

21

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

- 21.1 Heat Engines and the Second Law of Thermodynamics
- 21.2 Heat Pumps and Refrigerators
- 21.3 Reversible and Irreversible Processes
- 21.4 The Carnot Engine
- 21.5 Gasoline and Diesel Engines
- 21.6 Entropy
- 21.7 Entropy in Thermodynamic Systems
- 21.8 Entropy and the Second Law

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP21.1 Conceptualize** Review Section 21.4 and Figures 21.9 and 21.10 to make sure you understand the Carnot cycle.

Categorize We categorize the engine as operating in a Carnot cycle, allowing us to further categorize the processes in the cycle as adiabatic and isothermal.

Analyze (a) Let us begin by entering into the table the information provided in the problem statement:

Point in Cycle	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B			
C		24.0	
D		15.0	

Because process $A \rightarrow B$ is isothermal, we know that $T_B = 720$ K. Let us now work our way around the cycle. We begin from point D so that we can take advantage of Equations 20.37 and 20.39 for adiabatic processes. For the adiabatic process $D \rightarrow A$, we can write

$$P_D V_D^\gamma = P_A V_A^\gamma \rightarrow P_D = P_A \left(\frac{V_A}{V_D} \right)^\gamma = (1\,400 \text{ kPa}) \left(\frac{10.0 \text{ L}}{15.0 \text{ L}} \right)^{5/3} = 712 \text{ kPa}$$

$$T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1} \rightarrow T_D = T_A \left(\frac{V_A}{V_D} \right)^{\gamma-1} = (720 \text{ K}) \left(\frac{10.0 \text{ L}}{15.0 \text{ L}} \right)^{2/3} = 549 \text{ K}$$

Because process $C \rightarrow D$ is isothermal, we know that $T_C = 549$ K. Now use the ideal gas law (Equation 18.9) to find the pressure at C :

$$PV = nRT$$

$$\rightarrow P_C = \frac{nRT_C}{V_C} = \frac{(2.34 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(549 \text{ K})}{24.0 \text{ L}} \left(\frac{1 \text{ L}}{10^{-3} \text{ m}^3} \right)$$

$$= 445 \text{ kPa}$$

Now use Equations 20.39 and 20.37 again for the adiabatic process $B \rightarrow C$:

$$T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1} \rightarrow V_B = V_C \left(\frac{T_C}{T_B} \right)^{1/(\gamma-1)} = (24.0 \text{ L}) \left(\frac{549 \text{ K}}{720 \text{ K}} \right)^{3/2} = 16.0 \text{ L}$$

$$P_B V_B^\gamma = P_C V_C^\gamma \rightarrow P_B = P_C \left(\frac{V_C}{V_B} \right)^\gamma = (445 \text{ kPa}) \left(\frac{24.0 \text{ L}}{16.0 \text{ L}} \right)^{5/3} = 875 \text{ kPa}$$

Entering these numbers in the table, we have:

Point in Cycle	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b) We begin by entering zero for the change in internal energy for the isothermal processes and for the heat in the adiabatic processes:

Process in Cycle	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A \rightarrow B$ (isothermal)			0
$B \rightarrow C$ (adiabatic)	0		
$C \rightarrow D$ (isothermal)			0
$D \rightarrow A$ (adiabatic)	0		

Use Equation 21.12 to find the work done in the isothermal processes:

$$\begin{aligned}
 W_{A \rightarrow B} &= nRT_A \ln \left(\frac{V_A}{V_B} \right) \\
 &= (2.34 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(720 \text{ K}) \ln \left(\frac{10.0 \text{ L}}{16.0 \text{ L}} \right) = -6.58 \text{ kJ}
 \end{aligned}$$

$$\begin{aligned}
 W_{C \rightarrow D} &= nRT_C \ln \left(\frac{V_C}{V_D} \right) \\
 &= (2.34 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(549 \text{ K}) \ln \left(\frac{24.0 \text{ L}}{15.0 \text{ L}} \right) = 5.02 \text{ kJ}
 \end{aligned}$$

Now, from the first law of thermodynamics, for an isothermal process,

$$\Delta E_{\text{int}} = 0 = W + Q \rightarrow Q = -W \quad (\text{isothermal})$$

Therefore,

$$Q_{A \rightarrow B} = 6.58 \text{ kJ}$$

$$Q_{C \rightarrow D} = -5.02 \text{ kJ}$$

For the adiabatic processes, we can use Equation 20.26 to find the change in internal energy:

$$\begin{aligned} \Delta E_{\text{int}, B \rightarrow C} &= nC_V (T_C - T_B) = \frac{3}{2} nR (T_C - T_B) \\ &= \frac{3}{2} (2.34 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) (549 \text{ K} - 720 \text{ K}) = -4.99 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \Delta E_{\text{int}, D \rightarrow A} &= nC_V (T_A - T_D) = \frac{3}{2} nR (T_A - T_D) \\ &= \frac{3}{2} (2.34 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) (720 \text{ K} - 549 \text{ K}) = 4.99 \text{ kJ} \end{aligned}$$

Now, from the first law of thermodynamics, for an adiabatic process,

$$\Delta E_{\text{int}} = W + 0 \rightarrow W = \Delta E_{\text{int}} \quad (\text{adiabatic})$$

Therefore,

$$W_{B \rightarrow C} = -4.99 \text{ kJ}$$

$$W_{D \rightarrow A} = 4.99 \text{ kJ}$$

Entering these numbers in the table, we have the following:

Process in Cycle	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A \rightarrow B$	6.58	-6.58	0
$B \rightarrow C$	0	-4.99	-4.99
$C \rightarrow D$	-5.02	5.02	0
$D \rightarrow A$	0	4.99	4.99

(c) From the table in part (b), we find, using Equation 21.2,

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W}{|Q_h|} = \frac{-(-6.58 \text{ kJ} - 4.99 \text{ kJ} + 5.02 \text{ kJ} + 4.99 \text{ kJ})}{6.58 \text{ kJ}} = 0.237 = \boxed{23.7\%}$$

(d) From the table in part (a), we find, using Equation 21.8,

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{549 \text{ K}}{720 \text{ K}} = 0.237 = \boxed{23.7\%}$$

Finalize Notice that we obtained the same efficiency in parts (c) and (d). That must be true, since both tables refer to the same engine, which happens to be a Carnot engine.

Answers: (a)

Point in Cycle	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b)

Process in Cycle	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A \rightarrow B$	6.58	−6.58	0
$B \rightarrow C$	0	−4.99	−4.99
$C \rightarrow D$	−5.02	5.02	0
$D \rightarrow A$	0	4.99	4.99

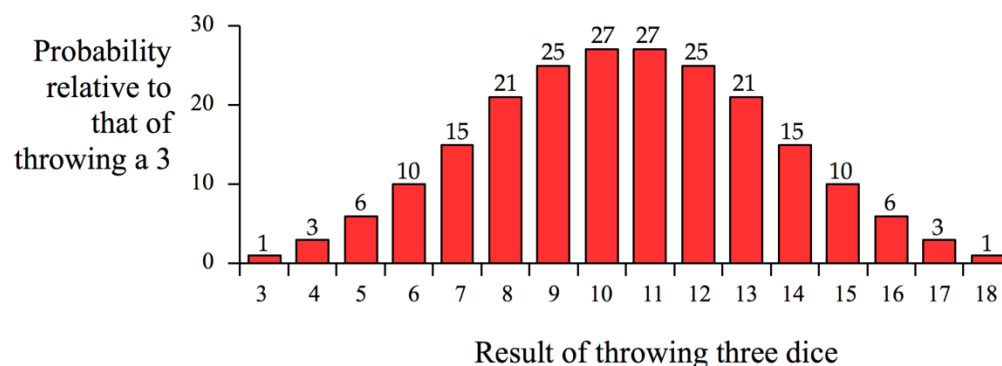
(c) 23.7% (d) 23.7%

***TP21.2 Conceptualize** For each die, the probability of a certain number to be face-up is $1/6$.

Categorize We need to use simple statistics to answer the questions in the problem, using the equal-probability situation noted in the Conceptualize step.

Analyze (a) For three dice, there are 16 possible macrostates, ranging from 3 (three 1s) to 18 (three 6s). The probability of a certain macrostate is related to the number of microstates. For example, for a macrostate of 5, there are six possible microstates: 1–1–3, 1–2–2, 1–3–1, 2–1–2, 2–2–1, and 3–1–1.

Following this process for all possible macrostates generates the probability graph for three dice as follows:

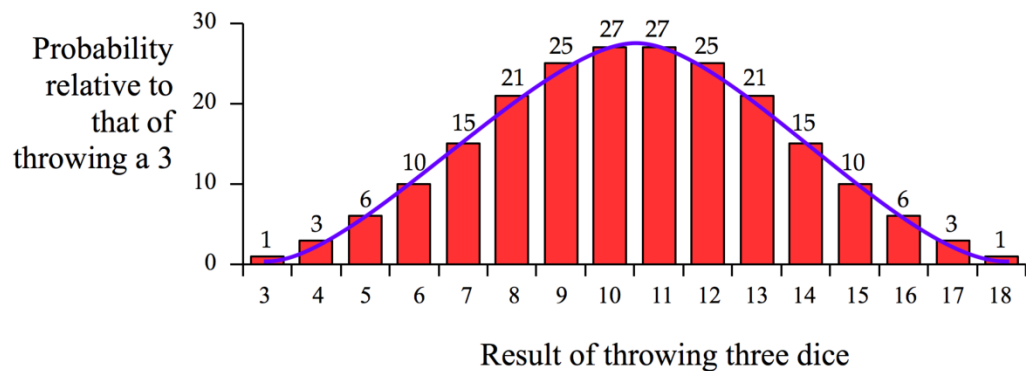
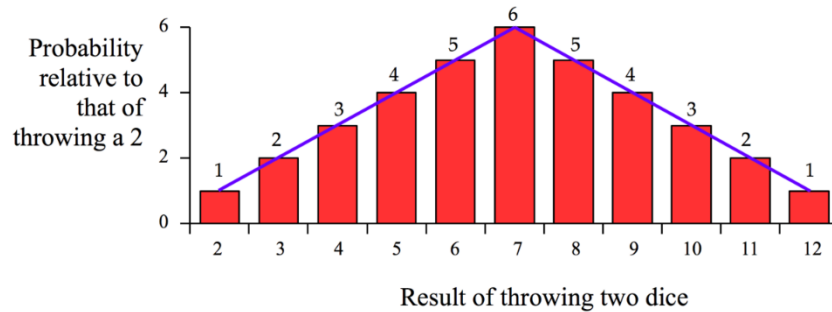


The histogram should have a similar shape.

(b) The things to notice from the histogram or the probability graph as the number of dice increases are the following:

- The graph continues to be symmetric about the midpoint.
- The probability of the middle value rises sharply as we add dice: by only adding one die (from two dice to three dice), the probability of the middle value rises from six times that of the lowest value to 27 times that of the lowest value. (It rises to 146 times the lowest probability for four dice.)
- Think about an envelope curve—that is, a curve that is drawn through the tops of all the bars. An envelope curve for *one* die is a flat, horizontal straight line: the probability is the same for all possible results 1 through 6! The

envelope curve is two straight lines for two dice, but is a curved line for three dice:



Notice that the result at which the probability maximizes is 7 for two dice and 10.5 for three dice. (The results 10 and 11 have the same probability.) Because the curves are symmetric, we can argue that, for n dice, the range of values for the result is from n to $6n$, and the result R_{\max} where the probability maximizes is the average of these values:

$$R_{\max} = \frac{n + 6n}{2} = 3.5n$$

For $n = 2$, this gives $R_{\max} = 7$; for $n = 3$, this gives $R_{\max} = 10.5$.

After noticing these behaviors, one can predict that increasing the number of dice causes the envelope curve to rapidly grow tall and skinny. Therefore, a

probability graph for Avogadro's number of dice would *very* tall and *very* skinny and might look something like this:



These graphs have shown the transition from probability to *determinism*. For two dice, one is governed very much by probability, although one can make some vague predictions, as is done in the game of craps. With Avogadro's number of dice, we have determinism: we can predict with great certainty the outcome of throwing this many dice.

(c) The shaking of the table would randomize the dice so that the result would be that represented by the vertical line in the final graph in part (b). The middle of a probability curve for dice can be seen to be the average of the lowest and highest possible values, so the number obtained when all the numbers on the faces are added up is

$$R_{\max} = 3.5N_A = 3.5(6.022 \times 10^{23}) = 2.108 \times 10^{24}$$

(d) It is *extremely* unlikely that all the dice would end up with a 1 showing. It is *not* impossible, but there are so many other possibilities that the probability of all upward faces showing 1 is essentially zero.

(e) When studying thermodynamic systems, we don't consider systems of one, two, or three particles. We consider systems with numbers of molecules

similar to Avogadro's number. Once the dice are thrown, the result is fixed and stays fixed, unlike a physical system, which changes from moment to moment. The shaking of the table represents time evolution in a physical system. The shaking of the table in part (c) represents a physical system evolving from a very low probability (low entropy) macrostate to a much higher probability (high entropy) macrostate. In part (d), once the high probability (high entropy) state has been achieved, it is extremely unlikely for the system to return to a low probability (low entropy) state. If the table is continuously shaken and then evaluated, the total number on the upward faces of the dice will always be very close to $3.5N_A$.

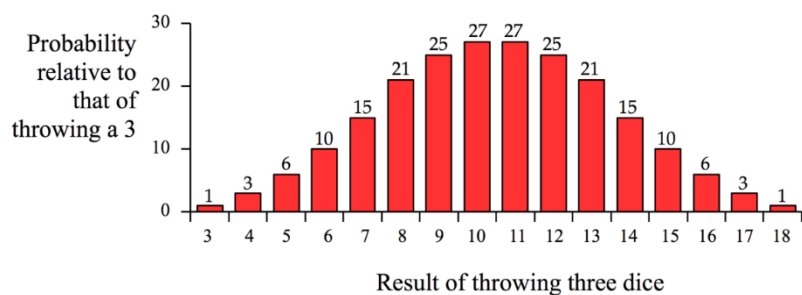
Finalize In part (c), can we actually do this experiment? Assuming we could gather Avogadro's number of dice and that we had a table large enough and strong enough to support them, how long would it take to set up Avogadro's number of dice with a 1 on the upper face?

Let's assume we could turn two dice per second so that the face with a 1 is the upper face. The number of dice that have to be turned is $(5/6)N_A$, since, by chance, $1/6$ of the dice will already have their 1 face turned upward. The time interval required to turn the rest of the dice so their 1 face is turned upward is

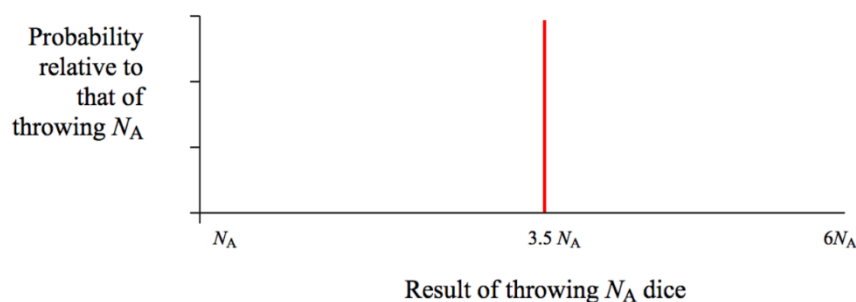
$$\Delta t = \frac{\frac{5}{6}N_A}{2 \text{ s}^{-1}} = \frac{\frac{5}{6}(6.022 \times 10^{23})}{2 \text{ s}^{-1}} = 2.51 \times 10^{23} \text{ s} \left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}} \right) = 7.94 \times 10^{15} \text{ yr}$$

This time interval is over half a million times the age of the Universe. That should suggest that the experiment cannot be carried out! A flat, square table to hold the dice would have to be almost 8 Gm on a side, larger than the diameter of the Earth. Estimating about 0.4 g per die, the mass of Avogadro's number of dice is over 2×10^{19} kg, about the mass of a small asteroid.

Answers: (a)



(b)



(c) 2.108×10^{24} (d) 0 (e) Answers will vary.

***TP21.3** The entropy change can be calculated using the result of Example 21.7, modified to reflect the phase change of boiling:

$$\Delta S = \frac{L_{v(\text{kg})}m}{T_b} = \frac{L_{v(\text{kg})}nM}{T_b} \rightarrow \frac{\Delta S}{n} = \frac{L_{v(\text{kg})}M}{T_b} = \frac{L_{v(\text{mol})}}{T_b}$$

where m is the mass of the sample, $L_{v(\text{kg})}$ is the latent heat of vaporization in J/kg, $L_{v(\text{mol})}$ is the latent heat of vaporization in J/mol, M is the molar mass, and T_b is the boiling temperature. The expanded table below shows the results.

	L_v (kJ/mol)	Boiling Point (°C)	Boiling Point (K)	ΔS per mole (J/mol · K)
<i>Polar compounds</i>				

HF	25.2	19.7	292.7	86.1
HCl	16.2	-84.8	188.2	86.1
HI	19.8	-35.6	237.4	83.4
H ₂ O	40.7	100	373	109.1
<i>Nonpolar compounds</i>				
C ₃ H ₈	19.0	-42.1	230.9	82.3
C ₄ H ₁₀	22.4	-0.50	272.5	82.2
<i>Elements</i>				
Hg	54.7	357	630	86.8
Pb	178	1 749	2 022	88.0
Cl ₂	20.4	-34.0	239	85.4
Br ₂	30.0	58.8	331.8	90.4

All but one of the entries in the rightmost column are in the range of 82 J/mol · K to 91 J/mol · K. This general pattern is called *Trouton's rule*. It is related to the fact that entropy is related to the number of ways of locating particles. One mole of each of the substances above has the same number of particles. And those particles are all doing the same thing: changing from the liquid state to the vapor state. Therefore, it is not surprising that the entropy change has similar values for many of these substances.

There is an outlier. Water has a higher value, 109 J/mol · K. Water molecules exhibit hydrogen bonding, so they are not as free to move around in the liquid as in other liquid substances. There are fewer ways of locating the water molecules, so the initial entropy of the system is lower than expected. Therefore, the entropy change is larger when water vaporizes because the hydrogen bonds must be broken *and* the molecules must break into the vapor from the liquid surface.

Other substances also deviate from Trouton's rule, with values both higher and lower than the typical range of values. The reasons for the deviation vary from one substance to another.

Answers: All but one of the entries have a value for ΔS between 82 and 91 J/mol · K

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 21.1 Heat Engines and the Second Law of Thermodynamics

P21.1 (a) We have $e = \frac{W_{\text{eng}}}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} \rightarrow \frac{|Q_c|}{|Q_h|} = 1 - e \rightarrow |Q_h| = \frac{|Q_c|}{1 - e}$

With $|Q_c| = 8\,000\text{ J}$, we have $|Q_h| = \frac{|Q_c|}{1 - e} = \frac{8\,000\text{ J}}{1 - 0.250} = \boxed{10.7\text{ kJ}}$

(b) The work per cycle is

$$W_{\text{eng}} = |Q_h| - |Q_c| = 2\,667\text{ J}$$

From the definition of output power,

$$P = \frac{W_{\text{eng}}}{\Delta t}$$

we have the time for one cycle:

$$\Delta t = \frac{W_{\text{eng}}}{P} = \frac{2\,667\text{ J}}{5\,000\text{ J/s}} = \boxed{0.533\text{ s}}$$

P21.2 (a) The efficiency of a heat engine is $e = W_{\text{env}}/|Q_h|$, where W_{env} is the work done by the engine and $|Q_h|$ is the energy absorbed from the higher temperature reservoir. Thus, if $W_{\text{env}} = |Q_h|/4$, the efficiency is $e = 1/4 = \boxed{0.25\text{ or }25\%}$.

(b) From conservation of energy, the energy exhausted to the lower temperature reservoir is $|Q_c| = |Q_h| - W_{\text{env}}$. Therefore, if $W_{\text{env}} = |Q_h|/4$, we

$$\text{have } |Q_c| = 3|Q_h|/4 \text{ or } |Q_c|/|Q_h| = 3/4.$$

P21.3 The energy to melt a mass Δm_{Hg} of Hg is $|Q_c| = m_{\text{Hg}}L_f$. The energy absorbed to freeze Δm_{Al} of aluminum is $|Q_h| = m_{\text{Al}}L_f$. The efficiency is

$$\begin{aligned} e &= 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{\Delta m_{\text{Hg}}L_{\text{Hg}}}{\Delta m_{\text{Al}}L_{\text{Al}}} = 1 - \frac{(15.0 \text{ g})(1.18 \times 10^4 \text{ J/kg})}{(1.00 \text{ g})(3.97 \times 10^5 \text{ J/kg})} \\ &= 0.554 = \boxed{55.4\%} \end{aligned}$$

Section 21.2 Heat Pumps and Refrigerators

P21.4 (a) The work done on the refrigerant in each cycle is

$$W = Q_H - Q_L = 625 \text{ kJ} - 550 \text{ kJ} = \boxed{75.0 \text{ kJ}}$$

(b) The coefficient of performance of a refrigerator is:

$$\text{COP} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

Solving numerically:

$$\text{COP} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{550 \text{ kJ}}{625 \text{ kJ} - 550 \text{ kJ}} = \boxed{7.33}$$

P21.5 (a) The energy use by the freezer each day is

$$\begin{aligned} W &= P \cdot \Delta t = \left(457 \frac{\cancel{\text{kWh}}}{\cancel{\text{y}}} \right) \left(\frac{3.60 \times 10^6 \text{ J}}{1 \cancel{\text{kWh}}} \right) \left(\frac{1 \cancel{\text{y}}}{365 \text{ d}} \right) \cdot (1 \text{ d}) \\ &= \boxed{4.51 \times 10^6 \text{ J}} \end{aligned}$$

(b) From the definition of the coefficient of performance for a refrigerator, $(\text{COP})_R = |Q_c|/W$, the thermal energy removed from the cold space each day is

$$|Q_c| = (\text{COP})_R \cdot W = 6.30(4.51 \times 10^6 \text{ J}) = \boxed{2.84 \times 10^7 \text{ J}}$$

- (c) The water must be cooled 20.0°C before it will start to freeze, so the thermal energy that must be removed from mass m of water to freeze it is $|Q_c| = mc_w|\Delta T| + mL_f$. The mass of water that can be frozen each day is then

$$\begin{aligned} m &= \frac{|Q_c|}{c_w|\Delta T| + L_f} = \frac{2.84 \times 10^7 \text{ J}}{(4186 \text{ J/kg} \cdot ^\circ\text{C})(20.0^\circ\text{C}) + 3.33 \times 10^5 \text{ J/kg}} \\ &= \boxed{68.1 \text{ kg}} \end{aligned}$$

- P21.6** (a) The coefficient of performance of a heat pump is

$$\text{COP}_{h.p.} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L}$$

Because work (energy) is power times time ($W = P\Delta t$), the equation above may be rearranged to obtain the heat added to the home:

$$\begin{aligned} Q_H &= \text{COP} \cdot W = \text{COP} \cdot P\Delta t \\ &= (4.20)(1.75 \times 10^3 \text{ J/s})(3600 \text{ s}) = \boxed{2.65 \times 10^7 \text{ J}} \end{aligned}$$

- (b) The coefficient of performance of a refrigerator or air conditioner is

$$\text{COP}_{\text{refr.}} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

and can be written in terms of the coefficient of performance of a heat pump because:

$$\begin{aligned} W &= Q_H - Q_L: \\ \text{COP}_{\text{refr.}} &= \frac{Q_L}{W} = \frac{Q_H - W}{Q_H - Q_L} = \frac{Q_H}{Q_H - Q_L} + \frac{W}{Q_H - Q_L} \end{aligned}$$

Where the first term on the far right is identically the coefficient of

performance of the heat pump, and the second term is identically one (because $W = Q_H - Q_L$). Thus,

$$\begin{aligned}\text{COP}_{\text{refr.}} &= \frac{Q_H}{Q_H - Q_L} + \frac{W}{Q_H - Q_L} = \text{COP}_{\text{h.p.}} - 1 = (4.20) - 1 \\ &= \boxed{3.20}\end{aligned}$$

Section 21.4 The Carnot Engine

P21.7 We use the Carnot expression for maximum possible efficiency, and the definition of efficiency to find the useful output. The engine is a steam turbine in an electric generating station with

$$T_c = 430^\circ\text{C} = 703 \text{ K} \quad \text{and} \quad T_h = 1\,870^\circ\text{C} = 2\,143 \text{ K}$$

$$(a) \quad e_c = \frac{\Delta T}{T_h} = \frac{1\,440 \text{ K}}{2\,143 \text{ K}} = 0.672 = \boxed{67.2\%}$$

$$(b) \quad e = W_{\text{eng}}/|Q_h| = 0.420 \quad \text{and} \quad |Q_h| = 1.40 \times 10^5 \text{ J}$$

for one second of operation, so

$$W_{\text{eng}} = 0.420|Q_h| = 5.88 \times 10^4 \text{ J}$$

and the power is

$$P = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88 \times 10^4 \text{ J}}{1 \text{ s}} = \boxed{58.8 \text{ kW}}$$

P21.8 The efficiency of a Carnot engine operating between these temperatures is

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{273 \text{ K}}{293 \text{ K}} = 0.0683 = 6.83\%$$

Therefore, there is no way that the inventor's engine can have an efficiency of $0.110 = 11.0\%$.

P21.9 We wish to evaluate $\text{COP} = |Q_c|/W$ for a refrigerator, which is a Carnot engine run in reverse. For a Carnot engine,

$$\left. \begin{aligned} |Q_h| &= |Q_c| + W \\ e &= \frac{W}{|Q_h|} = \frac{W}{|Q_c| + W} \end{aligned} \right\} \rightarrow \frac{1}{e} = \frac{|Q_c| + W}{W} = \frac{|Q_c|}{W} + 1$$

which gives

$$\text{COP} = \frac{|Q_c|}{W} = \frac{1}{e} - 1$$

Therefore,

$$\text{COP} = \frac{1}{e} - 1 = \frac{1}{0.350} - 1 = \boxed{1.86}.$$

P21.10 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and

$$W = |Q_h| - |Q_c| = |Q_c| \left[\frac{|Q_h|}{|Q_c|} - 1 \right]$$

The text shows that for a Carnot cycle (and only for a reversible cycle),

$$\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}. \text{ Therefore, } W = \frac{T_h - T_c}{T_c} |Q_c|.$$

(b) We have the definition of the coefficient of performance for a

refrigerator, $\text{COP} = \frac{|Q_c|}{W}$. Using the result from part (a), this becomes

$$\text{COP} = \frac{T_c}{T_h - T_c}.$$

P21.11 (a) The absolute temperature of the cold reservoir is

$T_c = 20.0^\circ + 273 = 293 \text{ K}$. If the Carnot efficiency is to be

$e_c = 0.650$, it is necessary that

$$1 - \frac{T_c}{T_h} = 0.650 \quad \text{or} \quad \frac{T_c}{T_h} = 0.350 \quad \text{and} \quad T_h = \frac{T_c}{0.35}$$

Thus,

$$T_h = \frac{293 \text{ K}}{0.35} = 837 \text{ K} \quad \text{or} \quad T_h = 837 - 273 = \boxed{564^\circ\text{C}}$$

- (b) No. A real engine will always have an efficiency *less* than the Carnot efficiency because it operates in an irreversible manner.

P21.12 The Carnot summer efficiency is

$$e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 \text{ K} + 20.0^\circ\text{C})}{(273 \text{ K} + 350^\circ\text{C})} = 0.530$$

And in winter,

$$e_{c,w} = 1 - \frac{283}{623} = 0.546$$

Then the actual winter efficiency is

$$0.320 \left(\frac{0.546}{0.530} \right) = \boxed{0.330} \quad \text{or} \quad \boxed{33.0\%}$$

***P21.13 Conceptualize** In Figure 21.2, imagine the hot reservoir to be the 20.0°C water at the surface of the ocean. The cold reservoir is the water at a lower depth at a temperature of 5.00°C .

Categorize The problem involves calculations using the model of a heat engine described in Section 21.1, along with some relatively simple energy calculations.

Analyze (a) Use Equation 21.8 to find the maximum possible efficiency, the Carnot efficiency, and substitute numerical values:

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{273 + 5.00}{273 + 20.0} = 0.0512 = \boxed{5.12\%}$$

(b) Use Equation 21.2, solve for Q_h , differentiate with respect to time, so that you can evaluate power, and substitute numerical values:

$$e = \frac{W_{\text{eng}}}{|Q_h|} \rightarrow |Q_h| = \frac{W_{\text{eng}}}{e} \rightarrow \frac{d|Q_h|}{dt} = \frac{1}{e} \frac{dW_{\text{eng}}}{dt}$$

$$\rightarrow |P_h| = \frac{P_{\text{eng}}}{e} = \frac{75.0 \text{ MW}}{0.0512} = 1.46 \times 10^3 \text{ MW} \left(\frac{10^6 \text{ J/s}}{1 \text{ MW}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right)$$

$$= \boxed{5.27 \times 10^{12} \text{ J/h}}$$

(c) To find the number of homes served by the system, divide the power output of the engine by the typical power use of a home:

$$N_{\text{homes}} = \frac{P_{\text{eng}}}{P_{\text{home}}} = \frac{75.0 \text{ MW}}{950 \text{ kWh/mo}} \left(\frac{10^3 \text{ kW}}{1 \text{ MW}} \right) \left(\frac{30 \text{ d}}{1 \text{ mo}} \right) \left(\frac{24 \text{ h}}{1 \text{ d}} \right) = \boxed{5.68 \times 10^4} \text{ homes}$$

(d) Applying Equation 16.38 to light, the intensity of sunlight on the surface is

$$I_{\text{absorbed}} = \frac{P_{\text{absorbed}}}{A} \quad (1)$$

For the energy absorbed from sunlight to match the energy required for the engine, we must have

$$P_{\text{absorbed}} = P_h \quad (2)$$

Where P_h is the power input to the engine. Combining Equations (1) and (2), we have

$$I_{\text{absorbed}} = \frac{P_h}{A} \rightarrow A = \frac{P_h}{I_{\text{absorbed}}} \quad (3)$$

Substitute numerical values:

$$A = \frac{5.27 \times 10^{12} \text{ J/h}}{650 \text{ W/m}^2 (0.5)} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = \boxed{4.50 \times 10^6 \text{ m}^2}$$

where the factor of 0.5 in the denominator represents the fact that the area on the ocean only has daylight for 12 hours a day. This area is a little under two square miles.

(e) The area of the Earth's ocean surface can be looked up online and found to be about $3.6 \times 10^{14} \text{ m}^2$. Divide this area by the area A required for one engine to find the number of engines that could be operated:

$$N_{\text{engines}} = \frac{A_{\text{ocean}}}{A} = \frac{3.6 \times 10^{14} \text{ m}^2}{4.50 \times 10^6 \text{ m}^2} = 8 \times 10^7 \text{ engines}$$

Multiply this by the number of homes served by one engine:

$$N_{\text{total homes}} = N_{\text{engine}} N_{\text{homes}} = (8 \times 10^7)(5.68 \times 10^4) = 5 \times 10^{12} \text{ homes}$$

The result is five trillion homes, which is much larger than the population of the Earth, a few billion. Therefore, this calculation shows that there is plenty of ocean surface area to operate the engines and supply the Earth's electricity needs.

(f) These calculations assumed that the engines operated with Carnot efficiency. That will not be true in practice; the actual efficiency will be much lower. The calculation also includes only electricity use in homes; it does not include electricity use in industry and transportation. The temperature distribution in the ocean is in balance now, *with* the solar input on the surface and *without* the use of the engines. What will happen to the temperature distribution if we use some of the warmer water to run engines and transport that energy by electricity to the land surface? How would that affect ocean life? Another consideration is free passage in shipping lanes. If the ocean is crowded with mechanical systems associated with these engines, how does that affect passing ships? And then there are myriad political and economic questions

regarding the use of these systems by various countries. If the system is used by only one country, can they operate it in the open ocean, or only within their 12-mile limit? What about landlocked countries that have no ocean surface available? If the system is used by several countries, which country is responsible for maintenance, repair, etc.? This will be a major concern with 180 million engines operating, as found in part (e). While the calculations we have performed may look promising, the introduction of such a system is fraught with difficulties and would need to be approached very carefully.

Finalize Notice the important difference discussed in part (f) between physics calculations and real-world considerations.

Answers: (a) 5.12% (b) 5.27×10^{12} J/h (c) 5.68×10^4 (d) $4.50 \times 10^6 \text{ m}^2$ (e) yes (f) numerically, yes; feasibly, probably not

P21.14 (a) $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{350 \text{ K}}{500 \text{ K}} = \boxed{0.300}$

(b) We differentiate $e_c = 1 - T_c/T_h$ to find

$$\frac{de_c}{dT_h} = 0 - T_c(-1)T_h^{-2} = \frac{T_c}{T_h^2} = \frac{350 \text{ K}}{(500 \text{ K})^2} = \boxed{1.40 \times 10^{-3} \text{ K}^{-1}}$$

(c) We differentiate $e_c = 1 - T_c/T_h$ to find

$$\frac{de_c}{dT_c} = 0 - \frac{1}{T_h} = -\frac{1}{500 \text{ K}} = \boxed{-2.00 \times 10^{-3} \text{ K}^{-1}}$$

(d) $\boxed{\text{No. The derivative in part (c) depends only on } T_h.}$

P21.15 (a) “The actual efficiency is two thirds the Carnot efficiency” reads as an equation

$$\frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{|Q_c| + W_{\text{eng}}} = \frac{2}{3} \left(1 - \frac{T_c}{T_h} \right) = \frac{2}{3} \frac{T_h - T_c}{T_h}$$

All the T 's represent absolute temperatures. Then

$$\frac{|Q_c| + W_{\text{eng}}}{W_{\text{eng}}} = \frac{1.5 T_h}{T_h - T_c} \rightarrow \frac{|Q_c|}{W_{\text{eng}}} = \frac{1.5 T_h}{T_h - T_c} - 1 = \frac{1.5 T_h - T_h + T_c}{T_h - T_c}$$

$$|Q_c| = W_{\text{eng}} \frac{0.5 T_h + T_c}{T_h - T_c} \rightarrow \frac{|Q_c|}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t} \frac{0.5 T_h + T_c}{T_h - T_c}$$

$$\frac{Q_c}{\Delta t} = 1.40 \left(\frac{0.5 T_h + 383}{T_h - 383} \right), \text{ where } Q_c / \Delta t \text{ is in megawatts and } T \text{ is in kelvins.}$$

(b) The exhaust power decreases as the firebox temperature increases.

$$\begin{aligned} \text{(c)} \quad \frac{|Q_c|}{\Delta t} &= (1.40 \text{ MW}) \left(\frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}} \right) \\ &= (1.40 \text{ MW}) \left(\frac{0.5(1073 \text{ K}) + 383 \text{ K}}{1073 \text{ K} - 383 \text{ K}} \right) = \boxed{1.87 \text{ MW}} \end{aligned}$$

(d) We require

$$\begin{aligned} \frac{|Q_c|}{\Delta t} &= \frac{1}{2} (1.87 \text{ MW}) = (1.40 \text{ MW}) \left(\frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}} \right) \\ \frac{0.5 T_h + 383 \text{ K}}{T_h - 383 \text{ K}} &= 0.666 \\ 0.5 T_h + 383 \text{ K} &= 0.666 T_h - 255 \text{ K} \\ T_h &= 638 \text{ K} / 0.166 = \boxed{3.84 \times 10^3 \text{ K}} \end{aligned}$$

(e) The minimum possible heat exhaust power is approached as the firebox temperature goes to infinity, and it is $|Q_c| / \Delta t =$

$1.40 \text{ MW}(0.5/1) = 0.700 \text{ MW}$. The heat exhaust power cannot be as small as $(1/4)(1.87 \text{ MW}) = 0.466 \text{ MW}$. So no answer exists. The energy exhaust cannot be that small.

P21.16 (a)
$$e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{e_1 Q_{1h} + e_2 Q_{2h}}{Q_{1h}}$$

Now, $Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng1}} = Q_{1h} - e_1 Q_{1h}$,

so

$$e = \frac{e_1 Q_{1h} + e_2 (Q_{1h} - e_1 Q_{1h})}{Q_{1h}} = \boxed{e_1 + e_2 - e_1 e_2}$$

(b)
$$e = e_1 + e_2 - e_1 e_2 = 1 - \frac{T_i}{T_h} + 1 - \frac{T_c}{T_i} - \left(1 - \frac{T_i}{T_h}\right) \left(1 - \frac{T_c}{T_i}\right)$$

$$= 2 - \frac{T_i}{T_h} - \frac{T_c}{T_i} - 1 + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}$$

(c) The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency. No improvement in net efficiency has resulted.

(d) With $W_{\text{eng2}} = W_{\text{eng1}}$, $e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{2W_{\text{eng1}}}{Q_{1h}} = 2e_1$

$$1 - \frac{T_c}{T_h} = 2 \left(1 - \frac{T_i}{T_h}\right)$$

$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$

$$2T_i = T_h + T_c$$

$$\boxed{T_i = \frac{1}{2}(T_h + T_c)}$$

(e)
$$e_1 = e_2 = 1 - \frac{T_i}{T_h} = 1 - \frac{T_c}{T_i}$$

$$T_i^2 = T_c T_h$$

$$\boxed{T_i = (T_h T_c)^{1/2}}$$

P21.17 The coefficient of performance of the device is

$$\text{COP} = 0.100 \text{ COP}_{\text{Carnot cycle}}$$

$$\text{or } \frac{|Q_h|}{W} = 0.100 \left(\frac{|Q_h|}{W} \right)_{\text{Carnot cycle}} = 0.100 \left(\frac{1}{\text{Carnot efficiency}} \right)$$

$$\frac{|Q_h|}{W} = 0.100 \left(\frac{T_h}{T_h - T_c} \right) = 0.100 \left(\frac{293 \text{ K}}{293 \text{ K} - 268 \text{ K}} \right) = \boxed{1.17}$$

Thus, 1.17 joules of energy enter the room by heat for each joule of work done.

Section 21.5 Gasoline and Diesel Engines

P21.18 Compression ratio = 6.00, $\gamma = 1.40$

(a) Efficiency of an Otto engine: $e = 1 - \left(\frac{V_2}{V_1} \right)^{\gamma-1}$

$$e = 1 - \left(\frac{1}{6.00} \right)^{0.400} = \boxed{51.2\%}$$

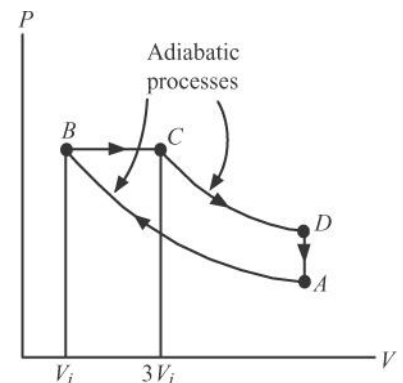
(b) If actual efficiency $e' = 15.0\%$, the fraction of fuel wasted is (assuming complete combustion of the air-fuel mixture) $e - e' = \boxed{36.2\%}$.

P21.19 The energy transferred by heat over the paths CD and BA is zero since they are adiabatic.

Over path BC : $Q_{BC} = nC_P (T_C - T_B) > 0$

Over path DA : $Q_{DA} = nC_V (T_A - T_D) < 0$

Therefore, $|Q_c| = |Q_{DA}|$ and $Q_h = Q_{BC}$.



The efficiency is then

ANS. FIG. P21.19

$$e = 1 - \frac{|Q_c|}{Q_h} = 1 - \frac{(T_D - T_A)C_V}{(T_C - T_B)C_P} = 1 - \frac{1}{\gamma} \left(\frac{T_D - T_A}{T_C - T_B} \right)$$

Section 21.6 Entropy

P21.20 (a) The table is shown in TABLE P21.20 below.

(b) On the basis of the table, the most probable recorded result of a toss is

2 heads and 2 tails

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

TABLE P21.20

P21.21 Each marble is returned to the bag before the next is drawn, so the probability of drawing a red one is the same as drawing a green one.

(a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3

1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

TABLE P21.21(a)

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

TABLE P21.21(b)

Section 21.7 Entropy in Thermodynamic Systems

P21.22 The hot water has negative energy input by heat, given by $Q = mc\Delta T$. The surrounding room has positive energy input of this same number of joules, which we can write as $Q_{\text{room}} = (mc|\Delta T|)_{\text{water}}$. Imagine the room absorbing this energy reversibly by heat, from a stove at 20.001°C . Then its entropy increase

is Q_{room}/T :

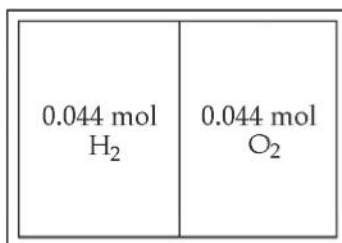
$$\Delta S = \frac{Q_r}{T} = \frac{mc_w |\Delta T|}{T} = \frac{(0.125 \text{ kg})(4186 \text{ J/kg} \cdot ^\circ\text{C})(80^\circ\text{C})}{293 \text{ K}} = \boxed{143 \text{ J/K}}$$

P21.23 The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{\frac{1}{2}(1500 \text{ kg})(20.0 \text{ m/s})^2}{293 \text{ K}} = \boxed{1.02 \text{ kJ/K}}$$

P21.24 Each gas expands into the other half of the container as though the other gas were not there; therefore, consider each gas to undergo a free expansion process in which its volume doubles. From Equation 21.17, the entropy change is twice that for a single gas:

$$\begin{aligned}\Delta S &= 2 \left[nR \ln \left(\frac{V_f}{V_i} \right) \right] \\ &= 2 \left[(0.0440)(8.314 \text{ J/mol} \cdot \text{K})(\ln 2) \right] \\ &= \boxed{0.507 \text{ J/K}}\end{aligned}$$



ANS. FIG. P21.24

P21.25 The change in entropy is given by

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} mc \frac{dT}{T}$$

Here T means the absolute temperature. We would ordinarily think of dT as the change in the Celsius temperature, but one Celsius degree of temperature change is the same size as one kelvin of change, so dT is also the change in absolute T .

$$\begin{aligned}\Delta S &= mc \ln T \Big|_{T_i}^{T_f} = mc \ln \left(\frac{T_f}{T_i} \right) \\ &= (0.250 \text{ kg})(4186 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{353 \text{ K}}{293 \text{ K}} \right) = \boxed{195 \text{ J/K}}\end{aligned}$$

P21.26 We take data from Tables 19.1 and 19.2, and we assume a constant specific heat for each phase. As the ice is warmed from -12.0°C to 0°C , its entropy increases by

$$\begin{aligned}\Delta S &= \int_i^f \frac{dQ}{T} = \int_{261 \text{ K}}^{273 \text{ K}} \frac{mc_{\text{ice}} dT}{T} = mc_{\text{ice}} \int_{261 \text{ K}}^{273 \text{ K}} T^{-1} dT = mc_{\text{ice}} \ln T \Big|_{261 \text{ K}}^{273 \text{ K}} \\ \Delta S &= (0.0279 \text{ kg})(2090 \text{ J/kg} \cdot ^\circ\text{C})(\ln 273 \text{ K} - \ln 261 \text{ K}) \\ &= (0.0279 \text{ kg})(2090 \text{ J/kg} \cdot ^\circ\text{C}) \left[\ln \left(\frac{273 \text{ K}}{261 \text{ K}} \right) \right] \\ \Delta S &= 2.62 \text{ J/K}\end{aligned}$$

As the ice melts its entropy change is

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(0.0279 \text{ kg})(3.33 \times 10^5 \text{ J/kg})}{273 \text{ K}} = 34.0 \text{ J/K}$$

As liquid water warms from 273 K to 373 K,

$$\begin{aligned}\Delta S &= \int_i^f \frac{mc_{\text{liquid}} dT}{T} = mc_{\text{liquid}} \ln \left(\frac{T_f}{T_i} \right) \\ &= (0.0279 \text{ kg})(4186 \text{ J/kg} \cdot ^\circ\text{C}) \ln \left(\frac{373 \text{ K}}{273 \text{ K}} \right) = 36.5 \text{ J/K}\end{aligned}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + mc_{\text{steam}} \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{(0.0279 \text{ kg})(2.26 \times 10^6 \text{ J/kg})}{373 \text{ K}} + (0.0279 \text{ kg})(2010 \text{ J/kg} \cdot ^\circ\text{C}) \ln\left(\frac{388 \text{ K}}{373 \text{ K}}\right)$$

$$= 169 \text{ J/K} + 2.21 \text{ J/K}$$

The total entropy change is

$$\Delta S_{\text{tot}} = (2.62 + 34.0 + 36.5 + 169 + 2.21) \text{ J/K} = \boxed{244 \text{ J/K}}$$

For steam at constant pressure, the molar specific heat in Table 19.1 implies a specific heat of $(35.4 \text{ J/mol} \cdot \text{K}) \left(\frac{1 \text{ mol}}{0.018 \text{ kg}} \right) = 1970 \text{ J/kg} \cdot \text{K}$, nearly agreeing with $2010 \text{ J/kg} \cdot \text{K}$.

Section 21.8 Entropy and the Second Law

P21.27 The change in entropy of a reservoir is $\Delta S = Q_r/T$, where Q_r is the energy absorbed ($Q_r > 0$) or expelled ($Q_r < 0$) by the reservoir, and T is the absolute temperature of the reservoir.

(a) For the hot reservoir: $\Delta S_h = \frac{-2.50 \times 10^3 \text{ J}}{725 \text{ K}} = \boxed{-3.45 \text{ J/K}}$

(b) For the cold reservoir: $\Delta S_c = \frac{+2.50 \times 10^3 \text{ J}}{310 \text{ K}} = \boxed{+8.06 \text{ J/K}}$

(c) For the Universe:

$$\Delta S_U = \Delta S_h + \Delta S_c = -3.45 \text{ J/K} + 8.06 \text{ J/K} = \boxed{+4.61 \text{ J/K}}$$

P21.28 The change in entropy of a reservoir is $\Delta S = Q_r/T$, where Q_r is the energy absorbed ($Q_r > 0$) or expelled ($Q_r < 0$) by the reservoir, and T is the absolute temperature of the reservoir.

(a) Energy is transferred from the hot reservoir by heat: $|Q_h| = -Q$, and

$$\Delta S_h = -\frac{Q}{T_h}.$$

(b) Energy is transferred to the cold reservoir by heat: $|Q_c| = +Q$, and

$$\Delta S_c = \frac{Q}{T_c}.$$

(c) For the Universe, $\Delta S_U = \Delta S_h + \Delta S_c = Q \left(\frac{1}{T_c} - \frac{1}{T_h} \right)$.

P21.29 Sitting here writing, I convert chemical energy from molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2\,500 \text{ kcal/d} = \left(\frac{2\,500 \times 10^3 \text{ cal}}{86\,400 \text{ s}} \right) \left(\frac{4.186 \text{ J}}{1 \text{ cal}} \right) = 120 \text{ W}$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120 \text{ W}}{293 \text{ K}} = 0.4 \text{ W/K} \sim \boxed{1 \text{ W/K}}$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P21.30 The conversion of gravitational potential energy into kinetic energy as the

water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount $mg y$, to the water from a stove at a temperature infinitesimally above 20.0°C . Then,

$$\begin{aligned}\frac{\Delta S}{\Delta t} &= \int \frac{dQ_r/\Delta t}{T} = \frac{Q/\Delta t}{T} = \frac{mg y/\Delta t}{T} \\ &= \frac{(5\,000\text{ m}^3/\text{s})(1\,000\text{ kg/m}^3)(9.80\text{ m/s}^2)(50.0\text{ m})}{293\text{ K}} \\ &= \boxed{8.36 \times 10^6\text{ J/K} \cdot \text{s}}\end{aligned}$$

Additional Problems

P21.31 $|Q_h| = 3W$, and for an engine, $|Q_h| = W + |Q_c| = 3W \rightarrow |Q_c| = 2W$.

(a) $e = \frac{W}{|Q_h|} = \frac{W}{3W} = \boxed{\frac{1}{3}}$

(b) $\frac{|Q_c|}{|Q_h|} = \frac{2W}{3W} = \boxed{\frac{2}{3}}$

P21.32 (a) $\left(10.0 \frac{\text{Btu/h}}{\text{W}}\right)\left(\frac{1\,055\text{ J}}{1\text{ Btu}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)\left(\frac{1\text{ W}}{1\text{ J/s}}\right) = \boxed{2.93}$

(b) The energy extracted by heat from the cold side divided by required work input is by definition the coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With an EER of 5,

$$5 \frac{\text{Btu}}{\text{h} \cdot \text{W}} = \frac{10\,000\text{ Btu/h}}{P}$$

which gives

$$P = \frac{10\,000 \text{ Btu/h}}{5 \text{ Btu/h} \cdot \text{W}} = 2\,000 \text{ W} = 2.00 \text{ kW}$$

Energy purchased is $P\Delta t = (2.00 \text{ kW})(1\,500 \text{ h}) = 3.00 \times 10^3 \text{ kWh}$.

$$\text{Cost} = (3.00 \times 10^3 \text{ kWh})(0.170 \text{ \$/kWh}) = \$510:$$

With EER 5, \$510

With EER 10,

$$10 \frac{\text{Btu}}{\text{h} \cdot \text{W}} = \frac{10\,000 \text{ Btu/h}}{P}$$

$$\rightarrow P = \frac{10\,000 \text{ Btu/h}}{10 \text{ Btu/h} \cdot \text{W}} = 1\,000 \text{ W} = 1.00 \text{ kW}$$

Energy purchased is $P\Delta t = (1.00 \text{ kW})(1\,500 \text{ h}) = 1.50 \times 10^3 \text{ kWh}$

$$\text{Cost} = (1.50 \times 10^3 \text{ kWh})(0.170 \text{ \$/kWh}) = \$255:$$

With EER 10, \$255

Thus, the cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

P21.33 (a) For the constant volume process AB ,

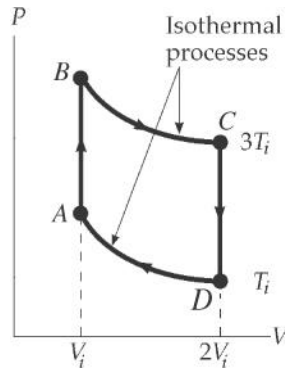
$$Q_{AB} = \Delta E_{\text{int}, AB} = \frac{3}{2} nR\Delta T = \frac{3}{2} nR(3T_i - T_i) = \boxed{3nRT_i}$$

(b) For an isothermal process, $Q = nRT \ln\left(\frac{V_2}{V_1}\right)$.

$$\text{Therefore, for process } BC, \quad Q_{BC} = \boxed{3nRT_i \ln 2}.$$

(c) For the constant volume process CD ,

$$Q_{CD} = \Delta E_{\text{int}, CD} = \frac{3}{2} nR \Delta T = \frac{3}{2} nR (T_i - 3T_i) = \boxed{-3nRT_i}$$



ANS. FIG. P21.33

(d) For an isothermal process DA , $Q_{DA} = nRT_i \ln \frac{1}{2} = \boxed{-nRT_i \ln 2}$.

(e) $Q_h = Q_{AB} + Q_{BC} = 3nRT_i + 3nRT_i \ln 2 = \boxed{3nRT_i (1 + \ln 2)}$

(f) Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \text{ and } W_{\text{eng}} = Q$$

and work done by the engine is

$$\begin{aligned} W = Q &= Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} \\ &= 3nRT_i + 3nRT_i \ln 2 - 3nRT_i - nRT_i \ln 2 \\ W &= \boxed{2nRT_i \ln 2} \end{aligned}$$

(g) The efficiency is

$$e_c = \frac{W_{\text{eng}}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2 \ln 2}{3(1 + \ln 2)} = \boxed{0.273}$$

P21.34 (a) The coefficient of performance of an air conditioner is defined as

$$(\text{COP})_{\text{ac}} = \frac{|Q_c|}{W} = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{1}{|Q_h|/|Q_c| - 1}$$

But when a device operates on the Carnot cycle, $|Q_h|/|Q_c| = T_h/T_c$. Thus, the coefficient of performance for a Carnot heat pump would be

$$(\text{COP})_{\text{ac}} = \frac{1}{T_h/T_c - 1} = \boxed{\frac{T_c}{T_h - T_c}}$$

(b) From the result of part (a) above, we observe that the COP of a Carnot air conditioner would increase if the temperature difference $T_h - T_c$ becomes smaller.

(c) If $T_c = 20^\circ + 273 = 293 \text{ K}$ and $T_h = 40^\circ + 273 = 313 \text{ K}$, the COP of a Carnot heat pump would be

$$(\text{COP})_{\text{ac,C}} = \frac{T_c}{T_h - T_c} = \frac{293 \text{ K}}{313 \text{ K} - 293 \text{ K}} = \boxed{14.6}$$

P21.35 (a) Let state i represent the gas before its compression and state f afterwards, $V_f = \frac{V_i}{8}$. For a diatomic ideal gas, $C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$, and

$$\gamma = \frac{C_p}{C_v} = 1.40. \text{ Next,}$$

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma = P_i 8^{1.40} = 18.4 P_i$$

$$P_i V_i = nRT_i$$

$$P_f V_f = \frac{18.4 P_i V_i}{8} = 2.30 P_i V_i = 2.30 nRT_i = nRT_f$$

$$\text{so } T_f = 2.30 T_i$$

$$\begin{aligned} \Delta E_{\text{int}} &= nC_v \Delta T = n \frac{5}{2} R (T_f - T_i) = \frac{5}{2} nR (1.30 T_i) = \frac{5}{2} (1.30 P_i V_i) \\ &= \frac{5}{2} (1.30) (1.013 \times 10^5 \text{ N/m}^2) (0.120 \times 10^{-3} \text{ m}^3) = 39.4 \text{ J} \end{aligned}$$

Since the process is adiabatic, $Q = 0$ and $\Delta E_{\text{int}} = Q + W$ gives

$$W = \boxed{39.4 \text{ J}}$$

(b) The moment of inertia of the wheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(5.10 \text{ kg})(0.0850 \text{ m})^2 = 0.0184 \text{ kg} \cdot \text{m}^2$$

We want the flywheel to do work 39.4 J, so the work on the flywheel should be -39.4 J :

$$\begin{aligned} K_{\text{rot } i} + W &= K_{\text{rot } f} \\ \frac{1}{2}I\omega_i^2 - 39.4 \text{ J} &= 0 \\ \omega_i &= \left[\frac{2(39.4 \text{ J})}{0.0184 \text{ kg} \cdot \text{m}^2} \right]^{1/2} = \boxed{65.4 \text{ rad/s} = 625 \text{ rev/min}} \end{aligned}$$

(c) Now we want $W = 0.05K_{\text{rot } i}$:

$$\begin{aligned} 39.4 \text{ J} &= 0.05 \left[\frac{1}{2}(0.0184 \text{ kg} \cdot \text{m}^2)\omega_i^2 \right] \\ \omega_i &= \left(\frac{2(789 \text{ J})}{0.0184 \text{ kg} \cdot \text{m}^2} \right)^{1/2} = \boxed{293 \text{ rad/s} = 2.79 \times 10^3 \text{ rev/min}} \end{aligned}$$

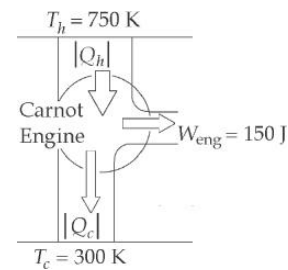
P21.36 For the Carnot engine,

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300 \text{ K}}{750 \text{ K}} = 0.600$$

$$\text{Also, } e_c = \frac{W_{\text{eng}}}{|Q_h|},$$

$$\text{so } |Q_h| = \frac{W_{\text{eng}}}{e_c} = \frac{150 \text{ J}}{0.600} = 250 \text{ J}$$

$$\text{and } |Q_c| = |Q_h| - W_{\text{eng}} = 250 \text{ J} - 150 \text{ J} = 100 \text{ J}.$$



ANS. FIG. P21.36

$$(a) \quad |Q_h| = \frac{W_{\text{eng}}}{e_s} = \frac{150 \text{ J}}{0.700} = \boxed{214 \text{ J}}$$

$$|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_s} - 150 \text{ J} = \boxed{64.3 \text{ J}}$$

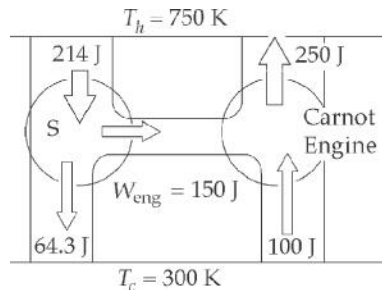
- (b) When engine S delivers 150 J of work to the Carnot engine, the Carnot engine transfers 250 J to the firebox while engine S takes 214 J from the firebox:

$$|Q_{h,\text{net}}| = -\frac{W_{\text{eng}}}{e_s} + 250 \text{ J} = \boxed{35.7 \text{ J}}$$

and the Carnot engine removes 100 J from the environment while engine S returns 64.3 J:

$$|Q_{c,\text{net}}| = |64.3 \text{ J} - 100 \text{ J}| = \boxed{35.7 \text{ J}}$$

The total energy the firebox puts out equals the total energy transferred to the environment.



ANS. FIG. P21.36(a–b)

- (c) The net flow of energy by heat from the cold to the hot reservoir without work input is impossible.

(d) For engine S: $|Q_{c,S}| = |Q_{h,S}| - W_{\text{eng S}} = \frac{W_{\text{eng S}}}{e_S} - W_{\text{eng S}}$

so work output is $W_{\text{eng S}} = \frac{|Q_{c,S}|}{\frac{1}{e_S} - 1} = \frac{100 \text{ J}}{\frac{1}{0.700} - 1} = \boxed{233 \text{ J}}$

and energy input to engine S is

$$|Q_{h,S}| = |Q_{c,S}| + W_{\text{eng S}} = 233 \text{ J} + 100 \text{ J} = \boxed{333 \text{ J}}$$

(e) Engine S contributes 150 J out of 233 J to running the Carnot engine:

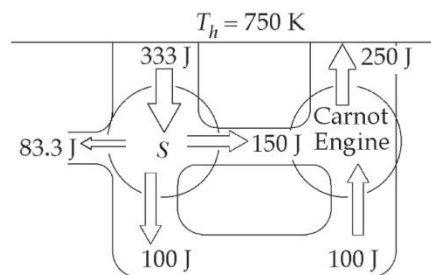
$$|Q_{h,\text{net}}| = |Q_{h,S}| - 250 \text{ J} = 333 \text{ J} - 250 \text{ J} = \boxed{83.3 \text{ J}}$$

This is the net energy lost by the firebox.

(f) The remaining work output is

$$W_{\text{net}} = W_{\text{eng S}} - 250 \text{ J} = 233 \text{ J} - 150 \text{ J} = \boxed{83.3 \text{ J}}$$

(g) $|Q_{c,\text{net}}| = \boxed{0}$



ANS. FIG. P21.36(e–g)

(h) The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

(i) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0$.

For the reservoirs, $\Delta S_h = -\frac{|Q_h|}{T_h}$ and $\Delta S_c = +\frac{|Q_c|}{T_c}$.

Thus,

$$\begin{aligned}\Delta S_{\text{total}} &= \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3 \text{ J}}{750 \text{ K}} + \frac{0}{300 \text{ K}} \\ &= \boxed{-0.111 \text{ J/K}}\end{aligned}$$

- (j) A decrease in total entropy is impossible.

Challenge Problem

P21.37 (a) For the isothermal process AB , the work on the gas is

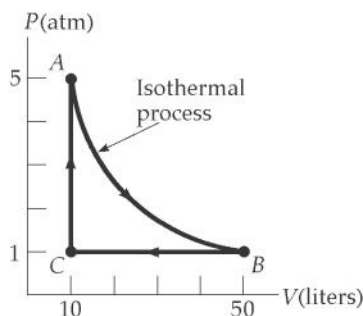
$$\begin{aligned}W_{AB} &= -P_A V_A \ln\left(\frac{V_B}{V_A}\right) \\ W_{AB} &= -5(1.013 \times 10^5 \text{ Pa})(10.0 \times 10^{-3} \text{ m}^3) \ln\left(\frac{50.0 \text{ L}}{10.0 \text{ L}}\right) \\ W_{AB} &= -8.15 \times 10^3 \text{ J}\end{aligned}$$

where we have used $1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ and

$$1.00 \text{ L} = 1.00 \times 10^{-3} \text{ m}^3.$$

$$\begin{aligned}W_{BC} &= -P_B \Delta V = -(1.013 \times 10^5 \text{ Pa})[(10.0 - 50.0) \times 10^{-3}] \text{ m}^3 \\ &= +4.05 \times 10^3 \text{ J}\end{aligned}$$

$$W_{CA} = 0 \text{ and } W_{\text{eng}} = -W_{AB} - W_{BC} = 4.10 \times 10^3 \text{ J} = \boxed{4.10 \text{ kJ}}$$



ANS. FIG. P21.37

- (b) Since AB is an isothermal process, $\Delta E_{\text{int}, AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15 \times 10^3 \text{ J}$.

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$.

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(50.0 \times 10^{-3} \text{ m}^3)}{R} = \frac{5.06 \times 10^3}{R}$$

Also, $T_C = \frac{P_C V_C}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(10.0 \times 10^{-3} \text{ m}^3)}{R} = \frac{1.01 \times 10^3}{R}$

$$Q_{CA} = nC_V \Delta T = 1.00 \left(\frac{3}{2} R \right) \left(\frac{5.06 \times 10^3 - 1.01 \times 10^3}{R} \right) = 6.08 \text{ kJ}$$

so the total energy absorbed by heat is

$$Q_{AB} + Q_{CA} = 8.15 \text{ kJ} + 6.08 \text{ kJ} = \boxed{1.42 \times 10^4 \text{ J}}$$

(c) $Q_{BC} = nC_P \Delta T = \frac{5}{2} (nR \Delta T) = \frac{5}{2} P_B \Delta V_{BC}$

$$|Q_{BC}| = \left| \frac{5}{2} (1.013 \times 10^5) [(10.0 - 50.0) \times 10^{-3}] \right| = |-1.01 \times 10^4 \text{ J}|$$

$$= \boxed{1.01 \times 10^4 \text{ J}}$$

(d) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB} + Q_{CA}} = \frac{4.10 \times 10^3 \text{ J}}{1.42 \times 10^4 \text{ J}} = 0.289 \quad \text{or} \quad \boxed{28.9\%}$

(e) A Carnot engine operating between $T_{\text{hot}} = T_A = 5060/R$ and $T_{\text{cold}} = T_C = 1010/R$ has $e_c = 1 - T_c/T_h = 1 - 1/5 = \boxed{80.0\%}$. The efficiency of the cycle is much lower than that of a Carnot engine operating between the same temperature extremes.

P21.38 (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding, it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i , and T_i

represent the state of the gas before the isothermal expansion. Let P_C, V_C , and T_i represent the state after this process, so that $P_i V_i = P_C V_C$. Let $P_i, 3V_i$, and T_f represent the state after the adiabatic compression.

Then
$$P_C V_C^\gamma = P_i (3V_i)^\gamma$$

Substituting
$$P_C = \frac{P_i V_i}{V_C}$$

gives
$$P_i V_i V_C^{\gamma-1} = P_i (3^\gamma V_i^\gamma)$$

Then
$$V_C^{\gamma-1} = 3^\gamma V_i^{\gamma-1} \quad \text{and} \quad \frac{V_C}{V_i} = 3^{\gamma/(\gamma-1)}$$

The work output in the isothermal expansion is

$$\begin{aligned} W &= \int_i^C P dV = nRT_i \int_i^C V^{-1} dV \\ &= nRT_i \ln \left(\frac{V_C}{V_i} \right) = nRT_i \ln \left(3^{\gamma/(\gamma-1)} \right) = nRT_i \left(\frac{\gamma}{\gamma-1} \right) \ln 3 \end{aligned}$$

This is also the input heat, so the entropy change is

$$\Delta S = \frac{Q}{T} = nR \left(\frac{\gamma}{\gamma-1} \right) \ln 3$$

Since $C_p = \gamma C_v = C_v + R$,

we have $(\gamma - 1)C_v = R, C_v = \frac{R}{\gamma - 1}$

and $C_p = \frac{\gamma R}{\gamma - 1}$.

Then the result is $\boxed{\Delta S = nC_p \ln 3}$.

- (b) The pair of processes considered here carries the gas from the initial state in P21.46 to the final state here. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in P21.46 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to P21.46 and P21.38(a) must be the same.

P21.39 We recognize that $T_c = T_1$ and $T_h = T_2$, and $Q_c = 350 \text{ J}$ and $Q_h = -1\,000 \text{ J}$.

$$\Delta S_{\text{hot}} = \frac{Q_h}{T_h} = -\frac{|Q_h|}{T_2} = \frac{-1000 \text{ J}}{600 \text{ K}}$$

$$\Delta S_{\text{cold}} = \frac{Q_c}{T_c} = \frac{|Q_c|}{T_1} = \frac{+750 \text{ J}}{350 \text{ K}}$$

$$(a) \quad \Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = -\frac{1\,000 \text{ J}}{600 \text{ K}} - \frac{750 \text{ J}}{350 \text{ K}} = \boxed{0.476 \text{ J/K}}$$

$$(b) \quad e_C = 1 - \frac{T_1}{T_2} = 1 - \frac{350 \text{ K}}{600 \text{ K}} = 0.417$$

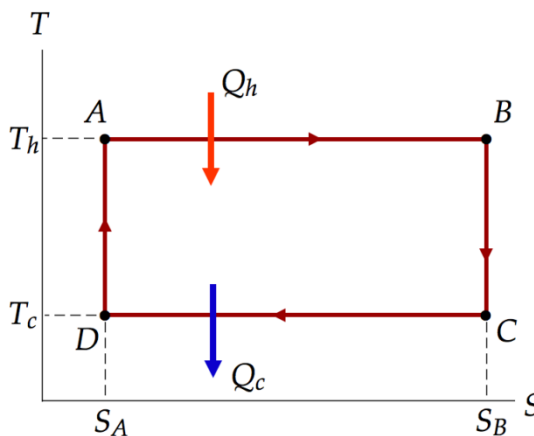
$$W_{\text{eng}} = e_C |Q_h| = 0.417(1\,000 \text{ J}) = \boxed{417 \text{ J}}$$

$$\begin{aligned} (c) \quad \Delta W &= W_C - W_{\text{real}} \\ &= e_C |Q_h| - (|Q_h| - |Q_c|) \\ &= \left(1 - \frac{T_c}{T_h}\right) |Q_h| - (|Q_h| - |Q_c|) \\ &= \left(|Q_h| - \frac{T_c}{T_h} |Q_h|\right) - (|Q_h| - |Q_c|) \\ &= |Q_c| - \frac{T_c}{T_h} |Q_h| = T_c \left(\frac{|Q_c|}{T_c} - \frac{|Q_h|}{T_h}\right) \\ &= T_c \Delta S_U = T_1 \Delta S_U \end{aligned}$$

***P21.40 Conceptualize** You may not be familiar with a TS diagram, but if you compare the phrase to “ PV diagram,” it must be the same type of graph, but with temperature T on the vertical axis and entropy S on the horizontal axis.

Categorize The problem statement tells us to restrict our attention to two types of cycles, the Carnot cycle and the Otto cycle.

Analyze (a) Let's begin with the Carnot cycle, whose PV diagram is shown in Figure 21.10. We begin at point A in the figure and follow the four processes in the cycle. The process $A \rightarrow B$ is isothermal. On a TS diagram, this process must be represented by a horizontal line, since the temperature remains fixed. Because energy enters the gas during this process, the entropy of the gas increases, so the horizontal line must be directed to the right. For the reversible adiabatic process $B \rightarrow C$, the entropy of the gas does not change, so the process must be represented by a vertical line at a fixed entropy on the TS diagram. The line is directed downward because the temperature drops from T_h to T_c . For the isothermal process $C \rightarrow D$, we have a horizontal line directed to the left as energy leaves the gas by heat and the entropy drops. Finally, process $D \rightarrow A$ is represented by a horizontal line directed upward and closing the path. Therefore, the TS diagram you should prepare for the presentation slide looks like this:



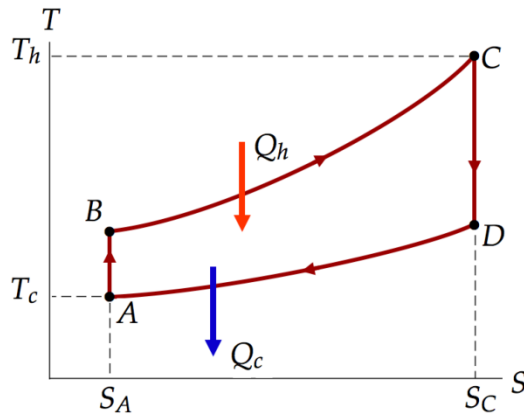
You might also want to add the following information to the slide. From Equation 21.12,

$$dS = \frac{dQ_r}{T} \rightarrow dQ_r = T dS \rightarrow Q_r = \int T dS \quad (1)$$

Therefore, the area under the curve on a TS diagram is the heat if the process is reversible. In the diagram above for the reversible Carnot engine, heat Q_h is the rectangular area underneath the upper horizontal line and heat Q_c is the rectangular area underneath the lower horizontal line. The difference between the two heats is the area surrounded by the rectangular path and, according to Equation 21.1, is the work done by the engine. Therefore, the work done by the engine is the area enclosed by the path on both a PV diagram and a TS diagram!

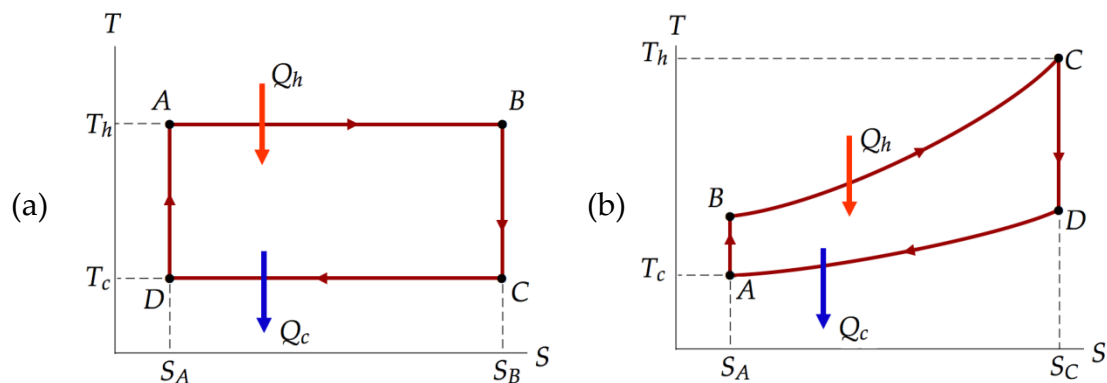
(b) Let's now consider the Otto cycle, shown in the PV diagram in Figure 21.12. The processes $O \rightarrow A$ and $A \rightarrow O$ are difficult to represent on a TS diagram. (See the Finalize step for more information.) So we will focus on just the cycle $A \rightarrow B \rightarrow C \rightarrow D$.

We begin at point A in the figure and follow the four processes in the cycle. The process $A \rightarrow B$ is adiabatic. On a TS diagram, this process must be represented by a vertical line, since the entropy of the gas remains fixed. Because the temperature of the gas increases during this process, the vertical line must be directed upward. For the isovolumetric process $B \rightarrow C$, in which the spark plug fires, the temperature and entropy of the gas both increase, so the process is represented by a curved line directed toward the upper right on the TS diagram. For the adiabatic process $C \rightarrow D$, we have a vertical line directed downward as the temperature drops but the entropy remains fixed. Finally, process $D \rightarrow A$ is another isovolumetric process that occurs as the exhaust valve is opened, closing the path. Therefore, the TS diagram you should prepare for the presentation slide looks like this:



Finalize Consider the statement made above that the processes $O \rightarrow A$ and $A \rightarrow O$ are difficult to represent on a TS diagram. Actually, they are difficult on a PV diagram, too. Figure 21.12 shows the pressure remaining constant and the volume changing as the fuel mixture is drawn into the cylinder in $O \rightarrow A$ and then expelled in $A \rightarrow O$. But it appears that the temperature increases in $O \rightarrow A$, since O is on a lower isotherm. This is not accurate; the temperature of the fuel mixture does not change as it is brought into the cylinder. The internal energy in the cylinder does indeed increase, because more gas is brought into the cylinder, but the change is in n , the number of moles of gas in the cylinder, not the temperature T . This confusion is due to the fact that an automobile engine does not quite match the definition of a heat engine because it doesn't take the *same* sample of gas and run it through multiple cycles. It brings in a sample of gas, runs it through *one* cycle, and then expels it. The next cycle is run on a *new* sample of gas. In a true heat engine, there is no intake and exhaust of the gas during the cycle; the same gas always remains within the engine.

Answers:



***P21.41 Conceptualize** In Figure 21.2, the warm reservoir is the steam. The cold reservoir is the water being rejected into the stream. The work output is represented by the 1.00 GW of electrical power.

Categorize In part (a) we are told to categorize the heat engine in the power plant as operating in a Carnot cycle.

Analyze From Equation 19.4, we can relate energy going into the stream to the flow rate of the stream and the temperature increase of the water in the stream:

$$Q = mc\Delta T_{\text{water}} \rightarrow \frac{Q}{\Delta t} = \left(\frac{\Delta m}{\Delta t} \right) c \Delta T_{\text{water}}$$

where c is the specific heat of water. Solve for the temperature change of the water:

$$\Delta T_{\text{water}} = \frac{\frac{Q_c}{\Delta t}}{\left(\frac{\Delta m}{\Delta t} \right) c} = \frac{P_c}{\left(\frac{\Delta m}{\Delta t} \right) c} \quad (1)$$

where we have recognized the energy going into the stream as that rejected from the power plant. Now, from Equation 21.1,

$$W_{\text{eng}} = |Q_h| - |Q_c| \rightarrow |Q_c| = |Q_h| - W_{\text{eng}} \quad (2)$$

Use Equation 21.2 to substitute for Q_h :

$$|Q_c| = \frac{W_{\text{eng}}}{e} - W_{\text{eng}} = W_{\text{eng}} \left(\frac{1}{e} - 1 \right) \quad (3)$$

Imagine energy exchanges that take place over a time interval Δt , so we can express Equation (3) with the power:

$$P_c = P_{\text{eng}} \left(\frac{1}{e} - 1 \right) \quad (4)$$

Because the power plant is operating at Carnot efficiency, use Equation 21.8 for the efficiency:

$$P_c = P_{\text{eng}} \left(\frac{T_h}{T_h - T_c} - 1 \right) = P_{\text{eng}} \left(\frac{T_c}{T_h - T_c} \right) \quad (5)$$

Substitute Equation (5) into Equation (1):

$$\Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t} \right)_c} \left(\frac{T_c}{T_h - T_c} \right) \quad (6)$$

Substitute numerical values:

$$\Delta T_{\text{water}} = \frac{1.00 \times 10^9 \text{ W}}{(6.00 \times 10^4 \text{ kg/s})(4186 \text{ J/kg} \cdot ^\circ\text{C})} \left(\frac{300 \text{ K}}{500 \text{ K} - 300 \text{ K}} \right) = \boxed{5.97 \text{ K}}$$

(b) Let's respond to this question algebraically. We can follow the analysis in part (a) through Equation (4), but cannot substitute the Carnot efficiency as we did in Equation (5).

Substitute Equation (4) into Equation (1):

$$\Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right) c} \left(\frac{1}{e} - 1\right) \quad (7)$$

The efficiency e in this equation is smaller than that for a Carnot engine.

Imagine reducing e in Equation (7) to a smaller value. This will *increase* the change in the water temperature, making it higher than that found in part (a).

(c) Substitute numerical values into Equation (7):

$$\Delta T_{\text{water}} = \frac{1.00 \times 10^9 \text{ W}}{(6.00 \times 10^4 \text{ kg/s})(4186 \text{ J/kg} \cdot ^\circ\text{C})} \left(\frac{1}{0.150} - 1\right) = \boxed{22.6 \text{ K}}$$

Finalize The conclusion in part (b) makes sense conceptually. If the engine is operating at a lower efficiency, less energy is leaving as the useful electrical output of the engine, and more is being rejected to the stream, raising the water temperature further. The change in water temperature in part (c) is much higher than that claimed by the utility, and could be damaging to the fish in the stream.

Answers: (a) 5.97 K (b) higher (c) 22.6 K

***P21.42 Conceptualize** In Figure 21.2, the warm reservoir is the steam. The cold reservoir is the water being rejected into the stream. The work output is represented by P .

Categorize In part (a) we are told to categorize the heat engine in the power plant as operating in a Carnot cycle.

Analyze From Equation 19.4, we can relate energy going into the stream to the flow rate of the stream and the temperature increase of the water in the stream:

$$Q = mc\Delta T_{\text{water}} \rightarrow \frac{Q}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) c\Delta T_{\text{water}}$$

where c is the specific heat of water. Solve for the temperature change of the water:

$$\Delta T_{\text{water}} = \frac{\frac{Q_c}{\Delta t}}{\left(\frac{\Delta m}{\Delta t}\right)_c} = \frac{P_c}{\left(\frac{\Delta m}{\Delta t}\right)_c} \quad (1)$$

where we have recognized the energy going into the stream as that rejected from the power plant. Now, from Equation 21.1,

$$W_{\text{eng}} = |Q_h| - |Q_c| \rightarrow |Q_c| = |Q_h| - W_{\text{eng}} \quad (2)$$

Use Equation 21.2 to substitute for Q_h :

$$|Q_c| = \frac{W_{\text{eng}}}{e} - W_{\text{eng}} = W_{\text{eng}} \left(\frac{1}{e} - 1 \right) \quad (3)$$

Imagine energy exchanges that take place over a time interval Δt , so we can express Equation (3) with the power:

$$P_c = P_{\text{eng}} \left(\frac{1}{e} - 1 \right) \quad (4)$$

Because the power plant is operating at Carnot efficiency, use Equation 21.8 for the efficiency:

$$P_c = P_{\text{eng}} \left(\frac{T_h}{T_h - T_c} - 1 \right) = P_{\text{eng}} \left(\frac{T_c}{T_h - T_c} \right) \quad (5)$$

Substitute Equation (5) into Equation (1):

$$\boxed{\Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right)_c} \left(\frac{T_c}{T_h - T_c} \right)} \quad (6)$$

(b) We can follow the analysis in part (a) through Equation (4), but cannot substitute the Carnot efficiency as we did in Equation (5). Substitute Equation (4) into Equation (1):

$$\Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right)_c} \left(\frac{1}{e} - 1\right) \quad (7)$$

(c) The efficiency e in this equation is smaller than that for a Carnot engine. This will *increase* the change in the water temperature, making it **higher** than that found in part (a).

Finalize The conclusion in part (c) makes sense conceptually. If the engine is operating at a lower efficiency, less energy is leaving as the useful electrical output of the engine, and more is being rejected to the stream, raising the water temperature further. The change in water temperature in part (b) is much higher than that claimed by the utility, and could be damaging to the fish in the stream.

$$\text{Answers: (a) } \Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right)_c} \left(\frac{T_c}{T_h - T_c}\right) \quad \text{(b) } \Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right)_c} \left(\frac{1}{e} - 1\right)$$

(c) higher

P21.43 (a) water: $T_{\text{water}} = 35.0^\circ\text{F} \rightarrow \frac{5}{9}(35.0 - 32.0)^\circ\text{C}$
 $\rightarrow (1.67 + 273.15) \text{ K} = 274.82 \text{ K}$

body: $T_{\text{body}} = 98.6^\circ\text{F} \rightarrow \frac{5}{9}(98.6 - 32.0)^\circ\text{C}$
 $\rightarrow (37.0 + 273.15) \text{ K} = 310.15 \text{ K}$

$$\begin{aligned}
\Delta S_{\text{cold water}} &= \int \frac{dQ}{T} = m_w c \times \int_{T_{\text{water}}}^{T_{\text{body}}} \frac{dT}{T} = m_w c \times \ln \left(\frac{T_{\text{body}}}{T_{\text{water}}} \right) \\
\Delta S_{\text{body}} &= -\frac{|Q|}{T_{\text{body}}} = -\frac{m_w c (T_{\text{body}} - T_{\text{water}})}{T_{\text{body}}} \\
\Delta S_{\text{system}} &= \Delta S_{\text{cold water}} + \Delta S_{\text{body}} \\
&= (0.454 \text{ kg})(4 \text{ 186 J/kg} \cdot \text{K}) \times \ln \left(\frac{310.15}{274.82} \right) \\
&\quad - (0.454 \text{ kg})(4 \text{ 186 J/kg} \cdot \text{K}) \frac{(310.15 - 274.82)}{310.15} = \boxed{13.4 \text{ J/K}}
\end{aligned}$$

(b) Conservation of energy, $Q_{\text{hot}} = -Q_{\text{cold}}$, gives

$$\begin{aligned}
m_w c (T_F - T_{\text{water}}) &= -m_{\text{Ath}} c (T_F - T_{\text{body}}) \\
m_w (T_F - T_{\text{water}}) &= -m_{\text{Ath}} (T_F - T_{\text{body}}) \\
m_w T_F - m_w T_{\text{water}} &= -m_{\text{Ath}} T_F + m_{\text{Ath}} T_{\text{body}} \\
(m_w + m_{\text{Ath}}) T_F &= m_w T_{\text{water}} + m_{\text{Ath}} T_{\text{body}}
\end{aligned}$$

Solving for T_F ,

$$\begin{aligned}
T_F &= \frac{m_w T_{\text{water}} + m_{\text{Ath}} T_{\text{body}}}{m_w + m_{\text{Ath}}} \\
&= \frac{(0.454 \text{ kg})(274.82 \text{ K}) + (70.0 \text{ kg})(310.15 \text{ K})}{0.454 \text{ kg} + 70.0 \text{ kg}} \\
&= 309.92 \text{ K} = \boxed{310 \text{ K}}
\end{aligned}$$

(c) $\Delta S = \Delta S'_{\text{ice water}} + \Delta S'_{\text{body}}$

$$\begin{aligned}
&= m_w c \times \ln \left(\frac{T_F}{T_{\text{water}}} \right) + m_{\text{Ath}} c \times \ln \left(\frac{T_F}{T_{\text{body}}} \right) \\
&= (0.454 \text{ kg})(4 \text{ 186 J/kg} \cdot \text{K}) \ln \left(\frac{309.92}{274.82} \right) \\
&\quad + (70.0 \text{ kg})(4 \text{ 186 J/kg} \cdot \text{K}) \ln \left(\frac{309.92}{310.15} \right) \\
&= \boxed{11.1 \text{ J/K}}
\end{aligned}$$

(d) Smaller by less than 1%

P21.44 When energy enters a substance by heat, we describe the process with Equation 20.4, $Q = mc\Delta T$. This is a reversible process; if energy leaves the substance, the temperature drops down again. Therefore, the entropy change for one of the samples of water is

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc dT}{T} = mc \ln \left(\frac{T_f}{T_i} \right)$$

Consequently, the entropy change for both samples of water is

$$\begin{aligned} \Delta S_{\text{total}} &= \Delta S_{\text{hot}} + \Delta S_{\text{cold}} \\ &= mc \ln \left(\frac{T_f}{T_{hi}} \right) + mc \ln \left(\frac{T_f}{T_{ci}} \right) = mc \ln \left[\left(\frac{T_f}{T_{hi}} \right) \left(\frac{T_f}{T_{ci}} \right) \right] \\ (1.00 \text{ kg})(4186 \text{ J/kg} \cdot ^\circ\text{C}) \ln \left[\left(\frac{293 \text{ K}}{303 \text{ K}} \right) \left(\frac{293 \text{ K}}{283 \text{ K}} \right) \right] &= 4.88 \text{ J/K} \end{aligned}$$

This is *not* zero. While the statements about energy transfer by heat are true, the mixing process is irreversible. After the water has come to equilibrium, it will not spontaneously separate again into warm and cool water. Therefore, there is an entropy increase of the mixture during this irreversible process.

P21.45 (a) $W = \int_{V_i}^{V_f} PdV = nRT \int_{V_i}^{2V_i} \frac{dV}{V} = (1.00)RT \ln \left(\frac{2V_i}{V_i} \right) = \boxed{RT \ln 2}$

(b) Yes.

(c) No. The second law refers to an engine operating in a cycle, whereas this problem involves only a single process.

$$\begin{aligned} \text{P21.46} \quad \Delta S &= \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_p dT}{T} = nC_p \int_i^f T^{-1} dT = nC_p \ln T \Big|_{T_i}^{T_f} = nC_p (\ln T_f - \ln T_i) \\ &= nC_p \ln \left(\frac{T_f}{T_i} \right) \end{aligned}$$

$$\Delta S = nC_p \ln \left(\frac{PV_f}{nR} \frac{nR}{PV_i} \right) = nC_p \ln \left[\frac{P(3V_i)}{nR} \frac{nR}{PV_i} \right] = \boxed{nC_p \ln 3}$$

Challenge Problem

P21.47 The quantity of gas is

$$n = \frac{P_A V_A}{RT_A} = \frac{(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.0205 \text{ mol}$$

(a) In process $A \rightarrow B$,

$$P_B = P_A \left(\frac{V_A}{V_B} \right)^\gamma = (100 \times 10^3 \text{ Pa})(8.00)^{1.40} = 1.84 \times 10^6 \text{ Pa}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{(1.84 \times 10^6 \text{ Pa})(500 \times 10^{-6} \text{ m}^3 / 8.00)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 673 \text{ K}$$

$$V_A/V_B = 8.00 \rightarrow V_B = V_A/8.00 = 500/8 = 62.5 \text{ cm}^3$$

State C:

$$V_C = V_B$$

$$\begin{aligned} P_C &= \frac{nRT_C}{V_C} = \frac{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K})}{62.5 \times 10^{-6} \text{ m}^3} \\ &= 2.79 \times 10^6 \text{ Pa} \end{aligned}$$

State D:

$$V_D = V_A$$

In process $C \rightarrow D$:

$$P_D = P_C \left(\frac{V_C}{V_D} \right)^\gamma = (2.79 \times 10^6 \text{ Pa}) \left(\frac{1}{8.00} \right)^{1.40} = 1.52 \times 10^5 \text{ Pa}$$

$$T_D = \frac{P_D V_D}{nR} = \frac{(1.52 \times 10^5 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 445 \text{ K}$$

TABLE P21.47 (a) tabulates these results:

	T (K)	P (kPa)	V (cm ³)
A	293	100	500
B	673	1.84×10^3	62.5
C	1023	2.79×10^3	62.5
D	445	152	500

TABLE P21.47 (a)

(b) In the adiabatic process $A \rightarrow B$, $Q = 0$,

$$\begin{aligned} \Delta E_{\text{int}, A \rightarrow B} &= \frac{5}{2} nR (T_B - T_A) \\ &= \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(673 \text{ K} - 293 \text{ K}) \\ &= 162 \text{ J} \end{aligned}$$

$$\text{and } \Delta E_{\text{int}, AB} = 162 \text{ J} = Q - W_{\text{out}} = 0 - W_{\text{out}} \rightarrow W_{AB} = -162 \text{ J}$$

In the isovolumetric process $B \rightarrow C$, $W = 0$

$$\begin{aligned}
\Delta E_{\text{int}, B \rightarrow C} &= \frac{5}{2} n R (T_C - T_B) \\
&= \frac{5}{2} (0.020 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) (1023 \text{ K} - 673 \text{ K}) \\
&= 149 \text{ J} \\
\Delta E_{\text{int}, B \rightarrow C} &= 149 \text{ J} = Q - W_{\text{out}} = Q - 0 \rightarrow Q_{BC} = 149 \text{ J}
\end{aligned}$$

In the adiabatic process $C \rightarrow D$, $Q = 0$

$$\begin{aligned}
\Delta E_{\text{int}, C \rightarrow D} &= \frac{5}{2} n R (T_D - T_C) \\
&= \frac{5}{2} (0.020 \text{ mol}) (8.314 \text{ J/mol} \cdot \text{K}) (445 \text{ K} - 1023 \text{ K}) \\
&= -246 \text{ J} \\
\Delta E_{\text{int}, C \rightarrow D} &= -246 \text{ J} = Q - W_{\text{out}} = 0 - W_{\text{out}} \rightarrow W_{CD} = 246 \text{ J}
\end{aligned}$$

For the entire cycle, $\Delta E_{\text{int}, \text{net}} = \frac{5}{2} n R \Delta T = 0$:

$$\begin{aligned}
W_{\text{eng}} &= -162 \text{ J} + 0 + 246.3 \text{ J} + 0 = 84.3 \text{ J} \\
\Delta E_{\text{int}} &= Q_{\text{net}} + W_{\text{eng}} = 0 \rightarrow Q_{\text{net}} = -W_{\text{eng}} = 84.3 \text{ J}
\end{aligned}$$

TABLE P21.47 (b) tabulates these results:

	Q	W_{eng}	ΔE_{int}
$A \rightarrow B$	0	-162	162
$B \rightarrow C$	149	0	149
$C \rightarrow D$	0	246	-246
$D \rightarrow A$	-65.0	0	-65.0
ABC	84.3	84.3	0

TABLE P21.47 (b)

(c) From $B \rightarrow C$, the input energy is $Q_h = \boxed{149 \text{ J}}$.

(d) From $D \rightarrow A$, the energy exhaust is $|Q_c| = \boxed{65.0 \text{ J}}$.

(e) From ABCDA, $W_{\text{eng}} = \boxed{84.3 \text{ J}}$.

(f) The efficiency is: $e = \frac{W_{\text{eng}}}{Q_h} = \frac{84.3 \text{ J}}{149 \text{ J}} = \boxed{0.565}$

(g) Let f represent the angular speed of the crankshaft. Then $\frac{f}{2}$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1\,000 \text{ J/s} = \left(\frac{f}{2}\right)(84.3 \text{ J/cycle})$$

$$f = \frac{2\,000 \text{ J/s}}{84.3 \text{ J/cycle}} = 23.7 \text{ rev/s} = \boxed{1.42 \times 10^3 \text{ rev/min}}$$



ANSWERS TO QUICK-QUIZZES

1. (i) (c) (ii) (b)

2. (d)

3. C, B, A

4. (a) one (b) six

5. (a)

6. false (The adiabatic process must be *reversible* for the entropy change to be equal to zero.



ANSWERS TO EVEN-NUMBERED PROBLEMS

- P21.2** (a) 0.25 or 25%; (b) $|Q_c|/|Q_h| = 3/4$
- P21.4** (a) 75.0 kJ (b) 7.33
- P21.6** (a) 2.65×10^7 J; (b) 3.20
- P21.8** The efficiency of a Carnot engine operating between these temperatures is 6.83%. Therefore, there is no way that the inventor's engine can have an efficiency of $0.110 = 11.0\%$.
- P21.10** (a) See P21.10 (a) for the full solution; (b) See P21.10 (b) for the full solution.
- P21.12** 0.330 or 33.0%
- P21.14** (a) 0.300; (b) $1.40 \times 10^{-3} \text{ K}^{-1}$; (c) $-2.00 \times 10^{-3} \text{ K}^{-1}$; (d) No. The derivative in part (c) depends only on T_h
- P21.16** (a) See P21.16 (a) for full explanation; (b) $1 - \frac{T_c}{T_h}$; (c) The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency. No improvement in net efficiency has resulted; (d) $T_i = \frac{1}{2}(T_h + T_c)$;
(e) $T_i = (T_h T_c)^{1/2}$
- P21.18** (a) 51.2%; (b) 36.2%
- P21.20** (a) See TABLE P21.20; (b) 2 heads and 2 tails
- P21.22** 143 J/K
- P21.24** 0.507 J/K
- P21.26** 244 J/K

P21.28 (a) $\Delta S_h = -\frac{Q}{T_h}$; (b) $\Delta S_c = -\frac{Q}{T_c}$; (c) $Q\left(\frac{1}{T_c} - \frac{1}{T_h}\right)$

P21.30 $8.36 \times 10^6 \text{ J/K} \cdot \text{s}$

P21.32 (a) 2.93; (b) $(\text{COP})_{\text{refrigerator}}$; (c) with EER 5, \$510, with EER 10, \$255; Thus, the cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

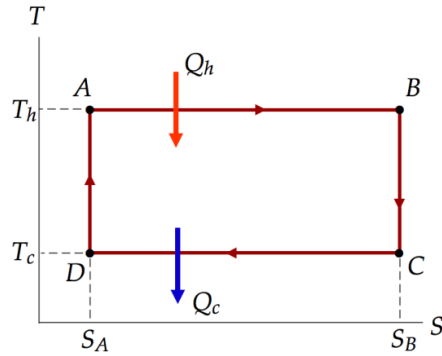
P21.34 (a) $\frac{T_c}{T_h - T_c}$; (b) smaller; (c) 14.6

P21.36 (a) 214 J and 64.3 J; (b) 35.7 J and 35.7 J. The total energy the firebox puts out equals to the total energy transferred to the environment; (c) The net flow of energy by heat from the cold to the hot reservoir without work input is possible; (d) $W_{\text{eng S}} = 233 \text{ J}$, $|Q_{h,S}| = 333 \text{ J}$; (e) 83.3 J; (f) 83.3 J; (g) 0; (h) The output of 83.8 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible; (i) -0.111 J/K ; (j) A decrease in total entropy is impossible.

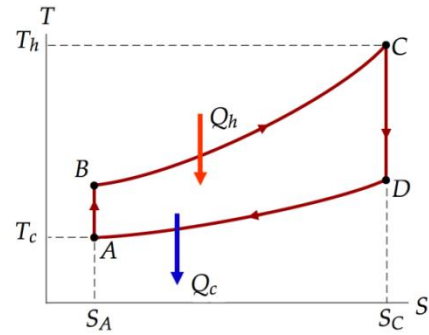
P21.38 (a) $\Delta S = nC_p \ln 3$; (b) The pair of processes considered here carries the gas from the initial state in P21.46 to the final state here. Entropy is a function of state. Entropy change does not depend on path. Therefore, the entropy change in P21.46 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to P21.46 and P21.38 (a) must be the same.

P21.40

(a)



(b)



P21.42

$$(a) \Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right)_C} \left(\frac{T_c}{T_h - T_c} \right) \quad (b) \Delta T_{\text{water}} = \frac{P_{\text{eng}}}{\left(\frac{\Delta m}{\Delta t}\right)_C} \left(\frac{1}{e} - 1 \right) \quad (c) \text{ higher}$$

P21.44

The computed change in entropy is 4.88 J/K, which is *not* zero. While the statements about energy transfer by heat are true, the mixing process is irreversible. After the water has come to equilibrium, it will not spontaneously separate again into warm and cool water. Therefore, there is an entropy increase of the mixture during the irreversible process.

P21.46

$$nC_p \ln 3$$