6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP6.1** **Conceptualize** If you take the curve too fast, the maximum force of static friction may not be enough to provide the centripetal acceleration of the crate of eggs and it may begin to slip and strike the side of the pickup. If you hit

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the brakes, the crate of eggs might start sliding forward and hit the front of the bed of the truck. What should you do?

Categorize If you go around the curve, the crate of eggs is modeled as a *particle in uniform circular motion*. If you hit the brakes, the crate is modeled as a *particle under a net force*. Let's assume that the acceleration during braking is constant.

Analyze Let's first convert the speed to metric units so we have all values in consistent units:

$$v_i = 45.0 \text{ mi/ h}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = 20.1 \text{ m/ s}$$

(i) If the truck goes around the curve, the particle in uniform circular motion model gives us

$$f_s = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{f_s r}{m}}$$

If the force of static friction is at its maximum value, this equation will give us the highest speed at which the truck can take the curve without the crate slipping. From the particle in equilibrium model in the vertical direction, the normal force on the crate is equal to its weight. Therefore,

$$v_{\text{max}} = \sqrt{\frac{f_{s,\text{max}} r}{m}} = \sqrt{\frac{\mu_s n r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

Substitute numerical values:

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$$v_{\max} = \sqrt{(0.600)(9.80\ \text{m/s}^2)(35.0\ \text{m})} = 14.3\ \text{m/s} \quad (1)$$

Your speed of 20.1 m/s is much higher than this, so you clearly do not want to continue around the curve at your initial speed.

(ii) Let's apply enough braking pressure to the pedal to stop in the distance of 15.0 m, so that we do not enter the curve. Modeling the crate as a particle under a net force, with the only force in the horizontal direction being that due to the friction force,

$$F_x = ma_x \quad \rightarrow \quad f_s = ma_x$$

Let's suppose you are slowing down at the maximum magnitude of acceleration possible such that the crate does not slip. Then we can replace the friction force with – ${}_s n$ (the friction force is toward the back of the truck to keep the eggs from sliding forward):

$$-\mu_s n = ma_{\max} \quad \rightarrow \quad -\mu_s mg = ma_{\max} \quad \rightarrow \quad a_{\max} = -\mu_s g$$

Substituting numerical values:

$$a_{\max} = -(0.600)(9.80\ \text{m/s}^2) = -5.88\ \text{m/s}^2 \quad (2)$$

Any acceleration of magnitude larger than 5.88 m/s^2 will cause the eggs to slide. Now model the crate as a particle under constant acceleration and find the acceleration necessary to reduce the speed of the crate to zero. From Equation 2.17,

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \quad \rightarrow \quad a_x = \frac{-v_{xi}^2}{2(x_f - x_i)} \quad (3)$$

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Substitute numerical values:

$$a_x = \frac{-(20.1\text{ m/ s})^2}{2(15.0\text{ m})} = -13.5\text{ m/ s}^2 \quad (4)$$

The magnitude of acceleration in Equation (4) required to stop is larger than the maximum value found in Equation (2), so the eggs will slip as you slow down if you try this.

(iii) Suppose you want to slow down just enough so you enter the curve at the maximum speed for the curve found above in (i). Then Equation (3) becomes

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right) \quad \rightarrow \quad a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} \qquad (5)$$

Substitute numerical values:

$$a_x = \frac{(14.3\text{ m/ s})^2 - (20.1\text{ m/ s})^2}{2(15.0\text{ m})} = -6.65\text{ m/ s}^2$$

This value is still larger in magnitude than the safe acceleration in Equation (2).

Finalize Therefore, *none* of the suggested actions can help. There is nothing you can do; the eggs are doomed! Pay more attention!

Answer: None of the options can save the eggs.

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***TP6.2** *Answers:* (a) Typical periods will be about 4 s and typical radii will be about 7–8 m, leading to centripetal accelerations ranging from about 17 to 20 m/s^2. (b) The centripetal acceleration of a rider at the top of the ride is larger than that due to gravity, $g = 9.80\ m/s^2$. (c) Because the centripetal acceleration of the wall behind the rider is larger than that due to gravity, the wall actually pushes downward with a normal force on the rider. The rider is accelerating downward at that point faster than he would if simply falling.

***TP6.3** Typical periods will be about 2 s and typical radii will be about 5 m. The minimal static friction force between the wall and a rider must just balance the weight of a rider: $f_s = mg$. Furthermore, the normal force inward from the wall must provide the centripetal acceleration: $n = mr\quad^2$. Combining these equations, using $f_s = \quad_s n$, we find that $\mu_s = \dfrac{gT^2}{4\pi^2 r}$. Using typical values, the minimum coefficient of static friction should come out to be about 0.2. A coefficient larger than this value can easily be attained by covering the walls of the ride with rough-textured cloth.

Answer: ~ 0.2

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 6.1 Extending the Particle in Uniform Circular Motion Model

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P6.1 (a) The force acting on the electron in the Bohr model of the hydrogen atom is directed radially inward and is equal to

$$F = \frac{mv^2}{r} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(2.20\times10^{6}\text{ m/s}\right)^2}{0.529\times10^{-10}\text{ m}}$$
$$= \boxed{8.33\times10^{-8}\text{ N inward}}$$

(b)

$$a = \frac{v^2}{r} = \frac{\left(2.20\times10^{6}\text{ m/s}\right)^2}{0.529\times10^{-10}\text{ m}} = \boxed{9.15\times10^{22}\text{ m/s}^2\text{ inward}}$$

P6.2 (a) The astronaut's orbital speed is found from Newton's second law, with

$$\sum F_y = ma_y:\ mg_{\text{moon}}\text{ down} = \frac{mv^2}{r}\text{ down}$$

solving for the velocity gives

$$v = \sqrt{g_{\text{moon}}r} = \sqrt{\left(1.52\text{ m/s}^2\right)\left(1.7\times10^{6}\text{ m} + 100\times10^{3}\text{ m}\right)}$$
$$v = \boxed{1.65\times10^{3}\text{ m/s}}$$

(b) To find the period, we use $v = \dfrac{2\pi r}{T}$ and solve for T:

$$T = \frac{2\pi\left(1.8\times10^{6}\text{ m}\right)}{1.65\times10^{3}\text{ m/s}} = \boxed{6.84\times10^{3}\text{ s}} = 1.90\text{ h}$$

P6.3 (a) The car's speed around the curve is found from

$$v = \frac{235\text{ m}}{36.0\text{ s}} = 6.53\text{ m/s}$$

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This is the answer to part (b) of this problem. We calculate the radius of the curve from $\frac{1}{4}(2\pi r) = 235\ \text{m}$, which gives $r = 150$ m.

The car's acceleration at point B is then

$$\begin{aligned}
\vec{\mathbf{a}}_r &= \left(\frac{v^2}{r}\right) \text{ toward the center} \\
&= \frac{(6.53\ \text{m/s})^2}{150\ \text{m}} \text{ at } 35.0° \text{ north of west} \\
&= \left(0.285\ \text{m/s}^2\right)\left(\cos 35.0°\left(-\hat{\mathbf{i}}\right) + \sin 35.0°\,\hat{\mathbf{j}}\right) \\
&= \boxed{\left(-0.233\hat{\mathbf{i}} + 0.163\hat{\mathbf{j}}\right)\ \text{m/s}^2}
\end{aligned}$$

(b) From part (a), $v = \boxed{6.53\ \text{m/s}}$

(c) We find the average acceleration from

$$\begin{aligned}
\vec{\mathbf{a}}_{\text{avg}} &= \frac{\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right)}{\Delta t} \\
&= \frac{\left(6.53\hat{\mathbf{j}} - 6.53\hat{\mathbf{i}}\right)\ \text{m/s}}{36.0\ \text{s}} \\
&= \boxed{\left(-0.181\hat{\mathbf{i}} + 0.181\hat{\mathbf{j}}\right)\ \text{m/s}^2}
\end{aligned}$$

P6.4 In $\sum F = m\frac{v^2}{r}$, both m and r are unknown but remain constant.

Symbolically, write

$$\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\ \text{m/s})^2 \text{ and } \sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\ \text{m/s})^2$$

Therefore, $\sum F$ is proportional to v^2 and increases by a factor of

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$\left(\frac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130\ \text{N}) = \boxed{215\ \text{N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.5 We neglecting relativistic effects. With 1 u = 1.661 x 10^{-27} kg, and from Newton's second law, we obtain

$$\begin{aligned} F &= ma_c = \frac{mv^2}{r} \\ &= \left(2\times 1.661\times 10^{-27}\ \text{kg}\right)\frac{\left(2.998\times 10^7\ \text{m/s}\right)^2}{(0.480\ \text{m})} \\ &= \boxed{6.22\times 10^{-12}\ \text{N}} \end{aligned}$$

P6.6 We solve for the tensions in the two strings:

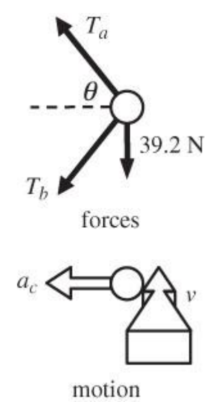
$$F_g = mg = (4.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 39.2\ \text{N}$$

The angle θ is given by

$$\theta = \sin^{-1}\left(\frac{1.50\ \text{m}}{2.00\ \text{m}}\right) = 48.6°$$

The radius of the circle is then

$$r = (2.00\ \text{m})\cos 48.6° = 1.32\ \text{m}$$



ANS. FIG. P6.6

Applying Newton's second law,

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$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a \cos 48.6° + T_b \cos 48.6° = \frac{(4.00 \text{ kg})(3.00 \text{ m/s})^2}{1.32 \text{ m}}$$

$$T_a + T_b = \frac{27.27 \text{ N}}{\cos 48.6°} = 41.2 \text{ N} \qquad \textbf{[1]}$$

$$\sum F_y = ma_y: \quad T_a \sin 48.6° - T_b \sin 48.6° - 39.2 \text{ N} = 0$$

$$T_a - T_b = \frac{39.2 \text{ N}}{\sin 48.6°} = 52.3 \text{ N} \qquad \textbf{[2]}$$

To solve simultaneously, we add the equations in T_a and T_b:

$$(T_a + T_b) + (T_a - T_b) = 41.2 \text{ N} + 52.3 \text{ N}$$

$$T_a = \frac{93.8 \text{ N}}{2} = 46.9 \text{ N}$$

This means that $T_b = 41.2 \text{ N} - T_a = -5.7 \text{ N}$, which we may interpret as meaning the lower string pushes rather than pulls!

$\boxed{\text{The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.}}$

To answer the **What if?**, we go back to equation [2] above and substitute mg for the weight of the object. Then,

$$\sum F_y = ma_y: \quad T_a \sin 48.6° - T_b \sin 48.6° - mg = 0$$

$$T_a - T_b = \frac{(4.00 \text{ kg})g}{\sin 48.6°} = 5.33g$$

We then add this equation to equation [2] to obtain

$$(T_a + T_b) + (T_a - T_b) = 41.2 \text{ N} + 5.33g$$

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or $$T_a = 20.6\text{ N} + 2.67g \text{ and } T_b = 41.2\text{ N} - T_a = 41.2\text{ N} - 2.67g$$

For this situation to be possible, T_b must be > 0, or $g < 7.72\text{ m/s}^2$. This is certainly the case on the surface of the Moon and on Mars.

***P6.7 Conceptualize** You may have been on this ride and may be familiar with the situation. Figure P6.5 shows a rider on the ride. The rider is moving in a circular path, along with the section of wall just behind her.

Categorize Consider a time interval during which the rotation speed of the ride is constant. Then the rider can be modeled as a *particle under a net force* and a *particle in uniform circular motion* in the horizontal direction, and a *particle in equilibrium* in the vertical direction.

Analyze A free-body diagram for the rider appears below.

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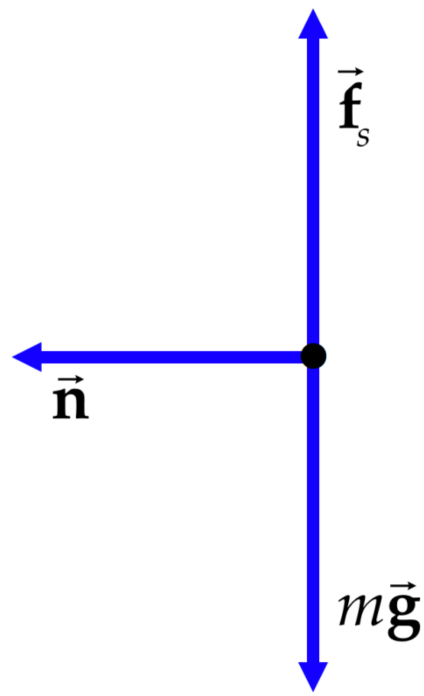

Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = 0 \quad \rightarrow \quad f_s - mg = 0 \quad \rightarrow \quad f_s = mg \qquad (1)$$

Apply the particle under a net force model in the horizontal direction, with the acceleration being centripetal:

$$\sum F_x = ma \quad \rightarrow \quad n = m\frac{v^2}{R} \qquad (2)$$

Divide equation (2) by Equation (1):

$$\frac{n}{f_s} = \frac{\left(m\dfrac{v^2}{R}\right)}{mg} = \frac{v^2}{gR} \qquad (3)$$

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Let us consider the limiting case in which the ride is rotating at the right angular speed to just keep the rider pinned to the wall. If the ride were to operate just a bit more slowly, the rider would begin to slide downward. Therefore, we are looking at the situation of impending motion for static friction, so that

$$f_s = \mu_s n \quad (4)$$

Substituting Equation (4) into Equation (3) and solving for v gives

$$\frac{n}{(\mu_s n)} = \frac{v^2}{gR} \quad \rightarrow \quad \frac{1}{\mu_s} = \frac{v^2}{gR} \quad \rightarrow \quad v = \sqrt{\frac{gR}{\mu_s}} \quad (5)$$

We wish to find the angular speed, so let us evaluate this parameter. From Equation 4.24, and using Equation (5), we find

$$\omega = \frac{v}{R} = \frac{\left(\sqrt{\frac{gR}{\mu_s}}\right)}{R} = \sqrt{\frac{g}{\mu_s R}} \quad (6)$$

(a) Now, what if the heavy person enters the ride? The mass m of the person does not appear in Equation (6), so the desired angular speed is independent of the mass of the person. You do not have to change the speed of the ride.

(b) How about the person wearing the slippery satin workout outfit? The slippery outfit results in a lower coefficient of friction between the outfit and the wall. In Equation (6), we see that reducing the coefficient of friction increases the

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minimum angular speed to keep the person pinned to the wall. Therefore, the ride must be spun faster.

Finalize In practice, the interior wall of the cylinder is lined with a rough material to minimize any effects of variations in clothing, and the ride is spun with a significantly higher angular speed than that found numerically from Equation (6).

Answers: (a) no (b) yes

***P6.8** **Conceptualize** In Example 6.7, we determined that a pendulum can be used as an accelerometer. Try running along a straight line while hanging an object on a string from your finger as a pendulum. If you accelerate in the forward direction, the pendulum will deviate backward. The faster you accelerate, the larger is the angle of deviation. In the opening storyline for this chapter, we used a pendulum to exhibit the existence of centripetal acceleration on the Mad Tea Party ride. For our car rounding a curve on the freeway in this problem, we will see the pendulum deviating toward the outside of the curve, opposite to the direction of the centripetal acceleration of the pendulum bob.

Categorize The pendulum bob can be modeled as a *particle under a net force* and a *particle in uniform circular motion* in the horizontal direction, and a *particle in equilibrium* in the vertical direction.

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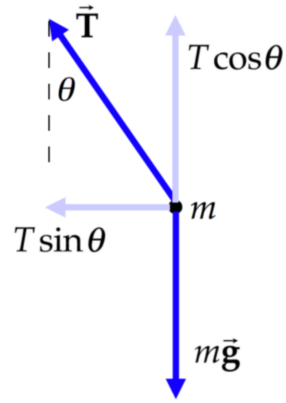
Analyze A free-body diagram for the pendulum bob appears below, looking along the direction of travel of the car.



Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = T\cos\theta - mg = 0 \quad \rightarrow \quad T\cos\theta = mg \qquad (1)$$

Apply the particle under a net force model in the horizontal direction, with the acceleration being centripetal:

$$\sum F_x = T\sin\theta = ma \quad \rightarrow \quad T\sin\theta = m\frac{v^2}{r} \qquad (2)$$

where v is the speed of the car and, therefore, of the pendulum bob, and r is the radius of the circular path of the pendulum bob and, therefore, the radius of curvature of the roadway. Divide Equation (2) by Equation (1) and solve for r:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m\dfrac{v^2}{r}}{mg} \quad \rightarrow \quad r = \frac{v^2}{g\tan\theta} \qquad (3)$$

Substitute the numerical values given in the experiment:

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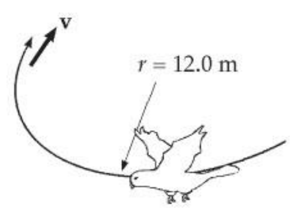
$$r = \frac{(23.0\text{ m/ s})^2}{(9.80\text{ m/ s}^2)\tan 15.0□} = 201\text{ m}$$

Finalize The radius of curvature is 34% larger than the state regulation, so you can testify that the curve was *not* too tight for the driver to negotiate at 65 mi/h. The accident must have been caused by something else.

Answer: The radius of curvature is larger than 150 m, so the driver is not justified in his claim as to faulty design of the roadway.

Section 6.2 Non uniform Circular Motion

P6.9 (a) The hawk's centripetal acceleration is



ANS. FIG. P6.9

$$a_c = \frac{v^2}{r} = \frac{(4.00\text{ m/s})^2}{12.0\text{ m}} = \boxed{1.33\text{ m/s}^2}$$

(b) The magnitude of the acceleration vector is

$$a = \sqrt{a_c^2 + a_t^2}$$
$$= \sqrt{(1.33\text{ m/s}^2)^2 + (1.20\text{ m/s}^2)^2} = \boxed{1.79\text{ m/s}^2}$$

at an angle

$$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{1.33\text{ m/s}^2}{1.20\text{ m/s}^2}\right) = \boxed{48.0°\text{ inward}}$$

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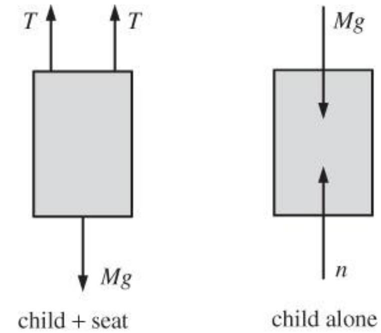
P6.10 We first draw a force diagram that shows the forces acting on the child-seat system and apply Newton's second law to solve the problem. The child's path is an arc of a circle, since the top ends of the chains are fixed. Then at the lowest point the child's motion is changing in direction: He moves with centripetal acceleration even as his speed is not changing and his tangential acceleration is zero.



ANS. FIG. P6.10

(a) ANS. FIG. P6.10 shows that the only forces acting on the system of child + seat are the tensions in the two chains and the weight of the boy:

$$\sum F = F_{\text{net}} = 2T - mg = ma = \frac{mv^2}{r}$$

with

$$F_{\text{net}} = 2T - mg = 2(350\text{ N}) - (40.0\text{ kg})(9.80\text{ m/s}^2) = 308\text{ N}$$

solving for v gives

$$v = \sqrt{\frac{F_{\text{net}}r}{m}} = \sqrt{\frac{(308\text{ N})(3.00\text{ m})}{40.0\text{ kg}}} = \boxed{4.81\text{ m/s}}$$

(b) The normal force from the seat on the child accelerates the child in the same way that the total tension in the chain accelerates the child-seat system. Therefore, $n = 2T = \boxed{700\text{ N}}$.

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P6.11 See the forces acting on seat (child) in ANS. FIG. P6.10.

(a) $\sum F = 2T - Mg = \dfrac{Mv^2}{R}$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

$$\boxed{v = \sqrt{(2T - Mg)\left(\frac{R}{M}\right)}}$$

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

$$\boxed{n = Mg + \frac{Mv^2}{R}}$$

P6.12 (a) Consider radial forces on the object, taking inward as positive.

$$\sum F_r = ma_r: \quad T - mg\cos\theta = \frac{mv^2}{r}$$

Solving for the tension gives

$$\begin{aligned} T &= mg\cos\theta + \frac{mv^2}{r} \\ &= (0.500\text{ kg})(9.80\text{ m/s}^2)\cos 20.0° \\ &\qquad + (0.500\text{ kg})(8.00\text{ m/s})^2/2.00\text{ m} \\ &= 4.60\text{ N} + 16.0\text{ N} = \boxed{20.6\text{ N}} \end{aligned}$$

(b) We already found the radial component of acceleration,

$$a_r = \frac{v^2}{r} = \frac{(8.00\text{ m/s})^2}{2.00\text{ m}} = \boxed{32.0\text{ m/s}^2\text{ inward}}$$

Consider the tangential forces on the object:

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$$\sum F_t = ma_t: \quad mg\sin\theta = ma_t$$

Solving for the tangential component of acceleration gives

$$a_t = g\sin\theta = \left(9.80 \text{ m/s}^2\right)\sin 20.0°$$
$$= \boxed{3.35 \text{ m/s}^2 \text{ downward tangent to the circle}}$$

(c) The magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(32.0 \text{ m/s}^2\right)^2 + \left(3.35 \text{ m/s}^2\right)^2} = 32.2 \text{ m/s}^2$$

at an angle of

$$\tan^{-1}\left(\frac{3.35 \text{ m/s}^2}{32.0 \text{ m/s}^2}\right) = 5.98°$$

Thus, the acceleration is

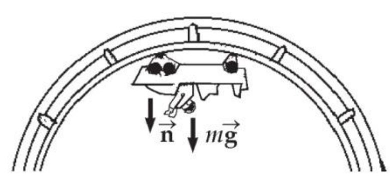
$$\boxed{32.2 \text{ m/s}^2 \text{ inward and below the cord at } 5.98°}$$

(d) $\boxed{\text{No change.}}$

(e) $\boxed{\text{If the object is swinging down it is gaining speed, and if the object is swinging up it is losing speed, but the forces are the same; therefore, its acceleration is regardless of the direction of swing.}}$

P6.13 (a) $a_c = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_c} = \frac{\left(13.0 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{8.62 \text{ m}}$$



ANS. FIG. P6.13

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(b) Let n be the force exerted by the rail.

Newton's second law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg,\ \text{downward}}$$

(c) $a_c = \frac{v^2}{r}$, or $a_c = \frac{(13.0\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{8.45\ \text{m/s}^2}$

(d) If the force exerted by the rail is n_1,

then $n_1 + Mg = \frac{Mv^2}{r} = Ma_c$

$n_1 = M(a_c - g)$ which is < 0 since a_c = 8.45 m/s^2

> Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars.

> In a teardrop-shaped loop, the radius of curvature r decreases, causing the centripetal acceleration to increase. The speed would decrease as the car rises (because of gravity), but the overall effect is that the required centripetal force increases, meaning the normal force increases--there is less danger if not wearing a seatbelt.

Section 6.3 Motion in Accelerated Frames

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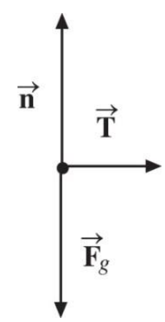
P6.14 (a) From $\sum F_x = Ma$, we obtain

$$a = \frac{T}{M} = \frac{18.0\text{ N}}{5.00\text{ kg}} = \boxed{3.60\text{ m/s}^2} \text{ to the right}$$

(b) If v = const, $a = 0$, so $\boxed{T = 0}$. (This is also an equilibrium situation.)

(c) $\boxed{\text{Someone in the car (noninertial observer) claims that the forces on the mass along } x \text{ are } T \text{ and a fictitious force } (-Ma).}$

(d) $\boxed{\text{Someone at rest outside the car (inertial observer) claims that } T \text{ is the only force on } M \text{ in the } x \text{ direction.}}$



ANS. FIG. P6.14

P6.15 The scale reads the upward normal force exerted by the floor on the passenger. The maximum force occurs during upward acceleration (when starting an upward trip or ending a downward trip). The

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minimum normal force occurs with downward acceleration. For each respective situation,

$$\sum F_y = ma_y \quad \text{becomes for starting} \quad +591\text{ N} - mg = +ma$$

$$\text{and for stopping} \quad +391\text{ N} - mg = -ma$$

where a represents the magnitude of the acceleration.

(a) These two simultaneous equations can be added to eliminate a and solve for mg:

$$+\,591\text{ N} - mg + 391\text{ N} - mg = 0$$

or $\quad 982\text{ N} - 2mg = 0$

$$F_g = mg = \frac{982\text{ N}}{2} = \boxed{491\text{ N}}$$

(b) From the definition of weight, $m = \dfrac{F_g}{g} = \dfrac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Substituting back gives $\quad +591\text{ N} - 491\text{ N} = (50.1\text{ kg})a$, or

$$a = \frac{100\text{ N}}{50.1\text{ kg}} = \boxed{2.00\text{ m/s}^2}$$

P6.16 Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g+a),\ f_k = \mu_k m(g+a)$$
$$\sum F_x = ma_x: \quad -\mu_k m(g+a) = ma_x$$

The motion across the floor is described by

$$L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g+a)t^2$$

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We solve for μ_k:

$$vt - L = \frac{1}{2}\mu_k(g+a)t^2$$

$$\boxed{\mu_k = \frac{2(vt-L)}{(g+a)t^2}}$$

P6.17 The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.120\text{ m})}{7.25\text{ s}} = 0.104\ \text{m/s}$$

The top layer of water feels a downward force of gravity *mg* and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104\ \text{m/s})^2}{0.12\text{ m}} = m\,9.01\times10^{-2}\ \text{m/s}^2$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\left(\frac{0.090\,1\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) = \boxed{0.527°}$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

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Section 6.4 Motion in the Presence of Resistive Forces

P6.18 With 100 km/h = 27.8 m/s, the resistive force is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}(0.250)\left(1.20\ \text{kg/m}^3\right)\left(2.20\ \text{m}^2\right)\left(27.8\ \text{m/s}\right)^2$$
$$= 255\ \text{N}$$
$$a = -\frac{R}{m} = -\frac{255\ \text{N}}{1\ 200\ \text{kg}} = \boxed{-0.212\ \text{m/s}^2}$$

P6.19 (a) Since the window is vertical, the normal force is horizontal and is given by $n = 4.00$ N. To find the vertical component of the force, we note that the force of kinetic friction is given by

$$f_k = \mu_k n = 0.900(4.00\ \text{N}) = 3.60\ \text{N upward}$$

to oppose downward motion. Newton's second law then becomes

$$\sum F_y = ma_y: \quad +3.6\ \text{N} - (0.16\ \text{kg})(9.8\ \text{m/s}^2) + P_y = 0$$
$$P_y = -2.03\ \text{N} = \boxed{2.03\ \text{N down}}$$

(b) Now, with the increased downward force, Newton's second law gives

$$\sum F_y = ma_y:$$
$$+3.60\ \text{N} - (0.160\ \text{kg})(9.80\ \text{m/s}^2) - 1.25(2.03\ \text{N})$$
$$= 0.160\ \text{kg}\ a_y$$

then

$$a_y = -0.508\ \text{N}/0.16\ \text{kg} = -3.18\ \text{m/s}^2 = \boxed{3.18\ \text{m/s}^2\ \text{down}}$$

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(c) At terminal velocity,

$$\sum F_y = ma_y: \quad +(20.0\ \text{N}\cdot\text{s/m})v_T - (0.160\ \text{kg})(9.80\ \text{m/s}^2) - 1.25(2.03\ \text{N}) = 0$$

Solving for the terminal velocity gives

$$v_T = 4.11\ \text{N}/(20\ \text{N}\cdot\text{s/m}) = \boxed{0.205\ \text{m/s down}}$$

P6.20 (a) The acceleration of the Styrofoam is given by

$$a = g - Bv$$

When $v = v_T$, $a = 0$ and $g = Bv_T \to B = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,

$$v_T = \frac{h}{\Delta t} = \frac{1.50\ \text{m}}{5.00\ \text{s}} = 0.300\ \text{m/s}$$

Then

$$\boxed{B = \frac{g}{v_T} = \frac{9.80\ \text{m/s}^2}{0.300\ \text{m/s}} = 32.7\ \text{s}^{-1}}$$

(b) At $t = 0$, $v = 0$, and $a = g = \boxed{9.80\ \text{m/s}^2\ \text{down}}$

(c) When $v = 0.150$ m/s,

$$\begin{aligned} a &= g - Bv \\ &= 9.80\ \text{m/s}^2 - \left(32.7\ \text{s}^{-1}\right)(0.150\ \text{m/s}) \\ &= \boxed{4.90\ \text{m/s}^2\ \text{down}} \end{aligned}$$

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P6.21 We have a particle under a net force in the special case of a resistive force proportional to speed, and also under the influence of the gravitational force.

(a) The speed v varies with time according to Equation 6.6,

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

where $v_T = mg/b$ is the terminal speed. Hence,

$$b = \frac{mg}{v_T} = \frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) To find the time interval for v to reach $0.632v_T$, we substitute $v = 0.632v_T$ into Equation 6.6, giving

$$0.632v_T = v_T\left(1 - e^{-bt/m}\right) \qquad \text{or} \qquad 0.368 = e^{-(1.47t/0.003\,00)}$$

Solve for t by taking the natural logarithm of each side of the equation:

$$\ln(0.368) = -\frac{1.47\,t}{3.00\times10^{-3}} \qquad \text{or} \qquad -1 = -\frac{1.47\,t}{3.00\times10^{-3}}$$

or $$t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$$

(c) At terminal speed, $R = v_T b = mg$. Therefore,

$$R = v_T b = mg = \left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{2.94\times10^{-2}\ \text{N}}$$

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P6.22 We start with Newton's second law,

$$\sum F = ma$$

substituting,

$$-kmv^2 = m\frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_i}^{v} v^{-2}\,dv$$

integrating both sides gives

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_i}^{v} = -\frac{1}{v} + \frac{1}{v_i}$$

$$\frac{1}{v} = \frac{1}{v_i} + kt = \frac{1+v_i kt}{v_i}$$

$$\boxed{v = \frac{v_i}{1+v_i kt}}$$

P6.23 In $R = \frac{1}{2}D\rho A v^2$, we estimate that the coefficient of drag for an open palm is D = 1.00, the density of air is $\rho = 1.20\ \text{kg/m}^3$, the area of an open palm is $A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60\times 10^{-2}\ \text{m}^2$, and v = 29.0 m/s (65 miles per hour). The resistance force is then

$$R = \frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60\times 10^{-2}\ \text{m}^2\right)(29.0\ \text{m/s})^2 = 8.07\ \text{N}$$

or $\quad R \sim \boxed{10^1\ \text{N}}$

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Additional Problems

P6.24 Because the car travels at a constant speed, it has no tangential acceleration, but it does have centripetal acceleration because it travels along a circular arc. The direction of the centripetal acceleration is toward the center of curvature, and the direction of velocity is tangent to the curve.

<u>Point A</u>

direction of velocity: $\boxed{\text{East}}$

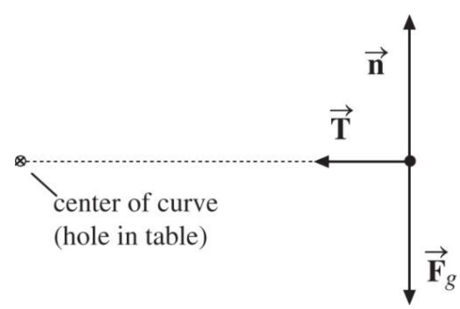
direction of the centripetal acceleration: $\boxed{\text{South}}$

<u>Point B</u>

direction of velocity: $\boxed{\text{South}}$

direction of the centripetal acceleration: $\boxed{\text{West}}$

P6.25 The free-body diagram of the rock is shown in ANS. FIG. P6.25. Take the x direction inward toward the center of the circle. The mass of the rock does not change. We know when $r_1 = 2.50$ m, $v_1 = 20.4$ m/s, and $T_1 = 50.0$ N. To find T_2 when $r_2 = 1.00$ m, and $v_2 = 51.0$ m/s, we use Newton's second law in the



ANS. FIG. P6.25

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horizontal direction:

$$\sum F_x = ma_x$$

In both cases,

$$T_1 = \frac{mv_1^2}{r_1} \quad \text{and} \quad T_2 = \frac{mv_2^2}{r_2}$$

Taking the ratio of the two tensions gives

$$\frac{T_2}{T_1} = \frac{v_2^2}{v_1^2}\frac{r_1}{r_2} = \left(\frac{51.0\text{ m/s}}{20.4\text{ m/s}}\right)^2\left(\frac{2.50\text{ m}}{1.00\text{ m}}\right) = 15.6$$

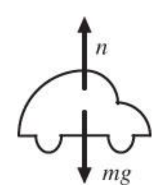
then

$$T_2 = 15.6T_1 = 15.6(50.0\text{ N}) = \boxed{781\text{ N}}$$

We assume the tension in the string is not altered by friction from the hole in the table.

P6.26 (a) We first convert the speed of the car to SI units:

$$v = (30\ \text{km/h})\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)\left(\frac{1\,000\text{ m}}{1\text{ km}}\right) = 8.33\ \text{m/s}$$



ANS. FIG. P6.26

Newton's second law in the vertical direction then gives

$$\sum F_y = ma_y: \qquad +n - mg = -\frac{mv^2}{r}$$

Solving for the normal force,

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$$n = m\left(g - \frac{v^2}{r}\right)$$
$$= (1\,800\text{ kg})\left[9.80\text{ m/s}^2 - \frac{(8.33\text{ m/s})^2}{20.4\text{ m}}\right]$$
$$= \boxed{1.15 \times 10^4\text{ N up}}$$

(b) At the maximum speed, the weight of the car is just enough to provide the centripetal force, so $n = 0$. Then $mg = \frac{mv^2}{r}$ and

$$v = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(20.4\text{ m})} = \boxed{14.1\text{ m/s}} = 50.9\text{ km/h}$$

P6.27 (a) The free-body diagram in ANS. FIG. P6.26 shows the forces on the car in the vertical direction. Newton's second law then gives

$$\sum F_y = ma_y = \frac{mv^2}{R}$$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $\qquad mg = \frac{mv^2}{R}$

Then, $v = \boxed{\sqrt{gR}}$

A more gently curved bump, with larger radius, allows the car to have a higher speed without leaving the road. This speed is proportional to the square root of the radius.

P6.28 The free-body diagram for the object is shown in ANS. FIG. P6.28. The object travels in a circle of radius $r = L\cos\theta$ about the vertical rod.

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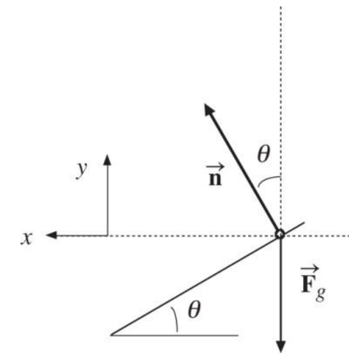
Taking inward toward the center of the circle as the positive x direction, we have

$$\sum F_x = ma_x: \quad n\sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = ma_y:$$

$$n\cos\theta - mg = 0 \rightarrow n\cos\theta = mg$$



ANS. FIG. P6.28

Dividing, we find

$$\frac{n\sin\theta}{n\cos\theta} = \frac{mv^2/r}{gr} \qquad \rightarrow \qquad \tan\theta = \frac{v^2}{gr}$$

Solving for v gives

$$v^2 = gr\tan\theta$$
$$v^2 = g(L\cos\theta)\tan\theta$$
$$\boxed{v = (gL\sin\theta)^{1/2}}$$

P6.29 Let v_i represent the speed of the object at time 0. We have

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{b}{m}\int_i^t dt \qquad \ln v\Big|_{v_i}^{v} = -\frac{b}{m}t\Big|_i^t$$

$$\ln v - \ln v_i = -\frac{b}{m}(t-0) \qquad \ln(v/v_i) = -\frac{bt}{m}$$

$$v/v_i = e^{-bt/m} \qquad \boxed{v = v_i e^{-bt/m}}$$

From its original value, the speed decreases rapidly at first and then more and more slowly, asymptotically approaching zero.

In this model the object keeps losing speed forever. It travels a finite distance in stopping.

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The distance it travels is given by

$$\int_0^r dr = v_i \int_0^t e^{-bt/m} dt$$

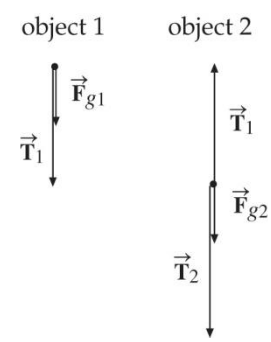
$$r = -\frac{m}{b} v_i \int_0^t e^{-bt/m} \left(-\frac{b}{m} dt \right) = -\frac{m}{b} v_i e^{-bt/m} \Big|_0^t$$
$$= -\frac{m}{b} v_i \left(e^{-bt/m} - 1 \right) = \frac{m v_i}{b} \left(1 - e^{-bt/m} \right)$$

As t goes to infinity, the distance approaches $\frac{m v_i}{b}(1-0) = m v_i / b$.

P6.30 The radius of the path of object 1 is twice that of object 2. Because the strings are always "collinear," both objects take the same time interval to travel around their respective circles; therefore, the speed of object 1 is twice that of object 2.

The free-body diagrams are shown in ANS. FIG. P6.30. We are given $m_1 = 4.00$ kg, $m_2 = 3.00$ kg,
$v = 4.00$ m/s, and $\ell = 0.500$ m.

Taking down as the positive direction, we have



Object 1: $T_1 + m_1 g = \frac{m_1 v_1^2}{r_1}$, where $v_1 = 2v$, $r_1 = 2\ell$.

Object 2: $T_2 - T_1 + m_2 g = \frac{m_2 v_2^2}{r_2}$, where $v_2 = v$, $r_2 = 2\ell$.

ANS. FIG. P6.30

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(a) From above:

$$T_1 = \frac{m_1 v_1^2}{r_1} - m_1 g = m_1\left(\frac{v_1{}^2}{r_1} - g\right)$$

$$T_1 = (4.00\text{ kg})\left[\frac{[2(4.00\text{ m/s})]^2}{2(0.500\text{ m})} - 9.80\text{ m/s}^2\right]$$

$$T_1 = 216.8\text{ N} = \boxed{217\text{ N}}$$

(b) From above:

$$T_2 = T_1 + \frac{m_2 v_2^2}{r_2} - m_2 g$$

$$T_2 = T_1 + m_2\left(\frac{v_2{}^2}{r_2} - g\right)$$

$$T_2 = T_1 + (3.00\text{ kg})\left[\frac{(4.00\text{ m/s})^2}{0.500\text{ m}} - 9.80\text{ m/s}^2\right]$$

$$T_2 = 216.8\text{ N} + 66.6\text{ N} = 283.4\text{ N} = \boxed{283\text{ N}}$$

(c) $\boxed{\text{From above, } T_2 > T_1 \text{ always, so string 2 will break first.}}$

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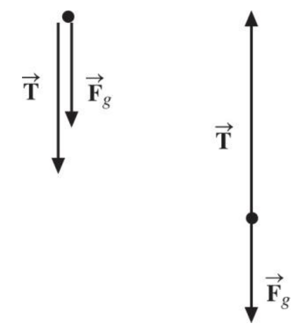
P6.31 (a) At each point on the vertical circular path, two forces are acting on the ball (see ANS. FIG. P6.31):

(1) $\boxed{\text{The downward gravitational force}}$ with constant magnitude $F_g = mg$

(2) $\boxed{\text{The tension force in the string, always directed toward the center of the path}}$



ANS. FIG. P6.31

(b) ANS. FIG. P6.31 shows the forces acting on the ball when it is at the highest point on the path (left-hand diagram) and when it is at the bottom of the circular path (right-hand diagram). Note that the gravitational force has the same magnitude and direction at each point on the circular path. The tension force varies in magnitude at different points and is always directed toward the center of the path.

(c) At the top of the circle, $F_c = mv^2/r = T + F_g$, or

$$T = \frac{mv^2}{r} - F_g = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

$$= (0.275\ \text{kg})\left[\frac{(5.20\ \text{m/s})^2}{0.850\ \text{m}} - 9.80\ \text{m/s}^2\right] = \boxed{6.05\ \text{N}}$$

(d) At the bottom of the circle, $F_c = mv^2/r = T - F_g = T - mg$, and solving for the speed gives

$$v^2 = \frac{r}{m}(T - mg) = r\left(\frac{T}{m} - g\right) \quad \text{and} \quad v = \sqrt{r\left(\frac{T}{m} - g\right)}$$

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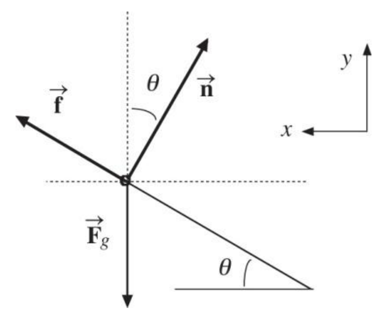
If the string is at the breaking point at the bottom of the circle, then $T = 22.5$ N, and the speed of the object at this point must be

$$v = \sqrt{(0.850\text{ m})\left(\frac{22.5\text{ N}}{0.275\text{ kg}} - 9.80\text{ m/s}^2\right)} = \boxed{7.82\text{ m/s}}$$

P6.32 The free-body diagram is shown on the right, where it is assumed that friction points up the incline, otherwise, the child would slide down the incline. The net force is directed left toward the center of the circular path in which the child travels. The radius of this path is $R = d\cos\theta$.



ANS. FIG. P6.46

Three forces act on the child, a normal force, static friction, and gravity. The relations of their force components are:

$$\sum F_x: \quad f_s \cos\theta - n\sin\theta = mv^2/R \qquad \textbf{[1]}$$

$$\sum F_y: \quad f_s \sin\theta + n\cos\theta - mg = 0 \rightarrow$$
$$f_s \sin\theta + n\cos\theta = mg \qquad [2]$$

Solve for the static friction and normal force.

To solve for static friction, multiply equation [1] by cos θ and equation [2] by sin θ and add:

$$\cos\theta\left[f_s\cos\theta - n\sin\theta\right] + \sin\theta\left[f_s\sin\theta - n\cos\theta\right]$$
$$= \cos\theta\left(\frac{mv^2}{R}\right) + \sin\theta(mg)$$

$$f_s = mg\sin\theta + \left(\frac{mv^2}{R}\right)\cos\theta$$

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To solve for the normal force, multiply equation [1] by –sin θ and equation [2] by cos θ and add:

$$-\sin\theta\left[f_s\cos\theta - n\sin\theta\right] + \cos\theta\left[f_s\sin\theta - n\cos\theta\right] = -\sin\theta\left(\frac{mv^2}{R}\right) + \cos\theta(mg)$$

$$n = mg\cos\theta - \left(\frac{mv^2}{R}\right)\sin\theta$$

In the above, we have used $\sin^2\theta + \cos^2\theta = 1$.

If the above equations are to be consistent, static friction and the normal force must satisfy the condition $f_s \le \mu_s n$; this means

$$(mg)\sin\theta + (mv^2/R)\cos\theta \le \mu_s[(mg)\cos\theta - (mv^2/R)\sin\theta] \rightarrow$$
$$v^2(\cos\theta + \mu_s\sin\theta) \le g\,R(\mu_s\cos\theta - \sin\theta)$$

Using this result, and that $R = d\cos\theta$, we have the requirement that

$$v \le \sqrt{\frac{gd\cos\theta(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}}$$

If this condition cannot be met, if v is too large, the physical situation cannot exist.

The values given in the problem are d = 5.32 m, μ_s = 0.700, θ = 20.0°, and v = 3.75 m/s. Check whether the given value of v satisfies the above condition:

$$\sqrt{\frac{(9.80\ \text{m/s}^2)(5.32\ \text{m})\cos 20.0°\left[(0.700)\cos 20.0° - \sin 20.0°\right]}{(\cos 20.0° + 0.700\sin 20.0°)}}$$
$$= 3.62\ \text{m/s}$$

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$\boxed{\text{The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline.}}$

P6.33 (a) We first convert miles per hour to feet per second:

$$v = (300\ \text{mi/h})\left(\frac{88.0\ \text{ft/s}}{60.0\ \text{mi/h}}\right) = 440\ \text{ft/s} \text{ at the top of the loop}$$

and $v = 450$ mi/h $= 660$ ft/s at the bottom of the loop.

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160\ \text{lb} + \left(\frac{160\ \text{lb}}{32.0\ \text{ft/s}^2}\right)\frac{(660\ \text{ft/s})^2}{1\,200\ \text{ft}} = \boxed{1\,975\ \text{lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = 160\ \text{lb} - \left(\frac{160\ \text{lb}}{32.0\ \text{ft/s}^2}\right)\frac{(440\ \text{ft/s})^2}{1\,200\ \text{ft}} = \boxed{-647\ \text{lb}}$$

Since the plane is upside down, the seat exerts this downward force as a normal force.

(c) $\boxed{\text{When } F_g' = 0, \text{ then } mg = \dfrac{mv^2}{R}. \text{ If we vary the aircraft's } R \text{ and } v \text{ such that this equation is satisfied, then the pilot feels weightless.}}$

P6.34 (a) The forces acting on the ice cube are the Earth's gravitational force, straight down, and the basin's normal force, upward and inward at 35.0° with the vertical. We choose the x and y axes to be horizontal and vertical, so that the acceleration is purely in the x

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direction. Then

$$\sum F_x = ma_x: \quad n\sin 35° = mv^2/R$$

$$\sum F_y = ma_y: \quad n\cos 35° - mg = 0$$

Dividing eliminates the normal force:

$$n\sin 35.0°/n\cos\ 35.0° = mv^2/Rmg$$

$$\tan 35.0° = v^2/Rg$$

$$\boxed{v = \sqrt{Rg\tan 35.0°} = \sqrt{\left(6.86\ \text{m/s}^2\right)R}}$$

(b) $\boxed{\text{The mass is unnecessary.}}$

(c) The answer to (a) indicates that the speed is proportional to the square root of the radius, so $\boxed{\text{increasing the radius will make the required speed increase.}}$

(d) The period of revolution is given by

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg\tan 35.0°}} = \left(2.40\ \text{s}/\sqrt{\text{m}}\right)\sqrt{R}$$

$\boxed{\text{When the radius increases, the period increases.}}$

(e) $\boxed{\text{On a larger circle, the ice cube's speed is proportional to } \sqrt{R} \text{ but the distance it travels is proportional to } R\text{, so the time interval required is proportional to } R/\sqrt{R} = \sqrt{R}.}$

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P6.35 (a) $\boxed{\text{The only horizontal force on the car is the force of friction, with a maximum value determined by the surface roughness (described by the coefficient of static friction) and the normal force (here equal to the gravitational force on the car).}}$

(b) From Newton's second law in one dimension,

$$\sum F_x = ma_x: \quad -f = ma \quad \rightarrow \quad a = -\frac{f}{m} = \left(v^2 - v_0^2\right)/2\left(x - x_0\right)$$

solving for the stopping distance gives

$$x - x_0 = \frac{m\left(v^2 - v_0^2\right)}{2f} = \frac{(1\ 200\ \text{kg})\left[0^2 - (20.0\ \text{m/s})^2\right]}{2(-7\ 000\ \text{N})} = \boxed{34.3\ \text{m}}$$

(c) Newton's second law now gives

$$f = \frac{mv^2}{r}$$

or $$r = \frac{mv^2}{f} = \frac{(1\ 200\ \text{kg})(20.0\ \text{m/s})^2}{7\ 000\ \text{N}} = \boxed{68.6\ \text{m}}$$

A top view shows that you can avoid running into the wall by turning through a quarter-circle, if you start at least this far away from the wall.

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(d) Braking is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car, and the stopping distance would be longer.

(e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

P6.36 Take the positive x axis up the hill. Newton's second law in the x direction then gives

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$

from which we obtain

$$a = \frac{T}{m}\sin\theta - g\sin\phi \qquad \textbf{[1]}$$

In the y direction,

$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$

Solving for the tension gives

$$T = \frac{mg\cos\phi}{\cos\theta} \qquad \textbf{[2]}$$

Substituting for T from [2] into [1] gives

$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

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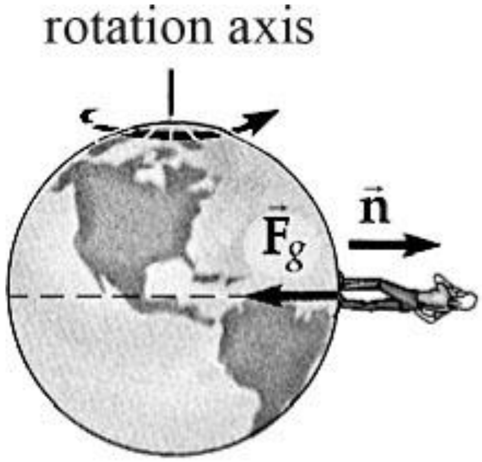
P6.37 (a) The gravitational force exerted by the planet on



ANS. FIG. P6.37

the person is

$$mg = (75.0\text{ kg})(9.80\text{ m/s}^2)$$

$$= \boxed{735\text{ N}}\text{ down}$$

Let n represent the force exerted on the person by a scale, which is an upward force whose size is her "apparent weight." The true weight is mg down. For the person at the equator, summing up forces on the object in the direction towards the Earth's center gives $\sum F = ma$:

$$mg - n = ma_c$$

where $a_c = v^2/R_E = 0.033\,7\text{ m/s}^2$

is the centripetal acceleration directed toward the center of the Earth.

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Thus, we can solve part (c) before part (b) by noting that

$$n = m(g - a_c) < mg$$

(c) or $mg = n + ma_c > n$.

$\boxed{\text{The gravitational force is greater. The normal force is smaller, just as one experiences at the top of a moving ferris wheel.}}$

(b) If $m = 75.0$ kg and $g = 9.80$ m/s^2, at the equator we have

$$n = m(g - a_c) = (75.0\text{ kg})(9.800\text{ m/s}^2 - 0.033\,7\text{ m/s}^2) = \boxed{732\text{ N}}$$

P6.38 (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$,

we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}}$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension pulls at an angle of less than 90° to the direction of the inward-spiraling velocity,

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producing forward tangential acceleration as well as inward radial acceleration of the puck.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

P6.39 (a) $v = v_i + kx$ implies the acceleration is

$$\boxed{a = \frac{dv}{dt} = 0 + k\frac{dx}{dt} = +kv}$$

(b) The total force is

$$\sum F = ma = m(+kv)$$

As a vector, the force is parallel or antiparallel to the velocity:

$$\boxed{\sum \vec{\mathbf{F}} = km\vec{\mathbf{v}}}$$

(c) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially.

(d) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

P6.40 (a)

t(s)	d (m)	t(s)	d (m)
1.00	4.88	11.0	399
2.00	18.9	12.0	452
3.00	42.1	13.0	505
4.00	43.8	14.0	558

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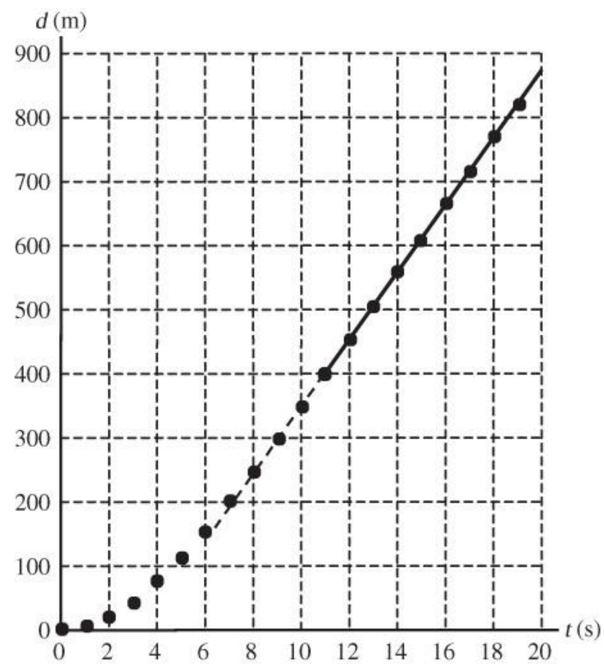
5.00	112	15.0	611
6.00	154	16.0	664
7.00	199	17.0	717
8.00	246	18.0	770
9.00	296	19.0	823
10.0	347	20.0	876

(b)



(c) A straight line fits the points from $t = 11.0$ s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\ \text{m} - 399\ \text{m}}{20.0\ \text{s} - 11.0\ \text{s}} = \boxed{53.0\ \text{m/s}}$$

P6.41 (a) If the car is about to slip *down* the incline, *f* is directed up the

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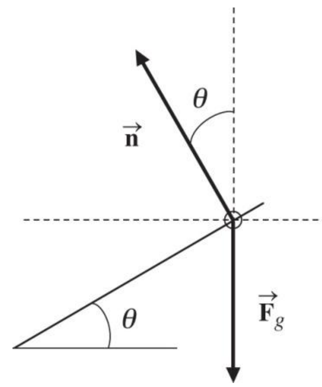
incline.

$$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$$

where $f = \mu_s n$. Substituting,

$$n = \frac{mg}{\cos\theta\left(1+\mu_s \tan\theta\right)}$$

and $f = \dfrac{\mu_s mg}{\cos\theta\left(1+\mu_s \tan\theta\right)}$



ANS. FIG. P6.41

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\text{min}}^2}{R}$ yields

$$\boxed{v_{\text{min}} = \sqrt{\frac{Rg\left(\tan\theta - \mu_s\right)}{1+\mu_s \tan\theta}}}$$

When the car is about to slip *up* the incline, *f* is directed down the incline. Then,

$$\sum F_y = n\cos\theta - f\sin\theta - mg = 0, \text{ with } f = \mu_s n$$

This yields

$$n = \frac{mg}{\cos\theta\left(1-\mu_s \tan\theta\right)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta\left(1-\mu_s \tan\theta\right)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{\text{max}}^2}{R}$, which gives

$$\boxed{v_{\text{max}} = \sqrt{\frac{Rg\left(\tan\theta + \mu_s\right)}{1-\mu_s \tan\theta}}}$$

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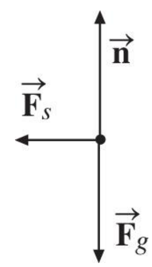
(b) If $v_{\min} = \sqrt{\dfrac{Rg(\tan\theta - \mu_s)}{1 + \mu_s \tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

P6.42 There are three forces on the child, a vertical normal force, a horizontal force (combination of friction and a horizontal force from a seat belt), and gravity.

$$\sum F_x: \quad F_s = mv^2/R$$

$$\sum F_y: \quad n - mg = 0 \to n = mg$$



ANS. FIG. P6.42

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{(mv^2/R)^2 + (mg)^2}$$

with a direction of

$$\theta = \tan^{-1}\left[\frac{mg}{mv^2/R}\right] = \tan^{-1}\left[\frac{gR}{v^2}\right] \text{ above the horizontal}$$

For $m = 40.0$ kg and $R = 10.0$ m:

$$F_{\text{net}} = \sqrt{\left[\frac{(40.0\text{ kg})(3.00\text{ m/s})^2}{10.0\text{ m}}\right]^2 + \left[(40.0\text{ kg})(9.80\text{ m/s}^2)\right]^2}$$

$$\boxed{F_{\text{net}} = 394\text{ N}}$$

direction: $\theta = \tan^{-1}\left[\dfrac{(9.80\text{ m/s}^2)(10.0\text{ m})}{(3.00\text{ m/s})^2}\right] \to \boxed{\theta = 84.7°}$

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P6.43 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$, where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$.

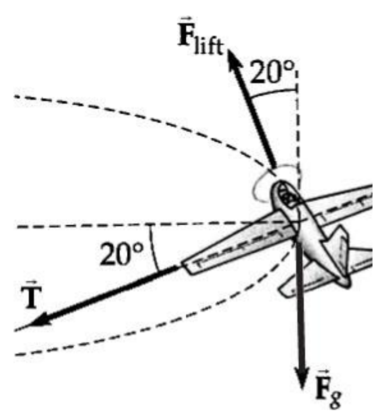
Thus, $v^2 = \pi R g$ and $v = \boxed{\sqrt{\pi R g}}$.

(b) The putty is dislodged when F, the force holding it to the wheel, is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

P6.44 The plane's acceleration is toward the center of the circle of motion, so it is horizontal. The radius of the circle of motion is (60.0 m) cos 20.0° = 56.4 m and the acceleration is

$$a_c = \frac{v^2}{r} = \frac{(35\ \text{m/s})^2}{56.4\ \text{m}}$$
$$= 21.7\ \text{m/s}^2$$



ANS. FIG. P6.44

We can also calculate the weight of the airplane:

$$F_g = mg$$
$$= (0.750\ \text{kg})(9.80\ \text{m/s}^2)$$
$$= 7.35\ \text{N}$$

We define our axes for convenience. In this case, two of the forces—one of them our force of interest—are directed along the 20.0° line. We

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define the x axis to be directed in the $+\vec{T}$ direction, and the y axis to be directed in the direction of lift. With these definitions, the x component of the centripetal acceleration is

$$a_{cx} = a_c \cos 20.0°$$

and $\qquad \sum F_x = ma_x \quad$ yields $\quad T + F_g \sin 20.0° = ma_{cx}$

Solving for T,

$$T = ma_{cx} - F_g \sin 20.0°$$

Substituting,

$$T = (0.750 \text{ kg})(21.7 \text{ m/s}^2) \cos 20.0° - (7.35 \text{ N}) \sin 20.0°$$

Computing,

$$T = 15.3 \text{ N} - 2.51 \text{ N} = \boxed{12.8 \text{ N}}$$

Challenge Problems

P6.45 We find the terminal speed from

$$v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] \qquad \textbf{[1]}$$

where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$: $\qquad v \to v_T = \dfrac{mg}{b}$

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At $t = 5.54$ s: $$0.500v_T = v_T\left[1-\exp\left(\frac{-b(5.54\text{ s})}{9.00\text{ kg}}\right)\right]$$

Solving,

$$\exp\left(\frac{-b(5.54\text{ s})}{9.00\text{ kg}}\right) = 0.500$$

$$\frac{-b(5.54\text{ s})}{9.00\text{ kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\text{ kg})(0.693)}{5.54\text{ s}} = 1.13\ \text{kg/s}$$

(a) From $v_T = \dfrac{mg}{b}$, we have

$$v_T = \frac{(9.00\text{ kg})(9.80\ \text{m/s}^2)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$$

(b) We substitute $0.750v_T$ on the left-hand side of equation [1]:

$$0.750v_T = v_T\left[1-\exp\left(\frac{-1.13t}{9.00\text{ s}}\right)\right]$$

and solve for t:

$$\exp\left(\frac{-1.13t}{9.00\text{ s}}\right) = 0.250$$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\text{ s}}$$

(c) We differentiate equation [1] with respect to time,

$$\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1-\exp\left(-\frac{bt}{m}\right)\right]$$

then, integrate both sides

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$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] dt$$

$$\begin{aligned} x - x_0 &= \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Bigg|_0^t \\ &= \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right) - 1\right] \end{aligned}$$

At $t = 5.54$ s,

$$\begin{aligned} x &= (9.00\text{ kg})(9.80\text{ m/s}^2)\left(\frac{5.54\text{ s}}{1.13\text{ kg/s}}\right) \\ &\quad + \left(\frac{(9.00\text{ kg})^2(9.80\text{ m/s}^2)}{(1.13\text{ kg/s})^2}\right)[\exp(-0.693) - 1] \\ x &= 434\text{ m} + 626\text{ m}(-0.500) = \boxed{121\text{ m}} \end{aligned}$$

P6.46 (a) From Problem 6.33,

$$\begin{aligned} v &= \frac{dx}{dt} = \frac{v_i}{1 + v_i kt} \\ \int_0^x dx &= \int_0^t v_i \frac{dt}{1 + v_i kt} = \frac{1}{k}\int_0^t \frac{v_i k dt}{1 + v_i kt} \\ x\big|_0^x &= \frac{1}{k}\ln(1 + v_i kt)\Big|_0^t \\ x - 0 &= \frac{1}{k}[\ln(1 + v_i kt) - \ln 1] \end{aligned}$$

$$\boxed{x = \frac{1}{k}\ln(1 + v_i kt)}$$

(b) We have $\ln(1 + v_i kt) = kx$

$$1 + v_i kt = e^{kx} \quad \text{so} \quad v = \frac{v_i}{1 + v_i kt} = \frac{v_i}{e^{kx}} = \boxed{v_i e^{-kx} = v}$$

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P6.47 Let the x axis point eastward, the y axis upward, and the z axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80 \text{ m/s}^2)(285 \text{ m})}{\sin 96.0°}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \dfrac{1}{2}a_y t^2$ as

$$0 = (53.0 \text{ m/s})(\sin 48.0°)t - (4.90 \text{ m/s}^2)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $$v_{xi} = \frac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \frac{2\pi(6.37\times 10^6 \text{ m})\cos 35.0°}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285 \text{ m}}{2\pi(6.37\times 10^6 \text{ m})}\right)(360°)$$
$$= 2.56\times 10^{-3} \text{ degrees}$$

The final latitude is then

$$\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$$

The cup is moving eastward at a speed

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$$v_{xf} = \frac{2\pi R_e \cos\phi_f}{86\ 400\ \text{s}}$$

which is larger than the eastward velocity of the tee by

$$\begin{aligned}
\Delta v_x = v_{xf} - v_{xi} &= \left(\frac{2\pi R_e}{86\ 400\ \text{s}}\right)\left[\cos\phi_f - \cos\phi_i\right] \\
&= \left(\frac{2\pi R_e}{86\ 400\ \text{s}}\right)\left[\cos(\phi - \Delta\phi) - \cos\phi_i\right] \\
&= \left(\frac{2\pi R_e}{86\ 400\ \text{s}}\right)\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]
\end{aligned}$$

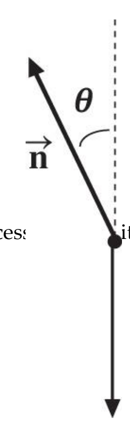
Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and

$$\Delta v_x \approx \left(\frac{2\pi R_e}{86\ 400\ \text{s}}\right)\sin\phi_i \sin\Delta\phi$$

$$\begin{aligned}
\Delta v_x &\approx \left[\frac{2\pi\left(6.37\times10^6\ \text{m}\right)}{86\ 400\ \text{s}}\right]\sin 35.0° \sin 0.002\ 56° \\
&= \boxed{1.19\times10^{-2}\ \text{m/s}}
\end{aligned}$$

(d) $\Delta x = (\Delta v_x)t = \left(1.19\times10^{-2}\ \text{m/s}\right)(8.04\ \text{s}) = 0.095\ 5\ \text{m} = \boxed{9.55\ \text{cm}}$

P6.48 (a) We let R represent the radius of the hoop and T represent the period of its rotation. The bead moves in a circle with radius $r = R\sin\theta$ at a speed of



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$$v = \frac{2\pi r}{T} = \frac{2\pi R \sin\theta}{T}$$

The normal force has an inward radial component of $n\sin\theta$ and an upward component of $n\cos\theta$.

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

ANS. FIG. P6.48

or

$$n = \frac{mg}{\cos\theta}$$

Then $\sum F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$$

which reduces to $\dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \quad \Rightarrow \quad \theta = 0°$ **[1]**

and $\cos\theta = \dfrac{gT^2}{4\pi^2 R}$ **[2]**

If R = 15.0 cm and T = 0.450 s, the second solution yields

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.450\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 0.335 \quad \text{or} \quad \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions: $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

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(b) At this slower rotation, solution [2] above becomes

$$\cos\theta = \frac{\left(9.80\ \text{m/s}^2\right)\left(0.850\ \text{s}\right)^2}{4\pi^2\left(0.150\ \text{m}\right)} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$.

(c) $\boxed{\text{There is only one solution for (b) because the period is too large.}}$

(d) $\boxed{\begin{array}{l}\text{The equation that the angle must satisfy has two solutions} \\ \text{whenever } 4\pi^2 R > gT^2 \text{ but only the solution } 0° \text{ otherwise.} \\ \text{The loop's rotation must be faster than a certain threshold} \\ \text{value in order for the bead to move away from the lowest} \\ \text{position. Zero is always a solution for the angle.}\end{array}}$

(e) $\boxed{\begin{array}{l}\text{From the derivation of the solution in (a), there are never more} \\ \text{than two solutions.}\end{array}}$

P6.49 At a latitude of 35°, the centripetal acceleration of a plumb bob is directed at 35° to the local normal, as can be seen from the following diagram below at left.

Therefore, if we look at a diagram of the forces on the plumb bob and its acceleration with the local normal in a vertical orientation, we see the second diagram in ANS. FIG. P6.49:

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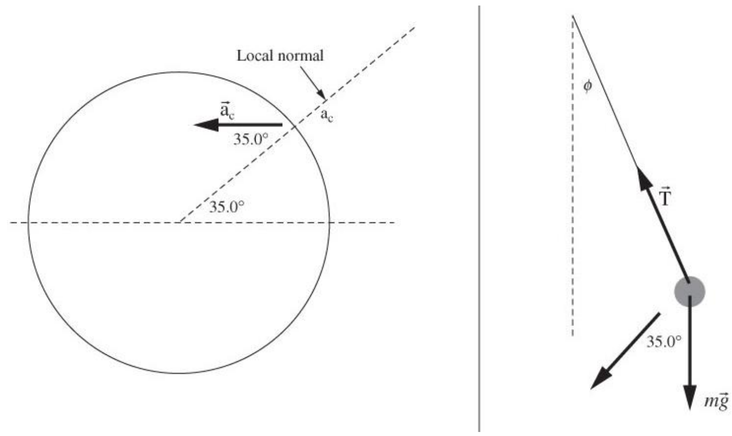

ANS. FIG. P6.49

We first find the centripetal acceleration of the plumb bob. The first figure shows that the radius of the circular path of the plumb bob is R cos 35.0°, where R is the radius of the Earth. The acceleration is

$$a_c = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 R\cos 35.0^\circ}{T^2}$$
$$= \frac{4\pi^2\left(6.37\times 10^6\ \text{m}\right)\cos 35.0^\circ}{(86\ 400\ \text{s})^2} = 0.027\ 6\ \text{m/s}^2$$

Apply the particle under a net force model to the plumb bob in both x and y directions in the second diagram:

$$x:\ T\sin\phi = ma_c\sin 35.0^\circ$$
$$y:\ mg - T\cos\phi = ma_c\cos 35.0^\circ$$

Divide the equations:

$$\tan\phi = \frac{a_c\sin 35.0^\circ}{g - a_c\cos 35.0^\circ}$$

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$$\tan\phi = \frac{(0.027\ 6\ \text{m/s}^2)\sin 35.0°}{9.80\ \text{m/s}^2 - (0.027\ 6\ \text{m/s}^2)\cos 35.0°} = 1.62 \times 10^{-3}$$

$$\phi = \tan^{-1}(1.62 \times 10^{-3}) = \boxed{0.092\ 8°}$$

***P6.50 Conceptualize** The ball is moving through air, so the resistive force will cause it to slow down as it approaches home plate. This is very hard to see in the short time interval between the pitcher's mound and home plate, but it does happen.

Categorize Because the speed of the baseball through the air is high, we use Equation 6.7 to characterize the resistive force on the baseball. As the ball moves through the air, it is modeled as a *particle under a net force,* where the net force varies in time, because it varies with speed.

Analyze Using Newton's second law, with the resistive force on the baseball described by Equation 6.7, we have

$$\sum F_x = ma \quad \rightarrow \quad -\tfrac{1}{2}D\rho A v^2 = ma \quad (1)$$

Solve Equation (1) for a:

$$a = -\frac{D\rho A}{2m}v^2 = -kv^2 \quad (2)$$

where we have defined k as

$$k = \frac{D\rho A}{2m} \quad (3)$$

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Now, let's express *a* as a derivative and use the chain rule as suggested in the Hint:

$$\frac{dv}{dt} = -kv^2 \quad \rightarrow \quad \frac{dv}{dx}\frac{dx}{dt} = -kv^2 \quad \rightarrow \quad \frac{dv}{dx}v = -kv^2 \quad \rightarrow \quad \frac{dv}{dx} = -kv \quad (4)$$

Rearrange the differential equation and integrate:

$$\frac{dv}{v} = -kdx \quad \rightarrow \quad \int_{v_i}^{v} \frac{dv}{v} = -k\int_0^x dx \quad \rightarrow \quad \ln\frac{v}{v_i} = -kx \quad \rightarrow \quad v = v_i e^{-kx} \quad (5)$$

Considering Equation 6.10, we recognize the parameters that form the value of *k* as appearing in the expression for the terminal speed:

$$v_T = \sqrt{\frac{2mg}{D\rho A}} = \sqrt{\frac{g}{k}} \quad (6)$$

Therefore, *k* is given by

$$k = \frac{g}{v_T^2} \quad (7)$$

The equation for the speed of the baseball at any time, then, can be expressed as

$$v = v_i e^{-\frac{g}{v_T^2}x} \quad (8)$$

Substitute numerical values, using the terminal velocity found in Table 6.1:

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$$v = (40.2\ \mathrm{m/s})e^{-\frac{9.80\ \mathrm{m/s^2}}{(43\ \mathrm{m/s})^2}(18.3\ \mathrm{m})} = 36.5\ \mathrm{m/s}$$

Finalize We were able to simplify this problem because we have the terminal speed of a baseball given in Table 6.1. If we didn't have that value, we could express Equation (5) as

$$v = v_i e^{-\frac{D\rho A}{2m}x}$$

This equation would also work, but we would need to determine the density of air as well as parameters associated with the baseball: its drag coefficient, mass, and cross-sectional area

Answer: 36.5 m/s

ANSWERS TO QUICK-QUIZZES

1. **(i)** (a) **(ii)** (b)

2. **(i)** Because the speed is constant, the only direction the force can have is that of the centripetal acceleration. The force is larger at Ⓒ than at Ⓐ because the radius at Ⓒ is smaller. There is no force at Ⓑ because the wire is straight. **(ii)** In addition to the forces in the centripetal direction in part (a), there are now tangential forces to provide the tangential acceleration. The tangential force is the same at all three points because the tangential acceleration is constant.

3. (c)

4. (a)

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ANSWERS TO EVEN-NUMBERED PROBLEMS

P6.2 (a) 1.65×10^3 m/s; (b) 6.84×10^3 s

P6.4 215 N, horizontally inward

P6.6 The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.

P6.8 The radius of curvature is larger than 150 m, so the driver is not justified in his claim as to faulty design of the roadway.

P6.10 (a) 4.81 m/s; (b) 700 N

P6.12 (a) 20.6 N; (b) 32.0 m/s^2 inward, 3.35 m/s^2 downward tangent to the circle; (c) 32.2 m/s^2 inward and below the cord at 5.98°; (d) no change; (e) acceleration is regardless of the direction of swing

P6.14 (a) 3.60 m/s^2; (b) $T = 0$; (c) noninertial observer in the car claims that the forces on the mass along x are T and a fictitious force ($-Ma$); (d) inertial observer outside the car claims that T is the only force on M in the x direction

P6.16 $\dfrac{2(vt-L)}{(g+a)t^2}$

P6.18 0.212 m/s^2, opposite the velocity vector

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P6.20 (a) 32.7 s^{-1}; (b) 9.80 m/s^2 down; (c) 4.90 m/s^2 down

P6.22 (a) 1975 lb (b) -647 lb

(c) When $F_g' = 0$, then $mg = mv^2/R$. If we vary the aircraft's R and v such that this equation is satisified, then the pilot feels weightless.

P6.24 (a) At A, the velocity is eastward and the acceleration is southward.

(b) At B, the velocity is southward and the acceleration is westward.

P6.26 (a) 1.15×10^4 N up; (b) 14.1 m/s

P6.28 See Problem 6.28 for full derivation.

P6.30 (a) 217 N; (b) 283 N; (c) $T_2 > T_1$ always, so string 2 will break first

P6.32 The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline

P6.34 (a) $v = \sqrt{Rg\tan 35.0°} = \sqrt{(6.86\ \text{m/s}^2)R}$; (b) the mass is unnecessary; (c) increasing the radius will make the required speed increase; (d) when the radius increases, the period increases; (e) the time interval required is proportional to $R/\sqrt{R} = \sqrt{R}$

P6.36 $g(\cos\varphi \tan\theta - 2\sin\varphi)$

P6.38 (a) m_2g; (b) m_2g; (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$; (d) The puck will spiral inward, gaining speed as it does so; (e) The puck will spiral outward, slowing down as it does so

P6.40 (a) See table in P6.40 (a); (b) See graph in P6.60 (b); (c) 53.0 m/s

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P6.42 84.7°

P6.44 12.8 N

P6.46 (a) $x = \dfrac{1}{k}\ln\left(1 + v_i kt\right)$; (b) $v = v_i e^{-kx}$

P6.48 (a) θ = 70.4° and θ = 0°; (b) θ = 0°; (c) the period is too large; (d) Zero is always a solution for the angle; (e) there are never more than two solutions

P6.50 36.5 m/s

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