22

Electric Fields

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP22.1** **Conceptualize** Imagine the electron in Figure 22.21 is replaced by a nanoparticle. Figure 22.21 demonstrates the curved path followed by the nanoparticle.

Categorize Each nanoparticle is modeled as a *particle in an electric field,* a *particle under constant velocity* in the direction parallel to the plates, a *particle under constant acceleration* in the direction perpendicular to the plates, and a *particle under a net force* perpendicular to the plates.

Analyze From Equation 22.8, the electric force on a nanoparticle is in the downward direction in Figure 22.21, with magnitude

$$F_e = |q|E = eE \qquad (1)$$

From the particle under a net force model in the vertical direction,

$$\sum F_y = ma_y \quad \rightarrow \quad F_e + mg = ma_y \quad \rightarrow \quad a_y = \frac{eE + mg}{m} = \frac{eE}{m} + g \quad (2)$$

From the particle under constant acceleration model, the position of the nanoparticle in the y direction at any time is

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}a_y t^2 = y_i - \tfrac{1}{2}\left(\frac{eE}{m} + g\right)t^2 \qquad (3)$$

where we have used Equation (2) to substitute for the acceleration. From the particle under constant velocity model, the position of the nanoparticle in the horizontal direction is

$$x_f = x_i + v_x t \qquad (4)$$

Suppose we define the initial position of the nanoparticle as just below the left end of the upper plate, so that the nanoparticle enters the field at the highest and leftmost point possible in Figure 22.21. Lets call this point the origin, so Equations (3) and (4) become

$$y_f = -\tfrac{1}{2}\left(\frac{eE}{m} + g\right)t^2 \qquad (5)$$

$$x_f = v_x t \qquad (6)$$

Solve Equation (6) for t and substitute into Equation (5):

$$y_f = -\tfrac{1}{2}\left(\frac{eE}{m} + g\right)\left(\frac{x_f}{v_x}\right)^2 \qquad (7)$$

Substitute numerical values to find the vertical position of the nanoparticle as it arrives at position of the end of the plate:

$$y_f = -\tfrac{1}{2}\left(\frac{\left(1.60 \square 10^{-19}\ \text{C}\right)\left(2.00 \square 10^{4}\ \text{N/C}\right)}{6.50\times 10^{-16}\ \text{kg}} + 9.80\ \text{m/s}^2\right)\left(\frac{1.00\ \text{m}}{30.0\ \text{m/s}}\right)^2$$

$$= -8.18\times 10^{-3}\ \text{m} = -8.18\ \text{mm}$$

You are embarrassed because no nanoparticles exit your device. The plates are only separated by 8.00 mm, so if the nanoparticles are deflected downward by 8.18 mm from the top edge, $\boxed{\text{they strike the bottom plate before exiting the parallel plates}}$.

Finalize Notice in this problem that the acceleration caused by the electric force is comparable in size to that of the gravitational force, so both forces must be included in Equation (2), unlike in Example 22.8. If you left out the gravitational force in your calculation for this problem, your prediction would show that the nanoparticles do indeed leave the plates!

Answer: The nanoparticles strike the lower plate and do not exit the device.

***TP22.2** *Answers:* (a) + (b) + (c) – (d) – (e) – (f) – (g) – (h) + (i) + (j) – (k) The two materials that are farthest apart on the list are in part (e), silicone and cotton. They are 20 entries apart. However, choice (c), paper and PVC, represents two materials separated by 19 entries. Paper is a little higher into the positive range than is cotton, so it might be a toss-up as to which gives the greater amount of charge transfer.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 22.1 Properties of Electric Charges

P22.1 (a) The charge due to loss of one electron is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}$$

The mass of an average neutral hydrogen atom is 1.007 9 u.

Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

$$\text{mass}=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$$

(c) Gain of one electron: charge of $\text{Cl}^{-}=\boxed{1.60\times10^{-19}\text{ C}}$

$$\text{mass}=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$$

(d) Loss of two electrons: charge of $\text{Ca}^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=$

$$\boxed{+3.20\times10^{-19}\text{ C}}$$

$$\text{mass}=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)$$
$$=\boxed{6.65\times10^{-26}\text{ kg}}$$

(e) Gain of three electrons: charge of $\text{N}^{3-}=3\left(-1.60\times10^{-19}\text{ C}\right)=$

$$\boxed{-4.80\times10^{-19}\text{ C}}$$

$$\text{mass}=14.007\left(1.66\times10^{-27}\text{ kg}\right)+3\left(9.11\times10^{-31}\text{ kg}\right)$$
$$=\boxed{2.33\times10^{-26}\text{ kg}}$$

(f) Loss of four electrons: charge of $N^{4+} = 4\left(1.60\times10^{-19}\ \text{C}\right) =$

$\boxed{+6.40\times10^{-19}\ \text{C}}$

$$\begin{aligned}\text{mass} &= 14.007\left(1.66\times10^{-27}\ \text{kg}\right) - 4\left(9.11\times10^{-31}\ \text{kg}\right)\\ &= \boxed{2.32\times10^{-26}\ \text{kg}}\end{aligned}$$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom. $\text{Charge} = 7\left(1.60\times10^{-19}\ \text{C}\right) = \boxed{1.12\times10^{-18}\ \text{C}}$

$$\begin{aligned}\text{mass} &= 14.007\left(1.66\times10^{-27}\ \text{kg}\right) - 7\left(9.11\times10^{-31}\ \text{kg}\right)\\ &= \boxed{2.32\times10^{-26}\ \text{kg}}\end{aligned}$$

(h) Gain of one electron: charge $= \boxed{-1.60\times10^{-19}\ \text{C}}$

$$\begin{aligned}\text{mass} &= \left[2\left(1.007\ 9\right) + 15.999\right]1.66\times10^{-27}\ \text{kg} + 9.11\times10^{-31}\ \text{kg}\\ &= \boxed{2.99\times10^{-26}\ \text{kg}}\end{aligned}$$

Section 22.3 Coulomb's Law

P22.2 (a) The two ions are both singly charged, $|q| = 1e$, one positive and one negative. Thus,

$$\begin{aligned}|F| &= \frac{k_e|q_1||q_2|}{r^2} = \frac{k_e e^2}{r^2}\\ &= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(0.500\times10^{-9}\ \text{m}\right)^2}\\ &= \boxed{9.21\times10^{-10}\ \text{N}}\end{aligned}$$

(b) No. The electric force depends only on the magnitudes of the two charges and the distance between them.

P22.3 The electric force is given by

$$F = k_e \frac{q_1 q_2}{(r_{12})^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \frac{(+40 \text{ C})(-40 \text{ C})}{(2\ 000 \text{ m})^2}$$
$$= -3.60 \times 10^6 \text{ N (attractive)} = \boxed{3.60 \times 10^6 \text{ N downward}}$$

P22.4 Suppose each person has mass 70 kg. In terms of elementary charges, each person consists of precisely equal numbers of protons and electrons and a nearly equal number of neutrons. The electrons comprise very little of the mass, so for each person we find the total number of protons and neutrons, taken together:

$$(70 \text{ kg}) \left(\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}} \right) = 4 \times 10^{28} \text{ u}$$

Of these, nearly one half, 2×10^{28}, are protons, and 1% of this is 2×10^{26}, constituting a charge of $(2 \times 10^{26})(1.60 \times 10^{-19} \text{ C}) = 3 \times 10^{7}$ C.

Thus, Feynman's force has magnitude

$$F = \frac{k_e q_1 q_1}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3 \times 10^7 \text{ C})^2}{(0.5 \text{ m})^2} \sim \boxed{10^{26} \text{ N}}$$

where we have used a half-meter arm's length. According to the particle in a gravitational field model, if the Earth were in an externally-produced uniform gravitational field of magnitude 9.80 m/s^2, it would weigh $F_g = mg = (6 \times 10^{24} \text{ kg})(10 \text{ m/s}^2) \sim 10^{26}$ N.

Thus, the forces are of the same order of magnitude.

P22.5 (a) $|F| = \dfrac{k_e |q_1||q_2|}{r^2}$

$$F = \frac{k_e e^2}{r^2} = \frac{\left(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(7.50\times 10^{-9}\ \mathrm{C}\right)\left(4.20\times 10^{-9}\ \mathrm{C}\right)}{(1.80\ \mathrm{m})^2}$$

$$= \boxed{8.74\times 10^{-8}\ \mathrm{N}}$$

(b) The charges are like charges. $\boxed{\text{The force is repulsive.}}$

***P22.6** **Conceptualize** The Earth and the Moon are far apart compared to their radii, so they can be modeled as point charges.

Categorize With the modeling proposed in the Conceptualize step, this problem becomes a simple calculation of an electric force between two charged particles.

Analyze Using Equation 22.1, the magnitude of the electric force between the Earth and the Moon is

$$F_e = k_e \frac{q_{\text{Earth}} q_{\text{Moon}}}{r^2} = \left(8.99 \square 10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.00 \square 10^5\ \mathrm{C}\right)^2}{\left(3.84 \square 10^8\,\mathrm{m}\right)^2} = 610\ \mathrm{N}$$

This is a very small force on a planetary scale. In comparison, the gravitational force between the Earth and the Moon is

$$F_g = G\frac{m_{\text{Earth}} m_{\text{Moon}}}{r^2}$$

$$= \left(6.67 \square 10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.98 \square 10^{24}\ \mathrm{kg}\right)\left(7.36 \square 10^{22}\ \mathrm{kg}\right)}{\left(3.84 \square 10^8\,\mathrm{m}\right)^2} = 1.99 \square 10^{20}\ \mathrm{N}$$

$\boxed{\text{This force is 18 orders of magnitude larger than the electric force we calculated.}}$

Finalize Because of its small magnitude, the electric force cannot have any influence on the Moon when compared to gravitational forces. That is likely what the professor had raised his hand about. Check your calculations before giving a presentation!

Answer: The electric force is 18 orders of magnitude smaller than the gravitational force.

P22.7 (a) Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}(-\hat{\mathbf{i}}), \text{ where } d = 1.50 \text{ m}$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of

$$x = 0.634d = 0.634(1.50 \text{ m}) = \boxed{0.951 \text{ m}}$$

(b) $\boxed{\text{Yes, if the third bead has positive charge.}}$ The equilibrium would be stable because if charge Q were displaced either to the left or right on the rod, the new net force would be opposite to the direction Q has been displaced, causing it to be pushed back to its equilibrium position.

P22.8 (a) Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e q_1 Q}{x^2}\hat{\mathbf{i}} + \frac{k_e q_2 Q}{(d-x)^2}(-\hat{\mathbf{i}})$$

The net force will be zero if $\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2}$:

$$\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2} \quad \rightarrow \quad (d-x)^2 = x^2\left(\frac{q_2}{q_1}\right) \quad \rightarrow \quad d - x = x\sqrt{\frac{q_2}{q_1}}$$

because $d > x$. Thus,

$$d - x = x\sqrt{\frac{q_2}{q_1}} \quad \rightarrow \quad d = x + x\frac{\sqrt{q_2}}{\sqrt{q_1}} = x\left(\frac{\sqrt{q_1}+\sqrt{q_2}}{\sqrt{q_1}}\right)$$

$$\rightarrow \quad x = \boxed{\frac{\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}d}$$

(b) $\boxed{\text{Yes, if the third bead has positive charge.}}$ The equilibrium would be stable because if charge Q were displaced either to the left or right on the rod, the new net force would be opposite to the direction Q has been displaced, causing it to be pushed back to its equilibrium position.

P22.9 (a) $$F = \frac{k_e e^2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(0.529\times10^{-10}\ \text{m}\right)^2} = \boxed{8.22\times10^{-8}\ \text{N}}$$

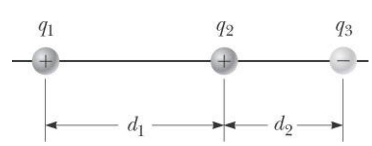
toward the other particle.

(b) We have $F = \dfrac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{\left(8.22\times10^{-8}\ \text{N}\right)\left(0.529\times10^{-10}\ \text{m}\right)}{9.11\times10^{-31}\ \text{kg}}}$$

$$= \boxed{2.19\times10^6\ \text{m/s}}$$

P22.10 The forces are as shown in ANS. FIG. P22.10.



ANS. FIG. P22.10

$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00\times10^{-6}\ \text{C})(1.50\times10^{-6}\ \text{C})}{(3.00\times10^{-2}\ \text{m})^2}$$
$$= 89.9\ \text{N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00\times10^{-6}\ \text{C})(2.00\times10^{-6}\ \text{C})}{(5.00\times10^{-2}\ \text{m})^2}$$
$$= 43.2\ \text{N}$$

$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(1.50\times10^{-6}\ \text{C})(2.00\times10^{-6}\ \text{C})}{(2.00\times10^{-2}\ \text{m})^2}$$
$$= 67.4\ \text{N}$$

(a) The net force on the 6 μC charge is

$$F_{(6\mu\text{C})} = F_1 - F_2 = \boxed{46.7\ \text{N to the left}}$$

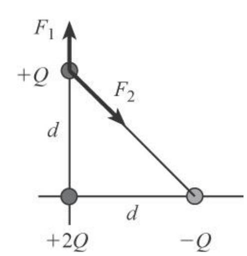
(b) The net force on the 1.5 μC charge is

$$F_{(1.5\mu\text{C})} = F_1 + F_3 = \boxed{157\ \text{N to the right}}$$

(c) The net force on the –2 μC charge is

$$F_{(-2\mu\text{C})} = F_2 + F_3 = \boxed{111\ \text{N to the left}}$$

P22.11 The force due to the first charge is given by



ANS. FIG. P22.11

$$\vec{\mathbf{F}}_1 = \frac{k_e Q(2Q)}{d^2}\hat{\mathbf{j}} = \frac{k_e Q^2}{d^2}\left[2\hat{\mathbf{j}}\right]$$

and the force due to the second charge is given by

$$\vec{\mathbf{F}}_2 = \frac{k_e Q(Q)}{(d^2+d^2)}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}\right] = \frac{k_e Q^2}{d^2}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{2\sqrt{2}}\right]$$

thus the total force on the point charge +Q located at $x = 0$

and $y = d$ is

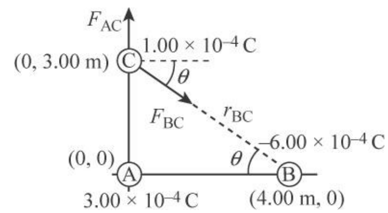
$$\vec{\mathbf{F}}_1+\vec{\mathbf{F}}_2=\frac{k_eQ^2}{d^2}\left[2\hat{\mathbf{j}}\right]+\frac{k_eQ^2}{d^2}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{2\sqrt{2}}\right]=\boxed{k_e\frac{Q^2}{d^2}\left[\frac{1}{2\sqrt{2}}\hat{\mathbf{i}}+\left(2-\frac{1}{2\sqrt{2}}\right)\hat{\mathbf{j}}\right]}$$

P22.12 Charge C is attracted to charge B and repelled by charge A, as shown in

ANS. FIG. P22.12. In the sketch,

$$r_{BC}=\sqrt{(4.00\text{ m})^2+(3.00\text{ m})^2}=5.00\text{ m}$$

and



ANS. FIG. P22.12

$$\theta=\tan^{-1}\left(\frac{3.00\text{ m}}{4.00\text{ m}}\right)=36.9^\circ$$

(a) $(F_{AC})_x=\boxed{0}$

(b) $(F_{AC})_y=|F_{AC}|=k_e\dfrac{|q_A||q_C|}{r_{AC}^2}$

$$(F_{AC})_y=(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-4}\text{ C})(1.00\times10^{-4}\text{ C})}{(3.00\text{ m})^2}$$
$$=\boxed{30.0\text{ N}}$$

(c) $|F_{BC}|=k_e\dfrac{|q_B||q_C|}{r_{BC}^2}$

$$=(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(6.00\times10^{-4}\text{ C})(1.00\times10^{-4}\text{ C})}{(5.00\text{ m})^2}$$
$$=\boxed{21.6\text{ N}}$$

(d) $(F_{BC})_x=|F_{BC}|\cos\theta=(21.6\text{ N})\cos(36.9^\circ)=\boxed{17.3\text{ N}}$

(e) $(F_{BC})_y=-|F_{BC}|\sin\theta=-(21.6\text{ N})\sin(36.9^\circ)=\boxed{-13.0\text{ N}}$

(f) $(F_R)_x=(F_{AC})_x+(F_{BC})_x=0+17.3\text{ N}=\boxed{17.3\text{ N}}$

(g) $\left(F_R\right)_y = \left(F_{\text{AC}}\right)_y + \left(F_{\text{BC}}\right)_y = 30.0 - 13.0\ \text{N} = \boxed{17.0\ \text{N}}$

(h) $F_R = \sqrt{\left(F_R\right)_x^2 + \left(F_R\right)_y^2} = \sqrt{(17.3\ \text{N})^2 + (17.0\ \text{N})^2} = 24.3\ \text{N}$

Both components are positive, placing the force in the first quadrant:

$$\varphi = \tan^{-1}\left[\frac{\left(F_R\right)_y}{\left(F_R\right)_x}\right] = \tan^{-1}\left(\frac{17.0\ \text{N}}{17.3\ \text{N}}\right) = 44.5°$$

Therefore, $\vec{\mathbf{F}}_R = \boxed{24.3\ \text{N at } 44.5° \text{ above the } +x \text{ direction}}$.

P22.13 Each charge exerts a force of magnitude $\dfrac{k_e qQ}{(d/2)^2 + x^2}$ on the negative charge $-Q$: the top charge exerts its force directed upward and to the left, and bottom charge exerts its force directed downward and to the left, each at angle $\theta = \tan^{-1}\left(\dfrac{d}{2x}\right)$, respectively, above and below the x axis. The two positive charges together exert a net force:

$$\begin{aligned}\vec{\mathbf{F}} &= -2\frac{k_e qQ}{(d/2)^2 + x^2}\cos\theta\hat{\mathbf{i}} \\ &= -2\left[\frac{k_e qQ}{\left(d^2/4 + x^2\right)}\right]\left[\frac{x}{\left(d^2/4 + x^2\right)^{1/2}}\right]\hat{\mathbf{i}} \\ &= \left[\frac{-2xk_e qQ}{\left(d^2/4 + x^2\right)^{3/2}}\right]\hat{\mathbf{i}} = m\vec{\mathbf{a}}\end{aligned}$$

or for $x \ll \dfrac{d}{2}$, $\vec{\mathbf{a}} \approx -\left(\dfrac{2k_e qQ}{md^3/8}\right)\vec{\mathbf{x}} \quad \rightarrow \quad \vec{\mathbf{a}} \approx -\left(\dfrac{16k_e qQ}{md^3}\right)\vec{\mathbf{x}}$

(a) $\boxed{\text{The acceleration of the charge is equal to a negative constant times its displacement from equilibrium, as in } \vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}, \text{ so we have Simple Harmonic Motion with } \omega^2 = \frac{16k_e qQ}{md^3}.}$

(b) $\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{16k_e qQ}{md^3} \rightarrow T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(c) $v_{\max} = \omega A = \boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$

P22.14 Each of the dust particles is a particle in equilibrium. Express this mathematically for one of the particles:

$$\sum \vec{\mathbf{F}} = 0 \quad \rightarrow \quad F_e - F_g = 0 \quad \rightarrow \quad F_e = F_g$$

where we have recognized that the gravitational force is attractive and the electric force is repulsive, so the forces on one particle are in opposite directions. Substitute for the forces from Coulomb's law and Newton's law of universal gravitation, and solve for q, the unknown charge on each dust particle:

$$k_e\frac{q^2}{r^2} = G\frac{m^2}{r^2} \quad \rightarrow \quad q = \sqrt{\frac{G}{k_e}}\, m$$

Substitute numerical values:

$$q = \sqrt{\frac{6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}{8.987\ 6 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}}}\left(1.00 \times 10^{-9}\ \mathrm{kg}\right)$$
$$= 8.61 \times 10^{-20}\ \mathrm{C}$$

This is about half of the smallest possible free charge, the charge of the electron. No such free charge exists. Therefore, the forces cannot balance. Even if the charge on each dust particle is due to one electron, the net force will be repulsive and the particles will move apart.

Section 22.4 Analysis Model: Particle in a Field (Electric)

P22.15 For equilibrium, $\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_g$ or $q\vec{\mathbf{E}} = -mg\left(-\hat{\mathbf{j}}\right)$. Thus,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}}.$$

(a) For an electron,

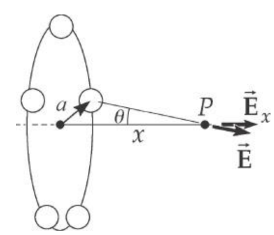
$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{-1.60\times10^{-19}\text{ C}}\hat{\mathbf{j}}$$
$$= \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\hat{\mathbf{j}}}$$

(b) For a proton, which is 1 836 times more massive than an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{-1.60\times10^{-19}\text{ C}}\hat{\mathbf{j}}$$
$$= \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\hat{\mathbf{j}}}$$

P22.16 (a) One of the charges creates at P a field

$$\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} = \frac{\left(k_eQ/n\right)}{a^2+x^2}\hat{\mathbf{i}}$$



ANS. FIG. P22.16

at an angle θ to the x axis as shown in ANS. FIG. P22.16. When all the charges produce the field, for $n > 1$, by symmetry the components perpendicular to the x axis add to zero.

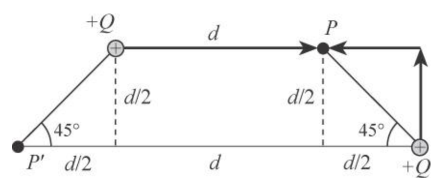
The total field is then

$$\vec{\mathbf{E}} = nE_x\hat{\mathbf{i}} = n\left(\frac{k_e(Q/n)\hat{\mathbf{i}}}{a^2+x^2}\cos\theta\right) = \boxed{\frac{k_eQx\hat{\mathbf{i}}}{\left(a^2+x^2\right)^{3/2}}}$$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P22.17 (a) See ANS. FIG. P22.17(a). The distance from the +Q charge on the upper left is d, and the distance from the +Q charge on the lower right to point P is

$$\sqrt{(d/2)^2+(d/2)^2}$$



ANS. FIG. P22.17(a)

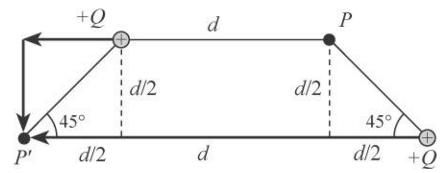
The total electric field at point P is then

$$\vec{\mathbf{E}}_P = k_e\frac{Q}{d^2}\hat{\mathbf{i}} + k_e\frac{Q}{\left[(d/2)^2+(d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)$$

$$= k_e\left[\frac{Q}{d^2}\hat{\mathbf{i}} + \frac{Q}{d^2/2}\left(\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)\right]$$

$$= \boxed{k_e\frac{Q}{d^2}\left[\left(1-\sqrt{2}\right)\hat{\mathbf{i}}+\sqrt{2}\hat{\mathbf{j}}\right]}$$

(b) See ANS. FIG. P22.17(b). The distance from the +Q charge on the lower right to point P′ is $2d$, and the distance from the $+Q$ charge on the upper right to point P' is



ANS. FIG. P22.17(b)

$$\sqrt{(d/2)^2 + (d/2)^2}$$

The total electric field at point P' is then

$$\vec{\mathbf{E}}_{P'} = k_e \frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}\right) + k_e \frac{Q}{(2d)^2}\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{E}}_{P'} = -k_e\left[\frac{Q}{d^2/2}\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) + \frac{Q}{4d^2}\left(-\hat{\mathbf{i}}\right)\right]$$

$$= -k_e \frac{Q}{4d^2}\left[\frac{8}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) + \left(\hat{\mathbf{i}}\right)\right]$$

$$\vec{\mathbf{E}}_{P'} = \boxed{-k_e \frac{Q}{4d^2}\left[\left(1 + 4\sqrt{2}\right)\hat{\mathbf{i}} + 4\sqrt{2}\hat{\mathbf{j}}\right]}$$

P22.18 The first charge creates at the origin a field $\dfrac{k_e Q}{a^2}$ to the right. Both charges are on the x axis, so the total field cannot have a vertical component, but it can be either to the right or to the left. If the total field at the origin is to the right, then q must be negative:

ANS. FIG. P22.18

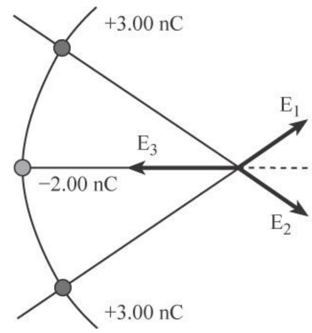
$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\hat{\mathbf{i}} \quad \rightarrow \quad q = -9Q$$

In the alternative, if the total field at the origin is to the left,

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\left(-\hat{\mathbf{i}}\right) \quad \rightarrow \quad q = +27Q$$

$$\boxed{\begin{array}{l}\text{The field at the origin can be to the right, if the unknown charge is } -9Q,\\ \text{or the field can be to the left, if and only if the unknown charge is } +27Q.\end{array}}$$

P22.19 Call $Q = 3.00$ nC and $q = |-2.00 \text{ nC}| = 2.00$ nC, and $r = 4.00 \text{ cm} = 0.040\ 0$ m. Then,



ANS. FIG. P22.19

$$E_1 = E_2 = \frac{k_e Q}{r^2} \text{ and } E_3 = \frac{k_e q}{r^2}$$

Then,

$$E_y = 0$$

$$E_x = E_{total} = 2\frac{k_e Q}{r^2}\cos 30.0° - \frac{k_e q}{r^2}$$

$$E_x = \frac{k_e}{r^2}\left(2Q\cos 30.0° - q\right)$$

$$E_x = \left[\frac{8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{(0.040\ 0 \text{ m})^2}\right]$$

$$\times\left[2\left(3.00\times 10^{-9}\text{ C}\right)\cos 30.0° - 2.00\times 10^{-9}\text{ C}\right]$$

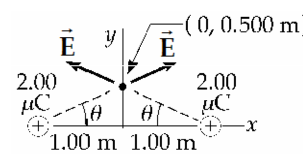
$$= 1.80\times 10^4 \text{ N/C}$$

(a) $\boxed{1.80\times 10^4 \text{ N/C to the right}}$

(b) The electric force on a point charge placed at point P is

$$F = qE = \left(-5.00\times 10^{-9}\text{ C}\right)E = \boxed{-8.98\times 10^{-5}\text{ N (to the left)}}$$

P22.20 (a) The distance from each charge to the point at $y = 0.500$ m is



ANS. FIG. P22.20

$$d = \sqrt{(1.00 \text{ m})^2 + (0.500 \text{ m})^2} = 1.12 \text{ m}$$

the magnitude of the electric field from each charge at that point is then given by

$$E = \frac{k_e q}{r^2} = \frac{\left(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times 10^{-6}\text{ C}\right)}{(1.12 \text{ m})^2} = 14\ 400 \text{ N/C}$$

The x components of the two fields cancel and the y components add, giving

$$E_x = 0 \text{ and } \quad E_y = 2(14\ 400 \text{ N/C})\sin 26.6° = 1.29\times 10^4 \text{ N/C}$$

so $\boxed{\vec{\mathbf{E}} = 1.29\times 10^4\,\hat{\mathbf{j}} \text{ N/C}}$.

(b) The electric force at this point is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-3.00\times 10^{-6} \text{ C})(1.29\times 10^4 \text{ N/C}\hat{\mathbf{j}})$$
$$= \boxed{-3.86\times 10^{-2}\,\hat{\mathbf{j}} \text{ N}}$$

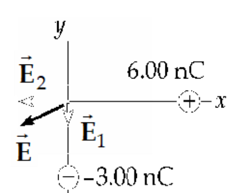
P22.21 (a) The electric field at the origin due to each of the charges is given by

$$\vec{\mathbf{E}}_1 = \frac{k_e|q_1|}{r_1^2}(-\hat{\mathbf{j}})$$
$$= \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times 10^{-9} \text{ C})}{(0.100 \text{ m})^2}(-\hat{\mathbf{j}})$$
$$= -(2.70\times 10^3 \text{ N/C})\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2 = \frac{k_e|q_2|}{r_2^2}(-\hat{\mathbf{i}})$$
$$= \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(6.00\times 10^{-9} \text{ C})}{(0.300 \text{ m})^2}(-\hat{\mathbf{i}})$$
$$= -(5.99\times 10^2 \text{ N/C})\hat{\mathbf{i}}$$

and their sum is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_1 = \boxed{-(5.99\times 10^2 \text{ N/C})\hat{\mathbf{i}} - (2.70\times 10^3 \text{ N/C})\hat{\mathbf{j}}}$$



ANS. FIG. P22.21

(b) The vector electric force is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (5.00\times 10^{-9} \text{ C})(-599\hat{\mathbf{i}} - 2\ 700\hat{\mathbf{j}}) \text{ N/C}$$

$$\vec{\mathbf{F}} = \left(-3.00\times10^{-6}\,\hat{\mathbf{i}} - 13.5\times10^{-6}\,\hat{\mathbf{j}}\right)\text{ N} = \boxed{\left(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}}\right)\ \mu\text{N}}$$

P22.22 The electric field at any point x is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{[x-(-a)]^2} = \frac{k_e q(4ax)}{\left(x^2 - a^2\right)^2}$$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{4a\left(k_e q\right)}{x^3}}$.

Section 22.5 Electric Field Lines

P22.23 (a) The electric field has the general appearance shown in ANS. FIG. P22.23 below.

(b) It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second panel of ANS. FIG. P22.23 indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(c) You may need to review vector addition in Chapter 1. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}.$$

As shown in the bottom panel of

ANS. FIG. P22.23,

$$\vec{\mathbf{E}}_1 = k_e \frac{q}{a^2}$$

to the right and upward at 60°, and

$$\vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}$$

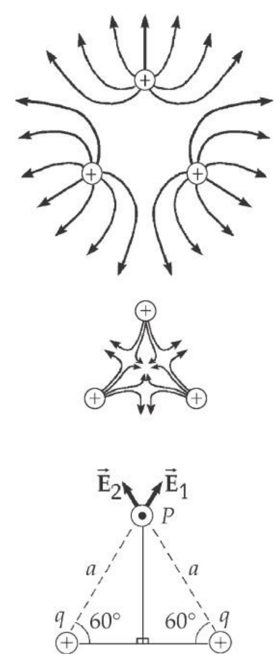
to the left and upward at 60°. So,

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right] = \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 22.6 Motion of a Charged Particle in a Uniform Electric Field

P22.24 (a) We obtain the acceleration of the proton from the particle under a net force model, with $F = qE$ representing the electric force:



ANS. FIG. P22.23

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(640\ \text{N/C}\right)}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{6.14 \times 10^{10}\ \text{m/s}^2}$$

(b) The particle under constant acceleration model gives us

$v_f = v_i + at$, from which we obtain

$$t = \frac{v_f - 0}{a} = \frac{1.20 \times 10^6\ \text{m/s}}{6.14 \times 10^{10}\ \text{m/s}^2} = \boxed{19.5\ \mu\text{s}}$$

(c) Again, from the particle under constant acceleration model,

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(6.14 \times 10^{10}\ \text{m/s}^2)(19.5 \times 10^{-6}\ \text{s})^2$$
$$= \boxed{11.7\ \text{m}}$$

(d) The final kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(1.20 \times 10^6\ \text{m/s})^2 = \boxed{1.20 \times 10^{-15}\ \text{J}}$$

P22.25 $\vec{\mathbf{E}}$ is directed along the y direction; therefore, $a_x = 0$ and $x = v_{xi}t$.

(a) $$t = \frac{x}{v_{xi}} = \frac{0.050\ 0\ \text{m}}{4.50 \times 10^5\ \text{s}} = 1.11 \times 10^{-7}\ \text{s} = \boxed{111\ \text{ns}}$$

(b) $$a_y = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(9.60 \times 10^3\ \text{N/C}\right)}{1.67 \times 10^{-27}\ \text{kg}} = 9.21 \times 10^{11}\ \text{m/s}^2$$

$$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2:$$

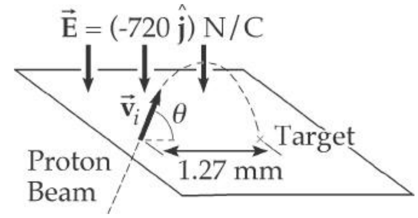
$$y_f = \frac{1}{2}\left(9.21 \times 10^{11}\ \text{m/s}^2\right)\left(1.11 \times 10^{-7}\ \text{s}\right)^2$$
$$= 5.68 \times 10^{-3}\ \text{m} = \boxed{5.67\ \text{mm}}$$

(c) $$v_x = 4.50 \times 10^5\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_y t = \left(9.21 \times 10^{11}\ \text{m/s}^2\right)\left(1.11 \times 10^{-7}\ \text{s}\right) = 1.02 \times 10^5\ \text{m/s}$$

$$\vec{\mathbf{v}} = \boxed{\left(450\hat{\mathbf{i}} + 102\hat{\mathbf{j}}\right)\text{ km/s}}$$

P22.26 (a) $\boxed{\text{Particle under constant velocity}}$



ANS. FIG. P22.26

(b) $\boxed{\text{Particle under constant acceleration}}$

(c) The vertical acceleration caused by the electric force is constant and downward; therefore, $\boxed{\text{the proton moves in a parabolic path just like a projectile in a gravitational field.}}$

(d) We may neglect the effect of the acceleration of gravity on the proton because the magnitude of the vertical acceleration caused by the electric force is

$$a_y = \frac{eE}{m_p} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(720\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 6.90\times10^{10}\text{ m/s}^2$$

which is much greater than that of gravity.

Replacing acceleration g in Equation 4.20 with eE/m_P, we have

$$R = \frac{v_i^2\sin 2\theta}{eE/m_p} = \boxed{\frac{m_p v_i^2\sin 2\theta}{eE}}$$

(e) $$R = \frac{m_p v_i^2\sin 2\theta}{eE} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.55\times10^{3}\text{ m/s}\right)^2\sin 2\theta}{\left(1.60\times10^{-19}\text{ C}\right)\left(720\text{ N/C}\right)}$$

$$= 1.27\times10^{-3}\text{ m}$$

which gives $\sin 2\theta = 0.961$, or

$$\theta = \boxed{36.9°} \quad\text{or}\quad 90.0° - \theta = \boxed{53.1°}$$

(f) $\Delta t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i \cos\theta}$

If $\theta = 36.9°$, $\Delta t = \boxed{166 \text{ ns}}$. If $\theta = 53.1°$, $\Delta t = \boxed{221 \text{ ns}}$.

***P22.27** **Conceptualize** Imagine the electron in Figure 22.21 is replaced by an ink drop. Figure 22.21 demonstrates the curved path followed by the ink drop. Now rotate the entire figure clockwise by 90° to represent the fact that the ink drops are fired downward into the region between the plates.

Categorize Each ink drop is modeled as a *particle in field (electric),* a *particle under constant velocity* in the direction parallel to the plates, a *particle under constant acceleration* perpendicular to the plates, and a *particle under a net force* perpendicular to the plates.

Analyze From Equation 22.8, the electric force on an ink drop is in the horizontal direction in the rotated version of Figure 22.21, with magnitude

$$F_e = qE \qquad (1)$$

From the particle under a net force model in the horizontal direction,

$$\sum F_x = ma_x \quad \rightarrow \quad F_e = ma_x \quad \rightarrow \quad a_x = \frac{qE}{m} \qquad (2)$$

From the particle under constant acceleration model, the position of the ink drop in the *x* direction at any time is

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 = x_i + \tfrac{1}{2}\left(\frac{qE}{m}\right)t^2 \qquad (3)$$

where we have used Equation (2) to substitute for the acceleration. From the particle under constant velocity model, the position of the ink drop in the vertical direction is

$$y_f = y_i + v_y t \qquad (4)$$

Define $y_i = 0$ as the point at which the ink drop enters the top of the parallel plates. Incorporate this idea into Equation (4) and solve for t:

$$t = \frac{y_f}{v_y} \qquad (5)$$

Solve Equation (3) for the horizontal deflection of an ink drop and substitute for the time from Equation (5):

$$\Delta x = x_f - x_i = \tfrac{1}{2}\left(\frac{qE}{m}\right)t^2 = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{y_f}{v_y}\right)^2 \qquad (6)$$

We want a particular deflection Δx at the instant when the ink drop arrives at the lower end of the plates, so that $y_f = -\ell$:

$$\Delta x = \tfrac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{-\ell}{v_y}\right)^2 \qquad (7)$$

Solve Equation (7) for the required charge:

$$q = 2\left(\frac{m}{E}\right)\left(\frac{v_y}{-\ell}\right)^2 \Delta x$$

Substitute numerical values:

$$q = 2\left(\frac{1.25\times10^{-11}\text{ kg}}{6.35\times10^{4}\text{ N/ C}}\right)\left(\frac{-18.5\text{ m/ s}}{-0.022\,5\text{ m}}\right)^2 (0.000\,17\text{ m}) = \boxed{4.52\times10^{-14}\text{ C}}$$

Finalize The sign of the charge is not important, since we were only provided with a magnitude of the deflection, with no direction indicated.

Answer: 4.52×10^{-14} C

***P22.28** **Conceptualize** The electric field between the plates will cause the path of the electrons to curve downward after being projected in the direction shown in Figure P22.28. In a very strong electric field, the downward force on the electrons will cause them to strike the lower plate. For a certain value of the electric field, the electrons will follow a trajectory and exit the apparatus by just missing the right edge of the lower plate.

Categorize The electrons will be modeled as *particles in a field (electric), particles under constant acceleration* in the vertical direction, and *particles under constant velocity* in the horizontal direction.

Analyze (a) The largest *positive* angle relative to the x axis at which the electrons leave the apparatus, in the absence of an electric field, is the same as the initial angle of projection of the electrons into the apparatus, and can be determined from Figure P22.28:

$$\tan\theta_i = \frac{d}{\ell} \quad \rightarrow \quad \theta_i = \tan^{-1}\left(\frac{d}{\ell}\right) \qquad (1)$$

where the subscript i indicates the *initial* angle at which the electrons enter the apparatus. The largest *negative* angle relative to the x axis at which the electrons leave the apparatus is determined by the curve that the electrons follow when there is an electric field between the plates.

Because the magnitude of the electric force on the electron is given by Equation 22.8, $F_e = qE$, which is constant like the gravitational force on a particle, given by Equation 5.5, $F_g = mg$, we expect the path of the electron between the plates to be parabolic, as discussed for a projectile in a gravitational field in Chapter 4. For the critical situation in which the electron just misses the right edge of the lower plate, the trajectory is similar to that of the projectile in Figure 4.8. Based on that figure, because the electron returns to the same vertical position from which it began, its angle as it arrives at that position is the same as the initial angle, but with a negative value. Therefore, the range of values of the angles of the electrons leaving the apparatus is

$$-\theta_i < \theta < \theta_i \quad \rightarrow \quad -\tan^{-1}\left(\frac{d}{\ell}\right) < \theta < \tan^{-1}\left(\frac{d}{\ell}\right) \qquad (1)$$

Substitute numerical values:

$$-\tan^{-1}\left(\frac{0.030\,0\text{ m}}{0.500\text{ m}}\right) < \theta < \tan^{-1}\left(\frac{0.030\,0\text{ m}}{0.500\text{ m}}\right) \quad \rightarrow \quad \boxed{-3.43^\circ < \theta < 3.43^\circ}$$

(b) Now, let's find the electric field that will cause just the right trajectory to give us the maximum possible deviation angle for the electron. Combine the particle in a field model, Equation 22.8, and the particle under a net force model to determine the vertical acceleration of the electron:

$$\sum F_y = ma_y \quad \rightarrow \quad qE = ma_y \quad \rightarrow \quad a_y = -\frac{eE}{m_e} \qquad (2)$$

Because everything on the right side of Equation (2) is constant, we can model the electron as a particle under constant acceleration in the vertical direction. Therefore, the vertical position of the electron at any time is given by the vertical component of Equation 4.9:

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \qquad (3)$$

Define the left edge of the lower plate as the origin, and substitute for the acceleration from Equation (2):

$$y_f = (v_i \sin\theta_i)t - \frac{eE}{2m_e}t^2 \qquad (4)$$

Because there is no force on the electron in the horizontal direction, we can model the electron as a particle under constant velocity in that direction. Therefore, the horizontal position of the electron at any time is given by the horizontal component of Equation 4.9:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 = 0 + (v_i \cos\theta_i)t + \tfrac{1}{2}(0)t^2 = (v_i \cos\theta_i)t \qquad (5)$$

Let us imagine the electron follows a parabolic trajectory and exits the apparatus at the right edge of the lower plate, which is defined as $x_f = \ell$ and $y_f = 0$. Then Equations (4) and (5) become

$$0 = (v_i \sin\theta_i)t - \frac{eE}{2m_e}t^2 \quad \rightarrow \quad v_i \sin\theta_i = \frac{eE}{2m_e}t \qquad (6)$$

$$\ell = (v_i \cos\theta_i)t \qquad (7)$$

Solve Equation (7) for t, substitute the result into Equation (6), and then solve the resulting equation for the electric field E:

$$v_i \sin\theta_i = \frac{eE}{2m_e}\left(\frac{\ell}{v_i \cos\theta_i}\right) \quad \rightarrow \quad E = \frac{2m_e v_i^2}{e\ell}\sin\theta_i \cos\theta_i \qquad (8)$$

Substitute for the sine and cosine from Figure P22.28:

$$E = \frac{2m_e v_i^2}{e\ell}\left(\frac{d}{\sqrt{d^2+\ell^2}}\right)\left(\frac{\ell}{\sqrt{d^2+\ell^2}}\right) = \frac{2m_e v_i^2 d}{e(d^2+\ell^2)}$$

Substitute numerical values:

$$E = \frac{2(9.11\times10^{-31}\text{ kg})(5.00\times10^{6}\text{ m/s})^2(0.030\,0\text{ m})}{(1.602\times10^{-19}\text{ C})\left[(0.030\,0\text{ m})^2 + (0.500\text{ m})^2\right]} = \boxed{34.0\text{ N/C}}$$

Finalize Solve Equation (8) for the plate length ℓ:

$$E = \frac{2m_e v_i^2}{e\ell}\sin\theta_i\cos\theta_i \quad \rightarrow \quad \ell = \frac{2v_i^2\sin\theta_i\cos\theta_i}{\left(\dfrac{eE}{m_e}\right)}$$

Recognize the combination $2\sin\theta_i\cos\theta_i = \sin 2\theta_i$, and substitute for the denominator from Equation (2):

$$\ell = \frac{v_i^2\sin 2\theta_i}{-a_y}$$

Compare this equation to the range equation, Equation 4.20. The equations are the same, recognizing that $a_y = -g$ for the gravitational situation described by Equation 4.20! The path of the electron is a parabolic projectile, just like a thrown projectile in a gravitational field. Because the electrons arrive at the bottom plate just at its right edge, the length ℓ of the plate is the same as the range R of the gravitational projectile.

Answers: (a) $-3.43° < \theta < 3.43°$ (b) 34.0 N/C

Additional Problems

P22.29 The electric field is given by the sum of the fields due to each of the n particles:

$$\vec{\mathbf{E}} = \sum \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{k_e q}{a^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(2a)^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) + \cdots$$

$$= \frac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

$$= \boxed{-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$$

P22.30 The positive charge, call it q, is 50.0 cm – 20.9 cm = 29.1 cm from charge Q. The force on q from the –3.00 nC charge balances the force on q from the $-Q$ charge:

$$\frac{k_e(3.00\text{ nC})q}{(0.209\text{ m})^2} = \frac{k_e Q q}{(0.291\text{ m})^2}$$

which then gives

$$Q = (3.00\text{ nC})\left(\frac{0.291\text{ m}}{0.209\text{ m}}\right)^2 = \boxed{5.82\text{ nC}}$$

P22.31 (a) Take up the incline as the positive x direction. Newton's second law along the incline gives

$$\sum F_x = -mg\sin\theta + |Q|E = 0$$

solving for the electric field gives

$$E = \boxed{\frac{mg}{|Q|}\sin\theta}$$

(b) The electric force must be up the incline, so the electric field must point down the incline because the charge is negative.

$$E = \frac{mg}{|Q|}\sin\theta = \frac{(5.40\times10^{-3})(9.80)}{|7.00\times10^{-6}|}\sin 25.0^\circ$$

$$= \boxed{3.19\times10^3\text{ N/C, down the incline}}$$

P22.32 The downward electric force on the 0.800 μC charge is balanced by the upward spring force:

$$\frac{k_e q_1 q_2}{r^2} = kx$$

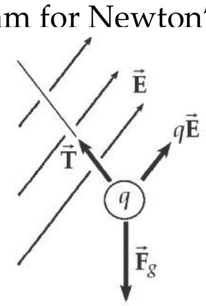
solving for the spring constant gives

$$k = \frac{k_e q_1 q_2}{x r^2}$$

$$= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.800 \times 10^{-6} \text{ C})(0.600 \times 10^{-6} \text{ C})}{(0.0350 \text{ m})(0.0500 \text{ m})^2}$$

$$= \boxed{49.3 \text{ N/m}}$$

P22.33 ANS. FIG. P22.33 shows the free-body diagram for Newton's second law gives



Free Body Diagram

ANS. FIG. P22.33

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{T}} + q\vec{\mathbf{E}} + \vec{\mathbf{F}}_{\mathbf{g}} = 0$$

We are given

$E_x = 3.00$ □ 10^5 N/C

and $E_y = 5.00$ □ 10^5 N/C

Applying Newton's second law or the first condition for equilibrium in the x and y directions,

$$\sum F_x = qE_x - T \sin 37.0° = 0 \qquad \textbf{[1]}$$

$$\sum F_y = qE_y + T \cos 37.0° - mg = 0 \qquad \textbf{[2]}$$

(a) We solve for T from equation [1]:

$$T = \frac{qE_x}{\sin 37.0°}$$

and substitute into equation [2] to obtain

$$q = \frac{mg}{E_y + \dfrac{E_x}{\tan 37.0^\circ}}$$

$$= \frac{(1.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{5.00 \times 10^5\ \text{N/C} + \left(\dfrac{3.00 \times 10^5\ \text{N/C}}{\tan 37.0^\circ}\right)}$$

$$q = \boxed{1.09 \times 10^{-8}\ \text{C}}$$

(b) Using the above result for q in equation [1], we find that the tension is

$$T = \frac{qE_x}{\sin 37.0^\circ} = \frac{(1.09 \times 10^{-8}\ \text{C})(3.00 \times 10^5\ \text{N/C})}{\sin 37.0^\circ}$$
$$= \boxed{5.44 \times 10^{-3}\ \text{N}}$$

P22.34 This is the general version of the preceding problem . The known quantities are A, B, m, g, and θ. The unknowns are q and T.

Refer to ANS. FIG. P22.33 above. The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 51.

Again, from Newton's second law,

$$\sum F_x = -T\sin\theta + qA = 0 \qquad \textbf{[1]}$$

and $$\sum F_y = +T\cos\theta + qB - mg = 0 \qquad \textbf{[2]}$$

(a) Substituting $T = \dfrac{qA}{\sin\theta}$ into equation [2], we obtain

$$\frac{qA\cos\theta}{\sin\theta} + qB = mg$$

Isolating q on the left,

$$\boxed{q = \frac{mg}{(A\cot\theta + B)}}$$

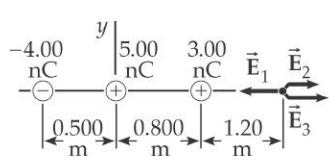
(b) Substituting this value into equation [1], we obtain

$$\boxed{T = \frac{mgA}{(A\cos\theta + B\sin\theta)}}$$

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 24. If you find this problem more difficult than problem 24, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

P22.35 (a) Refer to ANS. FIG. P22.35(a). The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2})(-4.00\times 10^{-9}\ \mathrm{C})}{(2.50\ \mathrm{m})^2}\hat{\mathbf{i}}$$
$$= -5.75\hat{\mathbf{i}}\ \mathrm{N/C}$$



ANS. FIG. P22.35(a)

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge, are

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(5.00\times10^{-9}\ \mathrm{C}\right)}{(2.00\ \mathrm{m})^2}\hat{\mathbf{i}}$$
$$= 11.2\ \mathrm{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(3.00\times10^{-9}\ \mathrm{C}\right)}{(1.20\ \mathrm{m})^2}\hat{\mathbf{i}} = 18.7\ \mathrm{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 = \boxed{24.2\ \mathrm{N/C\ in}\ +x\ \mathrm{direction}}$$

(b) In this case, referring to ANS. FIG. P22.35 (b),

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46\ \mathrm{N/C})\left(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2\ \mathrm{N/C})\left(+\hat{\mathbf{j}}\right)$$
$$\vec{\mathbf{E}}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81\ \mathrm{N/C})\left(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}}\right)$$

The components of the resultant electric field are

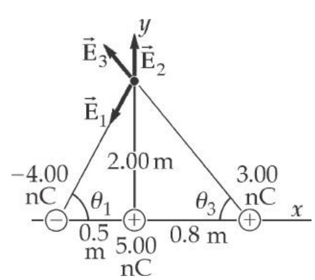
$$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}}\ \mathrm{N/C} \quad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}}\ \mathrm{N/C}$$

then, the magnitude of the resultant electric field is

$$E_R = \boxed{9.42\ \mathrm{N/C}}$$

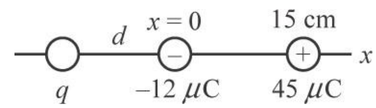
and is directed at

$$\theta = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{8.43\ \mathrm{N/C}}{4.21\ \mathrm{N/C}}\right) = \boxed{63.4^\circ\ \text{above}\ -x\ \text{axis}}$$



ANS. FIG. P22.35(b)

P22.36 (a) The two given charges exert equal size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x > 15.0$ cm, it would exert a stronger force on the 45.0-μC charge than on the –12.0-μC charge, and could not produce equilibrium for both. Thus the third charge must be at $x = -d < 0$.



ANS. FIG. P22.36

$\boxed{\text{It is possible in just one way.}}$

(b) The equilibrium of the third charge requires

$$\frac{k_e q(12.0\ \mu\text{C})}{d^2} = \frac{k_e q(45.0\ \mu\text{C})}{(15.0\text{ cm}+d)^2} \rightarrow \left(\frac{15.0\text{ cm}+d}{d}\right)^2 = \frac{45.0}{12.0} = 3.75$$

Solving,

$$15.0\text{ cm} + d = 1.94d \qquad \rightarrow \qquad d = 16.0\text{ cm}$$

The third charge is at $\boxed{x = -16.0\text{ cm}}$.

(c) The equilibrium of the –12.0-μC charge requires

$$\frac{k_e q(12.0\ \mu\text{C})}{(16.0\text{ cm})^2} = \frac{k_e(45.0\ \mu\text{C})(12.0\ \mu\text{C})}{(15.0\text{ cm})^2}$$

solving,

$$q = \boxed{+51.3\ \mu\text{C}}$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P22.37 We model the spheres as particles. They have different charges. They exert on each other forces of equal magnitude. They have equal masses, so their strings make equal angles θ with the vertical. We define r as the distance between the centers of the two spheres. We find r from

$$\sin\theta = \frac{r/2}{40.0 \text{ cm}}$$

from which we obtain

$$r = (80.0 \text{ cm})\sin\theta$$

Now let T represent the string tension. We have, from the particle under a net force model,

$$\sum F_x = 0: \quad \frac{k_e q_1 q_2}{r^2} - T\sin\theta = 0 \quad \rightarrow \quad \frac{k_e q_1 q_2}{r^2} = T\sin\theta \qquad \textbf{[1]}$$

$$\sum F_y = 0: \quad T\cos\theta - mg = 0 \quad \rightarrow \quad mg = T\cos\theta \qquad \textbf{[2]}$$

Dividing equation [1] by [2] to eliminate T gives

$$\frac{k_e q_1 q_2}{r^2 mg} = \tan\theta = \frac{r/2}{\sqrt{(40.0 \text{ cm})^2 - r^2/4}}$$

Clearing the fractions,

$$k_e q_1 q_2 \sqrt{(80.0 \text{ cm})^2 - r^2} = mgr^3$$

Substituting numerical values gives

$$\begin{aligned}&\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(200\times10^{-9} \text{ C}\right)\left(300\times10^{-9} \text{ C}\right)\\ &\quad\times\sqrt{(0.800 \text{ m})^2 - r^2} = \left(2.40\times10^{-3} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)r^3\end{aligned}$$

Suppressing units,

$$(0.800)^2 - r^2 = 1\,901\, r^6$$

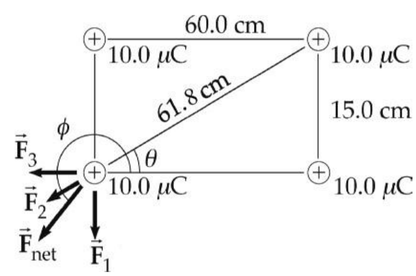
Instead of attempting to solve this equation, we instead home in on a solution by trying values, tabulated below:

r	$0.640 - r^2 - 1901\, r^6$
0	+0.64
0.5	–29.3
0.2	+0.48
0.3	–0.84
0.24	+0.22
0.27	–0.17
0.258	+0.013
0.259	–0.001

Thus the distance to three digits is 0.259 m = $\boxed{2.59 \text{ cm.}}$

P22.38 The magnitude of the electric force is given by $F = \dfrac{k_e q_1 q_2}{r^2}$. The angle θ in ANS. FIG. P22.38 is found from

$$\theta = \tan^{-1}\left(\frac{15.0}{60.0}\right) = 14.0°$$



ANS. FIG. P22.38

$$F_1 = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \times 10^{-6} \text{ C})^2}{(0.150 \text{ m})^2}$$
$$= 40.0 \text{ N}$$

$$F_2 = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \times 10^{-6} \text{ C})^2}{(0.618 \text{ m})^2} = 2.35 \text{ N}$$

$$F_3 = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \times 10^{-6} \text{ C})^2}{(0.600 \text{ m})^2} = 2.50 \text{ N}$$

$$F_x = -F_3 - F_2\cos 14.0° = -2.50 - 2.35\cos 14.0° = -4.78\text{ N}$$
$$F_y = -F_1 - F_2\sin 14.0° = -40.0 - 2.35\sin 14.0° = -40.5\text{ N}$$

(a) $F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78\text{ N})^2 + (-40.5\text{ N})^2} = \boxed{40.8\text{ N}}$

(b) $\tan\phi = \dfrac{F_y}{F_x} = \dfrac{-40.5\text{ N}}{-4.78\text{ N}} \quad \rightarrow \phi = \boxed{263°}$

P22.39 Charge Q resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e Q^2}{L^2} = k(L - L_i)$$

Solving for Q, we find

$$Q = L\sqrt{\frac{k(L-L_i)}{k_e}} = (0.500\text{ m})\sqrt{\frac{(100\text{ N/m})(0.500\text{ m} - 0.400\text{ m})}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}}$$
$$= \boxed{1.67\times 10^{-5}\text{C}}$$

P22.40 Charge Q resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e Q^2}{L^2} = k(L - L_i)$$

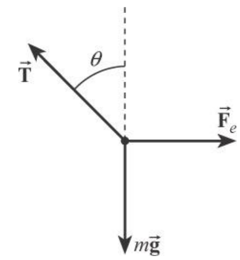
Solving for Q, we find

$$Q = \boxed{L\sqrt{\frac{k(L-L_i)}{k_e}}}$$

P22.41 Consider the free-body diagram of the rightmost charge given in ANS. FIG. P22.41. Newton's second law then gives

$$\sum F_y = 0 \;\Rightarrow\; T\cos\theta = mg \quad \text{or} \quad T = \frac{mg}{\cos\theta}$$

and



ANS. FIG. P22.41

$$\sum F_x = 0$$

$$\Rightarrow\; F_e = T\sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta$$

But,

$$F_e = \frac{k_e q^2}{r_1^2} + \frac{k_e q^2}{r_2^2} = \frac{k_e q^2}{(L\sin\theta)^2} + \frac{k_e q^2}{(2L\sin\theta)^2} = \frac{5k_e q^2}{4L^2\sin^2\theta}$$

Thus,

$$\frac{5k_e q^2}{4L^2\sin^2\theta} = mg\tan\theta \text{ or } q = \sqrt{\frac{4L^2 mg\sin^2\theta\tan\theta}{5k_e}}$$

If $\theta = 45°$, $m = 0.100$ kg, and $L = 0.300$ m, then

$$q = \sqrt{\frac{4(0.300\text{ m})^2(0.100\text{ kg})(9.80\text{ m/s}^2)\sin^2(45.0°)\tan(45.0°)}{5(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}$$

or $$q = 1.98\times10^{-6}\text{ C} = \boxed{1.98\ \mu\text{C}}$$

P22.42 Use Figure 22.21 for guidance on the physical setup of this problem. Let the electron enter at the origin of coordinates at the left end and just under the upper plate, which we choose to be negative so that the electron accelerates downward. The electron is a particle under constant velocity in the horizontal direction:

$$x_f = v_{xi}t$$

The electron is a particle under constant acceleration in the vertical direction:

$$y_f = \tfrac{1}{2}a_y t^2$$

Eliminate t between the equations:

$$y_f = \tfrac{1}{2}a_y\left(\frac{x_f}{v_{xi}}\right)^2 \quad \to \quad y_f = \left(\frac{a_y}{2v_{xi}^{\ 2}}\right)x_f^{\ 2}$$

Substitute for the acceleration of the particle in terms of the electric force:

$$y_f = \left(\frac{-eE}{2v_{xi}^{\ 2}\ m_e}\right)x_f^{\ 2}$$

Substitute numerical values, letting the final horizontal position be at the right end of the plates:

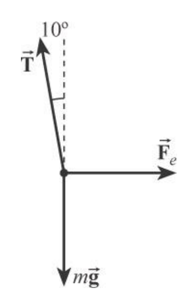
$$\begin{aligned} y_f &= \left[\frac{-\left(1.60\times10^{-19}\ \text{C}\right)\left(200\ \text{N/C}\right)}{2\left(3.00\times10^{6}\ \text{m/s}\right)^2\left(9.11\times10^{-31}\ \text{kg}\right)}\right]\left(0.200\ \text{m}\right)^2 \\ &= -0.078\ 1\ \ \text{m} \end{aligned}$$

Therefore, when the electron leaves the plates, its final position is well below that of the lower plate, which is at position $y = -1.50$ cm = –0.015 m. Consequently, because we have let the electron enter the field at as high a position as possible, the electron will strike the lower plate long before it reaches the end, regardless of where it enters the field.

P22.43 The charges are q and $2q$. The magnitude of the repulsive force that one charge exerts on the other is

$$F_e = 2k_e\frac{q^2}{r^2}$$

From Figure P22.43 in the textbook, observe that



ANS. FIG. P22.43

the distance separating the two spheres is

$$r = d + 2L\sin 10°$$

From the free-body diagram of one sphere given in ANS. FIG. P22.43, observe that

$$\sum F_y = 0 \;\Rightarrow\; T\cos 10° = mg \quad \text{or} \quad T = mg/\cos 10°$$

and

$$\sum F_x = 0 \;\Rightarrow\; F_e = T\sin 10° = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10°$$

Thus,

$$2k_e\frac{q^2}{r^2} = mg\tan 10° \qquad \rightarrow \qquad 2k_e\frac{q^2}{\left(d + 2L\sin 10°\right)^2} = mg\tan 10°$$

or

$$\begin{aligned} q &= \sqrt{\frac{mg\left(d + 2L\sin\theta\right)^2\tan 10°}{2k_e}} \\ &= \sqrt{\frac{(0.015\text{ kg})(9.80\text{ m/s}^2)\left[0.0300\text{ m} + 2(0.0500\text{ m})\sin 10°\right]^2\tan 10°}{2\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)}} \\ &= 5.69\times 10^{-8}\text{ C} \end{aligned}$$

giving $\boxed{1.14\times 10^{-7}\text{ C on one sphere and } 5.69\times 10^{-8}\text{ C on the other.}}$

P22.44 (a) The bowl exerts a normal force on each bead, directed along the radius line at angle θ above the horizontal. Consider the free-body diagram shown in ANS. FIG. P22.44 for the bead on the left side of the bowl:

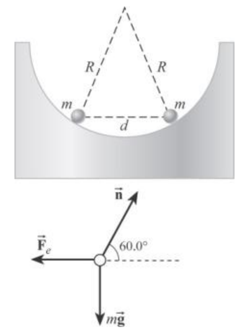
$$\sum F_y = n\sin\theta - mg = 0 \;\rightarrow\; n = \frac{mg}{\sin\theta}$$

Also,

$$\sum F_x = -F_e + n\cos\theta = 0$$

which gives

$$F_e = n\cos\theta = \left(\frac{mg}{\sin\theta}\right)\cos\theta = \frac{mg}{\tan\theta}$$



ANS. FIG. P22.44

The electric force is

$$F_e = \frac{k_e q^2}{d^2}$$

And from ANS. FIG. P22.44,

$$\tan\theta = \frac{\sqrt{R^2 - (d/2)^2}}{(d/2)} = \frac{\sqrt{4R^2 - d^2}}{d}$$

Therefore,

$$F_e = \frac{k_e q^2}{d^2} = \frac{mg}{\tan\theta} = \frac{mg}{\sqrt{4R^2 - d^2}/d} \quad \rightarrow \quad q = \boxed{\left(\frac{mgd^3}{k_e\sqrt{4R^2 - d^2}}\right)^{1/2}}$$

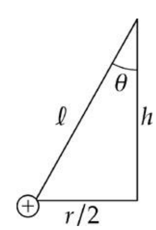
(b) As $d \rightarrow 2R$, $\sqrt{4R^2 - d^2} \rightarrow 0$; therefore, $\boxed{q \rightarrow \infty}$.

P22.45 (a) From the $2Q$ charge we have

$$F_e - T_2\sin\theta_2 = 0 \text{ and } mg - T_2\cos\theta_2 = 0$$

Combining these we find

$$\frac{F_e}{mg} = \frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$$



ANS. FIG. P22.45

From the Q charge we have

$$F_e = T_1\sin\theta_1 = 0 \text{ and } mg - T_1\cos\theta_1 = 0$$

Combining these we find

$$\frac{F_e}{mg} = \frac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \tan\theta_1 \text{ or } \boxed{\theta_2 = \theta_1}$$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_e Q^2}{r^2}$. If we assume θ is small then $\tan\theta \approx \dfrac{r/2}{\ell}$.

Substitute expressions for F_e and tan θ into either equation found in part (a) and solve for r. $\dfrac{F_e}{mg} = \tan\theta$, then $\dfrac{2k_e Q^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r \approx \left(\dfrac{4k_e Q^2 \ell}{mg}\right)^{1/3}$.

***P22.46** **Conceptualize** The amount of charge necessary to counteract the gravitational force on a human is very large. When two such charged humans are close together, there is a large repulsive force between them, which will cause them to accelerate away from each other.

Categorize A single human is modeled as a *particle in a field (electric)* for the hovering experiment, as well as a *particle in equilibrium* between the electric and gravitational forces. When two humans are hovering, each is a particle in the electric field of the other, and the repulsive force between them is represented by Coulomb's law. Then they can each also be modeled as a *particle under a net force.*

Analyze Let's first look at the hovering of a single experimental subject. Model the subject as a particle in equilibrium, with the gravitational force downward and the electric force, given by Equation 22.8, upward:

$$\sum F_y = 0 \quad \rightarrow \quad qE - mg = 0 \quad \rightarrow \quad q = \frac{mg}{E} \qquad (1)$$

which is the charge necessary to be applied to the body to make the experimental subject hover. Now imagine that there are two hovering subjects with the same mass and charge. Modeling them as particles, they will feel a repulsive force according to Coulomb's law in Equation 22.1:

$$F_e = k_e \frac{q^2}{r^2} = k_e \frac{\left(\frac{mg}{E}\right)^2}{r^2} = k_e \left(\frac{mg}{Er}\right)^2 \qquad (2)$$

Now model one of the experimental subjects as a particle under a net force in the horizontal direction. The only force in this direction is the electrical repulsion from the other subject, so we can find the horizontal acceleration of this experimental subject:

$$\sum F_x = ma \quad \rightarrow \quad F_e = ma \quad \rightarrow \quad a = \frac{F_e}{m} = \frac{k_e \left(\frac{mg}{Er}\right)^2}{m} = k_e m \left(\frac{g}{Er}\right)^2 \qquad (3)$$

Substitute numerical values given in the problem, using the fact that the experimental subjects are working 1.00 m apart:

$$a = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(85.0 \text{ kg}\right)\left[\frac{9.80 \text{ m/s}^2}{\left(130 \text{ N/C}\right)\left(1.00 \text{ m}\right)}\right]^2 = \boxed{4.34 \times 10^9 \text{ m/s}^2}$$

Finalize This is a *tremendous* acceleration, millions of times larger than that for gravity. The experimental subjects would be lucky to survive this experiment. You should advise the inventor that he should have warned the experimental subjects about this possibility, and the most likely outcome of a trial would be that he will be ruled responsible for the injuries to his experimental subjects.

Answer: 4.34×10^9 m/s²

P22.47 (a) The total non-contact force on the cork ball is:

$$F = qE + mg = m\left(g + \frac{qE}{m}\right)$$

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g + qE/m}} \\ &= 2\pi\sqrt{\frac{0.500 \text{ m}}{9.80 \text{ m/s}^2 + \left[\dfrac{(2.00\times10^{-6} \text{ C})(1.00\times10^{5} \text{ N/C})}{1.00\times10^{-3} \text{ kg}}\right]}} \\ &= \boxed{0.307 \text{ s}} \end{aligned}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get

$$T = 2\pi\sqrt{\frac{L}{qE/m}}$$

$$T = 2\pi\sqrt{\frac{0.500 \text{ m}}{\left[\dfrac{(2.00\times10^{-6} \text{ C})(1.00\times10^{5} \text{ N/C})}{1.00\times10^{-3} \text{ kg}}\right]}} = 0.314 \text{ s}$$

(a 2.28% difference).

Challenge Problems

P22.48 First, we use unit vectors to find the total electric field at point A produced by the 7 other charges.

source charge	vector field components	equivalent field
(1) lower left, front:	$\vec{\mathbf{E}}_1 = \frac{k_e q}{r_1^2}\hat{\mathbf{r}}_1 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$
(2) lower right, front:	$\vec{\mathbf{E}}_2 = \frac{k_e q}{r_2^2}\hat{\mathbf{r}}_2 = \frac{k_e q}{s^2}\hat{\mathbf{k}}$	$\frac{k_e q}{s^2}\hat{\mathbf{k}}$
(3) lower right, back:	$\vec{\mathbf{E}}_3 = \frac{k_e q}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{k}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{k}}\right)$
(4) lower left, back:	$\vec{\mathbf{E}}_4 = \frac{k_e q}{r_4^2}\hat{\mathbf{r}}_4 = \frac{k_e q}{s^2+s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$	$\left(\frac{1}{3\sqrt{3}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$
(5) upper right, back	$\vec{\mathbf{E}}_5 = \frac{k_e q}{r_5^2}\hat{\mathbf{r}}_5 = \frac{k_e q}{s^2}\hat{\mathbf{i}}$	$\frac{k_e q}{s^2}\hat{\mathbf{i}}$
(6) upper left, back	$\vec{\mathbf{E}}_6 = \frac{k_e q}{r_6^2}\hat{\mathbf{r}}_6 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$
(7) upper right, front	$\vec{\mathbf{E}}_7 = \frac{k_e q}{r_7^2}\hat{\mathbf{r}}_7 = \frac{k_e q}{s^2}\hat{\mathbf{j}}$	$\frac{k_e q}{s^2}\hat{\mathbf{j}}$
total field $\vec{\mathbf{E}}_{\text{total}} = \frac{k_e q}{s^2}\left[1+\frac{2}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$		

Notice that because of symmetry, the components of the field have the same magnitude.

(a) At point A,

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}_{\text{total}} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$$
$$= \frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$$

$$\rightarrow \boxed{F_x = F_y = F_z = 1.90 k_e \frac{q^2}{s^2}}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29 \frac{k_e q^2}{s^2}}$

(c) $\boxed{\text{away from the origin}}$

P22.49 (a) The two charges create fields of equal magnitude, both with outward components along the x axis and with upward and downward y components that add to zero. The net field is then

$$\vec{\mathbf{E}} = \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} + \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} = 2\frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}}$$
$$= \frac{2(8.99 \times 10^9)\left(52 \times 10^{-9}\right)x\ \hat{\mathbf{i}}}{\left[(0.25)^2 + x^2\right]^{3/2}}$$

$$\boxed{\vec{\mathbf{E}} = \frac{935x}{\left(0.0625 + x^2\right)^{3/2}}\hat{\mathbf{i}} \text{ where } \vec{\mathbf{E}} \text{ is in newtons per coulomb and } x \text{ is in meters.}}$$

(b) At $x = 0.36$ m,

$$\vec{\mathbf{E}} = \frac{935(0.36)\ \hat{\mathbf{i}}}{(0.0625 + (0.36)^2)^{3/2}} = \boxed{4.00\ \text{kN/C}\ \hat{\mathbf{i}}}$$

(c) We solve $1\,000 = (935\,x)(0.0625 + x^2)^{-3/2}$ by tabulating values for the field function:

x	$(935\,x)(0.0625 + x^2)^{-3/2}$
0	0
0.01	597
0.02	1 185
0.1	4 789
0.2	5 698
0.36	4 000
0.9	1 032
1	854
∞	0

We see that there are two points where $E = 1\,000$ N/C. We home in on them to determine their coordinates as (to three digits)

$\boxed{x = 0.016\,8 \text{ m and } x = 0.916 \text{ m.}}$

(d) The table in part (c) shows that

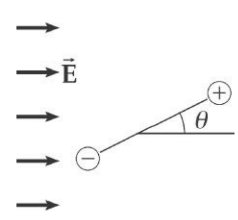
$\boxed{\text{nowhere is the field so large as } 16\,000 \text{ N/C.}}$

P22.50 (a) The electrostatic forces exerted on the two charges result in a net torque

$$\tau = -2Fa\sin\theta = -2Eqa\sin\theta$$

For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have

$$\tau = -Ep\theta$$



ANS. FIG. P22.50

The torque produces an angular acceleration given by

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

where the moment of inertia of the dipole is $I = 2ma^2$

Combining the two expressions for torque, we have

$$\frac{d^2\theta}{dt^2} = -\left(\frac{Ep}{I}\right)\theta$$

This equation can be written in the form $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ which is the standard equation characterizing simple harmonic motion, with

$$\omega^2 = \frac{Ep}{I} = \frac{E(2qa)}{2ma^2} = \frac{qE}{ma}$$

The frequency of oscillation is $f = \omega / 2\pi$, so

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{qE}{ma}}}$$

(b) If the masses are unequal, the dipole will oscillate about its center of mass (CM). Assume mass m_2 is greater than mass m_1, and treat the *center* of the dipole as being at the origin of an x axis, so that mass m_1 is at $x = -a$ and mass m_2 is at $x = +a$. The coordinate of the CM of the dipole is then

$$x_{\text{cm}} = \frac{m_2 a - m_1 a}{m_1 + m_2} = a\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

relative to the center of the dipole. Notice that the moment of inertia of the dipole about its *center* is

$$I = m_1 a^2 + m_2 a^2$$

but its center is a distance x_{cm} from its CM. By the parallel-axis theorem, the moment of inertia of the dipole about its *center* is related to its moment about its CM thus:

$$I = m_1 a^2 + m_2 a^2 = I_{\text{CM}} + \left(m_1 + m_2\right) x_{\text{cm}}{}^2$$

therefore,

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - \left(m_1 + m_2\right) x_{\text{cm}}{}^2$$

The moment of inertia of the dipole about its CM is then

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - \left(m_1 + m_2\right) a^2 \left(\frac{m_2 - m_1}{m_1 + m_2}\right)^2$$

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - a^2 \frac{\left(m_2 - m_1\right)^2}{\left(m_1 + m_2\right)}$$

$$I_{\text{CM}} = \frac{\left(m_1 + m_2\right)\left(m_1 a^2 + m_2 a^2\right) - \left(m_2{}^2 a^2 - 2 m_1 m_2 a^2 + m_1{}^2 a^2\right)}{\left(m_1 + m_2\right)}$$

$$I_{\text{CM}} = \frac{\left(m_1{}^2 a^2 + 2 m_1 m_2 a^2 + m_2{}^2 a^2\right) - \left(m_2{}^2 a^2 - 2 m_1 m_2 a^2 + m_1{}^2 a^2\right)}{\left(m_1 + m_2\right)}$$

$$I_{\text{CM}} = \frac{4 m_1 m_2 a^2}{\left(m_1 + m_2\right)}$$

Therefore, from part (a),

$$\omega^2 = \frac{Ep}{I_{\text{CM}}} = \frac{E\left(2qa\right)}{\left[\dfrac{4 m_1 m_2 a^2}{\left(m_1 + m_2\right)}\right]} = \frac{qE\left(m_1 + m_2\right)}{2 m_1 m_2 a} = \left(2\pi f\right)^2$$

and

$$f = \frac{1}{2\pi}\sqrt{\frac{qE\left(m_1 + m_2\right)}{2 m_1 m_2 a}}$$

ANSWERS TO QUICK-QUIZZES

1. (a), (c), (e)

2. (e)

3. (b)

4. (a)

5. *A*, *B*, *C*

ANSWERS TO EVEN-NUMBERED PROBLEMS

P22.2 (a) 9.21×10^{-10} N; (b) No. The electric force depends only on the magnitudes of the two charges and the distance between them.

P22. 4 $\sim 10^{26}$ N

P22. 6 The electric force is 18 orders of magnitude smaller than the gravitational force.

P22. 8 (a) $\dfrac{\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}\,d$; (b) Yes, if the third bead has a positive charge.

P22.10 (a) 46.7 N to the left; (b) 157 N to the right; (c) 111 N to the left

P22.12 (a) 0; (b) 30.0 N; (c) 21.6 N; (d) 17.3 N; (e) –13.0 N; (f) 17.3 N; (g) 17.0 N; (h) 24.3 N at 44.5° above the $+x$ direction

P22. 14 The unknown charge on each dust particle is about half of the smallest possible free charge, the charge of the electron. No such free charge exists. Therefore, the forces cannot balance.

P22. 16 (a) $\dfrac{k_e Q x \hat{\mathbf{i}}}{\left(a^2 + x^2\right)^{3/2}}$; (b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P22. 18 The field at the origin can be to the right, if the unknown charge is $-9Q$, or the field can be to the left, if and only if the unknown charge is $+27Q$.

P22. 20 (a) $1.29 \times 10^4 \hat{\mathbf{j}}$ N/C; (b) $-3.86 \times 10^{-2} \hat{\mathbf{j}}$ N

P22. 22 $\dfrac{4a(k_e q)}{x^3}$

P22. 24 (a) 6.13×10^{10} m/s^2 (b) 1.96×10^{25} s (c) 11.7 m (d) 1.20×10^{-15} J

P22. 26 (a) Particle under constant velocity; (b) Particle under constant acceleration; (c) the proton moves in a parabolic path just like a projectile in a gravitational field; (d) $\dfrac{m_p v_i^2 \sin 2\theta}{eE}$; (e) 36.9° or 53.1°; (f) 166 ns or 221 ns

P22. 28 (a) $-3.43° < \theta < 3.43°$ (b) 34.0 N/C

P22. 30 5.81 nC

P22. 32 49.3 N/m

P22. 34 (a) $q = \dfrac{mg}{(A\cot\theta + B)}$; (b) $T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}$

P22. 36 (a) It is possible in just one way; (b) $x = -16.0$ cm; (c) $+51.3\mu\text{C}$

P22. 38 (a) 40.9 N; (b) 263°

P22. 40 $\dfrac{2k_eQ}{3\sqrt{3}a^2} = \dfrac{Q}{6\sqrt{3}\pi \in_0 a^2}$

P22. 42 See P22.42 for complete solution

P22. 44 (a) $\left(\dfrac{mgd^3}{k_e\sqrt{4R^2 - d^2}}\right)^{1/2}$; (b) $q \to \infty$

P22. 46 4.34×10^9 m/s^2

P22. 48 (a) $\rightarrow \boxed{F_x = F_y = F_z = 1.90k_e \dfrac{q^2}{s^2}}$ (b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\dfrac{k_e q^2}{s^2}}$

(c) $\boxed{\text{away from the origin}}$

P22. 50 (a) $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{qE}{ma}}}$ (b) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{qE(m_1 + m_2)}{2m_1 m_2 a}}$