40

Quantum Mechanics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP40.1** **Conceptualize** The proton in the nucleus is constrained to move in a small region of space. Is the particle in a box a good model?

Categorize This problem involves the *quantum particle under boundary conditions* model, applied to the particle in a one-dimensional box.

Analyze (a) Using Equation 40.14, find the energy difference between a general state described by quantum number n and the $n = 1$ state of the particle in a box:

$$\Delta E = \frac{h^2}{8mL^2}\left(n^2 - 1\right) \qquad (1)$$

Substitute numerical values for a proton in the box:

$$\Delta E = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.673 \times 10^{-27}\ \text{kg}\right)\left(10.0 \times 10^{-15}\ \text{m}\right)^2}\left(4-1\right)$$
$$= 9.84 \times 10^{-13}\ \text{J}\left(\frac{1\ \text{MeV}}{1.602 \times 10^{-13}\ \text{J}}\right) = \boxed{6.14\ \text{MeV}}$$

(b) Set the energy difference equal to the photon energy and solve for the wavelength:

$$\Delta E = hf = \frac{hc}{\lambda} \quad \rightarrow \quad \lambda = \frac{hc}{\Delta E} \qquad (2)$$

Substitute numerical values:

$$\lambda = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{9.84 \times 10^{-13}\ \text{J}} = 2.02 \times 10^{-13}\ \text{m} = \boxed{202\ \text{fm}}$$

(c) Consulting Figure 33.13, we find that this wavelength lies deeply in the range of $\boxed{\text{gamma radiation}}$.

Finalize While the particle in a box is a reasonable first step in modeling the particles in the nucleus, reality is complicated by other effects, such as, for example, *spin-orbit coupling*. Section 43.3 discusses more details about nuclear models.]

Answers: (a) 6.14 MeV (b) 202 fm (c) gamma radiation

***TP40.2 Conceptualize** The wave functions for the particle in a box are shown in

Figure 40.4a. Figure 40.4b shows the probability of finding the particle *at* any given position. In this problem, we are trying to find the probability that the particle is anywhere to the *left* of a given position.

Categorize This problem involves the *quantum particle under boundary conditions* model, applied to the particle in a one-dimensional box.

Analyze (a) Use the probability expression in Equation 40.6 to find the probability that the particle is between $x = 0$ and $x = \ell$:

$$P_{0\ell} = \int_0^{\ell} |\psi|^2 dx \qquad (1)$$

Substitute the wave function for the particle in a box from Equation 40.13, using $n = 1$ for the ground state:

$$P_{0\ell} = \int_0^{\ell} \left[\sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \right]^2 dx \qquad (2)$$

Carry out the integration in Equation (2):

$$\begin{aligned} P_{0\ell} &= \frac{2}{L}\int_0^{\ell} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L}\int_0^{\ell} \tfrac{1}{2}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right] dx \\ &= \frac{1}{L}\int_0^{\ell} dx - \frac{1}{L}\int_0^{\ell} \cos\left(\frac{2\pi x}{L}\right) dx = \frac{\ell}{L} - \frac{1}{L}\left[\frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{\ell} \\ &= \frac{\ell}{L} - \frac{1}{2\pi}\left[\sin\left(\frac{2\pi \ell}{L}\right) - 0\right] = \boxed{\frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi \ell}{L}\right)} \qquad (2) \end{aligned}$$

(b) Let's test the function at $\ell = 0$:

$$P_{0\ell}(\ell = 0) = \frac{(0)}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi(0)}{L}\right) = 0$$

This is a reasonable result. If $\ell = 0,$ there is no part of the box to the left of ℓ because we are at the left edge of the box, and the probability must be zero.

Now, for $\ell = \frac{1}{2}L$,

$$P_{0\ell}\left(\ell = \tfrac{1}{2}L\right) = \frac{\left(\frac{1}{2}L\right)}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi\left(\frac{1}{2}L\right)}{L}\right) = \tfrac{1}{2} - 0 = \tfrac{1}{2}$$

This is a reasonable result. At the middle of the box, due to the symmetry of the wave function, the probability should be one-half that the particle is to the left of the middle.

Now, for $\ell = L$,

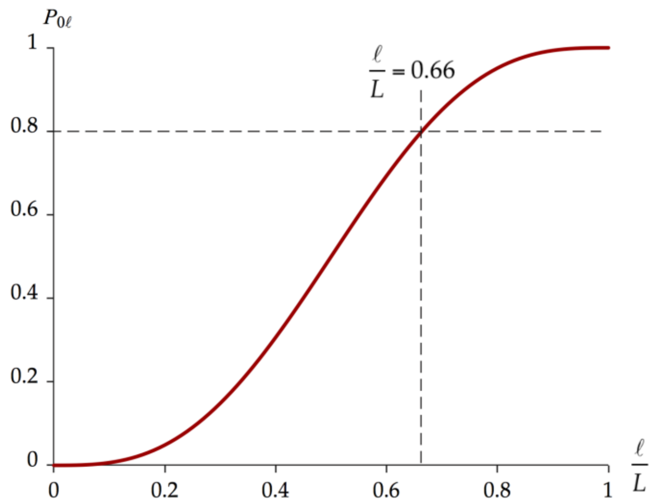
$$P_{0\ell}\left(\ell = L\right) = \frac{(L)}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi(L)}{L}\right) = 1 - 0 = 1$$

This is also reasonable. The value of $\ell = L$ puts us at the right edge of the box, so the entire box is to the left of ℓ. The particle must be in the box *somewhere*, so the probability is 1.

(c) For the probability to be four times as great for the particle to be to the left of ℓ than to the right, then the probability must be $P_{0\ell} = 0.800$ ($\frac{4}{5}$ to the left, $\frac{1}{5}$ to the right). Substitute this result into Equation (2):

$$0.800 = \frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi\ell}{L}\right)$$

Uh-oh. This is a transcendental equation. While you might try to solve this with various techniques, here's another idea. Let's graph Equation (2) and find the value of ℓ/L for which the probability is 0.800:



The graph gives a value of $\ell = \boxed{0.66L}$, where we have kept only two significant figures due to the uncertainty in reading the graph.

Finalize A more detailed analysis of part (c) shows that $P_{0\ell} = 0.800$ at $\ell = 0.663\,683\,5L$.]

Answers: (a) $\dfrac{\ell}{L} - \dfrac{1}{2\pi}\sin\left(\dfrac{2\pi\ell}{L}\right)$ (b) $P_{0\ell}(\ell = 0) = 0$, $P_{0\ell}\left(\ell = \frac{1}{2}L\right) = \frac{1}{2}$, and $P_{0\ell}(\ell = L) = 1$ (c) $\ell = 0.66L$

***TP40.3 Conceptualize** Review Section 40.4 to be sure you understand the particle in a finite well. Because the wave function does not go to zero at the ends of the well as it does for the infinite well, the evaluation of the quantized energies of the particle will not be as simple.

Categorize The electron in the finite well is modeled as a *quantum particle under boundary conditions.*

Analyze (a) The cosine function in the suggested solution is symmetric around $x = 0$ while the sine function is antisymmetric. Therefore, for symmetric solutions, we set $F = 0$:

$$\psi_{\mathrm{I}} = Ae^{Cx} \qquad \text{for } x < -\frac{L}{2}$$

$$\psi_{\mathrm{II}} = G\cos kx \qquad \text{for } -\frac{L}{2} < x < \frac{L}{2}$$

$$\psi_{\mathrm{III}} = Be^{-Cx} \qquad \text{for } x > \frac{L}{2}$$

Apply the boundary conditions for the wave functions:

$$\psi_{\mathrm{I}} = \psi_{\mathrm{II}} \quad \rightarrow \quad Ae^{-CL/2} = G\cos\left(-\frac{kL}{2}\right) = G\cos\left(\frac{kL}{2}\right) \qquad (1)$$

$$\psi_{\mathrm{II}} = \psi_{\mathrm{III}} \quad \rightarrow \quad G\cos\left(\frac{kL}{2}\right) = Be^{-CL/2} \qquad (2)$$

The right side of Equation (1) is the same as the left side of Equation (2), so we see that $A = B$ for symmetric solutions.

Apply the boundary conditions for the derivatives of the wave functions:

$$\frac{d\psi_{\mathrm{I}}}{dx} = \frac{d\psi_{\mathrm{II}}}{dx} \quad \rightarrow \quad ACe^{-CL/2} = -Gk\sin\left(-\frac{kL}{2}\right) = Gk\sin\left(\frac{kL}{2}\right) \qquad (3)$$

$$\frac{d\psi_{\mathrm{II}}}{dx} = \frac{d\psi_{\mathrm{III}}}{dx} \quad \rightarrow \quad -Gk\sin\left(\frac{kL}{2}\right) = -BCe^{-CL/2} = -ACe^{-CL/2} \qquad (4)$$

Equations (3) and (4) are the same. Divide Equation (3) by Equation (2), noting that $B = A$:

$$\frac{ACe^{-CL/2}}{Ae^{-CL/2}} = \frac{Gk\sin\left(\frac{kL}{2}\right)}{G\cos\left(\frac{kL}{2}\right)} \quad \rightarrow \quad C = k\tan\left(\frac{kL}{2}\right) \tag{5}$$

Now consider the antisymmetric solutions. Antisymmetric solutions involve a sine function. Therefore, we set $G = 0$ in the general solutions, and antisymmetric solutions are given by

$$\begin{aligned} \psi_{\mathrm{I}} &= Ae^{Cx} && \text{for } x < -\frac{L}{2} \\ \psi_{\mathrm{II}} &= F\sin kx && \text{for } -\frac{L}{2} < x < \frac{L}{2} \\ \psi_{\mathrm{III}} &= Be^{-Cx} && \text{for } x > \frac{L}{2} \end{aligned}$$

Apply the boundary conditions for the wave functions:

$$\psi_{\mathrm{I}} = \psi_{\mathrm{II}} \quad \rightarrow \quad Ae^{-CL/2} = F\sin\left(-\frac{kL}{2}\right) = -F\sin\left(\frac{kL}{2}\right) \tag{6}$$

$$\psi_{\mathrm{II}} = \psi_{\mathrm{III}} \quad \rightarrow \quad F\sin\left(\frac{kL}{2}\right) = Be^{-CL/2} \tag{7}$$

The right side of Equation (6) is the negative of the left side of Equation (7), so we see that $A = -B$ for antisymmetric solutions.

Apply the boundary conditions for the derivatives of the wave functions:

$$\frac{d\psi_{\mathrm{I}}}{dx} = \frac{d\psi_{\mathrm{II}}}{dx} \quad \rightarrow \quad ACe^{-CL/2} = Fk\cos\left(-\frac{kL}{2}\right) = Fk\cos\left(\frac{kL}{2}\right) \tag{8}$$

$$\frac{d\psi_{\mathrm{II}}}{dx} = \frac{d\psi_{\mathrm{III}}}{dx} \quad \rightarrow \quad Fk\cos\left(\frac{kL}{2}\right) = -BCe^{-CL/2} = ACe^{-CL/2} \tag{9}$$

Equations (8) and (9) are the same. Divide Equation (8) by Equation (7), noting that $B = -A$:

$$\frac{ACe^{-CL/2}}{-Ae^{-CL/2}} = \frac{Fk\cos\left(\frac{kL}{2}\right)}{F\sin\left(\frac{kL}{2}\right)} \quad \rightarrow \quad C = -k\cot\left(\frac{kL}{2}\right) \tag{10}$$

Equations (5) and (10) are the requested equations.

(b) Use the definitions of a and b given in the problem and then use Equations 40.17 and 40.29:

$$a^2 + b^2 = \left(\frac{CL}{2}\right)^2 + \left(\frac{kL}{2}\right)^2 = \left(\frac{L}{2}\right)^2 \left(C^2 + k^2\right)$$

$$= \left(\frac{L}{2}\right)^2 \left\{\left[\frac{\sqrt{2m_e(U-E)}}{\hbar}\right]^2 + \left[\frac{\sqrt{2m_eE}}{\hbar}\right]^2\right\}$$

$$= \left(\frac{L}{2}\right)^2 \left[\frac{2m_e(U-E)}{\hbar^2} + \frac{2m_eE}{\hbar^2}\right] = \left(\frac{L}{2}\right)^2 \left[\frac{2m_eU}{\hbar^2}\right] = \left(\frac{L}{\hbar}\sqrt{\frac{m_eU}{2}}\right)^2$$

Therefore,

$$a^2 + b^2 = r^2 \tag{11}$$

where

$$r = \frac{L}{\hbar}\sqrt{\frac{m_eU}{2}} \tag{12}$$

(c) From the definitions of a and b in part (b), we see that

$$C = \frac{2a}{L} \qquad k = \frac{2b}{L} \tag{13}$$

Therefore, Equations (5) and (10) become

$$\frac{2a}{L} = \frac{2b}{L}\tan\left[\left(\frac{2b}{L}\right)\left(\frac{L}{2}\right)\right] \rightarrow a = b\tan b \rightarrow \sqrt{r^2 - b^2} = b\tan b \tag{14}$$

$$\frac{2a}{L} = -\frac{2b}{L}\cot\left[\left(\frac{2b}{L}\right)\left(\frac{L}{2}\right)\right] \rightarrow a = -b\cot b \rightarrow \sqrt{r^2 - b^2} = -b\cot b \tag{15}$$

(d) In the definition of b in part (b), substitute for k from Equation 40.17:

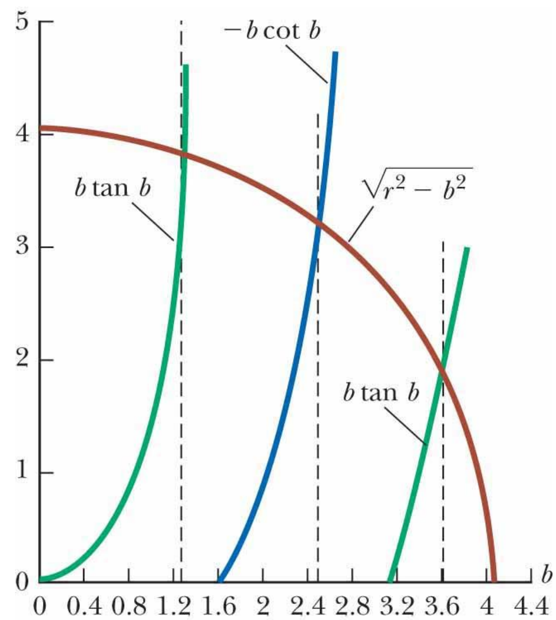
$$b = \frac{kL}{2} \quad \rightarrow \quad b_E = \frac{\sqrt{2m_e E}}{\hbar}\frac{L}{2}$$

$$\rightarrow \quad E = \frac{2\hbar^2 b_E^2}{m_e L^2} = \frac{b_E^2}{\left(\frac{L}{\hbar}\sqrt{\frac{m_e}{2}}\right)^2} = \frac{b_E^2}{\left(\frac{r}{\sqrt{U}}\right)^2} = \frac{b_E^2}{r^2}U \tag{16}$$

(e) We first evaluate the value of r for the data provided in the problem statement, using Equation (12):

$$r = \frac{L}{\hbar}\sqrt{\frac{m_e U}{2}} = \frac{0.500\text{ nm}}{1.054\,6\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right)$$
$$\times\sqrt{\frac{\left(9.109\times10^{-31}\text{ kg}\right)\left(10.0\text{ eV}\right)}{2}\left(\frac{1.602\times10^{-19}\text{ J}}{1\text{ eV}}\right)}$$
$$= 4.05$$

Using a spreadsheet or graphing software and the value of r, graph the left side of Equation (14) and (15) versus b. On the same graph, graph the right sides of Equations (14) and (15) versus b, keeping only positive values. This graph appears below.



Notice the circular shape of the red line, representing Equation (11). The intersections of the red circle with the green and blue lines represent possible states of the system.

(f) The vertical black lines show points where the curves cross. Reading from the horizontal axis, we find

$$b_{E,1} = 1.26$$
$$b_{E,2} = 2.48$$
$$b_{E,3} = 3.605$$

(g) Because there are three points at which the curves cross, there are three quantized energies for the electron in the well.

(h) Using Equation (16), we have

$$E = \frac{b_E^2}{r^2}U$$

Substitute numerical values, reading r from the intersection of the red curve with the x or y axes, or evaluating it from Equation (12):

$$E_1 = \frac{b_{E,1}^2}{r^2}U = \frac{(1.26)^2}{(4.05)^2}(10.0 \text{ eV}) = 0.968 \text{ eV}$$

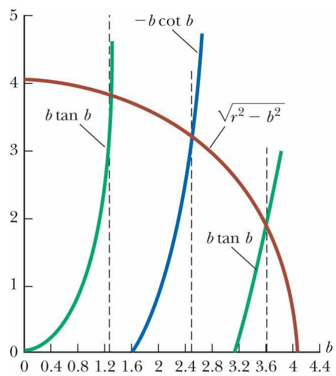
$$E_2 = \frac{b_{E,2}^2}{r^2}U = \frac{(2.48)^2}{(4.05)^2}(10.0 \text{ eV}) = 3.75 \text{ eV}$$

$$E_3 = \frac{b_{E,3}^2}{r^2}U = \frac{(3.605)^2}{(4.05)^2}(10.0 \text{ eV}) = 7.92 \text{ eV}$$

Finalize This was a long problem, but we succeeded in finding the quantized energies! If we find the first three quantized energies of an *infinite* well of the same width, we find them to be 1.50 eV, 6.02 eV, and 13.5 eV. The finite well clearly lowers the quantized energies and restricts them to a finite number, three in this case. We can explain the lowering of energy conceptually by noticing that the wavelengths of the sinusoidal waves inside the well in Figure 40.7a are longer than those in Figure 40.4, because the wave function extends beyond the boundaries. A longer wavelength corresponds to a lower energy.]

Answers:

(e)



ANS.FIG. TP40.3

(f) 1.26, 2.48. 3.605 (g) three (h) 0.968 eV, 3.75 eV, 7.92 eV

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 40.1 The Wave Function

P40.1 (a) The wave function,

$$\psi(x) = Ae^{i(5\times10^{10}x)} = A\cos\left(5\times10^{10}x\right) + iA\sin\left(5\times10^{10}x\right)$$

will go through one full cycle between $x_1 = 0$ and $(5.00 \times 10^{10})x_2 = 2\pi$. The wavelength is then

$$\lambda = x_2 - x_1 = \frac{2\pi}{5.00\times10^{10}\ \text{m}^{-1}} = \boxed{1.26\times10^{-10}\ \text{m}}$$

To say the same thing, we can inspect $Ae^{i(5\times10^{10}x)}$ to see that the wave number is $k = 5.00 \times 10^{10}\ \text{m}^{-1} = 2\pi/\lambda$.

(b) Since $\lambda = h/p$, the momentum is

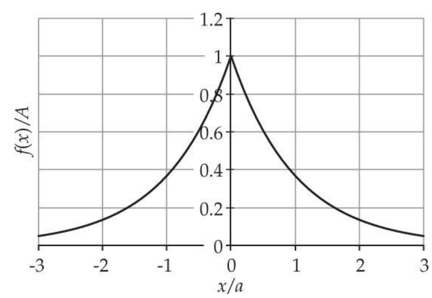
$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{1.26 \times 10^{-10}\ \text{m}} = \boxed{5.27 \times 10^{-24}\ \text{kg} \cdot \text{m/s}}$$

(c) The electron's kinetic energy is

$$K = \frac{1}{2}mu^2 = \frac{p^2}{2m}$$
$$= \frac{\left(5.27 \times 10^{-24}\ \text{kg} \cdot \text{m/s}\right)^2}{2\left(9.11 \times 10^{-31}\ \text{kg}\right)}\left(\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}}\right) = \boxed{95.3\ \text{eV}}$$

[We use u to represent the speed of a particle with mass in chapters 38, 39, and 40.]

P40.2 (a) See ANS. FIG. P40.2 for a graph of $\dfrac{f(x)}{A} = e^{-|x|/a}$ for the range $-3 < \dfrac{x}{a} < 3$.



ANS. FIG. P40.2

(b) Normalization requires

$$\int_{\text{all space}} |\psi|^2\, dx = 1:$$

$$\int_{-\infty}^{\infty} A^2 e^{-2|x|/a}dx = 2\int_0^{\infty} A^2 e^{-2|x|/a}dx = 1$$

$$-aA^2 e^{-2|x|/a}\Big|_0^{\infty} = aA^2 = 1 \quad \rightarrow \quad A = \boxed{\frac{1}{\sqrt{a}}}$$

(c) $$P = \int_{-a}^{a} \frac{e^{-2|x|/a}}{a}dx = 2\int_0^{a} \frac{e^{-2|x|/a}}{a}dx = -e^{-2x/a}\Big|_0^{a} = -e^{-2} + 1 = \boxed{0.865}$$

P40.3 The probability is given by

$$P = \int_{-a}^{a} |\psi(x)|^2 = \int_{-a}^{a} \frac{a}{\pi(x^2 + a^2)}dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\Big|_{-a}^{a}$$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{\frac{1}{2}}$$

Section 40.2 Analysis Model: Quantum Particle Under Boundary Conditions

P40.4 The energy of the photon is

$$E = \frac{hc}{\lambda} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{6.06\ \text{mm}}\left(\frac{1\ \text{mm}}{10^6\ \text{nm}}\right) = 2.05 \times 10^{-4}\,\text{eV}$$

The allowed energies of the proton in the box are

$$\begin{aligned} E_n &= \left(\frac{h^2}{8mL^2}\right)n^2 \\ &= \left[\frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.673 \times 10^{-27}\ \text{kg}\right)\left(1.00 \times 10^{-9}\ \text{m}\right)^2}\right]\left(\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}}\right)n^2 \\ &= \left(2.05 \times 10^{-4}\ \text{eV}\right)n^2 \end{aligned}$$

The smallest possible energy for a transition between states is from $n = 1$ to $n = 2$, which has energy

$$\Delta E_n = \left(2.05 \times 10^{-4}\,\text{eV}\right)\left(2^2 - 1^2\right) = 6.14 \times 10^{-4}\,\text{eV}$$

The photon does not have enough energy to cause this transition. The photon energy would be sufficient to cause a transition from $n = 0$ to $n = 1$, but the $n = 0$ state does not exist for the particle in a box.

P40.5 (a) The energy of a quantum particle confined to a line segment is

$$E_n = \frac{h^2 n^2}{8mL^2}$$

Here we have for the ground state

$$\begin{aligned} E_1 &= \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2 (1)^2}{8\left(1.67 \times 10^{-27}\text{kg}\right)\left(2.00 \times 10^{-14}\text{m}\right)^2} \\ &= 8.22 \times 10^{-14}\text{J} = \boxed{0.513\ \text{MeV}} \end{aligned}$$

and for the first and second excited states, which are states 2 and 3,

$$E_2 = 4E_1 = \boxed{2.05\ \text{MeV}} \quad \text{and} \quad E_3 = 9E_1 = \boxed{4.62\ \text{MeV}}$$

(b) They do; the MeV is the natural unit for energy radiated by an atomic nucleus.

Stated differently: Scattering experiments show that an atomic nucleus is a three-dimensional object always less than 15 fm in diameter. This one-dimensional box 20 fm long is a good model in energy terms.

P40.6 The ground state energy of a particle (mass m) in a 1-dimensional box of width L is $E_1 = \dfrac{h^2}{8mL^2}$.

(a) For a proton (m = 1.67 × 10^{-27} kg) in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{-10}\ \text{m}\right)^2}$$
$$= 8.22\times10^{-22}\ \text{J} = \boxed{5.13\times10^{-3}\ \text{eV}}$$

(b) For an electron (m = 9.11 × 10^{-31} kg) in the same size box:

$$E_1 = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.00\times10^{-10}\ \text{m}\right)^2}$$
$$= 1.51\times10^{-18}\ \text{J} = \boxed{9.41\ \text{eV}}$$

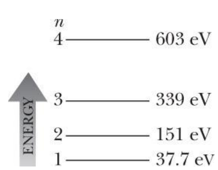
(c) $\boxed{\text{The electron has a much higher energy because it is much less massive.}}$

P40.7 (a) From Equation 40.14, the allowed energy levels of an electron in a box is

$$E_n = \left(\frac{h^2}{8m_eL^2}\right)n^2 \qquad n = 1, 2, 3, \ldots$$

Substituting numerical values,

$$E_n = \left[\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.100\times10^{-9}\ \text{m}\right)^2}\right]n^2$$
$$= \left(6.02\times10^{-18}\ \text{J}\right)n^2 = (37.7\ \text{eV})n^2$$



P40.8 (a) The classical kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(4.00\times10^{-3}\text{ kg}\right)\left(1.00\times10^{-3}\text{ m/s}\right)^2$$
$$= \boxed{2.00\times10^{-9}\text{ J}}$$

(a) The length L can be found from

$$E = \left(\frac{h^2}{8mL^2}\right)n^2$$

Solving,

$$L = n\sqrt{\frac{h^2}{8mE}} = 2\sqrt{\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(4.00\times10^{-3}\text{ kg}\right)\left(2.00\times10^{-9}\text{ J}\right)}}$$
$$= \boxed{1.66\times10^{-28}\text{ m}}$$

(c) No. The length of the box would have to be much smaller than the size of a nucleus ($\sim 10^{-14}$ m) to confine the particle.

P40.9 (a) From $\Delta x\Delta p \geq \frac{\hbar}{2}$, with $\Delta x = L$, $\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2L}$, so the uncertainty in momentum must be at least $\Delta p \approx \boxed{\frac{\hbar}{2L}}$.

(b) Its energy is all kinetic, so

$$E = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} \approx \boxed{\frac{\hbar^2}{8mL^2}} = \frac{h^2}{(4\pi)^2 8mL^2}$$

(c) Compare the result of part (b) to the result $h^2/8mL^2$ for the wave function as a standing wave. This estimate is too low by $4\pi^2 \approx 40$ times, but it correctly displays the pattern of dependence of the energy on the mass and on the length of the well.

P40.10 Normalization requires $\int_{\text{all space}} |\psi|^2 dx = 1$:

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = \int_0^L A^2 \frac{1-\cos\left[2(\pi x/L)\right]}{2}dx = 1$$

$$= \frac{A^2}{2}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^L = 1$$

$$= \frac{A^2}{2}\left[L - \frac{L}{2\pi}\sin 2\pi\right]_0^L = \frac{A^2 L}{2} = 1$$

$$A = \sqrt{\frac{2}{L}}$$

P40.11 (a) The expectation value is $\langle x \rangle = \int_0^L \psi^* x\psi\, dx$:

$$\langle x \rangle = \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left\{\frac{1-\cos\left[2(\pi x/L)\right]}{2}\right\}dx$$

$$= \frac{1}{L}\int_0^L x\left(1-\cos\frac{4\pi x}{L}\right)dx$$

From integral tables, we find that

$$\langle x \rangle = \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{\frac{L}{2}}$$

(b) The probability of finding the particle in the range $0.490L \le x \le 0.510L$ is

$$P = \frac{2}{L}\int_{0.490L}^{0.510L} \sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_{0.490L}^{0.510L} \frac{1-\cos\left[2(2\pi x/L)\right]}{2}dx$$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{0.490L}^{0.510L}$$

$$= 0.020 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times 10^{-5}}$$

(c) The probability of finding the particle in the range $0.240L \le x \le 0.260L$ is

$$P = \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$$

(d) $\boxed{\text{In the } n = 2 \text{ graph in the text's Figure 40.4(b), it is more probable to find the particle either near } x = L/4 \text{ or } x = 3L/4 \text{ than at the center, where the probability density is zero. Nevertheless, the symmetry of the distribution means that the average position is } x = L/2.}$

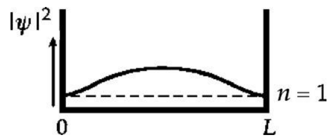
P40.12 (a) The most probable positions of the particle are $\boxed{x = L/4,\ L/2, \text{ and } 3L/4}$.

(b) $\boxed{\text{We look for } \sin(3\pi x/L) \text{ taking on its extreme values 1 and } -1 \text{ so that the squared wave function is as large as it can be. The result can also be found by studying Figure 40.4b.}}$ The most probable locations are at the antinodes of the standing wave pattern $n = 3$, which has three antinodes that are equally spaced, one at the center, and two a distance $L/4$ from either end.

P40.13 (a) The probability is

$$\begin{aligned}
P &= \int_0^{L/3} |\psi_1|^2\,dx = \frac{2}{L}\int_0^{L/3}\sin^2\left(\frac{\pi x}{L}\right)dx \\
&= \frac{2}{L}\int_0^{L/3}\frac{1-\cos\left[2(\pi x/L)\right]}{2}dx \\
&= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]\Bigg|_0^{L/3} = \frac{1}{L}\left[\frac{L}{3} - \frac{L}{2\pi}\sin\left(\frac{2\pi}{3}\right)\right] \\
&= \frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right) \\
&= \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}
\end{aligned}$$

(b) The probability density is symmetric about $x = \frac{L}{2}$. Thus, the probability of finding the particle between $x = \frac{2L}{3}$ and $x = L$ is the same, 0.196. Therefore, the probability of finding it in the range $\frac{L}{3} \le x \le \frac{2L}{3}$ is $P = 1.00 - 2(0.196) = \boxed{0.609}$.



ANS. FIG. P40.13(b)

***P40.14** **Conceptualize** The electron in the atom is constrained to move in a small region of space. Is the particle in a box a good model?

Categorize This problem involves the *quantum particle under boundary conditions* model, applied to the particle in a one-dimensional box.

Analyze Using Equation 40.14, find the energy difference between a general state described by quantum number n and the n = 1 state of the particle in a box and set the energy difference equal to the photon energy emitted by that transition:

$$\Delta E = hf = \frac{hc}{\lambda_n} = \frac{h^2}{8mL^2}\left(n^2 - 1\right) \qquad (1)$$

Solve Equation (1) for the length of the box:

$$L = \sqrt{\frac{h\lambda_n}{8mc}\left(n^2 - 1\right)} \qquad (2)$$

Substitute numerical values for the transition n = 2 to n = 1, using the wavelength given in the problem statement:

$$L = \sqrt{\frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(121.6 \times 10^{-9}\ \text{m}\right)}{8\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}\left(4-1\right)}$$

$$= 3.32 \times 10^{-10}\ \text{m} = \boxed{0.332\ \text{nm}}$$

(b) Solve Equation (2) for the wavelength:

$$L = \sqrt{\frac{h\lambda_n}{8mc}\left(n^2-1\right)} \quad \rightarrow \quad \lambda_n = \frac{8mcL^2}{h\left(n^2-1\right)} \qquad (3)$$

Substitute numerical values for the transition $n = 3$ to $n = 1$, using the box length found in part (a):

$$\lambda_3 = \frac{8\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)\left(3.32 \times 10^{-10}\ \text{m}\right)^2}{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(9-1\right)}$$

$$= 4.56 \times 10^{-8}\ \text{m} = \boxed{45.6\ \text{nm}}$$

변경된 필드 코드

The length of the box in part (a) is indeed very similar to atomic "sizes." However, the predicted wavelength in part (b) does not agree with the experimentally measured wavelength. And it gets worse. For the $n = 5$ to $n = 1$ transition, the particle-in-a-box model predicts a wavelength of 15.2 nm, while the measured wavelength is 95.0 nm. Take a look ahead at Figure 41.8, showing the quantized energy levels of the hydrogen atom. As energy goes up, the energy levels get closer together and approach a maximum value. Figure 40.5 for the particle in a box, however, shows energy levels getting *farther apart* as the energy goes up. This is a serious discrepancy that dooms the particle-in-a-box model for the hydrogen atom.

Finalize In some ways, the particle-in-a-box model for the hydrogen atom is similar to the Thomson model of the atom, which we addressed in Think–Pair–Share Problem 23.1. In that problem, we used a different model to estimate the size of the atom and obtained a similar result. But, like the particle-in-a-box model, the Thomson model does not give correct predictions beyond the size of the atom.]

Answers: (a) 0.332 nm (b) 45.6 nm, quite different from 102.6 nm

Section 40.3 The Schrödinger Equation

P40.15 (a) Given the function

$$\psi(x) = A\cos kx + B\sin kx$$

Its derivative with respect to x is

$$\frac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$$

And its second derivative is

$$\begin{aligned}\frac{\partial^2\psi}{\partial x^2} &= -k^2 A\cos kx - k^2 B\sin kx \\ &= -k^2(A\cos kx + B\sin kx) = -k^2\psi\end{aligned}$$

The Schrödinger equation is satisfied if

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi, \qquad \text{where} \quad U = 0:$$

$$-\frac{\hbar^2}{2m}\left(-k^2\psi\right) = E\psi \quad \rightarrow \quad \frac{\hbar^2 k^2}{2m}\psi = E\psi$$

This is true as an identity (functional equality) for all x if

$E = \dfrac{\hbar^2 k^2}{2m}$, which is true because $E = K + U = K + 0 = K$, and

$$\frac{\hbar^2 k^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\pi}\right)^2\left(\frac{2\pi}{\lambda}\right)^2 = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{p^2}{2m} = K$$

(b) From part (a), $E = \boxed{\dfrac{\hbar^2 k^2}{2m}}$.

P40.16 From $\psi = Ae^{i(kx - \omega t)}$ **[1]**

we evaluate

$$\frac{d\psi}{dx} = ikAe^{i(kx - \omega t)}$$

and $\dfrac{d^2\psi}{dx^2} = -k^2 Ae^{i(kx - \omega t)}$ **[2]**

We substitute equations [1] and [2] into the Schrödinger equation, so that Equation 40.15,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

becomes the test equation

$$\left(-\frac{\hbar^2}{2m}\right)\left(-k^2 Ae^{i(kx - \omega t)}\right) + 0 = EAe^{i(kx - \omega t)} \quad \textbf{[3]}$$

The wave function $\psi = Ae^{i(kx - \omega t)}$ is a solution to the Schrödinger equation if equation [3] is true. Both sides depend on A, x, and t in the same way, so we can cancel several factors, and determine that we have a solution if

$$\frac{\hbar^2 k^2}{2m} = E$$

But this is true for a nonrelativistic particle with mass in a region where the potential energy is zero, since

$$\frac{\hbar^2 k^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\pi}\right)^2\left(\frac{2\pi}{\lambda}\right)^2 = \underbrace{\frac{(h/\lambda)^2}{2m} = \frac{p^2}{2m}}_{\text{using de Broglie's equation}}$$

$$= \frac{m^2 u^2}{2m} = \tfrac{1}{2}mu^2 = \underbrace{K = K + U}_{\text{recall } U=0} = E$$

where K is the kinetic energy. Therefore, the given wave function does satisfy Equation 40.15.

P40.17 (a) Setting the total energy E equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$\left(\frac{\hbar^2}{2m}\right)\frac{d^2\psi}{dx^2} + U(x)\psi = 0$$

$$U(x) = \left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\psi(x) = Axe^{-x^2/L^2}$

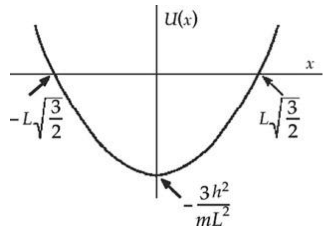
Then,

$$\frac{d^2\psi}{dx^2} = \left(4Ax^3 - 6AxL^2\right)\frac{e^{-x^2/L^2}}{L^4}$$

or $\dfrac{d^2\psi}{dx^2} = \dfrac{\left(4x^2 - 6L^2\right)}{L^4}\psi(x)$

and $\boxed{U(x) = \dfrac{\hbar^2}{2mL^2}\left(\dfrac{4x^2}{L^2} - 6\right)}$

(b) U(x) is sketched in ANS. FIG. P40.17(b).



ANS. FIG. P40.17(b)

P40.18 (a) These are standing wave patterns with nodes at the ends and n antinodes.

For $n = 1$, the wave function is

$$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$$

and the probability density is

$$P_1(x) = |\psi_1(x)|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$$

For $n = 2$, the wave function is

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

and the probability density is

$$P_2(x) = |\psi_2(x)|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$$

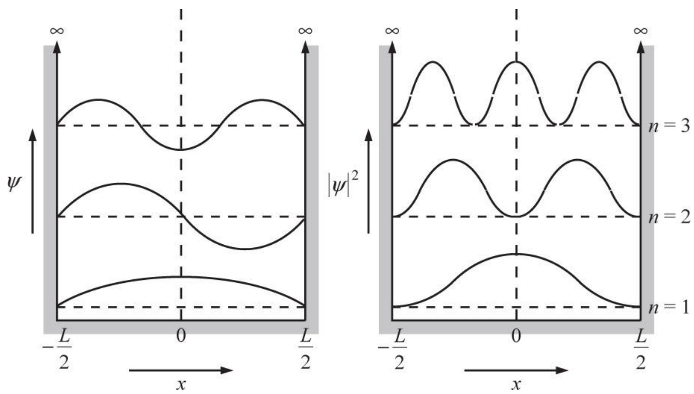
For $n = 3$, the wave function is

$$\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$$

and the probability density is

$$P_3(x) = |\psi_3(x)|^2 = \frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$$

(b) The wave functions and probability densities are shown in ANS. FIG. P40.18(b).

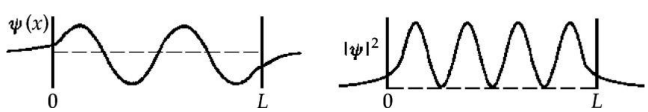


ANS. FIG. P40.18

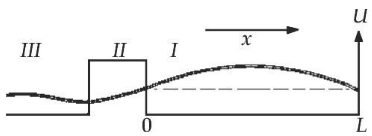
Section 40.4 A Particle in a Well of Finite Height

P40.19 (a) For $n = 4$, the wave function has two maxima and two minima (four extrema), as shown in the left-hand panel of ANS. FIG. P40.19.

(b) For $n = 4$, the probability function has four maxima. as shown in the right-hand panel of ANS. FIG. P40.19.



P40.20 (a) See ANS. FIG. P40.20(a).



ANS. FIG. P40.20 (a)

(b) The wavelength inside the box is $2L$. The wave function penetrates the wall, but the wavelength of the transmitted wave traveling to the left is the same, $\boxed{2L}$, because $U = 0$ on both sides of the wall, so the energy and momentum and, therefore, the wavelength, are the same.

Section 40.5 Tunneling Through a Potential Energy Barrier

P40.21 (a) $T = e^{-2CL}$, where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}$$
$$= \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)(5.00-4.50)\left(1.60\times10^{-19}\text{ J}\right)}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}$$
$$= 3.62\times10^{9}\text{ m}^{-1}$$

and $T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\text{ m}^{-1}\right)\left(950\times10^{-12}\text{ m}\right)\right]$
$$= \exp(-6.88) = \boxed{1.03\times10^{-3}}$$

(b) We require $e^{-2CL} = 10^{-6}$. Taking logarithms,

$$-2CL = \ln 10^{-6} = -6\ln 10$$
$$L = \frac{3\ln 10}{C} = \frac{3\ln 10}{3.62\times10^{9}\text{ m}^{-1}} = 1.91\times10^{-9}\text{ m} = \boxed{1.91\text{ nm}}$$

Section 40.6 Applications of Tunneling

P40.22 With transmission coefficient e^{-2CL}, the fractional change in transmission is

$$\frac{e^{-2(10.0/\text{nm})L} - e^{-2(10.0/\text{nm})(L+0.002\ 00\ \text{nm})}}{e^{-2(10.0/\text{nm})L}} = 1 - e^{-20.0(0.002\ 00)}$$

$$= 0.039\ 2 = \boxed{3.92\%}$$

Section 40.7 The Simple Harmonic Oscillator

P40.23 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator. From $E = \hbar\omega$, we have

$$\frac{hc}{\lambda} = \hbar\sqrt{\frac{k}{m}} = \frac{h}{2\pi}\sqrt{\frac{k}{m}}$$

or

$$\lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times 10^{8}\ \text{m/s}\right)\left(\frac{9.11\times 10^{-31}\ \text{kg}}{8.99\ \text{N/m}}\right)^{1/2}$$

$$= \boxed{600\ \text{nm}}$$

P40.24 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator, which is (from Equation 40.34)

$$E = \hbar\omega$$

$$\frac{hc}{\lambda} = \hbar\sqrt{\frac{k}{m}} = \frac{h}{2\pi}\sqrt{\frac{k}{m}}$$

$$\lambda = \boxed{2\pi c\sqrt{\frac{m}{k}}}$$

P40.25 (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all } x} |\psi|^2\, dx = 1$

becomes

$$1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2}\, dx = 2B^2 \int_0^{\infty} e^{-(m\omega/\hbar)x^2}\, dx$$
$$= 2B^2 \frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}} = B^2\sqrt{\frac{\pi\hbar}{m\omega}}$$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $\boxed{B = \left(\dfrac{m\omega}{\pi\hbar}\right)^{1/4}}$.

(b) For small δ, the probability of finding the particle in the range $-\dfrac{\delta}{2} < x < \dfrac{\delta}{2}$ is

$$\int_{-\delta/2}^{\delta/2} |\psi|^2\, dx \approx \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

P41.26 (a) The wave function is given by $\psi = Axe^{-bx^2}$, so

$$\frac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2 Ae^{-bx^2}$$

and

$$\frac{d^2\psi}{dx^2} = \left[-2bxAe^{-bx^2} - 4bxAe^{-bx^2}\right] + 4b^2x^3e^{-bx^2} = -6b\psi + 4b^2x^2\psi$$

Substitute into Equation 40.30:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}m\omega^2x^2\psi=E\psi$$

$$-\frac{\hbar^2}{2m}\left[-6b\psi+4b^2x^2\psi\right]+\frac{1}{2}m\omega^2x^2\psi=E\psi$$

$$\frac{3b\hbar^2}{m}\psi-\frac{2b^2\hbar^2}{m}x^2\psi=-\frac{1}{2}m\omega^2x^2\psi+E\psi$$

For this to be true as an identity, the coefficients of like terms must be the same for all values of x. So we must have both

$$\frac{2b^2\hbar^2}{m}=\frac{1}{2}m\omega^2 \quad\rightarrow\quad b^2=\frac{m^2\omega^2}{4\hbar^2} \quad \text{and} \quad \frac{3b\hbar^2}{m}=E$$

(b) Therefore, $\boxed{b=\frac{m\omega}{2\hbar}}$ and $E=\frac{3b\hbar^2}{m}=\boxed{\frac{3}{2}\hbar\omega}$

(c) The energy levels are $E_n=\left(n+\frac{1}{2}\right)\hbar\omega=\frac{3}{2}\hbar\omega$, so $n=1$, which corresponds to the $\boxed{\text{first excited state}}$.

P40.27 (a) With $\langle x\rangle=0$ and $\langle p_x\rangle=0$, the average value of x^2 is $(\Delta x)^2$ and the average value of p_x^2 is $(\Delta p_x)^2$. We know $\Delta x\geq\frac{\hbar}{2\Delta p_x}$.

The average of the energy is constant:

$$\langle E\rangle=\left\langle\frac{p_x^2}{2m}\right\rangle+\left\langle\frac{k}{2}x^2\right\rangle=\frac{\langle p_x^2\rangle}{2m}+\frac{k}{2}\langle x^2\rangle$$

$$E=\frac{(\Delta p_x)^2}{2m}+\frac{k}{2}(\Delta x)^2\geq\frac{(\Delta p_x)^2}{2m}+\frac{k}{2}\left(\frac{\hbar}{2\Delta p_x}\right)^2$$

$$E\geq\frac{(\Delta p_x)^2}{2m}+\frac{k\hbar^2}{8(\Delta p_x)^2}$$

We rewrite the last equation as $E \geq \dfrac{p_x^2}{2m} + \dfrac{k\hbar^2}{8p_x^2}$

(b) To minimize E as a function of $(\Delta p_x)^2$, we require

$$\frac{d}{d\left[(\Delta p_x)^2\right]}\left[\frac{(\Delta p_x)^2}{2m} + \frac{k\hbar^2}{8(\Delta p_x)^2}\right] = 0$$

$$\frac{1}{2m} + \frac{k\hbar^2}{8}(-1)\frac{1}{(\Delta p_x)^4} = 0$$

Then

$$\frac{k\hbar^2}{8(\Delta p_x)^4} = \frac{1}{2m} \rightarrow (\Delta p_x)^2 = \left(\frac{2mk\hbar^2}{8}\right)^{1/2} = \frac{\hbar\sqrt{mk}}{2}$$

and

$$E \geq \frac{(\Delta p_x)^2}{2m} + \frac{k\hbar^2}{8(\Delta p_x)^2} = \frac{\hbar\sqrt{mk}}{2(2m)} + \frac{k\hbar^2 2}{8\hbar\sqrt{mk}}$$

$$= \frac{\hbar}{4}\sqrt{\frac{k}{m}} + \frac{\hbar}{4}\sqrt{\frac{k}{m}} = \frac{\hbar}{2}\sqrt{\frac{k}{m}}$$

Therefore, $E_{\min} = \dfrac{\hbar}{2}\sqrt{\dfrac{k}{m}} = \boxed{\dfrac{\hbar\omega}{2}}$

***P40.28** **Conceptualize** Be sure you are clear on the notion of *degeneracy*. The states we have seen in this chapter have a degeneracy of 1—that is, a given energy level is associated with only *one* state. In Chapter 41, we will see that the hydrogen atom has energy levels with higher degeneracies.

Categorize We model the system as a *quantum harmonic oscillator*, using the suggestion that the three-dimensional oscillator is a combination of three one-dimensional oscillators.

Analyze Let's begin with the lowest possible energy. For each one-dimensional oscillator, the lowest value of n is $n = 0$. Therefore, the lowest energy for the three-dimensional oscillator is found by using the quantum numbers

$$\left(n_x, n_y, n_z\right) = (0,0,0)$$

From the expression for the energy, this gives us

$$E_{000} = \left(0+0+0+\tfrac{3}{2}\right)\hbar\omega = \boxed{\tfrac{3}{2}\hbar\omega}$$

There is only one way to achieve this energy, so the degeneracy is $\boxed{1}$. Higher energies will be found by increasing the value of $n_x + n_y + n_z$. Because each of the values of n is an integer, the sum must be an integer. Let's let the value of this sum be the next highest possible value $n_x + n_y + n_z = 1$. We see that there are three ways to do this:

$$\left(n_x, n_y, n_z\right) = (1,0,0),(0,1,0),(0,0,1)$$

All three of these choices give an energy of

$$E_{100} = E_{010} = E_{001} = \left(1+\tfrac{3}{2}\right)\hbar\omega = \boxed{\tfrac{5}{2}\hbar\omega}$$

Therefore, the degeneracy of this energy level is $\boxed{3}$. Let's increase the sum $n_x + n_y + n_z$ by 1 again: $n_x + n_y + n_z = 2$. There are six ways to do it:

$$\left(n_x, n_y, n_z\right) = (1,1,0),(1,0,1),(0,1,1),(2,0,0),(0,2,0),(0,0,2)$$

All six of these choices give an energy of

$$E_{110} = E_{101} = E_{011} = \left(2+\tfrac{3}{2}\right)\hbar\omega = \boxed{\tfrac{7}{2}\hbar\omega}$$

Therefore, the degeneracy of this energy level is $\boxed{6}$. Let's increase the sum $n_x + n_y + n_z$ by 1 again so that $n_x + n_y + n_z = 3$. Aha, we find several ways to do this:

$$\begin{aligned}(n_x, n_y, n_z) = &(1,1,1),\\ &(1,2,0),(2,1,0),\\ &(1,0,2),(2,0,1),\\ &(0,1,2),(0,2,1),\\ &(3,0,0),(0,3,0),(0,0,3)\end{aligned}$$

All of these choices give an energy of

$$E_{\text{all}} = \left(3+\tfrac{3}{2}\right)\hbar\omega = \boxed{\tfrac{9}{2}\hbar\omega}$$

Counting up the possibilities, we see that this energy level has a degeneracy of $\boxed{10}$.

Finalize You can probably see that the degeneracies will rise rapidly, corresponding to the large number of ways that you can add three integers to arrive at the same sum.]

Answers: energy, $\frac{3}{2}\hbar\omega$, degeneracy 1; energy, $\frac{5}{2}\hbar\omega$, degeneracy 3; energy, $\frac{7}{2}\hbar\omega$, degeneracy 6; energy, $\frac{9}{2}\hbar\omega$, degeneracy 10

P40.29 Equation 40.32 is $\psi = Be^{-(m\omega/2\hbar)x^2}$, so

$$\frac{d\psi}{dx} = -\left(\frac{m\omega}{\hbar}\right)x\psi \quad \text{and} \quad \frac{d^2\psi}{dx^2} = \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi$$

Substitute into Equation 40.30:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}m\omega^2x^2\psi=E\psi$$

$$-\frac{\hbar^2}{2m}\left[\left(\frac{m\omega}{\hbar}\right)^2x^2\psi+\left(-\frac{m\omega}{\hbar}\right)\psi\right]+\frac{1}{2}m\omega^2x^2\psi=E\psi$$

$$\cancel{-\frac{1}{2}m\omega^2x^2\psi}+\left(\frac{\hbar\omega}{2}\right)\psi+\cancel{\frac{1}{2}m\omega^2x^2\psi}=E\psi$$

$$\left(\frac{\hbar\omega}{2}\right)\psi=E\psi$$

which is satisfied provided that $E=\dfrac{\hbar\omega}{2}$.

P40.30 (a) For the center of mass to be fixed, $m_1u_1+m_2u_2=0$. Then

$$u=|u_1|+|u_2|=|u_1|+\frac{m_1}{m_2}|u_1|=\frac{m_2+m_1}{m_2}|u_1|$$

and

$$|u_1|=\frac{m_2u}{m_1+m_2}$$

Also,

$$u=\frac{m_2}{m_1}|u_2|+|u_2|=\left(\frac{m_2+m_1}{m_1}\right)|u_2| \quad\rightarrow\quad |u_2|=\frac{m_1u}{m_1+m_2}$$

Substitute for $|u_1|$ and $|u_2|$:

$$\begin{aligned}\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2+\frac{1}{2}kx^2&=\frac{1}{2}\frac{m_1m_2^2u^2}{(m_1+m_2)^2}+\frac{1}{2}\frac{m_2m_1^2u^2}{(m_1+m_2)^2}+\frac{1}{2}kx^2\\&=\frac{1}{2}\frac{m_1m_2(m_1+m_2)}{(m_1+m_2)^2}u^2+\frac{1}{2}kx^2\\&=\frac{1}{2}\mu u^2+\frac{1}{2}kx^2\end{aligned}$$

(b) Because the total energy is constant

$$\frac{d}{dx}\left(\frac{1}{2}\mu u^2 + \frac{1}{2}kx^2\right) = 0$$

$$0 = \frac{1}{2}\mu 2u\frac{du}{dx} + \frac{1}{2}k2x = \mu\frac{dx}{dt}\frac{du}{dx} + kx = \mu\frac{du}{dt} + kx = \mu a + kx$$

$$\mu a = -kx$$

$$a = -\frac{kx}{\mu}$$

This is the condition for simple harmonic motion; the acceleration of the equivalent particle is a negative constant times the displacement from equilibrium.

(c) By identification with $a = -\omega^2 x$,

$$\omega = \sqrt{\frac{k}{\mu}} = 2\pi f \text{ and } \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}}$$

Additional Problems

P40.31 (a) From Equation 41.4 for $\psi(x) = Ae^{ikx}$, the first and second derivatives are

$$\frac{d}{dx}\left(Ae^{ikx}\right) = ikAe^{ikx} \quad \text{and} \quad \frac{d^2\psi}{dx^2} = -k^2 Ae^{ikx}$$

Then

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -\frac{\hbar^2}{2m}\left(-k^2 Ae^{ikx}\right) = \frac{\hbar^2 k^2}{2m}\left(Ae^{ikx}\right)$$
$$= \frac{1}{2m}\left(\frac{h}{\cancel{2\pi}}\right)^2\left(\frac{\cancel{2\pi}}{\lambda}\right)^2\left(Ae^{ikx}\right)$$
$$= \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2\left(Ae^{ikx}\right) = \frac{p^2}{2m}\psi = K\psi$$

(b) For $\psi(x) = A\sin\left(\dfrac{2\pi x}{\lambda}\right) = A\sin kx$,

$$\frac{d}{dx}(A\sin kx) = Ak\cos kx \quad \text{and} \quad \frac{d^2\psi}{dx^2} = -Ak^2\sin kx.$$

Then, similarly to the proof in part (a),

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -\frac{\hbar^2}{2m}\left(-Ak^2\sin kx\right) = \frac{\hbar^2 k^2}{2m}\left(Ak^2\sin kx\right) = \frac{p^2}{2m}\psi$$
$$= K\psi$$

P40.32 If we had $n = 0$ for a quantum particle in a box, its momentum would be zero. The uncertainty in its momentum would be zero. The uncertainty in its position would not be infinite, but just equal to the width of the box. Then the uncertainty product would be zero, to violate the uncertainty principle. The contradiction shows that the quantum number cannot be zero. In its ground state the particle has some nonzero zero-point energy.

P40.33 $T = e^{-2CL}$, where $C = \dfrac{\sqrt{2m(U-E)}}{\hbar}$ and where m is in kilograms, and U and E are in joules.

(a) We compute

$$C = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left[\left(0.010\ 0\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)\right]}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 5.12\times10^{8}\ \text{m}^{-1}$$

Then,

$$2CL = 2\left(5.12\times10^{8}\ \text{m}^{-1}\right)\left(0.100\times10^{-9}\ \text{m}\right) = 0.102$$

and $T = e^{-0.102} = \boxed{0.903}$

(b) We compute

$$C = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left[\left(1.00\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)\right]}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 5.12\times10^{9}\ \text{m}^{-1}$$

Then,

$$2CL = 2\left(5.12\times10^{9}\ \text{m}^{-1}\right)\left(0.100\times10^{-9}\ \text{m}\right) = 1.02$$

and $T = e^{-1.02} = \boxed{0.359}$

(c) We compute

$$C = \frac{\sqrt{2\left(6.65\times10^{-27}\ \text{kg}\right)\left[\left(1.00\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)\right]}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 4.37\times10^{14}\ \text{m}^{-1}$$

Then,

$$2CL = 2\left(4.37\times10^{14}\ \text{m}^{-1}\right)\left(1.00\times10^{-15}\ \text{m}\right) = 0.875$$

and $T = e^{-0.875} = \boxed{0.417}$

(d) We compute

$$2CL = 2\frac{\sqrt{2\left(8.00\ \text{kg}\right)\left(1.00\ \text{J}\right)}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}\left(0.020\ 0\ \text{m}\right) = 1.52\times10^{33}$$

Then,

$$T = e^{-1.52\times10^{33}} = e^{(\ln 10)(-1.52\times10^{33}/\ln 10)} = \boxed{10^{-6.59\times10^{32}}}$$

P40.34 From Equation 40.14, the energy levels of an electron in an infinitely deep potential well are proportional to n^2. If the energy of the ground state, $n = 1$, is $E_1 = 0.300$ eV, the energy levels of the states $n = 2$, 3, and 4 are

$$E_2 = 2^2(0.300 \text{ eV}) = 1.20 \text{ eV}$$
$$E_3 = 3^2(0.300 \text{ eV}) = 2.70 \text{ eV}$$
$$E_4 = 4^2(0.300 \text{ eV}) = 4.80 \text{ eV}$$

(a) For the transition from the $n = 3$ level to the $n = 1$ level, the electron loses energy

$$E = \frac{hc}{\lambda} = E_3 - E_1 = 2.70 \text{ eV} - 0.300 \text{ eV} = 2.40 \text{ eV}$$
$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV}\cdot\text{nm}}{2.40 \text{ eV}} = 517 \text{ nm}$$

(b) For the transition from level 2 to level 1,

$$E = 1.20 \text{ eV} - 0.300 \text{ eV} = 0.900 \text{ eV}$$

and

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.900 \text{ eV}} = 1\ 380 \text{ nm} = \boxed{1.38\ \mu\text{m}}$$

This photon, with wavelength greater than 700 nm, is in the $\boxed{\text{infrared}}$ region.

In like manner, we find

for 3 to 2: $\Delta E = 1.50$ eV, and $\lambda = \boxed{827 \text{ nm, infrared}}$

for 4 to 1: $\Delta E = 4.50$ eV, and $\lambda = \boxed{275 \text{ nm, ultraviolet}}$

for 4 to 2: $\Delta E = 3.60$ eV, and $\lambda = \boxed{344 \text{ nm, near ultraviolet}}$

for 4 to 3: $\Delta E = 2.10$ eV, and $\lambda = \boxed{590 \text{ nm, yellow-orange visible}}$

***P40.35** **Conceptualize** The reason that the picture slips into the configuration shown in Figure P40.35b is that it represents a lower gravitational potential energy of the picture–Earth system. Because of the lack of friction, as soon as the symmetric position in Figure P40.35a is disturbed, the wire slips on the nail, and gravity pulls the picture until it looks like Figure P40.35b.

Categorize The combination of the picture and the Earth is a two-state quantized isolated system.

Analyze You can raise the picture from the state in Figure P40.35b to that in Figure P40.35a by doing work on the picture. The appropriate reduction of Equation 8.2 is

$$\Delta U_g = W \qquad (1)$$

where W is the energy we must put into the system. Expand the term ΔU_g:

$$W = U_a - U_b \qquad (2)$$

where the subscripts a and b refer to the two parts of Figure P40.35. Evaluate the initial and final potential energies relative to $U_g = 0$ if the center of mass of the picture were at the nail, recognizing that the center of mass is below the nail by about half the *height* in configuration a and by about half the *diagonal* in configuration b:

$$W=-mg\left(\tfrac{1}{2}h\right)-\left[-mg\left(\tfrac{1}{2}\sqrt{h^2+\ell^2}\right)\right]=\boxed{\tfrac{1}{2}mgh\left(\sqrt{1+\frac{\ell^2}{h^2}}-1\right)}$$

Finalize What if the wire is very long? For example, suppose the picture hangs from a picture rail near the ceiling. Is there still a two-state system?]

Answer: $\tfrac{1}{2}mgh\left(\sqrt{1+\frac{\ell^2}{h^2}}-1\right)$

P40.36 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U=mgy=(0.02\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m})=0.023\,5\text{ J}$$

and

$$E=K=\frac{1}{2}mu^2=\frac{1}{2}(0.02\text{ kg})(0.8\text{ m/s})^2=0.006\,4\text{ J}$$

Then,

$$C=\frac{\sqrt{2m(U-E)}}{\hbar}=\frac{\sqrt{2(0.02\text{ kg})(0.017\,1\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}=2.5\times10^{32}\text{ m}^{-1}$$

and the transmission coefficient is

$$e^{-2CL}=e^{-2\left(2.5\times10^{32}\right)\left(2\times10^{-3}\right)}=e^{-10\times10^{29}}=e^{-2.30\left(4.3\times10^{29}\right)}$$
$$=10^{-4.3\times10^{29}}=\boxed{\sim10^{-10^{30}}}$$

P40.37 (a) From Equation 41.14, the allowed energy levels are

$$E_n=\left(\frac{h^2}{8mL^2}\right)n^2\qquad n=1,2,3,\ldots$$

The energy of the absorbed photon is

$$E = \Delta E_n = E_3 - E_1 = \left(\frac{h^2}{8m_eL^2}\right)(3)^2 - \left(\frac{h^2}{8m_eL^2}\right)(1)^2 = 8\left(\frac{h^2}{8m_eL^2}\right)$$

We determine the length of the box from

$$\frac{hc}{\lambda} = \frac{h^2}{m_eL^2} \quad \rightarrow \quad \boxed{L = \left(\frac{h\lambda}{m_ec}\right)^{1/2}}$$

(b) The energy lost during the $n = 3$ to $n = 2$ transition is

$$E' = E_3 - E_2 = \left(\frac{h^2}{8m_eL^2}\right)(3)^2 - \left(\frac{h^2}{8m_eL^2}\right)(2)^2 = 5\left(\frac{h^2}{8m_eL^2}\right)$$

The wavelength of the emitted photon is then

$$\frac{hc}{\lambda'} = \frac{5h^2}{8m_eL^2} = \frac{5h^2}{8\cancel{m_e}}\left(\frac{\cancel{m_e}c}{h\lambda}\right) \quad \rightarrow \quad \boxed{\lambda' = \frac{8}{5}\lambda}$$

P40.38 $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\psi|^2\,dx$

For a one-dimensional box of width L, from Equation 41.18,

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

$$\langle x^2\rangle = \frac{2}{L}\int_0^L x^2\sin^2\left(\frac{n\pi x}{L}\right)dx$$

With the substitution

$$y = \frac{n\pi x}{L} \quad \rightarrow \quad dy = \frac{n\pi}{L}dx$$

$$x = \frac{L}{n\pi}y \quad \rightarrow \quad dx = \frac{L}{n\pi}dy$$

the integral becomes (from integral tables)

$$\langle x^2 \rangle = \frac{2}{L}\left(\frac{L}{n\pi}\right)^3 \int_0^{n\pi} x^2 \sin^2 y\, dy$$

$$= \frac{2L^2}{(n\pi)^3}\left[\frac{y^3}{6} - \left(\frac{y^2}{4} - \frac{1}{8}\right)\sin 2y - \frac{y}{4}\cos 2y\right]\Bigg|_0^{n\pi}$$

$$= \frac{2L^2}{(n\pi)^3}\left[\frac{(n\pi)^3}{6} - \frac{n\pi}{4}\cos 2(n\pi)\right]$$

$$= \frac{2L^2}{(n\pi)^3}\left[\frac{(n\pi)^3}{6} - \frac{n\pi}{4}\right] = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

P40.39 (a) The requirements that $\frac{n\lambda}{2} = L$ and $p = \frac{h}{\lambda} = \frac{nh}{2L}$ are still valid.

From the relativistic energy of the particle,

$$E = \sqrt{(pc)^2 + (mc^2)^2} \Rightarrow E_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2}$$

its kinetic energy is therefore

$$K_n = E_n - mc^2 = \boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2} - mc^2}$$

(b) Taking $L = 1.00 \times 10^{-12}$ m, $m = 9.11 \times 10^{-31}$ kg, and $n = 1$, we find

$$K_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2} - mc^2$$

$$= \left\{\left[\frac{(1)(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{2(1.00\times10^{-12}\ \text{m})}\right]^2 + \left[(9.11\times10^{-31}\ \text{kg})\left(2.998\times10^{8}\ \frac{\text{m}}{\text{s}}\right)^2\right]^2\right\}^{1/2}$$

$$- (9.11\times10^{-31}\ \text{kg})\left(2.998\times10^{8}\ \frac{\text{m}}{\text{s}}\right)^2$$

$$= \boxed{4.68\times10^{-14}\ \text{J}}$$

(c) The particle's nonrelativistic energy is

$$E_1 = \frac{h^2}{8mL^2} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.00\times10^{-12}\ \text{m}\right)^2} = 6.02\times10^{-14}\ \text{J}$$

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

P40.40 Looking at Figure 40.7, we see that wavelengths for a particle in a finite well are longer than those for a particle in an infinite well. Therefore, the energies of the allowed states should be lower for a finite well than for an infinite well. As a result, the photons from the source have too much energy to be absorbed or, equivalently, the photons have a frequency that is too high. In order to lower their apparent frequency using the Doppler shift, the source would have to move *away* from the particle in the finite square well, not *toward* it.

***P40.41** **Conceptualize** Classically, as long as $E > U$, the particles would all pass the step and travel with a reduced speed. Quantum mechanically, a change in the potential results in both transmission and reflection, just like a change in the medium causes both transmission and reflection for mechanical waves.

Categorize The particles are modeled as quantum particles, encountering a potential step.

Analyze Because the problem asks for energies, let us modify the reflection coefficient to be expressed in terms of energies. We first divide the numerator and denominator of the reflection coefficient by

k_1^2:

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{\left(1 - \frac{k_2}{k_1}\right)^2}{\left(1 + \frac{k_2}{k_1}\right)^2} \qquad (1)$$

Now, evaluate the ratio of k values:

$$\frac{k_2}{k_1} = \frac{\frac{\sqrt{2m(E-U)}}{\hbar}}{\frac{\sqrt{2mE}}{\hbar}} = \sqrt{\frac{E-U}{E}} = \sqrt{1 - \frac{U}{E}} = \sqrt{1-f} \qquad (2)$$

where f is the ratio of energies, U/E. Substitute Equation (2) into Equation (1):

$$R = \frac{\left(1 - \sqrt{1-f}\right)^2}{\left(1 + \sqrt{1-f}\right)^2} \qquad (3)$$

(a) If half of the incident particles reflect from the step, we must have $R = \frac{1}{2}$. Therefore, from Equation (3),

$$\frac{1}{2} = \frac{\left(1 - \sqrt{1-f}\right)^2}{\left(1 + \sqrt{1-f}\right)^2} \quad \rightarrow \quad \left(1 - \sqrt{1-f}\right)^2 = \tfrac{1}{2}\left(1 + \sqrt{1-f}\right)^2 \qquad (4)$$

Take the square root of both sides of Equation (4), choosing the positive root:

$$1 - \sqrt{1-f} = \frac{1}{\sqrt{2}}\left(1 + \sqrt{1-f}\right) \quad \rightarrow \quad \sqrt{1-f} = \frac{1-\sqrt{2}}{1+\sqrt{2}} = -0.172 \qquad (5)$$

Solve for f:

$$f = 1 - (-0.172)^2 = 0.971$$

Now, because $f = U/E$, we have

$$f = \frac{U}{E} \quad \rightarrow \quad E = \frac{U}{f} = \frac{U}{0.971} = \boxed{1.03U}$$

(b) The fraction *a* by which the kinetic energy of the particles changes upon being transmitted through the step is

$$a = \frac{E - U}{E}$$

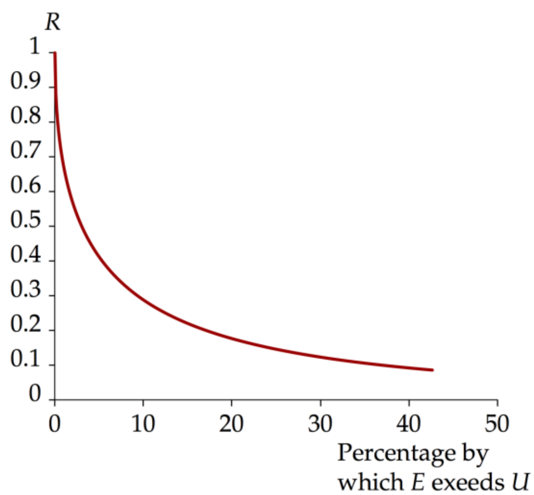
Substitute the value of *E* in part (a):

$$a = \frac{1.03U - U}{1.03U} = 0.029\,4$$

Set this result equal to the ratio of kinetic energies of the particles in terms of their speeds:

$$0.029\,4 = \frac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \frac{v_2^2}{v_1^2} \quad \rightarrow \quad \frac{v_2}{v_1} = \sqrt{0.029\,4} = \boxed{0.172}$$

Finalize Notice that the particle energy must be close to the potential height for a significant number of particles to be reflected. In part (a), to reflect 50% of the particles, the incident energy is only 3% larger than the barrier height. At much higher energies, a small fraction of the particles is reflected. The graph below shows that the reflection coefficient falls rapidly as the energy *E* increases.



The remarkable thing is that *any* particles are reflected. Imagine rolling a bowling ball in the street toward a disabled ramp in a sidewalk, with plenty of energy for the ball to roll up on the sidewalk, and have it reflect from the ramp! This is just one example of the surprising nature of quantum mechanics.]

Answers: (a) 1.03U (b) 0.172

P40.42 (a) Taking $L_x = L_y = L$, we see that the expression for E becomes

$$\boxed{E = \frac{h^2}{8m_eL^2}\left(n_x^2 + n_y^2\right)}$$

The general form of the wave function is

$$\psi \sim \sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$$

For a normalizable wave function, neither n_x nor n_y can be zero, otherwise $\psi = 0$.

(b) The ground state corresponds to $\boxed{n_x = n_y = 1}$.

(c) The energy of the ground state is

$$E_{1,\,1} = \frac{h^2}{8m_e L^2}\left(1^2 + 1^2\right) = \boxed{\frac{h^2}{4m_e L^2}}$$

(d) For the first excited state, $\boxed{n_x = 1 \text{ and } n_y = 2, \text{ or } n_x = 2 \text{ and } n_y = 1}$.

(e) For the second excited state, $\boxed{n_x = 2 \text{ and } n_y = 2}$.

(f) The second excited state, corresponding to $n_x = 2$, $n_y = 2$, has an energy of

$$E_{2,\,2} = \frac{h^2}{8m_e L^2}\left(2^2 + 2^2\right) = \boxed{\frac{h^2}{m_e L^2}}$$

(g) The energy difference between the ground state and the second excited state is

$$\Delta E = E_{2,\,2} - E_{1,\,1} = \frac{h^2}{m_e L^2} - \frac{h^2}{4m_e L^2} = \boxed{\frac{3h^2}{4m_e L^2}}$$

(h) $\Delta E = \dfrac{3h^2}{4m_e L^2} = \dfrac{hc}{\lambda} \quad \rightarrow \quad \lambda = \boxed{\dfrac{4m_e cL^2}{3h}}$

P40.43 (a) For a particle with wave function

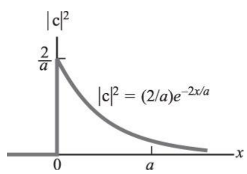
$$\psi(x) = \begin{cases} \sqrt{\dfrac{2}{a}}\, e^{-x/a} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

The probability densities are

$$|\psi(x)|^2 = 0 \qquad \text{for } x < 0$$

and $\left|\psi^2(x)\right| = \frac{2}{a}e^{-2x/a}$ for $x > 0$.

ANS. FIG. P40.43. shows a sketch of the probability density for this particle.



ANS. FIG. P40.43

(b) The probability is obtained from

$$\text{Prob}(x<0) = \int_{-\infty}^{0} \left|\psi(x)\right|^2 dx = \int_{-\infty}^{0} (0)\,dx = \boxed{0}$$

(a) For the wave function to be normalized, we require

$$\int_{-\infty}^{\infty} \left|\psi(x)\right|^2 dx = \int_{-\infty}^{0} |\psi|^2 dx + \int_{0}^{\infty} |\psi|^2 dx = 1$$

Performing the integration gives

$$\int_{-\infty}^{0} 0dx + \int_{0}^{\infty}\left(\frac{2}{a}\right)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

(b) The probability is obtained from

$$\text{Prob}(0<x<a) = \int_0^a |\psi|^2 dx = \int_0^a \left(\frac{2}{a}\right)e^{-2x/a} dx = -e^{-2x/a}\Big|_0^a$$

$$= 1 - e^{-2} = \boxed{0.865}$$

Challenge Problems

P40.44 (a) The potential energy of the system is given by

$$U = \frac{e^2}{4\pi \in_0 d}\left[\left(-1+\frac{1}{2}-\frac{1}{3}\right)+\left(-1+\frac{1}{2}\right)+(-1)\right] = \frac{(-7/3)e^2}{4\pi \in_0 d}$$

$$= \boxed{-\frac{7k_e e^2}{3d}}$$

(b) There are two electrons, each with minimum energy E_1. From Equation 41.14, the total energy is

$$K = 2E_1 = \frac{2h^2}{8m_e(3d)^2} = \boxed{\frac{h^2}{36m_e d^2}}$$

(c) The total energy of the system is

$$E = K + U = \frac{h^2}{36m_e d^2} - \frac{7k_e e^2}{3d}$$

For a minimum, we require $\frac{dE}{d(d)} = 0$. Differentiating,

$$\frac{dE}{d(d)} = 0$$

$$\frac{d}{d(d)}\left(\frac{h^2}{36m_e d^2} - \frac{7k_e e^2}{3d}\right) = 0$$

$$(-2)\frac{h^2}{36m_e d^3} - (-1)\frac{7k_e e^2}{3d^2} = 0$$

$$\frac{h^2}{18m_e d^3} = \frac{7k_e e^2}{3d^2}$$

$$d = \frac{3h^2}{7(18m_e)k_e e} = \frac{h^2}{42m_e k_e e^2}$$

Substituting numerical values,

$$d = \frac{\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)^2}{(42)\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}$$
$$= 4.99\times10^{-11}\ \mathrm{m} = \boxed{49.9\ \mathrm{pm}}$$

(d) The lithium spacing is d and the number of atoms N in volume V is related by $Nd^3 = V$, and the density is $\frac{Nm}{V}$, where m is the mass of one atom. We have:

$$\text{density} = \frac{Nm}{V} = \frac{Nm}{Nd^3} = \frac{m}{d^3}$$

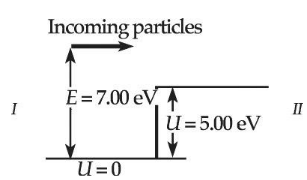
From which we obtain

$$d = \left(\frac{m}{\text{density}}\right)^{1/3} = \left[\frac{\frac{6.94\ \mathrm{g}}{\mathrm{mol}}\left(\frac{1\ \mathrm{mol}}{6.022\times10^{23}\ \text{atoms}}\right)}{0.530\ \frac{\mathrm{g}}{\mathrm{cm^3}}}\right]^{1/3}$$
$$= 2.79\times10^{-8}\ \mathrm{cm} = 2.79\times10^{-10}\ \mathrm{mm} = 279\ \mathrm{pm}$$

$$\boxed{\begin{array}{l}\text{The lithium interatomic spacing of 280 pm is 5.59 times larger.}\\ \text{Therefore, it is of the same order of magnitude as the interatomic}\\ \text{spacing } 2d \text{ here.}\end{array}}$$

P40.45 (a) The claim is that Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$



ANS. FIG. P40.45(a)

has the solutions

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \quad \text{[region } I\text{]}$$

$$\psi_2 = Ce^{ik_2x} \qquad \text{[region } II\text{]}$$

Check that the solution for region *I* satisfies Schrödinger's equation:

$$\frac{\partial^2\psi_1}{\partial x^2} = -\frac{2m}{\hbar^2}E\psi_1$$

$$\frac{\partial^2}{\partial x^2}\left(Ae^{ik_1x}\right) + \frac{\partial^2}{\partial x^2}\left(Be^{-ik_1x}\right) = -\frac{2m}{\hbar^2}E\left(Ae^{ik_1x} + Be^{-ik_1x}\right)$$

$$-k_1^2\left(Ae^{ik_1x}\right) - k_1^2\left(Be^{-ik_1x}\right) = -\frac{2m}{\hbar^2}E\left(Ae^{ik_1x} + Be^{-ik_1x}\right)$$

$$-k_1^2\left(Ae^{ik_1x} + Be^{-ik_1x}\right) = -\frac{2m}{\hbar^2}E\left(Ae^{ik_1x} + Be^{-ik_1x}\right)$$

The last line is true if $k_1^2 = \frac{2m}{\hbar^2}E$, which it is because

$$E = \frac{p^2}{2m} = \frac{\left(\hbar k_1\right)^2}{2m} \rightarrow k_1 = \frac{\sqrt{2mE}}{\hbar}$$

Therefore, the equation is satisfied in region *I*.

Check that the solution for region *II* satisfies Schrödinger's equation:

$$\frac{\partial^2\psi_2}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi_2$$

$$\frac{\partial^2}{\partial x^2}\left(Ce^{ik_2x}\right) = -\frac{2m}{\hbar^2}(E-U)\left(Ce^{ik_2x}\right)$$

$$-k_2^2\left(Ce^{ik_2x}\right) = -\frac{2m}{\hbar^2}(E-U)\left(Ce^{ik_2x}\right)$$

The last line is true if $k_2^2 = \frac{2m}{\hbar^2}(E-U)$, which it is because

$$E = \frac{p^2}{2m} + U = \frac{\left(\hbar k_2\right)^2}{2m} \rightarrow k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$$

Therefore, the equation is satisfied in region *II*. We apply boundary conditions. Matching functions and derivatives at $x = 0$, we find that

$$(\psi_1)_0 = (\psi_2)_0 \quad \text{gives} \quad A + B = C,$$

and $$\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0 \quad \text{gives} \quad k_1(A - B) = k_2 C.$$

Then $B = \dfrac{1 - k_2/k_1}{1 + k_2/k_1}A$ and $C = \dfrac{2}{1 + k_2/k_1}A$.

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability

$$R = \frac{B^2}{A^2} = \frac{(1 - k_2/k_1)^2}{(1 + k_2/k_1)^2} = \boxed{\frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}}$$

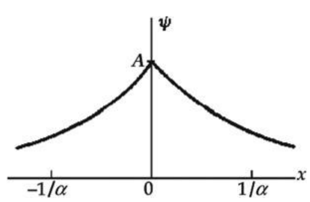
(b) With $E = 7.00$ eV and $U = 5.00$ eV:

$$\frac{k_2}{k_1} = \sqrt{\frac{E - U}{E}} = \sqrt{\frac{2.00 \text{ eV}}{7.00 \text{ eV}}} = 0.535$$

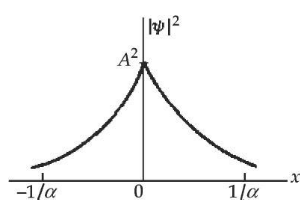
The reflection probability is $R = \dfrac{(1 - 0.535)^2}{(1 + 0.535)^2} = \boxed{0.092\ 0}$.

(c) The probability of transmission is $T = 1 - R = \boxed{0.908}$.

P40.46 (a) and (b) The Wave functions are shown in ANS. FIG. P40.46(a) and ANS. FIG. P40.46(b).



ANS. FIG. P40.46(a)



ANS. FIG. P40.46(b)

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$.

(d) Since ψ is symmetric,

$$\int_{-\infty}^{\infty} |\psi|^2 \, dx = 2\int_{0}^{\infty} |\psi|^2 \, dx = 1$$

or $$2A^2 \int_0^{\infty} e^{-2\alpha x} \, dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1.$$

This gives $\boxed{A = \sqrt{\alpha}}$.

(e) The probability of finding the particle between $-1/2\alpha$ and $+1/2\alpha$ is

$$P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{a}\right)^2 \int_{x=0}^{1/2\alpha} e^{-2\alpha x} \, dx = \left(\frac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha} - 1\right)$$
$$= \left(1 - e^{-1}\right) = \boxed{0.632}$$

P40.47 (a) Recall from Section 40.7 that the potential energy of a harmonic oscillator is $\frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$. We can find the energy of the oscillator E by substituting the wave function into the Schrödinger equation.

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi \quad \to \quad \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2 \psi = E\psi$$

From $\psi = Bxe^{-(m\omega/2\hbar)x^2}$, we have

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(-\frac{m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2}$$

$$= Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2}$$

$$- B\left(\frac{m\omega}{\hbar}\right)x^2\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2 x^3e^{-(m\omega/2\hbar)x^2}$$

Substituting the above into the Schrödinger equation, we have

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

$$\frac{-\hbar^2}{2m}\left[-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2 x^3e^{-(m\omega/2\hbar)x^2}\right]$$

$$+\frac{1}{2}m\omega^2x^2\left[Bxe^{-(m\omega/2\hbar)x^2}\right]$$

$$= E\left[Bxe^{-(m\omega/2\hbar)x^2}\right]$$

$$\left(\frac{3\hbar\omega}{2}\right)\left[Bxe^{-(m\omega/2\hbar)x^2}\right] + \cancel{\left(-\frac{1}{2}m\omega^2x^2\right)\left[Bxe^{-(m\omega/2\hbar)x^2}\right]}$$

$$+ \cancel{\left(\frac{1}{2}m\omega^2x^2\right)\left[Bxe^{-(m\omega/2\hbar)x^2}\right]}$$

$$= E\left[Bxe^{-(m\omega/2\hbar)x^2}\right]$$

$$\left(\frac{3\hbar\omega}{2}\right)\left(Bxe^{-(m\omega/2\hbar)x^2}\right) = E\left(Bxe^{-(m\omega/2\hbar)x^2}\right)$$

The last line is true if $\boxed{E = \frac{3\hbar\omega}{2}}$.

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) ψ is maximized if

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2 e^{-(m\omega/2\hbar)x^2} = 0$$

$$1-\left(\frac{m\omega}{\hbar}\right)x^2 = 0$$

which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$.

(d) We require $\int\limits_{-\infty}^{\infty} |\psi|^2\,dx = 1$:

$$1 = \int\limits_{-\infty}^{\infty} B^2 x^2 e^{-(m\omega/\hbar)x^2}\,dx = 2B^2 \int x^2 e^{-(m\omega/\hbar)x^2}\,dx$$

$$= 2B^2 \frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = B^2 \frac{\pi^{1/2}}{2}\left(\frac{\hbar}{m\omega}\right)^{3/2}$$

Then,

$$B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$$

(e) At $x = 2(\hbar/m\omega)^{1/2}$, the potential energy is

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2\left(\frac{4\hbar}{m\omega}\right) = 2\hbar\omega$$

This is larger than the total energy $\frac{3\hbar\omega}{2}$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) The actual probability is given by

$$P = |\psi|^2\,dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2\delta$$

$$P=\delta B^2x^2e^{-(m\omega/\hbar)x^2}=\delta\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)x^2}$$

$$=\delta\frac{2}{\pi^{1/2}}\left(\frac{m^{3/2}\omega^{3/2}}{\hbar^{3/2}}\right)\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)}=\boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}}$$

P40.48 (a) To find the normalization constant, we note that $\int_0^L|\psi|^2\,dx=1$, or

$$A^2\int_0^L\left[\sin^2\left(\frac{\pi x}{L}\right)+16\sin^2\left(\frac{2\pi x}{L}\right)+8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right]dx=1$$

Noting that

$$\int_0^L\sin^2\left(\frac{\pi x}{L}\right)dx=\int_0^L\frac{1-\cos\left[2(\pi x/L)\right]}{2}dx$$

$$=\left[\frac{x}{2}-\frac{L}{\pi}\frac{\sin(2\pi x/L)}{2}\right]_0^L=\frac{L}{2}$$

the integral becomes

$$\int_0^L|\psi|^2\,dx=A^2\left[\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)+8\int_0^L\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right]$$

$$1=A^2\left\{\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)\right.$$

$$\left.+8\int_0^L\sin\left(\frac{\pi x}{L}\right)\left[2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)\right]dx\right\}$$

$$1=A^2\left[\frac{17L}{2}+16\int_0^L\sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right]$$

$$1=A^2\left[\frac{17L}{2}+\frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Bigg|_{x=0}^{x=L}\right]=A^2\left(\frac{17L}{2}\right)$$

$$\rightarrow\ \boxed{A=\sqrt{\frac{2}{17L}}}$$

(b) To determine the relationship between A and B, we note that $\int_{-a}^{a} |\psi|^2\, dx = 1$. Therefore,

$$\int_{-a}^{a} \left[|A|^2 \cos^2\left(\frac{\pi x}{2a}\right) + |B|^2 \sin^2\left(\frac{\pi x}{a}\right) + 2|A||B| \cos\left(\frac{\pi x}{2a}\right) \sin\left(\frac{\pi x}{a}\right) \right] dx = 1$$

Noting that

$$\int_{-a}^{a} \sin^2\left(\frac{\pi x}{2a}\right) dx = \int_{-a}^{a} \frac{1 - \cos\left[2(\pi x/2a)\right]}{2} dx = \left[\frac{x}{2} + \frac{2L}{\pi} \frac{\sin(\pi x/a)}{2}\right]_{-a}^{a} = a$$

and

$$\int_{-a}^{a} \cos^2\left(\frac{\pi x}{2a}\right) dx = \int_{-a}^{a} \frac{1 + \cos\left[2(\pi x/2a)\right]}{2} dx = \left[\frac{x}{2} + \frac{2L}{\pi} \frac{\sin(\pi x/a)}{2}\right]_{-a}^{a} = a$$

the integral becomes

$$|A|^2 a + |B|^2 a + \int_{-a}^{a} \left[2|A||B| \cos\left(\frac{\pi x}{2a}\right) \sin\left(\frac{\pi x}{a}\right) \right] dx = 1$$

The third term is:

$$2|A||B| \int_{-a}^{a} \cos\left(\frac{\pi x}{2a}\right) \left[2\sin\left(\frac{\pi x}{2a}\right) \cos\left(\frac{\pi x}{2a}\right) \right] dx$$

$$= 4|A||B| \int_{-a}^{a} \cos^2\left(\frac{\pi x}{2a}\right) \sin\left(\frac{\pi x}{2a}\right) dx$$

$$= \frac{8a|A||B|}{3\pi} \cos^3\left(\frac{\pi x}{2a}\right)\Big|_{-a}^{a} = 0$$

so the whole integral is

$$a\left(|A|^2+|B|^2\right)=1, \text{ giving } \boxed{|A|^2+|B|^2=\frac{1}{a}}$$

ANSWERS TO QUICK-QUIZZES

1. (d)

2. (i) (a) (ii) (d)

3. (c)

4. (a), (c), (f)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P40.2 (a) See ANS. FIG. P40.2; (b) $\dfrac{1}{\sqrt{a}}$; (c) 0.865

P40.4 The photon does not have the smallest possible energy to cause the transition between states $n = 1$ to $n = 2$.

P40.6 (a) 5.13×10^{-3} eV; (b) 9.41 eV; (c) The electron has a much higher energy because it is much less massive.

P40.8 (a) 2.00×10^{-9} J; (b) 1.66×10^{-28} m; (c) No. The length of the box would have to be much smaller than the size of a nucleus (~10^{-14} m) to confine the particle.

P40.10 See P40.10 for full explanation.

P40.12 (a) $x = L/4$, $L/2$, and $3\,L/4$; (b) We look for $\sin(3\pi x/L)$ taking on its extreme values 1 and –1 so that the squared wave function is as large as it can be. The result can also be found by studying Figure 40.12b.

P40.14 (a) 0.332 nm (b) 45.6nm, quite different from 102.6 nm

P40.16 See P40.16 for full explanation.

P40.18 (a) $n = 1$: $\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$; $P_1(x) = |\psi_1(x)|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$,

$n = 2$: $\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$; $P_2(x) = |\psi_2(x)|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$,

$n = 3$: $\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$; $P_3(x) = |\psi_3(x)|^2 = \frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$;

(b) See ANS FIG. P40.18(b).

P40.20 (a) See ANS. FIG. P40.20, (b) $\boxed{2L}$

P40.22 3.92%

P40.24 $2\pi c\sqrt{\frac{m}{k}}$

P40.26 (a) See P40.26 (a) for full explanation; (b) $b = \frac{m\omega}{2\hbar}$ and $\frac{3}{2}\hbar\omega$;

(c) first excited state

P40.28 energy, $\frac{3}{2}\hbar\omega$, degeneracy 1; energy, $\frac{5}{2}\hbar\omega$, degeneracy 3; energy, $\frac{7}{2}\hbar\omega$, degeneracy 6; energy, $\frac{9}{2}\hbar\omega$, degeneracy 10

P40.30 (a) See P40.30(a) for full explanation; (b) See P40.30(b) for full explanation; (c) $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$

P40.32 See P40.32 for full explanation

P40.34 (a) See P40.34(a) for full proof; (b) For 2 to 1, $\lambda = 1.38\ \mu$m, infrared; For 3 to 2, $\lambda = 827$ nm, infrared; For 4 to 1, $\lambda = 275$ nm, ultraviolet; For 4 to 2, λ = 344 nm, near ultraviolet; For 4 to 3, $\lambda = 590$ nm, yellow-orange visible.

P40.36 (a) See P40.36(a) for full explanation; (b) $b = \dfrac{m\omega}{2\hbar}$ and $\dfrac{3}{2}\hbar\omega$; (c) first excited state

P40.38 $\sim 10^{-10^{30}}$

P40.40 Looking at Figure 40.4, we see that wavelengths for a particle in a finite well are longer than those for a particle in an infinite well. Therefore, the energies of the allowed states should be lower for a finite well than for an infinite well. As a result, the photons from the source have too much energy to be absorbed or, equivalently, the photons have a frequency that is too high. In order to lower their apparent frequency using the Doppler shift, the source would have to move *away* from the particle in the finite square well, not *toward* it.

P40.42 (a) $E = \dfrac{h^2}{8m_eL^2}\left(n_x^2 + n_y^2\right)$; (b) $n_x = n_y = 1$; (c) $\dfrac{h^2}{4m_eL^2}$; (d) $n_x = 1$ and $n_y = 2$, or $n_x = 2$ and $n_y = 1$; (e) $n_x = 2$ and $n_y = 2$; (f) $\dfrac{h^2}{m_eL^2}$; (g) $\dfrac{3h^2}{4m_eL^2}$; (h) $\dfrac{4m_ecL^2}{3h}$

P40.44 (a) $-\dfrac{7k_ee^2}{3d}$; (b) $\dfrac{h^2}{36m_ed^2}$; (c) 49.9 pm; (d) The lithium interatomic spacing of 280 pm is 5.59 times larger. Therefore, it is of the same order of magnitude as the interatomic spacing $2d$ here.

P40.46 (a) See ANS. FIG. P40.46(a); (b) See ANS. FIG. P40.46(b); (c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$; (d) $A = \sqrt{\alpha}$; (e) 0.632

P40.48 (a) $A = \sqrt{\dfrac{2}{17L}}$; (b) $|A|^2 + |B|^2 = \dfrac{1}{a}$