43

Nuclear Physics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO THINK-PAIR-SHARE ACTIVITIES

***TP43.1** **Conceptualize** Imagine the technical difficulties, and the cost, of extracting the uranium from the ocean water for Group (i) and for isolating the deuterium in the water from Group (ii). These considerations would have to be taken into account if a decision had to be made about which source to go after. We will ignore those considerations for now, however, and just determine the *existing* energy, without regard for how much of it is feasibly *available*.

Categorize We focus on the energy calculations for fission and fusion reactions in Sections 43.8 and 43.10.

Analyze Let's begin with Group (i). We first find the amount of uranium in the ocean, based on the concentration density $\rho = 3.00 \times 10^{-6}$ kg/m^3. Using Equation 1.1,

$$m_{\mathrm{U}} = \rho V \qquad (1)$$

where V is the volume of the ocean. Evaluate the volume of the ocean, which will be needed by both groups, using the information given about the ocean in the problem statement:

$$V = \tfrac{2}{3}\left(4\pi R_E^2 d\right) \qquad (2)$$

where d is the average depth of the ocean. Substitute numerical values:

$$V = \tfrac{2}{3}\left[4\pi\left(6.37\times10^{6}\ \mathrm{m}\right)^2\left(4.00\times10^{3}\ \mathrm{m}\right)\right] = 1.360\times10^{18}\ \mathrm{m}^3$$

Modify Equation (1) to represent the mass of *fissionable* uranium-235, using the fraction given in the problem statement:

$$m_{^{235}\mathrm{U}} = \left(0.007\,00\right)\rho V \qquad (3)$$

The energy that could be theoretically released from the fission of every $^{235}\mathrm{U}$ nucleus in the ocean is

$$E = N(208\text{ MeV}) = \left(\frac{m_{^{235}\mathrm{U}}\text{ in ocean}}{M_{^{235}\mathrm{U}}}\right)(208\text{ MeV}) = \left(\frac{(0.007\,00)\rho V}{M_{^{235}\mathrm{U}}}\right)(208\text{ MeV})$$

where we have used the energy released per event from Example 43.9, N is the number of uranium-235 atoms in the ocean, and $M_{^{235}\mathrm{U}}$ is the atomic mass of uranium-235. Substitute numerical values:

$$E = \left(\frac{(0.007\,00)(3.00\times10^{-6}\text{ kg/ m}^3)(1.360\times10^{18}\text{ m}^3)}{(235\text{ u})(1.66\times10^{-27}\text{ kg/ u})}\right)(208\text{ MeV})\left(\frac{1.602\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$$
$$= \boxed{2.44\times10^{24}\text{ J}}$$

Now, let's consider the information for Group (ii). The statement "of all the hydrogen in the oceans, 0.030 0% of the mass is deuterium" can be expressed mathematically as

$$m_{\text{Deuterium}} = (0.000\,300)m_{\mathrm{H}} \qquad (4)$$

where m_{H} is the mass of all the hydrogen in the ocean. The mass of all the hydrogen in the ocean can be expressed as

$$m_{\mathrm{H}} = \left(\frac{2M_{\mathrm{H}}}{M_{\mathrm{H_2O}}}\right)m_{\mathrm{H_2O}} \qquad (5)$$

where the uppercase Ms are molecular masses and $m_{\mathrm{H_2O}}$ is the mass of all the water in the ocean. Using Equation 1.1,

$$m_{\mathrm{H_2O}} = \rho_{\mathrm{H_2O}}V \qquad (6)$$

where V is the volume of the ocean. Combining Equations (4), (5), and (6), we have

$$m_{\text{Deuterium}} = (0.000\,300)\left(\frac{2M_{\text{H}}}{M_{\text{H}_2\text{O}}}\right)\rho_{\text{H}_2\text{O}}V \tag{7}$$

Now the number N of deuterons in this mass of deuterium is

$$N = \frac{m_{\text{Deuterium}}}{M_{^2\text{H}}} \tag{8}$$

Because each fusion event requires two deuterons, the number of fusion events possible is

$$N_{\text{events}} = \frac{N}{2} = \frac{m_{\text{Deuterium}}}{2M_{^2\text{H}}} \tag{9}$$

The energy released per fusion event from the reaction assumed in the problem statement can be determined from Equation 43.30:

$$\begin{aligned} Q &= \left(M_{^2\text{H}} + M_{^2\text{H}} - M_{^4\text{He}}\right)c^2 \\ &= (2.014\,102\text{ u} + 2.014\,102\text{ u} - 4.002\,603\text{ u})(931.494\text{ MeV/ u}) \\ &= 23.8\text{ MeV} \end{aligned}$$

Therefore, the total energy released from the fusion of all deuterium in the ocean is, combining Equations (7) and (9),

$$E = N_{\text{events}}Q = \frac{m_{\text{Deuterium}}}{2M_{^2\text{H}}}Q = (0.000\,300)\left(\frac{M_{\text{H}}}{M_{\text{H}_2\text{O}}M_{^2\text{H}}}\right)\rho_{\text{H}_2\text{O}}VQ$$

(10)

Substitute numerical values:

$$E = (0.000\,300)\left[\frac{1.007\,9\text{ u}}{(18.015\,2\text{ u})(2.014\,102\text{ u})(1.66\times10^{-27}\text{ kg/ u})}\right]$$
$$\times(1\,000\text{ kg/ m}^3)(1.360\times10^{18}\text{ m}^3)(23.8\text{ MeV})\left(\frac{1.602\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$$
$$= \boxed{2.60\times10^{31}\text{ J}}$$

Finalize There is more energy in the oceans for fusion than fission by seven orders of magnitude! That's the good news. The bad news is that fusion reactors are not yet a reality. If we do end up developing fusion reactors, we have a vast amount of energy available in seawater. At the current rate of world energy usage, the energy available for fusion in the oceans would last for 100 million years.]

Answers: Group (i): 2.44×10^{24} J; Group (ii): 2.60×10^{31} J

***TP43.2** **Conceptualize** At first glance, we might say Patient A received twice as much as Patient B. But, reading onward, we see that the types of radiation are different and the mass of tissue irradiated is different. Maybe it's not so clear cut.

Categorize We use the information in Section 43.11 on radiation dosage.

Analyze (a) Calculate first the dose in sieverts received by each patient using Equation 43.36:

Patient A:

$$\text{Dose} = (2.0\text{ Gy})\left(\frac{100\text{ rad}}{1\text{ Gy}}\right)(10) = 2\,000\text{ rem}\left(\frac{1\text{ Sv}}{100\text{ rem}}\right) = 20\text{ Sv}$$

Patent B:

$$\text{Dose} = (1.0\ \text{Gy})\left(\frac{100\ \text{rad}}{1\ \text{Gy}}\right)(18) = 1\,800\ \text{rem}\left(\frac{1\ \text{Sv}}{100\ \text{rem}}\right) = 18\ \text{Sv}$$

Now, account for the mass of tissue involved to see which patient received the most energy:

Patient A: $E_{\text{A}} = 20\ \text{Sv}(0.022\ \text{kg}) = 0.44\ \text{J}$

Patent B: $E_{\text{B}} = 18\ \text{Sv}(0.030\ \text{kg}) = 0.54\ \text{J}$

Therefore, $\boxed{\text{patient B}}$ received more energy than Patient A.

(b) Determine the ratio:

$$\frac{E_{\text{B}}}{E_{\text{A}}} = \frac{0.54\ \text{J}}{0.44\ \text{J}} = \boxed{1.23}$$

Finalize Therefore, Patient B received about 23% more radiation in terms of biological effectiveness than Patient A. It is interesting to point out that the radiation energy aimed at Patient A was larger than for Patient B: 2.0 Gy(0.022 kg) = 0.044 J compared to 1.0 Gy(0.030 kg) = 0.030 J. But, because of the RBE factor, the biological effectiveness was larger for Patient B.]

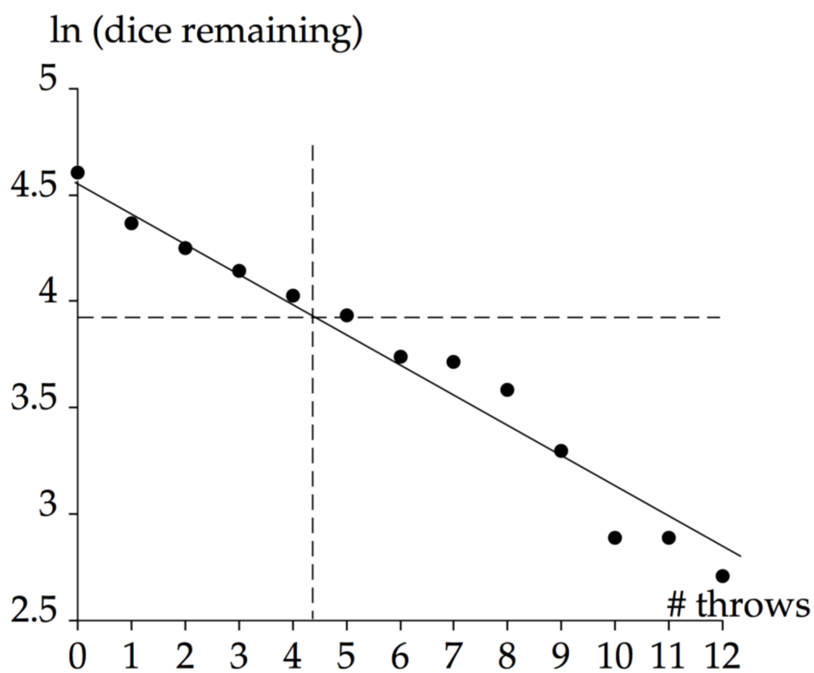
Answers: (a) Patient B (b) 1.023

***TP43.3** (a) The decay constant λ, which is the probability of decay per throw, is one-sixth, because that is the probability of throwing a one on any die. Therefore, from Equation 43.8, the theoretical half-life is

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1/\ 6} = \boxed{4.16\ \text{throws}}$$

As a result, when starting with 100 dice, somewhere between the 4th and 5th throws, the number of dice should pass from above 50 to below 50.

(b) Because 100 is a small number when dealing with statistics, this may not occur every time in practice. For example, the graph below shows some sample data. There is clearly a great deal of scatter in the data points. The experimental half-life, indicated by the dashed line, is about 4.3 throws. A better agreement between theory and experiment would result from a larger number of dice, say, 10 000, but that would be experimentally cumbersome! If you have some coding experience, you can set up a Monte Carlo simulation for 10 000 dice and test the results.



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Answer: (a) 4.16 throws (b) Answers will vary.

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SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 43.1 Some Properties of Nuclei

P43.1 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons. So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

(a) $$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim 10^{28}\text{ protons}},$$

(b) and $\boxed{\sim 10^{28}\text{ neutrons}}$.

(c) The electron number is precisely equal to the proton number, $\boxed{\sim 10^{28}\text{ electrons}}$.

P43.2 (a) Approximate nuclear radii are given by $r = r_0A^{1/3}$. Thus, if a nucleus of atomic number A has a radius approximately two-thirds that of ${}^{230}_{88}\text{Ra}$, we should have

$$r = r_0A^{1/3} = \frac{2}{3}r_0(230)^{1/3}$$

or $$A = \frac{2^3}{3^3}(230) = \frac{8}{27}(230) \approx \boxed{68}$$

(b) $\boxed{\text{One possible nucleus is } {}^{68}_{30}\text{Zn}}$.

(c) $\boxed{\text{Isotopes of other elements to the left and right of zinc in the periodic table (from manganese to bromine) may have the same mass number.}}$

P43.3 (a) The electric potential energy between two protons is

$$U = k_e \frac{q_1 q_2}{r} = k_e \frac{e^2}{r}$$

$$= \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{4.00\times10^{-15}\ \text{m}}\right]$$

$$\times\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{MeV}}{10^{6}\ \text{eV}}\right)$$

$$= \boxed{0.360\ \text{MeV}}$$

(b) $\boxed{\text{Figure P43.8 shows the highest point in the curve at about 4 MeV, a factor of ten higher than the value in (a).}}$

P43.4 We obtain the alpha particle's momentum from

$$E_\alpha = 7.70\ \text{MeV} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} \quad\rightarrow\quad mv = \sqrt{2mE_\alpha}$$

(a) The de Broglie wavelength of the alpha particle is (mass from Table 43.1)

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}}$$

$$= \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(6.64\times10^{-27}\ \text{kg}\right)\left(7.70\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}}$$

$$= 5.18\times10^{-15}\ \text{m} = \boxed{5.18\ \text{fm}}$$

(b) Since $\boxed{\lambda \text{ is much less than the distance of closest approach,}}$ the alpha particle may be considered a particle.

P43.5 (a) Let V represent the volume of the tank. The number of molecules present is

$$N = nN_A = \frac{PV}{RT} = \frac{\left(1.013\times10^5\ \text{N/m}^2\right)V}{(8.315\ \text{J/mol}\cdot\text{K})(273\ \text{K})}\left(6.022\times10^{23}\right)$$
$$= \left(2.69\times10^{25}\ \text{m}^{-3}\right)V$$

The volume of one molecule is

$$2\left(\frac{4}{3}\pi r^3\right) = \frac{8\pi}{3}\left(\frac{1.00\times10^{-10}\ \text{m}}{2}\right)^3 = 1.047\times10^{-30}\ \text{m}^3$$

The volume of all the molecules is

$$\left(2.69\times10^{25}\ \text{m}^{-3}\right)V\left(1.047\times10^{-30}\ \text{m}^3\right) = 2.82\times10^{-5}\ V$$

So the fraction of the volume occupied by the hydrogen molecules is $\boxed{2.82\times10^{-5}.}$ An atom is precisely one half of a molecule.

(b) The fraction occupied by the nucleus is found from

$$\frac{\text{nuclear volume}}{\text{atomic volume}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(d/2)^3} = \left(\frac{r}{d/2}\right)^3$$
$$= \left(\frac{1.20\times10^{-15}\ \text{m}}{0.500\times10^{-10}\ \text{m}}\right)^3 = \boxed{1.38\times10^{-14}}$$

In linear dimension, the nucleus is small inside the atom in the way a fat strawberry is small inside the width of the Grand Canyon. In terms of volume, the nucleus is *really* small.

Section 43.2 Nuclear Binding Energy

***P43.6** **Conceptualize** Both nuclei have the same total number of nucleons, A = 15. We expect the binding energy to be greater for ${}^{15}_{7}\mathrm{N}$, because there will be fewer protons repelling each other.

Categorize We categorize this problem as a substitution problem.

Use Equation 43.2 to find a theoretical value of the binding energy for each nucleus:

$$E_b(\mathrm{N}) = \left[Z_\mathrm{N} M(\mathrm{H}) + N_\mathrm{N} m_n - M\left({}^{15}_{7}\mathrm{N}\right)\right] \times 931.494 \text{ MeV/ u} \tag{1}$$

$$E_b(\mathrm{O}) = \left[Z_\mathrm{O} M(\mathrm{H}) + N_\mathrm{O} m_n - M\left({}^{15}_{8}\mathrm{O}\right)\right] \times 931.494 \text{ MeV/ u} \tag{2}$$

Subtract Equation (2) from Equation (1):

$$\Delta E_b = \left[(Z_\mathrm{N} - Z_\mathrm{O}) M(\mathrm{H}) + (N_\mathrm{N} - N_\mathrm{O}) m_n - M\left({}^{15}_{7}\mathrm{N}\right) + M\left({}^{15}_{8}\mathrm{O}\right)\right] \times 931.494 \text{ MeV/ u}$$

Substitute numerical values:

$$\Delta E_b = \begin{bmatrix} (7-8)(1.007\,825 \text{ u}) + (8-7)(1.008\,665 \text{ u}) & \\ & -15.000\,109 \text{ u} + 15.003\,065 \text{ u} \end{bmatrix} \times 931.494 \text{ MeV/ u}$$
$$= \boxed{3.54 \text{ MeV}}$$

The binding energy does indeed turn out to be higher for nitrogen.]

Answer: 3.54 MeV

P43.7 We use Equation 43.2,

$$E_b(\mathrm{MeV}) = \left[ZM(\mathrm{H}) + Nm_n - M\left({}^{A}_{Z}\mathrm{X}\right)\right](931.494 \text{ MeV/u})$$

Then, for ${}^{23}_{11}\mathrm{Na}$,

$$E_b\left({}^{23}_{11}\mathrm{Na}\right) = \left[11M(\mathrm{H}) + 12m_n - M\left({}^{23}_{11}\mathrm{Na}\right)\right](931.494\ \mathrm{MeV/u})$$
$$= \left[11(1.007\,825\ \mathrm{u}) + 12(1.008\,665\ \mathrm{u}) - 22.989\,769\ \mathrm{u}\right]$$
$$\times(931.494\ \mathrm{MeV/u})$$
$$= 186.565\ \mathrm{MeV}$$

and $$\frac{E_b}{A} = \frac{186.565\ \mathrm{MeV}}{23} = 8.11\ \mathrm{MeV}$$

For ${}^{23}_{12}\mathrm{Mg}$,

$$E_b = E_b\left({}^{23}_{12}\mathrm{Mg}\right)$$
$$= \left[12M(\mathrm{H}) + 11m_n - M\left({}^{23}_{12}\mathrm{Mg}\right)\right](931.494\ \mathrm{MeV/u})$$
$$= \left[12(1.007\,825\ \mathrm{u}) + 11(1.008\,665\ \mathrm{u}) - 22.994\,124\ \mathrm{u}\right]$$
$$\times(931.494\ \mathrm{MeV/u})$$
$$= 181.726\ \mathrm{MeV}$$

and $$\frac{E_b}{A} = \frac{181.726\ \mathrm{MeV}}{23} = 7.90\ \mathrm{MeV}$$

The difference is

$$\frac{\Delta E_b}{A} = \frac{E_b\left({}^{23}_{11}\mathrm{Na}\right) - E_b\left({}^{23}_{12}\mathrm{Mg}\right)}{A}$$
$$= 8.11\ \mathrm{MeV} - 7.90\ \mathrm{MeV} = 0.210\ \mathrm{MeV}$$

The binding energy per nucleon is greater for ${}^{23}_{11}\mathrm{Na}$ by $\boxed{0.210\ \mathrm{MeV}}$. There is less proton repulsion in ${}^{23}_{11}\mathrm{Na}$; it is the more stable nucleus.

P43.8 We find the mass difference, $\Delta M = Zm_\mathrm{H} + Nm_\mathrm{n} - M$, and then the binding energy per nucleon, $\dfrac{E_b}{A} = \dfrac{\Delta M(931.5)}{A}$, in units of MeV. The results are tabulated below

Nuclei	Z	N	M in u	ΔM in u	$\frac{E_b}{A}$ in MeV
^{55}Mn	25	30	54.938 050	0.517 5	8.765
^{56}Fe	26	30	55.934 942	0.528 46	8.790
^{59}Co	27	32	58.933 200	0.555 35	8.768

$\boxed{\therefore\ ^{56}\text{Fe has a greater } \frac{E_b}{A} \text{ than its neighbors.}}$

P43.9 (a) The isobar with the highest neutron-to-proton ratio is $\boxed{^{139}_{55}\text{Cs}}$; the ratio is $\frac{N}{Z} = \frac{A - Z}{Z} = \frac{139 - 55}{55} = \frac{84}{55} = 1.53$

(b) $\boxed{^{139}_{57}\text{La}}$ is stable, so has the largest binding energy per nucleon (8.378 MeV).

(c) The isobars are close in Figure 43.6, the plot of binding energy per nucleon versus mass number, and there is not much detail, so we may assume they have about the same binding energy, or missing mass. However, neutrons have more mass than protons, so the isobar with more neutrons (thus, fewer protons) should be more massive: $\boxed{^{139}_{55}\text{Cs}}$.

P43.10 (a) The radius of the ^{40}Ca nucleus is,

$$R = r_0 A^{1/3} = \left(1.20 \times 10^{-15}\ \text{m}\right)(40)^{1/3} = 4.10 \times 10^{-15}\ \text{m}$$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_eQ^2}{5R} = \frac{3\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[20\left(1.602\times10^{-19}\ \text{C}\right)\right]^2}{5\left(4.10\times10^{-15}\ \text{m}\right)}$$
$$= 1.35\times10^{-11}\ \text{J} = \boxed{84.2\ \text{MeV}}$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ ($Z = 20$, $N = A - Z = 20$) is (using Equation 43.2 and masses from Table 43.2),

$$E_b = \left[20\left(1.007\,825\ \text{u}\right) + 20\left(1.008\,665\ \text{u}\right) - 39.962\,591\ \text{u}\right] \times\left(931.5\ \text{MeV/u}\right)$$
$$= \boxed{342\ \text{MeV}}$$

(c) $\boxed{\text{The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.}}$

Section 43.3 Nuclear Models

P43.11 The curve of binding energy shows that a heavy nucleus of mass number $A = 200$ has binding energy about

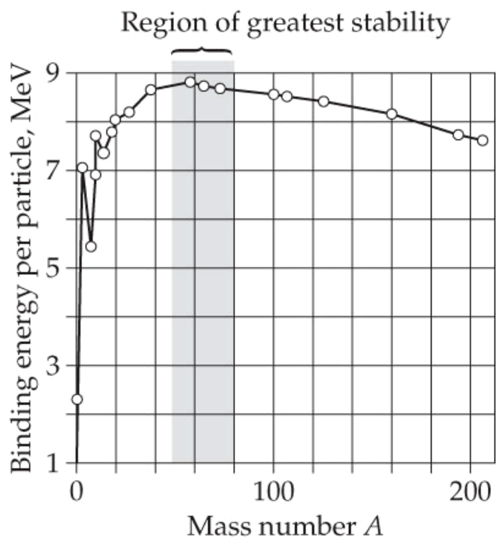
$$\left(7.8\frac{\text{MeV}}{\text{nucleon}}\right)(200\ \text{nucleons}) \approx 1.56\ \text{GeV}$$

Thus, it is less stable than its potential fission products, two middleweight nuclei of $A = 100$, together having binding energy

$$2(8.7\ \text{MeV/nucleon})(100\ \text{nucleons}) \approx 1.74\ \text{GeV}$$

Fission then releases about

$$1.74\ \text{GeV} - 1.56\ \text{GeV}\ \boxed{\sim 200\ \text{MeV}}$$



ANS. FIG. P43.11

P43.12 (a) $\boxed{\text{Nucleons on the surface have fewer neighbors with which to interact. The surface term is negative to reduce the estimate from the volume term, which assumes that all nucleons have the same number of neighbors.}}$

(b) The volume to surface ratio for a sphere of radius r is

$$\frac{\text{Volume}}{\text{Area}} = \frac{(4/3)\pi r^3}{4\pi r^2} = \boxed{\frac{1}{3}r}$$

The volume to surface ratio for a cube of side length L is

$$\frac{\text{Volume}}{\text{Area}} = \frac{L^3}{6L^2} = \boxed{\frac{1}{6}L}$$

$\boxed{\text{The sphere has a larger ratio to its characteristic length, so it would represent a larger binding energy and be more plausible for a nuclear shape.}}$

Section 43.4 Radioactivity

P43.13 According to Equation 43.7, the time dependence of the decay rate is $R = R_0 e^{-\lambda \Delta t}$. From this equation we can derive a relation between the change in decay rate over the time interval Δt to the decay constant. We start with $R = R_0 e^{-\lambda \Delta t}$. Then, rearranging and taking the natural log of both sides gives

$$e^{-\lambda \Delta t} = \frac{R}{R_0} \quad \rightarrow \quad \ln\left(e^{-\lambda \Delta t}\right) = \ln\left(\frac{R}{R_0}\right)$$

or

$$-\lambda \Delta t = \ln\left(\frac{R}{R_0}\right) = -\ln\left(\frac{R_0}{R}\right)$$

Solving,

$$\lambda = \frac{1}{\Delta t}\ln\left(\frac{R_0}{R}\right)$$

Now, because $\lambda = \dfrac{\ln 2}{T_{1/2}}$, we can relate the time interval Δt to the half-life:

$$\lambda = \frac{1}{\Delta t}\ln\left(\frac{R_0}{R}\right) \quad \rightarrow \quad \frac{\ln 2}{T_{1/2}} = \frac{1}{(\ln 2)\Delta t}\ln\left(\frac{R_0}{R}\right)$$

$$\frac{1}{T_{1/2}} = \frac{1}{(\ln 2)\Delta t}\ln\left(\frac{R_0}{R}\right)$$

$$T_{1/2} = \frac{(\ln 2)\Delta t}{\ln\left(R_0/R\right)}$$

***P43.14** **Conceptualize** ^{60}Co decays by beta and gamma processes. The gamma rays are used medically. As the sample sits on the shelf, nuclei are decaying, so that the activity of the sample decreases. At some point, there is not enough activity for the sample to be effective for fighting the cancer cells.

Categorize The sample undergoes radioactive decay according to the principles outlined in Section 43.4.

Analyze We first find the number of years that have elapsed since the sample was delivered:

$$N = (4\text{ yr})\left(\frac{365.25\text{ d}}{1\text{ yr}}\right) - 31\text{ d} = 1\,430\text{ d}\left(\frac{1\text{ yr}}{365.25\text{ d}}\right) = 3.915\text{ yr} \qquad (1)$$

Now use Equation 43.7 to find the fraction of the original nuclide still available:

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(\frac{\ln 2}{T_{1/2}}\right)t} = e^{-\left(\frac{\ln 2}{5.27\text{ yr}}\right)(3.915\text{ yr})} = 0.598 = 59.8\%$$

This is just under the threshold of 60.0%, so $\boxed{\text{it is probably best to discard this sample}}$.

Finalize In Equation (1), we recognized that the period included a leap year by indicating that each year has 365.25 days. We subtracted 31 days because the time interval began on January 31 and ended on December 31.]

Answer: Dispose of the sample.

P43.15 The number of nuclei that decay during the interval will be

$$\Delta N = N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$$

First we find the decay constant λ:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{64.8\text{ h}} = 0.010\ 7\text{ h}^{-1} = 2.97\times 10^{-6}\text{ s}^{-1}$$

Now we find N_0:

$$\begin{aligned} N_0 &= \frac{R_0}{\lambda} = \frac{(40.0\ \mu\text{Ci})}{2.97\times 10^{-6}\text{ s}^{-1}}\left(\frac{3.70\times 10^4\text{ s}^{-1}}{\mu\text{Ci}}\right) \\ &= 4.98\times 10^{11}\text{ nuclei} \end{aligned}$$

Substituting in these values,

$$N_1 - N_2 = \left(4.98\times 10^{11}\right)\left[e^{-(\ln 2/64.8\text{ h})(10.0\text{ h})} - e^{-(\ln 2/64.8\text{ h})(12.0\text{ h})}\right]$$

$$N_1 - N_2 = \boxed{9.47\times 10^9\text{ nuclei}}$$

P43.16 The number of nuclei that decay during the interval will be

$$N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$$

We wish to write this expression in terms of the half-life $T_{1/2}$ and the initial decay rate R_0. First, from the definition of λ, we have

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad \rightarrow \quad e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$$

Now we find N_0:

$$N_0 = \frac{R_0}{\lambda} = \frac{R_0 T_{1/2}}{\ln 2}$$

Substituting in these expressions, we find that

$$N_1 - N_2 = \frac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\frac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$$

P43.17 (a) From Equation 43.6, the fraction remaining at $t = 5.00$ yr will be

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-t \ln 2 / T_{1/2}} = e^{-(5.00\text{ yr})\ln 2/(12.33\text{ yr})} = \boxed{0.755}$$

(b) At $t = 10.0$ yr,

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-t \ln 2 / T_{1/2}} = e^{-(10.0\text{ yr})\ln 2/(12.33\text{ yr})} = \boxed{0.570}$$

(c) At $t = 123.3$ yr,

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-t \ln 2 / T_{1/2}} = e^{-(123.3\text{ yr})\ln 2/(12.33\text{ yr})} = e^{-10 \ln 2} = \boxed{9.766 \times 10^{-4}}$$

(d) No. The decay model depends on large numbers of nuclei. After some long but finite time, only one undecayed nucleus will remain. It is likely that the decay of this final nucleus will occur before infinite time.

P43.18 (a) $\frac{dN_2}{dt}$ = rate of change of N_2

= rate of production of N_2 – rate of decay of N_2

= rate of decay of N_1 – rate of decay of N_2

$= \lambda_1 N_1 - \lambda_2 N_2$

(b) From the trial solution,

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

$$\therefore \frac{dN_2}{dt} = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t}\right) \qquad \textbf{[1]}$$

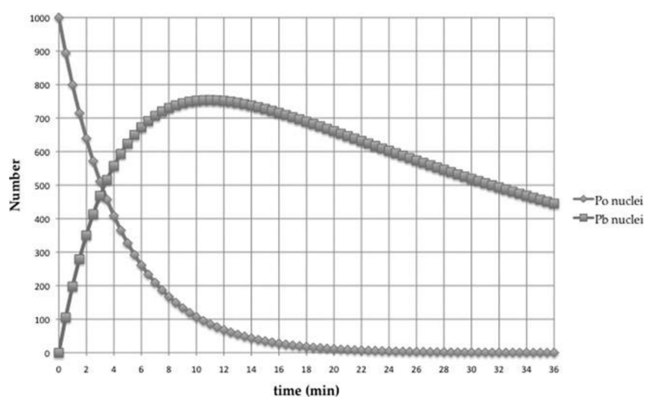
$$\frac{dN_2}{dt}+\lambda_2 N_2=\frac{N_{10}\lambda_1}{\lambda_1-\lambda_2}\left(\cancel{-\lambda_2 e^{-\lambda_2 t}}+\lambda_1 e^{-\lambda_1 t}\right)$$
$$+\frac{N_{10}\lambda_1}{\lambda_1-\lambda_2}\left(\cancel{\lambda_2 e^{-\lambda_2 t}}-\lambda_2 e^{-\lambda_1 t}\right)$$
$$=\frac{N_{10}\lambda_1}{\lambda_1-\lambda_2}(\lambda_1-\lambda_2)e^{-\lambda_1 t}=\lambda_1 N_1$$

So $\dfrac{dN_2}{dt}=\lambda_1 N_1-\lambda_2 N_2$ as required.

(c) The functions plotted in ANS. FIG. P43.18(c) are

Po nuclei: $N_1(t)=1\,000e^{-(\ln 2/3.10\text{ min})t}$

Pb nuclei: $N_2(t)=1\,130.8\left[e^{-(\ln 2/26.8\text{ min})t}-e^{-(\ln 2/3.10\text{ min})t}\right]$



ANS. FIG. P43.18(c)

(d) From the graph, $t_m \approx \boxed{10.9\text{ min}}$

(e) From equation [1], $\dfrac{dN_2}{dt}=0$ if

$$\lambda_2 e^{-\lambda_2 t}=\lambda_1 e^{-\lambda_1 t}$$

$$\therefore e^{(\lambda_1-\lambda_2)t}=\frac{\lambda_1}{\lambda_2}$$

Thus, $t=\boxed{t_m=\dfrac{\ln(\lambda_1/\lambda_2)}{\lambda_1-\lambda_2}}$.

(f) With $\lambda_1 = \ln 2/(3.10\text{ min})$, $\lambda_2 = \ln 2/(26.8\text{ min})$, this formula gives

$$t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$$

$$= \frac{\ln\left[\dfrac{\ln 2/(3.10\text{ min})}{\ln 2/(26.8\text{ min})}\right]}{\left(\dfrac{\ln 2}{3.10\text{ min}} - \dfrac{\ln 2}{26.8\text{ min}}\right)} = \frac{\ln\left(\dfrac{26.8\text{ min}}{3.10\text{ min}}\right)}{\ln 2\left(\dfrac{1}{3.10\text{ min}} - \dfrac{1}{26.8\text{ min}}\right)}$$

$$= \boxed{10.9\text{ min}}$$

This result is in agreement with the result of part (d).

Section 43.5 The Decay Processes

P43.19 Atomic masses are given in Table 43.2.

(a) For this e^+ decay,

$$Q = (M_X - M_Y - 2m_e)c^2$$
$$= [39.962\,591\text{ u} - 39.963\,999\text{ u} - 2(0.000\,549\text{ u})] \times (931.5\text{ MeV/u})$$
$$Q = -2.33\text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For this alpha decay,

$$Q = (M_X - M_Y - 2m_e)c^2$$
$$= [97.905\,287\text{ u} - 4.002\,603\text{ u} - 93.905\,088\text{ u}] \times (931.5\text{ MeV/u})$$
$$Q = -2.24\text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For this alpha decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2$$
$$= \left[143.910\,083\text{ u} - 4.002\,603\text{ u} - 139.905\,434\text{ u}\right] \times \left(931.5\text{ MeV/u}\right)$$
$$Q = 1.91\text{ MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

P43.20 Total Z and A are conserved.

(a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z = 0$ and $A = 0$. Keeping the total values of Z and A for the system conserved requires $Z = 28$ and $A = 65$ for X. With this atomic number it must be nickel, and the nucleus must be in an excited state, so X is $\boxed{{}^{65}_{28}\text{Ni}^*}$.

(b) An alpha particle, $\alpha = {}^4_2\text{He}$, has $Z = 2$ and $A = 4$. Total initial Z is 84, and total initial A is 215, so for X we require

$$Z = 84 = Z_X + 2 \quad \rightarrow \quad Z_X = 82 \quad \rightarrow \quad \text{Pb, and}$$

$$A = 215 = A_X + 4 \quad \rightarrow \quad A_X = 211, \quad \rightarrow \quad \text{X is } \boxed{{}^{211}_{82}\text{Pb}}.$$

(c) A positron, $e^+ = {}^0_1e$, has charge the same as a nucleus with $Z = 1$. A neutrino, ${}^0_0\nu$, has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z = 26 + 1 + 0 = 27$; so, X is Co. And $A = 55 + 0 + 0 = 55$: X is $\boxed{{}^{55}_{27}\text{Co}}$.

P43.21 (a) The reaction for one particle is $\boxed{e^- + p \rightarrow n + \nu}$.

(b) For nuclei, ${}^{15}_8\text{O} + e^- \rightarrow {}^{15}_7\text{N} + \nu$.

Add seven electrons to both sides to obtain

$$^{15}_{8}\text{O atom} \rightarrow {}^{15}_{7}\text{N atom} + \nu$$

From Table 44.2 of atomic masses,

$$\begin{aligned} Q &= (15.003\ 065\ \text{u} - 15.000\ 109\ \text{u})(931.5\ \text{MeV/u}) \\ &= \boxed{2.75\ \text{MeV}} \end{aligned}$$

P43.22 (a) The decay constant is λ = ln2/10 h = 0.0693/h. The number of parent nuclei is given by $N_P = N_{P,0}\,e^{-\lambda t} = \left(1.00\times 10^6\right)e^{-0.0693t}$, where t is in hours.

The number of daughter nuclei is equal to the number of missing parent nuclei,

$$\boxed{N_d = N_{P,0} - N_{P,0}\,e^{-\lambda t} = \left(1.00\times 10^6\right)\left(1 - e^{-0.0693t}\right), \text{ where } t \text{ is in hours.}}$$

(b) $\boxed{\text{The number of daughter nuclei starts from zero at } t = 0. \text{ The number of stable product nuclei always increases with time and asymptotically approaches } 1.00\times 10^6 \text{ as } t \text{ increases without limit.}}$

(c) $\boxed{\text{The minimum number of daughter nuclei is zero at } t = 0. \text{ The maximum number of daughter nuclei asymptotically approaches } 1.00\times 10^6 \text{ as } t \text{ increases without limit.}}$

(d) The rate of change is

$$\frac{dN_d}{dt} = \left(1.00\times 10^6\right)\left(0 + 0.0693\ e^{-0.0693t}\right) = 6.93\times 10^4 e^{-0.0693t}$$

where $\frac{dN_d}{dt}$ is in decays per hour and t is in hours. $\boxed{\text{The rate of change has its maximum value, } 6.93 \times 10^4 \text{ h}^{-1}\text{, at } t = 0\text{, after which the rate decreases more and more, approaching zero as } t \text{ increases without limit.}}$

P43.23 (a) The number of carbon atoms in the sample is

$$N_C = \left(\frac{0.021\ 0\text{ g}}{12.0\text{ g/mol}}\right)\left(\frac{6.02\times 10^{23}\text{ atoms}}{\text{mol}}\right) = \boxed{1.05\times 10^{21}}$$

(b) 1 in 7.70×10^{11} carbon atoms is a ^{14}C atom. Then,

$$(N_0)_{\text{C-14}} = 1.05\times 10^{21}\left(\frac{1}{7.70\times 10^{11}}\right) = \boxed{1.37\times 10^{9}}$$

(c) The decay constant for ^{14}C is

$$\lambda_{\text{C-14}} = \frac{\ln 2}{5\,730\text{ yr}} = 1.21\times 10^{-4}\text{ yr}^{-1}\left(\frac{1\text{ yr}}{3.16\times 10^{7}\text{ s}}\right)$$
$$= \boxed{3.83\times 10^{-12}\text{ s}^{-1}}$$

(d) We use $R = \lambda N = \lambda N_0 e^{-\lambda t}$. At $t = 0$,

$$R_0 = \lambda N_0 = \left(3.83\times 10^{-12}\text{ s}^{-1}\right)\left(1.37\times 10^{9}\right)\left[\frac{7(86\ 400\text{ s})}{1\text{ week}}\right]$$
$$= \boxed{3.17\times 10^{3}\text{ decays/week}}$$

(e) At time t, $R = \dfrac{837}{0.880} = \boxed{951\text{ decays/week}}$.

(f) Taking logarithms,

$$\ln\frac{R}{R_0} = -\lambda t \quad \text{so} \quad t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

and

$$t = \frac{-1}{1.21\times10^{-4}\ \text{yr}^{-1}}\ln\left(\frac{951}{3.17\times10^{3}}\right) = \boxed{9.95\times10^{3}\ \text{yr}}$$

Section 43.6 Natural Radioactivity

P43.24 The number of radon atoms remaining is

$$N = N_0 e^{-(\ln 2)t/T_{1/2}}$$

And the fraction remaining is

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$$

(a) With $T_{1/2} = 3.82$ d and $t = 7.00$ d,

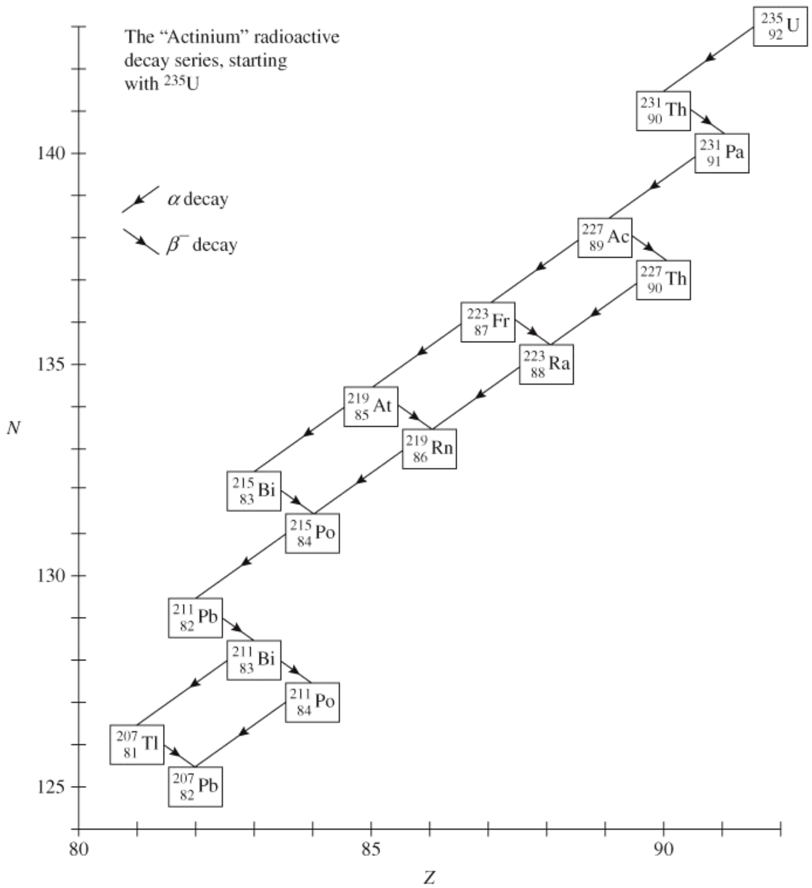
$$\frac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$$

(b) When $t = 1.00$ yr $= 365.25$ d,

$$\frac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$$

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

P43.25 We find the chemical name by looking up Z in a periodic table. The values in the shaded boxes (^{235}U and ^{207}Pb) in Figure P43.25 were given; all others have been filled in as part of the solution shown in ANS. FIG. P43.25 below.



ANS. FIG. P43.25

Section 43.7 Nuclear Reactions

P43.26 (a) $\boxed{{}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \to {}^{198}_{79}\text{Au}^{*} \to {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}}$

Note the conservation of baryon number (which you can think of as nucleon census number and call mass number in this chapter) in the superscripts: 197 + 1 = 198 + 0. Note the conservation of charge in the subscripts: 79 + 0 = 80 – 1.

(b) Consider adding 79 electrons:

$${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \to {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$$

Then,

$$Q = \left[M_{^{197}\text{Au}} + m_n - M_{^{198}\text{Hg}}\right]c^2$$
$$Q = [196.966\ 552\ \text{u} + 1.008\ 665\ \text{u} - 197.966\ 752\ \text{u}] \times (931.5\ \text{MeV/u})$$
$$= \boxed{7.89\ \text{MeV}}$$

P43.27 Total A and total Z are conserved.

(a) For X, $A = 24 + 1 - 4 = 21$ and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{{}^{21}_{10}\text{Ne}}$.

(b) $A = 235 + 1 - 90 - 2 = 144$ and $Z = 92 + 0 - 38 - 0 = 54$,

so X is $\boxed{{}^{144}_{54}\text{Xe}}$.

(c) $A = 2 - 2 = 0$ and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, it is accompanied by a neutrino:

$$X + X' = \boxed{{}^{0}_{1}\text{e}^{+} + \nu}$$

Section 43.8 Nuclear Fission

P43.28 Three different fission reactions are possible:

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \to {}^{90}_{38}\text{Sr} + {}^{144}_{54}\text{Xe} + 2\,{}^{1}_{0}\text{n} \qquad \boxed{{}^{144}_{54}\text{Xe}}$$

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \to {}^{90}_{38}\text{Sr} + {}^{143}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n} \qquad \boxed{{}^{143}_{54}\text{Xe}}$$

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \to {}^{90}_{38}\text{Sr} + {}^{142}_{54}\text{Xe} + 4\,{}^{1}_{0}\text{n} \qquad \boxed{{}^{142}_{54}\text{Xe}}$$

P43.29 First, the thorium is bombarded:

$${}^{1}_{0}\text{n} + {}^{232}_{90}\text{Th} \to {}^{233}_{90}\text{Th}$$

Then, the thorium decays by beta emission:

$$^{233}_{90}\text{Th} \rightarrow {}^{233}_{91}\text{Pa} + {}^{0}_{-1}\text{e} + \bar{\nu}$$

Protactinium-233 has more neutrons than the more stable protactinium-231, so it too decays by beta emission:

$$^{233}_{91}\text{Pa} \rightarrow {}^{233}_{92}\text{U} + {}^{0}_{-1}\text{e} + \bar{\nu}$$

Section 43.9 Nuclear Reactors

P43.30 (a) For a sphere: $V = \frac{4}{3}\pi r^3 \rightarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so

$$\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \frac{3}{r} = \left(\frac{36\pi}{V}\right)^{1/3} = \boxed{4.84V^{-1/3}}$$

(b) For a cube: $V = \ell^3 \rightarrow \ell = V^{1/3}$, so

$$\frac{A}{V} = \frac{6\ell^2}{\ell^3} = \frac{6}{\ell} = \boxed{6V^{-1/3}}$$

(c) For a parallelepiped: $V = 2a^3 \rightarrow a = \left(\frac{V}{2}\right)^{1/3}$, so

$$\frac{A}{V} = \frac{(2a^2 + 8a^2)}{2a^3} = \frac{5}{a} = 5\left(\frac{2}{V}\right)^{1/3} = \left(\frac{250}{V}\right)^{1/3} = \boxed{6.30V^{-1/3}}$$

(d) The answers show that the sphere has the smallest surface area for a given volume and the brick has the greatest surface area of the three. Therefore, $\boxed{\text{The sphere has minimum leakage and the parallelepiped has maximum leakage.}}$

P43.31 (a) Do not think of the "reserve" as being held in reserve. We are depleting it as fast as we choose. The remaining current balance of irreplaceable ^{235}U is 0.7% of the whole mass of uranium:

$$(0.007\ 00)\left(4.40\times10^{6}\text{ tons}\right)\left(\frac{10^{3}\text{ kg}}{1\text{ ton}}\right)\left(\frac{10^{3}\text{ g}}{1\text{ kg}}\right)=\boxed{3.08\times10^{10}\text{ g}}$$

(b) The number of moles of ^{235}U in the reserve is

$$n=\frac{m}{M}=\frac{3.08\times10^{10}\text{ g}}{235\text{ g/mole}}=\boxed{1.31\times10^{8}\text{ mole}}$$

(c) The number of moles found in part (b) corresponds to

$$N=nN_{\text{A}}=\left(1.31\times10^{8}\text{ mole}\right)\left(\frac{6.02\times10^{23}\text{ atom}}{1\text{ mole}}\right)\left(\frac{1\text{ nucleus}}{1\text{ atom}}\right)$$
$$=\boxed{7.89\times10^{31}\text{ nuclei}}$$

(d) We imagine each nucleus as fissioning, to release

$$\left(7.89\times10^{31}\text{ fissions}\right)\left(\frac{200\text{ MeV}}{1\text{ fission}}\right)\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)=\boxed{2.52\times10^{21}\text{ J}}$$

(e) The definition of power is represented by $P=(\text{energy converted})/\Delta t$, so we have

$$\Delta t=\frac{\text{energy}}{P}=\frac{2.52\times10^{21}\text{ J}}{1.5\times10^{13}\text{ J/s}}=\left(1.68\times10^{8}\text{ s}\right)\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right)$$
$$=\boxed{5.33\text{ yr}}$$

(f) Fission is not sufficient to supply the entire world with energy at a price of \$130 or less per kilogram of uranium.

P43.32 Assuming that the impossibility is *not* that he can have this control over the process (which, as far as we know presently, *is* impossible), let's see what else might be wrong. The reaction can be written

$$ {}_{0}^{1}\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{57}^{141}\text{La} + {}_{35}^{94}\text{Br} + n\left({}_{0}^{1}\text{n}\right) $$

where n is the number of neutrons released in the fission reaction. By balancing the equation for electric charge and number of nucleons, we find that $n = 1$. If one incoming neutron results in just one outgoing neutron, the possibility of a chain reaction is not there, so this nuclear reactor will not work.

P43.33 (a) Since $K = p^2/2m$, we have

$$ \begin{aligned} p = \sqrt{2mK} &= \sqrt{2m\left(\frac{3}{2}k_B T\right)} \\ &= \sqrt{3(1.675\times10^{-27}\text{ kg})(1.38\times10^{-23}\text{ J/K})\,(300\text{ K})} \\ &= \boxed{4.56\times10^{-24}\text{ kg}\cdot\text{m/s}} \end{aligned} $$

(b) The de Broglie wavelength of the particle is

$$ \lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4.56\times10^{-24}\text{ kg}\cdot\text{m/s}} = 1.45\times10^{-10}\text{ m} = \boxed{0.145\text{ nm}} $$

(c) This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

Section 43.10 Nuclear Fusion

***P43.34** **Conceptualize** The Sun has a mass of 1.99×10^{30} kg, from Table 13.2, and the total power output of 3.85×10^{26} W is easily found online.

Categorize This problem involves a calculation of the time interval knowing the power and the total energy.

Analyze (a) From the definition of power, find the time interval for a certain amount of energy to be transformed:

$$P = \frac{\Delta E}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{\Delta E}{P} \qquad (1)$$

If the Sun is formed entirely from gasoline, let's find the available energy:

$$\Delta E = \left(1.3 \times 10^{8}\ \text{J/ gal}\right)\left(\frac{1\ \text{gal}}{3.785\ \text{L}}\right)\left(\frac{1\ \text{L}}{10^{-3}\ \text{m}^{3}}\right)\left(\tfrac{4}{3}\pi R_S^{3}\right)$$

where R_S is the radius of the Sun. Using the value from Table 13.2, we have

$$\begin{aligned}\Delta E &= \left(1.3 \times 10^{8}\ \text{J/ gal}\right)\left(\frac{1\ \text{gal}}{3.785\ \text{L}}\right)\left(\frac{1\ \text{L}}{10^{-3}\ \text{m}^{3}}\right)\left(\tfrac{4}{3}\pi\right)\left(6.96 \times 10^{8}\ \text{m}\right)^{3} \\ &= 4.85 \times 10^{37}\ \text{J}\end{aligned}$$

Substitute numerical values into Equation (1):

$$\Delta t = \frac{4.85 \times 10^{37}\ \text{J}}{3.85 \times 10^{26}\ \text{W}} = 1.26 \times 10^{11}\ \text{s} = \boxed{4.0 \times 10^{3}\ \text{yr}}$$

(b) Now imagine that the Sun is made up entirely of protons. The effect of the fusion reaction is to combine four protons into a helium nucleus, so the energy released in this reaction is

$$\begin{aligned}Q &= \left[4M_p - M_{^4\text{He}}\right]c^2 \\ &= \left[4\left(1.007\ 825\ \text{u}\right) - 4.002\ 603\ \text{u}\right]\left(931.494\ \text{MeV/ u}\right) \\ &= 26.73\ \text{MeV}\left(\frac{1.602 \times 10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = 4.28 \times 10^{-12}\ \text{J}\end{aligned}$$

Now, the number of possible reactions is equal to the number of protons divided by 4, since we need four for each reaction:

$$N_{\text{reactions}} = \frac{N_{\text{protons}}}{4} = \frac{m}{4M_{\text{H}}} = \frac{1.99 \times 10^{30}\ \text{kg}}{4\left(1.67 \times 10^{-27}\ \text{kg}\right)} = 2.98 \times 10^{56}$$

Therefore, the total available energy output is

$$\Delta E = N_{\text{reactions}} Q = \left(2.98 \times 10^{56}\right)\left(4.28 \times 10^{-12}\ \text{J}\right) = 1.28 \times 10^{45}\ \text{J}$$

Substitute numerical values into Equation (1):

$$\Delta t = \frac{1.28 \times 10^{45}\ \text{J}}{3.85 \times 10^{26}\ \text{W}} = 3.3 \times 10^{18}\ \text{s} = \boxed{1.0 \times 10^{11}\ \text{yr}}$$

Finalize In part (a), the Sun only lasts for 4 000 years. We know that the Sun has been around longer than that! In part (b), we have a more reasonable estimate of 100 billion years, although this value is too high because only the central core of the Sun has the required temperatures and densities for nuclear fusion to occur. The current age of the Sun is estimated to be about 4.6 billion years, and we appear to be about halfway through its lifetime.]

Answers: (a) 4.0×10^3 yr (b) 1.0×10^{11} yr

P43.35 (a) The radius of a nucleus with mass number A is $r = aA^{1/3}$, where $a = 1.2$ fm. The distance of closest approach is equal to the center to center distance of the two nuclei:

$$\begin{aligned} r_f &= r_{\text{D}} + r_{\text{T}} = \left(1.20 \times 10^{-15}\ \text{m}\right)\left[(2)^{1/3} + (3)^{1/3}\right] \\ &= 3.24 \times 10^{-15}\ \text{m} = \boxed{3.24\ \text{fm}} \end{aligned}$$

(b) At this distance, the electric potential energy is

$$U_f = \frac{k_e e^2}{r_f} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{3.24\times10^{-15}\ \mathrm{m}}$$
$$= 7.10\times10^{-14}\ \mathrm{J} = \boxed{444\ \mathrm{keV}}$$

(c) Conserving momentum, $m_{\mathrm{D}}v_i = \left(m_{\mathrm{D}} + m_{\mathrm{T}}\right)v_f$ or

$$v_f = \left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)v_i = \boxed{\frac{2}{5}v_i}$$

(d) To find the minimum initial kinetic energy of the deuteron, we use $K_i + U_i = K_f + U_f$, where $U_i = 0$ because the deuteron starts from very far away (infinity), and with the result from part (c),

$$K_i + 0 = \frac{1}{2}\left(m_{\mathrm{D}} + m_{\mathrm{T}}\right)v_f^2 + U_f$$
$$K_i = \frac{1}{2}\left(m_{\mathrm{D}} + m_{\mathrm{T}}\right)\left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)^2 v_i^2 + U_f$$

With some re-arrangement, we have

$$K_i = \left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)\left(\frac{1}{2}m_{\mathrm{D}}v_i^2\right) + U_f = \left(\frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)K_i + U_f$$

or

$$\left(1 - \frac{m_{\mathrm{D}}}{m_{\mathrm{D}} + m_{\mathrm{T}}}\right)K_i = U_f$$

solving for the initial kinetic energy then gives

$$K_i = U_f\left(\frac{m_{\mathrm{D}} + m_{\mathrm{T}}}{m_{\mathrm{T}}}\right) = \frac{5}{3}\left(444\ \mathrm{keV}\right) = \boxed{740\ \mathrm{keV}}$$

(e) The nuclei can fuse $\boxed{\text{possibly by tunneling}}$ through the potential energy barrier.

P43.36 (a) We assume that the nuclei are stationary at closest approach, so that the electrostatic potential energy equals the total energy E. Then, from the isolated system model,

$$K_f + U_f = K_i + U_i \quad \rightarrow \quad U_f = E$$

then,

$$\frac{k_e(Z_1 e)(Z_2 e)}{r_{\min}} = E$$

$$E = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2 Z_1 Z_2}{1.00 \times 10^{-14} \text{ m}}\left(\frac{1 \text{ keV}}{1.60 \times 10^{-16} \text{ J}}\right)$$
$$= (144 \text{ keV})Z_1 Z_2$$

or $\boxed{E = 144 Z_1 Z_2 \text{ where } E \text{ is in keV.}}$

(b) $\boxed{\text{The energy is proportional to each atomic number.}}$

(c) $\boxed{\text{Take } Z_1 = 1 \text{ and } Z_2 = 59 \text{ or vice versa. This choice minimizes the product } Z_1 Z_2.}$ If extra cleverness is allowed, take $Z_1 = 0$ and $Z_2 = 60$: use neutrons as the bombarding particles. A neutron is a nucleon but not an atomic nucleus.

(d) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = (2.30 \times 10^{-14} \text{ J})\left(\frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}}\right)$$
$$= \boxed{144 \text{ keV for both, according to this model.}}$$

Section 43.10 in the text gives more accurate values for the critical ignition temperatures, of about 52 keV for D-D fusion and 6 keV for D-T fusion. The nuclei can fuse by tunneling. A triton moves more slowly than a deuteron at a given temperature. Then D-T collisions last longer than D-D collisions and have much greater tunneling probabilities.

P43.37 (a) Taking $m \approx 2m_p$ for deuterons, we have

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$$

The root-mean-square speed is

$$v_{rms} = \sqrt{\frac{3k_B T}{2m_p}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\text{ J/K}\right)\left(4.00 \times 10^{8}\text{ K}\right)}{2\left(1.67 \times 10^{-27}\text{ kg}\right)}}$$
$$= \boxed{2.23 \times 10^{6}\text{ m/s}}$$

(b) The confinement time in the absence of confinement measures is

$$\Delta t = \frac{x}{v} = \frac{0.100\text{ m}}{2.23 \times 10^{6}\text{ m/s}} \sim 10^{-7}\text{ s}$$

P43.38 (a) By adding $1 + 6 = 7$ and $1 + 12 = 13$, we have ${}^{1}_{1}\text{H} + {}^{12}_{6}\text{C} \to {}^{13}_{7}\text{N} + \gamma$ so nucleus A is $\boxed{{}^{13}\text{N}.}$

(b) Now $13 - 0 = 13$ and $7 - 1 = 6$, so the positron decay is ${}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + {}^{0}_{1}\text{e} + \nu$ and nucleus B is $\boxed{{}^{13}\text{C}.}$

(c) Similarly, we have ${}^{1}_{1}\text{H} + {}^{13}_{6}\text{C} \to {}^{14}_{7}\text{N} + \gamma$ and nucleus C is $\boxed{{}^{14}\text{N}.}$

(d) The hydrogen nuclei keep piling on like rugby players after a tackle. We have ${}^{1}_{1}\text{H} + {}^{14}_{7}\text{N} \to {}^{15}_{8}\text{O} + \gamma$ and nucleus D is $\boxed{{}^{15}\text{O}.}$

(e) Now ${}^{15}_{8}\text{O} \to {}^{15}_{7}\text{N} + {}^{0}_{1}\text{e} + \nu$, so nucleus E is $\boxed{{}^{15}\text{N}.}$

(f) We calculate $15 + 1 - 4 = 12$ and $7 + 1 - 2 = 6$ to identify ${}^{1}_{1}\text{H} + {}^{15}_{7}\text{N} \to {}^{12}_{6}\text{C} + {}^{4}_{2}\text{He}$ and nucleus F is $\boxed{{}^{12}\text{C}.}$

(g) $\boxed{\text{The original carbon-12 nucleus is returned.}}$ One carbon nucleus can participate in the fusions of colossal numbers of hydrogen nuclei, four after four. Carbon is a catalyst.

The two positrons immediately annihilate with electrons according to ${}^{0}_{1}\text{e} + {}^{0}_{-1}\text{e} \rightarrow 2\gamma$. The overall reaction, obtained by adding all eight reactions, can be represented as

$$ {}^{1}_{1}\text{H} + {}^{12}_{6}\text{C} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + 2\,{}^{0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He} + {}^{12}_{6}\text{C} + 7\gamma + 2\nu $$

This simplifies to $\boxed{4\left({}^{1}_{1}\text{H}\right) + 2\,{}^{0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He} + 2\nu}$. The net reaction is identical to the net reaction in the proton–proton cycle which predominates in the Sun. In energy terms the reaction can be considered as $4\left({}^{1}_{1}\text{H atom}\right) \rightarrow {}^{4}_{2}\text{He atom} + 26.7\text{ MeV}$, where the Q value of energy output was computed in Chapter 38.

Section 43.11 Biological Radiation Damage

P43.39 (a) The number of x-ray images made per year is (assuming a 2-week vacation)

$$ n = (8\ \text{x-ray/d})(5\ \text{d/wk})(50\ \text{wk/yr}) = 2.0 \times 10^{3}\ \text{x-ray/yr} $$

The average dose per photograph is

$$ \frac{5.0\ \text{rem/yr}}{2.0 \times 10^{3}\ \text{x-ray/yr}} = 2.5 \times 10^{-3}\ \text{rem/x-ray} = \boxed{2.5\ \text{mrem/x-ray}} $$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The ration dose of 5.0 rem/yr received as a result of the job to background is

$$\frac{5.0\ \text{rem/yr}}{0.13\ \text{rem/yr}} = 38$$

$\boxed{\text{The technician's occupational exposure is high compared to background radiation—it is 38 times 0.13 rem/yr.}}$

P43.40 Assume all the energy from the x-ray machine is absorbed by the water and that no energy leaves the cup of water by heat or thermal radiation. The energy input to the cup and the temperature of the water are related by

$$T_{ER} = mc\Delta T$$

Because the power input P is equal to $T_{ER}/\Delta T$, we have

$$P\Delta t = mc\Delta T \quad \to \quad \Delta t = \frac{mc\Delta T}{P}$$

where we have solved for the time interval required to raise the temperature of the water. We note that the temperature of the water will increase until it is 100°C, after which the latent heat of vaporization of $L_v = 2.26 \times 10^6$ J/kg would have to be added to boil the water. For the purposes of this problem, we limit ourselves to increasing the temperature of the water to 100°C. Substituting numerical values gives

$$\Delta t = \frac{m(4\ 186\ \text{J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C})}{(10.0\ \text{rad/s})(1 \times 10^{-2}\ \text{J/kg})m} = 2.09 \times 10^6\ \text{s} = 24.2\ \text{d}$$

Therefore, it would take over 24 days just to increase the water's temperature to 100°C, and much longer to boil it, and this technique will not work for a 20-minute coffee break!

P43.41 The nuclei initially absorbed are (mass from Table 43.2)

$$N_0 = \left(1.00\times10^{-9}\ \text{g}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei/mol}}{89.9\ \text{g/mol}}\right) = 6.70\times10^{12}$$

The number of decays in time t is

$$\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$$

At the end of 1 year,

$$\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left\{1-\exp\left[\left(\frac{-\ln 2}{29.1\ \text{yr}}\right)1.00\ \text{yr}\right]\right\}$$
$$= 1.58\times10^{11}$$

The energy deposited is

$$E = \left(1.58\times10^{11}\right)(1.10\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 0.027\,7\ \text{J}$$

Thus, the dose received is

$$\text{Dose} = \left(\frac{0.027\,7\ \text{J}}{70.0\ \text{kg}}\right) = \boxed{3.96\times10^{-4}\ \text{J/kg}} = 0.039\,6\ \text{rad}$$

Section 43.12 Uses of Radiation from the Nucleus

P43.42 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \quad \text{so} \quad dN = (R - \lambda N)dt.$$

The variables are separable.

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt: \quad -\frac{1}{\lambda}\ln\left(\frac{R - \lambda N}{R}\right) = t$$

so $\ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t$ and $\left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t}$.

Therefore $1 - \frac{\lambda}{R}N = e^{-\lambda t} \quad \rightarrow \quad N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$.

(b) The maximum number of radioactive nuclei would be $\boxed{\frac{R}{\lambda}}$.

P43.43 (a) The number of photons is $\frac{10^4 \text{ MeV}}{1.04 \text{ MeV}} = 9.62 \times 10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4}N_0 = 1.92 \times 10^4$ and $N_0 = 2.56 \times 10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is 2.56×10^6 $\boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{65}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is $N_{63} = 0.691\,7\, N_{Cu}$ and $N_{65} = 0.308\,3\, N_{Cu}$. Therefore,

$$\frac{N_{63}}{N_{65}} = \frac{0.691\,7}{0.308\,3}$$

or $N_{63} = \left(\frac{0.6917}{0.3083}\right)N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56 \times 10^6\right) = 5.75 \times 10^6$

The total mass of copper present is then

$$m_{\text{Cu}} = (62.93\text{ u})N_{63} + (64.93\text{ u})N_{65}$$

$$m_{\text{Cu}} = \left[(62.93\text{ u})(5.75\times10^{6}) + (64.93\text{ u})(2.56\times10^{6})\right] \times (1.66\times10^{-24}\text{ g/u})$$

$$= 8.77\times10^{-16}\text{ g} \quad \boxed{\sim 10^{-15}\text{ g}}$$

Section 43.13 Nuclear Magnetic Resonance and Magnetic Resonance Imaging

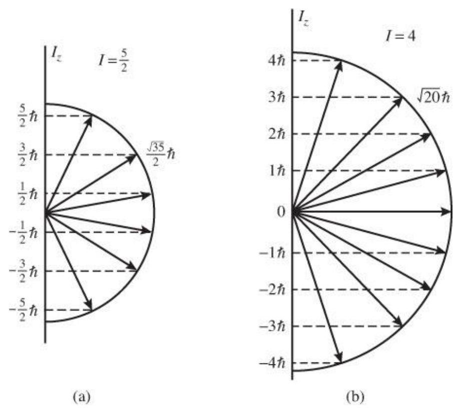
P43.44 It is the quantum particle under boundary conditions model that is behind the general rules: With angular momentum quantum number I, the magnitude of the angular momentum must be $\sqrt{I(I+1)}\,\hbar$. Whether I is an integer or a half-integer, the allowed values for one component of angular momentum being measured range from $+I\hbar$ to $+(I-1)\hbar$ to … to $-I\hbar$. Conditions that the wave function for a quantum particle must satisfy, for self-consistency under rotations in three-dimensional space, impose these requirements. We call a component being measured the z component. It can be measured more directly, as in a nuclear magnetic resonance experiment, or less directly, as from the way the angular momentum influences the intrinsic energy levels of a system and the number of available states within an energy level.

(a) With $I = 5/2$, the magnitude of the angular momentum is

$$\sqrt{I(I+1)}\,\hbar = \sqrt{\tfrac{5}{2}\left(\tfrac{5}{2}+1\right)}\,\hbar = \sqrt{35}\,\hbar/2$$
$$= 2.958\ 04(6.626\times10^{-34}\text{ J}\cdot\text{s})/2\pi$$
$$= 3.119\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$$

The z component can take the values $+5\hbar/2$, $+3\hbar/2$, $+\hbar/2$, $-\hbar/2$, $-3\hbar/2$, and $-5\hbar/2$. These identifications are shown in ANS. FIG. P43.44(a).

(b) Similarly, with $I = 4$, the magnitude of the angular momentum of a nucleus is $\sqrt{I(I+1)}\ \hbar = \sqrt{4(4+1)}\ \hbar = \sqrt{20}\ \hbar$ and its z component must have one of the nine values $+4\hbar$, $+3\hbar$, $+2\hbar$, $+\hbar$, 0, $-\hbar$, $-2\hbar$, $-3\hbar$, $-4\hbar$, as shown in ANS. FIG. P43.44(b).



ANS. FIG. P43.44

Additional Problems

P43.45 (a) $\boxed{\text{The process cannot occur because energy input would be required}}$. Note that the mass of the proton is less than the sum of the masses of the neutron and positron (electron):

$$\begin{aligned} &m_n + m_{e^+} > m_p \\ &1.008\ 665\ \text{u} + 0.000\ 549\ \text{u} \\ &1.009\ 214\ \text{u} > 1.007\ 276\ \text{u} \end{aligned}$$

Therefore, the reaction $\text{p} \to \text{n} + \text{e}^+ + \nu$ would violate the law of conservation of energy.

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the parent nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei $^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + \text{e}^{+} + \nu$ to obtain the reaction for neutral atoms $^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + \text{e}^{+} + \text{e}^{-} + \nu$.

$$Q = \left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{\text{e}^{+}} - m_{\text{e}^{-}} - m_{\nu}\right]c^2$$
$$Q = \left[13.005\,739\ \text{u} - 13.003\,355\ \text{u} - 2\left(5.49 \times 10^{-4}\ \text{u}\right) - 0\right] \times \left(931.5\ \text{MeV/u}\right)$$
$$= \boxed{1.20\ \text{MeV}}$$

P43.46 The proposed reaction can be written as

$$^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \quad \to \quad {}^{1}_{1}\text{H} + {}^{12}_{6}\text{C}$$

While electric charge is conserved (5 + 2 = 1 + 6), the number of nucleons is not (10 + 4 ≠ 1 + 12). Therefore, this reaction cannot occur.

P43.47 (a) The energy released by the $^{1}_{1}\text{H} + {}^{11}_{5}\text{B} \to 3\left({}^{4}_{2}\text{He}\right)$ reaction is

$$Q = \left[M_{^1_1\text{H}} + M_{^{11}_{5}\text{B}} - 3M_{^4_2\text{He}}\right]c^2$$

$$Q = \left[1.007\,825\ \text{u} + 11.009\,305\ \text{u} - 3\left(4.002\,603\ \text{u}\right)\right] \times \left(931.5\ \text{MeV/u}\right)$$
$$= \boxed{8.68\ \text{MeV}}$$

(b) $\boxed{\text{The particles must have enough kinetic energy to overcome their mutual electrostatic repulsion so that they can get close enough to fuse.}}$

P43.48 We check the Q value of this reaction:

$$Q = [238.050\ 788\ \text{u} - 237.051\ 144\ \text{u} - 1.007\ 825\ \text{u}] \times (931.5\ \text{MeV/u})$$
$$= -7.62\ \text{MeV}$$

$\boxed{\text{The Q value of this hypothetical decay is calculated to be } -7.62 \text{ MeV, which means you would have to add this much energy to the } ^{238}\text{U nucleus to make it emit a proton.}}$

P43.49 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy for the nucleus-photon system gives

$$\Delta E = hf + E_r \qquad \textbf{[1]}$$

where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \qquad \textbf{[2]}$$

Since system momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \qquad \textbf{[3]}$$

Hence, E_r can be expressed as $E_r = \dfrac{(hf)^2}{2Mc^2}$.

When $hf << Mc^2$, we can make the approximation that $hf \approx \Delta E$,

so $E_r \approx \dfrac{(\Delta E)^2}{2Mc^2}$.

(b) $E_r = \frac{(\Delta E)^2}{2Mc^2}$, where $\Delta E = 0.014\ 4\ \text{MeV}$

and $Mc^2 = (57\ \text{u})(931.5\ \text{MeV/u}) = 5.31\times 10^4\ \text{MeV}$.

Therefore,

$$E_r = \frac{\left(1.44\times 10^{-2}\ \text{MeV}\right)^2}{2\left(5.31\times 10^4\ \text{MeV}\right)} = 1.95\times 10^{-9}\ \text{MeV} = \boxed{1.95\times 10^{-3}\ \text{eV}}$$

P43.50 (a) If we assume all the ^{87}Sr came from ^{87}Rb, then $N = N_0 e^{-\lambda t}$ yields

$$t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right)$$

where $N = N_{\text{Rb-87}}$

and $N_0 = N_{\text{Sr-87}} + N_{\text{Rb-87}}$.

$$t = \frac{\left(4.75\times 10^{10}\ \text{yr}\right)}{\ln 2}\ln\left(\frac{1.82\times 10^{10} + 1.07\times 10^{9}}{1.82\times 10^{10}}\right) = \boxed{3.91\times 10^{9}\ \text{yr}}$$

(b) It could be $\boxed{\text{no older.}}$ The rock could be younger if some ^{87}Sr were originally present. We must make some assumption about the original quantity of radioactive material. In part (a) we assumed that the rock originally contained no strontium.

P43.51 (a) For the electron capture,

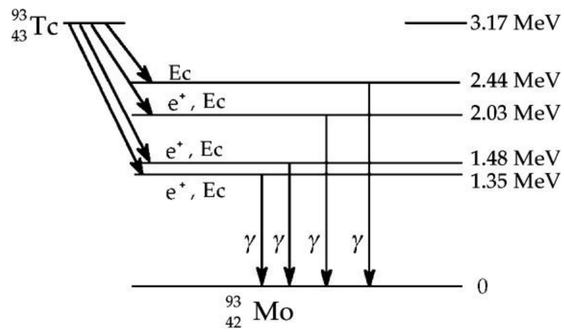
$$^{93}_{43}\text{Tc} + {}^{0}_{-1}\text{e} \rightarrow {}^{93}_{42}\text{Mo} + \gamma$$

For positron emission,

$$^{93}_{43}\text{Tc} \rightarrow {}^{93}_{42}\text{Mo} + {}^{0}_{+1}\text{e} + \gamma$$

The daughter nucleus in both forms of decay is $\boxed{^{93}_{42}\text{Mo}}$.

(b) We usually calculate the Q value under the assumption that the daughter nucleus is in its ground state, but for these decays, the Q value gives the upper limit of energy available to the daughter nucleus to be above its ground state.



ANS. FIG. P43.51

For electron capture, the disintegration energy is

$$Q = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}} \right] c^2$$

$$Q = [92.910\,2\text{ u} - 92.906\,8\text{ u}](931.5\text{ MeV/u})$$
$$= 3.17\text{ MeV} > 2.44\text{ MeV}$$

so $\boxed{\text{electron capture}}$ provides enough energy for $^{93}_{42}\text{Mo}$ to be in $\boxed{\text{all levels}}$ above its ground state.

For e^+ emission, the disintegration energy is

$$Q' = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}} - 2m_e \right] c^2.$$

$$Q' = [92.910\ 2\text{ u} - 92.906\ 8\text{ u} - 2(0.000\ 549\text{ u})](931.5\text{ MeV/u})$$
$$= 2.14\text{ MeV}$$

so $\boxed{e^+\text{ emission}}$ does not supply enough energy for $^{93}_{42}\text{Mo}$ to be in the 4.22 MeV state, $\boxed{\text{only 1.35 MeV, 1.48 MeV, and 1.35 MeV}}$ above ground (see ANS. FIG. P43.51).

P43.52 We check the Q value of the ^{57}Co nuclei decay by e^+:

$$^{57}_{27}\text{Co} \to {}^{57}_{26}\text{Fe} + {}^{0}_{+1}\text{e} + \bar{\nu}$$

Mass values appear in Table 43.2. For this reaction,

$$\begin{aligned} Q &= \left[56.936\,291 - 56.935\,394 - 2(0.000\,549)\right]\text{u}(931.5\ \text{MeV/u}) \\ &= -0.187\ \text{MeV} \end{aligned}$$

$\boxed{\text{The nucleus } ^{57}\text{Co cannot decay by } e^+ \text{ emission because the } Q \text{ value is } -0.187\ \text{MeV.}}$

P43.53 (a) With m_n and v_n as the mass and speed of the neutrons, Equation 9.24 for elastic collisions becomes for the two collisions, after making appropriate notational changes,

$$v_1 = \left(\frac{2m_n}{m_n + m_1}\right)v_n, \text{ and } v_2 = \left(\frac{2m_n}{m_n + m_2}\right)v_n$$

Solving,

$$(m_n + m_2)v_2 = (m_n + m_1)v_1 = 2m_n v_n$$

$$m_n(v_2 - v_1) = m_1 v_1 - m_2 v_2 \qquad \to \qquad m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b) We obtain the neutron mass from

$$m_n = \frac{(1\ \text{u})(3.30\times10^7\ \text{m/s}) - (14\ \text{u})(4.70\times10^6\ \text{m/s})}{4.70\times10^6\ \text{m/s} - 3.30\times10^7\ \text{m/s}} = \boxed{1.16\ \text{u}}$$

P43.54 (a) We treat the collision of the two particles a and X as a perfectly inelastic collision: the kinetic energy that is converted into internal energy supplies the missing energy Q, permitting the conversion of the particles into Y and b.

Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation for the projectile-target system:

$$M_\mathrm{a} v_\mathrm{a} = \left(M_\mathrm{a} + M_\mathrm{X}\right) v_\mathrm{c}$$

The initial energy is

$$E_i = \frac{1}{2} M_\mathrm{a} v_\mathrm{a}^2$$

The final kinetic energy is:

$$\begin{aligned} E_f &= \frac{1}{2}\left(M_\mathrm{a} + M_\mathrm{X}\right) v_c^2 = \frac{1}{2}\left(M_\mathrm{a} + M_\mathrm{X}\right)\left[\frac{M_\mathrm{a} v_\mathrm{a}}{M_\mathrm{a} + M_\mathrm{X}}\right]^2 \\ &= \left[\frac{M_\mathrm{a}}{M_\mathrm{a} + M_\mathrm{X}}\right] E_i \end{aligned}$$

From this, we see that E_f is always less than E_i and the change in energy, $E_f - E_i$, is given by

$$E_f - E_i = \left[\frac{M_\mathrm{a}}{M_\mathrm{a} + M_\mathrm{X}} - 1\right] E_i = -\left[\frac{M_\mathrm{X}}{M_\mathrm{a} + M_\mathrm{X}}\right] E_i$$

This loss of kinetic energy in the isolated system corresponds to an increase in mass-energy during the reaction. Thus, the absolute value of this kinetic energy change is equal to $-Q$ (remember that Q is negative in an endothermic reaction). The initial kinetic energy E_i is the threshold energy E_th. Therefore,

$$-Q = \left[\frac{M_\mathrm{X}}{M_\mathrm{a} + M_\mathrm{X}}\right] E_\mathrm{th}$$

or $$E_{\text{th}} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -Q\left[1 + \frac{M_a}{M_X}\right].$$

(b) We first calculate the Q value for the reaction:

$$Q = \left[M_{\text{N-14}} + M_{\text{He-4}} - M_{\text{O-17}} - M_{\text{H-1}}\right]c^2$$

$$Q = [14.003\ 074\ \text{u} + 4.002\ 603\ \text{u} - 16.999\ 132\ \text{u} - 1.007\ 825\ \text{u}] \times (931.5\ \text{MeV/u})$$

$$= -1.19\ \text{MeV}$$

Then,

$$E_{\text{th}} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\ 603\ \text{u}}{14.003\ 074\ \text{u}}\right]$$
$$= \boxed{1.53\ \text{MeV}}$$

P43.55 We have the following information: $N_X(0) = 2.50N_Y(0)$, $N_X(3\ \text{d}) = 4.20N_Y(3\ \text{d})$, and $T_{1/2\,Y} = 1.60\ \text{d}$. The nuclei decay exponentially:

$$N_X(3\ \text{d}) = 4.20N_Y(3\ \text{d})$$
$$N_X(0)e^{-\lambda_X(3\ \text{d})} = 4.20N_Y(0)e^{-\lambda_Y(3\ \text{d})} = 4.20\frac{N_X(0)}{2.50}e^{-\lambda_Y(3\ \text{d})}$$
$$e^{(3\ \text{d})\lambda_X} = \frac{2.5}{4.2}e^{(3\ \text{d})\lambda_Y}$$

Taking the natural logarithm of both sides,

$$(3\ \text{d})\lambda_X = \ln\left(\frac{2.5}{4.2}\right) + (3\ \text{d})\lambda_Y$$
$$(3\ \text{d})\frac{0.693}{T_{1/2\,X}} = \ln\left(\frac{2.5}{4.2}\right) + (3\ \text{d})\frac{0.693}{1.60\ \text{d}} = 0.781$$

The half-life of X is $T_{1/2\,X} = \boxed{2.66\ \text{d}}$

P43.56 We have the following information: $\dfrac{N_X(0)}{N_Y(0)} = r_1$, $\dfrac{N_X(\Delta t)}{N_Y(\Delta t)} = r_2$, and $T_{1/2\,Y} = T_Y$. The nuclei decay exponentially:

$$N_X(\Delta t) = r_2 N_Y(\Delta t)$$

$$N_X(0)e^{-\lambda_X \Delta t} = r_2 N_Y(0)e^{-\lambda_Y \Delta t} = \left(\frac{r_2}{r_1}\right)N_X(0)e^{-\lambda_Y \Delta t}$$

$$e^{-\Delta t \lambda_X} = \frac{r_2}{r_1}e^{-\Delta t \lambda_Y}$$

Taking the natural logarithm of both sides,

$$-\Delta t \lambda_X = \ln\left(\frac{r_2}{r_1}\right) - \Delta t \lambda_Y$$

$$\Delta t \frac{\ln 2}{T_X} = -\ln\left(\frac{r_2}{r_1}\right) + \Delta t \frac{\ln 2}{T_Y} = \ln\left(\frac{r_1}{r_2}\right) + \Delta t \frac{\ln 2}{T_Y}$$

$$\frac{1}{T_X} = \frac{\ln(r_1/r_2)}{\Delta t \ln 2} + \frac{1}{T_Y} = \frac{T_Y \ln(r_1/r_2) + \Delta t \ln 2}{T_Y \Delta t \ln 2} = \frac{\ln\left[2(r_1/r_2)^{T_Y/\Delta t}\right]}{T_Y \ln 2}$$

The half-life of X is $T_X = \boxed{\dfrac{T_Y \ln 2}{\ln\left[2(r_1/r_2)^{T_Y/\Delta t}\right]}}$

P43.57 (a) Subtracting the background counts, the decay counts are $N_1 = 372 - 5(15) = 297$ in the first 5.00 min interval and $N_2 = 337 - 5(15) = 262$ in the second. The midpoints of the time intervals are separated by $T = 5.00$ min. We use $R = R_0 e^{-\lambda t}$, taking $t = T$ and identifying $R_0 = N_1/T = 297/5$ min and $R = N_2/T = 262/5$ min. We have then

$$\frac{N_2}{T} = \left(\frac{N_1}{T}\right)e^{-\lambda T} \quad \text{or} \quad \frac{262}{5\text{ min}} = \left(\frac{297}{5\text{ min}}\right)e^{-(\ln 2/T_{1/2})(5.00\text{ min})}$$

which gives

$$e^{-(\ln 2/T_{1/2})T} = \frac{N_2}{N_1} \quad \text{or} \quad e^{-(\ln 2/T_{1/2})(5.00\ \text{min})} = \frac{262}{297}$$

Solving,

$$-\frac{\ln 2}{T_{1/2}}T = \ln\left(\frac{N_2}{N_1}\right) \quad \text{or} \quad -\frac{\ln 2}{T_{1/2}}T = \ln\left(\frac{262}{297}\right)$$

The half-life is then

$$T_{1/2} = \frac{-\ln 2}{\ln(N_2/N_1)}T = \frac{-\ln 2}{\ln(262/297)}(5.00\ \text{min}) = \boxed{27.6\ \text{min}}$$

NOTE: If it seems questionable to set instantaneous decay rates equal to average decay rates, to let $R_0 = N_1/T$ and $R = N_2/T$, see the Alternate Solution to (a) below. The results are the same.

(b) The average count rate is about

$$\frac{1}{2}\left(\frac{262}{5\ \text{min}} + \frac{297}{5\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \sim 1\ \text{s}^{-1}$$

but the counts are randomly spaced in time, meaning some counts near the beginning and end of each 5.00-min interval should or should not have been counted. Let's assume that the count incidence could vary by as much as 5 seconds, so we shall assume a count uncertainty of ±5. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right) = -\frac{\ln 2}{T_{1/2}}(5.00\ \text{min}),\ \text{giving}\ (T_{1/2})_{\text{min}} = 21.5\ \text{min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right)=-\frac{\ln 2}{T_{1/2}}(5.00\ \text{min}),\ \text{yielding}\ \left(T_{1/2}\right)_{\max}=38.7\ \text{min}$$

Thus, the half-life is about

$$T_{1/2}=\left(\frac{38.5+21.7}{2}\right)\pm\left(\frac{38.5-21.7}{2}\right)\ \text{min}$$
$$=(30\pm 8)\ \text{min}=\boxed{30\ \text{min}\pm 27\%}$$

<u>Alternate Solution to (a)</u> The amount of the radioactive sample at time t is $N=N_0e^{-\lambda t}$, where we do not know N_0. The number of decay counts between $t = 0$ and $t = T$ are

$$N_1=N_0\left(1-e^{-\lambda T}\right)=297$$

and the number of decay counts between $t = 0$ and $t = 2T$ are

$$N_1+N_2=N_0\left(1-e^{-\lambda 2T}\right)=297+262=559$$

To eliminate N_0, we consider the ratio of the counts:

$$r=\frac{N_1+N_2}{N_1}=\frac{N_0\left(1-e^{-\lambda 2T}\right)}{N_0\left(1-e^{-\lambda T}\right)}=\frac{559}{297}$$
$$r=\frac{\left(1-e^{-\lambda 2T}\right)}{\left(1-e^{-\lambda T}\right)}=\frac{\left(1-e^{-\lambda T}\right)\left(1+e^{-\lambda T}\right)}{\left(1-e^{-\lambda T}\right)}=1+e^{-\lambda T}$$

solving,

$$e^{-\lambda T}=r-1=\frac{N_1+N_2}{N_1}-1=\frac{N_2}{N_1}\quad\rightarrow\quad e^{-\left(\ln 2/T_{1/2}\right)T}=\frac{N_2}{N_1}$$

which leads to the same result as above, $T_{1/2}=\dfrac{-\ln 2}{\ln\left(N_2/N_1\right)}T$.

P43.58 (a) To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1v_1 = m_2v_2 \quad \text{or} \quad v_2 = \left(\frac{m_1}{m_2}\right)v_1$$

The kinetic energies after the break-up are then

$$K_1 = \frac{1}{2}m_1v_1^2 \quad \text{and} \quad K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right)K_1$$

The fraction of the total kinetic energy carried off by m_1 is

$$\frac{K_1}{K_{\text{tot}}} = \frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + (m_1/m_2)K_1} = \frac{m_2}{m_1 + m_2}$$

and the fraction carried off by m_2 is

$$\frac{K_2}{K_{\text{tot}}} = 1 - \frac{K_1}{K_{\text{tot}}} = 1 - \frac{m_2}{m_1 + m_2} = \frac{m_1}{m_1 + m_2}$$

(b) The disintegration energy is

$$Q = (236.045\,562\text{ u} - 86.920\,711\text{ u} - 148.934\,370\text{ u}) \times (931.5\text{ MeV/u})$$

$$= 177.4\text{ MeV} = \boxed{177\text{ MeV}}$$

(c) Immediately after fission, this Q-value is the total kinetic energy of the fission products. From part (a),

$$\frac{K_1}{K_{\text{tot}}} = \frac{m_2}{m_1 + m_2} = \frac{K_{\text{Br}}}{Q}$$

Then,

$$K_{\text{Br}} = Q\frac{m_{\text{La}}}{m_{\text{Br}} + m_{\text{La}}} = (177.4\text{ MeV})\left(\frac{149\text{ u}}{87\text{ u} + 149\text{ u}}\right) = \boxed{112.0\text{ MeV}}$$

and $K_{La} = Q - K_{Br} = 177.4 \text{ MeV} - 112.0 \text{ MeV} = \boxed{65.4 \text{ MeV}}$

(d) The speed of the fragments is given by

$$v_{Br} = \sqrt{\frac{2K_{Br}}{m_{Br}}} = \sqrt{\frac{2(112\times 10^6 \text{ eV})(1.60\times 10^{-19} \text{ J/eV})}{(87 \text{ u})(1.66\times 10^{-27} \text{ kg/u})}}$$
$$= 1.58\times 10^7 \text{ m/s} = \boxed{15.8 \text{ Mm/s}}$$

and

$$v_{La} = \sqrt{\frac{2K_{La}}{m_{La}}} = \sqrt{\frac{2(65.4\times 10^6 \text{ eV})(1.60\times 10^{-19} \text{ J/eV})}{(149 \text{ u})(1.66\times 10^{-27} \text{ kg/u})}}$$
$$= 9.20\times 10^6 \text{ m/s} = \boxed{9.30 \text{ Mm/s}}$$

P43.59 The complete fissioning of 1.00 gram of ^{235}U releases

$$Q = \left(\frac{1.00 \text{ g}}{235 \text{ grams/mol}}\right)\left(\frac{6.02\times 10^{23} \text{ atoms}}{\text{mol}}\right) \times \left(\frac{200 \text{ MeV}}{\text{fission}}\right)\left(\frac{1.60\times 10^{-13} \text{ J}}{\text{MeV}}\right)$$
$$= 8.20\times 10^{10} \text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 19 of text for values), then

$$Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$
$$Q = m\left[(4\ 186 \text{ J/kg °C})(80.0\text{ °C}) + 2.26\times 10^6 \text{ J/kg} + (2\ 010 \text{ J/kg °C})(300\text{ °C})\right]$$

Therefore, $m = \dfrac{8.20\times 10^{10} \text{ J}}{3.20\times 10^6 \text{ J/kg}} = \boxed{2.56\times 10^4 \text{ kg}}$

P43.60 When mass m of ^{235}U undergoes complete fission, releasing energy E

per fission event, the total energy released is

$$Q = \left(\frac{m}{M_{\text{U-235}}}\right) N_{\text{A}} E$$

where N_A is Avogadro's number. If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then

$$Q = m_w \left[c_w \left(100°\text{C} - T_c\right) + L_v + c_s \left(T_h - 100°\text{C}\right) \right]$$

where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w = \frac{Q}{\left[c_w \left(100°\text{C} - T_c\right) + L_v + c_s \left(T_h - 100°\text{C}\right) \right]}$$
$$= \boxed{\frac{m N_{\text{A}} E}{M_{\text{U-235}} \left[c_w \left(100 - T_c\right) + L_v + c_s \left(T_h - 100\right) \right]}}$$

P43.61 (a) $Q_{\text{I}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}}\right]c^2$, and

$$Q_{\text{II}} = \left[M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}} + M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} + M_{\text{D}} - M_{\text{E}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$${}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{\;\,0}_{-1}\text{e} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{\;\,0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He} + 2\nu$$

$$4\,{}^{1}_{1}\text{H} + 2\,{}^{\;\,0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He} + 2\nu$$

Adding two electrons to each side gives

$$4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + 2\nu$$

Thus,

$$\begin{aligned} Q_{\text{net}} &= \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 \\ &= [4(1.007\ 825\ \text{u}) - 4.002\ 603\ \text{u}](931.5\ \text{MeV/u}) \\ &= \boxed{26.7\ \text{MeV}} \end{aligned}$$

P43.62 (a) From the definition of the volume of a cube and the definition of mass density, we have $V = \ell^3 = \dfrac{m}{\rho}$, so

$$\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{70.0\ \text{kg}}{19.1\times 10^3\ \text{kg/m}^3}\right)^{1/3} = 0.154\ \text{m} = \boxed{15.4\ \text{cm}}$$

(b) We add 92 electrons to both sides of the given nuclear reaction. Then it becomes

$${}^{238}_{92}\text{U atom} \to 8\,{}^4_2\text{He atom} + {}^{206}_{82}\text{Pb atom}$$

The Q value of this reaction is

$$\begin{aligned} Q &= \left[M_{{}^{238}_{92}\text{U}} - 8M_{{}^4_2\text{He}} - M_{{}^{206}_{82}\text{Pb}}\right]c^2 \\ &= [238.050\,783 - 8(4.002\,603) - 205.974\,449](931.5\ \text{MeV/u}) \\ Q &= \boxed{51.7\ \text{MeV}} \end{aligned}$$

(c) The number of decays per second is the decay rate R, and the energy released in each decay is Q. Then the energy released per unit time interval is $P = QR$.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00\times10^4\ \text{g}}{238\ \text{g/mol}}\right)(6.02\times10^{23}\ \text{nuclei/mol})$$
$$= 1.77\times10^{26}\ \text{nuclei}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47\times10^9\ \text{yr}} = 1.55\times10^{-10}\ \frac{1}{\text{yr}}$$

and the rate of decays is then

$$R = \lambda N = \left(1.55\times10^{-10}\ \frac{1}{\text{yr}}\right)(1.77\times10^{26}\ \text{nuclei})$$
$$= 2.75\times10^{16}\ \text{decays/yr}$$

so, $P = QR = (51.7\ \text{MeV})(2.75\times10^{16}\ \text{yr}^{-1})(1.60\times10^{-13}\ \text{J/MeV})$
$$= \boxed{2.27\times10^5\ \text{J/yr}}$$

(e) We know that

dose in rem = dose in rad × RBE

or

5.00 rem/yr = (dose in rad/yr)(1.10)

giving

(dose in rad/yr) = 4.55 rad/yr

The allowed whole-body dose is then

$$(70.0\ \text{kg})(4.55\ \text{rad/yr})\left(\frac{10^{-2}\ \text{J/kg}}{1\ \text{rad}}\right) = \boxed{3.18\ \text{J/yr}}$$

P43.63 (a) We have 1.00 kg – (1.00 kg)(0.007 20) – (1.00 kg)(0.000 0500) = 0.993 kg of ^{238}U, comprising

$$N = (0.993\ \text{kg})\left(\frac{6.02\times10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.238\ \text{kg}}\right)$$
$$= 2.51\times10^{24}\ \text{nuclei}$$

with activity

$$R = \lambda N = \frac{\ln 2}{4.47\times10^{9}\ \text{yr}}\left(2.51\times10^{24}\ \text{nuclei}\right)$$
$$\times\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70\times10^{10}\ \text{s}^{-1}}\right)$$
$$= \boxed{3.3\times10^{-4}\ \text{Ci}} = 330\ \mu\text{Ci}$$

We have (1.00 kg)(0.007 20) = 0.007 2 kg of ^{235}U, comprising

$$N = (0.007\ 2\ \text{kg})\left(\frac{6.02\times10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.235\ \text{kg}}\right)$$
$$= 1.84\times10^{22}\ \text{nuclei}$$

with activity

$$R = \lambda N = \frac{\ln 2}{7.04\times10^{8}\ \text{yr}}\left(1.84\times10^{22}\ \text{nuclei}\right)$$
$$\times\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70\times10^{10}\ \text{s}^{-1}}\right)$$
$$= 1.6\times10^{-5}\ \text{Ci} = \boxed{16\ \mu\text{Ci}}$$

We have (1.00 kg)(0.000 0500) = 5.00 × 10^{-5} kg of ^{234}U, comprising

$$N = (5.00\times10^{-5}\ \text{kg})\left(\frac{6.02\times10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.234\ \text{kg}}\right)$$
$$= 1.29\times10^{20}\ \text{nuclei}$$

with activity

$$R = \lambda N = \frac{\ln 2}{2.44 \times 10^5 \text{ yr}}\left(1.29 \times 10^{20} \text{ nuclei}\right)$$

$$\times\left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right)\left(\frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ s}^{-1}}\right)$$

$$= \boxed{3.1 \times 10^{-4} \text{ Ci}} = 310\ \mu\text{Ci}$$

(b) The total activity is (330 + 16 + 310) μCi = 656 μCi, so the fractional contributions are, respectively, 330/656 = $\boxed{50\%}$, 16/656 = $\boxed{2.4\%}$, and 310/656 = $\boxed{47\%}$

(c) It is dangerous, notably if the material is inhaled as a powder. With precautions to minimize human contact, however, microcurie sources are routinely used in laboratories.

P43.64 (a) From the given equation, the ratio of the two intensities is

$$\frac{I_2}{I_1} = \frac{I_0 e^{-\mu_2 x}}{I_0 e^{-\mu_1 x}} = \boxed{e^{-(\mu_2 - \mu_1)x}}$$

(b) Substituting numerical values into the equation in part (a) gives

$$\frac{I_{50}}{I_{100}} = \exp\left[-\left(5.40 \text{ cm}^{-1} - 41.0 \text{ cm}^{-1}\right)(0.100 \text{ cm})\right] = e^{3.56} = \boxed{35.2}$$

(c) Here, x = 10.0 mm = 1.00 cm, and

$$\frac{I_{50}}{I_{100}} = \exp\left[-\left(5.40 \text{ cm}^{-1} - 41.0 \text{ cm}^{-1}\right)(1.00 \text{ cm})\right] = e^{35.6}$$

$$= \boxed{2.89 \times 10^{15}}$$

Thus, a 1.00-cm-thick aluminum plate has essentially removed the long-wavelength x-rays from the beam.

P43.65 (a) The number of Pu nuclei in 1.00 kg is

$$\frac{6.02 \times 10^{23} \text{ nuclei/mol}}{239.05 \text{ g/mol}}(1\,000 \text{ g}) = 2.52 \times 10^{24} \text{ nuclei}$$

The total energy is

$$\left(25.2 \times 10^{23} \text{ nuclei}\right)\left(\frac{1 \text{ fission}}{\text{nucleus}}\right)\left(\frac{200 \text{ MeV}}{\text{fission}}\right) = 5.04 \times 10^{26} \text{ MeV}$$

$$E = \left(5.04 \times 10^{26} \text{ MeV}\right)\left(4.44 \times 10^{-20} \text{ kWh/MeV}\right)$$
$$= \boxed{2.24 \times 10^{7} \text{ kWh}}$$

or 22 million kWh.

(b) $$E = \Delta m c^2 = \left(3.016\,049 \text{ u} + 2.014\,102 \text{ u} - 4.002\,603 \text{ u} - 1.008\,665 \text{ u}\right) \times \left(931.5 \text{ MeV/u}\right)$$

$$E = \boxed{17.6 \text{ MeV for each D-T fusion}}$$

(c) $$E_n = (\text{total number of D nuclei})(17.6 \text{ MeV})\left(4.44 \times 10^{-20} \text{ kWh/MeV}\right)$$

$$E_n = \left(\frac{6.02 \times 10^{23}}{\text{mol}}\right)\left(\frac{1\,000 \text{ g}}{2.014 \text{ g/mol}}\right)(17.6 \text{ MeV}) \times \left(4.44 \times 10^{-20} \text{ kWh/MeV}\right)$$
$$= \boxed{2.34 \times 10^{8} \text{ kWh}}$$

(d) $$E_n = \left(\text{the number of C atoms in 1.00 kg}\right) \times \left(\frac{4.20 \text{ eV}}{\text{kg}}\right)$$

$$E_n = \left(\frac{6.02 \times 10^{26}}{12 \text{ g}}\right)\left(4.20 \times 10^{-6} \text{ MeV}\right)\left(4.44 \times 10^{-20} \text{ kWh/MeV}\right)$$
$$= \boxed{9.36 \text{ kWh}}$$

(e) $\boxed{\text{Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.}}$

Challenge Problems

P43.66 The electric charge density in the sphere is

$$\rho = \frac{Ze}{(4/3)\pi R^3}$$

Using Gauss's Law inside the sphere,

$$E \cdot 4\pi r^2 = \left(\frac{(4/3)\pi r^3}{\epsilon_0}\right)\frac{Ze}{(4/3)\pi R^3}:$$

or $$E = \left(\frac{1}{4\pi \epsilon_0}\frac{Ze}{R^3}\right)r \qquad (r \le R)$$

Outside the sphere, the field is

$$E = \frac{1}{4\pi \epsilon_0}\frac{Ze}{r^2} \qquad (r \ge R)$$

We now find the electrostatic energy

$$U = \int_{r=0}^{\infty}\left(\frac{1}{2}\epsilon_0 E^2\right)4\pi r^2\,dr$$

$$U = \frac{1}{2}\in_0 \int_0^R \left[\left(\frac{Ze}{4\pi \in_0 R^3}\right) r\right]^2 4\pi r^2\,dr + \frac{1}{2}\in_0 \int_R^\infty \left[\frac{1}{4\pi \in_0}\frac{Ze}{r^2}\right]^2 4\pi r^2\,dr$$

$$= 2\pi \in_0 \left(\frac{Ze}{4\pi \in_0}\right)^2 \int_0^R \left[\frac{r^2}{R^6}\right] r^2\,dr + 2\pi \in_0 \left(\frac{Ze}{4\pi \in_0}\right)^2 \int_R^\infty \left[\frac{1}{r^4}\right] r^2 dr$$

$$= \frac{Z^2e^2}{8\pi \in_0}\left[\int_0^R \frac{r^4}{R^6}\,dr + \int_R^\infty \frac{dr}{r^2}\right] = \frac{Z^2e^2}{8\pi \in_0}\left[\left(\frac{R^5}{5R^6}\right)\Bigg|_0^R - \left(\frac{1}{r}\right)\Bigg|_R^\infty\right]$$

$$= \frac{Z^2e^2}{8\pi \in_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right] = \frac{3}{20}\frac{Z^2e^2}{\pi \in_0 R} = \frac{3}{5}\left(\frac{1}{4\pi \in_0}\right)\frac{Z^2e^2}{R}$$

or $$U = \boxed{\frac{3}{20}\frac{Z^2e^2}{\pi \in_0 R} = \frac{3k_eZ^2e^2}{5R}}$$

ANSWERS TO QUICK-QUIZZES

1. **(i)** (b) **(ii)** (a) **(iii)** (c)

2. (e)

3. (b)

4. (c)

5. (b)

6. (a), (b)

7. (d)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P43.2 (a) 68; (b) ${}^{68}_{30}\text{Zn}$; (c) Isotopes of other elements to the left and right of zinc in the periodic table (from manganese to bromine) may have the same mass number.

P43.4 (a) 5.18 fm; (b) λ is much less than the distance of closest approach

P43.6 3.54 MeV

P43.8 ${}^{56}\text{Fe}$ has a greater $\dfrac{E_b}{A}$ than its neighbors

P43.10 (a) 84.2 MeV; (b) 342 MeV; (c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

P43.12 (a) Nucleons on the surface have fewer neighbors with which to interact. The surface term is negative to reduce the estimate from the volume term, which assumes that all nucleons have the same number of neighbors; (b) sphere, $\dfrac{1}{3}r$, cube, $\dfrac{1}{6}L$. The sphere has a larger ratio to its characteristic length, so it would represent a larger binding energy and be more plausible for a nuclear shape.

P43.14 Dispose of the sample.

P43.16 $\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)$

P43.18 (a) See P43.18(a) for full explanation; (b) See P43.18(b) for full explanation; (c) See ANS. FIG. P43.18(c); (d) 10.9 min; (e) $t_m = \dfrac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$; (f) 10.9 min

P43.20 (a) ${}^{65}_{28}\text{Ni}^*$; (b) ${}^{211}_{82}\text{Pb}$; (c) ${}^{55}_{27}\text{Co}$

P43.22 (a) $N_d = N_{P,0} - N_{P,0}e^{-\lambda t} = \left(1.00\times10^6\right)\left(1-e^{-0.0693t}\right)$, where t is in hours; (b) The number of daughter nuclei starts from zero at $t = 0$. The number of stable product nuclei always increases with time and asymptotically approaches 1.00×10^6 as t increases without limit; (c) The minimum number of daughter nuclei is zero at $t = 0$. The maximum number of daughter nuclei asymptotically approaches 1.00×10^6 as t increases without limit; (d) The rate of change has its maximum value, $6.93 \times 10^4\ \text{h}^{-1}$, at $t = 0$, after which the rate decreases more and more, approaching zero as t increases without limit.

P43.24 (a) 0.281; (b) 1.65×10^{-29}; (c) Radon is continuously created.

P43.26 (a) ${}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{79}\text{Au}^* \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}$; (b) 7.89 MeV

P43.28 ${}^{144}_{54}\text{Xe}$, ${}^{143}_{54}\text{Xe}$, ${}^{142}_{54}\text{Xe}$

P43.30 (a) $4.84V^{-1/3}$ (b) $6V^{-1/3}$ **(c)** $6.30V^{-1/3}$ (d) $\boxed{\text{The sphere has minimum leakage and the parallelepiped has maximum leakage}}$

P43.32 See P43.32 for full explanation

P43.34 (a) 4.0×10^3 yr (b) 1.0×10^{11} yr

P43.36 (a) $E = 144Z_1Z_2$ where E is in keV; (b) The energy is proportional to each atomic number; (c) Take $Z_1 = 1$ and $Z_2 = 59$ or vice versa. This choice minimizes the product $Z_1 Z_2$; (d) 144 keV for both, according to this model

P43.38 (a) ${}^{13}_{7}\mathrm{N}$; (b) ${}^{13}_{6}\mathrm{C}$; (c) ${}^{14}_{7}\mathrm{N}$; (d) ${}^{15}_{8}\mathrm{O}$; (e) ${}^{15}_{7}\mathrm{N}$; (f) ${}^{12}_{6}\mathrm{C}$; (g) The original carbon-12 nucleus is returned so the overall reaction is

$4\left({}^{1}_{1}\mathrm{H}\right) \rightarrow {}^{4}_{2}\mathrm{He}$.

P43.40 It would take over 24 days to raise the temperature of the water to 100°C and even longer to boil it, so this technique will not work for a 20-minute coffee break!

P43.42 (a) See P45.44(a) for full explanation; (b) $\dfrac{R}{\lambda}$

P43.44 (a) See ANS. FIG P43.44 (a); (b) See ANS. FIG P43.44 (b)

P43.46 See ANS. P43.46 for full explanation

P43.48

The Q value of this hypothetical decay is calculated to be ? .62 MeV, which means you would have to add this much energy to the ${}^{238}\mathrm{U}$ nucleus to make it emit a proton.

P43.50 (a) 3.91×10^{9} yr; (b) no older

P43.52 The nucleus ${}^{57}\mathrm{Co}$ cannot decay by e^{+} emission because the Q value is ? .187 MeV.

P43.54 (a) See ANS. P43.54(a) for full explanation; (b) 1.53 MeV

P43.56 $\dfrac{T_{\mathrm{Y}} \ln 2}{\ln\left[2\left(r_1/r_2\right)^{T_{\mathrm{Y}}/\Delta t}\right]}$

P43.58 (a) See P43.58(a) for full explanation; (b) 177 MeV; (c) $K_{\mathrm{Br}} = 112.0$ MeV, $K_{\mathrm{La}} = 65.4$ MeV; (d) $v_{\mathrm{Br}} = 15.8$ Mm/s, $v_{\mathrm{La}} = 9.30$ Mm/s

P43.60 $\dfrac{mN_{\mathrm{A}}E}{M_{\text{U-235}}\left[c_w\left(100 - T_c\right) + L_v + c_s\left(T_h - 100\right)\right]}$

P43.62 (a) 15.4 cm; (b) 51.7 MeV; (c) The number of decays per second is the decay rate *R*, and the energy released in each decay is *Q*. Then the energy released per unit time interval is $P = QR$; (d) 2.27×10^5 J/yr; (e) 3.18 J/yr

P43.64 (a) $e^{-(\mu_2 - \mu_1)x}$; (b) 35.2; (c) 2.89×10^{15}

P43.66 $\dfrac{3}{20}\dfrac{Z^2e^2}{\pi \in_0 R} = \dfrac{3k_eZ^2e^2}{5R}$