 4 interior diagonal pairs separated by $\sqrt{3}s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\frac{k_e q^2}{s}}$$

P24.20 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\frac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

Solving for the speed gives

$$v = \boxed{\sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

Section 24.4 Obtaining the Value of the Electric Field from the Electric Potential

P24.21 For a general expression for the potential on the y-axis, replace the a with y. The y component of the electric field is

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

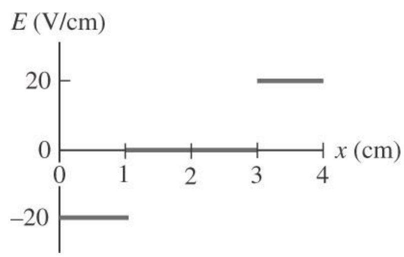
P24.22 $E_x = -\dfrac{\partial V}{\partial x} \quad \rightarrow \quad E_x = -\dfrac{\Delta V}{\Delta x} = -\text{ (slope of line)}$

The sign indicates the direction of the x component of the field.

$x = 0$ to 1 cm: $\quad E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{20\text{ V} - 0}{1\text{ cm}} = -20\text{ V/cm}$

$x = 1$ to 3 cm: $\quad E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0}{2\text{ cm}} = 0\text{ V/m}$

$x = 3$ to 4 cm: $\quad E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0 - 20\text{ V}}{1\text{ cm}} = +20\text{ V/cm}$

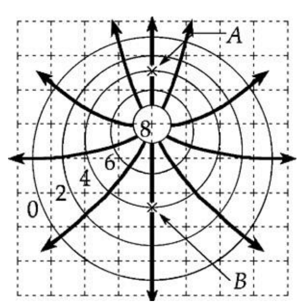


ANS. FIG. P24.22

P24.23 (a) $\boxed{E_A > E_B \text{ since } E = \dfrac{\Delta V}{\Delta s}}$

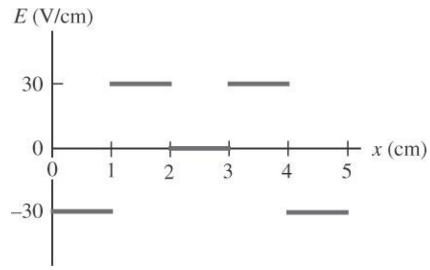
(b) $E_B = -\dfrac{\Delta V}{\Delta s} = -\dfrac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) ANS. FIG. P24.23 shows a sketch of the field lines.



ANS. FIG. P24.23

P24.24 $E_x = -\dfrac{\partial V}{\partial x} \quad \rightarrow \quad E_x = -\dfrac{\Delta V}{\Delta x} = -$ (slope of line)



ANS. FIG. P24.24

The sign indicates the direction of the x component of the field.

$x = 0$ to 1 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{30\text{ V} - 0}{1\text{ cm}} = -30\text{ V/cm}$$

$x = 1$ to 2 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{0 - 30\text{ V}}{2\text{ cm}} = 30\text{ V/m}$$

$x = 2$ to 3 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{0}{1\text{ cm}} = 0\text{ V/cm}$$

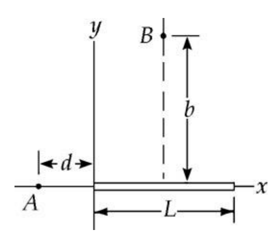
$x = 3$ to 4 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{-30\text{ V} - 0}{1\text{ cm}} = +30\text{ V/cm}$$

$x = 4$ to 5 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{0 - (-30\text{ V})}{1\text{ cm}} = -30\text{ V/cm}$$

Section 24.5 Electric Potential Due to Continuous Charge Distributions

P24.25 (a) As a linear charge density, λ has units of C/m. So $\alpha = \lambda/x$ must have units of C/m^2:

$$[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}} \cdot \left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$$



ANS. FIG. P24.25

(b) Consider a small segment of the rod at location x and of length dx. The amount of charge on it is $\lambda\, dx = (\alpha x)\, dx$. Its distance from A is $d + x$, so its contribution to the electric potential at A is

$$dV = k_e \frac{dq}{r} = k_e \frac{\alpha x\, dx}{d + x}$$

Relative to $V = 0$ infinitely far away, to find the potential at A we must integrate these contributions for the whole rod, from $x = 0$ to $x = L$. Then

$$V = \int_{\text{all } q} dV = \int_0^L \frac{k_e \alpha x}{d + x}\, dx.$$

To perform the integral, make a change of variables to

$u = d + x$, $du = dx$, $u(\text{at } x = 0) = d$, and $u(\text{at } x = L) = d + L$:

$$V = \int_d^{d+L} \frac{k_e \alpha (u - d)}{u}\, du = k_e \alpha \int_d^{d+L} du - k_e \alpha d \int_d^{d+L} \left(\frac{1}{u}\right) du$$

$$\begin{aligned} V &= k_e \alpha u \Big|_d^{d+L} - k_e \alpha d \ln\, u \Big|_d^{d+L} \\ &= k_e \alpha (d + L - d) - k_e \alpha d \left[\ln(d + L) - \ln\, d\right] \end{aligned}$$

$$V = \boxed{k_e \alpha \left[L - d \ln\left(1 + \frac{L}{d}\right)\right]}$$

P24.26 $$V = \int \frac{k_e dq}{r} = k_e \int \frac{\alpha x dx}{\sqrt{b^2 + \left(L/2 - x\right)^2}}$$

Let $z = \dfrac{L}{2} - x$. Then $x = \dfrac{L}{2} - z$, and $dx = -dz$.

$$\begin{aligned} V &= k_e \alpha \int \frac{\left(L/2 - z\right)\left(-dz\right)}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{\sqrt{b^2 + z^2}} + k_e \alpha \int \frac{z dz}{\sqrt{b^2 + z^2}} \\ &= -\frac{k_e \alpha L}{2} \ln\left(z + \sqrt{z^2 + b^2}\right) + k_e \alpha \sqrt{z^2 + b^2} \end{aligned}$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\left(\frac{L}{2} - x\right) + \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\right]_0^L + k_e \alpha \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\Bigg|_0^L$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{(L/2)^2 + b^2}}{L/2 + \sqrt{(L/2)^2 + b^2}}\right]$$

$$+ k_e \alpha \left[\sqrt{\left(\frac{L}{2} - L\right)^2 + b^2} - \sqrt{\left(\frac{L}{2}\right)^2 + b^2}\right]$$

$$V = \boxed{-\frac{k_e \alpha L}{2} \ln\left[\frac{\sqrt{b^2 + (L^2/4)} - L/2}{\sqrt{b^2 + (L^2/4)} + L/2}\right]}$$

P24.27 $V = k_e \int\limits_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int\limits_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_R^{3R} \frac{\lambda dx}{x}$

$$V = -k_e \lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x\Big|_R^{3R}$$

$$V = k_e \lambda \ln \frac{3R}{R} + k_e \lambda \pi + k_e \lambda \ln 3 = \boxed{k_e \lambda (\pi + 2 \ln 3)}$$

***P24.28** **Conceptualize** We can divide the letter D into two distributions of charge: a semicircle and a straight rod. We have not investigated the potential due to a semicircle or at a point straight out from the center of a rod in the textbook. Are we in trouble?

Categorize The potential at a given point along the midline will be a superposition of those due to two continuous charge distributions.

Analyze First, the ring. In Example 24.5, we found the potential along the axis of a full ring. Because potential is a scalar, however, we do not need to worry about vectors, and we realize that the potential of a half ring is just half that of a full ring:

$$V_{\text{half ring}} = \tfrac{1}{2}V_{\text{ring}} = \frac{k_e Q}{2\sqrt{R^2 + x^2}} \qquad (1)$$

where x is the distance along the midline from the center of the ring.

Now, what about the rod? In Example 24.7, we found the potential at a point along a line directed perpendicularly from the endpoint of a rod. Because potential is a scalar, however, we do not need to worry about vectors, and we realize that the rod in Figure P24.28 is just two rods, one directed to the left of the midline and the other to the right. Therefore, the potential due to the rod is just twice that found in Example 24.7:

$$V_{\text{rod}} = 2V_{\text{half rod}} = 2k_e \frac{Q}{R} \ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right) \qquad (2)$$

where R is the length of the "half rod" in Example 24.7.

The total potential at a position x along the midline is the sum of Equations (1) and (2):

$$\begin{aligned} V_{\text{total}} &= V_{\text{half ring}} + V_{\text{rod}} \\ &= \boxed{\frac{k_e Q}{2\sqrt{R^2 + x^2}} + 2k_e \frac{Q}{R} \ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right)} \end{aligned}$$

Finalize Notice how quickly we can solve this problem. We did not need to do any new calculations, other than multiplying or dividing by 2. We just needed to recognize that the distributions in Figure P24.28 could be represented in terms of the geometries we had already studied in Examples 24.5 and 24.7.

Answer: $$\frac{k_e Q}{2\sqrt{R^2+x^2}} + 2k_e \frac{Q}{R}\ln\left(\frac{R+\sqrt{x^2+R^2}}{x}\right)$$

Section 24.6 Conductors in Electrostatic Equilibrium

P24.29 $\boxed{\text{No.}}$ A conductor of any shape forms an equipotential surface. If the conductor is a sphere of radius R, and if it holds charge Q, the electric field at its surface is $E = k_eQ/R^2$ and the –potential of the surface is $V = k_eQ/R$; thus, if we know E and R, we can find V. However, if the surface varies in shape, there is no clear way to relate electric field at a point on the surface to the potential of the surface.

P24.30 Let's calculate the electric field just outside the surface:

$$\begin{aligned} E &= k_e \frac{q}{r^2} = \left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{40.0 \times 10^{-9}\text{ C}}{(0.15\text{ m})^2}\right] \\ &= 1.60 \times 10^4 \text{ N} = 16.0 \text{ kN/C} \end{aligned}$$

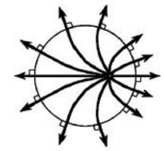
This should be the value of the electric field at the peak of the curve in Figure P24.30. We see, however, that the peak in the figure occurs at about 6.5 kN/C. Therefore, it is not possible that this figure represents the electric field for the given situation.

P24.31 The surface area is $A = 4\pi a^2$. The field is then

$$E = \frac{k_e Q}{a^2} = \frac{Q}{4\pi \epsilon_0 a^2} = \frac{Q}{A\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$$

It is not equal to $\sigma/2\epsilon_0$. At a point just outside, the uniformly charged surface looks just like a uniform flat sheet of charge. The distance to the field point is negligible compared to the radius of curvature of the surface.

P24.32 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



ANS. FIG. P24.32

P24.33 The fields are equal. The equation $E = \dfrac{\sigma_{\text{conductor}}}{\epsilon_0}$ suggested in the chapter for the field outside the aluminum looks different from the equation $E = \dfrac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \dfrac{Q}{2A}$. The glass carries charge only on area A, with $\sigma_{\text{insulator}} = \dfrac{Q}{A}$. The two fields are $\dfrac{Q}{2A\epsilon_0}$, the same in magnitude, and both are perpendicular to the plates, vertically upward if Q is positive.

P24.34 (a) $\vec{\mathbf{E}} = \boxed{0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.00 \times 10^{-6}\ \text{C})}{(0.030\,0\ \text{m})^2} = 7.99 \times 10^7\ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{79.9\ \text{MN/C radially outward}}$$

(c) $\vec{\mathbf{E}} = \boxed{0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.00 \times 10^{-6}\ \text{C})}{(0.070\,0\ \text{m})^2} = 7.34 \times 10^6\ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{7.34\ \text{MN/C radially outward}}$$

P24.35 For points on the surface and outside, the sphere of charge behaves like a charged particle at its center, both for creating field and potential.

(a) Inside a conductor when charges are not moving, the electric field is zero and the potential is uniform, the same as on the surface, and $E = \boxed{0}$.

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{0.140 \text{ m}} = \boxed{1.67 \text{ MV}}$$

(b) $$E = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{(0.200 \text{ m})^2}$$

$$= \boxed{5.84 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{0.200 \text{ m}} = \boxed{1.17 \text{ MV}}$$

(c) $$E = \frac{k_e q}{R^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{(0.140 \text{ m})^2}$$

$$= \boxed{11.9 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

P24.36 (a) Inside surface: consider a cylindrical gaussian surface of arbitrary length ℓ within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \quad \rightarrow \quad 0 = \frac{\left(\lambda + \lambda_{\text{inner}}\right)\ell}{\epsilon_0}$$

so $\lambda_{\text{inner}} = \boxed{-\lambda}$.

(b) Outside surface: consider a cylindrical gaussian surface of arbitrary length ℓ outside the metal. The total charge within the gaussian surface is

$$q_{\text{wire}} + q_{\text{cylinder}} = q_{\text{wire}} + \left(q_{\text{inner surface}} + q_{\text{outer surface}}\right)$$
$$\lambda\ell + 2\lambda\ell = \lambda\ell + \left(-\lambda\ell + \lambda_{\text{outer}}\ell\right) \quad \rightarrow \quad \lambda_{\text{outer}} = \boxed{3\lambda}$$

(c) Gauss's law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E 2\pi r \ell = \frac{3\lambda \ell}{\epsilon_0} \quad \rightarrow \quad E = 2\frac{3\lambda}{4\pi \epsilon_0 r} = \boxed{6k_e \frac{\lambda}{r}, \text{ radially outward}}$$

Additional Problems

P24.37 From Equation 24.13, solve for the separation distance of the electron and proton:

$$U = k_e \frac{q_1 q_2}{r_{12}} \quad \rightarrow \quad r_{12} = k_e \frac{q_1 q_2}{U} = -k_e \frac{e^2}{U}$$

The separation distance r_{12} between the electron and proton is the same as the radius r of the orbit of the electron. Substitute numerical values:

$$\begin{aligned} r &= -\left(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2\right) \frac{\left(1.6 \times 10^{-19} \ \text{C}\right)^2}{-13.6 \ \text{eV}} \left(\frac{1 \ \text{eV}}{1.6 \times 10^{-19} \ \text{J}}\right) \\ &= 1.06 \times 10^{-10} \ \text{m} \end{aligned}$$

Set this equal to $r = n^2(0.052\ 9 \text{ nm})$ and solve for n:

$$r = n^2\left(0.052\ 9 \ \text{nm}\right) = 1.06 \times 10^{-10} \ \text{m} = 0.106 \text{ nm}$$

Which gives $n = 1.42$. Because n is not an integer, this is not possible. Therefore, the energy given cannot be possible for an allowed state of the atom.

P24.38 (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$\Delta V = Ed = \left(3 \times 10^6 \ \text{V/m}\right)\left(5 \times 10^{-3} \ \text{m}\right) = 1.5 \times 10^4 \ \text{V} \ \boxed{\sim 10^4 \ \text{V}}$$

(b) The area of your skin is perhaps 1.5 m^2, so model your body as a sphere with this surface area. Its radius is given by $1.5\ \text{m}^2 = 4\pi r^2$, $r = 0.35$ m. We require that you are at the potential found in part (a), with $V = \frac{k_e q}{r}$. Then,

$$q = \frac{Vr}{k_e} = \frac{1.5\times 10^4\ \text{V}(0.35\ \text{m})}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$q = 5.8\times 10^{-7}\ \text{C}\ \boxed{\sim 10^{-6}\ \text{C}}$$

P24.39 (a) The exact potential is

$$+\frac{k_e q}{r+a} - \frac{k_e q}{r-a} = +\frac{k_e q}{3a+a} - \frac{k_e q}{3a-a} = \frac{k_e q}{4a} - \frac{2k_e q}{4a} = \boxed{-\frac{k_e q}{4a}}$$

(b) The approximate expression $-2k_e qa/x^2$ gives

$$-2k_e qa/(3a)^2 = -k_e q/4.5a$$

Compare the exact to the approximate solution:

$$\frac{1/4 - 1/4.5}{1/4} = \frac{0.5}{4.5} = 0.111.$$

$$\boxed{\text{The approximate expression } -2k_e qa/x^2 \text{ gives } -k_e q/4.5a, \text{ which is different by only 11.1\%.}}$$

P24.40 From Example 24.6, the potential along the x axis of a ring of charge of radius R is

$$V = \frac{k_e Q}{\sqrt{R^2 + x^2}}$$

Therefore, the potential at the center of the ring is

$$V = \frac{k_e Q}{\sqrt{R^2 + (0)^2}} = \frac{k_e Q}{R}$$

When we place the point charge Q at the center of the ring, the electric potential energy of the charge–ring system is

$$U = QV = Q\left(\frac{k_e Q}{R}\right) = \frac{k_e Q^2}{R}$$

Now, apply Equation 8.2 to the isolated system of the point charge and the ring with initial configuration being that with the point charge at the center of the ring and the final configuration having the point charge infinitely far away and moving with its highest speed:

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \left(\frac{1}{2}mv_{\max}^2 - 0\right) + \left(0 - \frac{k_e Q^2}{R}\right) = 0$$

Solve for the maximum speed:

$$v_{\max} = \left(\frac{2k_e Q^2}{mR}\right)^{1/2}$$

Substitute numerical values:

$$\begin{aligned} v_{\max} &= \left(\frac{2\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(50.0 \times 10^{-6} \text{ C}\right)^2}{(0.100 \text{ kg})(0.500 \text{ m})}\right)^{1/2} \\ &= 30.0 \text{ m/s} \end{aligned}$$

Therefore, even if the charge were to accelerate to infinity, it would only achieve a maximum speed of 30.0 m/s, so it cannot strike the wall of your laboratory at 40.0 m/s.

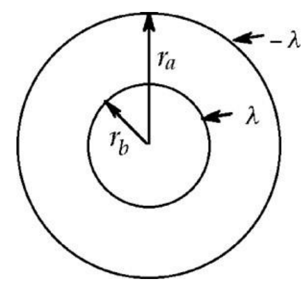
P24.41 We obtain the electric potential at P by integrating:

$$V = k_e \int_a^{a+L} \frac{\lambda dx}{\sqrt{x^2+b^2}} = k_e \lambda \ln\left[x + \sqrt{(x^2+b^2)}\right]_a^{a+L}$$

$$= \boxed{k_e \lambda \ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]}$$

P24.42 (a) $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is

$$E = \frac{\lambda}{2\pi \in_0 r} = \frac{2k_e\lambda}{r}$$



ANS. FIG. P24.42

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r} dr = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)$$

or $\boxed{\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)}$.

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right)$$

The field at r is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}$$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$.

P24.43 (a) The positive plate by itself creates a field

$$E = \frac{\sigma}{2\epsilon_0} = \frac{36.0\times10^{-9}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 2.03\ \text{kN/C}$$

away from the positive plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field $\boxed{4.07\ \text{kN/C}}$ in the space between.

(b) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$\Delta V = V - 0 = -\int_{x_i}^{x_f} E_x dx = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})dx$$

The potential difference between the plates is

$$V = (4.07\times10^3\ \text{N/C})(0.120\ \text{m}) = \boxed{488\ \text{V}}$$

(c) The positive proton starts from rest and accelerates from higher to lower potential. Taking $V_i = 488$ V and $V_f = 0$, by energy conservation, we find the proton's final kinetic energy.

$$(K+qV)_i = (K+qV)_f \rightarrow K_f = qV_i$$

$$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$qV_i = (1.60\times10^{-19}\ \text{C})(488\ \text{V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\ \text{J}}$$

(d) From the kinetic energy of part (c),

$$K = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(7.81\times10^{-17}\ \text{J}\right)}{1.67\times10^{-27}\ \text{kg}}} = 3.06\times10^5\ \text{m/s} = \boxed{306\ \text{km/s}}$$

(e) Using the constant-acceleration equation, $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$,

$$a = \frac{v_f^2 - v_i^2}{2\left(x_f - x_i\right)} = \frac{\left(3.06\times10^5\ \text{m/s}\right)^2 - 0}{2(0.120\ \text{m})}$$

$$= \boxed{3.90\times10^{11}\ \text{m/s}^2}\ \text{toward the negative plate}$$

(f) The net force on the proton is given by Newton's second law:

$$\sum F = ma = \left(1.67\times10^{-27}\ \text{kg}\right)\left(3.90\times10^{11}\ \text{m/s}^2\right)$$

$$= \boxed{6.51\times10^{-16}\ \text{N}}\ \text{toward the negative plate}$$

(g) The magnitude of the electric field is

$$E = \frac{F}{q} = \frac{6.51\times10^{-16}\ \text{N}}{1.60\times10^{-19}\ \text{C}} = \boxed{4.07\ \text{kN/C}}$$

(h) They are the same.

P24.44 (a) Inside the sphere, $\boxed{E_x = E_y = E_z = 0}$.

(b) Outside,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

$$= -\left[0 + 0 + E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2x)\right]$$

$$E_x = \boxed{3E_0 a^3 xz\left(x^2+y^2+z^2\right)^{-5/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2 + y^2 + z^2\right)^{-3/2}\right)$$

$$= -E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2 + y^2 + z^2\right)^{-5/2} 2y$$

$$E_y = \boxed{3E_0 a^3 yz\left(x^2 + y^2 + z^2\right)^{-5/2}}$$

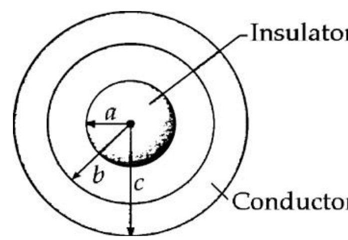
$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left[V_0 - E_0 z + E_0 a^3 z\left(x^2 + y^2 + z^2\right)^{-3/2}\right]$$

$$= E_0 - E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2 + y^2 + z^2\right)^{-5/2}(2z) - E_0 a^3\left(x^2 + y^2 + z^2\right)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3\left(2z^2 - x^2 - y^2\right)\left(x^2 + y^2 + z^2\right)^{-5/2}}$$

P24.45 Choose as each gaussian surface a concentric sphere of radius r. The electric field will be perpendicular to its surface, and will be uniform in strength over its surface. The density of charge in the insulating sphere is

$$\rho = Q / \left(\tfrac{4}{3}\pi a^3\right)$$



ANS. FIG. P24.45

(a) The sphere of radius $r < a$ encloses charge

$$q_{\text{in}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{Q\left(\frac{r}{R}\right)^3}$$

(b) Applying Gauss's law to this sphere reveals the magnitude of the field at its surface.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}\left(\frac{r}{a}\right)^3 \rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{a^3} = \boxed{k_e\frac{Qr}{a^3}}$$

(c) For a sphere of radius r with $a < r < b$, the whole insulating sphere is enclosed, so the charge within is Q: $q_{\text{in}} = \boxed{Q}$.

(d) Gauss's law for this sphere becomes:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{4\pi \epsilon_0}\frac{Q}{r^2} = \boxed{k_e \frac{Q}{r^2}}$$

(e) For $b \le r \le c$, $\boxed{E = 0}$ because there is no electric field inside a conductor.

(f) For $b \le r \le c$, we know $E = 0$. Assume the inner surface of the hollow sphere holds charge Q_{inner}. By Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$0 = \frac{Q + Q_{\text{inner}}}{\epsilon_0} \rightarrow Q_{\text{inner}} = \boxed{-Q}$$

(g) The total charge on the hollow sphere is zero; therefore, charge on the outer surface is opposite to that on the inner surface:

$$Q_{\text{outer}} = -Q_{\text{inner}} = \boxed{+Q}$$

(h) A surface of area A holding charge Q has surface charge $\sigma = q/A$. The solid, insulating sphere has small surface charge because its total charge Q is uniformly distributed throughout its volume. The $\boxed{\text{inner surface of radius } b}$ has smaller surface area, and therefore larger surface charge, than the outer surface of radius c.

P24.46 (a) The field is zero within the metal of the shell. The exterior electric field lines end at equally spaced points on the outer surface because the surface of the conductor is an equipotential surface. The charge on the outer surface is distributed uniformly. Its amount is given by

$$EA = Q/\epsilon_0$$

Solving for the charge Q gives

$$Q = -(890\ \text{N/C})\ 4\pi\ (0.750\ \text{m})^2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)$$
$$= -55.7\ \text{nC}$$

$\boxed{\text{The charge on the exterior surface is } -55.7\ \text{nC distributed uniformly.}}$

(b) For the net charge of the shell to be zero, the shell must carry +55.7 nC on its inner surface, induced there by –55.7 nC in the cavity within the shell. The charge in the cavity could have any distribution and give any corresponding distribution to the charge on the inner surface of the shell. $\boxed{\text{The charge on the interior surface is +55.7 nC. It can have any distribution.}}$ For example, a large positive charge might be within the cavity close to its topmost point, and a slightly larger negative charge near its easternmost point. The inner surface of the shell would then have plenty of negative charge near the top and even more positive charge centered on the eastern side.

(c) See the comments in (b). $\boxed{\text{The charge within the shell is −55.7 nC. It can have any distribution.}}$ For example, the charge could be distributed on the surface of an insulator of arbitrary shape.

P24.47 We have

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$$

(a) Solving for the charge Q on the insulating sphere, we write, for the region $a < r < b$,

$$Q = \epsilon_0\, E\left(4\pi r^2\right)$$
$$= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(-3.60\times10^3\ \text{N/C}\right)4\pi(0.100\ \text{m})^2$$
$$= -4.00\times10^{-9}\ \text{C} = \boxed{-4.00\ \text{nC}}$$

(b) We take Q' to be the net charge on the hollow sphere. For $r > c$,

$$Q + Q' = \epsilon_0 E\left(4\pi r^2\right)$$
$$= \left(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}\right)\left(2.00 \times 10^2\ \mathrm{N/C}\right) \times 4\pi(0.500\ \mathrm{m})^2$$
$$= 5.56 \times 10^{-9}\ \mathrm{C}$$

so

$$Q' = +9.56 \times 10^{-9}\ \mathrm{C} = \boxed{+9.56\ \mathrm{nC}}$$

(c) For $b < r < c$, $E = 0$; therefore, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = q_{\text{in}}/\epsilon_0 = 0$ implies $q_{\text{in}} = Q + Q_1 = 0$, where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00\ \mathrm{nC}}$.

(d) Let Q_2 be the total charge on the outer surface of the hollow sphere; then,

$$Q' = Q_1 + Q_2 \rightarrow Q_2 = Q' - Q_1 = 9.56\ \mathrm{nC} - 4.00\ \mathrm{nC} = \boxed{+5.56\ \mathrm{nC}}$$

Challenge Problems

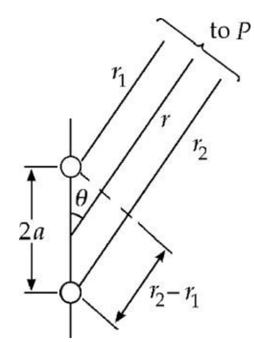
P24.48 (a) The total potential is

$$V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}\left(r_2 - r_1\right)$$

From the figure, for $r >> a$, $r_2 - r_1 \approx 2a\cos\theta$.

Note that r_1 is approximately equal to r_2. Then

$$V \approx \frac{k_e q}{r_1 r_2} 2a\cos\theta \approx \frac{k_e p\cos\theta}{r^2}$$



ANS. FIG. P24.48

(b) $E_r = -\dfrac{\partial V}{\partial r} = \boxed{\dfrac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the θ component of the gradient is $-\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$. Therefore,

$$E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$$

(c) For $r >> a$, $\theta = 90°$: $E_r(90°) = 0$, $E_\theta(90°) = \frac{k_e p}{r^3}$

For $r >> a$, $\theta = 0°$: $E_r(0°) = \frac{2k_e p}{r^3}$, $E_\theta(0°) = 0$

$\boxed{\text{Yes,}}$ these results are reasonable.

(d) $\boxed{\text{No,}}$ because as $r \to 0$, $E \to \infty$. The magnitude of the electric field between the charges of the dipole is not infinite.

(e) Substituting $r_1 \approx r_2 \approx r = (x^2 + y^2)^{1/2}$ and $\cos\,\theta = \frac{y}{\left(x^2+y^2\right)^{1/2}}$ into

$V = \frac{k_e p\cos\theta}{r^2}$ gives $\boxed{V = \frac{k_e py}{\left(x^2+y^2\right)^{3/2}}}$.

(f) $\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e pxy}{\left(x^2+y^2\right)^{5/2}}}$ and $\boxed{E_y = -\frac{\partial V}{\partial y} = \frac{k_e p\left(2y^2 - x^2\right)}{\left(x^2+y^2\right)^{5/2}}}$

P24.49 For an element of area which is a ring of radius r and width dr, the incremental potential is given by $dV = \frac{k_e dq}{\sqrt{r^2+x^2}}$, where

$$dq = \sigma dA = Cr\left(2\pi r dr\right)$$

The electric potential is then given by

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2 dr}{\sqrt{r^2+x^2}}$$

From a table of integrals,

$$\int \frac{r^2 dr}{\sqrt{r^2+x^2}} = \frac{r}{2}\sqrt{r^2+x^2} - \frac{x^2}{2}\ln\left(r+\sqrt{r^2+x^2}\right)$$

The potential then becomes, after substituting and rearranging,

$$V = C(2\pi k_e)\int_0^R \frac{r^2 dr}{\sqrt{r^2+x^2}}$$

$$= \boxed{\pi k_e C\left[R\sqrt{R^2+x^2} + x^2 \ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

P24.50 For the given charge distribution,

$$V(x,\ y,\ z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$$

where $\quad r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$

and $\quad r_2 = \sqrt{x^2+y^2+z^2}$

The surface on which $V(x, y, z) = 0$ is given by

$$k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0 \qquad \text{or} \qquad 2r_1 = r_2$$

This gives:

$$4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$$

which may be written in the form:

$$x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0 \qquad [1]$$

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 - a^2 = 0$$

or $$x^2+y^2+z^2+(-2x_0)x+(-2y_0)y+(-2z_0)z + \left(x_0^2+y_0^2+z_0^2-a^2\right)=0 \qquad [2]$$

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0=\frac{8}{3}R;\ -2y_0=0;\ -2z_0=0;\ x_0^2+y_0^2+z_0^2-a^2=\frac{4}{3}R^2$$

Thus, $x_0=-\frac{4}{3}R$, $y_0=z_0=0$, and $a^2=\left(\frac{16}{9}-\frac{4}{3}\right)R^2=\frac{4}{9}R^2$

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R,0,0\right)}$,

having a radius $\boxed{\frac{2}{3}R}$.

P24. 51 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $\frac{Qdx}{h}$ and, according to Example 24.5, creates potential

$$dV=\frac{k_eQdx}{h\sqrt{x^2+R^2}}$$

The whole stack of rings creates potential

$$V=\int_{\text{all charge}} dV=\int_d^{d+h}\frac{k_eQdx}{h\sqrt{x^2+R^2}}=\frac{k_eQ}{h}\ln\left(x+\sqrt{x^2+R^2}\right)\Bigg|_d^{d+h}$$

$$=\boxed{\frac{k_eQ}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $\frac{Qdx}{h}$ and charge-per-area $\frac{Qdx}{\pi R^2h}$.

According to Example 24.6, it creates potential

$$dV = 2\pi k_e \frac{Q dx}{\pi R^2 h}\left(\sqrt{x^2 + R^2} - x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_e Q}{R^2 h}\left(\sqrt{x^2 + R^2}\,dx - x\,dx\right)$$

$$= \frac{2k_e Q}{R^2 h}\left[\frac{1}{2}x\sqrt{x^2 + R^2} + \frac{R^2}{2}\ln\left(x + \sqrt{x^2 + R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$

$$V = \boxed{\begin{aligned}&\frac{k_e Q}{R^2 h}\Bigg[(d+h)\sqrt{(d+h)^2 + R^2} - d\sqrt{d^2 + R^2} \\ &\qquad - 2dh - h^2 + R^2 \ln\left(\frac{d + h + \sqrt{(d+h)^2 + R^2}}{d + \sqrt{d^2 + R^2}}\right)\Bigg]\end{aligned}}$$

ANSWERS TO QUICK-QUIZZES

1. (i) (b) **(ii)** (a)

2. Ⓑ to Ⓒ, Ⓒ to Ⓓ, Ⓐ to Ⓑ, Ⓓ to Ⓔ

3. (i) (c) **(ii)** (a)

4. (i) (a) **(ii)** (a)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P24. 2 (a) –2.31 kV; (b) Because a proton is more massive than an electron, a proton traveling at the same speed as an electron has more initial kinetic energy and requires a greater magnitude stopping potential; (c) $\Delta V_p / \Delta V_e = -m_p / m_e$

P24. 4 See P24.4 for full explanation.

P24. 6 (a) isolated; (b) electric potential energy and elastic potential energy; (c) $\frac{2QE}{k}$;

(d) Particle in equilibrium; (e) $\frac{QE}{k}$; (f) $\frac{d^2x'}{dt^2} = -\frac{kx'}{m}$;

(g) $2\pi\sqrt{\frac{m}{k}}$; (h) The period does not depend on the electric field. The electric field just shifts the equilibrium point for the spring, just like a gravitational field does for an object hanging from a vertical spring.

P24. 8 (a) 103 V; (b) -3.85×10^{-7} J, positive work must be done

P24. 10 (a) $-5.82 k_e \frac{e^2}{d}$ (b) See solution.

P24. 12 (a) 5.43 kV; (b) 6.08 kV; (c) 658 V

P24. 14 (a) $\frac{2}{\sqrt{(x/a)^2 + 1}}$; (b) See ANS. FIG. P24.14 (b).

P24. 16 $\Delta E_{\text{int}} = \frac{5k_e q^2}{9d}$

P24. 18 (a) $v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$

and $v_2 = \sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$; (b) faster than calculated in (a)

P24. 20 $v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}$

P24. 22 See ANS. FIG. P24.22.

P24. 24 See ANS. FIG. P24.24.

P24. 26 $-\frac{k_e \alpha L}{2} \ln\left[\frac{\sqrt{b^2 + \left(L^2/4\right)} - L/2}{\sqrt{b^2 + \left(L^2/4\right)} + L/2}\right]$

P24. 28 $\frac{k_e Q}{2\sqrt{R^2 + x^2}} + 2k_e \frac{Q}{R} \ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right)$

P24. 30 The electric field just outside the surface occurs at 16.0 kN/C. The peak in the figure occurs at about 6.5 kN/C. Therefore, it is not possible that this figure represents the electric field for the given situation.

P24. 32 See ANS. FIG. P24.32.

P24. 34 (a) 0 (b) 7.99 × 10^7 N/C (outward)(c) 0 (d) 7.34 ×10^6 N/C (outward)

P24. 36 (a) -λ (b) +3 λ (c) 6k_e λ/r radially outward

P24. 38 (a) ~10^4 V; (b) ~10^{-6} C

P24. 40 Even if the charge were to accelerate to infinity, it would only achieve a maximum speed of 30.0 m/s, so it cannot strike the wall of your laboratory at 40.0 m/s.

P24. 42 (a) $\Delta V = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right)$; (b) $E = \frac{\Delta V}{\ln\left(r_a/r_b\right)}\left(\frac{1}{r}\right)$

P24. 44 (a) $E_x = E_y = E_z = 0$; (b) $E_x = 3E_0 a^3 xz\left(x^2 + y^2 + z^2\right)^{-5/2}$,

$E_y = 3E_0 a^3 yz\left(x^2 + y^2 + z^2\right)^{-5/2}$, $E_z = E_0 + E_0 a^3\left(2z^2 - x^2 - y^2\right)\left(x^2 + y^2 + z^2\right)^{-5/2}$

P24. 46 (a) The charge on the exterior surface is –55.7 nC distributed uniformly; (b) The charge on the interior surface is +55.7 nC. It can have any distribution; (c) The charge within the shell is –55.7 nC. It can have any distribution.

P24. 48 (a) $V \approx \dfrac{k_e q}{r_1 r_2} 2a\cos\theta \approx \dfrac{k_e p\cos\theta}{r^2}$ (b) $E_\theta = -\dfrac{1}{r}\left(\dfrac{\partial V}{\partial \theta}\right) = \boxed{\dfrac{k_e p\sin\theta}{r^3}}$

(c) Yes (d) No (e) $\boxed{V = \dfrac{k_e py}{\left(x^2+y^2\right)^{3/2}}}$.

(f) $\boxed{E_x = -\dfrac{\partial V}{\partial x} = \dfrac{3k_e pxy}{\left(x^2+y^2\right)^{5/2}}}$ and $\boxed{E_y = -\dfrac{\partial V}{\partial y} = \dfrac{k_e p\left(2y^2-x^2\right)}{\left(x^2+y^2\right)^{5/2}}}$

P24. 50 $\left(-\dfrac{4}{3}R, 0, 0\right), \dfrac{2}{3}R$