38

Relativity

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO THINK-PAIR-SHARE ACTIVITIES

***TP38.1** Conceptualize The combined mass of the initial black holes is larger than that of the final black hole. The difference in mass is radiated away by gravitational waves. The black holes rotate around each other before combining, and then the actual merger of the two occurs in a very short time interval, leading to a tremendous power output.

Categorize This problem involves a combination of our understanding of power from Chapter 7 and our understanding of rest energy from this chapter.

Analyze From the definition of power, we can find the power radiated away by gravitational waves by determining the change in mass during the process. For the first detection,

$$P_1 = \frac{\Delta m_1 c^2}{\Delta t_1} = 50 P_{\text{stars}} \qquad (1)$$

and for the second,

$$P_2 = \frac{\Delta m_2 c^2}{\Delta t_2} = a P_{\text{stars}} \qquad (2)$$

where a is our desired factor. Divide Equation (2) by Equation (1) and solve for a:

$$\frac{\dfrac{\Delta m_2 c^2}{\Delta t_2}}{\dfrac{\Delta m_1 c^2}{\Delta t_1}} = \frac{a P_{\text{stars}}}{50 P_{\text{stars}}} \quad \rightarrow \quad a = 50 \frac{\Delta m_2}{\Delta m_1} \frac{\Delta t_1}{\Delta t_2}$$

Substitute numerical values, expressing the mass changes as multiples of M, the mass of the Sun:

$$a = 50 \frac{(14.2 + 7.5 - 20.8)M}{(29 + 36 - 62)M} \left(\frac{0.2 \text{ s}}{1.0 \text{ s}} \right) = \boxed{3}$$

where we have kept only one significant figure because we only have one for Δt_1.

Finalize While this is a much smaller ratio than that for the first detection, it is still a tremendous outpouring of energy in a short time interval.]

Answer: 3

*** TP38.2** **Conceptualize** We only have two data points, so we should not expect high precision. Notice, however, that the neutrino with the higher energy arrived first. That is consistent with the fact that a higher energy means that the first neutrino is moving *faster,* since both neutrinos have the same rest energy.

Categorize The neutrinos move as *particles under constant velocity.*

Analyze Let's look first at the arrival time data. If we call the early arriving neutrino A and the later one B, then the difference in arrival times is

$$\Delta t = t_B - t_A = \frac{D}{v_B} - \frac{D}{v_A} \qquad (1)$$

where we have used the particle under constant velocity model to find each time interval in terms of the neutrino speed and the distance D to the supernova. Solving Equation 38.8 for the speed of a particle, we can write Equation (1) as

$$\Delta t = \left(\frac{D}{c\sqrt{1 - \dfrac{1}{\gamma_B^2}}} - \frac{D}{c\sqrt{1 - \dfrac{1}{\gamma_A^2}}} \right) = \frac{D}{c}\left(\frac{1}{\sqrt{1 - \dfrac{1}{\gamma_B^2}}} - \frac{1}{\sqrt{1 - \dfrac{1}{\gamma_A^2}}} \right)$$

$$= \frac{D}{c}\left[\left(1 - \frac{1}{\gamma_B^2}\right)^{-1/2} - \left(1 - \frac{1}{\gamma_A^2}\right)^{-1/2} \right]$$

Because the neutrinos travel with very high speed, the value of γ is high, and we can use the binomial expansion for each term, keeping only the first two terms:

$$\Delta t = \frac{D}{c}\left[\left(1+\tfrac{1}{2}\frac{1}{\gamma_B^2}\right)-\left(1+\tfrac{1}{2}\frac{1}{\gamma_A^2}\right)\right] = \frac{D}{2c}\left(\frac{1}{\gamma_B^2}-\frac{1}{\gamma_A^2}\right) \qquad (2)$$

Now, let's look at the energy data. We assume the detected neutrinos are of the same particle species, and therefore have the same mass. Equation 38.26 tells us that

$$\frac{E_B}{\gamma_B c^2} = \frac{E_A}{\gamma_A c^2} \quad \rightarrow \quad \gamma_B = \frac{E_B}{E_A}\gamma_A \qquad (3)$$

Substitute Equation (3) into Equation (2):

$$\Delta t = \frac{D}{2c}\left[\frac{1}{\left(\dfrac{E_B}{E_A}\gamma_A\right)^2} - \frac{1}{\gamma_A^2}\right] = \frac{D}{2c\gamma_A^2}\left(\frac{E_A^2}{E_B^2}-1\right) \qquad (4)$$

Solve Equation (4) for γ_A:

$$\gamma_A = \sqrt{\frac{D}{2c\Delta t}\left(\frac{E_A^2}{E_B^2}-1\right)} \qquad (5)$$

Use Equation 38.26 again to find the mass of the neutrino and substitute Equation (5):

$$m = \frac{E_A}{\gamma_A c^2} = \frac{E_A}{c^2\left[\sqrt{\dfrac{D}{2c\Delta t}\left(\dfrac{E_A^2}{E_B^2}-1\right)}\right]}$$

Substitute numerical values:

$$m = \frac{38\text{ MeV}}{c^2\left[\sqrt{\dfrac{1.64\times 10^5\text{ ly}}{2(1\text{ ly/ yr})(5.59\text{ s})}\left[\dfrac{(38\text{ MeV})^2}{(24\text{ MeV})^2}-1\right]}\left(\dfrac{3.16\times 10^7\text{ s}}{1\text{ yr}}\right)\right]}$$

$$= 4.5\times 10^{-5}\text{ MeV}/\,c^2 = \boxed{45\text{ eV}/\,c^2}$$

Finalize This is a very small mass. It is remarkable that we have been able to come up with an estimate of the neutrino mass from measurements on only two neutrinos. We assumed that the neutrinos came from the same starting point in the supernova, but that's highly unlikely. Therefore, *D* in the above calculation may be different for the two neutrinos, but we have no way of knowing the individual values. It is also unlikely that the neutrinos were emitted simultaneously, since the supernova process spans tens of seconds. Measurements on more neutrinos tighten up the precision for the calculation of the mass. Current experiments provide neutrino masses on the order of a few eV/c^2 and sometimes less than 1 eV/c^2.]

Answer: 45 eV/c^2

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 38.1 The Principle of Galilean Relativity

P38.1 In the laboratory frame of reference, Newton's second law is valid: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Laboratory observer 1 watches some object accelerate under applied forces. Call the instantaneous velocity of the object $\vec{\mathbf{v}}_1 = \vec{\mathbf{v}}_{O1}$

(the velocity of object O relative to observer 1 in laboratory frame) and its acceleration $\frac{d\vec{\mathbf{v}}_1}{dt} = \vec{\mathbf{a}}_1$. A second observer has instantaneous velocity $\vec{\mathbf{v}}_{21}$ relative to the first. In general, the velocity of the object in the frame of the second observer is

$$\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_{O2} = \vec{\mathbf{v}}_{O1} + \vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{21}$$

(a) If the relative instantaneous velocity $\vec{\mathbf{v}}_{21}$ of the second observer is *constant*, the second observer measures the acceleration

$$\vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = \frac{d\vec{\mathbf{v}}_1}{dt} = \vec{\mathbf{a}}_1$$

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces and masses as well. Thus, the second observer also confirms that $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

(b) If the second observer's frame is accelerating, then the instantaneous relative velocity $\vec{\mathbf{v}}_{21}$ is *not constant*. The second observer measures an acceleration of

$$\vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = \frac{d\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{21}\right)}{dt} = \vec{\mathbf{a}}_1 - \frac{d\left(\vec{\mathbf{v}}_{21}\right)}{dt} = \vec{\mathbf{a}}_1 - \vec{\mathbf{a}}',$$

where $\frac{d\left(\vec{\mathbf{v}}_{21}\right)}{dt} = \vec{\mathbf{a}}'$

The observer in the accelerating frame measures the acceleration of the mass as being $\vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_1 - \vec{\mathbf{a}}'$. If Newton's second law held for the accelerating frame, that observer would expect to find valid the relation $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2$, or $\vec{\mathbf{F}}_1 = m\vec{\mathbf{a}}_2$ (since $\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer finds that $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2 - m\vec{\mathbf{a}}'$, which is *not* Newton's second law.

P38.2 In the rest frame,

$$\begin{aligned} p_i &= m_1 v_{1i} + m_2 v_{2i} = (2\,000\text{ kg})(20.0\text{ m/s}) + (1\,500\text{ kg})(0\text{ m/s}) \\ &= 4.00\times 10^4\text{ kg}\cdot\text{m/s} \\ p_f &= (m_1 + m_2)v_f = (2\,000\text{ kg} + 1\,500\text{ kg})v_f \end{aligned}$$

Since $p_i = p_f$,

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{4.00\times 10^4\text{ kg}\cdot\text{m/s}}{2\,000\text{ kg} + 1\,500\text{ kg}} = 11.429\text{ m/s}$$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$\begin{aligned} v'_{1i} &= v_{1i} - v' = 20.0\text{ m/s} - (+10.0\text{ m/s}) = 10.0\text{ m/s} \\ v'_{2i} &= v_{2i} - v' = 0\text{ m/s} - (+10.0\text{ m/s}) = -10.0\text{ m/s} \\ v'_f &= 11.429\text{ m/s} - (+10.0\text{ m/s}) = 1.429\text{ m/s} \end{aligned}$$

Our initial momentum is then

$$\begin{aligned} p'_i &= m_1 v'_{1i} + m_2 v'_{2i} \\ &= (2\,000\text{ kg})(10.0\text{ m/s}) + (1\,500\text{ kg})(-10.0\text{ m/s}) \\ &= 5\,000\text{ kg}\cdot\text{m/s} \end{aligned}$$

and our final momentum has the same value:

$$\begin{aligned} p'_f &= (2\,000\text{ kg} + 1\,500\text{ kg})v'_f = (3\,500\text{ kg})(1.429\text{ m/s}) \\ &= 5\,000\text{ kg}\cdot\text{m/s} \end{aligned}$$

Section 38.4 Consequences of the Special Theory of Relativity

P38.3 (a) The length of the meter stick measured by the observer moving at speed $v = 0.900\,c$ relative to the meter stick is

$$L = L_p/\gamma = L_p\sqrt{1-(v/c)^2} = (1.00\text{ m})\sqrt{1-(0.900)^2} = \boxed{0.436\text{ m}}$$

(b) If the observer moves relative to Earth in the direction opposite the motion of the meter stick relative to Earth, the velocity of the observer relative to the meter stick is greater than that in part (a). The measured length of the meter stick will be $\boxed{\text{less than } 0.436\text{ m}}$ under these conditions, but so small it is unobservable.

P38.4 For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$.

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{L_P}{v} = \frac{4.60\text{ km}}{0.990c} = \left[\frac{4.60\times10^3\text{ m}}{0.990(3.00\times10^8\text{ m/s})}\right]$$
$$= 1.55\times10^{-5}\text{ s} = 15.5\ \mu\text{s}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}(15.5\ \mu\text{s}) = \boxed{2.18\ \mu\text{s}}$$

(b) In the muon's frame, the Earth is approaching the muon at speed $v = 0.990c$. During the time interval the muon exists, the Earth travels the distance

$$d = v\Delta t_p = v\frac{\Delta t}{\gamma} = v\frac{L_P}{\gamma v} = \frac{L_P}{\gamma}$$
$$= (4.60\times10^3\text{ m})\sqrt{1-(0.990)^2} = \boxed{649\text{ m}}$$

P38.5 The astronaut's measured time interval is a proper time in her reference frame. Therefore, according to an observer on Earth,

$$\Delta t = \gamma\,\Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{3.00\text{ s}}{\sqrt{1-(0.800)^2}} = \boxed{5.00\text{ s}}$$

P38.6 (a) The time interval between pulses as measured by the astronaut is a proper time:

$$\Delta t_p = \left(\frac{1 \text{ min}}{75.0 \text{ beats}} \right)$$

The time interval between pulses as measured by the Earth observer is then:

$$\Delta t = \gamma \Delta t_p = \frac{1}{\sqrt{1-(0.500)^2}} \left(\frac{1 \text{ min}}{75.0 \text{ beats}} \right) = 1.54 \times 10^{-2} \text{ min/beat}$$

Thus, the Earth observer records a pulse rate of

$$\frac{1}{\Delta t} = \frac{1}{\gamma \Delta t_p} = \sqrt{1-(0.500)^2} \left(\frac{75.0 \text{ beats}}{1 \text{ min}} \right) = \boxed{65.0 \text{ beats/min}}$$

(b) From part (a), the pulse rate is

$$\frac{1}{\Delta t} = \frac{1}{\gamma \Delta t_p} = \sqrt{1-(0.990)^2} \left(\frac{75.0 \text{ beats}}{1 \text{ min}} \right) = \boxed{10.5 \text{ beats/min}}$$

That is, the life span of the astronaut (reckoned by the duration of the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

P38.7 From the definition of γ,

$$\gamma = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}} = 1.010\ 0$$

we solve for the speed:

$$v = c\sqrt{1-\left(\frac{1}{\gamma}\right)^2} = c\sqrt{1-\left(\frac{1}{1.010\ 0}\right)^2} = \boxed{0.140c}$$

***P38.8** **Conceptualize** The driver is claiming an Earth-based version of the Doppler shift for light, which causes light from galaxies receding from us to experience a redshift. Because the driver is moving toward the light source, the wavelength should be shifted toward the blue end of the spectrum.

Categorize We use our understanding of the Doppler effect for light.

Analyze Solve Equation 38.10 for the relative speed between the driver and the traffic light when they are moving toward each other:

$$f' = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} f \quad \rightarrow \quad v = \frac{(f')^2 - f^2}{(f')^2 + f^2} c \tag{1}$$

Express Equation (1) in terms of wavelengths:

$$v = \frac{\left(\dfrac{c}{\lambda'}\right)^2 - \left(\dfrac{c}{\lambda}\right)^2}{\left(\dfrac{c}{\lambda'}\right)^2 + \left(\dfrac{c}{\lambda}\right)^2} c = \frac{\lambda^2 - (\lambda')^2}{\lambda^2 + (\lambda')^2} c \tag{2}$$

Substitute numerical values:

$$v = \frac{(650\text{ nm})^2 - (520\text{ nm})^2}{(650\text{ nm})^2 + (520\text{ nm})^2} c = 0.220c$$

A speed of 22% of the speed of light is well over the speed limit of any Earth-based road. $\boxed{\text{The driver's own testimony shows him blatantly violating any Earth-based speed limit; look for another defense.}}$

Finalize The speed found in the problem is equivalent to 6.59×10^7 m/s, or 1.47×10^8 mi/h. Such speeds are not possible with automobiles or other Earth-based vehicles, so it is clear that the Doppler shift of light is not a phenomenon with which we have to deal on an everyday basis.]

Answer: The driver's own testimony shows him blatantly violating any Earth-based speed limit; look for another defense.

P38.9 The spaceship is measured by the Earth observer to be length-contracted to

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{or} \qquad L^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Also, the contracted length is related to the time required to pass overhead by

$$L = v\Delta t \qquad \text{or} \qquad L^2 = v^2(\Delta t)^2 = \frac{v^2}{c^2}(c\Delta t)^2$$

Equating these two expressions gives $L_p^2 - L_p^2\dfrac{v^2}{c^2} = (c\Delta t)^2\dfrac{v^2}{c^2}$.

or $$\left[L_p^2 + (c\Delta t)^2\right]\frac{v^2}{c^2} = L_p^2$$

Using the given values $L_p = 300$ m and $\Delta t = 0.750\times 10^{-6}$ s, this becomes

$$\left(1.41\times 10^5 \text{ m}^2\right)\frac{v^2}{c^2} = 9.00\times 10^4 \text{ m}^2$$

giving $v = \boxed{0.800c}$

P38.10 The spaceship is measured by Earth observers to be of length L, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}}$$

and $L = v\Delta t$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Solving for v,

$$v^2\left(\Delta t^2 + \frac{L_p^2}{c^2}\right) = L_p^2$$

giving $\boxed{v = \dfrac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}}$

P38.11 (a) When the source moves away from an observer, the observed frequency is

$$f' = f\left(\frac{c+v}{c-v}\right)^{1/2} = f\left(\frac{c-v_S}{c+v_S}\right)^{1/2}$$

where $v = v_{\text{source}} = -v_S$ because the source is moving away from the observer.

When $v_s \ll c$, the binomial expansion gives

$$\left(\frac{c-v_S}{c+v_S}\right)^{1/2} = \left[1-\left(\frac{v_S}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_S}{c}\right)\right]^{-1/2}$$
$$\approx \left(1-\frac{v_S}{2c}\right)\left(1-\frac{v_S}{2c}\right) \approx \left(1-\frac{v_S}{c}\right)$$

So, $f' \approx f\left(1-\dfrac{v_S}{c}\right)$

The observed wavelength is found from $c = \lambda' f' = \lambda f$:

$$\lambda' = \frac{\lambda f}{f'} \approx \frac{\lambda f}{f(1-v_S/c)} = \frac{\lambda}{1-v_S/c}$$
$$\Delta\lambda = \lambda' - \lambda = \lambda\left(\frac{1}{1-v_S/c} - 1\right) = \lambda\left(\frac{v_S/c}{1-v_S/c}\right)$$

Since $1-\dfrac{v_S}{c}\approx 1$, $\boxed{\dfrac{\Delta\lambda}{\lambda}\approx\dfrac{v_S}{c}}$

(b) We use the equation from part (a) with the given values:

$$v_S = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\text{ nm}}{397\text{ nm}}\right) = \boxed{0.050\,4c}$$

P38.12 The relativistic density is

$$\frac{E_R}{c^2V} = \frac{\gamma mc^2}{c^2V} = \frac{\gamma m}{V} = \frac{m}{\left(L_p\right)^3\left[1-(u/c)^2\right]}$$
$$= \frac{8.00\text{ g}}{(1.00\text{ cm})^3\left[1-(0.900)^2\right]} = \boxed{42.1\text{ g/cm}^3}$$

P38.13 We find Cooper's speed from Newton's second law:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving,

$$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(6.37\times10^6\text{ m}+0.160\times10^6\text{ m}\right)}\right]^{1/2}$$
$$= 7.82\times10^3 = 7.82\text{ km/s}$$

Then the time period of one orbit is

$$T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53\times10^6\text{ m})}{7.82\times10^3\text{ m/s}} = 5.25\times10^3\text{ s}$$

(a) The time difference for 22 orbits is

$$\Delta t - \Delta t_p = (\gamma-1)\Delta t_p = \left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right](22T)$$

$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82 \times 10^3 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2$$
$$\times 22\left(5.25 \times 10^3 \text{ s}\right) = \boxed{39.2\,\mu s}$$

(b) For each one orbit Cooper aged less by

$$\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$$

The press report is $\boxed{\text{accurate to one digit}}$.

***P38.14 Conceptualize** The time interval to wait will involve two considerations: (1) the time interval for the exam will appear dilated to the professor on Earth *and* (2) there will be a time interval required for the radio signal to reach the spacecraft.

Categorize This problem will involve time dilation for the exam as well as the *particle under constant velocity* model applied to the radio signal.

Analyze The dilated time interval for the exam to be finished, as measured by the professor, is given by Equation 38.7:

$$\Delta t_{\text{exam}} = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

where Δt_p is the proper time interval of 2.00 hr for the students to take the exam. This time interval consists of two parts for the professor on Earth, the time interval to wait before sending the signal to stop the exam plus the time interval required for the signal to reach the spacecraft:

$$\Delta t_{\text{exam}} = \Delta t_{\text{wait}} + \Delta t_{\text{travel}} \qquad (2)$$

Therefore, the time interval that we want to determine to answer the question is

$$\Delta t_{\text{wait}} = \Delta t_{\text{exam}} - \Delta t_{\text{travel}} \qquad (3)$$

The travel time for the signal can be found from the particle under constant velocity model. The time interval is the ratio of the distance to the spacecraft when the signal arrives to the speed of light. The distance to the spacecraft, as measured by the professor, is the product of its speed and the dilated time interval for the exam:

$$\Delta t_{\text{travel}} = \frac{d}{c} = \frac{v\left(\Delta t_{\text{wait}} + \Delta t_{\text{travel}}\right)}{c} \quad \rightarrow \quad \Delta t_{\text{travel}} = \frac{v}{c-v}\Delta t_{\text{wait}} \qquad (4)$$

On the right-hand side of Equation (3), substitute Equation (4) for the second term and solve for Δt_{wait}:

$$\Delta t_{\text{wait}} = \Delta t_{\text{exam}} - \frac{v}{c-v}\Delta t_{\text{wait}} \quad \rightarrow \quad \Delta t_{\text{wait}} = \left(1 - \frac{v}{c}\right)\Delta t_{\text{exam}} \qquad (5)$$

Now substitute Equation (1) into Equation (5):

$$\Delta t_{\text{wait}} = \left(1 - \frac{v}{c}\right)\frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \frac{v}{c}\right)\frac{\Delta t_p}{\sqrt{\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)}}$$

$$= \Delta t_p\sqrt{\frac{c-v}{c+v}} \qquad (6)$$

Substitute numerical values:

$$\Delta t_{\text{wait}} = (2.00\text{ h})\sqrt{\frac{c - 0.960c}{c + 0.960c}} = \boxed{0.286\text{ h} = 17.1\text{ min}}$$

Finalize Notice that the waiting time is much less than the 2.00 h for the exam. The individual pieces of this waiting time are longer and are found from Equations (1) and (4):

$$\Delta t_{\text{exam}} = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.00\text{ h}}{\sqrt{1-\frac{(0.960c)^2}{c^2}}} = 7.14\text{ h}$$

$$\Delta t_{\text{travel}} = \frac{v}{c-v}\Delta t_{\text{wait}} = \frac{0.960c}{c-0.960c}(0.286\text{ h}) = 6.86\text{ h}$$

Of course, it would be a lot easier to just have a proctor traveling with the students give them 2.00 h before signaling them by voice to stop!]

Answer: 0.286 h = 17.1 min

P38.15 (a) The mirror is approaching the source. Let f_m be the frequency as seen by the mirror. Thus,

$$f_m = f\sqrt{\frac{c+v}{c-v}}$$

After reflection, the mirror acts as a source, approaching the receiver. If f' is the frequency of the reflected wave,

$$f' = f_m\sqrt{\frac{c+v}{c-v}}$$

Combining gives

$$\boxed{f' = \frac{c+v}{c-v}f}$$

(b) Using the above result, the beat frequency is

$$f_{\text{beat}} = f' - f = f' = \frac{c+v}{c-v}f - f = f\left(\frac{c+v}{c-v} - 1\right)$$

$$f_{\text{beat}} = f\left(\frac{c+v-(c-v)}{c-v}\right) = f\left(\frac{2v}{c-v}\right) \approx f\frac{2v}{c} = \frac{2v}{c/f}$$

$$f_{\text{beat}} = \frac{2v}{\lambda}$$

(c) The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{10.0\times10^{9}\ \text{Hz}} = 0.030\ 0\ \text{m}$$

The beat frequency is therefore,

$$f_{\text{beat}} = \frac{2v}{\lambda} = \frac{(2)(30.0\ \text{m/s})}{(0.030\ 0\ \text{m})} = 2\ 000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

(d) From part (b), $v = \dfrac{f_{\text{beat}}\lambda}{2}$, so

$$\Delta v = \frac{\Delta f_{\text{beat}}\lambda}{2} = \frac{(5.0\ \text{Hz})(0.030\ 0\ \text{m})}{2}$$
$$= \boxed{0.075\ 0\ \text{m/s} \approx 0.17\ \text{mi/h}}$$

Section 38.5 The Lorentz Transformation Equations

P38.16 Let Shannon be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0$, $t_1 = 0$, $x_2 = 0$, $t_2 = 3.00\ \mu$s. Let Kimmie be fixed in reference frame S′ and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9.00\ \mu$s.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$9.00\ \mu\text{s} = \frac{1}{\sqrt{1-v^2/c^2}}(3.00\ \mu\text{s} - 0)$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3}$$

$$\boxed{v = 0.943c}$$

(b) The coordinate separation of the events is

$$\begin{aligned}\Delta x' = x_2' - x_1' &= \gamma\left[(x_2 - x_1) - v(t_2 - t_1)\right]\\ &= 3\left[0 - (0.943c)(3.00\times 10^{-6}\ \text{s})\right]\left(\frac{3.00\times 10^{8}\ \text{m/s}}{c}\right)\\ &= -2.55\times 10^{3}\ \text{m}\end{aligned}$$

$$|\Delta x'| = \boxed{2.55\times 10^{3}\ \text{m}}$$

The later pulse is to the left of the origin.

P38.17 The rod's length perpendicular to the motion is the same in both the proper frame of the rod and in the frame in which the rod is moving–our frame:

$$\ell_y = \ell\sin\theta = \ell_{Py}$$

where ℓ_{Py} is the y component of the proper length.

We are given: $\ell = 2.00\ \text{m}$, and $\theta = 30.0°$, both measured in our reference frame. Also,

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.995^2}} \approx 10.0$$

As observed in our frame,

$$\ell_x = \ell\cos\theta = (2.00\text{ m})\cos 30.0° = 1.73\text{ m}$$

and $$\ell_y = \ell\sin\theta = (2.00\text{ m})\sin 30.0° = 1.00\text{ m}$$

ℓ_{Px} is a proper length, related to ℓ_x by $\ell_x = \dfrac{\ell_{Px}}{\gamma}$.

Therefore, $$\ell_{Px} = 10.0\ell_x = 17.3\text{ m}$$

and $$\ell_{Py} = \ell_y = 1.00\text{ m}$$

(a) $$\ell_P = \sqrt{(\ell_{Px})^2 + (\ell_{Py})^2} = \sqrt{\left(\frac{\ell_x}{\gamma}\right)^2 + (\ell_y)^2} = \boxed{17.4\text{ m}}$$

(b) In the proper frame,

$$\theta_2 = \tan^{-1}\left(\frac{\ell_{Py}}{\ell_{Px}}\right) = \tan^{-1}\left(\frac{\ell_y}{\gamma\ell_x}\right) = \tan^{-1}\left(\frac{\tan 30.0°}{\gamma}\right) = \boxed{3.30°}$$

P38.18 (a) $L_0^2 = L_{0x}^2 + L_{0y}^2$ and $L^2 = L_x^2 + L_y^2$.

Since the motion is in the x direction, the length of the rod in the y direction does not change: $L_y = L_{0y} = L_0\sin\theta_0$ and

$$L_x = L_{0x}\sqrt{1-\left(\frac{v}{c}\right)^2} = (L_0\cos\theta_0)\sqrt{1-\left(\frac{v}{c}\right)^2}$$

Thus,

$$L^2 = L_0^2\cos^2\theta_0\left[1-\left(\frac{v}{c}\right)^2\right] + L_0^2\sin^2\theta_0 = L_0^2\left[1-\left(\frac{v}{c}\right)^2\cos^2\theta_0\right]$$

or $$\boxed{L = L_0\left[1-\left(\frac{v}{c}\right)^2\cos^2\theta_0\right]^{1/2}}.$$

(b) $\tan\theta = \frac{L_y}{L_x} = \frac{L_{0y}}{L_{0x}\sqrt{1-(v/c)^2}} = \boxed{\gamma\tan\theta_0}$

P38.19 (a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$$\Delta x' = \gamma(\Delta x - v\Delta t):\ 0 = \gamma\left[2.00\ \text{m} - v\left(8.00\times10^{-9}\ \text{s}\right)\right]$$

$$v = \frac{2.00\ \text{m}}{8.00\times10^{-9}\ \text{s}} = \boxed{2.50\times10^{8}\ \text{m/s}}$$

so

$$\gamma = \frac{1}{\sqrt{1-\left(2.50\times10^{8}\ \text{m/s}\right)^2/\left(3.00\times10^{8}\ \text{m/s}\right)^2}} = 1.81$$

(b) Again from the Lorentz transformation, $x' = \gamma(x - vt)$:

$$x' = 1.81\left[3.00\ \text{m} - \left(2.50\times10^{8}\ \text{m/s}\right)\left(1.00\times10^{-9}\ \text{s}\right)\right] = \boxed{4.98\,\text{m}}$$

(c) $t' = \gamma\left(t - \frac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\ \text{s} - \frac{\left(2.50\times10^{8}\ \text{m/s}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)^2}(3.00\ \text{m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{s}}$$

Section 38.6 The Lorentz Velocity Transformation Equations

***P38.20** **Conceptualize** Place the S frame on the Earth. Place the S′ frame on the police spacecraft. An observer on the police spacecraft is measuring the speed of the driver's spacecraft.

Categorize We need to make a simple velocity transformation, so this problem is categorized as a substitution problem.

We want the speed of the spacecraft relative to the S frame, so we use Equation 38.18:

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

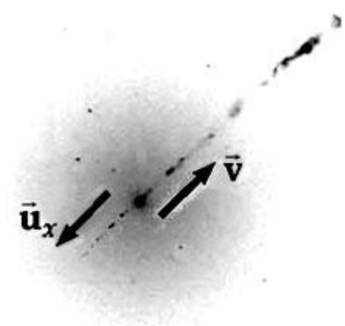
Substitute numerical values:

$$u_x = \frac{0.300c + 0.600c}{1 + \frac{(0.300c)(0.600c)}{c^2}} = \boxed{0.763c}$$

$\boxed{\text{The driver was traveling above the speed limit relative to the Earth}}$, so the lawyer cannot use these data to defend his client. He will need to search elsewhere for a defense.]

Answer: The driver was traveling at 0.763c relative to the Earth. He was speeding.

P38.21 Take the galaxy as the unmoving frame. Arbitrarily define the jet moving upward to be the object, and the jet moving downward to be the "moving" frame:



ANS. FIG. P38.21

$u'_x =$ velocity of other jet in frame of jet

$u_x =$ velocity of other jet in frame of galaxy center

$$= \quad 0.750c$$

$$v = \quad \text{speed of galaxy center in frame of jet} = -0.750c$$

From Equation 38.16, the speed of the upward-moving jet as measured from the downward-moving jet is

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.750c - (-0.750c)}{1 - (0.750c)(-0.750c)/c^2} = \frac{1.50c}{1 + 0.750^2}$$
$$= \boxed{0.960c}$$

P38.22 Let frame S be the Earth frame of reference. Then $v = -0.700c$.

The components of the velocity of the first spacecraft are

$$u_x = (0.600c)\cos 50.0° = 0.386c$$

and $$u_y = (0.600c)\sin 50.0° = 0.460c.$$

As measured from the S' frame of the second spacecraft,

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.386c - (-0.700c)}{1 - \left[(0.386c)(-0.700c)/c^2\right]}$$
$$= \frac{1.086c}{1.27} = 0.855c$$

and

$$u_y' = \frac{u_y}{\gamma\left(1 - u_x v/c^2\right)} = \frac{0.460c\sqrt{1 - (0.700)^2}}{1 - (0.386)(-0.700)}$$
$$= \frac{0.460c(0.714)}{1.27} = 0.258c$$

The magnitude of $\vec{\mathbf{u}}'$ is $\sqrt{(0.855c)^2 + (0.285c)^2} = \boxed{0.893c}$

and its direction is at $\tan^{-1}\left(\dfrac{0.258c}{0.855c}\right) = \boxed{16.8° \text{ above the } x' \text{ axis}}$.

Section 38.7 Relativistic Linear Momentum

P38.23 (a) $p = \gamma mu$; for an electron moving at 0.010 0*c*,

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-(0.010\,0)^2}} = 1.000\,05 \approx 1.00$$

Thus, $p = 1.00\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.010\,0\right)\left(3.00\times10^{8}\ \text{m/s}\right)$

$$p = \boxed{2.73\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(b) Following the same steps as used in part (a), we find at 0.500*c*,

$\gamma = 1.15$ and

$$p = \boxed{1.58\times10^{-22}\ \text{kg}\cdot\text{m/s}}$$

(c) At 0.900*c*, $\gamma = 2.29$ and

$$p = \boxed{5.64\times10^{-22}\ \text{kg}\cdot\text{m/s}}$$

P38.24 From the definition of relativistic linear momentum,

$$p = \frac{mu}{\sqrt{1-(u/c)^2}}$$

we obtain

$$1-\frac{u^2}{c^2} = \frac{m^2u^2}{p^2}$$

which gives:

$$1 = u^2\left(\frac{m^2}{p^2}+\frac{1}{c^2}\right)$$

or $\quad c^2 = u^2\left(\dfrac{m^2c^2}{p^2}+1\right) \quad$ and $\quad \boxed{u = \dfrac{c}{\sqrt{\left(m^2c^2/p^2\right)+1}}}$.

P38.25 (a) Classically,

$$p = mv = m(0.990c) = \left(1.67\times10^{-27}\ \text{kg}\right)(0.990)\left(3.00\times10^{8}\ \text{m/s}\right)$$
$$= \boxed{4.96\times10^{-19}\ \text{kg}\cdot\text{m/s}}$$

(b) By relativistic calculations,

$$p = \frac{mu}{\sqrt{1-(u/c)^2}} = \frac{m(0.990c)}{\sqrt{1-(0.990)^2}}$$
$$= \frac{(1.67\times10^{-27}\ \text{kg})(0.990)(3.00\times10^{8}\ \text{m/s})}{\sqrt{1-(0.990)^2}}$$
$$= \boxed{3.52\times10^{-18}\ \text{kg}\cdot\text{m/s}}$$

(c) $\boxed{\text{No}}$, neglecting relativistic effects at such speeds would introduce an approximate 86% error in the result.

P38.26 We can express the proportion relating the speeding fine to the excess momentum as $\dfrac{F}{\$80.0} = \dfrac{\left(p_u - p_{90\ \text{km/h}}\right)}{\left(p_{190\ \text{km/h}} - p_{90\ \text{km/h}}\right)}$, where F is the fine,

$p_u = \dfrac{mu}{\sqrt{1-(u/c)^2}}$ is the magnitude of the vehicle's momentum at speed u, and $c = 1.08 \times 10^9$ km/h. After substitution of the expression for momentum, the proportion becomes

$$\frac{F}{\$80.0} = \frac{\left[\dfrac{mu}{\sqrt{1-(u/c)^2}} - \dfrac{m(90.0\ \text{km/h})}{\sqrt{1-(90.0\ \text{km/h}/c)^2}}\right]}{\left[\dfrac{m(190.0\ \text{km/h})}{\sqrt{1-(190.0\ \text{km/h}/c)^2}} - \dfrac{m(90.0\ \text{km/h})}{\sqrt{1-(90.0\ \text{km/h}/c)^2}}\right]}$$
$$\approx \frac{\dfrac{u}{\sqrt{1-(u/c)^2}} - (90.0\ \text{km/h})}{100.0\ \text{km/h}}$$

(a) For $u = 1\ 090$ km/h,

$$\frac{F}{\$80.0} \approx \frac{\dfrac{(1\ 090\ \text{km/h})}{\sqrt{1-\left(1\ 090\ \text{km/h}/1.08\times 10^{9}\ \text{km/h}\right)^2}}-(90.0\ \text{km/h})}{100.0\ \text{km/h}}$$

$$\approx \frac{(1\ 090\ \text{km/h})-(90.0\ \text{km/h})}{100.0\ \text{km/h}} = \frac{1\ 000\ \text{km/h}}{100.0\ \text{km/h}} = 10$$

$$F = \boxed{\$800}$$

(b) For $u = 1\ 000\ 000\ 090$ km/h,

$$\frac{F}{\$80.0} \approx \left(\frac{1}{100\ \text{km/h}}\right)$$

$$\left[\frac{(1\ 000\ 000\ 090\ \text{km/h})}{\sqrt{1-\left(1\ 000\ 000\ 090\ \text{km/h}/1.08\times 10^{9}\ \text{km/h}\right)^2}} - (90.0\ \text{km/h})\right]$$

$$\frac{F}{\$80.0} \approx \frac{(2.648)(1\ 000\ 000\ 090\ \text{km/h})-(90.0\ \text{km/h})}{100.0\ \text{km/h}}$$

$$F = \boxed{\$2.12\times 10^{9}}$$

P38.27 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$,

or $$\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50\times 10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$$

or $$\frac{\left(1.67\times 10^{-27}\ \text{kg}\right)u_2}{\sqrt{1-\left(u_2/c\right)^2}} = \left(4.960\times 10^{-28}\ \text{kg}\right)c$$

Proceeding to solve, we find

$$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2 = 1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2}=1 \quad \text{and} \quad u_2 = \boxed{0.285c}$$

Section 38.8 Relativistic Energy

P38.28 (a) Using the classical equation,

$$K=\frac{1}{2}mu^2=\frac{1}{2}(78.0\text{ kg})\left(1.06\times10^5\text{ m/s}\right)^2=\boxed{4.38\times10^{11}\text{ J}}$$

(b) Using the relativistic equation, $K=\left(\dfrac{1}{\sqrt{1-(u/c)^2}}-1\right)mc^2$:

$$K=\left[\frac{1}{\sqrt{1-\left(\dfrac{1.06\times10^5}{2.998\times10^8}\right)^2}}-1\right](78.0\text{ kg})\left(3.00\times10^8\text{ m/s}\right)^2$$

$$=\boxed{4.38\times10^{11}\text{J}}$$

(c) When $\dfrac{u}{c}\ll1$, the binomial series expansion gives

$$\left[1-\left(\frac{u}{c}\right)^2\right]^{-1/2}\approx1+\frac{1}{2}\left(\frac{u}{c}\right)^2$$

Thus, $\left[1-\left(\frac{u}{c}\right)^2\right]^{-1/2}-1\approx\frac{1}{2}\left(\frac{u}{c}\right)^2$ and the relativistic expression for kinetic energy becomes $K\approx\frac{1}{2}\left(\frac{u}{c}\right)^2 mc^2=\frac{1}{2}mu^2$. That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results. In this situation the two kinetic energy values are experimentally indistinguishable. The fastest-moving macroscopic objects launched by human beings move sufficiently slowly compared to light that relativistic corrections to their energy are negligible.

P38.29 We use the equation $\Delta E=(\gamma_1-\gamma_2)mc^2$. For an electron, $mc^2=0.511$ MeV.

(a) $$\Delta E=\left(\sqrt{\frac{1}{(1-0.810)}}-\sqrt{\frac{1}{(1-0.250)}}\right)mc^2=\boxed{0.582\text{ MeV}}$$

(b) $$\Delta E=\left(\sqrt{\frac{1}{1-(0.990)^2}}-\sqrt{\frac{1}{1-0.810}}\right)mc^2=\boxed{2.45\text{ MeV}}$$

P38.30 The relativistic kinetic energy of an object of mass m and speed u is $K_r=\left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)mc^2$. The classical equation is $K_c=\frac{1}{2}mu^2$. Their ratio is

$$\frac{K_r}{K_c} = \frac{\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2}{\frac{1}{2}mu^2} = \frac{2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)}{u^2/c^2}$$

$$= 2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)\frac{1}{u^2/c^2}$$

$$\frac{K_r}{K_c} = 2\left(\frac{1}{\sqrt{1-(0.100)^2}} - 1\right)\frac{1}{(0.100)^2} = 1.007\ 56$$

For still smaller speeds the agreement will be still better.

P38.31 (a) To find the speed of the protons with $E = \gamma mc^2 = 400mc^2$, we write

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} \quad \rightarrow \quad u = c\sqrt{1-\frac{1}{\gamma^2}}$$

So, $u = c\sqrt{1-\frac{1}{(400)^2}} = \boxed{0.999\ 997c}$

(b) From Example 38.9, for a proton, $mc^2 = 938$ MeV. Then

$$K = (\gamma - 1)mc^2 = 399(938\text{ MeV}) = \boxed{3.74\times 10^5\text{ MeV}}$$

***P38.32 Conceptualize** Wouldn't it be wonderful if such a reactor existed? You would simply feed the matter and antimatter into the machine and they would annihilate, releasing their *entire* rest energy. In reality, of course, we don't have a supply of a large amount of antimatter. While antimatter is produced in particle accelerators and stored for future experiments, the total amount of antimatter produced in this way would not boil a cup of tea if annihilated completely with matter. Producing enough antimatter for the proposal in this problem would be horrendously cost-prohibitive, but let's carry on anyway!

Categorize The matter and antimatter would be modeled as an *isolated system* for *energy*, in which the rest energy transforms entirely to other forms, all of which can theoretically be used to supply the world's energy needs.

Analyze (a) Solve Equation 38.24 for the mass needed to transform the rest energy into other forms of energy:

$$E_R = mc^2 \quad \rightarrow \quad m = \frac{E_R}{c^2} \qquad (1)$$

Substitute numerical values for the transformed rest energy to be equal to the world's energy requirement for one year:

$$m = \frac{4.0 \times 10^{20}\ \text{J}}{\left(3.00 \times 10^{8}\ \text{m/s}\right)^2} = 4.4 \times 10^{3}\ \text{kg}$$

This is the total mass to be transformed. Half of this mass is matter and half is antimatter, so

$$m_{\text{matter}} = \boxed{2.2 \times 10^{3}\ \text{kg}}$$

$$m_{\text{antimatter}} = \boxed{2.2 \times 10^{3}\ \text{kg}}$$

(b) Now, let's address the 5.0-yr supply. The amount of mass of either matter or antimatter that will have to be stored is

$$m_{5.0\ \text{yr}} = (5.0\ \text{yr})\left(2.2 \times 10^{3}\ \text{kg/yr}\right) = 1.1 \times 10^{4}\ \text{kg}$$

Use Equation 1.1 to find the volume in which this mass must be stored:

$$\rho = \frac{m}{V} \quad \rightarrow \quad V = \frac{m}{\rho} = \frac{1.1 \times 10^{4}\ \text{kg}}{2\,700\ \text{kg/m}^3} = \boxed{4.1\ \text{m}^3}$$

Therefore, each of the matter and antimatter could be stored in a cubic container about 1.6 m on a side.

Finalize How realistic is this proposal? Not realistic at all, at least with today's technology and scientific understanding. The concept of generating energy with no waste is optimistic. Even if we imagine that waste energy in the form of internal energy could be used to warm houses and commercial buildings, how do we transport that energy to the site? How do we feed the matter and antimatter in at a slow enough rate so that the reactor does not explode? How can we store antimatter on a world made of matter?]

Answers: (a) 2.2×10^3 kg for each of matter and antimatter (b) 4.1 m^3 for each of matter and antimatter

P38.33 Given $E = 2mc^2$, where $mc^2 = 938$ MeV from Example 38.9. We use Equation 38.27:

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

$$\left(2mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$$

$$4\left(mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2 \quad \to \quad p^2c^2 = 3\left(mc^2\right)^2$$

Solving for the momentum then gives

$$p = \sqrt{3}\frac{\left(mc^2\right)}{c} = \sqrt{3}\frac{(938 \text{ MeV})}{c} = \boxed{1.62 \times 10^3 \text{ MeV}/c}$$

P38.34 (a) $E = 2.86 \times 10^5$ J leaves the system, so the final mass is $\boxed{\text{smaller}}$.

(b) The mass-energy relation says that $E = mc^2$. Therefore,

$$m = \frac{E}{c^2} = \frac{2.86 \times 10^5 \text{ J}}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} = \boxed{3.18 \times 10^{-12} \text{ kg}}$$

(c) $\boxed{\text{It is too small a fraction of 9.00 g to be measured}}$.

P38.35 From $K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$, we have

$$\frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{K+mc^2}{mc^2}$$

$$1 - \frac{u^2}{c^2} = \frac{m^2c^4}{\left(K+mc^2\right)^2}$$

$$\frac{u^2}{c^2} = 1 - \frac{\left(mc^2\right)^2}{\left(K+mc^2\right)^2}$$

$$u = c\left[1 - \left(\frac{mc^2}{K+mc^2}\right)^2\right]^{1/2}$$

(a) Electron: $u = c\left[1 - \left(\dfrac{0.511}{2.511}\right)^2\right]^{1/2} = \boxed{0.979c}$

(b) Proton: $u = c\left[1 - \left(\dfrac{938}{940}\right)^2\right]^{1/2} = \boxed{0.065\,2c}$

(c) $\dfrac{u_{\text{electron}}}{u_{\text{proton}}} = \dfrac{0.979c}{0.065\,2c} = \boxed{15.0}$

In this case the electron is moving relativistically, but the classical expression $\dfrac{1}{2}mv^2$ is accurate to two digits for the proton.

(d) Electron: $u = c\left[1 - \left(\dfrac{0.511}{2\,000.511}\right)^2\right]^{1/2} = \boxed{0.999\,999\,97c}$

Proton: $u = c\left[1 - \left(\dfrac{938}{2\,938}\right)^2\right]^{1/2} = \boxed{0.948c}$

Then,

$$\frac{u_{\text{electron}}}{u_{\text{proton}}}=\frac{c\left[1-\left(\dfrac{0.511}{2\,000.511}\right)^2\right]^{1/2}}{c\left[1-\left(\dfrac{938}{2\,938}\right)^2\right]^{1/2}}=\boxed{1.06}$$

As the kinetic energies of both particles become large, their speeds approach c. By contrast, classically the speed would become large without any finite limit.

P38.36 We are told to start from $E=\gamma mc^2$ and $p=\gamma mu$. Squaring both equations gives

$$E^2=\left(\gamma mc^2\right)^2 \qquad \text{and} \qquad p^2=\left(\gamma mu\right)^2$$

We choose to multiply the second equation by c^2 and subtract it from the first:

$$E^2-p^2c^2=\left(\gamma mc^2\right)^2-(\gamma mu)^2c^2$$

We factor to obtain

$$E^2-p^2c^2=\gamma^2\left[\left(mc^2\right)\left(mc^2\right)-\left(mc^2\right)\left(mu^2\right)\right]$$

Extracting the (mc^2) factors gives

$$E^2-p^2c^2=\gamma^2\left(mc^2\right)^2\left(1-\frac{u^2}{c^2}\right)$$

We substitute the definition of γ:

$$E^2-p^2c^2=\left(1-\frac{u^2}{c^2}\right)^{-1}\left(mc^2\right)^2\left(1-\frac{u^2}{c^2}\right)$$

The γ^2 factors divide out, leaving

$$E^2 - p^2c^2 = \left(mc^2\right)^2$$

P38.37 Let $m = 1.99 \times 10^{-26}$ kg, and $\vec{\mathbf{u}} = u\hat{\mathbf{i}} = 0.500c\hat{\mathbf{i}}.$ An isolated system of two particles of mass m and $m' = m/3$ collide with the respective velocities $\vec{\mathbf{u}}$ and $-\vec{\mathbf{u}}$, resulting in a particle with mass M and velocity $\vec{\mathbf{v}}_f = v_f\hat{\mathbf{i}}.$

By conservation of the x component of momentum (γmu):

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{2mu}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[1]}$$

By conservation of total energy (γmc^2):

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{4mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[2]}$$

To start solving, we divide the momentum equation [1] by the energy equation [2], giving $v_f = \dfrac{2u}{4} = \dfrac{u}{2}$.

Then, substituting the value of the final speed back into the energy equation [2], we get

$$\frac{Mc^2}{\sqrt{1-u^2/4c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$\frac{2Mc^2}{\sqrt{4-u^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}} = \frac{2\left(1.99\times 10^{-26}\ \text{kg}\right)\sqrt{4-(0.500)^2}}{3\sqrt{1-(0.500)^2}}$$

$$M = \boxed{2.97\times 10^{-26}\ \text{kg}}$$

P38.38 (a) By conservation of the x component of momentum (γmu):

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{2mu}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[1]}$$

By conservation of total energy (γmc^2):

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{4mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[2]}$$

To start solving, we divide the momentum equation [1] by the energy equation [2], giving $v_f = \frac{2u}{4} = \frac{u}{2}$. Then, substituting the value of the final speed back into the energy equation [2], we get

$$\frac{Mc^2}{\sqrt{1-u^2/4c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$\frac{2Mc^2}{\sqrt{4-u^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$\boxed{M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}}}$$

(b) As $u \to 0$, $M = \dfrac{2m\sqrt{4 - u^2/c^2}}{3\sqrt{1 - u^2/c^2}} \to \dfrac{2m\sqrt{4}}{3\sqrt{1}} = \boxed{\dfrac{4m}{3}}$

(c) $\boxed{\text{The answer to part (b) is in agreement with the classical result, which is the arithmetic sum of the masses of the two colliding particles.}}$

P38.39 The kinetic energy of the car is given by

$$K = (\gamma - 1)mc^2 = \left(\left(1 - u^2/c^2\right)^{-1/2} - 1\right)mc^2$$

We use the series expansion from Appendix B.5:

$$K = mc^2\left[1 + \left(-\frac{1}{2}\right)(-u^2/c^2) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2}(-u^2/c^2)^2 + \cdots - 1\right]$$

$$K = \frac{1}{2}mu^2 + \frac{3}{8}m\frac{u^4}{c^2} + \cdots$$

The actual kinetic energy, given by this relativistic equation, is $\boxed{\text{larger}}$ than the classical $\dfrac{1}{2}mu^2$.

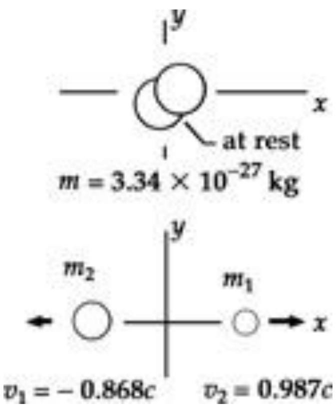
The difference, for m = 1 000 kg and u = 25 m/s, is

$$\frac{3}{8}m\frac{u^4}{c^2} = \frac{3}{8}(1\,000\text{ kg})\frac{(25\text{ m/s})^4}{(3.00 \times 10^8\text{ m/s})^2} = 1.6 \times 10^{-9}\text{ J}\ \boxed{\sim 10^{-9}\text{ J}}$$

P38.40 (a) The initial system is $\boxed{\text{isolated}}$.

(b) $\boxed{\text{Isolated system: conservation of energy, and isolated system: conservation of momentum.}}$

(c) We must conserve both energy and relativistic momentum of the system of fragments. With subscript 1 referring to the 0.987c particle and subscript 2 to the 0.868c particle,



ANS. FIG. P38.40

$$\gamma_1 = \frac{1}{\sqrt{1-(0.987)^2}} = \boxed{6.22} \text{ and}$$

$$\gamma_2 = \frac{1}{\sqrt{1-(0.868)^2}} = \boxed{2.01}$$

(d) Conservation of energy gives

$E_1 + E_2 = E_{\text{total}}$

which is

$$\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$$

or $\quad 6.22 m_1 + 2.01 m_2 = 3.34 \times 10^{-27} \text{ kg}$

This reduces to: $\boxed{3.09 m_1 + m_2 = 1.66 \times 10^{-27} \text{ kg}}$ **[1]**

(e) Since the final momentum of the system must equal zero, $p_1 = p_2$ gives

$$\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$$

or $\quad (6.22)(0.987c)m_1 = (2.01)(0.868c)m_2$

which becomes $\boxed{m_2 = 3.52 m_1}$ **[2]**

(f) Substituting [2] into [1] gives

$$3.09 m_1 + 3.52 m_1 = 1.66 \times 10^{-27} \text{ kg}$$

thus, $m_1 = \boxed{2.51 \times 10^{-28} \text{ kg}}$ and $m_2 = \boxed{8.84 \times 10^{-28} \text{ kg}}$

Section 38.9 The General Theory of Relativity

P38.41 (a) For the satellite, Newton's second law gives

$$\sum F = ma: \qquad \frac{GM_E m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$$

which gives

$$GM_E T^2 = 4\pi^2 r^3$$

Solving for the orbital radius,

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

$$r = \left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(43\,080\ \text{s}\right)^2}{4\pi^2}\right]^{1/3}$$

$$= \boxed{2.66\times10^7\ \text{m}}$$

(b) $$v = \frac{2\pi r}{T} = \frac{2\pi\left(2.66\times10^7\ \text{m}\right)}{43\,080\ \text{s}} = \boxed{3.87\times10^3\ \text{m/s}}$$

(c) From the relationship of frequency and period:

$$f = \frac{1}{T} \quad \to \quad df = -\frac{dT}{T^2} = -f\left(\frac{dT}{T}\right) \quad \to \quad \frac{df}{f} = -\frac{dT}{T}$$

We see the fractional decrease in frequency is equal in magnitude to the fractional change in period.

The small fractional decrease in frequency received is equal in magnitude to the fractional increase in period of the moving oscillator due to time dilation:

$$\frac{df}{f} = -\frac{dT}{T} = -\frac{\gamma\Delta t_P - \Delta t_P}{\Delta t_P} = -(\gamma - 1)$$

$$\frac{df}{f}=-\left(\frac{1}{\sqrt{1-(v/c)^2}}-1\right)=1-\frac{1}{\sqrt{1-(v/c)^2}}$$

$$\approx 1-\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right]=-\frac{1}{2}\left(\frac{v}{c}\right)^2$$

$$=-\frac{1}{2}\left[\left(\frac{3.87\times10^3\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2\right]=\boxed{-8.34\times10^{-11}}$$

(d) The orbit altitude is large compared to the radius of the Earth, so we must use

$$U_g=-\frac{GM_Em}{r}$$

The change in gravitational potential energy is

$$\Delta U_g=-\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)$$
$$\times\left(5.98\times10^{24}\ \text{kg}\right)m\left[\frac{1}{2.66\times10^7\ \text{m}}-\frac{1}{6.37\times10^6\ \text{m}}\right]$$
$$=\left(4.76\times10^7\ \text{J/kg}\right)m$$

Then

$$\frac{\Delta f}{f}=\frac{\Delta U_g}{mc^2}=\frac{\left(4.76\times10^7\ \text{J/kg}\right)\cancel{m}}{\cancel{m}\left(3.00\times10^8\ \text{m/s}\right)^2}=\boxed{+5.29\times10^{-10}}$$

(e) $-8.34\times10^{-11}+5.29\times10^{-10}=\boxed{+4.46\times10^{-10}}$

Additional Problems

P38.42 From the particle under constant speed model, find the travel time for Speedo from Goslo's reference frame:

$$\Delta t = \frac{d}{u} = \frac{2(50 \text{ ly})}{0.85c}\left(\frac{c \cdot \text{yr}}{\text{ly}}\right) = 118 \text{ yr}$$

Therefore, when Speedo arrives back on Earth, 118 years have passed and Goslo would have to be 158 years old. Furthermore, Speedo will be 102 years old. Perhaps future medical breakthroughs may extend the life expectancy to 158 years and beyond, but that is impossible at present.

P38.43 (a) Observers on Earth measure the distance to Andromeda to be

$$d = 2.00 \times 10^6 \text{ ly} = (2.00 \times 10^6 \text{ ly})c$$

The time for the trip, in Earth's frame of reference, is

$$\Delta t = \gamma \Delta t_p = \frac{30.0 \text{ yr}}{\sqrt{1-(v/c)^2}}$$

The required speed is then

$$v = \frac{d}{\Delta t} = \frac{(2.00 \times 10^6 \text{ ly})c}{(30.0 \text{ yr})\big/\sqrt{1-(v/c)^2}}$$

which gives, suppressing units,

$$(1.50 \times 10^{-5})(v/c) = \sqrt{1-(v/c)^2}$$

Squaring both sides of this equation and solving for v/c yields

$$\frac{v}{c} = \frac{1}{\sqrt{1+2.25 \times 10^{-10}}}$$

Then, the approximation $\dfrac{1}{\sqrt{1+x}}=1-\dfrac{x}{2}$ gives

$$\frac{v}{c}=1-\frac{2.25\times10^{-10}}{2}=\boxed{1-1.12\times10^{-10}}$$

P38.44 (a) We let H represent K/mc^2. Then,

$$H+1=\frac{1}{\sqrt{1-u^2/c^2}}$$

so $\quad 1-u^2/c^2=\dfrac{1}{H^2+2H+1}$

Solving,

$$\frac{u^2}{c^2}=1-\frac{1}{H^2+2H+1}=\frac{H^2+2H}{H^2+2H+1}$$

and $\quad \boxed{u=c\left(\dfrac{H^2+2H}{H^2+2H+1}\right)^{1/2}}$

(b) $\boxed{u \text{ goes to } 0 \text{ as } K \text{ goes to } 0.}$

(c) $\boxed{u \text{ approaches } c \text{ as } K \text{ increases without limit.}}$

(d) The acceleration is given by

$$a=\frac{du}{dt}=\frac{d}{dt}\left[c\left(\frac{H^2+2H}{H^2+2H+1}\right)^{1/2}\right]$$

$$a=c\frac{1}{2}\left(\frac{H^2+2H}{H^2+2H+1}\right)^{-1/2}$$
$$\times\left(\frac{\left[H^2+2H+1\right][2H+2]-\left[H^2+2H\right][2H+2]}{\left[H^2+2H+1\right]^2}\right)$$
$$\times\frac{d(K/mc^2)}{dt}$$

$$a = c\left(\frac{H^2+2H+1}{H^2+2H}\right)^{1/2}\left(\frac{H+1}{[H+1]^4}\right)\frac{P}{mc^2}$$
$$= \boxed{\frac{P}{mcH^{1/2}(H+2)^{1/2}(H+1)^2}}$$

where $P = \dfrac{dK}{dt}$.

(e) When H is small ($H \ll 1$), we have approximately

$$a = \frac{P}{mcH^{1/2}(2)^{1/2}(1)^2} = \frac{P}{mcH^{1/2}2^{1/2}} = \frac{P}{mc\left(\frac{K}{mc^2}\right)^{1/2}2^{1/2}}$$
$$= \frac{P}{(2mK)^{1/2}}$$

in agreement with the nonrelativistic case.

(f) When H is large the acceleration approaches

$$a = \frac{P}{mcH^{1/2}(H+2)^{1/2}(H+1)^2} \to \frac{P}{mcH^{1/2}(H)^{1/2}(H)^2} = \frac{P}{mcH^3}$$
$$= \frac{P}{mc\left(\frac{K}{mc^2}\right)^3} = \frac{m^2c^5P}{K^3}$$

(g) As energy is steadily imparted to the particle, the particle's acceleration decreases. It decreases steeply, proportionally to $1/K^3$ at high energy. In this way the particle's speed cannot reach or surpass a certain upper limit, which is the speed of light in vacuum.

P38.45 (a) From Problem 43,

$$\frac{v}{c} = \frac{1}{\sqrt{1+2.25\times10^{-10}}}$$

and

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{1}{\sqrt{1+2.25\times10^{-10}}}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{1}{1+2.25\times10^{-10}}\right)}}$$
$$= \boxed{6.67\times10^4}$$

(b) The astronaut's speed, from Problem 43, is

$$v = \frac{1}{\sqrt{1+2.25\times10^{-10}}}c$$

The time difference between the astronaut's trip and that of the beam of light is then

$$\Delta t = \frac{d}{v} - \frac{d}{c} = d\left(\frac{1}{v} - \frac{1}{c}\right) = \frac{d}{c}\left(\frac{c}{v} - 1\right) = \frac{d}{c}\left(\sqrt{1+x} - 1\right) \approx \frac{d}{c}\left(1 + \frac{x}{2} - 1\right)$$
$$= \frac{d}{c}\left(\frac{x}{2}\right)$$

Where $x = 2.25\times10^{-10}$. Substituting numerical values,

$$\Delta t = \frac{\left(2.00\times10^6 \text{ ly}\right)\left(9.46\times10^{15} \text{ m/ly}\right)}{3.00\times10^8 \text{ m/s}}\left(\frac{2.25\times10^{-10}}{2}\right)$$
$$= 7\ 095 \text{ s} = \boxed{1.96 \text{ h}}$$

P38.46 (a) From Equation 38.18, the speed of light in the laboratory frame is

$$u = \frac{v + \frac{c}{n}}{1 + \frac{v(c/n)}{c^2}} = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)}$$

(b) When v is much less than c we have

$$u = \frac{c}{n}\left(1+\frac{nv}{c}\right)\left(1+\frac{v}{nc}\right)^{-1} \approx \frac{c}{n}\left(1+\frac{nv}{c}\right)\left(1-\frac{v}{nc}\right)$$
$$\approx \frac{c}{n}\left(1+\frac{nv}{c}-\frac{v}{nc}\right) = \frac{c}{n}+v-\frac{v}{n^2}$$

(c) If light travels at speed c/n in the water, and the water travels at speed v, then the Galilean velocity transformation Equation 4.20 would indeed give $c/n + v$ for the speed of light in the moving water. The third term $-v/n^2$ does represent a relativistic effect that was observed decades before the Michelson-Morley experiment. It is a piece of twentieth-century physics that dropped into the nineteenth century. We could say that light is intrinsically relativistic.

(d) To take the limit as v approaches c we must go back to

$u = \dfrac{c\,(1+nv/c)}{n\,(1+v/nc)}$. As $v \to c$,

$$u \to \frac{c\,(1+nc/c)}{n\,(1+c/nc)} = \frac{c(1+n)}{n+1} = \boxed{c}$$

P38.47 The energy of the first fragment is given by

$$E_1^2 = p_1^2c^2 + \left(m_1c^2\right)^2 = \left(1.75\text{ MeV}\right)^2 + \left(1.00\text{ MeV}\right)^2$$

$$E_1 = 2.02\text{ MeV}$$

For the second,

$$E_2^2 = \left(2.00\text{ MeV}\right)^2 + \left(1.50\text{ MeV}\right)^2$$

$$E_2 = 2.50\text{ MeV}$$

(a) Energy is conserved, so the unstable object had $E = E_1 + E_2 = 4.52 \text{ MeV}$. Each component of momentum is conserved, so for the original object

$$p^2 = p_x^2 + p_y^2 = \left(\frac{1.75 \text{ MeV}}{c}\right)^2 + \left(\frac{2.00 \text{ MeV}}{c}\right)^2$$

Then, using Equation 38.27, we find the mass of the original object:

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

$$\left(4.52 \text{ MeV}\right)^2 = \left[\left(1.75 \text{ MeV}\right)^2 + \left(2.00 \text{ MeV}\right)^2\right] + \left(mc^2\right)^2$$

$$\boxed{m = \frac{3.65 \text{ MeV}}{c^2}}$$

(b) Now $E = \gamma mc^2$ gives

$$4.52 \text{ MeV} = \frac{1}{\sqrt{1 - u^2/c^2}} 3.65 \text{ MeV}$$

$$1 - \frac{u^2}{c^2} = 0.654 \quad \text{which gives} \quad \boxed{u = 0.589c}$$

P38.48 We find the speed of the electrons after accelerating through a potential difference ΔV from Equation 38.23:

$$K = e\Delta V = \left(\gamma - 1\right)mc^2 = \left(\frac{1}{\sqrt{1 - \left(u/c\right)^2}} - 1\right)mc^2$$

then

$$\frac{1}{\sqrt{1 - \left(u/c\right)^2}} = \frac{e\Delta V}{mc^2} + 1 = \frac{e\Delta V + mc^2}{mc^2}$$

or

$$1 - (u/c)^2 = \left(\frac{mc^2}{e\Delta V + mc^2}\right)^2$$

Solving,

$$\frac{u}{c} = \sqrt{1 - \left(\frac{\mathrm{m}}{\mathrm{e}\Delta\mathrm{V}/\mathrm{c}^2 + \mathrm{m}}\right)^2}$$

Substituting numerical values and suppressing units,

$$\frac{u}{c} = \sqrt{1 - \left[\frac{(9.11 \times 10^{-31}\ \mathrm{kg})}{\frac{(1.60 \times 10^{-19}\ \mathrm{C})(8.40 \times 10^4\ \mathrm{V})}{(3.00 \times 10^8\ \mathrm{m/s})^2} + 9.11 \times 10^{-31}\ \mathrm{kg}}\right]^2}$$

$$u = 0.512c$$

Because this speed is more than half the speed of light, there is no way to double its speed, regardless of the increased accelerating voltage. If the accelerating voltage is quadrupled to 336 kV, the speed of the electrons rises to $u = 0.798c$.

P38.49 (a) The speed of light in water is $c/1.33$, so the electron's speed is $1.10c/1.333$. Then

$$\gamma = \frac{1}{\sqrt{1 - (1.10/1.333)^2}} = 1.770$$

and the total energy is

$$E = \gamma mc^2 = 1.770(0.511\ \mathrm{MeV}) = \boxed{0.905\ \mathrm{MeV}}$$

(b) The electron's kinetic energy is

$$K = E - mc^2 = 0.905\ \mathrm{MeV} - 0.511\ \mathrm{MeV} = \boxed{0.394\ \mathrm{MeV}}$$

(c) The electron's momentum is found from

$$pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{\gamma^2 - 1}\ \ mc^2$$
$$= \sqrt{\gamma^2 - 1}(0.511\ \text{MeV}) = 0.747\ \text{MeV}$$

and

$$p = \boxed{0.747\frac{\text{MeV}}{c}} = \frac{0.747 \times 10^6\ \left(1.602 \times 10^{-19}\ \text{J}\right)}{3.00 \times 10^8\ \text{m/s}}$$
$$= \boxed{3.99 \times 10^{-22}\ \text{kg} \cdot \text{m/s}}$$

(d) From Figure 16.26, the angle between the particle (source of waves) and the shock wave is

$$\sin\theta = v/v_S$$

where v is the wave speed, which is the speed of light in water, and v_S is the source speed. Then

$$\sin\theta = v/v_S = 1/1.10 \quad \rightarrow \quad \theta = \boxed{65.4°}$$

P38.50 (a) Take $m = 1.00$ kg. The classical kinetic energy is

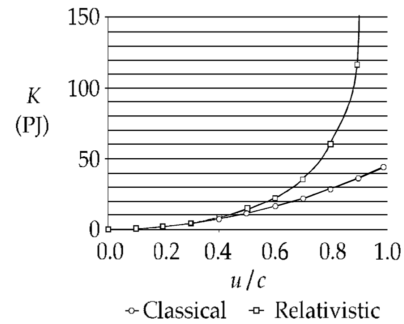
$$K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50 \times 10^{16}\ \text{J}\right)\left(\frac{u}{c}\right)^2$$

and the actual kinetic energy is

$$K_r = \left(\frac{1}{\sqrt{1-\left(u/c\right)^2}} - 1\right)mc^2 = \left(9.00 \times 10^{16}\ \text{J}\right)\left(\frac{1}{\sqrt{1-\left(u/c\right)^2}} - 1\right)$$

Using these expressions, we generate the graph in ANS. GRAPH P38.50.

$\dfrac{u}{c}$	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	$0.045\,3 \times 10^{16}$
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}



ANS. GRAPH P38.50

(b) $K_c = 0.990K_r$, when $\dfrac{1}{2}\left(\dfrac{u}{c}\right)^2 = 0.990\left[\dfrac{1}{\sqrt{1-(u/c)^2}} - 1\right]$, yielding

$$u = \boxed{0.115c}$$

(c) Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$.

(d) $K_c = 0.500K_r$ when $u = \boxed{0.786c}$.

P38.51 (a) Assuming the Sun-mass system is isolated, the energy (work) required to remove a mass m from the Sun's surface to infinity is equal to the change in potential energy of the system. If the work equals the rest energy mc^2, then

$$W = \Delta E = \Delta K + \Delta U = 0 + \left(U_f - U_i\right)$$

$$mc^2 = 0 - \left(-\frac{GM_s m}{R_g}\right)$$

$$mc^2 = \frac{GM_s m}{R_g} \quad \rightarrow \quad R_g = \frac{GM_s}{c^2}$$

(b) $$R_g = \frac{GM_s}{c^2} = \frac{\left(6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30} \text{ kg}\right)}{\left(3.00 \times 10^8 \text{ m/s}\right)^2}$$

$$R_g = 1.47 \times 10^3 \text{ m} = \boxed{1.47 \text{ km}}$$

P38.52 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0 \text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with

$$Mc^2 = 8.60 \times 10^{-9} \text{ J} = 5.38 \times 10^{10} \text{ eV} = 5.38 \times 10^7 \text{ keV}$$

Thus, $$\left(Mc^2 + K\right)^2 = \left(14.0 \text{ keV}\right)^2 + \left(Mc^2\right)^2$$

or

$$\left(1 + \frac{K}{Mc^2}\right)^2 = \left(\frac{14.0 \text{ keV}}{Mc^2}\right)^2 + 1$$

Because the term $\left(\frac{14.0 \text{ keV}}{Mc^2}\right)^2 \ll 1$, evaluating $\left(\frac{14.0 \text{ keV}}{Mc^2}\right)^2 + 1$ on a calculator gives 1.

We need to expand $\left(\frac{14.0\text{ keV}}{Mc^2}\right)^2 + 1$ using the Binomial Theorem:

$$1 + \frac{K}{Mc^2} = \sqrt{1 + \left(\frac{14.0\text{ keV}}{Mc^2}\right)^2} \approx 1 + \frac{1}{2}\left(\frac{14.0\text{ keV}}{Mc^2}\right)^2$$

$$K \approx \frac{(14.0\text{ keV})^2}{2Mc^2} = \frac{(14.0\times10^3\text{ eV})^2}{2(53.8\times10^9\text{ eV})} = \boxed{1.82\times10^{-3}\text{ eV}}$$

Challenge Problems

P38.53 (a) Take the two colliding protons as the system

$$E_1 = K + mc^2 \qquad E_2 = mc^2$$

$$E_1^2 = p_1^2c^2 + m^2c^4 \qquad p_2 = 0$$

In the final state,

$$E_f = K_f + Mc^2 = p_f^2c^2 + M^2c^4$$

By energy conservation, $E_1 + E_2 = E_f$, so

$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$

$$p_1^2c^2 + m^2c^4 + 2(K + mc^2)mc^2 + m^2c^4$$
$$= p_f^2c^2 + M^2c^4$$

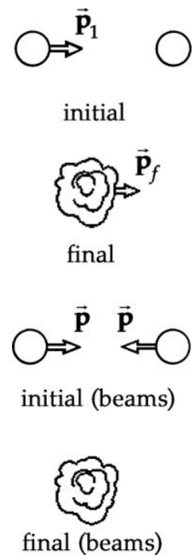
By conservation of momentum, $p_1 = p_f$, so

$$\cancel{p_1^2c^2} + m^2c^4 + 2(K + mc^2)mc^2 + m^2c^4$$
$$= \cancel{p_f^2c^2} + M^2c^4$$

and we have then

$$M^2c^4 = 2Kmc^2 + 4m^2c^4 = \frac{4Km^2c^4}{2mc^2} + 4m^2c^4$$

$$Mc^2 = 2mc^2\sqrt{1+\frac{K}{2mc^2}}$$



ANS. FIG. P38.53

(b) By contrast, for colliding beams we have, in the original state,

$$E_1 = K + mc^2 \qquad\qquad E_2 = K + mc^2$$

In the final state,

$$E_f = Mc^2$$

$$E_1 + E_2 = E_f:$$

$$K + mc^2 + K + mc^2 = Mc^2$$

$$Mc^2 = 2mc^2\left(1+\frac{K}{2mc^2}\right)$$

P38.54 (a) At any speed, the momentum of the particle is given by

$$p = \gamma mu = \frac{mu}{\sqrt{1-(u/c)^2}}$$

With Newton's law expressed as $F = qE = \frac{dp}{dt}$, we have

$$qE = \frac{d}{dt}\left[mu\left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]$$

$$qE = m\left(1-\frac{u^2}{c^2}\right)^{-1/2}\frac{du}{dt}+\frac{1}{2}mu\left(1-\frac{u^2}{c^2}\right)^{-3/2}\left(\frac{2u}{c^2}\right)\frac{du}{dt}$$

so $$\frac{qE}{m} = \frac{du}{dt}\left[\frac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$$

and $$\boxed{a = \frac{du}{dt} = \frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

(b) For u small compared to c, the relativistic expression reduces to the classical $a = \frac{qE}{m}$. As u approaches c, the acceleration approaches zero, so that the object can never reach the speed of light.

(c) We can use the result of (a) to find the velocity u at time t:

$$a = \frac{du}{dt} = \frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2} \quad \rightarrow \quad \int_0^u \frac{du}{\left(1-u^2/c^2\right)^{3/2}} = \int_0^t \frac{qE}{m}dt$$

$$\frac{u}{\left(1-u^2/c^2\right)^{1/2}} = \frac{qEt}{m}$$

$$u^2 = \left(\frac{qEt}{m}\right)^2\left(1-\frac{u^2}{c^2}\right)$$

$$\boxed{u = \frac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$$

Now, we can use this result to find position x at time t:

$$\frac{dx}{dt} = u = \frac{qEct}{\sqrt{m^2c^2 + q^2E^2t^2}}$$

$$x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2 + q^2E^2t^2}} = \frac{c}{qE}\sqrt{m^2c^2 + q^2E^2t^2}\Big|_0^t$$

$$\boxed{x = \frac{c}{qE}\left(\sqrt{m^2c^2 + q^2E^2t^2} - mc\right)}$$

P38.55 We choose to write down the answer to part (b) first.

(b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-\left(\frac{v}{c}\right)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$\frac{3.60\text{ ly}}{1.80c} = 2.00\text{ yr}$$

We see Tau Ceti as moving toward us at 0.800*c*, while its light approaches at 1.00*c*, only 0.200*c* faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at 0.200*c*. We calculate that it must have been opening for

$$\frac{3.60\text{ ly}}{0.200c} = 18.0\text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

ANSWERS TO QUICK-QUIZZES

1. (c)

2. (d)

3. (d)

4. (a)

5. (c)

6. (d)

7. **(i)** (c) **(ii)** (a)

8. (a) $m_3 > m_2 = m_1$ (b) $K_3 = K_2 > K_1$ (c) $U_2 > U_3 = U_1$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P38.2 See P38.2 for full explanation

P38.4 (a) 2.18 μs; (b) 649 m

P38.6 65.0 beats/min; (b) 10.5 beats/min

P38.8 The driver's own testimony shows him blatantly violating any Earth-based speed limit; look for another defense.

P38.10 $v = \dfrac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}$

P38.12 42.1 g/cm^3

P38.14 0.286 h = 17.1 min

P38.16 (a) $v = 0.943c$; (b) 2.55×10^3 m

P38.18 (a) $L = L_0\left[1-\left(\dfrac{v}{c}\right)^2\cos^2\theta_0\right]^{1/2}$; (b) $\gamma\tan\theta_0$

P38.20 The driver was traveling at $0.763c$ relative to the Earth. He was speeding.

P38.22 0.893c, 16.8° above the x' axis

P38.24

$$u = \frac{c}{\sqrt{\left(m^2c^2/p^2\right)+1}}$$

P38.26 (a) \$800; (b) $\$2.12 \times 10^9$

P38.28 (a) 4.38×10^{11} J; (b) 4.38×10^{11}; (c) See P39.48(c) for full explanation.

P38.30 See P38.30 for full explanation.

P38.32 (a) 2.2×10^3 kg for each of matter and antimatter (b) 4.1 m^3 for each of matter and antimatter

P38.34 (a) smaller; (b) 3.18×10^{-12} kg;

(c) It is too small a fraction of 9.00 g to be measured

P38.36 See P38.36 for full explanation.

P38.38 (a) $M = \dfrac{2m\sqrt{4 - u^2/c^2}}{3\sqrt{1 - u^2/c^2}}$; (b) $\dfrac{4m}{3}$; (c) The answer to part (b) is in agreement with the classical result, which is the arithmetic sum of the masses of the two colliding particles.

P38.40 (a) isolated; (b) isolated system: conservation of energy and isolated system: conservation of momentum; (c) 6.22 and 2.01;
(d) $3.09m_1 + m_2 = 1.66 \times 10^{-27}$ kg; (e) $m_2 = 3.52m_1$;
(f) $m_1 = 2.51 \times 10^{-28}$ kg and $m_2 = 8.84 \times 10^{-28}$ kg

P38.42 When Speedo arrives back on Earth, 118 years have passed, and Goslo would be 158 years old. That is impossible at the present time.

P38.44 (a) $u = c\left(\dfrac{H^2 + 2H}{H^2 + 2H + 1}\right)^{1/2}$; (b) u goes to 0 as K goes to 0; (c) u approaches c as K increases without limit; (d) $\dfrac{P}{mcH^{1/2}(H+2)^{1/2}(H+1)^2}$;
(e) See P39.74(e) for full explanation; (f) See P39.74(f) for full explanation; (g) As energy is steadily imparted to particle, the particle's acceleration decreases. It decreases steeply, proportionally to $1/K^3$ at high energy. In this way the particle's speed cannot reach or surpass a certain upper limit, which is the speed of light in vacuum.

P38.46 (a–c) See P38.46 for full explanation; (d) c

P38.48 See P38.48 for full explanation

P38.50 (a) See ANS GRAPH P38.50 (b) $0.115c$ (c) $0.257c$ (d) $0.786c$

P38.52 1.82×10^{-3} eV

P38.54 (a) $a = \dfrac{du}{dt} = \dfrac{qE}{m}\left(1 - \dfrac{u^2}{c^2}\right)^{3/2}$; (b) See P38.54 (b) for full explanation; (c)

$u = \dfrac{qEct}{\sqrt{m^2c^2 + q^2E^2t^2}}$, $x = \dfrac{c}{qE}\left(\sqrt{m^2c^2 + q^2E^2t^2} - mc\right)$