

20

The Kinetic Theory of Gases

CHAPTER OUTLINE

- 20.1 Molecular Model of an Ideal Gas
- 20.2 Molar Specific Heat of an Ideal Gas
- 20.3 The Equipartition of Energy
- 20.4 Adiabatic Processes for an Ideal Gas
- 20.5 Distribution of Molecular Speeds

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

- *TP20.1 Conceptualize** It is indeed true that the kinetic energy of any moving baseball will be transformed to internal energy. Some of this transformation will be direct: the friction of the ball rolling across the ground, for example; the ball will be warmer, and the grass will be warmer. Some will have an intermediate step. For example, some of the kinetic energy of a baseball will be carried away from the catcher's mitt by sound. But this sound will be absorbed by surfaces within the stadium and then transformed to internal energy.

Categorize Let's assume that the air conditioning system is not running and that all vents to the outside are closed. We model the interior of the stadium as an *isolated system for energy*. Therefore, Equation 8.2 becomes

$$\Delta K + \Delta E_{\text{int}} = 0 \rightarrow \Delta E_{\text{int}} = -\Delta K \quad (1)$$

In Equation (1), ΔK is the total change in kinetic energy of all balls in motion throughout the entire game. On the left, ΔE_{int} is the total internal energy change in the interior of the stadium due to stopping all those balls.

Let's assume that v is the average speed of a baseball after being pitched, thrown, or hit. Let's also assume that the ball is pitched, thrown, or hit N times during a game. Therefore,

$$\Delta K = N\left(0 - \frac{1}{2}mv^2\right) = -\frac{1}{2}Nmv^2 \quad (2)$$

Substitute Equation (2) into Equation (1):

$$\Delta E_{\text{int}} = -\left(-\frac{1}{2}Nmv^2\right) = \frac{1}{2}Nmv^2 \quad (3)$$

Now, solve Equation 20.26 for the temperature change associated with a change in internal energy and substitute Equation (3):

$$\Delta E_{\text{int}} = nC_V\Delta T \rightarrow \Delta T = \frac{\Delta E_{\text{int}}}{nC_V} = \frac{Nmv^2}{2nC_V} \quad (4)$$

Because the air inside the stadium is diatomic, we have

$$C_V = \frac{5}{2}R \quad (5)$$

Substitute Equation (5) into Equation (4):

$$\Delta T = \frac{Nmv^2}{2n\left(\frac{5}{2}R\right)} = \frac{Nmv^2}{5nR} \quad (6)$$

Use the ideal gas law to replace nR in Equation (7):

$$\Delta T = \frac{Nmv^2T}{5PV} \quad (7)$$

Let's look at some typical numerical values. The mass of a baseball is about $m = 145$ g. A typical average speed of a pitched, thrown, or hit baseball might be 45 m/s. The pressure P is atmospheric pressure, the temperature T might be 293 K. The volume of a baseball stadium? Well, let's model it as a box 100 m on a side horizontally and 60 m to the roof. Therefore, $V = 6.0 \times 10^5$ m³. Now, what about N ? Well, we'll have to make some estimates:

Pitches: MLB statistics show an average of 150 pitches per team per game:

$$N_{\text{pitch}} = (150 / \text{team})(2 \text{ teams}) = 300$$

Hits: Assume nine hits per team:

$$N_{\text{hit}} = 16$$

Fly balls: Assume nine fly balls per team, plus two throws per fly ball back to infield and pitcher:

$$N_{\text{fly}} = 54$$

Ground Balls: Assume nine ground balls per team:

$$N_{\text{ground}} = 18$$

Throws to first: Assume each ground ball is thrown to first and then back to the pitcher:

$$N_{\text{first}} = 36$$

After each pitch, there will be another stop of the ball in the pitcher's glove after the catcher throws it back to him. Let's assume 300 throws minus the number of hits, fly balls, and ground balls, for which the catcher does not throw the ball back to the pitcher:

$$N_{\text{catcher}} = N_{\text{pitch}} - N_{\text{hit}} - N_{\text{fly}} - N_{\text{ground}} = 300 - 16 - 54 - 18 = 212$$

What else? Well, there's the players' throwing the ball to each other for practice before each inning starts. There are the warm-up pitches before each inning. There's all the activity in the bullpen, with many pitches thrown if a pitcher needs to be replaced. Suppose we add another 1 000 stops of the baseball:

$$N_{\text{other}} = 1\,000$$

Adding these all together, we obtain,

$$N = 1\,836$$

Considering that we have probably underestimated the number of times the ball is stopped, let's call the number of stops of the ball and even 2 000.

Substitute numerical values into Equation (7):

$$\Delta T = \frac{(2\,000)(0.145\text{ kg})(45\text{ m/s})^2(293\text{ K})}{5(1.013 \times 10^5\text{ Pa})(6.0 \times 10^5\text{ m}^3)} = 6 \times 10^{-4}\text{ }^{\circ}\text{C}$$

where we have kept only one significant figure because of all the estimates we made.

This temperature increase in the air of the stadium is miniscule. It is clearly not necessary to worry about the warming of the stadium due

to stopped baseballs. Even if our estimate of the number of stopped balls is way off, the temperature change is insignificant.

Finalize Another way to consider this situation would be to go back to Equation (3) and find the power necessary to remove the extra internal energy caused by the stopped baseballs:

$$P = \frac{\Delta E_{\text{int}}}{\Delta t} = \frac{Nmv^2}{2\Delta t}$$

Substitute numerical values, assuming the time interval is that for an entire baseball game, let's say $\Delta t = 3 \text{ h}$:

$$P = \frac{(2\,000)(0.145 \text{ kg})(45 \text{ m/s})^2}{2(3 \text{ h})} \left(\frac{1 \text{ h}}{3\,600 \text{ s}} \right) = 27 \text{ W}$$

Your hair dryer operates at about 1 000 W. Scale this up and imagine the power requirements of a baseball stadium air conditioner. The 27-W power input from stopped baseballs is miniscule.

The primary consideration of stadium air conditioners is the energy released from the *spectators*. The basal metabolic rate of a human is on the order of 100 W. Multiply this by tens of thousands of spectators, and you have a power of millions of watts to be transferred out of the stadium. And if it's an exciting game, excited people transform energy from their food supplies at an even faster rate than 100 W. Tell your architect that *this* is where he needs to focus his attention when designing the cooling system for the stadium.

Answer: Answers will vary.

***TP20.2 Conceptualize** The warmer planets will have a higher average speed for the helium molecules. The escape speed, however, will depend on the mass and radius of the planet.

Categorize This is a substitution problem with a number of calculations, so it would be easiest to perform by setting it up on a spreadsheet.

Analyze Using Equation 20.44 for the average speed and Equation 13.22 along with Table 13.2 for the escape speed, we find the following:

Object	Surface Temperature (K) (from the Lunar and Planetary Institute)	v_{avg} (km/s)	v_{esc} (km/s)	Ratio $v_{\text{esc}}/v_{\text{avg}}$
Mercury	440	1.53	4.25	2.78
Venus	741	1.99	10.4	5.22
Earth	288	1.24	11.2	9.03
Mars	244	1.14	5.02	4.41
Ceres	173	0.515	0.959	0.54
Jupiter	165	0.937	60.2	64.3
Saturn	134	0.844	36.1	42.7
Uranus	77	0.640	21.4	33.4
Neptune	70	0.610	23.5	38.5
Pluto	40	0.461	1.18	2.56

(a) The objects with little to no atmosphere are Mercury, Ceres, and Pluto. We see that the values of the ratio vary from 0.54 to 2.78. (b) The objects with robust atmospheres but no helium are Venus, Earth, and Mars. We see that the values of the ratio vary from 4.41 to 9.03. (c) The objects with robust atmospheres and significant amounts of helium are Jupiter, Saturn, Uranus, and Neptune. We see that the values of the ratio vary from 33.4 to 64.3.

For little to no atmosphere, we see that the ratio is in the range of small numbers, say, 0–3. For these planets and dwarf planets (Mercury, Ceres, and Pluto), the average speed is large enough that a significant number of helium atoms have a speed higher than the escape velocity, and the helium bleeds off into space, along with hydrogen, which has an even higher average speed. Other gases also bleed into space, leaving little to no atmosphere. If the ratio is larger, in the range of 4–10, the planet (Venus, Earth, and Mars) is large enough to retain a significant atmosphere of heavier molecules, such as oxygen, nitrogen, and carbon dioxide, but enough lighter molecules, such as hydrogen and helium, still move fast enough to escape and those lighter molecules bleed off into space while the heavier molecules remain. When the ratio is a large number, such as above 30 for our solar system, the planets (Jupiter, Saturn, Uranus, and Neptune) have enough gravitational attraction to retain even the lightest molecules, so the atmospheres are primarily hydrogen and helium.

Finalize What do you predict for the ratio for the Earth’s Moon? For Titan, a satellite of Saturn? Io, a satellite of Jupiter?

Answers: (a) approximately 0–3 (b) approximately 4–10 (c) approximately 30–70

***TP20.3** If we denote a particular exam score as s_i , then the following equation provides the average score:

$$s_{\text{avg}} = \frac{1}{20} \sum_{i=1}^{20} s_i$$

Similarly, the rms average is found using

$$s_{\text{rms}} = \sqrt{\frac{1}{20} \sum_{i=1}^{20} s_i^2}$$

The table below shows the results of finding the straight mean and the rms values of the scores using these equations:

	Section 1 Exam Scores	Section 2 Exam Scores
	65	99
	65	95
	65	90
	65	85
	65	77
	65	75
	65	75
	65	73
	65	70
	65	67
	64	66
	64	65
	64	63
	64	58
	64	52
	64	49
	64	48
	64	35
	64	28
	64	20
Average:	64.5	64.5
rms Average:	64.5	67.7

Answers:

(a) Section 1: average = 64.5; rms average = 64.5

Section 2: average = 64.5; rms average = 67.7

(b) They are equal.

(c) They are the same, to four significant figures. The scores are tightly bunched around the average with a minimal, and binary, spread; there are ten numbers with a score 0.5 higher than the average and ten numbers with a score 0.5 below the average.

(d) The rms average is larger than the average.

(e) An rms average tends to weigh the larger numbers more heavily, because the numbers are all squared before averaging. Therefore, the rms average is always larger than the straight average.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 20.1 Molecular Model of an Ideal Gas

P20.1 The gas temperature must be that implied by $\frac{1}{2}m_0\overline{v^2} = \frac{3}{2}k_B T$ for a monatomic gas like helium.

$$T = \frac{2}{3} \left(\frac{\frac{1}{2}m_0\overline{v^2}}{k_B} \right) = \frac{2}{3} \left(\frac{3.60 \times 10^{-22} \text{ J}}{1.38 \times 10^{-23} \text{ J/K}} \right) = 17.4 \text{ K}$$

Now $PV = nRT$ gives

$$n = \frac{PV}{RT} = \frac{(1.20 \times 10^5 \text{ N/m}^2)(4.00 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(17.4 \text{ K})} = \boxed{3.32 \text{ mol}}$$

P20.2 $P = \frac{2}{3} \frac{N}{V} K_{avg}$ from the kinetic-theory account for pressure.

$$N = \frac{3}{2} \frac{PV}{K_{avg}}$$

$$n = \frac{N}{N_A} = \boxed{\frac{3}{2} \frac{PV}{K_{avg} N_A}}$$

P20.3 Use the equation describing the kinetic-theory account for pressure:

$$P = \frac{2N}{3V} \left(\frac{m_0\overline{v^2}}{2} \right). \text{ Then}$$

$$\overline{K} = \frac{m_0\overline{v^2}}{2} = \frac{3PV}{2N}, \text{ where } N = nN_A$$

$$\overline{K} = \frac{3PV}{2nN_A} = \frac{3(8.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(5.00 \times 10^{-3} \text{ m}^3)}{2(2 \text{ mol})(6.02 \times 10^{23} \text{ molecules/mol})}$$

$$\overline{K} = \boxed{5.05 \times 10^{-21} \text{ J}}$$

P20.4 The molar mass of diatomic oxygen is 32.0 g. The rms speed of oxygen molecules is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

and

$$\begin{aligned} p_{\text{rms}} &= mv_{\text{rms}} = \frac{M}{N_A} \sqrt{\frac{3RT}{M}} = \frac{1}{N_A} \sqrt{3RTM} \\ &= \frac{1}{6.02 \times 10^{23}} \sqrt{3(8.314 \text{ J/mol} \cdot \text{K})(350 \text{ K})(32.0 \times 10^{-3} \text{ kg})} \\ p_{\text{rms}} &= \boxed{2.78 \times 10^{-23} \text{ kg} \cdot \text{m/s}} \end{aligned}$$

P20.5 (a) From the ideal gas law,

$$PV = nRT = \frac{Nm_0v^2}{3}$$

The total translational kinetic energy is $\frac{Nm_0v^2}{2} = E_{\text{trans}}$:

$$\begin{aligned} E_{\text{trans}} &= \frac{3}{2}PV = \frac{3}{2}(3.00 \times 1.013 \times 10^5 \text{ Pa})(5.00 \times 10^{-3} \text{ m}^3) \\ &= \boxed{2.28 \text{ kJ}} \end{aligned}$$

$$(b) \quad \frac{m_0v^2}{2} = \frac{3k_B T}{2} = \frac{3RT}{2N_A} = \frac{3(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{2(6.02 \times 10^{23})} = \boxed{6.21 \times 10^{-21} \text{ J}}$$

P20.6 We use $1 \text{ u} = 1.66 \times 10^{-24} \text{ g}$.

$$(a) \quad \text{For He, } m_0 = 4.00 \text{ u} \left(\frac{1.66 \times 10^{-24} \text{ g}}{1 \text{ u}} \right) = \boxed{6.64 \times 10^{-27} \text{ kg}}$$

$$(b) \quad \text{For Fe, } m_0 = 55.9 \text{ u} \left(\frac{1.66 \times 10^{-24} \text{ g}}{1 \text{ u}} \right) = \boxed{9.28 \times 10^{-26} \text{ kg}}$$

$$(c) \quad \text{For Pb, } m_0 = 207 \text{ u} \left(\frac{1.66 \times 10^{-24} \text{ g}}{1 \text{ u}} \right) = \boxed{3.44 \times 10^{-25} \text{ kg}}$$

P20.7 To find the pressure exerted by the nitrogen molecules, we first calculated the average force exerted by the molecules:

$$\bar{F} = Nm_0 \frac{\Delta v}{\Delta t} = \frac{(5.00 \times 10^{23})[(4.65 \times 10^{-26} \text{ kg})2(300 \text{ m/s})]}{1.00 \text{ s}} = 14.0 \text{ N}$$

the pressure is then

$$P = \frac{\bar{F}}{A} = \frac{14.0 \text{ N}}{8.00 \times 10^{-4} \text{ m}^2} = \boxed{17.4 \text{ kPa}}$$

P20.8 (a) The volume occupied by this gas is

$$V = 7.00 \text{ L} (10^3 \text{ cm}^3 / 1 \text{ L}) (1 \text{ m}^3 / 10^6 \text{ cm}^3) = 7.00 \times 10^{-3} \text{ m}^3$$

Then, the ideal gas law gives

$$T = \frac{PV}{nR} = \frac{(1.60 \times 10^6 \text{ Pa})(7.00 \times 10^{-3} \text{ m}^3)}{(3.50 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} = \boxed{385 \text{ K}}$$

(b) The average kinetic energy per molecule in this gas is

$$\overline{KE}_{\text{molecule}} = \frac{3}{2} k_B T = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})(385 \text{ K}) = \boxed{7.97 \times 10^{-21} \text{ J}}$$

(c) You would need to know the mass of the gas molecule to find its average speed, which in turn requires knowledge of

the molecular mass of the gas.

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Sect ion 20.2 Molar Specific Heat of an Ideal Gas

P20.9

$$\Delta E_{\text{int}} = \frac{3}{2} n R \Delta T = \frac{3}{2} (3.00 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) (2.00 \text{ K}) = \boxed{74.8 \text{ J}}$$

***P20.10 Conceptualize** It certainly seems that adding energy to the air from the furnace would increase the internal energy of the air. But notice the phrasing: “the air *in the house*.”

Categorize We should be able to address this problem by relating the internal energy of the air to the temperature using the material in Section 20.3.

Analyze In Section 20.3, we find that the internal energy of a diatomic gas, such as the mixture of oxygen and nitrogen in air within the temperature range encountered in our households and daily lives, has contributions from translation and rotation, but not vibration.

Therefore, the internal energy of the air in the house is

$$E_{\text{int}} = \frac{5}{2} n R T \quad (1)$$

Use Equation (1) to set up a ratio of the final internal energy in the air to the initial internal energy:

$$\frac{E_{\text{int},f}}{E_{\text{int},i}} = \frac{\frac{5}{2} n_f R T_f}{\frac{5}{2} n_i R T_i} = \frac{n_f R T_f}{n_i R T_i} \quad (2)$$

Now, use the ideal gas law to replace the numerator and denominator of the fraction in Equation (2):

$$\frac{E_{\text{int},f}}{E_{\text{int},i}} = \frac{P_f V_f}{P_i V_i} \quad (3)$$

Now, let us impose the condition that we are considering the air *in the house*. Because of leaks around doors and windows, the air in the house is always in pressure equilibrium with the outside air. Therefore, the

pressure of the air in the house is always atmospheric pressure, regardless of the temperature setting on the thermostat: $P_f = P_i = P_0$. Turning up the thermostat has no effect on the size of the house, so the volume of air *in the house* is always the same: $V_f = V_i = V_{\text{house}}$. Therefore, Equation (3) becomes

$$\frac{E_{\text{int},f}}{E_{\text{int},i}} = 1$$

The statement made by your sister's client is true!

Finalize What's happening here? When the thermostat is turned up, the air warms and expands. Some of the air originally in the house leaks to the outside, so that the volume of the sample of air that was originally in the house is larger than it was initially. But we are asking about the air *in the house*, not the air that was *originally* in the house. The amount of air that leaks to the outside carries just as much internal energy out of the house as is added to the air inside the house by the furnace, so that the amount of internal energy in the air *in the house* remains fixed!

Answer: true

P20.11 $n = 1.00 \text{ mol}$, $T_i = 300 \text{ K}$

(a) Since $V = \text{constant}$, $W = \boxed{0}$.

(b) $\Delta E_{\text{int}} = Q + W = 209 \text{ J} + 0 = \boxed{209 \text{ J}}$

(c) $\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

$$\text{so } \Delta T = \frac{2\Delta E_{\text{int}}}{3nR} = \frac{2(209 \text{ J})}{3(1.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 16.8 \text{ K}$$

$$T = T_i + \Delta T = 300 \text{ K} + 16.8 \text{ K} = \boxed{317 \text{ K}}$$

P20.12 (a) Molar specific heat is $C_v = \frac{5}{2}R$.

Specific heat at constant volume per unit mass is given by

$$\begin{aligned} c_v &= \frac{C_v}{M} = \frac{5}{2}R \left(\frac{1}{M} \right) \\ &= \frac{5}{2}(8.314 \text{ J/mol} \cdot \text{K}) \left(\frac{1.00 \text{ mol}}{0.0289 \text{ kg}} \right) \\ &= 719 \text{ J/kg} \cdot \text{K} = \boxed{0.719 \text{ kJ/kg} \cdot \text{K}} \end{aligned}$$

(b) $m = Mn = M \left(\frac{PV}{RT} \right)$

$$m = (0.0289 \text{ kg/mol}) \left[\frac{(200 \times 10^3 \text{ Pa})(0.350 \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})} \right] = \boxed{0.811 \text{ kg}}$$

(c) We consider a constant-volume process where no work is done.

$$\begin{aligned} Q &= mc_v \Delta T \\ &= (0.811 \text{ kg})(0.719 \text{ kJ/kg} \cdot \text{K})(700 \text{ K} - 300 \text{ K}) \\ &= \boxed{233 \text{ kJ}} \end{aligned}$$

(d) We now consider a constant-pressure process where the internal energy of the gas is increased and work is done.

$$\begin{aligned} Q &= nC_p \Delta T = \frac{m}{M}(C_v + R)\Delta T = \frac{m}{M} \left(\frac{5}{2}R + R \right) \Delta T = \frac{m}{M} \left(\frac{7}{2}R \right) \Delta T \\ &= m \left(\frac{7}{5} \right) \left(\frac{\frac{5}{2}R}{M} \right) \Delta T = m \left[\left(\frac{7}{5} \right) \left(\frac{C_v}{M} \right) \right] \Delta T \end{aligned}$$

$$\rightarrow Q = (0.811 \text{ kg}) \left[\frac{7}{5} (0.719 \text{ kJ/kg} \cdot \text{K}) \right] (400 \text{ K}) = \boxed{327 \text{ kJ}}$$

P20.13 Consider 800 cm³ of tea (flavored water) at 90.0°C mixing with 200 cm³ of diatomic ideal gas at 20.0°C:

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or
$$m_{\text{air}} c_{P, \text{air}} (T_f - T_{i, \text{air}}) = -m_w c_w (\Delta T)_w$$

$$(\Delta T)_w = \frac{-m_{\text{air}} c_{P, \text{air}} (T_f - T_{i, \text{air}})}{m_w c_w} = \frac{-(\rho V)_{\text{air}} c_{P, \text{air}} (90.0^\circ\text{C} - 20.0^\circ\text{C})}{(\rho_w V_w) c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0°C.

The molar specific heat of air is $C_{P, \text{air}} = \frac{7}{2} R$.

So the specific heat per gram is

$$c_{P, \text{air}} = \frac{7}{2} \left(\frac{R}{M} \right) = \frac{7}{2} (8.314 \text{ J/mol} \cdot \text{K}) \left(\frac{1.00 \text{ mol}}{28.9 \text{ g}} \right) = 1.01 \text{ J/g} \cdot ^\circ\text{C}$$

and

$$(\Delta T)_w = - \frac{[(1.20 \times 10^{-3} \text{ g/cm}^3)(200 \text{ cm}^3)](1.01 \text{ J/g} \cdot ^\circ\text{C})(70.0^\circ\text{C})}{[(1.00 \text{ g/cm}^3)(800 \text{ cm}^3)](4.186 \text{ J/g} \cdot ^\circ\text{C})}$$

or
$$(\Delta T)_w \approx -5.05 \times 10^{-3} ^\circ\text{C}$$

The change of temperature for the water is between $10^{-3} ^\circ\text{C}$ and $10^{-2} ^\circ\text{C}$.

Section 20.3 The Equipartition of Energy

P20.14 (a)
$$E_{\text{int}} = Nf \left(\frac{k_B T}{2} \right) = f \left(\frac{nRT}{2} \right)$$

$$(b) \quad C_V = \frac{1}{n} \left(\frac{dE_{\text{int}}}{dT} \right) = \frac{1}{2} fR$$

$$(c) \quad C_P = C_V + R = \frac{1}{2} (f + 2)R$$

$$(d) \quad \gamma = \frac{C_P}{C_V} = \frac{f + 2}{f}$$

***P20.15 Conceptualize** As the tire flexes while the car is being driven, the air in the tire increases in temperature. As a result, air molecules will have a higher average speed.

Categorize We will model the air in the tire as an ideal gas and will assume the air molecules are diatomic (N_2 , O_2).

Analyze (a) Let us first find the final temperature of the air in the tire from the ideal gas law:

$$PV = nRT \quad \rightarrow \quad \frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i} \quad \rightarrow \quad T_f = \left(\frac{P_f}{P_i} \right) \left(\frac{V_f}{V_i} \right) T_i \quad (1)$$

We have an expression for the rms speed of molecules for a monatomic gas in Equation 20.22. The 3 in that equation comes from the fact that the internal energy of the gas is $\frac{3}{2} Nk_B T$, as in Equation 20.28. For a

diatomic gas, Equation 20.33 tells us that the internal energy is $\frac{5}{2} Nk_B T$.

Therefore, the rms speed of the molecules in the diatomic gas is Equation 20.22 with 3 replaced by 5. Setting up the ratio of the rms speed at the two situations gives us

$$\frac{v_{\text{rms},f}}{v_{\text{rms},i}} = \frac{\sqrt{\frac{5RT_f}{M}}}{\sqrt{\frac{5RT_i}{M}}} = \sqrt{\frac{T_f}{T_i}} \quad (2)$$

Substitute Equation (1) into Equation (2):

$$\frac{v_{\text{rms},f}}{v_{\text{rms},i}} = \sqrt{\frac{\left(\frac{P_f}{P_i}\right)\left(\frac{V_f}{V_i}\right)T_i}{T_i}} = \sqrt{\left(\frac{P_f}{P_i}\right)\left(\frac{V_f}{V_i}\right)}$$

Substitute numerical values, recognizing that the pressures given in the problem statement are *gauge* pressures:

$$\frac{v_{\text{rms},f}}{v_{\text{rms},i}} = \sqrt{\left(\frac{1.00 \text{ atm} + 1.95 \text{ atm}}{1.00 \text{ atm} + 1.65 \text{ atm}}\right)\left(\frac{1.0500V_i}{V_i}\right)} = \boxed{1.08}$$

(b) Now, what about the replacement of the air with argon? The difference is that argon is a monatomic gas. In Equation (2), the two occurrences of 5 would each become 3, but that would not change the result. The rms average speed would still increase by 11%.

Finalize The average speed of the molecules of air in the tires increases by 11% due to the operation of the car, whether air or argon. The *actual* values of the speed would be lower for the monatomic argon, according to Equation 20.22.

Answers: (a) 1.08 (b) no

P20.16 We must have the difference of molar specific heats given by Equation 20.31: $C_p - C_v = R$. The value of γ tells us that $C_p = 1.75C_v$, so

$$1.75C_v - C_v = R \quad \rightarrow \quad C_v = \frac{R}{0.75} = \frac{4}{3}R$$

The maximum possible value of $\gamma = 1 + \frac{R}{C_v} = 1.67$ occurs for the lowest possible value for $C_v = \frac{3}{2}R$. Therefore the claim of $\gamma = 1.75$ for the newly discovered gas cannot be true.

***P20.17 Conceptualize** Think about the air expanding and pushing the pellet down the barrel of the rifle. We want to find an initial pressure of the air. We also know the initial volume of the chamber, and the final volume is that of the chamber plus the volume of the rifle barrel. We know the relationship between pressure and volume for an adiabatic process on a gas, so it looks like we are in good shape regarding the thermodynamics. But how do we incorporate the desired speed of the pellet? It is clear that this problem is a combination of thermodynamics and mechanics.

Categorize The problem statement tells us to categorize the gas process as adiabatic. We also can identify the system of the air and the pellet as an *isolated system* for energy.

Analyze Let's begin with an energy approach. Because the process is adiabatic, $Q = 0$ for the system of the air and the pellet. No work is done on the system. (There is work done by the air on the pellet, but this is work done *within* the system.) We ignore any sound made by the pellet as it travels down the barrel, and there are no other transfers of energy. Therefore, the appropriate reduction of Equation 8.2 for the isolated system is

$$\Delta K + \Delta E_{\text{int}} = 0 \quad (1)$$

The kinetic energy in Equation (1) is that of the pellet. There will be some air that is expelled from the end of the rifle, but it is small in mass compared to the bullet, so we will ignore its kinetic energy.

The change in internal energy in Equation (1) is that of both the pellet and the air. The air will change temperature rapidly during the adiabatic expansion. Any change in the temperature of the pellet will occur by thermal conduction between the pellet and the air. Because the process occurs so quickly, however, there is only a very short time interval for this conduction to occur, so we can assume that the change in internal energy in Equation (1) is only that of the air.

Now, let's use Equation 20.26 to substitute for the internal energy change in Equation (1):

$$\Delta K + nC_v \Delta T = 0 \quad (2)$$

We know nothing about temperatures, but we do know something about volumes, so let's use Equation 20.39 to relate the temperature change to the initial and final volumes for the adiabatic process:

$$\begin{aligned} \Delta K &= -nC_v (T_f - T_i) \\ \rightarrow \Delta K &= -nC_v \left(\frac{T_i V_i^{\gamma-1}}{V_f^{\gamma-1}} - T_i \right) = -nC_v T_i \left[\left(\frac{V_i}{V_f} \right)^{\gamma-1} - 1 \right] \end{aligned} \quad (3)$$

Now, the final volume of the air is the initial volume plus the volume of the interior of the rifle barrel:

$$\Delta K = -nC_v T_i \left[\left(\frac{V_i}{V_i + AL} \right)^{\gamma-1} - 1 \right] \quad (4)$$

Air is diatomic, so we can use Equation 20.33 to substitute for the molar specific heat at constant volume and the appropriate value of γ :

$$\Delta K = -n\left(\frac{5}{2}R\right)T_i\left[\left(\frac{V_i}{V_i + AL}\right)^{\frac{7}{5}-1} - 1\right] = -\frac{5}{2}P_iV_i\left[\left(\frac{V_i}{V_i + AL}\right)^{\frac{2}{5}} - 1\right] \quad (5)$$

where we have used the ideal gas law to replace nRT_i with P_iV_i . Solve Equation (5) for the initial pressure, which is what we want to find:

$$P_i = \frac{\Delta K}{\frac{5}{2}V_i\left[1 - \left(\frac{V_i}{V_i + AL}\right)^{\frac{2}{5}}\right]} \quad (6)$$

Finally, substitute for the change in kinetic energy of the pellet:

$$P_i = \frac{\left(\frac{1}{2}mv^2 - 0\right)}{\frac{5}{2}V_i\left[1 - \left(\frac{V_i}{V_i + AL}\right)^{\frac{2}{5}}\right]} = \frac{mv^2}{5V_i\left[1 - \left(\frac{V_i}{V_i + AL}\right)^{\frac{2}{5}}\right]} \quad (7)$$

Substitute numerical values:

$$\begin{aligned} P_i &= \frac{(0.00110 \text{ kg})(120 \text{ m/s})^2}{5(12.0 \times 10^{-6} \text{ m}^3)\left\{1 - \left[\frac{12.0 \times 10^{-6} \text{ m}^3}{12.0 \times 10^{-6} \text{ m}^3 + (0.0300 \times 10^{-4} \text{ m}^2)(0.500 \text{ m})}\right]^{\frac{2}{5}}\right\}} \\ &= \boxed{5.74 \times 10^6 \text{ Pa}} \end{aligned}$$

Finalize This result corresponds to about 57 atm of pressure. This might seem high, but there are air rifles with significantly higher initial pressures. You will need to buy a pump that can provide 57 atm of pressure. Notice that the initial pressure goes up as the *square* of the

muzzle speed, so doubling the muzzle speed requires *four* times the initial pressure.

Answer: $5.74 \times 10^6 \text{ Pa}$

Section 20.4 Adiabatic Processes for an Ideal Gas

P20.18 (a) $P_i V_i^\gamma = P_f V_f^\gamma$ so $\frac{V_f}{V_i} = \left(\frac{P_i}{P_f} \right)^{1/\gamma} = \left(\frac{1.00}{20.0} \right)^{5/7} = \boxed{0.118}$.

(b) $\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = \left(\frac{P_f}{P_i} \right) \left(\frac{V_f}{V_i} \right) = (20.0)(0.118) \rightarrow \frac{T_f}{T_i} = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$.

(d) Since $\gamma = 1.40 = \frac{C_p}{C_v} = \frac{R + C_v}{C_v}$, $C_v = \frac{5}{2}R$

and $\Delta T = 2.35T_i - T_i = 1.35T_i$, then

$$\begin{aligned} \Delta E_{\text{int}} &= nC_v \Delta T \\ &= (0.0160 \text{ mol}) \left(\frac{5}{2} \right) (8.314 \text{ J/mol} \cdot \text{K}) [1.35(300 \text{ K})] \\ &= \boxed{135 \text{ J}} \end{aligned}$$

(e) $W = -Q + \Delta E_{\text{int}} = 0 + 135 \text{ J} = \boxed{+135 \text{ J}}$

P20.19 Combining $PV^\gamma = \text{constant}$ with the ideal gas law gives one of the textbook equations describing adiabatic processes, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$.

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (300 \text{ K}) \left(\frac{1}{2} \right)^{(1.40-1)} = \boxed{227 \text{ K}}$$

P20.20 Use Equation 20.38 for an adiabatic process to find the temperature of

the compressed fuel-air mixture at the end of the compression stroke, before ignition:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

which gives

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = (323 \text{ K})(14.5)^{1.40-1} = 941 \text{ K}$$

This is equivalent to 668°C, which is higher than the melting point of aluminum which is 660°C. Also, the temperature will rise much more when ignition occurs. The engine will melt when put into operation! Therefore, the claim of improved efficiency using an engine fabricated out of aluminum cannot be true.

P20.21 (a) $V_i = \pi \left(\frac{2.50 \times 10^{-2} \text{ m}}{2} \right)^2 (0.500 \text{ m}) = \boxed{2.45 \times 10^{-4} \text{ m}^3}$

(b) The quantity of air we find from $P_i V_i = nRT_i$:

$$n = \frac{P_i V_i}{RT_i} = \frac{(1.013 \times 10^5 \text{ Pa})(2.45 \times 10^{-4} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})}$$

$$n = \boxed{9.97 \times 10^{-3} \text{ mol}}$$

(c) Absolute pressure = gauge pressure + external pressure:

$$P_f = 101.3 \text{ kPa} + 800 \text{ kPa} = 901.3 \text{ kPa} = \boxed{9.01 \times 10^5 \text{ Pa}}$$

(d) Adiabatic compression: $P_i V_i^\gamma = P_f V_f^\gamma$

$$V_f = V_i \left(\frac{P_i}{P_f} \right)^{1/\gamma} = (2.45 \times 10^{-4} \text{ m}^3) \left(\frac{101.3}{901.3} \right)^{5/7}$$

$$V_f = \boxed{5.15 \times 10^{-5} \text{ m}^3}$$

(e) $P_f V_f = nRT_f$

$$T_f = T_i \frac{P_f V_f}{P_i V_i} = T_i \frac{P_f}{P_i} \left(\frac{P_i}{P_f} \right)^{1/\gamma} = T_i \left(\frac{P_i}{P_f} \right)^{(1/\gamma-1)}$$

$$T_f = 300 \text{ K} \left(\frac{101.3}{901.3} \right)^{(5/7-1)} = \boxed{560 \text{ K}}$$

- (f) The work done on the gas in compressing it is $W = \Delta E_{\text{int}} = nC_V \Delta T$:

$$\begin{aligned} \Delta E_{\text{int}} &= W = nC_V \Delta T \\ &= (9.97 \times 10^{-3} \text{ mol}) \frac{5}{2} (8.314 \text{ J/mol} \cdot \text{K}) (560 \text{ K} - 300 \text{ K}) \\ \Delta E_{\text{int}} &= \boxed{53.9 \text{ J}} \end{aligned}$$

- (g) The pump wall has outer diameter 25.0 mm + 2.00 mm + 2.00 mm = 29.0 mm, and volume

$$\begin{aligned} &\left[\pi (14.5 \times 10^{-3} \text{ m})^2 - \pi (12.5 \times 10^{-3} \text{ m})^2 \right] \\ &\quad \times (4.00 \times 10^{-2} \text{ m}) = \boxed{6.79 \times 10^{-6} \text{ m}^3} \end{aligned}$$

- (h) The mass of the pump is given by

$$\rho V = (7.86 \times 10^3 \text{ kg/m}^3) (6.79 \times 10^{-6} \text{ m}^3) = \boxed{53.3 \text{ g}}$$

- (i) Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The overall warming process is described by

$$\Delta E_{\text{int}} = W = nC_V \Delta T + mc \Delta T \rightarrow \Delta T = \frac{W}{nC_V + mc}$$

Suppressing the units of R ,

$$\begin{aligned} \Delta T &= \frac{53.9 \text{ J}}{(9.97 \times 10^{-3} \text{ mol}) \frac{5}{2} (8.314) + (0.0533 \text{ kg}) (448 \text{ J/kg} \cdot ^\circ\text{C})} \\ &= 2.24^\circ\text{C} = \boxed{2.24 \text{ K}} \end{aligned}$$

Section 20.5 Distribution of Molecular Speeds

P20.22 (a) $\frac{V_{\text{rms}, 35}}{V_{\text{rms}, 37}} = \frac{\sqrt{3RT / M_{35}}}{\sqrt{3RT / M_{37}}} = \left(\frac{37.0 \text{ g/mol}}{35.0 \text{ g/mol}} \right)^{1/2} = \boxed{1.03}$

(b) The lighter atom, $\boxed{{}^{35}\text{Cl}}$, moves faster.

P20.23 (a) From $v_{\text{avg}} = \sqrt{\frac{8k_B T}{\pi m_0}}$ we find the temperature as

$$T = \frac{\pi (6.64 \times 10^{-27} \text{ kg}) (1.12 \times 10^4 \text{ m/s})^2}{8 (1.38 \times 10^{-23} \text{ J/mol} \cdot \text{K})} = \boxed{2.37 \times 10^4 \text{ K}}$$

(b) $T = \frac{\pi (6.64 \times 10^{-27} \text{ kg}) (2.37 \times 10^3 \text{ m/s})^2}{8 (1.38 \times 10^{-23} \text{ J/mol} \cdot \text{K})} = \boxed{1.06 \times 10^3 \text{ K}}$

P20.24 In the Maxwell-Boltzmann speed distribution function take $\frac{dN_v}{dv} = 0$ to find

$$4\pi N \left(\frac{m_0}{2\pi k_B T} \right)^{3/2} \exp \left(-\frac{m_0 v^2}{2k_B T} \right) \left(2v - \frac{2m_0 v^3}{2k_B T} \right) = 0$$

and solve for v to find the most probable speed. Reject as solutions $v = 0$ and $v = \infty$. They describe minimally probable speeds.

Retain only $2 - \frac{m_0 v^2}{k_B T} = 0$.

Then, $v_{\text{mp}} = \sqrt{\frac{2k_B T}{m_0}}$.

P20.25 (a) From the Boltzmann distribution law, the number density of molecules with height y so that the gravitational potential energy

of the molecule-Earth system is $m_0 g y$ is $n_0 e^{-m_0 g y / k_B T}$. These are the molecules with height y , so this is the number per volume at height y as a function of y .

$$\begin{aligned}
 \text{(b)} \quad \frac{n(y)}{n_0} &= e^{-m_0 g y / k_B T} = e^{-M g y / N_A k_B T} = e^{-M g y / R T} \\
 &= e^{-\left(28.9 \times 10^{-3} \text{ kg/mol}\right)\left(9.8 \text{ m/s}^2\right)\left(11 \times 10^3 \text{ m}\right) / \left(8.314 \text{ J/mol} \cdot \text{K}\right)(293 \text{ K})} \\
 &= e^{-1.279} = \boxed{0.278}
 \end{aligned}$$

P20.26 It is convenient in the following to define $a = \frac{m_0 g}{k_B T}$.

(a) We calculate

$$\begin{aligned}
 \int_0^{\infty} e^{-m_0 g y / k_B T} dy &= \int_0^{\infty} e^{-a y} dy = \int_{y=0}^{\infty} e^{-a y} (-a dy) \left(-\frac{1}{a}\right) \\
 &= \left(-\frac{1}{a}\right) e^{-a y} \Big|_0^{\infty} = \left(-\frac{1}{a}\right)(0 - 1) = \frac{1}{a}
 \end{aligned}$$

Using Table B.6 in the appendix,

$$\int_0^{\infty} y e^{-a y} dy = \frac{1!}{(a)^2} = \left(\frac{1}{a}\right)^2$$

Then,

$$y_{\text{avg}} = \frac{\int_0^{\infty} y e^{-a y} dy}{\int_0^{\infty} e^{-a y} dy} = \frac{(1/a)^2}{1/a} = \frac{1}{a} = \frac{k_B T}{m_0 g}$$

$$\text{(b)} \quad y_{\text{avg}} = \frac{k_B T}{(M/N_A)g} = \frac{RT}{Mg} = \frac{(8.314 \text{ J/mol} \cdot \text{K})(283 \text{ K})}{(28.9 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)} = \boxed{8.31 \text{ km}}$$

Additional Problems

P20.27 (a) The average speed is given by $v_{\text{avg}} = \frac{\sum_i^N v_i}{N}$

which may be solved numerically for the values given.

Suppressing units,

$$\begin{aligned} v_{\text{avg}} &= \frac{\sum_i^N v_i}{N} \\ &= \frac{[(3.00) + (4.00) + (5.80) + (2.50) + (3.60) + (1.90) + (3.80) + (6.60)]}{8} \\ &= \frac{31.2 \text{ km/s}}{8} = \boxed{3.90 \text{ km/s}} \end{aligned}$$

(b) The rms speed of the molecules is given by $v_{\text{rms}} = \sqrt{\frac{\sum_i^N v_i^2}{N}}$

which may be solved numerically for the values given.

Suppressing units,

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{\sum_i^N v_i^2}{N}} \\ &= \sqrt{\frac{[(3.00)^2 + (4.00)^2 + (5.80)^2 + (2.50)^2 + (3.60)^2 + (1.90)^2 + (3.80)^2 + (6.60)^2]}{8}} \\ &= \sqrt{\frac{[(9.00) + (16.00) + (33.64) + (6.25) + (12.96) + (3.61) + (14.44) + (43.56)]}{8}} \\ &= \sqrt{\frac{139.46 \text{ km}^2 / \text{s}^2}{8}} = \sqrt{17.43 \text{ km}^2 / \text{s}^2} = \boxed{4.18 \text{ km/s}} \end{aligned}$$

P20.28 (a) For a pure metallic element, one atom is one molecule. Its energy can be represented as

$$\frac{1}{2} m_0 v_x^2 + \frac{1}{2} m_0 v_y^2 + \frac{1}{2} m_0 v_z^2 + \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2 + \frac{1}{2} k_z z^2$$

Its average value is

$$\frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T = 3k_B T$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/n = 3RT$, and the molar heat capacity at constant volume is $E_{\text{int}}/nT = 3R$.

(b) We calculate the specific heat from

$$3(8.314 \text{ J/mol} \cdot \text{K}) = \frac{3(8.314 \text{ J})}{(55.845 \times 10^{-3} \text{ kg}) \cdot \text{K}} = \boxed{447 \text{ J/kg} \cdot \text{K}}$$

This agrees with the tabulated value of 448 J/kg · °C within 0.3%.

(c) For gold,

$$3(8.314 \text{ J/mol} \cdot \text{K}) = \frac{3(8.314 \text{ J})}{(197 \times 10^{-3} \text{ kg}) \cdot \text{K}} = \boxed{127 \text{ J/kg} \cdot \text{K}}$$

This agrees with the tabulated value of 129 J/kg · °C within 2%.

P20.29 (a) $n = \frac{PV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(4.20 \text{ m})(3.00 \text{ m})(2.50 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 1.31 \times 10^3 \text{ mol}$

$$N = nN_A = (1.31 \times 10^3 \text{ mol})(6.02 \times 10^{23} \text{ molecules/mol})$$

$$N = \boxed{7.89 \times 10^{26} \text{ molecules}}$$

(b) $m = nM = (1.31 \times 10^3 \text{ mol})(0.0289 \text{ kg/mol}) = \boxed{37.9 \text{ kg}}$

(c) $\frac{1}{2}m_0 v^2 = \frac{3}{2}k_B T = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = \boxed{6.07 \times 10^{-21} \text{ J}}$

(d) For one molecule,

$$m_0 = \frac{M}{N_A} = \frac{0.0289 \text{ kg/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 4.80 \times 10^{-26} \text{ kg/molecule}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{2(6.07 \times 10^{-21} \text{ J/molecule})}{4.80 \times 10^{-26} \text{ kg/molecule}}} = \boxed{503 \text{ m/s}}$$

(e) $\boxed{0}$

(f) When the furnace operates, air expands and some of it leaves the room. The smaller mass of warmer air left in the room contains the same internal energy as the cooler air initially in the room.

P20.30 (a) The pressure increases as volume decreases (and vice versa), so dV/dP is always negative.

In equation form, $\frac{dV}{dP} < 0$ and $-\left(\frac{1}{V}\right)\left(\frac{dV}{dP}\right) > 0$

(b) For an ideal gas, $V = \frac{nRT}{P}$ and $\kappa_1 = -\frac{1}{V} \frac{d}{dP} \left(\frac{nRT}{P} \right)$

For isothermal compression, T is constant and the derivative gives us

$$\kappa_1 = -\frac{nRT}{V} \left(\frac{-1}{P^2} \right) = \frac{1}{P}$$

(c) For an adiabatic compression, $PV^\gamma = C$ (where C is a constant) and we evaluate dV/dP as follows:

$$\kappa_2 = -\left(\frac{1}{V}\right) \frac{d}{dP} \left(\frac{C}{P} \right)^{1/\gamma} = \left(\frac{1}{V\gamma} \right) \frac{C^{1/\gamma}}{P^{1/\gamma+1}} = \frac{V}{V\gamma P} = \frac{1}{\gamma P}$$

(d) $\kappa_1 = \frac{1}{P} = \frac{1}{(2.00 \text{ atm})} = \boxed{0.500 \text{ atm}^{-1}}$

- (e) For a monatomic ideal gas, $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$, so

$$\kappa_2 = \frac{1}{\gamma P} = \frac{1}{\frac{5}{3}(2.00 \text{ atm})} = \boxed{0.300 \text{ atm}^{-1}}$$

- P20.31** (a) The rms speed of molecules in a gas is related to the temperature by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

which can be rearranged and solved numerically for the temperature:

$$\begin{aligned} T &= \frac{mv_{\text{rms}}^2}{3k} = \frac{32(1.66 \times 10^{-27} \text{ kg})(535 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} \\ &= \boxed{367 \text{ K}} \end{aligned}$$

- (b) The rms speed is inversely related to the mass of the gas molecule (the mass is in the denominator of the square-root function above). The rms speed of nitrogen would be higher because the molar mass of nitrogen is less than that of oxygen.

- (c) The rms speed of the nitrogen molecules is:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(367 \text{ K})}{28(1.66 \times 10^{-27} \text{ kg})}} = \boxed{572 \text{ m/s}}$$

- P20.32** (a) Begin with Equation 16.35 and substitute the definition of bulk modulus from Equation 12.8:

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1}{\rho} \left(-V \frac{dP}{dV} \right)}$$

Now substitute using Equation 20.38 with the constant on the right hand side represented by K :

$$\begin{aligned} v &= \sqrt{\frac{-V}{\rho} \frac{d}{dV}(KV^{-\gamma})} = \sqrt{\frac{-KV}{\rho}(-\gamma V^{-\gamma-1})} = \sqrt{\frac{\gamma}{\rho}(KV^{-\gamma})} \\ &= \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma}{\rho} \frac{nRT}{V}} = \sqrt{\frac{\gamma(m/M)RT}{m}} = \sqrt{\frac{\gamma RT}{M}} \end{aligned}$$

$$(b) \quad v = \sqrt{\frac{1.40(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})}{0.0289 \text{ kg/mol}}} = \boxed{344 \text{ m/s}}$$

This agrees within 0.2% with the 343 m/s listed in Table 16.1.

$$(c) \quad \text{We use } k_B = \frac{R}{N_A} \text{ and } M = m_0 N_A: v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma k_B N_A T}{m_0 N_A}} = \sqrt{\frac{\gamma k_B T}{m_0}}$$

$$(d) \quad \text{The most probable molecular speed is } \sqrt{\frac{2k_B T}{m_0}}, \text{ the average speed is } \sqrt{\frac{8k_B T}{\pi m_0}}, \text{ and the rms speed is } \sqrt{\frac{3k_B T}{m_0}}.$$

The speed of sound is somewhat less than each measure of molecular speed. Sound propagation is orderly motion overlaid on the disorder of molecular motion.

P20.33

Sulfur dioxide is the gas with the greatest molecular mass of those listed. If the effective spring constants for various chemical bonds are comparable, SO_2 can then be expected to have low frequencies of atomic vibration. Vibration can be excited at lower temperature than for the other gases. Some vibration may be going on at 300 K. With more degrees of freedom for molecular motion, the material has higher specific heat.

P20.34

(a) For the adiabatic process $PV^\gamma = k$, a constant. The work is

$$W = -\int_i^f P dV = -k \int_{V_i}^{V_f} \frac{dV}{V^\gamma} = \frac{-kV^{1-\gamma}}{1-\gamma} \Big|_{V_i}^{V_f}$$

For k we can substitute $P_i V_i^\gamma$ and also $P_f V_f^\gamma$ to have

$$W = -\frac{P_f V_f^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1-\gamma} = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

(b) For an adiabatic process $\Delta E_{\text{int}} = Q + W$ and $Q = 0$. Therefore,

$$W = \Delta E_{\text{int}} = nC_V \Delta T = nC_V (T_f - T_i)$$

(c) The expressions are equal because $PV = nRT$ and $\gamma = (C_V + R)/C_V = 1 + R/C_V$ give $R = (\gamma - 1)C_V$, so $PV = n(\gamma - 1)C_V T$ and $PV/(\gamma - 1) = nC_V T$.

P20.35 (a) $\Delta E_{\text{int}} = Q + W = 0 + W \rightarrow W = nC_V (T_f - T_i)$

$$-2\,500 \text{ J} = (1 \text{ mol}) \left(\frac{3}{2} \right) (8.314 \text{ J/mol} \cdot \text{K}) (T_f - 500 \text{ K})$$

$$T_f = \boxed{300 \text{ K}}$$

(b) $P_i V_i^\gamma = P_f V_f^\gamma$

$$P_i \left(\frac{nRT_i}{P_i} \right)^\gamma = P_f \left(\frac{nRT_f}{P_f} \right)^\gamma$$

$$T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f}$$

$$P_f = P_i \left(\frac{T_f}{T_i} \right)^{\gamma/(\gamma-1)}$$

$$P_f = P_i \left(\frac{T_f}{T_i} \right)^{(5/3)(3/2)} = (3.60 \text{ atm}) \left(\frac{300}{500} \right)^{5/2} = \boxed{1.00 \text{ atm}}$$

P20.36 (a) The process is adiabatic:

$$\Delta E_{\text{int}} = Q + W = 0 + W \rightarrow W = nC_V(T_f - T_i)$$

For an ideal gas,

$$W = nC_V(T_f - T_i) = n \left(\frac{3}{2} \right) R(T_f - T_i)$$

Solving for final temperature, we get

$$T_f = \boxed{T_i + \frac{2}{3} \frac{W}{nR}}$$

$$(b) \quad P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_i \left(\frac{nRT_i}{P_i} \right)^\gamma = P_f \left(\frac{nRT_f}{P_f} \right)^\gamma \rightarrow T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\rightarrow \frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f} \rightarrow P_f = P_i \left(\frac{T_f}{T_i} \right)^{\gamma/(\gamma-1)}$$

$$\text{where } \gamma = \frac{C_p}{C_V} = \frac{5}{3} \text{ for an ideal gas, and } \frac{\gamma}{\gamma-1} = \frac{\frac{5}{3}}{\frac{5}{3}-1} = \frac{\frac{5}{3}}{\frac{2}{3}} = \frac{5}{2}.$$

Substituting this and the result from part (a) gives

$$P_f = P_i \left(\frac{T_i + \frac{2}{3} \frac{W}{nR}}{T_i} \right)^{5/2} = \boxed{P_i \left(1 + \frac{2}{3} \frac{W}{nRT_i} \right)^{5/2}}$$

P20.37 (a) The latent heat of evaporation per molecule is

$$2\,430\text{ J/g} = (2430\text{ J/g})\left(\frac{18.0\text{ g}}{1\text{ mol}}\right)\left(\frac{1\text{ mol}}{6.02 \times 10^{23}\text{ molecule}}\right)$$

$$= \boxed{7.27 \times 10^{-20}\text{ J/molecule}}$$

If the molecule is about to break free, we assume that it possesses the energy as translational kinetic energy.

- (b) Consider one gram of these molecules. From $K = \frac{1}{2}mv^2$ we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2\,430\text{ J})}{10^{-3}\text{ kg}}} = 2.20 \times 10^3\text{ m/s} = \boxed{2.20\text{ km/s}}$$

- (c) The total translational kinetic energy of an ideal gas is $\frac{3}{2}nrT$, so we have

$$(2\,430\text{ J/g})\left(\frac{18.0\text{ g}}{1\text{ mol}}\right) = \frac{3}{2}(1\text{ mol})(8.314\text{ J/mol} \cdot \text{K})T$$

which gives

$$T = \boxed{3.51 \times 10^3\text{ K}}$$

- (d) The evaporating particles emerge with much less kinetic energy, as negative work is performed on them by restraining forces as they leave the liquid. Much of the initial kinetic energy is used up in overcoming the latent heat of vaporization. There are also very few of these escaping at any moment in time.

P20.38 (a) Maxwell's speed distribution function is

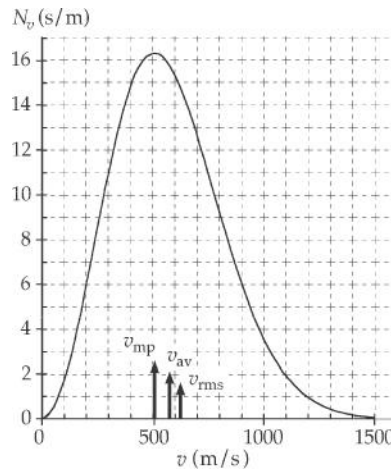
$$N_v = 4\pi N \left(\frac{m_0}{2\pi k_B T} \right)^{3/2} v^2 e^{-m_0 v^2 / 2k_B T}$$

$$\text{With } N = 1.00 \times 10^4, m_0 = \frac{M}{N_A} = \frac{0.032\text{ kg}}{6.02 \times 10^{23}} = 5.32 \times 10^{-26}\text{ kg},$$

$T = 500 \text{ K}$, and $k_B = 1.38 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$, this becomes

$$N_v = (1.71 \times 10^{-4}) v^2 e^{-(3.85 \times 10^{-6}) v^2}$$

ANS. FIG. P20.38(a) below is a plot of this function for the range $0 \leq v \leq 1500 \text{ m/s}$.



ANS. FIG. P20.38 (a)

- (b) The most probable speed occurs where N_v is a maximum.

From the graph, $v_{\text{mp}} \approx 510 \text{ m/s}$.

$$\begin{aligned} \text{(c)} \quad v_{\text{avg}} &= \sqrt{\frac{8k_B T}{\pi m_0}} \\ &= \sqrt{\frac{8(1.38 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(500 \text{ K})}{\pi(5.32 \times 10^{-26} \text{ kg})}} = \boxed{575 \text{ m/s}} \end{aligned}$$

Also,

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3k_B T}{m_0}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(500 \text{ K})}{5.32 \times 10^{-26} \text{ kg}}} \\ &= \boxed{624 \text{ m/s}} \end{aligned}$$

- (d) The fraction of particles in the range $300 \text{ m/s} \leq v \leq 600 \text{ m/s}$

is

$$\frac{\int_{300}^{600} N_v dv}{N}$$

where $N = 10^4$ and the integral of N_v is read from the graph as the area under the curve. This is approximately the area of a large rectangle 11 s/m high and 300 m/s wide [corners at (300, 0), (300, 11), (600, 11), and (600, 0)], plus a smaller rectangle 5.5 s/m high and 100 m/s wide [corners at (500, 11), (500, 16.6), (600, 16.5), and (600, 11)], plus a triangle 5.5 s/m high with a 200 m/s base [vertices at (300, 11), (500, 16.5), and (500, 11)]:

$$(11)(300) + (5.5)(100) + (1/2)(5.5)(200) = 4\,400$$

and the fraction is 0.44 or 44%.

P20.39 $N_v(v) = 4\pi N \left(\frac{m_0}{2\pi k_B T} \right)^{3/2} v^2 \exp\left(\frac{-m_0 v^2}{2k_B T} \right)$, where $\exp(x)$ represents e^x

Note that $v_{\text{mp}} = \left(\frac{2k_B T}{m_0} \right)^{1/2}$.

Thus, $N_v(v) = 4\pi N \left(\frac{m_0}{2\pi k_B T} \right)^{3/2} v^2 e^{(-v^2/v_{\text{mp}}^2)}$

and $\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{v}{v_{\text{mp}}} \right)^2 e^{(1-v^2/v_{\text{mp}}^2)}$.

For $v = \frac{v_{\text{mp}}}{50}$,

$$\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{1}{50}\right)^2 e^{\left[1-(1/50)^2\right]} = 1.09 \times 10^{-3}$$

The other values are computed similarly, with the following results:

	$\frac{v}{v_{\text{mp}}}$	$\frac{N_v(v)}{N_v(v_{\text{mp}})}$
(a)	$\frac{1}{50}$	1.09×10^{-3}
(b)	$\frac{1}{10}$	2.69×10^{-2}
(c)	$\frac{1}{2}$	0.529
(d)	1	1.00
(e)	2	0.199
(f)	10	1.01×10^{-41}
(g)	50	1.25×10^{-1082}

To find the last value, we note:

$$\begin{aligned}
 (50)^2 e^{1-2.500} &= 2.500 e^{-2.499} \\
 10^{\log 2.500} e^{(\ln 10)(-2.499/\ln 10)} &= 10^{\log 2.500} 10^{-2.499/\ln 10} = 10^{\log 2.500 - 2.499/\ln 10} \\
 &= 10^{-1.081.904} = 10^{0.096} \times 10^{-1.082}
 \end{aligned}$$

P20.40 (a) The energy of one molecule can be represented as

$$\frac{1}{2} m_0 v_x^2 + \frac{1}{2} m_0 v_y^2 + \frac{1}{2} m_0 v_z^2 + \frac{1}{2} I \omega_x^2 + \frac{1}{2} I \omega_z^2$$

Its average value is

$$\frac{1}{2} k_B T + \frac{1}{2} k_B T + \frac{1}{2} k_B T + \frac{1}{2} k_B T + \frac{1}{2} k_B T = \frac{5}{2} k_B T$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}} / n = \frac{5}{2}RT$.

And the molar heat capacity at constant volume is

$$E_{\text{int}} / nT = \boxed{\frac{5}{2}R}.$$

(b) The energy of one molecule can be represented as

$$\frac{1}{2}m_0v_x^2 + \frac{1}{2}m_0v_y^2 + \frac{1}{2}m_0v_z^2 + \frac{1}{2}I\omega_x^2 + \frac{1}{2}I\omega_z^2 + \frac{1}{2}I\omega_y^2$$

Its average value is

$$\frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T = 3k_B T$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/n = 3RT$.

And the molar heat capacity at constant volume is $E_{\text{int}}/nT = \boxed{3R}$.

(c) Let the modes of vibration be denoted by 1 and 2. The energy of one molecule can be represented as

$$\begin{aligned} \frac{1}{2}m_0(v_x^2 + v_y^2 + v_z^2) + \frac{1}{2}I\omega_x^2 + \frac{1}{2}I\omega_z^2 \\ + \left(\frac{1}{2}\mu v_{\text{rel}}^2 + \frac{1}{2}kx^2\right)_1 + \left(\frac{1}{2}\mu v_{\text{rel}}^2 + \frac{1}{2}kx^2\right)_2 \end{aligned}$$

Its average value is

$$\frac{3}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T = \frac{9}{2}k_B T$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}} / n = \frac{9}{2}RT$.

And the molar heat capacity at constant volume is

$$E_{\text{int}} / nT = \boxed{\frac{9}{2}R}.$$

(d) The energy of one molecule can be represented as

$$\begin{aligned} \frac{1}{2}m_0(v_x^2 + v_y^2 + v_z^2) + \frac{1}{2}I\omega_x^2 + \frac{1}{2}I\omega_z^2 + \frac{1}{2}I\omega_y^2 \\ + \left(\frac{1}{2}\mu v_{\text{rel}}^2 + \frac{1}{2}kx^2\right)_1 + \left(\frac{1}{2}\mu v_{\text{rel}}^2 + \frac{1}{2}kx^2\right)_2 \end{aligned}$$

Its average value is

$$\frac{3}{2}k_B T + \frac{3}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T = 5k_B T$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/n = 5RT$.

And the molar heat capacity at constant volume is $E_{\text{int}}/nT = \boxed{5R}$.

- (e) Measure the constant-volume specific heat of the gas as a function of temperature and look for plateaus on the graph. If the first jump goes from $\frac{3}{2}R$ to $\frac{5}{2}R$, the molecules can be diagnosed as linear. If the first jump goes from $\frac{3}{2}R$ to $3R$, the molecules must be nonlinear. The tabulated data at one temperature are insufficient for the determination. At room temperature some of the heavier molecules appear to be vibrating.

P20.41 (a) First find $\overline{v^2}$ as $\overline{v^2} = \frac{1}{N} \int_0^\infty v^2 N_v dv$. Let $a = \frac{m_0}{2k_B T}$.

$$\text{Then, } \overline{v^2} = \frac{[4N\pi^{-1/2}a^{3/2}]}{N} \int_0^\infty v^4 e^{-av^2} dv = [4a^{3/2}\pi^{-1/2}] \frac{3}{8a^2} \sqrt{\frac{\pi}{a}} = \frac{3k_B T}{m}$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{v^2} = \boxed{\sqrt{\frac{3k_B T}{m_0}}}$

(b) To find the average speed, we have

$$v_{\text{avg}} = \frac{1}{N} \int_0^\infty v N_v dv = \frac{(4Na^{3/2}\pi^{-1/2})}{N} \int_0^\infty v^3 e^{-av^2} dv = \frac{4a^{3/2}\pi^{-1/2}}{2a^2}$$

$$= \boxed{\sqrt{\frac{8k_B T}{\pi m_0}}}$$

P20.42 We want to evaluate $\frac{dP}{dV}$ for the function implied by

$PV = nRT = \text{constant}$, and also for the different function implied by

$PV^\gamma = \text{constant}$. We can use implicit differentiation:

From $PV = \text{constant}$ $P \frac{dV}{dV} + V \frac{dP}{dV} = 0 \rightarrow \left(\frac{dP}{dV} \right)_{\text{isotherm}} = -\frac{P}{V}$

From $PV^\gamma = \text{constant}$ $P\gamma V^{\gamma-1} + V^\gamma \frac{dP}{dV} = 0 \rightarrow \left(\frac{dP}{dV} \right)_{\text{adiabat}} = -\frac{\gamma P}{V}$

Therefore, $\left(\frac{dP}{dV} \right)_{\text{adiabat}} = \gamma \left(\frac{dP}{dV} \right)_{\text{isotherm}}$

The theorem is proved.

P20.43 (a) For sodium atoms (with a molar mass $M = 23.0 \text{ g/mol}$):

$$\frac{1}{2} m_0 v^2 = \frac{3}{2} k_B T$$

$$\frac{1}{2} \left(\frac{M}{N_A} \right) v^2 = \frac{3}{2} k_B T$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \text{ J/mol} \cdot \text{K})(2.40 \times 10^{-4} \text{ K})}{23.0 \times 10^{-3} \text{ kg}}}$$

$$= \boxed{0.510 \text{ m/s}}$$

$$(b) \quad \Delta t = \frac{d}{v_{\text{rms}}} = \frac{0.010 \text{ m}}{0.510 \text{ m/s}} = 19.6 \text{ ms} \approx \boxed{20 \text{ ms}}$$

- P20.44** (a) The effect of large centripetal acceleration is like the effect of a very high gravitational field on an atmosphere. The result is:

The larger-mass molecules settle to the outside while the region at smaller r has a higher concentration of low-mass molecules.

- (b) Consider a single kind of molecule, all of mass m_0 . To cause the centripetal acceleration of the molecules between r and $r + dr$, the inward force must increase with increasing distance from the center according to $\sum F_r = m_0 a_r$. Taking the positive direction toward the center of the centrifuge, we have

$$(P + dP)A - PA = n_v (m_0 A dr) (r\omega^2)$$

where $n_v = n_v(r) = N/V$, the number of molecules per unit volume, is an implicit function of r , and A is the area of any cylindrical shell of thickness dr and radius r . The equation reduces to

$$dP = n_v m_0 \omega^2 r dr \quad [1]$$

But also within any small cylindrical shell,

$$\begin{aligned} PV &= Nk_B T \rightarrow P = \left(\frac{N}{V} \right) k_B T \\ \rightarrow dP &= d \left(\frac{N}{V} \right) k_B T = d(n_v) k_B T = dn_v k_B T \end{aligned}$$

Therefore, equation [1] becomes

$$dn_V k_B T = n_V m_0 \omega^2 r dr \quad \rightarrow \quad \frac{dn_V}{n_V} = \frac{m_0 \omega^2}{k_B T} r dr$$

$$\text{giving } \int_{n_0}^n \frac{dn_V}{n_V} = \frac{m_0 \omega^2}{k_B T} \int_0^r r dr, \quad \text{where } n_V(r=0) = n_0.$$

Integrating, we find

$$\ln(n_V) \Big|_{n_0}^{n_V} = \frac{m_0 \omega^2}{k_B T} \left(\frac{r^2}{2} \right) \Big|_0^r \quad \rightarrow \quad \ln \left(\frac{n_V}{n_0} \right) = \frac{m_0 \omega^2}{2 k_B T} r^2$$

$$\text{and solving for } n \equiv n_V, \text{ we have } \boxed{n = n_0 e^{m_0 r^2 \omega^2 / 2 k_B T}}.$$

Challenge Problem

P20.45 (a) The average value of a collection of particle speeds is

$$v_{\text{avg}} = \frac{\sum_{i=1}^N v_i}{N}$$

Use this equation to find the average for the two speeds given in the problem:

$$v_{\text{avg}} = \frac{v_1 + v_2}{2} = \frac{av_{\text{avg}} + (2-a)v_{\text{avg}}}{2} = v_{\text{avg}}$$

(b) The rms average value of a collection of particle speeds is

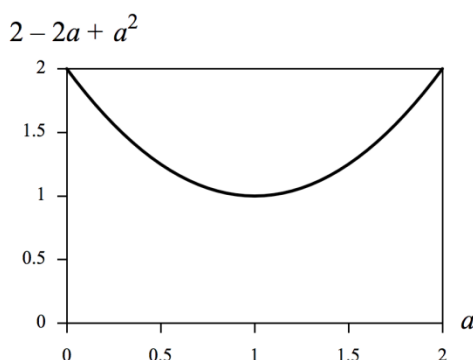
$$v_{\text{rms}} = \frac{\sum_{i=1}^N v_i^2}{N}$$

Use this equation to find the square of the rms average for the two speeds given in the problem:

$$\begin{aligned} v_{\text{rms}}^2 &= \frac{v_1^2 + v_2^2}{2} = \frac{(av_{\text{avg}})^2 + [(2-a)v_{\text{avg}}]^2}{2} \\ &= \frac{v_{\text{avg}}^2}{2} [a^2 + (4 - 4a + a^2)] \\ &= v_{\text{avg}}^2 (2 - 2a + a^2) \end{aligned}$$

$$v_{\text{rms}}^2 = v_{\text{avg}}^2 (2 - 2a + a^2) \quad [1]$$

- (c) The graph of $(2 - 2a + a^2)$ versus a appears below, over the range of possible values $0 \leq a \leq 2$.



ANS. FIG. P20.45 (c)

Because the factor $(2 - 2a + a^2)$ is generally larger than 1, equation [1] tells us that $v_{\text{rms}} > v_{\text{avg}}$ except at one point in the graph.

- (d) From the graph, we see that that $v_{\text{rms}} = v_{\text{avg}}$ when the factor $(2 - 2a + a^2) = 1$, which occurs at $\boxed{a = 1}$.

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ANSWERS TO QUICK-QUIZZES

1. (i) (b) (ii) (a)

2. (i) (a) (ii) (c)

3. (d)

4. (c)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P20.2 $\frac{3}{2} \frac{PV}{K_{avg} N_A}$

P20.4 $2.78 \times 10^{-23} \text{ kg} \cdot \text{m/s}$

P20.6 (a) $4.00 \text{ u} = 6.64 \times 10^{-27} \text{ kg}$ (b) $55.9 \text{ u} = 9.28 \times 10^{-26} \text{ kg}$ (c) $207 \text{ u} = 3.44 \times 10^{-25} \text{ kg}$

P20.8 (a) 385 K ; (b) $7.97 \times 10^{-21} \text{ J}$; (c) the molecular mass of the gas

P20.10 True

P20.12 (a) $0.719 \text{ kJ/kg} \cdot \text{K}$; (b) 0.811 kg ; (c) 233 kJ ; (d) 327 kJ

P20.14 (a) See P20.14 (a) for full explanation; (b) See P20.14 (b) for full explanation;

(c) See P20.14 (c) for full explanation

P20.16 The maximum possible value of $\gamma = 1 + \frac{R}{C_V} = 1.67$ occurs for the lowest possible value for $C_V = \frac{3}{2}R$. Therefore the claim of $\gamma = 1.75$ for the newly discovered gas cannot be true.

P20.18 (a) 0.118 (b) 2.35 (c) $Q = 0$ (d) 135 J (e) +135 J

P20.20

This is equivalent to 668 $^{\circ}\text{C}$, which is higher than the melting point of aluminum which is 660 $^{\circ}\text{C}$. Also, the temperature will rise much more when ignition occurs. The engine will melt when put into operation! Therefore, the claim of improved efficiency using an engine fabricated out of aluminum cannot be true.

P20.22 (a) 1.03; (b) ^{35}Cl

P20.24 See P20.24 for the full explanation.

P20.26 (a) See P20.26 (a) for the full explanation (b) 8.31 km

P20.28 (a) See P20.28(a) for the full explanation; (b) 447 J/kg \cdot $^{\circ}\text{C}$. This agrees with the tabulated value of 448 J/kg \cdot $^{\circ}\text{C}$ within 0.3%; (c) 127 J/kg \cdot $^{\circ}\text{C}$. This agrees with the tabulated value of 129 J/kg \cdot $^{\circ}\text{C}$ within 2%

P20.30 (a) pressure increases as volume decreases; (b) See P20.30(b) for full answer; (c) See P20.30(c) for full answer; (d) 0.500 atm $^{-1}$; (e) 0.300 atm $^{-1}$

P20.32 (a) See P20.32 (a) for full explanation; (b) This agrees within 0.2% with the 343 m/s listed in the Table 16.1; (c) See P20.32 (c) for full answer; (d) The speed of sound is somewhat less than each measure of molecular speed. Sound propagation is orderly motion overlaid on the disorder of molecular motion.

P20.34 (a) See P20.34 (a) for full explanation; (b) See P20.34 (b) for full explanation; (c) The expressions are equal because $PV = nRT$ and $\gamma = (C_V + R) / C_V = 1 + R/C_V$ give $R = (\gamma - 1)C_V$, so $PV = n(\gamma - 1)C_V T$ and $PV/(\gamma - 1) = nC_V T$.

- P20.36** (a) $T_i + \frac{2}{3} \frac{W}{nR}$; (b) $P_i \left(1 + \frac{2}{3} \frac{W}{nRT_i} \right)^{5/2}$
- P20.38** (a) See ANS. FIG. P20.36 (a); (b) $v_{\text{mp}} \approx 510 \text{ m/s}$; (c) 575 m/s, 624 m/s;
(d) 44%
- P20.40** (a) $\frac{5}{2}R$; (b) $3R$; (c) $\frac{9}{2}R$; (d) $5R$; (e) Measure the constant-volume specific heat of the gas as a function of temperature and look for plateaus on the graph. If the first jump goes from $\frac{3}{2}R$ to $\frac{5}{2}R$, the molecules can be diagnosed as linear. If the first jump goes from $\frac{3}{2}R$ to $3R$, the molecules must be nonlinear. The tabulated data at one temperature are insufficient for the determination. At room temperature some of the heavier molecules appear to be vibrating.
- P20.42** See P20.42 for full explanation.
- P20.44** (a) The larger-mass molecules settle to the outside; (b) $n = n_0 e^{m_0 r^2 \omega^2 / 2k_B T}$