31

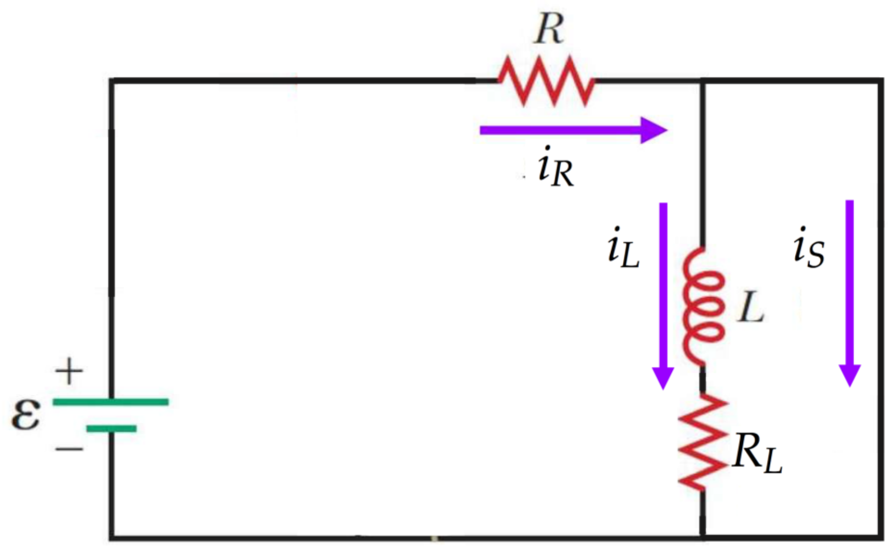
Inductance

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO THINK – PAIR – SHARE PROBLEMS

***TP31.1** **Conceptualize** Let's remove the complicating features of the various switches. With switch S_3 closed, the circuit is like that shown below, where we have explicitly indicated the resistance of the inductor in series with an ideal, resistance-free inductor. The path with the switch is a resistance-free short circuit across the inductor.



Categorize The circuit is an *RL* circuit, which can be analyzed with Kirchhoff's rules.

Analyze (a) Apply Kirchhoff's loop rule to the outer loop of the circuit:

$$\mathcal{E} - i_R R = 0 \qquad (1)$$

where the equation does not include i_S because that current is not carried by a circuit element and, therefore, does not generate a potential difference. Therefore, the current in the resistor R is constant in time:

$$i_R = \boxed{\frac{\mathcal{E}}{R}} \qquad (2)$$

(b) Now, apply Kirchhoff's loop rule to the right-hand loop of the circuit:

$$-i_L R_L - L\frac{di_L}{dt} = 0 \qquad (3)$$

Rearrange the equation in preparation for integration, and then integrate:

$$\frac{di_L}{i_L} = -\frac{R_L}{L}dt \quad \rightarrow \quad \int_{i_{L,0}}^{i_L} \frac{di_L}{i_L} = -\frac{R_L}{L}\int_0^t dt \quad \rightarrow \quad \ln\frac{i_L}{i_{L,0}} = -\frac{R_L}{L}t$$

$$\rightarrow \quad i_L = i_{L,0}e^{-\frac{R_L}{L}t} \qquad (4)$$

Just before switch S_3 is closed, the current in the left loop of the circuit is

$$i_{L,0} = \frac{\mathcal{E}}{R + R_L} \qquad (5)$$

Substituting Equation (5) into Equation (4),

$$i_L = \boxed{\frac{\mathcal{E}}{R + R_L}e^{-\frac{R_L}{L}t}} \qquad (6)$$

(c) Applying Kirchhoff's junction rule to the junction to the right of *R*, we see that

$$i_R - i_L - i_S = 0 \quad \rightarrow \quad i_S = i_R - i_L \qquad (7)$$

Substitute Equations (2) and (6) into Equation (7):

$$i_S = \boxed{\frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R + R_L}e^{-\frac{R_L}{L}t}} \qquad (8)$$

(d) Let *t* go to zero in Equation (8):

$$i_S(0) = \frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R + R_L}e^0 = \frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R + R_L} = \boxed{\frac{\mathcal{E}R_L}{R(R + R_L)}} \qquad (9)$$

Finalize The current in the resistor R is constant. Only the currents in the inductor and the switch change in time. Notice that, after a long time, the current in the switch (Equation (8)) is the same as that in the resistor R (Equation (2)). The inductor has an emf across it when the current is changing, but eventually the current reaches a steady state and the inductor has no effect on the circuit. Even though the battery is connected across the internal resistance of the inductor in this steady-state situation, the current in the inductor is zero because there is a zero-resistance wire connected in parallel with it. With no current in the inductor, there is no potential difference across the resistance of its windings.

Answers: (a) $\frac{\mathcal{E}}{R}$ (b) $\frac{\mathcal{E}}{R+R_L}e^{-\frac{R_L}{L}t}$ (c) $\frac{\mathcal{E}}{R}-\frac{\mathcal{E}}{R+R_L}e^{-\frac{R_L}{L}t}$

(d) $\frac{\mathcal{E}R_L}{R\left(R+R_L\right)}$

***TP31.2** **Conceptualize** Be sure you are clear on Figure 31.15 and what is happening in the circuit. When you throw the switch to position b, the battery is removed from the circuit and the circuit undergoes damped oscillations.

Categorize We categorize the circuit as an *RLC* circuit, so we can use the material discussed in Section 31.6.

Analyze Equation 31.26 gives the theoretical charge on the capacitor as a function of time. Therefore, the voltage across the capacitor is

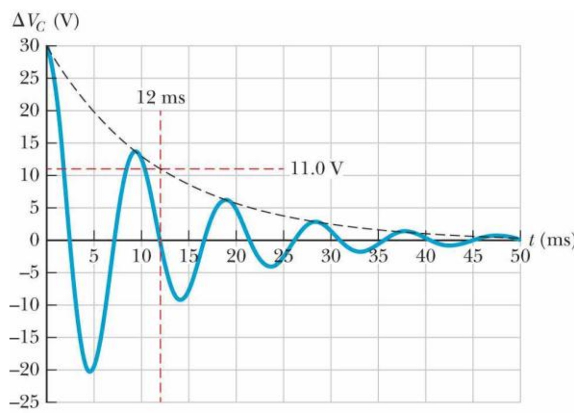
$$\Delta V_C = \frac{q}{C} = \frac{Q_{\text{max}}}{C}e^{-Rt/2L}\cos\omega_d t = \mathcal{E}e^{-Rt/2L}\cos\omega_d t \qquad (1)$$

where we have recognized that the maximum voltage on the capacitor was that of the battery when the switch was at position *a*.

(a) Because the voltage on the capacitor is ε at $t = 0$, we see from the graph that

$$\mathcal{E} = \boxed{30.0\text{ V}}$$

(b) To find the relationship between L and R, draw a smooth exponential curve following the peaks of the voltage curve, like in Figure 31.15a. For our data, the graph with the exponential curve added looks like Fig. ANS TP31.2a.



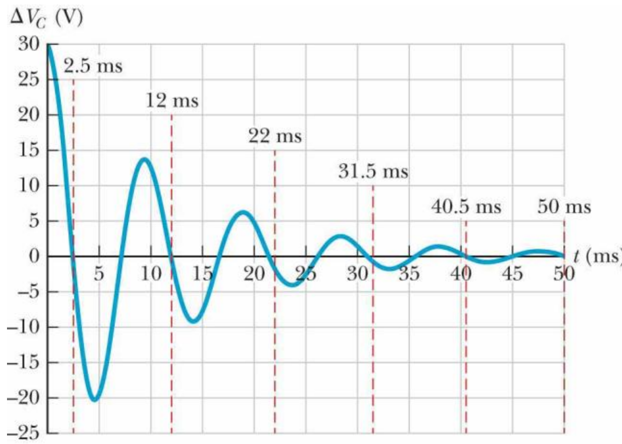
ANS. FIG. TP31.2 a

Also seen on the graph is a red line indicating 11.0 V, which is the voltage corresponding to εe^{-1}. Therefore, this voltage would occur (ignoring the cosine fluctuations) at the time $t = \tau$, where τ is the time constant. From the graph, we see, then, that the time constant is 12 ms. We keep only two significant figures due to the estimation necessary to read the graph. From the exponential factor in Equation 31.26, we see that the time constant is

$$\tau = 2\frac{L}{R} \qquad (2)$$

This process has allowed us to find something about the ratio of L to R, but we cannot find either one from this relationship alone.

Now, let's consider Equation 31.27, which also relates to L and R. This equation gives the frequency of the oscillations. Let's look back at the graph and find each crossing of the curve with the t axis that occurs after the curve has reached a maximum (see Fig. ANS TP31.2b):



ANS. FIG. TP31.2 b

Now let's tabulate these times and take the difference between adjacent times:

t (ms)	Δt (ms)
2.5	
12	9.5
22	10
31.5	9.5
40.5	9
50	9.5

From these data, it appears that the period of the oscillations is $T = 9.5$ ms.

Express Equation 31.27 in terms of the period of the oscillations and the time constant:

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \quad \rightarrow \quad \frac{2\pi}{T} = \left[\frac{1}{LC} - \left(\frac{1}{\tau}\right)^2\right]^{1/2} \qquad (3)$$

Solve Equation (3) for the inductance L:

$$L = \frac{1}{C\left[\left(\frac{2\pi}{T}\right)^2 + \frac{1}{\tau^2}\right]} \qquad (4)$$

Substitute numerical values:

$$L = \frac{1}{(15.0 \square 10^{-6}\ \text{F})\left[\left(\frac{2\pi}{9.5\times 10^{-3}\ \text{s}}\right)^2 + \frac{1}{(12\times 10^{-3}\ \text{s})^2}\right]} = \boxed{0.15\ \text{H}}$$

(c) Solve Equation (2) for the resistance:

$$\tau = 2\frac{L}{R} \quad \rightarrow \quad R = 2\frac{L}{\tau} \qquad (5)$$

Substitute numerical values:

$$R = 2\left(\frac{0.15\ \text{H}}{12\times 10^{-3}\ \text{s}}\right) = \boxed{25\ \Omega}$$

Finalize Your answers may differ from these a bit due to the reading of the graph, which is an estimation process.

Answers: Answer will vary due to estimation, but typical values are: (a) 30.0 V (b) 0.15 H (c) 25 Ω

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 31.1 Self-Induction and Inductance

P31.1 The self-induced emf at any instant is

$$\mathcal{E}_L = -L\frac{di}{dt}$$

Its average value is

$$\mathcal{E}_{L,\text{ave}} = -L\left(\frac{I_f - I_i}{t}\right) = (-2.00\text{ H})\left(\frac{0 - 0.500\text{ A}}{1.00\times 10^{-2}\text{ s}}\right)\left(\frac{\text{V}\cdot\text{s/A}}{1\text{ H}}\right)$$
$$= \boxed{+100\text{ V}}$$

P31.2 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(70.0)^2\,\pi(6.50\times 10^{-3}\text{ m})^2}{0.600\text{ m}}$$
$$= \boxed{1.36\ \mu\text{H}}$$

P31.3 From $|\mathcal{E}| = L\left(\frac{\Delta i}{\Delta t}\right)$, we have

$$L = \frac{\mathcal{E}}{(\Delta i/\Delta t)} = \frac{24.0\times 10^{-3}\text{ V}}{10.0\text{ A/s}} = 2.40\times 10^{-3}\text{ H}$$

From $L = \frac{N\Phi_B}{i}$, we have

$$\Phi_B = \frac{Li}{N} = \frac{(2.40\times 10^{-3}\text{ H})(4.00\text{ A})}{500}$$
$$= \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$$

P31.4 (a) $B = \mu_0 n i = (4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{450}{0.120\text{ m}}\right)(0.040\,0\text{ A}) = \boxed{188\ \mu\text{T}}$

(b) $$\Phi_B = BA = B\pi\left(\frac{15.0\times10^{-3}\,\text{m}}{2}\right)^2 = \boxed{3.33\times10^{-8}\ \text{T}\cdot\text{m}^2}$$

(c) $$L = \frac{N\Phi_B}{i} = \frac{450\Phi_B}{0.040\,0\ \text{A}} = \boxed{0.375\ \text{mH}}$$

(d) B and Φ_B are proportional to current; L is independent of current.

P31.5 The emf is given by

$$\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L\frac{di}{dt}$$

from which we obtain

$$di = -\frac{\mathcal{E}_0}{L}e^{-kt}dt$$

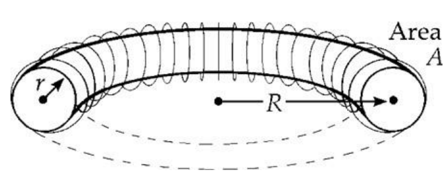
If we require $i \to 0$ as $t \to \infty$, the solution is $i = \frac{\mathcal{E}_0}{Lk}e^{-kt} = \frac{dq}{dt}$, so

$$Q = \int i\,dt = \int_0^\infty \frac{\mathcal{E}_0}{Lk}e^{-kt}\,dt = -\frac{\mathcal{E}_0}{Lk^2} \quad \rightarrow \quad \boxed{|Q| = \frac{\mathcal{E}_0}{Lk^2}}$$

P31.6 The inductance of a solenoid is $L = \frac{\mu_0 N^2 A}{\ell}$.

The long solenoid is bent into a circle of radius R, so its length $\ell \approx 2\pi R$; therefore, the inductance of the toroid is

$$L = \frac{\mu_0 N^2 A}{\ell} \approx \frac{\mu_0 N^2(\pi r^2)}{2\pi R} = \frac{1}{2}\mu_0 N^2\frac{r^2}{R}$$



ANS. FIG. P31.6

P31.7 Using the definition of self-inductance, $\mathcal{E} = -L\dfrac{di}{dt}$, we obtain

$$\mathcal{E} = -L\frac{d}{dt}\left(I_i \sin\omega t\right) = -L\omega\left(I_i \cos\omega t\right)$$
$$= -\left(10.0\times 10^{-3}\right)\left[2\pi(60.0)\right](5.00)\cos\omega t$$

$$\boxed{\mathcal{E} = -18.8\cos 120\pi t,\ \text{where } \mathcal{E} \text{ is in volts and } t \text{ is in seconds.}}$$

P31.8 The current change is linear, so $\mathcal{E} = -L\dfrac{di}{dt} = -L\dfrac{\Delta i}{\Delta t}$.

$t = 0$ to 4 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{-2.00\text{ mA}}{4.00\text{ ms}} = +2.00\text{ mV}$$

$t = 4$ to 8 ms:

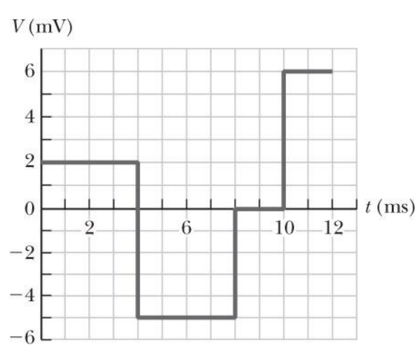
$$\mathcal{E} = -(4.00\text{ mH})\frac{+5.00\text{ mA}}{4.00\text{ ms}} = -5.00\text{ mV}$$

$t = 8$ to 10 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{0}{2.00\text{ ms}} = 0.00\text{ mV}$$

$t = 10$ to 12 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{-3.00\text{ mA}}{2.00\text{ ms}} = +6.00\text{ mV}$$



ANS. FIG. P31.8

***P31.9** **Conceptualize** You may have seen wire wrapped uniformly around cardboard tubes at a home store or hardware store. They represent ready-made inductors!

Categorize We can find the results using equations in Section 31.1, so we will categorize the problem as a substitution problem.

(a) Use Equation 31.4 to find the inductance:

$$L = \frac{\mu_0 N^2}{\ell} A = \frac{\mu_0 N^2}{\ell} \pi \left(\frac{d}{2}\right)^2$$
$$= \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/ A}\right)(580)^2}{0.360 \text{ m}} \pi \left(\frac{0.080\ 0 \text{ m}}{2}\right)^2 = 5.90 \times 10^{-3} \text{ H} = \boxed{5.90 \text{ mH}}$$

(b) Use Equation 31.1 to find the emf:

$$|\mathcal{E}| = L\frac{di}{dt} = (5.90 \text{ mH})(4.00 \text{ A/ s}) = 0.023\ 6 \text{ V} = \boxed{23.6 \text{ mV}}$$

Section 31.2 *RL* Circuits

P31.10 (a) The inductance of a solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 N^2 \pi r^2}{\ell} = \frac{\mu_0 (510)^2 \pi \left(8.00 \times 10^{-3} \text{ m}\right)^2}{0.140 \text{ m}}$$
$$= 4.69 \times 10^{-4} \text{ H} = \boxed{0.469 \text{ mH}}$$

(b) The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{4.69 \times 10^{-4} \text{ H}}{2.50\ \Omega} = 1.88 \times 10^{-4} \text{ s} = \boxed{0.188 \text{ ms}}$$

P31.11 ((a) Using $\tau = RC = \frac{L}{R}$, we get

$$R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00\text{ H}}{3.00\times 10^{-6}\text{ F}}} = 1.00\times 10^{3}\ \Omega = \boxed{1.00\text{ k}\Omega}.$$

(b) The time constant is

$$\begin{aligned}\tau &= RC = \left(1.00\times 10^{3}\ \Omega\right)\left(3.00\times 10^{-6}\text{ F}\right)\\ &= 3.00\times 10^{-3}\text{ s} = \boxed{3.00\text{ ms}}\end{aligned}$$

P31.12 Taking $\tau = \frac{L}{R}$, and $i = I_i e^{-t/\tau}$: $\frac{di}{dt} = I_i e^{-t/\tau}\left(-\frac{1}{\tau}\right)$

$iR + L\frac{di}{dt} = 0$ will be true if $I_i R\ e^{-t/\tau} + L\left(I_i e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = 0$

We have agreement because $\tau = \frac{L}{R}$.

P31.13 For the increasing current $i = \frac{\mathcal{E}}{R}\left(1 - e^{-Lt/R}\right)$. The final value is $\frac{\mathcal{E}}{R}$, so the condition on Δt is

$$\begin{aligned}0.800\frac{\mathcal{E}}{R} &= \frac{\mathcal{E}}{R}\left(1 - e^{-L\Delta t/R}\right)\\ e^{-L\Delta t/R} &= 0.200\\ e^{+L\Delta t/R} &= 5.00\\ \frac{L\Delta t}{R} &= \ln 5.00\\ \Delta t &= \frac{R\ln 5.00}{L}\end{aligned}$$

At the moment when the battery is removed, the current in the coil is quite precisely $\frac{\mathcal{E}}{R}$. During the decrease, $i = I_i e^{-Lt/R} = \frac{\mathcal{E}}{R}e^{-Lt/R}$.

(a) at $t = \Delta t = \dfrac{R \ln 5.00}{L}$,

$$\frac{i}{I_i} = e^{-L\Delta t/R} = 0.200 = \boxed{20.0\%}$$

(b) at $t = 2\Delta t$,

$$\frac{i}{I_i} = e^{-L2\Delta t/R} = \left(e^{-L\Delta t/R}\right)^2 = (0.200)^2 = 0.040\ 0 = \boxed{4.00\%}$$

***P31.14** **Conceptualize** When the switch is closed, current exists in both resistors and the inductor. The current is in the downward direction in Figure P31.14 in the lightbulb and the inductor. When the switch is opened, the inductor provides an emf in the opposite direction as the battery, so as to try to keep the current going. This current will exist in the lightbulb, causing it to glow, with the current being in the upward direction in Figure P31.14.

Categorize We model the circuit as an *RL* circuit.

Analyze (a) Imagine that the switch in Figure P31.14 has been closed a long time. The current is steady, so the inductor offers no resistance in the circuit, and the current in the inductor is the same as that in R_2:

$$I_i = \frac{\mathcal{E}}{R_2} \qquad (1)$$

Now imagine opening the switch. The circuit now is just the singleloop in Figure P31.14 containing the lightbulb, the resistor R_2, and the inductor. The inductor will provide an emf causing a time-varying current *i* in the clockwise direction. Apply Kirchhoff's loop rule to this branch, traveling in the clockwise direction:

$$-iR_1 - iR_2 - L\frac{di}{dt} = 0 \qquad (2)$$

Rearrange this equation to prepare for integration:

$$i(R_1 + R_2) = -L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{i} = -\frac{R_1 + R_2}{L}dt \qquad (3)$$

Integrate Equation (3):

$$\int_{I_i}^{i}\frac{di}{i} = -\frac{R_1 + R_2}{L}\int_0^t dt \quad \rightarrow \quad \ln i - \ln I_i = -\frac{R_1 + R_2}{L}t$$

$$\rightarrow \quad \ln\left(\frac{i}{I_i}\right) = -\frac{R_1 + R_2}{L}t \quad \rightarrow \quad i = I_i e^{-\frac{R_1+R_2}{L}t} \qquad (4)$$

Now we want the instantaneous power in the lightbulb to be its "normal" value, that is, its value when operating at 120 V, $P_{\text{normal}} = 40.0$ W at the moment the switch is opened:

$$P_{\text{normal}} = I_i^2 R_1 \qquad (5)$$

where we have recognized that the initial current in the lightbulb is the same as that in R_2, which is the same as the steady current in R_2 before the switch was opened. Substitute Equation (1) into Equation (5):

$$P_{\text{normal}} = \left(\frac{\mathcal{E}}{R_2}\right)^2 R_1 \quad \rightarrow \quad R_2 = \mathcal{E}\sqrt{\frac{R_1}{P_{\text{normal}}}} \qquad (6)$$

Evaluate R_1 from its normal operating conditions:

$$P_{\text{normal}} = \frac{(\Delta V_{\text{normal}})^2}{R_1} \quad \rightarrow \quad R_1 = \frac{(\Delta V_{\text{normal}})^2}{P_{\text{normal}}} \qquad (7)$$

Substitute Equation (7) into Equation (6):

$$R_2 = \mathcal{E}\sqrt{\frac{1}{P_{\text{normal}}}\left[\frac{\left(\Delta V_{\text{normal}}\right)^2}{P_{\text{normal}}}\right]} = \mathcal{E}\frac{\Delta V_{\text{normal}}}{P_{\text{normal}}} \tag{8}$$

Substitute numerical values:

$$R_2 = (12.0\text{ V})\frac{120\text{ V}}{40\text{ W}} = \boxed{36.0\ \Omega}$$

(b) The time constant for the circuit with the switch open can be found from Equation (4), and then we can find the inductance L:

$$\tau = \frac{L}{R_1 + R_2} \quad \rightarrow \quad L = \tau(R_1 + R_2) \tag{9}$$

Use Equation (4) to impose the requirement that the current drop to 50.0% of its initial value at 2.00 s:

$$i = I_i e^{-t/\tau} \quad \rightarrow \quad 0.500 I_i = I_i e^{-(2.00\text{ s})/\tau} \quad \rightarrow \quad \tau = 2.89\text{ s} \tag{10}$$

Substitute Equation (10) into Equation (9) and substitute numerical values of the resistances:

$$L = (2.89\text{ s})\left[\frac{\left(\Delta V_{\text{normal}}\right)^2}{P_{\text{normal}}} + 36.0\ \Omega\right] = (2.89\text{ s})\left[\frac{(120\text{ V})^2}{40.0\text{ W}} + 36.0\ \Omega\right]$$
$$= 1.14 \times 10^3\text{ H} = \boxed{1.14\text{ kH}}$$

Finalize This is a very large inductance, as would be expected to keep a large current going in an incandescent light bulb for a time interval of several seconds. In addition, the resistance of an incandescent light bulb does vary with temperature, an effect we did not address in this problem. You might suggest redesigning the demonstration apparatus to employ an LED as the light source.

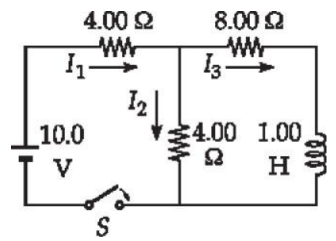
Answers: (a) 36.0 Ω (b) 1.14 kH

P31.15 Name the currents as shown in ANS. FIG. P31.15. By Kirchhoff's laws:

$$i_1 = i_2 + i_3 \qquad \textbf{[1]}$$

$$+10.0\text{ V} - 4.00i_1 - 4.00i_2 = 0 \qquad \textbf{[2]}$$

$$+10.0\text{ V} - 4.00i_1 - 8.00i_3 - (1.00)\frac{di_3}{dt} = 0 \qquad \textbf{[3]}$$



ANS. FIG. P31.15

From [1] and [2],

$$+10.0 - 4.00i_1 - 4.00i_1 + 4.00i_3 = 0$$

$$i_1 = 0.500i_3 + 1.25\text{ A}$$

Then [3] becomes $10.0\text{ V} - 4.00(0.500i_3 + 1.25\text{ A}) - 8.00i_3 - (1.00)\dfrac{di_3}{dt} = 0$

$$(1.00\text{ H})\left(\frac{di_3}{dt}\right) + (10.0\ \Omega)i_3 = 5.00\text{ V}$$

or $$5.00\text{ V} - (10.0\ \Omega)i_3 - (1.00\text{ H})\left(\frac{di_3}{dt}\right) = 0$$

which can be compared to the general form (Equation 31.6)

$$\varepsilon - iR - L\frac{di}{dt} = 0$$

which has the solution (from Equation 31.7) $i = \dfrac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$.

Thus, we have:

(a) $$i_3 = \left(\frac{5.00\text{ V}}{10.0\ \Omega}\right)\left[1 - e^{-(10.0\ \Omega)t/1.00\text{ H}}\right] = \boxed{(0.500\text{ A})\left[1 - e^{-10t/\text{s}}\right]}$$

(b) $$i_1 = 1.25 + 0.500 I_3 = \boxed{1.50\text{ A} - (0.250\text{ A})e^{-10t/\text{s}}}$$

P31.16 Refer to ANS. FIG. P31.15 above. Name the currents as shown. By Kirchhoff's laws:

$$i_1 = i_2 + i_3 \qquad \textbf{[1]}$$

$$\mathcal{E} - Ri_1 - Ri_2 = 0 \qquad \textbf{[2]}$$

$$\mathcal{E} - Ri_1 - 2Ri_3 - L\frac{di_3}{dt} = 0 \qquad \textbf{[3]}$$

From [1] and [2],

$$\mathcal{E} - Ri_1 - R(i_1 - i_3) = 0$$
$$\mathcal{E} - Ri_1 - Ri_1 + Ri_3 = 0$$

$$i_1 = \frac{1}{2}i_3 + \frac{\mathcal{E}}{2R}$$

Then [3] becomes

$$\mathcal{E} - R\left(\frac{1}{2}i_3 + \frac{\mathcal{E}}{2R}\right) - 2Ri_3 - L\frac{di_3}{dt} = 0$$

$$L\frac{di_3}{dt} + 2.5Ri_3 = \frac{\mathcal{E}}{2}$$
$$\frac{\mathcal{E}}{2} - 2.5Ri_3 - L\frac{di_3}{dt} = 0$$

which can be compared to the general form (Equation 31.6)

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

which has the solution (from Equation 31.7)

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

Thus, we have:

(a) $$i_3 = \left(\frac{\mathcal{E}/2}{2.5R}\right)\left[1 - e^{-2.5Rt/L}\right] = \boxed{\frac{\mathcal{E}}{5R}\left(1 - e^{-5Rt/2L}\right)}$$

(b) $$i_1 = \frac{1}{2}i_3 + \frac{\mathcal{E}}{2R} = \frac{1}{2}\left[\frac{\mathcal{E}}{5R}\left(1 - e^{-5Rt/2L}\right)\right] + \frac{\mathcal{E}}{2R}$$

$$i_1 = \frac{\mathcal{E}}{10R}\left(1 - e^{-5Rt/2L}\right) + \frac{5\mathcal{E}}{10R} = \boxed{\frac{\mathcal{E}}{10R}\left(6 - e^{-5Rt/2L}\right)}$$

P31.17 From Equation 31.7, $i = I_i\left(1 - e^{-t/\tau}\right)$. Therefore,

$$\frac{di}{dt} = -I_i\left(e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)$$

where

$$\tau = \frac{L}{R} = \frac{15.0\ \text{H}}{30.0\ \Omega} = 0.500\ \text{s}$$

Then, $$\frac{di}{dt} = \frac{R}{L}I_i e^{-t/\tau} \quad \text{with} \quad I_i = \frac{\mathcal{E}}{R}$$

(a) At $t = 0$,

$$\frac{di}{dt} = \frac{R}{L}I_i e^0 = \frac{\mathcal{E}}{L} = \frac{100\ \text{V}}{15.0\ \text{H}} = \boxed{6.67\ \text{A/s}}$$

(b) At $t = 1.50$ s,

$$\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-t/\tau} = (6.67\ \text{A/s})e^{-1.50/(0.500)} = (6.67\ \text{A/s})e^{-3.00}$$

$$= \boxed{0.332\ \text{A/s}}$$

P31.18 (a) For a series connection, both inductors carry equal currents at every instant, so $\frac{di}{dt}$ is the same for both. The voltage across the pair is

$$L_{\text{eq}}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} \quad \to \quad L_{\text{eq}} = L_1 + L_2$$

(b) For a parallel connection, the voltage across each inductor is the same for both.

$$L_{\text{eq}}\frac{di}{dt} = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt} = \Delta V_L$$

where the currents are related by $i = i_1 + i_2$. Therefore,

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{\Delta V_L}{L_{\text{eq}}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2} \quad \to \quad \frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

(c) $$L_{\text{eq}}\frac{di}{dt} + R_{\text{eq}}i = L_1\frac{di}{dt} + iR_1 + L_2\frac{di}{dt} + iR_2$$

Now i and $\frac{di}{dt}$ are independent quantities under our control, so functional equality requires both $L_{\text{eq}} = L_1 + L_2$ and $R_{\text{eq}} = R_1 + R_2$.

(d) Yes. The relations $\Delta V = L_{\text{eq}}\frac{di}{dt} + R_{\text{eq}}i = L_1\frac{di_1}{dt} + R_1 i_1 = L_2\frac{di_2}{dt} + R_2 i_2$, where $i = i_1 + i_2$ and $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$, must always be true.

We may choose to keep the currents constant in time. Then, from $i = i_1 + i_2$, we have

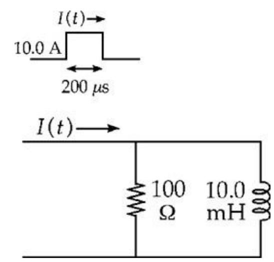
$$\frac{\Delta V_L}{R_{\text{eq}}} = \frac{\Delta V_L}{R_1} + \frac{\Delta V_L}{R_2} \quad \to \quad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We may choose to make the current oscillate so that at a given moment it is zero. Then, from $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$, as in part (b), we have $\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$.

P31.19 For $t \le 0$, the current in the inductor is $\boxed{\text{zero}}$.

For $0 \le t \le 200\ \mu\text{s}$, there will be current i_R in the resistor and I_L in the inductor so that $i = i_R + i_L = I_i = 10.0$ A. Assuming both currents are downward in ANS. FIG. P31.19, we apply Kirchhoff's loop rule going counterclockwise around the loop, and we find that

$$-i_R R + L\frac{di_L}{dt} = 0$$



ANS. FIG. P31.19

Using $I_i = i_R + i_L \ \rightarrow\ i_R = I_i - i_L$, we have

$$-(I_i - i_L)R + L\frac{di_L}{dt} = 0$$

Then,

$$L\frac{dI_L}{dt} = (I_{\max} - I_L)R$$

$$\int_0^I \frac{di_L}{(I_i - I_L)} = \int_0^t \frac{R}{L}dt$$

$$-\ln\frac{(I_i - i_L)}{I_i} = \frac{R}{L}t$$

which gives

$$i_L = I_i\left(1 - e^{-Rt/L}\right)$$

We see that $t = 0$, $i_L = 0$ as we expect because of the back emf induced in the inductor. With the time constant

$$\tau = \frac{L}{R} = \frac{(10.0 \text{ mH})}{(100\ \Omega)} = 1.00 \times 10^{-4} \text{ s}$$

we have

$$i_L = I_i\left(1 - e^{-t/\tau}\right) = \boxed{(10.0 \text{ A})\left(1 - e^{-10\,000t/\text{s}}\right)} \qquad (0 \le t \le 200\ \mu\text{s})$$

At $t = 200\ \mu$s, $i = (10.00 \text{ A})\left(1 - e^{-2.00}\right) = 8.65 \text{ A};$ thereafter, the current decays. The loop rule gives the same result,

$$-i_R R + L\frac{di_L}{dt} = 0$$

but now $i_R + i_L = 0 \rightarrow i_R = -i_L$, so we have

$$i_L R + L\frac{di_L}{dt} = 0 \rightarrow L\frac{di_L}{dt} = -i_L R$$

$$\int_{I_i}^{I} \frac{di_L}{i_L} = -\int_{200\ \mu\text{s}}^{t} \frac{R}{L}dt$$

$$\ln\frac{i_L}{I_i} = -\frac{R}{L}(t - 200\ \mu\text{s}) \rightarrow i_L = I_i e^{-R(t-200\mu\text{s})/L}$$

For $t = 200\ \mu$s, $i_i = 8.65$ A, and for $t \ge 200\ \mu\text{s}$,

$$i = (8.65 \text{ A})e^{-10\,000(t-200\ \mu\text{s})/\text{s}} = (8.65 \text{ A})e^{-10\,000t/\text{s}+2.00}$$

$$= \left(8.65e^{2.00} \text{ A}\right)e^{-10\,000t/\text{s}} = \boxed{(63.9 \text{ A})e^{-10\,000t/\text{s}}} \qquad (t \ge 200\ \mu\text{s})$$

Section 31.3 Energy in a Magnetic Field

P31.20 The inductance of the solenoid is

$$L = N\frac{\Phi_B}{i} = 200\frac{3.70 \times 10^{-4} \text{ Wb}}{1.75 \text{ A}} = 0.042\ 3 \text{ H}$$

The energy stored is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(0.042\ 3\ \text{H})(1.75\ \text{A})^2 = 0.064\ 8\ \text{J} = \boxed{64.8\ \text{mJ}}$$

P31.21 For a solenoid of length ℓ, the inductance is $L = \dfrac{\mu_0 N^2 A}{\ell}$.

Thus, since $U_B = \dfrac{1}{2}Li^2 = \dfrac{\mu_0 N^2 A i^2}{2\ell}$, the stored energy is

$$U_B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(68)^2 \pi (6.00 \times 10^{-3}\ \text{m})^2 (0.770\ \text{A})^2}{2\ (0.080\ 0)\ \text{m}}$$
$$= \boxed{2.44 \times 10^{-6}\ \text{J}}$$

P31.22 We compute the integral:

$$\int_0^\infty e^{-2Rt/L}\,dt = -\frac{L}{2R}\int_0^\infty e^{-2Rt/L}\left(\frac{-2R\,dt}{L}\right) = -\frac{L}{2R}e^{-2Rt/L}\Big|_0^\infty$$
$$= -\frac{L}{2R}\left(e^{-\infty} - e^0\right) = \frac{L}{2R}(0-1) = \boxed{\frac{L}{2R}}$$

P31.23 The current in the circuit at time *t* is $i = I_i\left(1 - e^{-t/\tau}\right)$, where $I_i = \dfrac{\mathcal{E}}{R}$, and

the energy stored in the inductor is $U_B = \dfrac{1}{2}Li^2$.

(a) As $t \to \infty$, $I \to I_i = \dfrac{\mathcal{E}}{R} = \dfrac{24.0\ \text{V}}{8.00\ \Omega} = 3.00$ A, and

$$U_B = \frac{1}{2}LI_i^2 = \frac{1}{2}(4.00\ \text{H})(3.00\ \text{A})^2 = \boxed{18.0\ \text{J}}$$

(b) At $t = \tau$, $I = I_i\left(1 - e^{-1}\right) = (3.00\ \text{A})(1 - 0.368) = 1.90$ A, and

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(4.00\ \text{H})(1.90\ \text{A})^2 = \boxed{7.19\ \text{J}}$$

P31.24 (a) $P = i\Delta V = (3.00\text{ A})(22.0\text{ V}) = \boxed{66.0\text{ W}}$

(b) $P = i\Delta V_R = i^2 R = (3.00\text{ A})^2 (5.00\ \Omega) = \boxed{45.0\text{ W}}$

(c) METHOD 1: We treat the real inductor as an ideal inductor (with no resistance) in series with an ideal resistor (with no inductance). When the current is 3.00 A, Kirchhoff's loop rule reads

$$+22.0\text{ V} - (3.00\text{ A})(5.00\ \Omega) - \Delta V_L = 0$$
$$\Delta V_L = 7.00\text{ V}$$

The power being stored in the inductor is

$$i\Delta V_L = (3.00\text{ A})(7.00\text{ V}) = \boxed{21.0\text{ W}}$$

METHOD 2: We do not treat the real inductor as an ideal inductor in series with an ideal resistor.

We wish to find the rate at which energy is being delivered to the inductor. As discussed in Section 31.3, $U_B = \frac{1}{2}Li^2 \to \frac{dU_B}{dt} = Li\frac{di}{dt}$.

We know L (0.040 0 H) and i (3.00 A); we need to evaluate the term $\frac{di}{dt}$. From Equations 31.7 and 31.8 (or Equation 31.9),

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \to \frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

because $\tau = \frac{L}{R}$. Also,

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \to e^{-t/\tau} = 1 - \frac{iR}{\mathcal{E}}$$

Therefore,

$$\frac{dU_B}{dt} = Li\frac{di}{dt} = Li\left(\frac{\mathcal{E}}{L}e^{-t/\tau}\right) = i\mathcal{E}e^{-t/\tau} = i\mathcal{E}\left(1 - \frac{iR}{\mathcal{E}}\right) = i(\mathcal{E} - iR)$$

When $i = 3.00$ A,

$$\frac{dU_B}{dt} = i(\mathcal{E} - iR) = (3.00\ \text{A})\left[22.0\ \text{V} - (3.00\ \text{A})(5.00\ \Omega)\right]$$
$$= \boxed{21.0\ \text{W}}$$

(d) The power supplied by the battery is equal to the sum of the power delivered to the internal resistance of the coil and the power stored in the magnetic field.

(e) Yes.

(f) Just after $t = 0$, the current is very small, so the power delivered to the internal resistance of the coil (iR^2) is nearly zero, but the rate of the change of the current is large, so the power delivered to the magnetic field (Ldi/dt) is large, and nearly all the battery power is being stored in the magnetic field. Long after the connection is made, the current is not changing, so no power is being stored in the magnetic field, and all the battery power is being delivered to the internal resistance of the coil.

Section 31.4 Mutual Inductance

P31.25 We use Equation 31.17, $|\mathcal{E}_2| = \left|-M\frac{di_1}{dt}\right|$, from which we obtain the mutual inductance:

$$M = \frac{|\mathcal{E}_2|}{|di_1/dt|} = \frac{0.096\ 0\ \text{V}}{1.20\ \text{A/s}} = 0.080\ 0\ \text{H} = \boxed{80.0\ \text{mH}}$$

P31.26 (a) The mutual inductance of the coils is

$$M = \frac{N_B\Phi_{BA}}{i_A} = \frac{700\left(90.0\times10^{-6}\,\text{Wb}\right)}{3.50\ \text{A}} = \boxed{18.0\ \text{mH}}$$

(b) The inductance of coil A is

$$L_A = \frac{\Phi_A}{i_A} = \frac{400\left(300\times10^{-6}\,\text{Wb}\right)}{3.50\text{ A}} = \boxed{34.3\text{ mH}}$$

(c) The emf induced in the other coil is

$$|\varepsilon_B| = \left|-M\frac{di_A}{dt}\right| = (18.0\text{ mH})\left(0.500\ \text{A/s}\right) = \boxed{9.00\text{ mV}}$$

P31.27 (a) Solenoid S_1 creates a nearly uniform field everywhere inside it, given by $B_1 = \mu_0 N_1 i/\ell$. The flux through one turn of solenoid S_2 is

$$\mu_0 \pi R_2^2 N_1 i/\ell$$

The emf induced in solenoid S_2 is

$$-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(di/dt)$$

The mutual inductance is

$$\boxed{M_{12} = \mu_0 \pi R_2^2 N_1 N_2/\ell}$$

(b) Solenoid S_2 creates a nearly uniform field everywhere inside it, given by $B_2 = \mu_0 N_2 i_2/\ell$ and nearly zero field outside. The flux through one turn of solenoid 1 is

$$\mu_0 \pi R_2^2 N_2 i_2/\ell$$

The emf induced in solenoid 1 is

$$-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(di_2/dt)$$

The mutual inductance is

$$\boxed{M_{12} = \mu_0 \pi R_2{}^2 N_1 N_2/\ell.}$$

(c) $\boxed{\text{They are the same.}}$

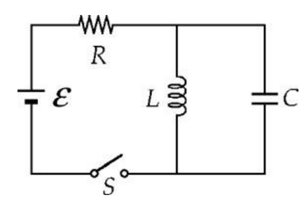
P31.28 (a) A current i in the large loop of radius R produces a magnetic field of magnitude $B = \dfrac{\mu_0 i}{2R}$ at its center. Because the radius of the small loop $r \ll R$, we may treat the flux through the small loop as being approximately $\Phi_B = BA\cos 0.00° = \left(\dfrac{\mu_0 i}{2R}\right)A = \dfrac{\mu_0 \pi r^2 i}{2R}$. The mutual inductance of the loops is then

$$M = \frac{\Phi_B}{i} = \frac{\mu_0 \pi r^2}{2R}$$

(b) $$M = \frac{\mu_0 \pi r^2}{2R} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.020\ 0\ \text{m})^2}{2(0.200\ \text{m})} = 3.95 \times 10^{-9}\ \text{H}$$
$$= \boxed{3.95\ \text{nH}}$$

Section 31.5 Oscillations in an *LC* Circuit

P31.29 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_i = \dfrac{\mathcal{E}}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the *LC* loop.



ANS. FIG. P31.29

We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\dfrac{1}{2}C(\Delta V)^2 = \dfrac{1}{2}LI_i^2$.

Then,

$$L = \frac{C(\Delta V)^2}{I_i^2} = \frac{C(\Delta V)^2 R^2}{\mathcal{E}^2} = \frac{(0.500 \times 10^{-6}\ \text{F})(150\ \text{V})^2(250\ \Omega)^2}{(50.0\ \text{V})^2}$$
$$= 0.281\ \text{H} = \boxed{281\ \text{mH}}$$

P31.30 Find the energy stored in the circuit:

$$U = \frac{Q_{\max}^2}{2C} = \frac{\left(200 \times 10^{-6}\ \text{C}\right)^2}{2\left(50.0 \times 10^{-6}\ \text{F}\right)} = 4.00 \times 10^{-4}\ \text{J} = 400\ \mu\text{J}$$

If the energy is split equally between the capacitor and inductor at some instant, the energy would be half this value, or 200 μJ. Therefore, there would be no time when each component stores 250 μJ.

P31.31 At different times, the maximum energy stored in the capacitor is equal to the maximum energy stored in the inductor.

$$\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \frac{1}{2}LI_i^2$$

so

$$\left(\Delta V_C\right)_{\max} = \sqrt{\frac{L}{C}}I_i = \sqrt{\frac{20.0 \times 10^{-3}\ \text{H}}{0.500 \times 10^{-6}\ \text{F}}}(0.100\ \text{A}) = \boxed{20.0\ \text{V}}$$

P31.32 At $t = 0$ the capacitor charge is at its maximum value, so $\phi = 0$ in

$$Q = Q_{\max}\cos(\omega t + \phi) = Q_{\max}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

Substituting the given information, the charge at 2 ms is

$$\begin{aligned} Q &= (105 \times 10^{-6}\ \text{C})\cos\left(\frac{2.00 \times 10^{-3}\ \text{s}}{\sqrt{(3.30\ \text{H})(840 \times 10^{-12}\ \text{F})}}\right) \\ &= (105 \times 10^{-6}\ \text{C})\cos(38.0\ \text{rad}) \\ &= 1.01 \times 10^{-4}\ \text{C} \end{aligned}$$

(a) Then the energy in the capacitor is

$$U_C = \frac{Q^2}{2C} = \frac{\left(1.01 \times 10^{-4}\ \text{C}\right)^2}{2\left(840 \times 10^{-12}\ \text{F}\right)} = \boxed{6.03\ \text{J}}$$

(c) The constant total energy is that originally of the capacitor:

$$U = \frac{Q_{\text{max}}^2}{2C} = \frac{\left(1.05 \times 10^{-4}\ \text{C}\right)^2}{2\left(840 \times 10^{-12}\ \text{F}\right)} = \boxed{6.56\ \text{J}}$$

(b) So the inductor's energy is the remaining

$$U_L = 6.56\ \text{J} - 6.03\ \text{J} = \boxed{0.529\ \text{J}}$$

Section 31.6 The *RLC* Circuit

P31.33 (a) The frequency of damped oscillations is given by Equation 31.27:

$$\begin{aligned}\omega_d &= \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \\ &= \sqrt{\frac{1}{\left(2.20\times10^{-3}\,\text{H}\right)\left(1.80\times10^{-6}\,\text{F}\right)} - \left(\frac{7.60}{2\left(2.20\times10^{-3}\,\text{H}\right)}\right)^2} \\ &= 1.58\times10^4\ \text{rad/s}\end{aligned}$$

Therefore, $f_d = \dfrac{\omega_d}{2\pi} = \dfrac{1.58\times10^4\ \text{rad/s}}{2\pi} = \boxed{2.51\ \text{kHz}}$.

(b) Critical damping occurs when $\omega_d = 0$, or when

$$R_c = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4\left(2.20\times10^{-3}\,\text{H}\right)}{1.80\times10^{-6}\,\text{F}}} = \boxed{69.9\ \Omega}$$

P31.34 We choose to call positive current clockwise in Figure 31.15. It drains charge from the capacitor according to $i = -\dfrac{dq}{dt}$. A clockwise trip around the circuit then gives

$$+\frac{q}{C} - iR - L\frac{di}{dt} = 0$$

or $$+\frac{q}{C} + \frac{dq}{dt}R + L\frac{d}{dt}\frac{dq}{dt} = 0.$$

P31.35 (a) The charge on the capacitor is given by Equation 31.26:

$$q = Q_{\max} e^{-Rt/2L} \cos \omega_d t \qquad \text{so} \qquad I_i \propto e^{-Rt/2L}$$

When the amplitude of the oscillation falls to 50.0% of its initial value, we have

$$0.500 = e^{-Rt/2L} \qquad \text{and} \qquad \frac{Rt}{2L} = -\ln(0.500)$$

Then,

$$t = -\frac{2L}{R}\ln(0.500) = \boxed{0.693\left(\frac{2L}{R}\right)}$$

(b) The initial energy of the circuit is $U_0 \propto Q_{\max}^2$. When $U = 0.500U_0$,

$$q = \sqrt{0.500}Q_{\max} = 0.707Q_{\max}$$

Then,

$$t = -\frac{2L}{R}\ln(0.707) = \boxed{0.347\left(\frac{2L}{R}\right)} \quad \text{(half as long as part (a))}$$

Additional Problems

P31.36 (a) Let Q represent the magnitude of the opposite charges on the plates of a parallel plate capacitor, the two plates having area A and separation d. The negative plate creates an electric field $\vec{\mathbf{E}} = \dfrac{Q}{2\in_0 A}$ toward itself. It exerts on the positive plate force $\vec{\mathbf{F}} = \dfrac{Q^2}{2\in_0 A}$ toward the negative plate. The total field between the plates is $\dfrac{Q}{\in_0 A}$. The energy density is

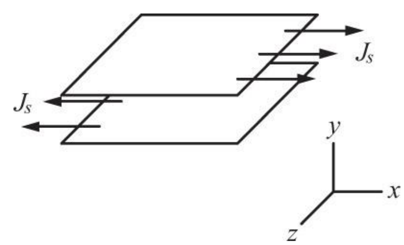
$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \frac{Q^2}{\epsilon_0^2 A^2} = \frac{Q^2}{2\epsilon_0 A^2}$. Modeling this as a negative or inward pressure, we have for the force on one plate

$F = PA = \frac{Q^2}{2\epsilon_0 A}$, in agreement with our first analysis.

(b) The lower of the two current sheets shown creates above it magnetic field $\vec{\mathbf{B}} = \frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$. Let ℓ and w represent the length and width of each sheet. The upper sheet carries current $J_s w$ and feels force

$$\vec{\mathbf{F}} = I\vec{\ell} \times \vec{\mathbf{B}} = J_s w\left[\ell\hat{\mathbf{i}} \times \left(-\frac{\mu_0 J_s}{2}\hat{\mathbf{k}}\right)\right] = \frac{\mu_0 w \ell J_s^2}{2}\hat{\mathbf{j}}$$

The force per area is $P = \frac{F}{\ell w} = \boxed{\frac{\mu_0 J_s^2}{2}}$.



ANS. FIG. P31.36 (b)

(c) Between the two sheets, each sheet contributes the same field, so the total magnetic field is $\frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) + \frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) = \mu_0 J_s \hat{\mathbf{k}}$, with magnitude $\boxed{B = \mu_0 J_s}$. Outside the space they enclose, the fields of the separate sheets are in opposite directions and add to $\boxed{\text{zero}}$.

(d) $u_B = \frac{1}{2\mu_0}B^2 = \frac{\mu_0^2 J_s^2}{2\mu_0} = \boxed{\frac{\mu_0 J_s^2}{2}}$

(e) $\boxed{\text{The energy density found in part (d) agrees with the magnetic pressure found in part (b).}}$

P31.37 The total energy equals the energy in the capacitor and inductor:

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}Li^2$$

so $$i = \sqrt{\frac{3Q^2}{4CL}}$$

The flux through each turn of the coil is

$$\Phi_B = \frac{Li}{N} = \boxed{\frac{q}{2N}\sqrt{\frac{3L}{C}}}$$

where N is the number of turns.

P31.38 (a) The inductor has no resistance, therefore it has voltage across it. It behaves as a $\boxed{\text{short circuit}}$.

(b) The battery sees an equivalent resistance

$$4.00\ \Omega + \left(\frac{1}{4.00\ \Omega} + \frac{1}{8.00\ \Omega}\right)^{-1} = 6.67\ \Omega$$

The battery current is

$$\frac{10.0\ \text{V}}{6.67\ \Omega} = 1.50\ \text{A}$$

The voltage across the parallel combination of resistors is

$$10.0\ \text{V} - (1.50\ \text{A})(4.00\ \Omega) = 4.00\ \text{V}$$

The current in the 8-Ω resistor and the inductor is

$$\frac{4.00\ \text{V}}{8.00\ \Omega} = \boxed{500\ \text{mA}}$$

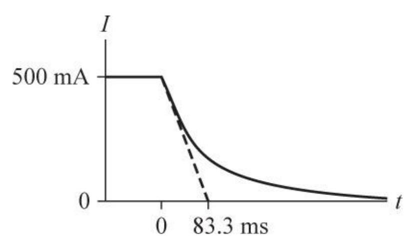
(c) The energy stored in the inductor for $t < 0$ is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(1.00\text{ H})(0.500\text{ A})^2 = \boxed{125\text{ mJ}}$$

(d) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch.

(e) See ANS. FIG. P31.38 (e). The current decreases from 500 mA toward zero, showing exponential decay with a time constant

$$\tau = \frac{L}{R} = \frac{1.00\text{ H}}{3(4.00\ \Omega)} = 0.083\ 3\text{ s} = 8.33\text{ ms}$$



ANS. FIG. P31.38(e)

P31.39 (a) At the center, $B \approx \dfrac{N\mu_0 i}{2R}$.

So the coil creates flux through itself

$$\Phi_B = BA\cos\theta = \frac{N\mu_0 i}{2R}\pi R^2\cos 0° = \frac{1}{2}N\mu_0\pi iR$$

The inductance is

$$L = N\frac{\Phi_B}{i} \approx N\left(\frac{N\mu_0\pi IR}{2i}\right) \approx \boxed{\frac{1}{2}\mu_0\pi N^2 R}$$

(b) To find the inductance of the circuit, we compute its radius from

$$2\pi R = 3(0.300\text{ m}) \quad \rightarrow \quad R = 0.143\text{ m}$$

Then, from the expression found in part(a), the inductance is

$$L \approx \frac{1}{2}\mu_0 \pi N^2 R = \frac{1}{2}\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\pi\left(1^2\right)(0.143\ \text{m})$$
$$= 2.83 \times 10^{-7}\ \text{H}$$
$$\boxed{L \sim 10^{-7}\ \text{H}}$$

(c) The time constant is

$$\tau = \frac{L}{R} \approx \frac{2.83 \times 10^{-7}\ \text{H}}{270\ \Omega} = 1.05 \times 10^{-9}\ \text{s} \approx \boxed{10^{-9}\ \text{s}} \qquad \boxed{\frac{L}{R} \sim 1\ \text{ns}}$$

P31.40 (a) Initially, the current is zero because of the emf induced in the coil resists an increase in the current. Just after the circuit is connected, the potential difference across the resistor is 0 and the emf across the coil is 24.0 V.

(b) After several seconds, the current has reached a steady value and does not change. After several seconds, the potential difference across the resistor is 24.0 V and that across the coil is 0.

(c) The resistor voltage and inductor voltage always add to 24 V. The resistor voltage increases monotonically, so the two voltages are equal to each other, both being 12.0 V, just once. The time is given by

$$V = iR = R\mathcal{E}/R(1 - e^{-Rt/L})$$

Substituting,

$$12\ \text{V} = 24\ \text{V}(1\ -\ e^{-6\Omega t/0.005\ \text{H}})$$
$$0.5 = e^{-1\ 200\ t} \rightarrow 1\ 200\ t = \ln\ 2 \quad \rightarrow \quad t = 0.578\ \text{ms}$$

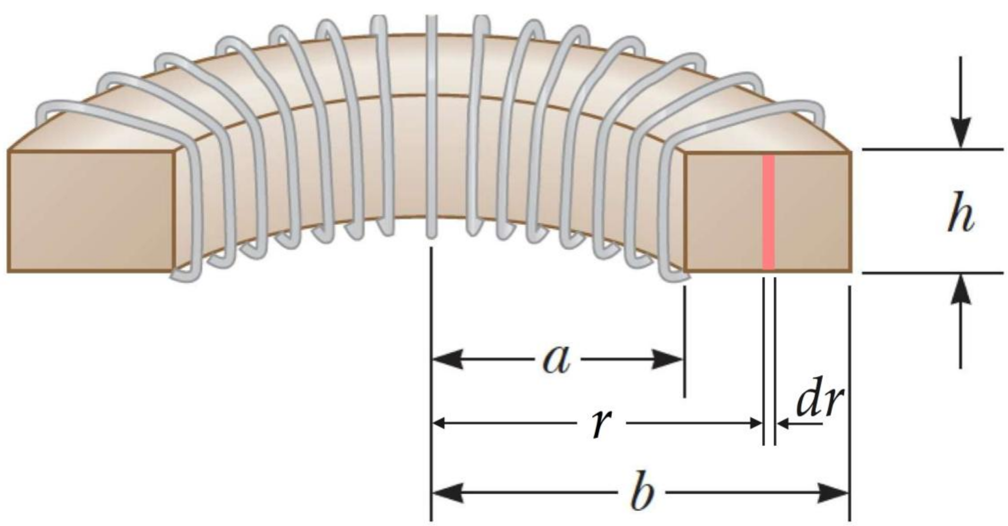
The two voltages are equal to each other, both being 12.0 V, just once, at 0.578 ms after the circuit is connected.

(d) There is now no battery in the circuit, so the current decays to zero. The resistor and inductor are in parallel because they have common connections on each side. $\boxed{\text{As the current decays the potential difference across the resistor is always equal to the emf across the coil.}}$

***P31.41** **Conceptualize** The inductance of the toroid will depend on the dimensions of the wooden ring. In turn, the energy stored in the inductor will depend on the inductance.

Categorize This problem involves a calculation of the inductance of a closelyspaced coil.

Analyze Identify a small element of the torus of radius *r* and thickness *dr*, as shown in the diagram, below:



From Example 29.6, we know that the magnetic field inside a toroid is given by

$$B = \frac{\mu_0 N i}{2\pi r} \qquad (1)$$

where *i* is the time-varying current in the inductor. From Equation 29.18, the magnetic flux through the rectangular cross section of the torus is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \qquad (2)$$

Recognize that the magnetic field is perpendicular to each cross-sectional element and substitute Equation (1) into Equation (2):

$$\Phi_B = \int B\,dA = \int \left(\frac{\mu_0 N i}{2\pi r}\right) h\,dr = \frac{\mu_0 N i h}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right) \tag{3}$$

Then, from Equation 31.2, the inductance of the toroidal inductor is

$$L = \frac{N\Phi_B}{i} = \frac{N}{i}\left[\frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right)\right] = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \tag{4}$$

Then, from Equation 31.12, the energy stored in the inductor is

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left[\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)\right]i^2 = \frac{\mu_0 N^2 h i^2}{4\pi}\ln\left(\frac{b}{a}\right) \tag{5}$$

Solve Equation (5) for the ratio b/a:

$$\frac{b}{a} = e^{\left(\frac{4\pi U_B}{\mu_0 N^2 h i^2}\right)}$$

Substitute numerical values:

$$\frac{b}{a} = e^{\left[\frac{4\pi\left(1.82\times10^{-4}\ \text{J}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(500)^2(0.010\,0\ \text{m})(2.00\ \text{A})^2}\right]} = \boxed{1.20}$$

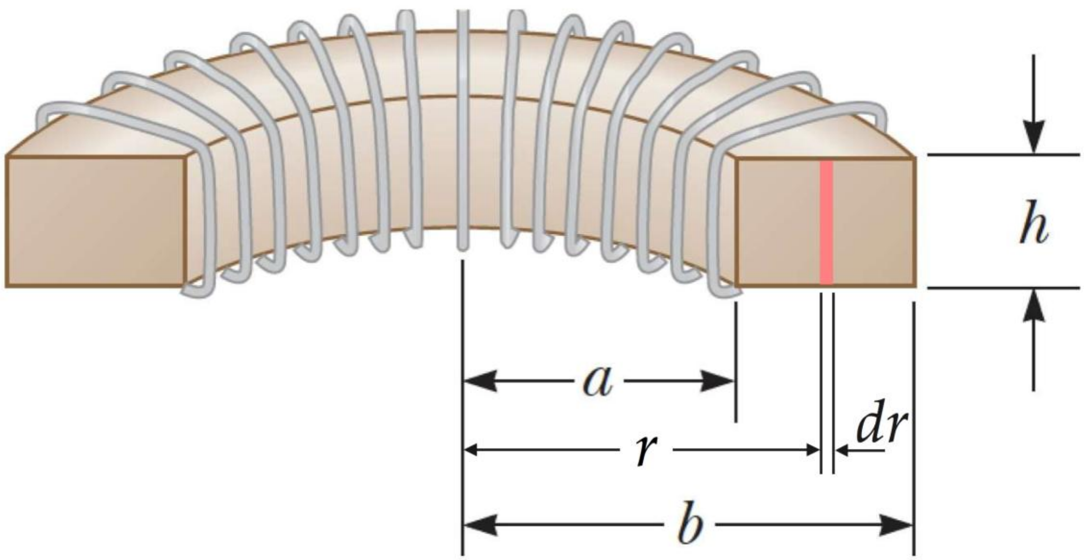
Finalize Notice that we found the ratio b/a without knowing either b or a. Therefore, you are free to choose an outer radius b that is convenient in size for your experiment. Then the result of the problem will determine the value of a to be used to cut the wooden ring.

*Answer:*1.20

***P31.42** **Conceptualize** The inductance of the toroid will depend on the dimensions of the wooden ring. In turn, the energy stored in the inductor will depend on the inductance.

Categorize This problem involves a calculation of the inductance of a closelyspaced coil.

Analyze Identify a small element of the torus of radius r and thickness dr, as shown in the diagram, below:



From Example 29.6, we know that the magnetic field inside a toroid is given by

$$B = \frac{\mu_0 Ni}{2\pi r} \quad (1)$$

where$i$ is the time-varying current in the inductor. From Equation 29.18, the magnetic flux through the rectangular cross section of the torus is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \quad (2)$$

Recognize that the magnetic field is perpendicular to each cross-sectional element and substitute Equation (1) into Equation (2):

$$\Phi_B = \int B\,dA = \int \left(\frac{\mu_0 N i}{2\pi r}\right) h\,dr = \frac{\mu_0 N i h}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right) \tag{3}$$

Then, from Equation 31.2, the inductance of the toroidal inductor is

$$L = \frac{N\Phi_B}{i} = \frac{N}{i}\left[\frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right)\right] = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \tag{4}$$

Then, from Equation 31.12, the energy stored in the inductor is

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left[\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)\right]i^2 = \frac{\mu_0 N^2 h i^2}{4\pi}\ln\left(\frac{b}{a}\right) \tag{5}$$

Solve Equation (5) for the ratio b/a:

$$\frac{b}{a} = e^{\left(\frac{4\pi U_B}{\mu_0 N^2 h i^2}\right)}$$

Finalize Notice that we found the ratio b/a without knowing either b or a. Therefore, you are free to choose an outer radius b that is convenient in size for your experiment. Then the result of the problem will determine the value of a to be used to cut the wooden ring.

Answer: $e^{\left(\frac{4\pi U_B}{\mu_0 N^2 h i^2}\right)}$

***P31.43** **Conceptualize** We see our familiar circuit elements: battery, resistor, inductor, an and capacitor, but in a combination that we have not seen before. Let's analyze the circuit carefully in the Categorize step.

Categorize With the switch in position a, the outer loop is an RL circuit. We've seen this before. Because the switch has been at position *a* for a long time, the current has reached its maximum value. When the switch is thrown to position *b*, the battery is removed and the right-hand loop is an *LC* circuit. We've seen this before, too.

Analyze Begin by finding the current in the circuit with the switch at position *a*:

$$I_{\max} = \frac{\mathcal{E}}{R} \qquad (1)$$

where we have recognized that the inductor has no effect on the circuit because the current is not changing. Now, we throw the switch to position *b*, creating an *LC* circuit. Energy will oscillate between the inductor and the capacitor. There will be no loss of energy because of the absence of resistance in the *LC* circuit, so

$$U_{E,\max} = U_{B,\max} \quad \rightarrow \quad \tfrac{1}{2}\frac{Q_{\max}^2}{C} = \tfrac{1}{2}LI_{\max}^2 \quad \rightarrow \quad Q_{\max} = I_{\max}\sqrt{LC} \qquad (2)$$

In the oscillations of the *LC* circuit, the capacitor begins with zero charge. Therefore, the appropriate solution to Equation 31.20 is

$$q = Q_{\max}\sin\omega t \qquad (3)$$

where ω is given by Equation 31.22:

$$\omega = \frac{1}{\sqrt{LC}} \qquad (4)$$

Combine Equations (1) to (4):

$$q = \left(\frac{\mathcal{E}}{R}\right)\sqrt{LC}\sin\left(\frac{t}{\sqrt{LC}}\right) \qquad (5)$$

Substitute numerical values:

$$q = \left(\frac{12.0\text{ V}}{10.0\ \Omega}\right)\sqrt{\left(2.00\times10^{-3}\text{ H}\right)\left(5.00\times10^{-6}\text{ F}\right)}$$

$$\times\sin\left[\frac{1.00\text{ s}}{\sqrt{\left(2.00\times10^{-3}\text{ H}\right)\left(5.00\times10^{-6}\text{ F}\right)}}\right]$$

$$= -3.67\times10^{-5}\text{ C} \qquad (6)$$

The negative sign on the answer indicates that the polarity of the capacitor is the opposite of the polarity that it has just after $t = 0$. Because our convention is to present the charge on the capacitor as a positive number, the charge on the capacitor is $\boldsymbol{q} =$ $\boxed{3.67\times10^{-5}\text{ C}}$.

Finalize In Figure P31.43, we see that the upper plate of the capacitor is charged positively when the switch is in position *a*. The current in the inductor is downward. When the switch is thrown to position *b*, the circuit consists only of the right-hand loop with the inductor and capacitor. The inductor attempts to maintain the clockwise current in the loop, thereby delivering positive charge to the lower plate of the capacitor. As a result, the polarity of the capacitor reverses, consistent with the negative sign in Equation (6).

Answer: 3.67×10^{-5}C

P31.44 We calculate the angular frequency of the circuit from Equation 31.27:

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

$$= \left[\frac{1}{\left(32.0\times10^{-3}\text{ H}\right)\left(500\times10^{-6}\text{ F}\right)} - \left(\frac{16.0\ \Omega}{2\left(32.0\times10^{-3}\text{ H}\right)}\right)^2\right]^{1/2}$$

$$= 0$$

The fact that the angular frequency at which the circuit oscillates is zero tells you that the circuit is critically damped. There will be no decaying oscillations. The critical resistance is given by

$$R_c = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4\left(32.0 \times 10^{-3}\ \text{H}\right)}{500 \times 10^{-6}\ \text{F}}} = 16.0\ \Omega$$

which is just the resistance that you are using for your experiment

P31.45 The emf across the inductor is given by

$$\mathcal{E} = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t} = -50\frac{\Delta i}{\Delta t}$$

where the rate of change of current $\frac{\Delta i}{\Delta t}$ is in amperes per second (A/s), and the induced emf $\mathcal{E}$ is in millivolts (mV).

Between $t = 0$ and $t = 1$ ms: $\frac{\Delta i}{\Delta t} = 2$ $\mathcal{E} = -100$ mV

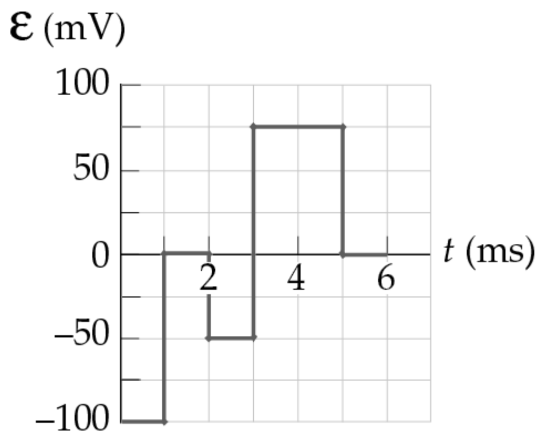
Between $t = 1$ ms and $t = 2$ ms: $\frac{\Delta i}{\Delta t} = 0$ $\mathcal{E} = 0$

Between $t = 2$ ms and $t = 3$ ms: $\frac{\Delta i}{\Delta t} = 1$ $\mathcal{E} = -50$ mV

Between $t = 3$ ms and $t = 5$ ms: $\frac{\Delta i}{\Delta t} = -\frac{3}{2}$ $\mathcal{E} = +75$ mV

Between $t = 5$ ms and $t = 6$ ms: $\frac{\Delta i}{\Delta t} = 0$ $\mathcal{E} = 0$

The graph of $\mathcal{E}$ is shown in ANS. FIG. P31.45.

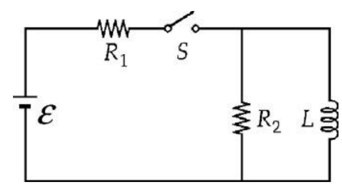


ANS. FIG. P31.45

P31.46 (a) $\boxed{i_1 = i_2 + i}$

(b) For the left-hand loop,

$$\boxed{\mathcal{E} - i_1 R_1 - i_2 R_2 = 0}$$



ANS. FIG. P31.46

(c) For the outside loop,

$$\boxed{\mathcal{E} - i_1 R_1 - L\frac{di}{dt} = 0}$$

(d) Substitute the equation for i_1 from part (a) into the equation in part (b):

$$\mathcal{E} - (i_2 + i)R_1 - i_2 R_2 = 0 \quad \rightarrow \quad i_2 = \frac{\mathcal{E} - iR_1}{R_1 + R_2}$$

Substitute the equation for i_1 from part (a) into the equation in part (c):

$$\mathcal{E} - (i_2 + i)R_1 - L\frac{di}{dt} = 0 \quad \rightarrow \quad i_2 = \frac{\mathcal{E} - L\dfrac{di}{dt}}{R_1} - i$$

Equate the two expressions for i_2:

$$\frac{\mathcal{E}-iR_1}{R_1+R_2}=\frac{\mathcal{E}-L\dfrac{di}{dt}}{R_1}-i$$

Expanding and solving,

$$\mathcal{E}-L\frac{di}{dt}=\left(\frac{\mathcal{E}-iR_1}{R_1+R_2}+i\right)R_1$$

$$=\left[\frac{\mathcal{E}-iR_1+i\left(R_1+R_2\right)}{R_1+R_2}\right]R_1=\left(\frac{\mathcal{E}+iR_2}{R_1+R_2}\right)R_1$$

$$L\frac{di}{dt}=\mathcal{E}-\left(\frac{\mathcal{E}+iR_2}{R_1+R_2}\right)R_1$$

$$=\frac{\mathcal{E}\left(R_1+R_2\right)-\left(\mathcal{E}+iR_2\right)R_1}{R_1+R_2}=\frac{\mathcal{E}\left(R_2\right)-\left(iR_2\right)R_1}{R_1+R_2}$$

And finally,

$$\mathcal{E}\frac{R_2}{R_1+R_2}-i\frac{R_1R_2}{R_1+R_2}-L\frac{di}{dt}=0$$

Calling $\mathcal{E}'=\mathcal{E}\dfrac{R_2}{R_1+R_2}$ and $R'=\dfrac{R_1R_2}{R_1+R_2}$, the equation for i can be written

$$\boxed{\mathcal{E}'-iR'-L\frac{di}{dt}=0}$$

(e) This is of the same form as the Equation 31.6 in the text for a simple *RL* circuit, so its solution is of the same form as Equation 31.7:

$$i=\frac{\mathcal{E}'}{R'}\left(1-e^{-R't/L}\right)$$

where

$$\frac{\mathcal{E}'}{R'} = \frac{\mathcal{E} R_2/(R_1 + R_2)}{R_1R_2/(R_1 + R_2)} = \frac{\mathcal{E}}{R_1}$$

Thus,

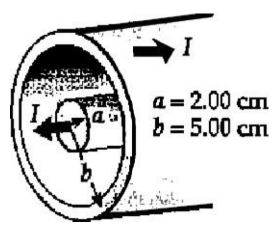
$$i = \frac{\mathcal{E}}{R_1}\left(1 - e^{-R't/L}\right) \quad \text{where} \quad R' = \frac{R_1R_2}{R_1 + R_2}$$

P31.47 Find the current in the cylinder.

$$P = i\Delta V \quad \rightarrow \quad i = \frac{P}{\Delta V} = \frac{1.00 \times 10^9 \text{ W}}{200 \times 10^3 \text{ V}} = 5.00 \times 10^3 \text{ A}$$

From Ampère's law,

$$B(2\pi r) = \mu_0 i_{\text{enclosed}} \quad \text{or} \quad B = \frac{\mu_0 i_{\text{enclosed}}}{2\pi r}$$



ANS. FIG. P31.47

(a) At $r = a = 0.020\,0$ m, $i_{\text{enclosed}} = 5.00 \times 10^3$ A and

$$B = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \times 10^3 \text{ A})}{2\pi(0.020\,0 \text{ m})} = 0.050\,0 \text{ T} = \boxed{50.0 \text{ mT}}$$

(b) At $r = b = 0.050\,0$ m, $i_{\text{enclosed}} = i = 5.00 \times 10^3$ A and

$$B = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \times 10^3 \text{ A})}{2\pi(0.050\,0 \text{ m})} = 0.020\,0 \text{ T} = \boxed{20.0 \text{ mT}}$$

(c) The energy density is $u_B = \dfrac{B^2}{2\mu_0}$:

$$U_B = \int u\,dV = \int_a^b \frac{[B(r)]^2 (2\pi r\ell dr)}{2\mu_0} = \frac{\mu_0 i^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$U_B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\times10^3\ \text{A})^2(1\,000\times10^3\ \text{m})}{4\pi} \times \ln\left(\frac{5.00\ \text{cm}}{2.00\ \text{cm}}\right)$$

$$= 2.29\times10^6\ \text{J} = \boxed{2.29\ \text{MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w.

It carries a current of $(5.00\times10^3\ \text{A})\left(\frac{w}{2\pi(0.050\,0\ \text{m})}\right)$

and experiences an outward force

$$F = i\ell B\sin\theta = \frac{(5.00\times10^3\ \text{A})w}{2\pi(0.050\,0\ \text{m})}\ell(20.0\times10^{-3}\ \text{T})\sin 90.0°$$

The pressure on it is

$$P = \frac{F}{A} = \frac{F}{w\ell} = \frac{(5.00\times10^3\ \text{A})(20.0\times10^{-3}\ \text{T})\cancel{w\ell}}{2\pi(0.050\,0\ \text{m})\cancel{w\ell}} = \boxed{318\ \text{Pa}}$$

P31.48 (a) The magnetic field inside the solenoid is given by $B = \frac{\mu_0 N i}{\ell}$:

$$B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1\,400)(2.00\ \text{A})}{1.20\ \text{m}}$$
$$= 2.93\times10^{-3}\ \text{T} = \boxed{2.93\ \text{mT}}$$

ANS. FIG. P31.48

(b) The energy density of the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(2.93\times10^{-3}\ \text{T})^2}{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = (3.42\ \text{J/m}^3)\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)$$
$$= 3.42\ \text{N/m}^2 = \boxed{3.42\ \text{Pa}}$$

(c) The supercurrents must be clockwise to produce a downward magnetic field to cancel the upward field of the current in the windings.

(d) The field of the windings is upward and radially outward around the top of the solenoid. It exerts a force radially inward and upward on each bit of the clockwise supercurrent. The total force on the supercurrents in the bar is upward. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi(1.10\times10^{-2}\ \text{m})^2\right] = 1.30\times10^{-3}\ \text{N} = \boxed{1.30\ \text{mN}}$

Note that we have not proved that energy density is pressure. In fact, it is not in some cases.

P31.49 From Ampère's law, the magnetic field at distance $r \le R$ is found as:

$$B(2\pi r) = \mu_0 j(\pi r^2) = \mu_0\left(\frac{i}{\pi R^2}\right)(\pi r^2),, \quad \text{or} \quad B = \frac{\mu_0 i r}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U_B}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\,dr) = \frac{\mu_0 i^2}{4\pi R^4}\int_0^R r^3\,dr = \frac{\mu_0 i^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 i^2}{16\pi}}$$

This is independent of the radius of the wire.

Challenge Problems

P31.50 (a) $\boxed{\text{It has a magnetic field, and it stores energy, so } L = \dfrac{2U_B}{i^2} \text{ is non-zero.}}$

(b) $\boxed{\text{Every field line goes through the rectangle between the conductors.}}$

(c) When the wires carry current i, magnetic flux passes through the rectangle bordered by the wires (surface to surface of the wires):

$$L = \frac{\Phi}{i} = \frac{1}{i}\int_{y=a}^{w-a} B\,dA$$

where y is measured from the center of the lower wire, dA is a rectangular area element of length x and width dy, and B is the magnitude of the net magnetic field generated by the upper and lower wires that passes through dA. The inductance is

$$L = \frac{1}{i}\int_{a}^{w-a}\left[\frac{\mu_0 i}{2\pi y} + \frac{\mu_0 i}{2\pi(w-y)}\right]x\,dy$$

We can simplify this calculation by noting that by the symmetry of the arrangement, each conductor contributes equally to the field that passes through the area between them. Thus, the total inductance of both wires is twice the inductance of one wire. The inductance due to the lower wire is

$$L_{\text{lower}} = \frac{1}{i}\int_{a}^{w-a}\frac{\mu_0 i}{2\pi y}x\,dy = \frac{\mu_0 x}{2\pi}\ln y\Big|_a^{w-a} = \frac{\mu_0 x}{2\pi}\left[\ln(w-a) - \ln a\right]$$
$$= \frac{\mu_0 x}{2\pi}\ln\left(\frac{w-a}{a}\right)$$

The inductance due to both wires is twice this: $L = \dfrac{\mu_0 x}{\pi}\ln\left(\dfrac{w-a}{a}\right)$.

P31.51 The total magnetic energy is the volume integral of the energy density,

$$u_B = \frac{B^2}{2\mu_0}$$

Because B changes with position, u_B is not constant. For $B = B_0\left(\frac{R}{r}\right)^2$,

$$u_B = \left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$$

Next, we set up an expression for the magnetic energy in a spherical shell of radius r and thickness dr. Such a shell has a volume $4\pi r^2 dr$, so the energy stored in it is

$$dU_B = u_B\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0^2 R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U_B = \boxed{\frac{2\pi B_0^2 R^3}{\mu_0}}$$

Substituting numerical values,

$$U_B = \frac{2\pi B_0^2 R^3}{\mu_0} = \frac{2\pi\left(5.00\times 10^{-5}\ \text{T}\right)^2\left(6.00\times 10^{6}\ \text{m}\right)^3}{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)}$$
$$= \boxed{2.70\times 10^{18}\ \text{J}}$$

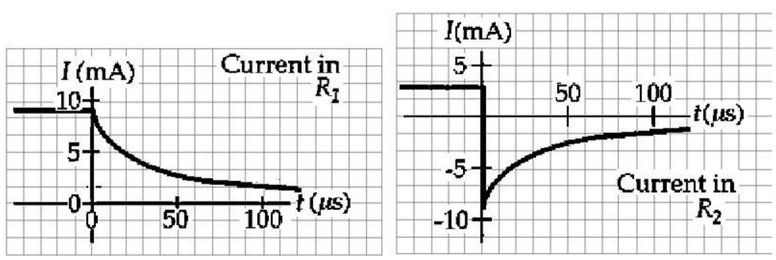
P31.52 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule going clockwise around this loop gives:

$$+\mathcal{E}-\left[(2.00+6.00)\times 10^{3}\ \Omega\right]\left(9.00\times 10^{-3}\ \text{A}\right)=0$$

$$\mathcal{E}=\boxed{72.0\ \text{V}}$$

(b) Starting at point a, the potential rises across the inductor then falls across resistors R_2 and R_1. The positive answer in part (a) means that $\boxed{\text{point b}}$ is the higher potential.

(c) The currents in R_1 and R_2 are shown in ANS. FIG. P31.52 (c).below. After $t = 0$, the current in R_1 decreases from an initial value of 9.00 mA according to $i = I_i e^{-Rt/L}$. Taking the original current direction as positive in each resistor, the current decreases from +9.00 mA (to the right) to zero in R_1. In R_2 the current jumps from +3.00 mA (downward) to –9.00 mA (upward) and then decreases in magnitude to zero. The time constant of each decay is 0.4 H/8 000 Ω = 50 μs. Thus we draw each current dropping to $1/e = 36.8\%$ of its original value = 3.3 μA at the 50 μs instant.



ANS. FIG. P31.52 (c)

(d) After the switch is opened, the current around the outer loop decays as

$$i = I_i e^{-Rt/L}$$

with $I_i = 9.00$ mA, $R = 8.00$ kΩ, and $L = 0.400$ H.

Thus, when the current has reached a value $i = 2.00$ mA, the elapsed time is:

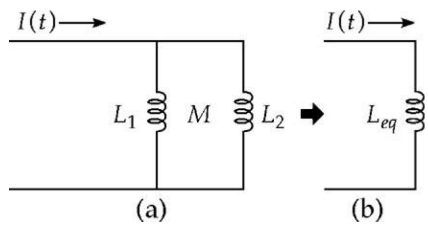
$$t = \left(\frac{L}{R}\right)\ln\left(\frac{I_i}{i}\right) = \left(\frac{0.400\text{ H}}{8.00\times 10^3\ \Omega}\right)\ln\left(\frac{9.00\text{ mA}}{2.00\text{ mA}}\right)$$
$$= 7.52\times 10^{-5}\text{ s} = \boxed{75.2\ \mu\text{s}}$$

P31.53 With $i = i_1 + i_2$, the voltage across the pair is:

$$\Delta V = -L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = -L_{\text{eq}}\frac{di}{dt} \qquad \textbf{[1]}$$

and

$$\Delta V = -L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = -L_{\text{eq}}\frac{di}{dt} \qquad \textbf{[2]}$$



ANS. FIG. P31.53

So, from [1], we have

$$-\frac{di_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{di_2}{dt}$$

which, when substituted into [2], gives

$$-L_2\frac{di_2}{dt} + M\left(\frac{(\Delta V)}{L_1} + \frac{M}{L_1}\frac{di_2}{dt}\right) = \Delta V$$

$$\left(-L_1L_2 + M^2\right)\frac{di_2}{dt} = \Delta V\left(L_1 - M\right) \qquad \textbf{[3]}$$

From [2], $-\dfrac{di_2}{dt} = \dfrac{\Delta V}{L_2} + \dfrac{M}{L_2}\dfrac{di_1}{dt}$, which, when substituted into [1], gives

$$-L_1\frac{di_1}{dt}+M\left(\frac{\Delta V}{L_2}+\frac{M}{L_2}\frac{di_1}{dt}\right)=\Delta V$$

$$\left(-L_1L_2+M^2\right)\frac{di_1}{dt}=\Delta V\left(L_2-M\right) \qquad \textbf{[4]}$$

Adding [3] to [4], we have

$$\left(-L_1L_2+M^2\right)\frac{di}{dt}=\Delta V\left(L_1+L_2-2M\right)$$

So, $$L_{\text{eq}}=-\frac{\Delta V}{di/dt}=\boxed{\frac{L_1L_2-M^2}{L_1+L_2-2M}}$$

ANSWERS TO QUICK-QUIZZES

1. (c), (f)
2. (i) (b) (ii) (a)
3. (a), (d)
4. (a)
5. (i) (b) (ii) (c)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P31.2 1.36 μH

P31.4 (a) 188 μT; (b) $3.33\times10^{-8}\ \text{T}\cdot\text{m}^2$; (c) 0.375 mH; (d) B and Φ_B are proportional to current: L is independent of current.

P31.6 See P31.6 for full explanation.

P31.8 See ANS. FIG. P31.8.

P31.10 (a) 0.469 mH (b) 0.188 ms

P31.12 See P31.12 for full explanation.

P31.14 (a) 36.0 Ω (b) 1.14 kH

P31.16 (a) $(0.500\text{ A})\left[1-e^{-10t/\text{s}}\right]$ (b) $1.50\text{ A}-(0.250\text{ A})e^{-10t/\text{s}}$

P31.18 (a) See P31.18 (a) for full explanation; (b) See P31.18 (b) for full explanation; (c) See P31.18 (c) for full explanation; (d) Yes. See P31.18 (d) for full explanation.

P31.20 64.8 mJ

P31.22 $\dfrac{L}{2R}$

P31.24 (a) 66.0 W; (b) 45.0 W; (c) 21.0 W; (d) The power supplied by the battery is equal to the sum of the power delivered to the internal resistance of the coil and the power stored in the magnetic field; (e) Yes; (f) Just after $t = 0$, the current is very small, so the power delivered to the internal resistance of the coil (iR^2) is nearly zero, but the rate of the change of the current is large, so the power delivered to the magnetic field (Ldi/dt) is large, and nearly all the battery power is being stored in the magnetic field. Long after the connection is made, the current is not changing, so no power is being stored in the magnetic field, and all the battery power is being delivered to the internal resistance of the coil.

P31.26 (a) 18.0 mH (b) 34.3 mH (c) 9.00 mV

P31.28 (a) See P31.28 (a) for full explanation; (b) 3.95 nH

P31.30 If the energy is split equally between the capacitor and inductor at some instant, the energy would be half this value, or 200 μJ. Therefore, there would be no time when each component stores 250 μJ.

P31.32 (a) 6.03 J; (b) 0.529 J; (c) 6.56 J

P31.34 See P31.34 for full explanation.

P31.36 (a) See P31.36(a) for full explanation; (b) $\frac{\mu_0 J_s^2}{2}$; (c) $B = \mu_0 J_s$, zero; (d) $\frac{\mu_0 J_s^2}{2}$; (e) The energy density found in part (d) agrees with the magnetic pressure found in part (b).

P31.38 (a) short circuit; (b) 500 mA; (c) 125 mJ; (d) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch; (e) See ANS FIG P31.38(e).

P31.40 (a) Just after the circuit is connected, the potential difference across the register is 0, and the emf across the coil is 24.0 V; (b) After several seconds, the potential difference across the resistor is 24.0 V and that across the coil is 0; (c) The two voltages are equal to each other, both being 12.0 V, just once, at 0.578 ms after the circuit is connected; (d) As the current decays, the potential difference across the resistor is always equal to the emf across the coil.

P31.42 $e^{\left(\frac{4\pi U_B}{\mu_0 N^2 h i^2}\right)}$

P31.44 See P31.44 for full explanation.

P31.46 (a) $i_1 = i_2 + i$; (b) $\mathcal{E} - i_1R_1 - i_2R_2 = 0$; (c) $\mathcal{E} - i_1R_1 - L\dfrac{di}{dt} = 0$;

(d) $\mathcal{E}' - iR' - L\dfrac{di}{dt} = 0$; (e) See P31.46(e) for full explanation.

P31.48 (a) 2.93 mT; (b) 3.42 Pa; (c) The supercurrents must be clockwise to produce a downward magnetic field to cancel the upward field of the current in the windings; (d) The field of the windings is upward and radially outward around the top of the solenoid. It exerts a force radially inward and upward on each bit of the clockwise supercurrent. The total force on the supercurrents in the bar is upward; (e) 1.30 mN

P31.50 (a) It has a magnetic field, and it stores energy, so $L = \dfrac{2U_B}{i^2}$ is non-zero;

(b) Every field line goes through the rectangle between the conductors;

(c) See P31.50 (c) for full explanation.

P31.52 (a) 72.0 V; (b) point b; (c) See ANS. FIG. P31.52 (c); (d) 75.2 μs