13

Universal Gravitation

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP13.1** **Conceptualize** Be sure you are clear on the material in Section 13.4 on Kepler's third law.

Categorize Because the moons are in an orbit around Pluto, they are each modeled as an *isolated system* for *angular momentum,* leading to the condition on their orbits that we know as Kepler's third law.

Analyze In the table below, we add a column for the ratio T^2/a^3.

Moon	Semimajor axis a (10^6 m)	Orbital period T (d)	Diameter (km)	T^2/a^3 (10^{-11} s^2/m^3)
Charon	17.54	6.387	1 208	5.64
Styx	42.66	20.16	~12	3.91
Nix	48.69	24.85	~40	3.99
Kerberos	57.78	32.17	~14	4.00
Hydra	64.74	38.20	~50	4.01

Notice that the four moons other than Charon have similar values of the ratio T^2/a^3, with variations arising in the fact that Pluto is so far away that accurate measurements are difficult. But the ratio for Charon is clearly different.

For a clue, look at the diameters of the moons. The four moons other than Charon have diameters small compared to that of Pluto (2 374 km). On the other hand, Charon has a diameter that is more than 50% of that of Pluto. As a result, Pluto and Charon can be considered to be a *double planet*, revolving around their center of mass. Consequently, Charon does *not* fit the Kepler's-third-law model of a small object in orbit around a much more massive object that we studied in Section 13.4.

Finalize The only widely agreed double-(dwarf-)planet in our solar system is the combination of Pluto and Charon. The mass ratio of Charon to Pluto is 0.117. A ratio of 1 would be a pure double planet: both objects with the same mass. Some argue that the Earth and the Moon qualify, but the mass ratio is 0.012 3, so other scientists do not adopt this combination as a double planet.

Answer: Charon's diameter is not small compared to Pluto's, so it does not meet the criteria for which Kepler's law was generated.

***TP13.2** **Conceptualize** Be sure you are clear on the material in Section 13.4 on Kepler's third law.

Categorize Because the moons are in an orbit around Jupiter, they are each modeled as an *isolated system* for *angular momentum,* leading to the condition on their orbits that we know as Kepler's third law.

Analyze In the table below, we add a column for the ratio T^2/a^3.

Moon	**Semimajor axis a (10^9 m)**	**Orbital period T (d)**	**Eccentricity**	**Inclination Angle**	**T^2/a^3 (10^{-16} s²/m³)**
Moons discovered by Galileo:					
Io	0.421 7	1.769 1	0.004 1	0.05	3.12
Europa	0.671 0	3.551 2	0.009 4	0.47	3.12
Ganymede	1.070 4	7.154 6	0.001 1	0.20	3.12
Callisto	1.882 7	16.689	0.007 4	0.20	3.12
Inner moons:					
Metis	0.127 7	0.294 8	0.000 02	0.06	3.12
Adrastea	0.128 7	0.298 3	0.001 5	0.03	3.12
Amalthea	0.181 4	0.498 2	0.003 2	0.37	3.11
Thebe	0.221 9	0.674 5	0.017 5	1.08	3.11
Outer moons:					
Themisto	7.393 2	129.87	0.215 5	45.8	3.12
Leda	11.187 8	240.82	0.167 3	27.6	3.09

Himalia	11.452 0	250.23	0.151 3	30.5	3.11
Lysithea	11.740 6	259.89	0.132 2	27.0	3.12
Elara	11.778 0	257.62	0.194 8	29.7	3.03
Dia	12.570 4	287.93	0.205 8	27.6	3.12
Carpo	17.144 9	458.62	0.273 5	56.0	3.12

(a) The values in the rightmost column are all close, remarkably close for objects that are so far away from the Earth.

(b) According to Equation 13.11, the ratio T^2/a^3 is inversely proportional to the mass of the central object. Because the mass of Jupiter is smaller than that of the Sun, the ratio T^2/a^3 will be higher.

(c) Yes. From Equation 13.11, we can build the ratio

$$\frac{K_J}{K_S} = \frac{\dfrac{4\pi^2}{GM_J}}{\dfrac{4\pi^2}{GM_S}} = \frac{M_S}{M_J} \quad \rightarrow \quad K_J = \frac{M_S}{M_J} K_S$$

Using masses and the value of K_S from Table 13.2, we find

$$K_J = \frac{1.989 \times 10^{30}\ \text{kg}}{1.90 \times 10^{27}\ \text{kg}} \left(2.97 \times 10^{-19}\ \text{s}^2/\text{m}^3\right) = 3.11 \times 10^{-16}\ \text{s}^2/\text{m}^3$$

This is remarkable agreement with the values in the table for Jupiter's moons.

Finalize The four large moons discovered by Galileo (Io, Europa, Ganymede, and Callisto) have identical ratios of T^2/a^3 to three significant figures. These moons have been observed for many years and there is dependable data for them. In addition, their eccentricities are small (0.001 1 to 0.009 4), so the orbits are almost circular. Furthermore, their inclination angles are all small, so they all revolve almost in the same plane. These moons are great candidates for obeying Kepler's third law.

The four inner moons (Metis, Adrastea, Amalthea, and Thebe) also have small eccentricities, so their orbits are almost circular. And their inclination angles are also small.

Now look at the outer moons. The eccentricities are high, as are the inclination angles. These are theorized to be captured moons; they did not arise at the same time as Jupiter, but were captured by its gravitational field as they passed by. As a result, their orbits are harder to measure accurately, and we see more variation in the ratio T^2/a^3 for the moons beyond Callisto. On the other hand, the ratios are remarkably close, given that these moons come from a different origin from the Galilean and inner moons.

If we move outward away from Jupiter, the moons become even stranger. For example, the 64th moon from the surface is Sponde, with an eccentricity of 0.443 1, an inclination angle of 154.372°, and a diameter of only 2 km. These data clearly show a captured moon; the inclination angle shows that the moon is revolving in the opposite direction around Jupiter compared to the Galilean and inner moons! Despite these surprising values, observational difficulties in making measurements on such a small object, and its captured origin, the value of T^2/a^3 for Sponde is 2.95×10^{-16} s^2/m^3, only 5.5% different from the Galilean moons.

Answers: (a) Kepler's third law is satisfied. (b) Answers will vary. (c) yes

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 13.1 Newton's Law of Universal Gravitation

P13.1 This is a direct application of the equation expressing Newton's law of gravitation:

$$F = \frac{GMm}{r^2} = \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{\left(1.50\ \mathrm{kg}\right)\left(15.0\times10^{-3}\ \mathrm{kg}\right)}{\left(4.50\times10^{-2}\ \mathrm{m}\right)^2}$$

$$= \boxed{7.41\times10^{-10}\ \mathrm{N}}$$

P13.2 (a) The Sun-Earth distance is 1.496×10^{11} m and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496 \times 10^{11}\ \mathrm{m} - 3.84 \times 10^{8}\ \mathrm{m} = 1.492 \times 10^{11}\ \mathrm{m}$$

The mass of the Sun, Earth, and Moon are

$$M_S = 1.99\times10^{30}\ \mathrm{kg}$$

$$M_E = 5.98\times10^{24}\ \mathrm{kg}$$

and $M_M = 7.36\times10^{22}\ \mathrm{kg}$

We have

$$F_{SM} = \frac{Gm_1m_2}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)\left(7.36\times10^{22}\ \mathrm{kg}\right)}{\left(1.492\times10^{11}\ \mathrm{m}\right)^2}$$

$$= \boxed{4.39\times10^{20}\ \mathrm{N}}$$

(b) $$F_{EM} = \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)\left(7.36\times10^{22}\ \mathrm{kg}\right)}{\left(3.84\times10^{8}\ \mathrm{m}\right)^2}$$

$$= \boxed{1.99\times10^{20}\ \mathrm{N}}$$

(c) $$F_{SE} = \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(1.496\times10^{11}\ \mathrm{m}\right)^2}$$

$$= \boxed{3.55\times10^{22}\ \mathrm{N}}$$

(d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.3 For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)(70\ \mathrm{kg})(70\ \mathrm{kg})}{(2\ \mathrm{m})^2}$$

$$\boxed{\sim 10^{-7}\ \mathrm{N}}$$

P13.4 Assume the masses of the sphere are the same. Using $F_g = \dfrac{Gm_1m_2}{r^2}$, we would find that the mass of a sphere is 1.22×10^5 kg! If the spheres have at most a radius of 0.500 m, the density of spheres would be at least $2.34 \times 10^5\ \mathrm{kg/m^3}$, which is ten times the density of the most dense element, osmium.

The situation is impossible because no known element could compose the spheres.

Section 13.2 Free-Fall Acceleration and the Gravitational Force

P13.5 (a) For the gravitational force on an object in the neighborhood of Miranda, we have

$$m_{\text{obj}}g = \frac{Gm_{\text{obj}}m_{\text{Miranda}}}{r_{\text{Miranda}}^2}$$

$$g = \frac{Gm_{\text{Miranda}}}{r_{\text{Miranda}}^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.68\times10^{19}\ \text{kg}\right)}{\left(242\times10^3\ \text{m}\right)^2}$$

$$= \boxed{0.076\,1\ \text{m/s}^2}$$

(b) We ignore the difference (of about 4%) in g between the lip and the base of the cliff. For the vertical motion of the athlete, we have

$$y_f = y_i + v_{yi} + \frac{1}{2}a_y t^2$$

$$-5\ 000\ \text{m} = 0 + 0 + \frac{1}{2}\left(-0.076\ 1\ \text{m/s}^2\right)t^2$$

$$t = \left(\frac{2\left(5\ 000\ \text{m}\right)\text{s}^2}{0.076\ 1\ \text{m}}\right)^{1/2} = \boxed{363\ \text{s}}$$

(c) $$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + (8.50\ \text{m/s})(363\ \text{s}) + 0 = \boxed{3.08\times10^3\ \text{m}}$$

We ignore the curvature of the surface (of about 0.7°) over the athlete's trajectory.

(d) $$v_{xf} = v_{xi} = 8.50\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_y t = 0 - \left(0.076\,1\ \text{m/s}^2\right)(363\ \text{s}) = -27.6\ \text{m/s}$$

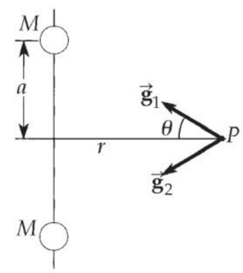
Thus $\vec{\mathbf{v}}_f = \left(8.50\hat{\mathbf{i}} - 27.6\hat{\mathbf{j}}\right)\ \text{m/s} = \sqrt{8.50^2 + 27.6^2}\ \text{m/s}$ at

$\tan^{-1}\left(\dfrac{27.6\ \text{m/s}}{8.50\ \text{m/s}}\right) = 72.9°$ below the x axis.

$$\boxed{\vec{\mathbf{v}}_f = 28.9\ \text{m/s at } 72.9° \text{ below the horizontal}}$$

Section 13.3 Analysis Model: Particle in a field (Gravitational)

P13.6 (a) $g_1 = g_2 = \dfrac{MG}{r^2 + a^2}$



ANS. FIG. P13.6

$g_{1y} = -g_{2y}$

$g_y = g_{1y} + g_{2y} = 0$

$g_{1x} = g_{2x} = g_2 \cos\theta$

$\cos\theta = \dfrac{r}{\left(a^2 + r^2\right)^{1/2}}$

$$\vec{\mathbf{g}} = 2g_{2x}\left(-\hat{\mathbf{i}}\right)$$

or $\vec{\mathbf{g}} = \boxed{\dfrac{2MGr}{\left(r^2 + a^2\right)^{3/2}} \text{ toward the center of mass}}$

(b) At $r = 0$, the fields of the two objects are equal in magnitude and opposite in direction, to add to zero.

(c) As $r \to 0$, $2MGr(r^2 + a^2)^{-3/2}$ approaches $2MG(0)/a^3 = 0$.

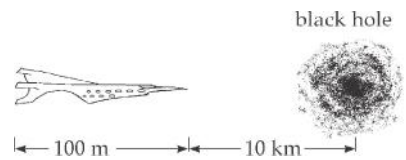
(d) When r is much greater than a, the angles the field vectors make with the x axis become smaller. At very great distances, the field vectors are almost parallel to the axis; therefore, they begin to look like the field vector from a single object of mass $2M$.

(e) As r becomes much larger than a, the expression approaches $2MGr(r^2 + 0^2)^{-3/2} = 2MGr/r^3 = 2MG/r^2$ as required.

P13.7 (a) $F = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\ \text{kg}\right)\left(10^3\ \text{kg}\right)\right]}{\left(1.00\times10^4\ \text{m}+50.0\ \text{m}\right)^2}$

$= \boxed{1.31\times10^{17}\ \text{N}}$

(b) $\Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$



ANS. FIG. P13.7

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

$$\Delta g = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left[100\left(1.99\times10^{30}\ \text{kg}\right)\right]\left[\left(1.01\times10^4\ \text{m}\right)^2-\left(1.00\times10^4\ \text{m}\right)^2\right]}{\left(1.00\times10^4\ \text{m}\right)^2\left(1.01\times10^4\ \text{m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\ \text{N/kg}}$$

Section 13.4 Kepler's Laws and the Motion of Planets

P13.8 The gravitational force on mass located at distance r from the center of the Earth is $F_g = mg = GM_Em/r^2$. Thus, the acceleration of gravity at this location is $g = GM_E/r^2$. If g = 9.00 m/s^2 at the location of the satellite, the radius of its orbit must be

$$r = \sqrt{\frac{GM_E}{g}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{9.00\ \text{m/s}^2}}$$
$$= 6.66\times10^6\ \text{m}$$

From Kepler's third law for Earth satellites, $T^2 = 4\pi^2 r^3 GM_E S$, the period is found to be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{\left(6.66\times10^6\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}}$$
$$= 5.41\times10^3\ \text{s}$$

or

$$T = \left(5.41\times10^3\ \text{s}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{1.50\ \text{h} = 90.0\ \text{min}}$$

***P13.9** **Conceptualize** The minimum-energy orbit described makes things relatively simple. Notice that the spacecraft leaves Earth orbit in a direction tangent to Earth orbit and then arrives at Mars in a direction tangent to Mars' orbit. Because the spacecraft travels from perihelion to aphelion, it makes exactly one-half of a full orbit to arrive at Mars.

Categorize Because the spacecraft is in an orbit around the Sun, it is modeled as an *isolated system* for *angular momentum,* leading to the condition on its orbit that we know as Kepler's third law.

Analyze (a) From Figure P13.9, we see that the *major* axis of the elliptical transfer orbit is the sum of the Sun–Earth distance and the Sun–Mars distance. Therefore, the *semimajor* axis is half this sum:

$$a = \frac{r_{\text{Earth}} + r_{\text{Mars}}}{2} \qquad (1)$$

From Kepler's third law, the period of the orbit is given by Equation 13.11. Because the transfer orbit is half of the full elliptical orbit, the transfer time interval is one-half of the period:

$$\Delta t_{\text{transfer}} = \frac{T}{2} = \frac{1}{2}\sqrt{\frac{4\pi^2}{GM_S}a^3} = \pi\sqrt{\frac{a^3}{GM_S}} \qquad (2)$$

Substitute Equation (1) into Equation (2):

$$\Delta t_{\text{transfer}} = \pi\sqrt{\frac{1}{GM_S}\left(\frac{r_{\text{Earth}} + r_{\text{Mars}}}{2}\right)^3} = \pi\sqrt{\frac{\left(r_{\text{Earth}} + r_{\text{Mars}}\right)^3}{8GM_S}} \qquad (3)$$

Substitute numerical values:

$$\Delta t_{\text{transfer}} = \pi\sqrt{\frac{\left(1.496\times10^{11}\text{ m} + 2.28\times10^{11}\right)^3}{8\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{ kg}^2\right)\left(1.989\times10^{30}\text{ kg}\right)}}$$

$$= 2.24\times10^{7}\text{ s}\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = \boxed{0.708\text{ yr} = 258\text{ d}}$$

(b) For the What If? question, a minimum-energy transfer orbit for reaching Venus, which is closer to the Sun than Earth, involves placing a spacecraft in a trajectory with its aphelion at Earth and perihelion at the arrival planet. The situation now is the reverse of that shown in the diagram, with the target planet closer to the Sun than the departure planet. Equation (3) becomes

$$\Delta t_{\text{transfer}} = \pi\sqrt{\frac{\left(r_{\text{Earth}} + r_{\text{Venus}}\right)^3}{8GM_S}} \qquad (4)$$

Substitute numerical values:

$$\Delta t_{\text{transfer}} = \pi\sqrt{\frac{\left(1.496\times10^{11}\text{ m} + 1.08\times10^{11}\right)^3}{8\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{ kg}^2\right)\left(1.989\times10^{30}\text{ kg}\right)}}$$

$$= 1.26\times10^{7}\text{ s}\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = \boxed{0.399\text{ yr} = 146\text{ d}}$$

The orbital periods of the Earth and Venus are in a ratio of 1.63 (See Table 13.2). Therefore, the launch windows for minimum energy orbits to Venus occur approximately once every 1.63 × 12 months = 19.6 months.

Finalize Your date is impressed. Figure P13.9 shows Earth and Mars on opposite sides of the Sun, but that is not the configuration that exists either at departure from Earth or arrival at Mars. Based on the periods of Mars and Earth, and the transfer time interval, impress your date further by arguing that Mars must be 43° ahead of Earth in its orbit when the spacecraft is launched so that Mars is in the right place when the spacecraft arrives at perihelion. Further impress your date by demonstrating that Earth is 75° ahead of Mars when the spacecraft arrives at Mars.

Answer: (a) 0.708 yr (b) 0.399 yr

P13.10 (a) The particle does possess angular momentum, because it is not headed straight for the origin.

(b) Its angular momentum is constant. There are no identified outside influences acting on the object.

(c) Since speed is constant, the distance traveled between t_A and t_B is equal to the distance traveled between t_C and t_D. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $\frac{1}{2}bv_0(t_B - t_A) = \frac{1}{2}bv_0(t_D - t_C)$

states that the particle's radius vector sweeps out equal areas in equal times.

P13.11 For an object in orbit about Earth, Kepler's third law gives the relation between the orbital period T and the average radius of the orbit ("semimajor axis") as

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3$$

We assume that the two given distances in the problem statements are the perigee and apogee, respectively.

Thus, if the average radius is

$$r = \frac{r_{\min} + r_{\max}}{2} = \frac{6\,670\text{ km} + 385\,000\text{ km}}{2}$$
$$= 1.96 \times 10^5\text{ km} = 1.96 \times 10^8\text{ m}$$

The period (time for a round trip from Earth to the Moon) would be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$
$$= 2\pi\sqrt{\frac{(1.96 \times 10^8\text{ m})^3}{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}}$$
$$= 8.63 \times 10^5\text{ s}$$

The time for a one-way trip from Earth to the Moon is then

$$\Delta t = \frac{1}{2}T = \left(\frac{8.63 \times 10^5\text{ s}}{2}\right)\left(\frac{1\text{ day}}{8.64 \times 10^4\text{ s}}\right) = \boxed{4.99\text{ d}}$$

P13.12 By conservation of angular momentum for the satellite, $r_p v_p = r_a v_a$, or

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2\,289\text{ km} + 6.37 \times 10^3\text{ km}}{459\text{ km} + 6.37 \times 10^3\text{ km}} = \frac{8\,659\text{ km}}{6\,829\text{ km}} = \boxed{1.27}$$

We do not need to know the period.

P13.13 The speed of a planet in a circular orbit is given by

$$\sum F = ma: \quad \frac{GM_{\text{sun}}m}{r^2} = \frac{mv^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{GM_{\text{sun}}}{r}}$$

(a) For Mercury, the speed is

$$v_M = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{5.79\times10^{10}\ \text{m}}}$$
$$= 4.79\times10^4\ \text{m/s}$$

and for Pluto,

$$v_p = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{5.91\times10^{12}\ \text{m}}}$$
$$= 4.74\times10^3\ \text{m/s}$$

With greater speed, $\boxed{\text{Mercury will eventually move farther from the Sun than Pluto.}}$

(b) With original distances r_P and r_M perpendicular to their lines of motion, they will be equally far from the Sun at time t, where

$$\sqrt{r_P^2 + v_P^2 t^2} = \sqrt{r_M^2 + v_M^2 t^2}$$
$$r_P^2 - r_M^2 = \left(v_M^2 - v_P^2\right)t^2$$
$$t = \sqrt{\frac{\left(5.91\times10^{12}\ \text{m}\right)^2 - \left(5.79\times10^{10}\ \text{m}\right)^2}{\left(4.79\times10^4\ \text{m/s}\right)^2 - \left(4.74\times10^3\ \text{m/s}\right)^2}}$$
$$= \sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}} = 1.24\times10^8\ \text{s} = \boxed{3.93\ \text{yr}}$$

P13.14 (a) In $T^2 = 4\pi^2 a^3/GM_{\text{central}}$ we take $a = 3.84 \times 10^8$ m.

$$M_{\text{central}} = \frac{4\pi^2 a^3}{GT^2}$$
$$= \frac{4\pi^2(3.84\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(27.3\times86\ 400\ \text{s})^2}$$
$$= \boxed{6.02\times10^{24}\ \text{kg}}$$

This is a little larger than 5.98×10^{24} kg.

(b)

The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it. The radius of the Moon's orbit is therefore a bit less than the Earth-Moon distance.

Section 13.5 Gravitational Potential Energy

P13.15 The enery required is equal to the change in gravitational potential energy of the object-Earth system:

$$U = -G\frac{Mm}{r} \text{ and } g = \frac{GM_E}{R_E^2} \text{ so that}$$

$$\Delta U = -GMm\left(\frac{1}{3R_E} - \frac{1}{R_E}\right) = \frac{2}{3}mgR_E$$

$$\Delta U = \frac{2}{3}(1\ 000\ \text{kg})(9.80\ \text{m/s}^2)(6.37\times 10^6\ \text{m}) = \boxed{4.17\times 10^{10}\ \text{J}}$$

P13.16 (a) Energy conservation of the object-Earth system from release to radius r:

$$\left(K+U_g\right)_{\text{altitude } h} = \left(K+U_g\right)_{\text{radius } r}$$

$$0 - \frac{GM_E m}{R_E + h} = \frac{1}{2}mv^2 - \frac{GM_E m}{r}$$

$$v = \left[2GM_E\left(\frac{1}{r} - \frac{1}{R_E + h}\right)\right]^{1/2} = -\frac{dr}{dt}$$

(b) $\int_i^f dt = \int_i^f -\frac{dr}{v} = \int_f^i \frac{dr}{v}$. The time of fall is, suppressing units,

$$\Delta t = \int_{R_E}^{R_E+h} \left[2GM_E \left(\frac{1}{r} - \frac{1}{R_E + h} \right) \right]^{-1/2} dr$$

$$\Delta t = \left(2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}\right)^{-1/2}$$

$$\times \int_{6.37\times10^6\ \text{m}}^{6.87\times10^6\ \text{m}} \left[\left(\frac{1}{r} - \frac{1}{6.87 \times 10^6\ \text{m}} \right) \right]^{-1/2} dr$$

We can enter this expression directly into a mathematical calculation program.

Alternatively, to save typing we can change variables to $u = \dfrac{r}{10^6}$.

Then

$$\Delta t = \left(7.977 \times 10^{14}\right)^{-1/2} \int_{6.37}^{6.87} \left(\frac{1}{10^6 u} - \frac{1}{6.87 \times 10^6} \right)^{-1/2} 10^6\, du$$

$$= 3.541 \times 10^{-8} \frac{10^6}{\left(10^6\right)^{-1/2}} \int_{6.37}^{6.87} \left(\frac{1}{u} - \frac{1}{6.87} \right)^{-1/2} du$$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall

$$\Delta t = 3.541 \times 10^{-8} \times 10^9 \times 9.596 = 339.8 = \boxed{340\ \text{s}}$$

P13.17 (a) Since the particles are located at the corners of an equilateral triangle, the distances between all particle pairs is equal to 0.300 m. The gravitational potential energy of the system is then

$$U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1 m_2}{r_{12}} \right)$$

$$U_{\text{Tot}} = -\frac{3\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.00 \times 10^{-3}\ \text{kg}\right)^2}{0.300\ \text{m}}$$

$$= \boxed{-1.67 \times 10^{-14}\ \text{J}}$$

(b)

$$\boxed{\text{Each particle feels a net force of attraction toward the midpoint between the other two. Each moves toward the center of the triangle with the same acceleration. They collide simultaneously at the center of the triangle.}}$$

Section 13.6 Energy Considerations in Planetary and Satellite Motion

P13.18 To obtain the orbital velocity, we use

$$\sum F = \frac{mMG}{R^2} = \frac{mv^2}{R}$$

or $$v = \sqrt{\frac{MG}{R}}$$

We can obtain the escape velocity from

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{mMG}{R}$$

or $$v_{\text{esc}} = \sqrt{\frac{2MG}{R}} = \boxed{\sqrt{2}v}$$

P13.19 To determine the energy transformed to internal energy, we begin by calculating the change in kinetic energy of the satellite. To find the initial kinetic energy, we use

$$\frac{v_i^2}{R_E + h} = \frac{GM_E}{(R_E + h)^2}$$

which gives

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{GM_E m}{R_E + h}\right)$$
$$= \frac{1}{2}\left[\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(500\ \text{kg})}{6.37\times10^{6}\ \text{m} + 0.500\times10^{6}\ \text{m}}\right]$$
$$= 1.45\times10^{10}\ \text{J}$$

Also, $\quad K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(500\ \text{kg})(2.00\times10^{3}\ \text{m/s})^2 = 1.00\times10^{9}\ \text{J}.$

The change in gravitational potential energy of the satellite-Earth system is

$$\Delta U = \frac{GM_E m}{R_i} - \frac{GM_E m}{R_f} = GM_E m\left(\frac{1}{R_i} - \frac{1}{R_f}\right)$$
$$= (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(500\ \text{kg}) \times(-1.14\times10^{-8}\ \text{m}^{-1})$$
$$= -2.27\times10^{9}\ \text{J}$$

The energy transformed into internal energy due to friction is then

$$\Delta E_{\text{int}} = K_i - K_f - \Delta U = (14.5 - 1.00 + 2.27)\times10^{9}\ \text{J}$$
$$= \boxed{1.58\times10^{10}\ \text{J}}$$

P13.20 For a satellite in an orbit of radius r around the Earth, the total energy of the satellite-Earth system is $E = -\frac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r = 2R_E$ to one of radius $r = 3R_E$, the required work is

$$W = \Delta E = -\frac{GM_E m}{2r_f} + \frac{GM_E m}{2r_i} = GM_E m\left[\frac{1}{4R_E} - \frac{1}{6R_E}\right] = \boxed{\frac{GM_E m}{12R_E}}$$

P13.21 For her jump on Earth,

$$\frac{1}{2}mv_i^2 = mgy_f \qquad \textbf{[1]}$$

which gives

$$v_i = \sqrt{2gy_f} = \sqrt{2(9.80\ \text{m/s})(0.500\ \text{m})} = 3.13\ \text{m/s}$$

We assume that she has the same takeoff speed on the asteroid. Here

$$\frac{1}{2}mv_i^2 - \frac{GM_A m}{R_A} = 0 + 0 \qquad \textbf{[2]}$$

The equality of densities between planet and asteroid,

$$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{M_A}{\frac{4}{3}\pi R_A^3}$$

implies

$$M_A = \left(\frac{R_A}{R_E}\right)^3 M_E \qquad \textbf{[3]}$$

Note also at Earth's surface

$$g = \frac{GM_E}{R_E^2} \qquad \textbf{[4]}$$

Combining the equations [2], [1], [3], and [4] by substitution gives

$$\frac{1}{2}v_i^2 = \frac{GM_A}{R_A}$$

$$\frac{1}{2}\left(2gy_f\right) = \frac{G}{R_A}\left(\frac{R_A}{R_E}\right)^3 M_E$$

$$\frac{GM_E}{R_E^2}y_f = \frac{GM_E R_A^2}{R_E^3}$$

$$R_A^2 = y_f R_E = (0.500\ \text{m})\left(6.37\times10^6\ \text{m}\right)$$

$$R_A = \boxed{1.78\times10^3\ \text{m}}$$

P13.22 (a) The escape velocity from the solar system, starting at Earth's orbit, is given by

$$v_{\text{solar escape}} = \sqrt{\frac{2M_{\text{Sun}}G}{R_{\text{Sun}}}}$$
$$= \sqrt{\frac{2(1.99\times10^{30}\text{ kg})(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)}{1.50\times10^{9}\text{ m}}}$$
$$= \boxed{42.1\text{ km/s}}$$

(b) Let x represent the variable distance from the Sun. Then,

$$v = \sqrt{\frac{2M_{\text{Sun}}G}{x}} \quad\rightarrow\quad x = \frac{v^2}{2M_{\text{Sun}}G}$$

If $v = \dfrac{125\,000\text{ km}}{3\,600\text{ s}} = 34.7\text{ m/s}$, then

$$x = \frac{v^2}{2M_{\text{Sun}}G} = \frac{(34.7\text{ m/s})^2}{2(1.99\times10^{30}\text{ kg})(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)}$$
$$= \boxed{2.20\times10^{11}\text{ m}}$$

Note that at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape.

P13.23 (a) Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket.

(b) The rocket has a gravitational potential energy with respect to Ganymede

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2)m_2(1.495\times10^{23}\text{ kg})}{(2.64\times10^{6}\text{ m})\text{kg}^2}$$
$$U_1 = (-3.78\times10^{6}\text{ m}^2/\text{s}^2)m_2$$

The rocket's gravitational potential energy with respect to

Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\right)m_2\left(1.90\times10^{27}\ \text{kg}\right)}{\left(1.071\times10^{9}\ \text{m}\right)\text{kg}^2}$$

$$U_2 = \left(-1.18\times10^{8}\ \text{m}^2/\text{s}^2\right)m_2$$

To escape from both requires

$$\frac{1}{2}m_2v_{\text{esc}}^2 = +\left[\left(3.78\times10^{6} + 1.18\times10^{8}\right)\ \text{m}^2/\text{s}^2\right]m_2$$

$$v_{\text{esc}} = \sqrt{2\times\left(1.22\times10^{8}\ \text{m}^2/\text{s}^2\right)} = \boxed{15.6\ \text{km/s}}$$

Additional Problems

P13.24 (a) When the rocket engine shuts off at an altitude of 250 km, we may consider the rocket to be beyond Earth's atmosphere. Then, its mechanical energy will remain constant from that instant until it comes to rest momentarily at the maximum altitude. That is, $KE_f + PE_f = KE_i + PE_i$, or

$$0 - \frac{GM_E\cancel{m}}{r_{\max}} = \frac{1}{2}\cancel{m}v_i^2 - \frac{GM_E\cancel{m}}{r_i} \qquad \text{or} \qquad \frac{1}{r_{\max}} = -\frac{v_i^2}{2GM_E} + \frac{1}{r_i}$$

With $r_i = R_E + 250\ \text{km} = 6.37\times10^6\ \text{m} + 250\times10^3\ \text{m} = 6.62\times10^6\ \text{m}$

and $v_i = 6.00\ \text{km/s} = 6.00\times10^3\ \text{m/s}$, this gives

$$\frac{1}{r_{\max}} = -\frac{\left(6.00\times10^{3}\ \text{m/s}\right)^2}{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)} + \frac{1}{6.62\times10^{6}\ \text{m}}$$
$$= 1.06\times10^{-7}\ \text{m}^{-1}$$

or $r_{\max} = 9.44\times10^6$ m. The maximum distance from Earth's surface is then

$$h_{max} = r_{max} - R_E = 9.44 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m} = \boxed{3.07 \times 10^6 \text{ m}}$$

(b) If the rocket were fired from a launch site on the equator, it would have a significant eastward component of velocity because of the Earth's rotation about its axis. Hence, compared to being fired from the South Pole, the rocket's initial speed would be greater, and $\boxed{\text{the rocket would travel farther from Earth}}$.

P13.25 To approximate the height of the sulfur, set $\dfrac{mv^2}{2} = mg_{Io}h$, with h = 70 000 m and $g_{Io} = \dfrac{GM}{r^2} = 1.79 \text{ m/s}^2$. This gives

$$v = \sqrt{2g_{Io}h} = \sqrt{2(1.79 \text{ m/s}^2)(70\,000 \text{ m})}$$
$$\approx 500 \text{ m/s} \ (\text{over } 1\,000 \text{ mi/h})$$

We can obtain a more precise answer from conservation of energy:

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2 = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(8.90 \times 10^{22} \text{ kg}) \times \left(\frac{1}{1.82 \times 10^6 \text{ m}} - \frac{1}{1.89 \times 10^6 \text{ m}}\right)$$

$$v = \boxed{492 \text{ m/s}}$$

***P13.26** **Conceptualize** Imagine an Earth-based example of material being flung from a rapidly rotating object, such as water from a truck tire in front of you while you drive on a rainy day. Many rotating amusement park rides have some structure behind you to provide centripetal acceleration so that *you* are not flung away from the ride!

Categorize We will model a small amount of material on the equator of a neutron star as a *particle in uniform circular motion.* We will also model the material as a *particle in a gravitational field.*

Analyze Let's set up a ratio of the acceleration $g_{\text{neutron star}}$ due to gravity of a bit of material at the surface of the neutron star to the centripetal acceleration a_c of that bit of material:

$$\frac{g_{\text{neutron star}}}{a_c} = \frac{G\dfrac{M_{\text{neutron star}}}{r^2_{\text{neutron star}}}}{r_{\text{neutron star}}\omega^2} = G\frac{M_{\text{neutron star}}}{r^3_{\text{neutron star}}\left(\dfrac{2\pi}{T}\right)^2} = G\frac{M_{\text{neutron star}}T^2}{4\pi^2 r^3_{\text{neutron star}}} \quad (1)$$

where we have used a neutron-star version of Equation 13.6 in the numerator and Equation 10.12 in the denominator. Evaluate this ratio for the neutron star with a typical radius and shortest period as described in the problem, using the midpoint of the mass range given:

$$\frac{g_{\text{neutron star}}}{a_c} = \left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\ \text{kg}^2\right)\frac{2\left(1.99 \times 10^{30}\ \text{kg}\right)\left(1.4 \times 10^{-3}\ \text{s}\right)^2}{4\pi^2\left(10 \times 10^3\ \text{m}\right)^3}$$

$$= 13.2$$

Therefore, even for the fastest-observed rotation of a neutron star, the value of $g_{\text{neutron star}}$ is 13.2 times as large as a_c. Therefore, the material on the equator of the rotating neutron star is nowhere near a situation where it is being flung off the surface.

Finalize We determined the ratio for the fastest-spinning neutron star. In Equation (1), the ratio is proportional to T^2. Therefore, for a neutron star spinning with a period of about 1 s, the ratio grows to on the order of seven orders of magnitude. Therefore, material is definitely not flung off the surface of a neutron star.

Answer: Mass of neutron star required is seven orders of magnitude less than the mass of the Sun.

***P13.27** **Conceptualize** By giving the golf ball more energy than it had when sitting on the space station, you have sent it into an elliptical orbit with a semimajor axis larger than the radius of its initial circular orbit. According to Kepler's third law, the larger semimajor axis corresponds to a longer period. You have hit the ball in just the right way that it comes back to after you have completed exactly n orbits.

Categorize Both the space station and the golf ball are in orbit around the Earth, so we use the orbital dynamics material we have learned in this chapter.

Analyze Based on the fact that you have completed an integral number of orbits when the ball returns, we have

$$\frac{T_b}{T_s} = n \qquad (1)$$

where b refers to the golf ball and s refers to the space station. We can use Kepler's third law to find a relationship between the semimajor axes of the ball and space station. From Equation 13.11,

$$T^2 = K_E a^3 \quad \rightarrow \quad \frac{a_b}{a_s} = \left(\frac{T_b}{T_s}\right)^{2/3} = n^{2/3} \quad \rightarrow \quad a_b = n^{2/3} a_s \qquad (2)$$

The energy of the golf-ball–Earth system after the ball is hit is, from Equation 13.20,

$$E_b = -G\frac{M_E m}{2a_b} \qquad (3)$$

We can also express this energy as the sum of the kinetic energy of the ball and the potential energy of the system, as in Equation 13.16:

$$E_b = \tfrac{1}{2}mv^2 - G\frac{M_E m}{h+R_E} \qquad (4)$$

Set these two expressions equal and noting that $a_s = h + R_E$, we can solve for the speed of the ball after it is hit:

$$-G\frac{M_E m}{2a_b} = \tfrac{1}{2}mv^2 - G\frac{M_E m}{a_s} \quad \rightarrow \quad v = \sqrt{GM_E\left(\frac{2}{a_s} - \frac{1}{a_b}\right)} \qquad (5)$$

Now incorporate Equation (2):

$$v = \sqrt{GM_E\left(\frac{2}{a_s} - \frac{1}{n^{2/3}a_s}\right)} = \sqrt{\frac{GM_E}{a_s}\left(2 - \frac{1}{n^{2/3}}\right)} = \sqrt{\frac{GM_E}{R_E+h}\left(2 - \frac{1}{n^{2/3}}\right)} \qquad (6)$$

This speed is relative to the *Earth*. The problem statement asked for the speed relative to the space station. Therefore, we must subtract the initial speed of the ball when it was just sitting on the station, which was found in Example 13.5:

$$v_{\text{rel}} = v - v_s = \sqrt{\frac{GM_E}{R_E+h}\left(2 - \frac{1}{n^{2/3}}\right)} - \sqrt{\frac{GM_E}{R_E+h}} = \sqrt{\frac{GM_E}{R_E+h}}\left(\sqrt{2 - \frac{1}{n^{2/3}}} - 1\right) \qquad (7)$$

Substitute numerical values, noting that $n = 2.00$ from the problem statement:

$$v_{\text{rel}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\ \text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{6.37\times10^{6}\ \text{m}+5.00\times10^{5}\ \text{m}}}\left(\sqrt{2-\frac{1}{(2.00)^{2/3}}}-1\right)$$

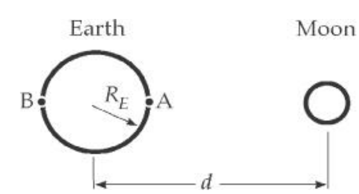
$$= \boxed{1.30\times10^{3}\ \text{m/ s}}$$

Finalize As noted in the problem statement, this is a very large speed. Let's look at what happens if the number of orbits you make is larger than $n = 2.00$ before the ball returns. In Equation (7), if n increases, convince yourself that v_{rel} also increases. That makes sense: the harder you hit the ball, the larger elliptical orbit it goes into and the more orbits you will make before the ball returns to you. For example, if you want to make $n = 10$ orbits before the ball returns, you must project it at 2.56×10^3 m/s. Can n be smaller than 2.00? The only integer smaller than 2.00 is 1.00. If you set $n = 1.00$ in Equation 7, the result is $v_{\text{rel}} = 0$. That makes sense. If it returns to you after you make one orbit, it was with you the entire time of the orbit: you didn't hit it!

Answer: 1.30×10^3 m/s

P13.28 If one uses the result $v = \sqrt{\frac{GM}{r}}$ and the relation $v = (2\pi r/T)$, one finds the radius of the orbit to be smaller than the radius of the Earth, so the spacecraft would need to be in orbit underground.

P13.29 The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is



ANS. FIG. P13.29

given by $a = G\dfrac{M_M}{d^2}$.

At the point A nearest the Moon,

$$a_+ = G\frac{M_M}{\left(d - R_E\right)^2}$$

At the point B farthest from the Moon,

$$a_- = G\frac{M_M}{\left(d + R_E\right)^2}$$

From the above, we have

$$\frac{\Delta g_M}{g} = \frac{\left(a_+ - a_-\right)}{g} = \frac{GM_M}{g}\left[\frac{1}{\left(d - R_E\right)^2} - \frac{1}{\left(d + R_E\right)^2}\right]$$

Evaluating this expression, we find across the planet

$$\begin{aligned}\frac{\Delta g_M}{g} &= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\ \text{kg}\right)}{9.80\ \text{m/s}^2} \\ &\quad\times\left[\frac{1}{\left(3.84\times10^{8}\ \text{m} - 6.37\times10^{6}\ \text{m}\right)^2} - \frac{1}{\left(3.84\times10^{8}\ \text{m} + 6.37\times10^{6}\ \text{m}\right)^2}\right] \\ &= \boxed{2.25\times10^{-7}}\end{aligned}$$

P13.30 (a) The only force acting on the astronaut is the normal force exerted on him by the "floor" of the cabin. The normal force supplies the centripetal force:

$$F_c = \frac{mv^2}{r} \qquad \text{and} \qquad n = \frac{mg}{2}$$

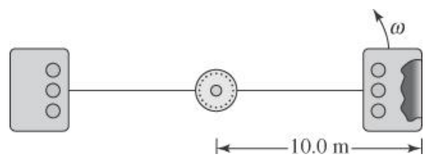
This gives

$$\frac{mv^2}{r} = \frac{mg}{2} \rightarrow v = \sqrt{\frac{gr}{2}}$$

$$v = \sqrt{\frac{(9.80\,\text{m/s}^2)(10.0\,\text{m})}{2}} \quad \rightarrow v = 7.00\text{ m/s}$$

Since $v = r\omega$, we have

$$\omega = \frac{v}{r} = \frac{7.00\text{ m/s}}{10.0\text{ m}} = \boxed{0.700\text{ rad/s}}$$



ANS. FIG. P13.30

(b) Because his feet stay in place on the floor, his head will be moving at the same tangential speed as his feet. However, his feet and his head are travelling in circles of different radii.

(c) If he stands up without holding on to anything with his hands, the only force on his body is radial. Because the wall of the cabin near the traveler's head moves in a smaller circle, it moves at a slower tangential speed than that of the traveler's head so his head moves toward the wall—if he is not careful, there could be a collision. This is an example of the Coriolis force investigated in Section 6.3. Holding onto to a rigid support with his hands will provide a tangential force to the traveler to slow the upper part of his body down.

P13.31 (a) Ignoring air resistance, the energy conservation for the object-Earth system from firing to apex is given by,

$$\left(K + U_g\right)_i = \left(K + U_g\right)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = 0 - \frac{GmM_E}{R_E + h}$$

where $\frac{1}{2}mv_{\text{esc}}^2 = \frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2 - \frac{1}{2}v_{\text{esc}}^2 = -\frac{1}{2}v_{\text{esc}}^2 \frac{R_E}{R_E + h}$$

$$v_{\text{esc}}^2 - v_i^2 = \frac{v_{\text{esc}}^2 R_E}{R_E + h}$$

$$\frac{1}{v_{\text{esc}}^2 - v_i^2} = \frac{R_E + h}{v_{\text{esc}}^2 R_E}$$

$$h = \frac{v_{\text{esc}}^2 R_E}{v_{\text{esc}}^2 - v_i^2} - R_E = \frac{v_{\text{esc}}^2 R_E - v_{\text{esc}}^2 R_E + v_i^2 R_E}{v_{\text{esc}}^2 - v_i^2}$$

$$h = \frac{R_E v_i^2}{v_{\text{esc}}^2 - v_i^2}$$

$$h = \frac{(6.37\times10^6\text{ m})(8.76\text{ km/s})^2}{(11.2\text{ km/s})^2 - (8.76\text{ km/s})^2} = \boxed{1.00\times10^7\text{ m}}$$

(b) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation:

$$v_i^2 = v_{\text{esc}}^2\left(1 - \frac{R_E}{R_E + h}\right)$$

$$= v_{\text{esc}}^2\left(\frac{h}{R_E + h}\right)$$

$$= (11.2\times10^3\text{ m/s})^2\left(\frac{2.51\times10^7\text{ m}}{6.37\times10^6\text{ m} + 2.51\times10^7\text{ m}}\right)$$

$$= 1.00\times10^8\text{ m}^2/\text{s}^2$$

$$v_i = \boxed{1.00\times10^4\text{ m/s}}$$

P13.32 (a) Ignoring air resistance, the energy conservation for the object-Earth system from firing to apex is given by,

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f$$

$$\frac{1}{2}mv_i^2-\frac{GmM_E}{R_E}=0-\frac{GmM_E}{R_E+h}$$

where $\frac{1}{2}mv_{\text{esc}}^2=\frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2-\frac{1}{2}v_{\text{esc}}^2=-\frac{1}{2}v_{\text{esc}}^2\frac{R_E}{R_E+h}$$

$$v_{\text{esc}}^2-v_i^2=\frac{v_{\text{esc}}^2R_E}{R_E+h}$$

$$\frac{1}{v_{\text{esc}}^2-v_i^2}=\frac{R_E+h}{v_{\text{esc}}^2R_E}$$

$$h=\frac{v_{\text{esc}}^2R_E}{v_{\text{esc}}^2-v_i^2}-R_E=\frac{v_{\text{esc}}^2R_E-v_{\text{esc}}^2R_E+v_i^2R_E}{v_{\text{esc}}^2-v_i^2}$$

$$\boxed{h=\frac{R_Ev_i^2}{v_{\text{esc}}^2-v_i^2}}$$

(b) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation. From (a) above, replacing v_i with v_f, we have

$$v_f^2=v_{\text{esc}}^2-v_{\text{esc}}^2\frac{R_E}{R_E+h}$$

$$v_f^2=v_{\text{esc}}^2\left(1-\frac{R_E}{R_E+h}\right)$$

$$\boxed{v_f=v_{\text{esc}}\sqrt{\frac{h}{R_E+h}}}$$

(c) With $v_i \ll v_{\text{esc}}$, $h\approx\frac{R_Ev_i^2}{v_{\text{esc}}^2}=\frac{R_Ev_i^2R_E}{2GM_E}$. But $g=\frac{GM_E}{R_E^2}$, so $h=\frac{v_i^2}{2g}$, in agreement with $0^2=v_i^2+2(-g)(h-0)$.

P13.33 (a) Let R represent the radius of the asteroid. Then its volume is $\frac{4}{3}\pi R^3$ and its mass is $\rho\frac{4}{3}\pi R^3$. For your orbital motion, $\sum F = ma$ gives

$$\frac{Gm_1m_2}{R^2} = \frac{m_2v^2}{R} \quad \rightarrow \quad \frac{G\rho 4\pi R^3}{3R^2} = \frac{v^2}{R}$$

solving for R,

$$\begin{aligned} R &= \left(\frac{3v^2}{G\rho 4\pi}\right)^{1/2} \\ &= \left[\frac{3(8.50\ \text{m/s})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1\,100\ \text{kg/m}^3)4\pi}\right]^{1/2} \\ &= \boxed{1.53\times10^4\ \text{m}} \end{aligned}$$

(b) $\rho\frac{4}{3}\pi R^3 = (1\,100\ \text{kg/m}^3)\frac{4}{3}\pi(1.53\times10^4\ \text{m})^3 = \boxed{1.66\times10^{16}\ \text{kg}}$

(c) $v = \dfrac{2\pi R}{T} \quad T = \dfrac{2\pi R}{v} = \dfrac{2\pi(1.53\times10^4\ \text{m})}{8.5\ \text{m/s}} = \boxed{1.13\times10^4\ \text{s}} = 3.15\ \text{h}$

(d) For an illustrative model, we take your mass as 90.0 kg and assume the asteroid is originally at rest. Angular momentum is conserved for the asteroid-you system:

$$\begin{aligned} &\sum L_i = \sum L_f \\ &0 = m_2vR - I\omega \\ &0 = m_2vR - \frac{2}{5}m_1R^2\frac{2\pi}{T_{\text{asteroid}}} \\ &m_2v = \frac{4\pi}{5}\frac{m_1R}{T_{\text{asteroid}}} \\ &T_{\text{asteroid}} = \frac{4\pi m_1R}{5m_2v} = \frac{4\pi(1.66\times10^{16}\ \text{kg})(1.53\times10^4\ \text{m})}{5(90.0\ \text{kg})(8.50\ \text{m/s})} \\ &\qquad = 8.37\times10^{17}\ \text{s} = 26.5\ \text{billion years} \end{aligned}$$

$\boxed{\text{Thus your running does not produce significant rotation of the asteroid if it is originally stationary and does not significantly affect any rotation it does have.}}$

This problem is realistic. Many asteroids, such as Ida and Eros, are roughly 30 km in diameter. They are typically irregular in shape and not spherical. Satellites such as Phobos (of Mars), Adrastea (of Jupiter), Calypso (of Saturn), and Ophelia (of Uranus) would allow a visitor the same experience of easy orbital motion. So would many Kuiper Belt objects.

P13.34 (a) $\boxed{\text{The two appropriate isolated system models are conservation of momentum and conservation of energy applied to the system consisting of the two spheres.}}$

(b) Applying conservation of momentum to the system, we find

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 + 0 = M\vec{\mathbf{v}}_{1f} + 2M\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{1f} = \boxed{-2\vec{\mathbf{v}}_{2f}}$$

(c) Applying conservation of energy to the system, we find

$$K_i + U_i + \Delta E = K_f + U_f$$

$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)v_{2f}^2 - \frac{GM(2M)}{4R}$$

$$\frac{1}{2}Mv_{1f}^2 = \frac{GM}{2R} - \frac{GM}{6R} - v_{2f}^2$$

$$v_{1f} = \boxed{\sqrt{\frac{2GM}{3R} - 2v_{2f}^2}}$$

(d) Combining the results for parts (b) and (c),

$$2v_{2f} = \sqrt{\frac{2GM}{3R} - 2v_{2f}^2}$$

$$6v_{2f}^2 = \frac{2GM}{3R}$$

$$v_2 = \boxed{\frac{1}{3}\sqrt{G\frac{M}{R}}} \qquad v_1 = \boxed{\frac{2}{3}\sqrt{G\frac{M}{R}}}$$

P13.35 (a) The free-fall acceleration produced by the Earth is

$$g = \frac{GM_E}{r^2} = GM_E r^{-2} \text{ (directed downward)}$$

Its rate of change is

$$\frac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$$

The minus sign indicates that g decreases with increasing height.

At the Earth's surface,

$$\boxed{\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}}$$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3}$$

Thus,

$$\boxed{|\Delta g| = \frac{2GM_E h}{R_E^3}}$$

(c) $$|\Delta g| = \frac{2\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)(6.00 \text{ m})}{\left(6.37 \times 10^6 \text{ m}\right)^3}$$

$$= \boxed{1.85 \times 10^{-5} \text{ m/s}^2}$$

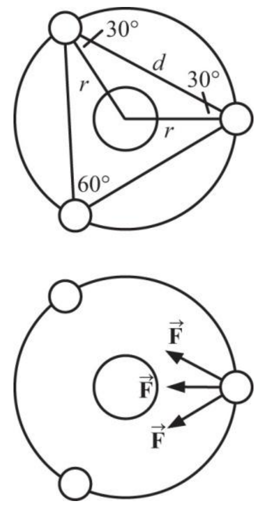
P13.36 The distance between the orbiting stars is

$d = 2r\cos 30° = \sqrt{3}r$ since $\cos 30° = \dfrac{\sqrt{3}}{2}$. The net inward force on one orbiting star is

$$\frac{Gmm}{d^2}\cos 30° + \frac{GMm}{r^2} + \frac{Gmm}{d^2}\cos 30° = \frac{mv^2}{r}$$

$$\frac{Gm2\cos 30°}{3r^2} + \frac{GM}{r^2} = \frac{4\pi^2 r^2}{rT^2}$$

$$G\left(\frac{m}{\sqrt{3}} + M\right) = \frac{4\pi^2 r^3}{T^2}$$



ANS. FIG. P13.36

solving for the period gives

$$T^2 = \frac{4\pi^2 r^3}{G\left(M + m/\sqrt{3}\right)}$$

$$T = 2\pi\left(\frac{r^3}{G\left(M + m/\sqrt{3}\right)}\right)^{1/2}$$

P13.37 (a) We find the period from

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(30\,000 \times 9.46 \times 10^{15}\text{ m}\right)}{2.50 \times 10^5\text{ m/s}} = 7 \times 10^{15}\text{ s}$$
$$= \boxed{2 \times 10^8\text{ yr}}$$

(b) We estimate the mass of the Milky Way from

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2\left(30\,000 \times 9.46 \times 10^{15}\text{ m}\right)^3}{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.13 \times 10^{15}\text{ s}\right)^2},$$
$$= 2.66 \times 10^{41}\text{ kg}$$

or $\boxed{\text{about } 10^{41}\text{ kg}}$

Note that this is the mass of the galaxy contained within the Sun's orbit of the galactic center. Recent studies show that the true mass of the galaxy, including an extended halo of dark matter, is at least an order of magnitude larger than our estimate.

(c) A solar mass is about 1×10^{30} kg: $10^{41}/10^{30} = 10^{11}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$.

P13.38 Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$
$$Gm\left(\frac{1}{2r} - \frac{1}{R}\right) = v^2 \quad \rightarrow \quad v = \left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2}$$

(a) The injected momentum is the final momentum of each sphere,

$$mv = m^{2/2}\left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2} = \boxed{\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

P13.39 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved. We use this to find the speed at aphelion:

$$mr_a v_a = mr_p v_p$$

and

$$v_a = v_p\left(\frac{r_p}{r_a}\right) = \left(3.027\times10^4 \text{ m/s}\right)\left(\frac{1.471}{1.521}\right) = \boxed{2.93\times10^4 \text{ m/s}}$$

(b) $K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}\left(5.98\times10^{24} \text{ kg}\right)\left(3.027\times10^4 \text{ m/s}\right)^2 = \boxed{2.74\times10^{33} \text{ J}}$

$$\begin{aligned} U_p &= -\frac{GmM}{r_p} \\ &= -\frac{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24} \text{ kg}\right)\left(1.99\times10^{30} \text{ kg}\right)}{1.471\times10^{11} \text{ m}} \\ &= \boxed{-5.40\times10^{33} \text{ J}} \end{aligned}$$

(c) Using the same form as in part (b),

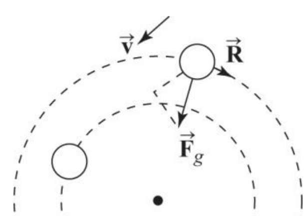
$$K_a = \boxed{2.57\times10^{33} \text{ J}} \text{ and } U_a = \boxed{-5.22\times10^{33} \text{ J}}$$

(d) Compare to find that

$$K_p + U_p = \boxed{-2.66\times10^{33} \text{ J}} \text{ and } K_a + U_a = \boxed{-2.65\times10^{33} \text{ J}}.$$

$\boxed{\text{They agree,}}$ with a small rounding error.

P13.40 For both circular orbits,



ANS. FIG. P13.40

$$\sum F = ma: \quad \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

(a) The original speed is

$$\begin{aligned} v_i &= \sqrt{\frac{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24} \text{ kg}\right)}{6.37\times10^6 \text{ m} + 2.00\times10^5 \text{ m}}} \\ &= \boxed{7.79\times10^3 \text{ m/s}} \end{aligned}$$

(b) The final speed is

$$v_i = \sqrt{\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})}{6.47\times10^{6}\ \mathrm{m}}}$$
$$= \boxed{7.85\times10^{3}\ \mathrm{m/s}}$$

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E}{r} = -\frac{GM_E m}{2r}$$

(c) Originally,

$$E_i = -\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})(100\ \mathrm{kg})}{2(6.57\times10^{6}\ \mathrm{m})}$$
$$= \boxed{-3.04\times10^{9}\ \mathrm{J}}$$

(d) Finally,

$$E_f = -\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})(100\ \mathrm{kg})}{2(6.47\times10^{6}\ \mathrm{m})}$$
$$= \boxed{-3.08\times10^{9}\ \mathrm{J}}$$

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04\times10^{9}\ \mathrm{J} - (-3.08\times10^{9}\ \mathrm{J}) = \boxed{4.69\times10^{7}\ \mathrm{J}}$$

(f) The only forces on the object are the backward force of air resistance R, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force,

$\boxed{\text{one component of the gravitational force pulls forward on the satellite}}$

to do positive work and make its speed increase.

P13.41 (a) The gravitational force exerted on m by the Earth (mass M_E) accelerates m according to $g_2 = \frac{GM_E}{r^2}$. The equal-magnitude force exerted on the Earth by m produces acceleration of the Earth given by $g_1 = \frac{Gm}{r^2}$. The acceleration of relative approach is then

$$\begin{aligned} g_2 + g_1 &= \frac{Gm}{r^2} + \frac{GM_E}{r^2} \\ &= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg} + m\right)}{\left(1.20\times10^{7}\ \text{m}\right)^2} \\ &= \boxed{\left(2.77\ \text{m/s}^2\right)\left(1 + \frac{m}{5.98\times10^{24}\ \text{kg}}\right)} \end{aligned}$$

(b) and (c) Here $m = 5$ kg and $m = 2000$ kg are both negligible compared to the mass of the Earth, so the acceleration of relative approach is just $\boxed{2.77\ \text{m/s}^2}$.

(d) Substituting $m = 2.00 \times 10^{24}$ kg into the expression for $(g_1 + g_2)$ above gives

$$g_1 + g_2 = \boxed{3.70\ \text{m/s}^2}$$

(e) $\boxed{\text{Any object with mass small compared to the Earth starts to fall with acceleration } 2.77\ \text{m/s}^2. \text{ As } m \text{ increases to become comparable to the mass of the Earth, the acceleration increases, and can become arbitrarily large. It approaches a direct proportionality to } m.}$

P13.42 From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Problem 38 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius R of the planet.

$$\sum F = ma: \quad \frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$G\rho V = \frac{R^2\left(4\pi^2 R^2\right)}{RT^2}$$

$$G\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi \quad \rightarrow \quad \boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

P13.43 For the Earth, $\sum F = ma: \quad \frac{GM_s m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$

Then $\quad GM_s T^2 = 4\pi^2 r^3$

Also, the angular momentum $L = mvr = m\frac{2\pi r}{T}r$ is a constant for the Earth. We eliminate $r = \sqrt{\frac{LT}{2\pi m}}$ between the equations:

$$GM_s T^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \quad \text{gives} \quad GM_s T^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}$$

Now the rates of change with time t are described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0$$

or

$$\frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \approx \frac{\Delta T}{\Delta t}$$

which gives

$$\Delta T \approx -\Delta t \frac{dM_s}{dt}\left(2\frac{T}{M_s}\right)$$

$$= -(5\,000 \text{ yr})\left(\frac{3.16\times 10^{7} \text{ s}}{1 \text{ yr}}\right)\left(-3.64\times 10^{9} \text{ kg/s}\right) \times\left(2\frac{1 \text{ yr}}{1.99\times 10^{30} \text{ kg}}\right)$$

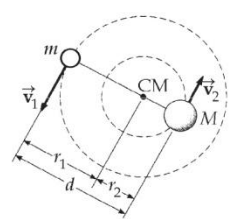
$$\Delta T = \boxed{5.78\times 10^{-10} \text{ s}}$$

P13.44 If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M+m} \qquad \text{and} \qquad Mr_2 = mr_1$$

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

$$mr_1\omega_1^2 = \frac{MGm}{d^2} \qquad \text{and} \qquad Mr_2\omega_2^2 = \frac{MGm}{d^2}$$



ANS. FIG. P13.44

Combining these two equations and using $d = r_1 + r_2$ gives

$$(r_1 + r_2)\omega^2 = \frac{(M+m)G}{d^2} \text{ with}$$

$$\omega_1 = \omega_2 = \omega$$

and

$$T = \frac{2\pi}{\omega}$$

we find

$$\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$$

Challenge Problem

P13.45 Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of

$$F_s = \frac{GM_s m}{(r_E - x)^2}$$

while the Earth exerts on it a radial outward force of

$$F_E = \frac{GM_E m}{x^2}$$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus, $F_S - F_E = \dfrac{GM_S m}{(r_E - x)^2} - \dfrac{GM_E m}{x^2} = \dfrac{mv^2}{(r_E - x)} = \dfrac{m}{(r_E - x)}\left[\dfrac{2\pi(r_E - x)}{T}\right]^2$

which reduces to $$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad \textbf{[1]}$$

Cleared of fractions, this equation would contain powers of x ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that x is 1.48×10^9 m by substituting it into the equation, along with the following data: $M_S = 1.99 \times 10^{30}$ kg,

$M_E = 5.974 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000$ yr $= 3.156 \times 10^7$ s.

With $x = 1.48 \times 10^9$ m, the result is

$$6.053 \times 10^{-3}\ \text{m/s}^2 - 1.82 \times 10^{-3}\ \text{m/s}^2 \approx 5.870\,8 \times 10^{-3}\ \text{m/s}^2$$

or $$5.870\,9 \times 10^{-3}\ \text{m/s}^2 \approx 5.870\,8 \times 10^{-3}\ \text{m/s}^2$$

$\boxed{\text{To three-digit precision, the solution is } 1.48 \times 10^9\,\text{m.}}$

As an equation of fifth degree, equation [1] has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. The twin satellites of NASA's STEREO mission, giving three-dimensional views of the Sun from orbital positions ahead of and trailing Earth, passed through these Lagrange points in 2009. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

ANSWERS TO QUICK-QUIZZES

1. (e)

2. (c)

3. (a)

4. (a) perihelion (b) aphelion (c) perihelion (d) all points

ANSWERS TO EVEN-NUMBERED PROBLEMS

P13.2 (a) 4.39×10^{20} N; (b) 1.99×10^{20} N; (c) 3.55×10^{22} N; (d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon.

P13.4 The situation is impossible because no known element could compose the spheres.

P13.6 (a) $\dfrac{2MGr}{\left(r^2+a^2\right)^{3/2}}$ toward the center of mass; (b) At $r = 0$, the fields of the two objects are equal in magnitude and opposite in direction, to add to zero; (c) As $r \to 0, 2MGr\left(r^2+a^2\right)^{-3/2}$ approaches $2MG(0)/a^3 = 0$; (d) When r is much greater than a, the angles the field vectors make with the x axis become smaller. At very great distances, the field vectors are almost parallel to the axis; therefore they begin to look like the field vector from a single object of mass $2M$; (e) As r becomes much larger than a, the expression approaches $2MGr\left(r^2+0^2\right)^{-3/2} = 2MGr/r^3 = 2MG/r^2$ as required.

P13.8 1.50 h or 90.0 min

P13.10 (a) The particle does posses angular momentum because it is not headed straight for the origin. (b) Its angular momentum is constant. There are no identified outside influences acting on the object. (c) See P13.10(c) for full explanation.

P13.12 1.27

P13.14 (a) 6.02×10^{24} kg; (b) The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it.

P13.16 (a) See P13.16 for full description; (b) 340 s

P13.18 $\sqrt{2}v$

P13.20 $\dfrac{GM_E m}{12R_E}$

P13.22 (a) 42.1 km/s; (b) 2.20×10^{11} m

P13.24 (a) 3.07×10^{6} m; (b) the rocket would travel farther from Earth

P13.26 For the typical data provided for a neutron star, the gravitational acceleration is an order of magnitude larger than the centripetal acceleration

P13.28 If one uses the result $v = \sqrt{\dfrac{GM}{r}}$ and the relation $v = (2\pi r/T)$, one finds the radius of the orbit to be smaller than the radius of the Earth, so the spacecraft would need to be in orbit underground.

P13.30 (a) 0.700 rad/s; (b) Because his feet stay in place on the floor, his head will be moving at the same tangential speed as his feet. However, his feet and his head are travelling in circles of different radii; (c) If he's not careful, there could be a collision between his head and the wall (see P13.30 for full explanation)

P13.32 (a) $h = \dfrac{R_E v_i^2}{v_{esc}^2 - v_i^2}$; (b) $v_f = v_{esc}\sqrt{\dfrac{h}{R_E + h}}$; (c) With $v_1 << v_{esc}, h \approx \dfrac{R_E v_i^2}{v_{esc}^2} = \dfrac{R_E v_i^2 R_E}{2GM_E}$. But $g = \dfrac{GM_E}{R_E^2}$, so $h = \dfrac{v_i^2}{2g}$ in agreement with $0^2 = v_i^2 + 2(-g)(h - 0)$.

P13.34 (a) The two appropriate isolated system models are conservation of momentum and conservation of energy applied to the system consisting of the two spheres; (b) $-2\vec{\mathbf{v}}_{2f}$; (c) $\sqrt{\dfrac{2GM}{3R}-2v_{2f}^2}$;

(d) $v_2=\dfrac{1}{3}\sqrt{G\dfrac{M}{R}}$, $v_1=\dfrac{2}{3}\sqrt{G\dfrac{M}{R}}$

P13.36 See P13.36 for the full answer.

P13.38 (a) $mv=\left[GM^3\left(\dfrac{1}{2r}-\dfrac{1}{R}\right)\right]^{1/2}$; (b) $2\left[GM^3\left(\dfrac{1}{2r}-\dfrac{1}{R}\right)\right]^{1/2}$

P13.40 (a) 7.79×10^3 m/s; (b) 7.85×10^3 m/s; (c) -3.04×10^9 J; (d) -3.08×10^9 J; (e) 4.69×10^7 J; (f) one component of the gravitational force pulls forward on the satellite

P13.42 See P13.42 for full description.

P13.44 See P13.44 for full description.