41

Atomic Physics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP41.1** **Conceptualize** Be sure you understand the material in Section 41.10, especially Figure 41.29. We will be building a laser with the same mirror construction as the one in that figure.

Categorize We will need to analyze a quantized energy-level diagram for the laser transitions, and apply the *wave under boundary conditions* model for the specific case of thin film interference for the mirror.

Analyze (a) In order to determine the wavelength for the $E_2 - E_1$ transition, we need to find the energy E_2. Looking at the transition marked "Green" in Figure TP41.1, we see that

$$\Delta E = \frac{hc}{\lambda_g} = E_4{}^* - E_2 \quad \rightarrow \quad E_2 = E_4{}^* - \frac{hc}{\lambda_g} \tag{1}$$

where λ_g is the wavelength of the green light. Now, the wavelength of the $E_2 - E_1$ transition is

$$\lambda_{2-1} = \frac{hc}{\Delta E_{2-1}} = \frac{hc}{E_2 - 0} = \frac{hc}{E_2} \tag{2}$$

Substitute Equation (1) into Equation (2):

$$\lambda_{2-1} = \frac{hc}{E_4{}^* - \dfrac{hc}{\lambda_g}} = \frac{1}{\dfrac{E_4{}^*}{hc} - \dfrac{1}{\lambda_g}} \tag{3}$$

Substitute numerical values:

$$\lambda_{2-1} = \frac{1}{\dfrac{20.66\text{ eV}}{1\,240\text{ eV}\cdot\text{nm}} - \dfrac{1}{543.0\text{ nm}}} = \boxed{67.5\text{ nm}}$$

(b) The silicon dioxide film is between layers of titanium dioxide. Therefore, Equation 36.12 applies to *constructive* interference:

$$2nt = \left(m_g + \tfrac{1}{2}\right)\lambda_g \qquad (4)$$

where m_g is an order number for green light, for which we want constructive interference, so that the mirror reflects the green light back into the cavity. Equation 36.13 applies to *destructive* interference:

$$2nt = m_r\lambda_r \qquad (5)$$

where m_r is an order number for red light, for which we want destructive interference, so that the mirror transmits the red light so that does not grow in strength. In both Equation (4) and (5), n is the index of refraction of the silicon dioxide and t is the thickness of the silicon dioxide film that reflects green light and transmits red light. Divide Equation (5) by Equation (4):

$$\frac{2nt}{2nt} = \frac{m_r\lambda_r}{\left(m_g + \tfrac{1}{2}\right)\lambda_g} \quad \rightarrow \quad 1 = \frac{m_r(632.8\text{ nm})}{\left(m_g + \tfrac{1}{2}\right)(543.0\text{ nm})} = 1.165\frac{m_r}{m_g + \tfrac{1}{2}}$$

$$\rightarrow \quad m_r = \frac{m_g + \tfrac{1}{2}}{1.165} \qquad (6)$$

Now, m_g must be a nonnegative integer beginning at 0. Lets evaluate the right side of Equation (6) for various values of m_g and see if the result is an integer for m_r:

m_g	$\dfrac{m_g + \frac{1}{2}}{1.165}$
0	0.429
1	1.29
2	2.15
3	3.00

Ah-ha! We see that a film of thickness such that $m_g = 3$ and $m_r = 3$ will reflect green light and transmit red light. Therefore, from Equation (5),

$$t = \frac{m_r \lambda_r}{2n} = \frac{3(632.8 \text{ nm})}{2(1.458)} = \boxed{651 \text{ nm}}$$

Finalize The wavelength in part (a) is in the ultraviolet region of the spectrum. Perhaps some eye shielding might be necessary to avoid danger from this radiation.]

Answers: (a) 67.5 nm (b) 651 nm

***TP41.2**

Answer:

Shell	n	**4**															
Subshell	λ	0	1			2					3						
Orbital	m_ℓ	0	1	0	–1	2	1	0	–1	–2	3	2	1	0	–1	–2	–3
	m_s	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 41.1 Atomic Spectra of Gases

P41.1 (a) The wavelengths in the Lyman series of hydrogen are given by

$$\frac{1}{\lambda} = R_{\text{H}}\left(1 - \frac{1}{n^2}\right)$$

where $n = 2, 3, 4, \ldots$, and the Rydberg constant is $R_{\text{H}} = 1.097\ 373\ 2 \times 10^7\ \text{m}^{-1}$. This can also be written as

$$\lambda = \left(\frac{1}{R_H}\right)\left(\frac{n^2}{n^2-1}\right)$$

therefore, the first three wavelengths in this series are

$$\lambda_1 = \frac{1}{1.097\,373\,2\times 10^7\ \text{m}^{-1}}\left(\frac{2^2}{2^2-1}\right) = 1.215\times 10^{-7}\ \text{m}$$
$$= \boxed{121.5\ \text{nm}}$$

$$\lambda_2 = \frac{1}{1.097\,373\,2\times 10^7\ \text{m}^{-1}}\left(\frac{3^2}{3^2-1}\right) = 1.025\times 10^{-7}\ \text{m}$$
$$= \boxed{102.5\ \text{nm}}$$

$$\lambda_3 = \frac{1}{1.097\,373\,2\times 10^7\ \text{m}^{-1}}\left(\frac{4^2}{4^2-1}\right) = 9.720\times 10^{-8}\ \text{m}$$
$$= \boxed{97.20\ \text{nm}}$$

(b) These wavelengths are all in the $\boxed{\text{ultraviolet}}$ of the spectrum.

P41.2 (a) The fifth excited state must lie above the second excited state by the photon energy

$$E_{52} = hf = \frac{hc}{\lambda} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^8\ \text{m/s})}{520\times 10^{-9}\ \text{m}}$$
$$= 3.82\times 10^{-19}\ \text{J}$$

The sixth excited state exceeds the second in energy by

$$E_{62} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^8\ \text{m/s})}{410\times 10^{-9}\ \text{m}} = 4.85\times 10^{-19}\ \text{J}$$

Then the sixth excited state is above the fifth by

$$(4.85-3.82)\times 10^{-19}\ \text{J} = 1.03\times 10^{-19}\ \text{J}$$

In the 6 to 5 transition the atom emits a photon with the infrared wavelength

$$\lambda = \frac{hc}{E_{65}} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{1.03 \times 10^{-19}\ \text{J}}$$
$$= 1.94 \times 10^{-6}\ \text{m} = \boxed{1.94\ \mu\text{m}}$$

P41.3 (a) Denote the energy level n of the atom by E_n. For the transition $m \to 1$, the energy of the emitted photon and its wavelength λ_{m1} are related thus:

$$\Delta E_{m1} = E_m - E_1 = \frac{hc}{\lambda_{m1}}$$

For the transition $n \to 1$, the energy of the emitted photon and its wavelength λ_{n1} are related similarly:

$$\Delta E_{n1} = E_n - E_1 = \frac{hc}{\lambda_{n1}}$$

Therefore, for the transition $m \to n$, the energy of the emitted photon and its wavelength λ_{mn} (where m is the higher state, so $\lambda_{n1} > \lambda_{m1}$) can be related as

$$\Delta E_{mn} = E_m - E_n = \frac{hc}{\lambda_{mn}}$$
$$\Delta E_{mn} = (E_m - E_1) - (E_n - E_1) = \frac{hc}{\lambda_{mn}}$$
$$= \frac{hc}{\lambda_{m1}} - \frac{hc}{\lambda_{n1}} = \frac{hc}{\lambda_{mn}} \qquad \to \qquad \frac{1}{\lambda_{mn}} = \frac{1}{\lambda_{m1}} - \frac{1}{\lambda_{n1}}$$

This result may be written as $\boxed{\lambda_{mn} = \left|\frac{1}{1/\lambda_{m1} - 1/\lambda_{n1}}\right|}$.

(b) Multiply the result of part (a) by 2π and apply the definition $k_{ij} = 2\pi/\lambda_{ij}$:

$$2\pi\left(\frac{1}{\lambda_{mn}} = \left|\frac{1}{\lambda_{m1}} - \frac{1}{\lambda_{n1}}\right|\right) \quad \rightarrow \quad \boxed{k_{mn} = \left|k_{m1} - k_{n1}\right|}$$

Section 41.2 Early Models of the Atom

P41.4 According to a classical model, the electron moving as a particle in uniform circular motion about the proton in the hydrogen atom experiences a force $k_e e^2/r^2$; and from Newton's second law, $F = ma$, its acceleration is $k_e e^2/m_e r^2$.

(a) Using the fact that the Coulomb constant is $k_e = \dfrac{1}{4\pi \in_0}$, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{F}{m_e} = \frac{e^2}{4\pi \in_0 r^2 m_e} \quad \rightarrow \quad m_e v^2 = \frac{e^2}{4\pi \in_0 r}$$

The total energy is

$$E = K + U = \frac{m_e v^2}{2} - \frac{e^2}{4\pi \in_0 r} = -\frac{e^2}{8\pi \in_0 r}$$

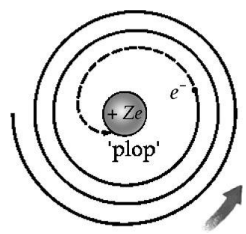
Substitute the expressions for E and a into the relation for $\dfrac{dE}{dt}$:

$$\frac{dE}{dt} = \frac{-1}{6\pi \in_0} \frac{e^2 a^2}{c^3}$$

$$\frac{e^2}{8\pi \in_0 r^2} \frac{dr}{dt} = \frac{-e^2}{6\pi \in_0 c^3}\left(\frac{e^2}{4\pi \in_0 r^2 m_e}\right)^2$$

Therefore, $\frac{dr}{dt} = -\frac{e^4}{12\pi^2 \epsilon_0^2 \; m_e^2 c^3}\left(\frac{1}{r^2}\right).$

(b) From the result of part (a), we have



ANS. FIG. P41.4

$$
\begin{aligned}
T &= \int_0^T dt = -\int_{2.00\times 10^{-10}\ \text{m}}^{0} \frac{12\pi^2 \epsilon_0^2 \; r^2 m_e^2 c^3}{e^4} dr \\
&= \int_0^{2.00\times 10^{-10}\ \text{m}} \frac{12\pi^2 \epsilon_0^2 \; r^2 m_e^2 c^3}{e^4} dr \\
&= \frac{12\pi^2 \epsilon_0^2 \; m_e^2 c^3}{e^4} \frac{r^3}{3}\bigg|_0^{2.00\times 10^{-10}} \\
&= \frac{12\pi^2 \left(8.85\times 10^{-12}\ \text{C}\right)^2 \left(9.11\times 10^{-31}\ \text{kg}\right)^2 \left(3.00\times 10^{8}\ \text{m/s}\right)^3}{\left(1.60\times 10^{-19}\ \text{C}\right)^4} \\
&\qquad \times \frac{\left(2.00\times 10^{-10}\ \text{m}\right)^3}{3} \\
&= 8.46\times 10^{-10}\ \text{s} = \boxed{0.846\ \text{ns}}
\end{aligned}
$$

Since atoms last much longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

Section 41.3 Bohr's Model of the Hydrogen Atom

P41.5 The allowed energy levels of the hydrogen atom are given by

$$E_n = -\frac{13.6\text{ eV}}{n^2} \text{ where } n = 1, 2, 3\ldots$$

A transition in which a lower state n_i absorbs a photon of energy ΔE results in a higher state n_f, and energy is conserved:

$$E_i + \Delta E = E_f$$

or

$$\Delta E = E_f - E_i = -\frac{13.6\text{ eV}}{n_f^2} - \left(-\frac{13.6\text{ eV}}{n_i^2}\right) = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

(a) For the transition $n_i = 2$ to $n_f = 5$,

$$\Delta E = 13.6\text{ eV}\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = \boxed{2.86\text{ eV}}$$

(b) For the transition $n_i = 4$ to $n_f = 6$,

$$\Delta E = 13.6\text{ eV}\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = \boxed{0.472\text{ eV}}$$

P41.6 From the equation just above Equation 41.9 in the text, $\frac{1}{2}m_e v^2 = \frac{k_e e^2}{2r}$, we have

$$v^2 = \frac{k_e e^2}{m_e r}$$

and using

$$r_n = \frac{n^2 \hbar^2}{m_e k_e e^2}$$

we obtain

$$v_n^2 = \frac{k_e e^2}{m_e\left(n^2\hbar^2/m_e k_e e^2\right)}$$

or

$$v_n = \frac{k_e e^2}{n\hbar}$$

P41.7 (a) The longest wavelength implies lowest frequency and smallest energy. The electron makes a transition from $n = 3$ to $n = 2$:

$$\Delta E = -\frac{13.6\text{ eV}}{3^2} + \frac{13.6\text{ eV}}{2^2} = \boxed{1.89\text{ eV}}$$

(b) The photon's wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{(1.89\text{ eV})} = \boxed{656\text{ nm}}$$

This is the red Balmer-alpha line, which gives its characteristic color to the chromosphere of the Sun and to photographs of the Orion nebula.

(c) The shortest wavelength implies highest frequency and greatest energy. The electron makes a transition from $n = \infty$ to $n = 2$:

$$\Delta E = -\frac{13.6\text{ eV}}{\infty} + \frac{13.6\text{ eV}}{2^2} = \boxed{3.40\text{ eV}}$$

(d) The photon's wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{3.40\text{ eV}} = \boxed{365\text{ nm}}$$

(e) This is the Balmer series limit, $\boxed{365\text{ nm}}$, in the near ultraviolet.

P41.8 (a) The collection of excited atoms must make these six transitions to get back to state one: $4 \to 1$, $4 \to 2$, and $4 \to 3$; $3 \to 1$ and $3 \to 2$; $2 \to 1$. Thus, the absorbed photon changes the atomic state from 1 to 4:

$$E_1 + hf = E_4 \quad \to \quad hf = E_4 - E_1, \quad \text{where} \quad E_n = -\frac{13.6 \text{ eV}}{n^2}$$

The incoming photons have energy

$$hf = \Delta E = E_f - E_i = -0.850 \text{ eV} - (-13.6 \text{ eV}) = 12.75 \text{ eV} = \frac{hc}{\lambda}$$

and wavelength

$$\lambda = \frac{hc}{\Delta E} = \frac{1\ 240 \text{ eV} \cdot \text{nm}}{12.75 \text{ eV}} = \boxed{97.3 \text{ nm}}$$

(b) The longest of the six wavelengths corresponds to the lowest photon energy, emitted in the transition $4 \to 3$. By energy conservation, $E_4 = hf + E_3$ and

$$hf = E_4 - E_3 = 13.6 \text{ eV}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.661 \text{ eV} = \frac{hc}{\lambda}$$

which gives

$$\lambda = \frac{hc}{E_4 - E_3} = \frac{1\ 240 \text{ eV} \cdot \text{nm}}{0.661 \text{ eV}} = 1\ 876 \text{ nm} = \boxed{1.88\ \mu\text{m}}$$

(c) The wavelength is in the $\boxed{\text{infrared}}$ region of the spectrum.

(d) The wavelength is part of the $\boxed{\text{Paschen}}$ series, since the lower state has $n = 3$.

(e) The shortest wavelength emitted is from the transition $4 \to 1$, and it is the same as the wavelength absorbed: $\boxed{97.3 \text{ nm}}$.

(f) The wavelength is in the $\boxed{\text{ultraviolet}}$ region of the spectrum.

(g) The wavelength is part of the $\boxed{\text{Lyman}}$ series, since the lower state has $n = 1$.

P41.9 (a) From Equation 41.12,

$$r_n = n^2 a_0 = n^2(0.052\,9\text{ nm})$$

and $r_3 = (3)^2(0.052\,9\text{ nm}) = \boxed{0.476\text{ nm}}$

(b) Using Equation 41.8, we calculate the momentum of the electron:

$$\begin{aligned} m_e v_2 &= m_e\sqrt{\frac{k_e e^2}{m_e r_2}} = \sqrt{\frac{m_e k_e e^2}{r_2}} \\ &= \sqrt{\frac{(9.11\times10^{-31}\text{ kg})(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\text{ C})^2}{0.476\times10^{-9}\text{ m}}} \\ &= 6.64\times10^{-25}\text{ kg}\cdot\text{m/s} \end{aligned}$$

The de Broglie wavelength for the electron is

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{6.64\times10^{-25}\text{ kg}\cdot\text{m/s}} = 9.97\times10^{-10}\text{ m} = \boxed{0.997\text{ nm}}$$

P41.10 (a) From the Bohr theory, we find the speed of the electron:

$$L = m_e v r = n\hbar \qquad \rightarrow \qquad v = \frac{n\hbar}{m_e r}$$

The period of its orbital motion is $T = \dfrac{2\pi r}{v} = \dfrac{2\pi r m_e r}{n\hbar}$.

Substituting the orbital radius $r = \dfrac{n^2\hbar^2}{m_e k_e e^2}$, we find

$$T = \frac{2\pi m_e n^4\hbar^4}{n\hbar m_e^2 k_e^2 e^4} = \frac{2\pi\hbar^3}{m_e k_e^2 e^4}n^3$$

Thus we have the periods determined in terms of the ground-state period

$$t_0 = \frac{2\pi\hbar^3}{m_e k_e^2 e^4}$$

$$= \frac{2\pi(1.054\ 6\times 10^{-34}\ \text{J}\cdot\text{s})^3}{\left(9.11\times 10^{-31}\ \text{kg}\right)(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)^2(1.602\times 10^{-19}\ \text{C})^4}$$

$$= 1.52\times 10^{-16}\ \text{s} = 152\times 10^{-18}\ \text{s} = \boxed{152\ \text{as}}$$

(b) In the $n = 2$ state, the period is

$$T = t_0 n^3 = t_0 (2)^3 = 8t_0 = 1.22\times 10^{-15}\ \text{s}$$

The number of orbits completed in the excited state is

$$N = \frac{10\times 10^{-6}\ \text{s}}{T} = \frac{10\times 10^{-6}\ \text{s}}{1.22\times 10^{-15}\ \text{s}} = \boxed{8.23\times 10^9\ \text{revolutions}}$$

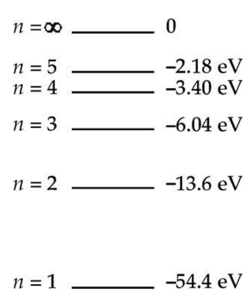
(c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so we can think of it as a long time.

P41.11 (a) The energy levels of a hydrogen-like ion whose charge number is Z are given by

$$E_n = (-13.6\ \text{eV})\frac{Z^2}{n^2}$$

Thus for helium ($Z = 2$), the energy levels are

$$\boxed{E_n = -\frac{54.4\ \text{eV}}{n^2} \quad n = 1,\ 2,\ 3,\ \ldots}$$



ANS. FIG. P41.11

The enegy level diagram for helium is shown in ANS. FIG. P41.11.

(b) For He^+, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state) is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\text{ eV})(2)^2}{(1)^2} = \boxed{54.4\text{ eV}}$$

Section 41.4 The Quantum Model of the Hydrogen Atom

P41.12 The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

All the factors in the given equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

For hydrogen, $\mu = \frac{m_p m_e}{m_p + m_e} \approx m_e$. The photon energy is $\Delta E = E_3 - E_2$.

Its wavelength is $\lambda = 656.3$ nm, where $\lambda = \frac{c}{f} = \frac{hc}{\Delta E}$.

(a) For positronium, $\mu = \dfrac{m_e m_e}{m_e + m_e} = \dfrac{m_e}{2}$,

so the energy of each level is one half as large as in hydrogen. The photon energy is inversely proportional to its wavelength, so for positronium,

$$\lambda_{32} = 2(656.3 \text{ nm}) = \boxed{1.31\ \mu\text{m}} \quad \text{(in the infrared region)}$$

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$, so the transition energy is $2^2 = 4$ times larger than hydrogen. Then,

$$\lambda_{32} = \left(\frac{656}{4}\right) \text{nm} = \boxed{164 \text{ nm}} \text{ (in the ultraviolet region)}$$

P41.13 (a) For this problem, refer to the equation from Problem 12, with $q_1 = q_2 = e$. For a particular transition from n_i to n_f,

$$\Delta E_{\text{H}} = -\frac{\mu_{\text{H}} k_e^{\,2} e^4}{2\hbar^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = \frac{hc}{\lambda_{\text{H}}}$$

and $\Delta E_{\text{D}} = -\dfrac{\mu_{\text{D}} k_e^{\,2} e^4}{2\hbar^2}\left(\dfrac{1}{n_f^{\,2}} - \dfrac{1}{n_i^{\,2}}\right) = \dfrac{hc}{\lambda_{\text{D}}}$,

where $\mu_{\text{H}} = \dfrac{m_e m_p}{m_e + m_p}$ and $\mu_{\text{D}} = \dfrac{m_e m_{\text{D}}}{m_e + m_{\text{D}}}$.

By division, $\dfrac{\Delta E_{\text{H}}}{\Delta E_{\text{D}}} = \dfrac{\mu_{\text{H}}}{\mu_{\text{D}}} = \dfrac{\lambda_{\text{D}}}{\lambda_{\text{H}}}$ or $\lambda_{\text{D}} = \left(\dfrac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}$. Then,

$$\boxed{\lambda_{\text{H}} - \lambda_{\text{D}} = \left(1 - \frac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}}$$

(b) $$\frac{\mu_H}{\mu_D} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_D}{m_e m_D}\right)$$
$$= \frac{(1.007\ 276\ \text{u})(0.000\ 549\ \text{u} + 2.013\ 553\ \text{u})}{(0.000\ 549\ \text{u} + 1.007\ 276\ \text{u})(2.013\ 553\ \text{u})}$$
$$= 0.999\ 728$$

$$\lambda_H - \lambda_D = (1 - 0.999\ 728)(656.3\ \text{nm}) = \boxed{0.179\ \text{nm}}$$

P41.14 (a) The uncertainty principle is represented by $\Delta x \Delta p \geq \frac{\hbar}{2}$.

Thus, if $\Delta x = r$, $\boxed{\Delta p \geq \frac{\hbar}{2r}}$.

(b) The minimum uncertainty would be attained only if the wave function had a particular (gaussian) waveform. We assume that the momentum uncertainty is just twice as large as its minimum possible value: $\Delta p = \frac{\hbar}{r}$. Then the kinetic energy is

$$K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \boxed{\frac{\hbar^2}{2m_e r^2}}$$

(c) The electric potential energy is $U = -\frac{k_e e^2}{r}$ so the total energy is

$$E = K + U \approx \boxed{\frac{\hbar^2}{2m_e r^2} - \frac{k_e e^2}{r}}.$$

(d) To minimize E as a function of r, we require

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{k_e e^2}{r^2} = 0 \quad \rightarrow \quad r = \boxed{\frac{\hbar^2}{m_e k_e e^2} = a_0} \text{ (the Bohr radius)}$$

(e) Then the energy is

$$E = \frac{\hbar^2}{2m_e}\left(\frac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2\left(\frac{m_e k_e e^2}{\hbar^2}\right) = -\frac{m_e k_e^2 e^4}{2\hbar^2}$$

Substituting numerical values,

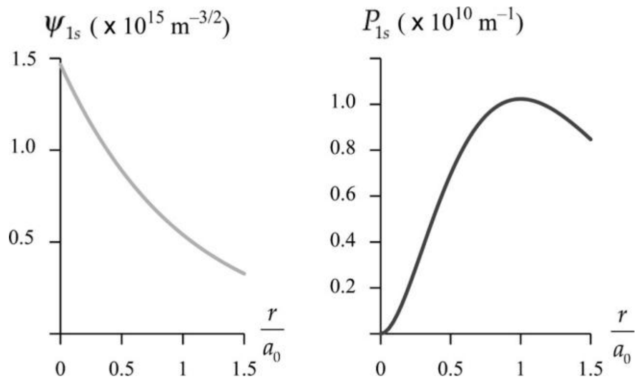
$$\begin{aligned}
E &= -\frac{m_e k_e^2 e^4}{2\hbar^2} = -\frac{m_e k_e^2 e^4}{2\hbar^2} \\
&= -\frac{\left(9.109\times10^{-31}\ \text{kg}\right)\left(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)^2\left(1.602\times10^{-19}\ \text{C}\right)^4}{2\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}/2\pi\right)^2} \\
&= -2.179\times10^{-18}\ \text{J}\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) \\
&= \boxed{-13.6\ \text{eV}}
\end{aligned}$$

(f) With our particular choice for the momentum uncertainty as double its minimum possible value, we find our results are in agreement with the Bohr theory.

Section 41.5 The Wave Functions for Hydrogen

P41.15 $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ is the ground state hydrogen wave function.

$P_{1s}(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}$ is the ground state radial probability distribution function. The plots are shown in ANS. FIG. P41.15.



ANS. FIG. P41.15

P41.16 (a) We first find the first and second derivatives of the wave function:

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad \rightarrow \quad \frac{2}{r}\frac{d\psi}{dr} = \frac{2}{r}\left(\frac{-1}{\sqrt{\pi a_0^5}} e^{-r/a_0}\right) = -\frac{2}{ra_0}\psi$$

and $\dfrac{d^2\psi}{dr^2} = \dfrac{1}{\sqrt{\pi a_0^7}} e^{-r/a_0} = \dfrac{1}{a_0^2}\psi$

Substitution into the Schrödinger equation to test the validity of the solution yields

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi \in_0 r}\psi = E\psi$$

But, from Equation 41.11, $a_0 = \dfrac{\hbar^2}{m_e k_e e^2} = \dfrac{\hbar^2(4\pi \in_0)}{m_e e^2}$, thus

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2}-\frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi \in_0 r}\psi = E\psi$$

$$-\frac{\hbar^2}{2m_e}\frac{1}{a_0^2}\psi + \left[\frac{\hbar^2}{m_e r}\frac{1}{a_0}\psi - \frac{e^2}{4\pi \in_0 r}\psi\right] = E\psi$$

$$-\frac{\hbar^2}{2m_e}\frac{1}{a_0^2}\psi + \left[\frac{\cancel{\hbar^2}}{\cancel{m_e} r}\frac{\cancel{m_e} e^2}{4\pi \in_0 \cancel{\hbar^2}} - \frac{e^2}{4\pi \in_0 r}\right]\psi = E\psi$$

$$-\frac{\hbar^2}{2m_e}\frac{1}{a_0^2}\psi = E\psi$$

The Schrödinger equation is satisfied if $E = -\dfrac{\hbar^2}{2m_e}\dfrac{1}{a_0^2}$.

(b) Substituting $a_0 = \dfrac{\hbar^2}{m_e k_e e^2}$ for one factor of a_0, we find that

$$E = -\frac{\hbar^2}{2m_e}\frac{1}{a_0^2} = -\frac{\cancel{\hbar^2}}{2\cancel{m_e}}\frac{1}{a_0}\frac{\cancel{m_e} k_e e^2}{\cancel{\hbar^2}} = \boxed{E = -\frac{k_e e^2}{2a_0}}$$

P41.17 (a) $\int |\psi|^2 dV = 4\pi \int_0^\infty |\psi|^2 r^2 dr = 4\pi\left(\dfrac{1}{\pi a_0^3}\right)\int_0^\infty r^2 e^{-2r/a_0} dr$. Using integral tables,

$$\int |\psi|^2 dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_0^\infty = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$$

so the wave function as given is normalized.

(b) $P_{a_0/2\to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 r^2 dr = 4\pi\left(\dfrac{1}{\pi a_0^3}\right)\int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0} dr$. Again, using integral tables,

$$P_{a_0/2\to 3a_0/2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2}$$

$$= -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right) - e^{-1}\left(\frac{5a_0^2}{4}\right)\right] = \boxed{0.497}$$

Section 41.6 Physical Interpretation of the Quantum Numbers

P41.18 (a) In the 3*d* subshell, $n = 3$ and $\ell = 2$, we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	–1	–1	–2	–2
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(a total of 10 states.)

(b) In the 3*p* subshell, $n = 3$ and $\ell = 1$, we have

n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	0	0	–1	–1
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(a total of 6 states.)

P41.19 (a) For a 3d state, $n = 3$ and $\ell = 2$. Therefore,

$$L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34} \text{ J} \cdot \text{s}$$

(b) m_ℓ can have the values −2, −1, 0, 1, and 2,

so $\boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

(c) Using the relation $\cos\theta = \frac{L_z}{L}$, we find the possible values of θ:

$$\boxed{145°,\ 114°,\ 90.0°,\ 65.9°,\ \text{and } 35.3°}$$

P41.20 (a) For $n = 1$, we have $\ell = 0$, $m_\ell = 0$, $m_s = \pm\frac{1}{2}$.

n	ℓ	m_ℓ	m_s
1	0	0	−1/2
1	0	0	+1/2

This yields $2n^2 = 2(1)^2 = \boxed{2}$ sets.

(b)

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	-1	±1/2
2	1	0	±1/2
2	1	+1	±1/2

For n-2, we have This yields $2n^2 = 2(2)^2 = \boxed{8}$ sets.

Note that the number is twice the number of m_ℓ values. Also, for each ℓ there are $(2\ell+1)$ different m_ℓ values. Finally, l can take on values ranging from 0 to $n - 1$.

So the general expression is $\text{number} = \sum_{\ell=0}^{n-1} 2(2\ell+1)$.

The series is an arithmetic progression like 2 + 6 + 10 + 14.

The sum is $\sum_{0}^{n-1} 4\ell + \sum_{0}^{n-1} 2 = 4\left[\dfrac{n^2 - n}{2}\right] + 2n = 2n^2$.

(c) $n = 3$: $2(1) + 2(3) + 2(5) = 2 + 6 + 10 = 18$ or $2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $2(1) + 2(3) + 2(5) + 2(7) = 32$ or $2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $32 + 2(9) = 32 + 18 = 50$ or $2n^2 = 2(5)^2 = \boxed{50}$

P41.21 (a) Modeling it as a solid sphere, the density of a proton is,

$$\rho = \frac{m}{V} = \frac{1.67\times10^{-27}\ \text{kg}}{(4/3)\pi\left(1.00\times10^{-15}\ \text{m}\right)^3} = \boxed{3.99\times10^{17}\ \text{kg/m}^3}$$

(b) The radius of an electron modelled as a solid sphere is,

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left[\frac{3\left(9.11\times10^{-31}\ \text{kg}\right)}{4\pi\left(3.99\times10^{17}\ \text{kg/m}^3\right)}\right]^{1/3}$$
$$= 8.17\times10^{-17}\ \text{m} = 81.7\times10^{-18}\ \text{m} = \boxed{8.17\ \text{am}}$$

(c) The moment of inertia of the spinning electron is

$$I = \frac{2}{5}mr^2 = \frac{2}{5}\left(9.11\times10^{-31}\ \text{kg}\right)\left(8.17\times10^{-17}\ \text{m}\right)^2$$
$$= 2.43\times10^{-63}\ \text{kg}\cdot\text{m}^2$$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore,

$$v = \frac{\hbar r}{2I} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(8.17\times10^{-17}\ \text{m})}{2\pi(2\times2.43\times10^{-63}\ \text{kg}\cdot\text{m}^2)}$$
$$= 1.77\times10^{12}\ \text{m/s} = \boxed{1.77\ \text{Tm/s}}$$

(d) It is 5.91×10^3c, which is huge compared with the speed of light and impossible.

***P41.22** **Conceptualize** Imagine the process of the photon leaving the two-state system and having just the right energy to allow the system to drop from the higher state to the lower state.

Categorize The electron is modeled as a *nonisolated system* for *energy*.

Analyze Write the appropriate reduction of Equation 8.2 for the time interval spanning the release of the photon:

$$\Delta U_B = T_{\text{ER}} \qquad (1)$$

Substitute for both sides of the equation. Evaluate the left side using Equations 28.19 and 41.33:

$$U_B = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}} = -\mu_z B$$
$$\rightarrow \quad \Delta U_B = -\Delta\mu_z B = -\left[\frac{e\hbar}{2m_e} - \left(-\frac{e\hbar}{2m_e}\right)\right]B = -2\mu_{\text{B}}B \qquad (2)$$

Use Equation 39.5 for the right side:

$$T_{\text{ER}} = -hf \qquad (3)$$

where the minus sign indicates that energy is leaving the system of the atom. Substitute Equations (2) and (3) into Equation (1) and solve for the magnetic field:

$$2\mu_B B = hf \quad \rightarrow \quad B = \frac{hf}{2\mu_B} = \frac{hc}{2\mu_B \lambda}$$

Substitute numerical values:

$$B = \frac{\left(6.626 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(3.00 \times 10^{8}\ \mathrm{m/s}\right)}{2\left(9.27 \times 10^{-24}\ \mathrm{J/T}\right)\left(0.21\ \mathrm{m}\right)} = \boxed{0.05\ \mathrm{T}}$$

Finalize We have kept only one significant figure for the magnetic field magnitude. The electron cannot be modeled as maintaining a fixed distance from the nucleus, so the magnetic field magnitude will vary over a range of values. What we have found is an approximation to the average value of the field. Look online for astronomical uses of 21-cm radiation.]

Answer: 0.05 T

P41.23 The 3*d* subshell has $n = 3$ and $\ell = 2$. Also, we have $s = 1$. Altogether we can have $n = 3$, $\ell = 2$, $m_\ell = -2, -1, 0, 1, 2$, $s = 1$, and $m_s = -1, 0, 1$, leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

The energy of the photon is

$$E_{ph} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{88.0\ \text{eV}} = 14.1\ \text{eV}$$

P41.24 The energy of the photon is

$$E_{ph} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{88.0\ \text{eV}} = 14.1\ \text{eV}$$

The maximum energy of the ejected photoelectron from the aluminum surface is

$$K_{\max} = E_{ph} - \phi = 14.1\ \text{eV} - 4.08\ \text{eV} = 10.0\ \text{eV}$$

where the work function ϕ for aluminum is found from Table 40.1. This electron energy is not enough to excite the hydrogen atom from its ground state to even the first exited state.

Section 41.7 The Exclusion Principle and the Periodic Table

P41.25 (a) $\boxed{\text{The 4s subshell}}$, for potassium and calcium, before the 3*d* subshell starts to fill for scandium through zinc.

(b) $\boxed{\text{We would expect } [\text{Ar}]3d^4 4s^2 \text{ to have lower energy, but } [\text{Ar}]3d^5 4s^1 \text{ has more unpaired spins and lower energy according to Hund's rule.}}$

(c) It is the ground-state configuration of $\boxed{\text{chromium}}$.

P41.26 Electronic configuration: sodium to argon

Orbitals 1*s*, 2*s*, and 2*p* are filled (and not shown).

	3*s*	3*p*			4*s*	
Na^{11}	⇑					$[1s^2 2s^2 2p^6]3s^1$

Mg^{12}	⇑⇓					$[1s^2 2s^2 2p^6]3s^2$
Al^{13}	⇑⇓	⇑				$[1s^2 2s^2 2p^6]3s^2 3p^1$
Si^{14}	⇑⇓	⇑	⇑			$[1s^2 2s^2 2p^6]3s^2 3p^2$
P^{15}	⇑⇓	⇑	⇑	⇑		$[1s^2 2s^2 2p^6]3s^2 3p^3$
S^{16}	⇑⇓	⇑⇓	⇑	⇑		$[1s^2 2s^2 2p^6]3s^2 3p^4$
Cl^{17}	⇑⇓	⇑⇓	⇑⇓	⇑		$[1s^2 2s^2 2p^6]3s^2 3p^5$
Ar^{18}	⇑⇓	⇑⇓	⇑⇓	⇑⇓		$[1s^2 2s^2 2p^6]3s^2 3p^6$
K^{19}	⇑⇓	⇑⇓	⇑⇓	⇑⇓	⇑	$[1s^2 2s^2 2p^6 3s^2 3p^6]4s^1$

P41.27 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the 1*s* electrons,

$$n = 1,\ \ell = 0,\ m_\ell = 0,\ m_s = +\frac{1}{2} \text{ and } -\frac{1}{2}$$

For the two 2*s* electrons,

$$n = 2,\ \ell = 0,\ m_\ell = 0,\ m_s = +\frac{1}{2} \text{ and } -\frac{1}{2}$$

For the four 2*p* electrons,

$$n = 2,\ \ell = 1,\ m_\ell = -1,\ 0,\ 1, \text{ and } m_s = +\frac{1}{2} \text{ and } -\frac{1}{2}$$

P41.28 (a) Note that the possible values for ℓ range from zero to $n - 1$.

$n+\ell$	1	2	3	4	5	6	7
subshell	1*s*	2*s*	2*p*, 3*s*	3*p*, 4*s*	3*d*, 4*p*, 5*s*	4*d*, 5*p*, 6*s*	4*f*, 5*d*, 6*p*, 7*s*

The order is $\boxed{1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s}$.

P41.29 (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$; possible values are $m_\ell = 1, 0, -1$ and $m_s = 1/2, -1/2$. The exclusion principle requires that the electrons cannot have identical sets of quantum numbers. The possible states are listed here in columns giving the other quantum numbers:

electron	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
one	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
two	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
one	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

electron	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
two	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are $6 \times 5 = \boxed{30}$ allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $6 \times 6 = \boxed{36}$ possible states, six for each electron independently.

Section 42.8 More on Atomic Spectra: Visible and X-Ray

P41.30 Some electrons can give all their kinetic energy $K_e = e\Delta V$ to the creation of a single photon of x-radiation, with

$$hf = \frac{hc}{\lambda} = e\Delta V$$

$$\lambda = \frac{hc}{e\Delta V} = \frac{(6.626\ 1 \times 10^{-34}\ \text{J}\cdot\text{s})(2.997\ 9 \times 10^{8}\ \text{m/s})}{(1.602\ 2 \times 10^{-19}\ \text{C})\Delta V}$$

$$= \boxed{\frac{1\ 240\ \text{nm}\cdot\text{V}}{\Delta V}}$$

P41.31 (a) For bismuth, $Z = 83$. Following Example 41.5, the electron in the M shell ($n = 3$) is shielded from the nuclear charge by one electron in the L shell ($n = 1$) and eight electrons in the K shell ($n = 2$). Its energy is

$$E_{\mathrm{M}} \approx -(Z-9)^2 \frac{13.6\text{ eV}}{(3)^2} = -13.6\text{ eV}\frac{(74)^2}{(3)^2}$$

The electrons in the L shell ($n = 2$) are shielded from the nuclear charge by one electron in the K shell, so (from page 1324)

$$E_{\mathrm{L}} \approx -(Z-1)^2 \frac{13.6\text{ eV}}{(2)^2} = -13.6\text{ eV}\frac{(82)^2}{(2)^2}$$

When the electron drops from the M to the L shell of the atom, it emits a photon of energy

$$\begin{aligned} E_{\text{photon}} &= E_{\mathrm{M}} - E_{\mathrm{L}} \approx 13.6\text{ eV}\left[-\frac{(74)^2}{(3)^2} + \frac{(82)^2}{(2)^2}\right] \\ &= 1.46\times 10^4\text{ eV} \approx \boxed{15\text{ keV}} \end{aligned}$$

(b) The wavelength of the emitted x-ray is given by

$$\begin{aligned} \lambda &= \frac{1.240\text{ keV}\cdot\text{nm}}{E} = \frac{1.240\text{ keV}\cdot\text{nm}}{15\text{ keV}} \\ &\approx 0.083\text{ nm} = \boxed{8.3\times 10^{-11}\text{ m}} \end{aligned}$$

P41.32 (a) For the 3p state, $E_n = \dfrac{-13.6\text{ eV }Z_{eff}^2}{n^2}$ becomes

$$-3.0\text{ eV} = \frac{-13.6\text{ eV }Z_{eff}^2}{3^2} \quad \text{so} \quad Z_{eff} = \boxed{1.4}$$

For the 3d state

$$-1.5\text{ eV} = \frac{-13.6\text{ eV }Z_{eff}^2}{3^2} \quad \text{so} \quad Z_{eff} = \boxed{1.0}$$

(b) When the outermost electron in sodium is promoted from the 3s state into a 3p state, its wave function still overlaps somewhat with the ten electrons below it. It therefore sees the $+11e$ nuclear charge not fully screened, and on the average moves in an electric field like that created by a particle with charge $+11e - 9.6e = 1.4e$. When this valence electron is lifted farther to a $3d$ state, it is essentially entirely outside the cloud of ten electrons below it, and moves in the field of a net charge $+11e - 10e = 1e$.

***P41.33** **Conceptualize** Review Section 41.8 on the production of x-rays when electrons strike a target.

Categorize This problem is a simple application of finding a wavelength of a photon with a certain energy, so we categorize the problem as a substitution problem.

The 35.0-kV accelerating voltage will create 35.0-keV electrons. The shortest wavelength of radiation from the x-ray machine will be that associated with an electron giving up all of its energy at once when it strikes the target, transforming this much energy to that of a photon. Find the wavelength of the photon associated with this energy:

$$\lambda = \frac{hc}{E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{35.0 \times 10^3\text{ eV}} = 0.035\,4\text{ nm} = 35.4\text{ pm}$$

This is the *minimum* wavelength that the doctor's machine can create. Therefore, the 30.0-pm radiation did not come from the doctor's office. It must be coming from some neighboring office that is not properly shielding its x-ray machine.]

Answer: Minimum wavelength from doctor's office is 35.4 pm. Radiation is coming from elsewhere.

P41.34 (a) All of the kinetic energy of an electron after its acceleration through a potential difference ΔV goes into producing a single photon:

$$E = \frac{hc}{\lambda} = e\Delta V \quad \rightarrow \quad \Delta V = \frac{hc}{e\lambda} = \frac{1240\ \cancel{e}\text{V}\cdot\text{nm}}{\cancel{e}\lambda} = \boxed{\frac{1240\ \text{V}\cdot\text{nm}}{\lambda}}$$

(b) The potential difference is inversely proportional to the wavelength.

(c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure.

(d) Yes, but it might be unlikely for a very high energy electron to stop in a single interaction to produce a high-energy gamma ray, and it might be difficult to observe the very low intensity radio waves produced as bremsstrahlung by low-energy electrons.

(e) The potential difference goes to infinity as the wavelength goes to zero.

(f) The potential difference goes to zero as the wavelength goes to infinity. \

Section 41.10 Lasers

P41.35 (a) The equilibrium ratio is

$$\frac{N_4{}^*}{N_3} = \frac{N_g e^{-E_3/k_BT}}{N_g e^{-E_2/k_BT}} = e^{-(E_3-E_2)/k_BT} = e^{-\Delta E/k_BT}$$

where the temperature T = 27.0 °C + 273.15 = 300.2 K, and the energy difference (from Figure P42.60) is

$$\Delta E = E_4{}^* - E_3 = 20.66 \text{ eV} - 18.70 \text{ eV} = 1.96 \text{ eV}$$

Substituting numerical values,

$$\frac{N_4{}^*}{N_3} = e^{-\Delta E/k_BT} = e^{-(1.96 \text{ eV})(1.602\times10^{-19} \text{ J/eV})/(1.381\times10^{-23} \text{ J/K})(300.2 \text{ K})}$$
$$= \boxed{1.26\times10^{-33}}$$

(b) Now, we require $\dfrac{N_4{}^*}{N_3} = e^{-\Delta E/k_BT} = 1.02$

where $\Delta E = E_4{}^* - E_3 = 1.96 \text{ eV}$

Thus,

$$\ln(1.02) = -\frac{(1.96 \text{ eV})(1.602\times10^{-19} \text{ J/eV})}{(1.381\times10^{-23} \text{ J/K})T}$$
$$T = \boxed{-1.15\times10^{6} \text{ K}}$$

(c) $\boxed{\text{The population inversion requires the temperature be negative.}}$ Because $\Delta E = E_4{}^* - E_3 > 0$, and in any real equilibrium state $T > 0$, the ratio $N_4{}^*/N_3 = e^{-\Delta E/k_BT} < 1$. Thus, a population inversion cannot happen in thermal equilibrium.

P41.36 (a) The distance between nodes is $\dfrac{\lambda}{2}$, so we require solutions to $35.124\ 103 \text{ cm} = \dfrac{N}{2}\lambda$, where N is an integer and λ is in the required range. The midpoint of the range is 632.809 10 nm, giving

$$N_{\text{trial}} = \frac{2(35.124\ 103\times10^{-2} \text{ m})}{632.809\ 1\times10^{9} \text{ m}} = 1\ 110\ 101.07$$

So we try $N = 1\ 110\ 101$, $1\ 110\ 102$, $1\ 110\ 100$, $1\ 110\ 103$, and so on:

$$\lambda_1 = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 101} = \boxed{632.809\ 14\ \text{nm}}$$

$$\lambda_2 = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 102} = \boxed{632.808\ 57\ \text{nm}}$$

$$\lambda_3 = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 100} = \boxed{632.809\ 71\ \text{nm}}$$

$$\lambda_{\text{trial}} = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 103} = 632.808\ 00\ \text{nm}$$

outside the range. Thus the laser light has just $\boxed{\text{three}}$ wavelength components.

(b) The rms speed is obtained from $\frac{1}{2}m_0 v^2 = \frac{3}{2}kT$. We use the periodic table for the mass of a neon atom. Then,

$$v = \sqrt{\frac{3kT}{m_0}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\ \text{J/K}\right)(393\ \text{K})}{20.18\ \text{u}}\left(\frac{1\ \text{u}}{1.66 \times 10^{-27}\ \text{kg}}\right)}$$

$$= \boxed{697\ \text{m/s}}$$

(c) For a neon atom moving toward one mirror at the rms speed as it emits, the Doppler shift is described by

$$f' = f\sqrt{\frac{c+v}{c-v}} = \frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{c+v}{c-v}}$$

$$\lambda' = \lambda\sqrt{\frac{c-v}{c+v}} = (632.809\ 1\ \text{nm})\sqrt{\frac{3 \times 10^8 - 697}{3 \times 10^8 + 697}} = 632.807\ 63\ \text{nm}$$

This is outside the given range. Many atoms are moving faster than the rms speed, so we should expect still more Doppler broadening of the resonance amplification peak.

Additional Problems

P41.37 The wave function for the 2*s* state is given by Equation 41.26:

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2 - \frac{r}{a_0}\right]e^{-r/2a_0}$$

(a) Taking $r = a_0 = 0.529 \times 10^{-10}$ m, we find

$$\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{0.529 \times 10^{-10}\ \text{m}}\right)^{3/2}[2-1]e^{-1/2}$$
$$= \boxed{1.57 \times 10^{14}\ \text{m}^{-3/2}}$$

(b) $$\left|\psi_{2s}(a_0)\right|^2 = \left(1.57 \times 10^{14}\ \text{m}^{-3/2}\right)^2 = \boxed{2.47 \times 10^{28}\ \text{m}^{-3}}$$

(c) Using Equation 41.24 and the result of part (b) gives

$$P_{2s}(a_0) = 4\pi a_0^2\left|\psi_{2s}(a_0)\right|^2 = \boxed{8.69 \times 10^{8}\ \text{m}^{-1}}$$

P41.38 From Equation 41.26,

$$\psi_{2s} = \frac{1}{4}(2\pi)^{-1/2}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0} = A\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

Differentiating gives

$$\frac{d\psi}{dr} = Ae^{-r/2a_0}\left(-\frac{2}{a_0} + \frac{r}{2a_0^2}\right)$$

Differentiating a second time gives,

$$\frac{d^2\psi}{dr^2} = \left(\frac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\frac{3}{2} - \frac{r}{4a_0}\right)$$

Substituting into Schrödinger's equation and dividing by $Ae^{-r/2a_0}$, we will have a solution if

$$-\frac{5}{4}\frac{\hbar^2}{m_e a_0^2}+\frac{k_e e^2}{a_0}+\frac{\hbar^2 r}{8m_e a_0^3}+\frac{2\hbar^2}{m_e a_0 r}-\frac{2k_e e^2}{r}=2E-\frac{Er}{a_0}$$

Now with $a_0=\frac{\hbar^2}{m_e e^2 k_e}$, this reduces to

$$-\frac{m_e e^4 k_e^2}{8\hbar^2}\left(2-\frac{r}{a_0}\right)=E\left(2-\frac{r}{a_0}\right)$$

This is true, so ψ_{2s} is a solution to the Schrödinger equation, provided $E=\frac{1}{4}E_1=-3.40\text{ eV}.$

P41.39 From Figure 41.20, a typical ionization energy is 8 eV. For internal energy to ionize most of the atoms we require

$$\frac{3}{2}k_B T=8\text{ eV}:$$

$$T=\frac{2\times 8\left(1.60\times 10^{-19}\text{ J}\right)}{3\left(1.38\times 10^{-23}\text{ J/K}\right)}\boxed{\sim\text{ between }10^4\text{ K and }10^5\text{ K}}$$

P41.40 The fact that there are five values of the *z* component of orbital angular momentum tells us that there are five values of m_ℓ, which, in turn, tells us that $\ell=2$. From Equation 41.28, we can find the maximum value of m_ℓ:

$$L_z = m_\ell\hbar \qquad\rightarrow\qquad m_\ell = \frac{L_z}{\hbar}=\frac{3.16\times 10^{-34}}{1.055\times 10^{-34}}=3$$

In order to have a maximum value of m_ℓ equal to 3, we need to have $\ell=3$, which is inconsistent with the first result.

P41.41 The expectation value of 1/*r* is found from

$$\left\langle\frac{1}{r}\right\rangle=\int_0^\infty\frac{4r^2}{a_0^3}e^{-2r/a_0}\frac{1}{r}dr=\frac{4}{a_0^3}\int_0^\infty re^{-(2/a_0)r}\,dr=\frac{4}{a_0^3}\frac{1}{\left(\frac{2}{a_0}\right)^2}=\boxed{\frac{1}{a_0}}$$

We compare this to $\frac{1}{\langle r\rangle} = \frac{1}{\frac{3a_0}{2}} = \frac{2}{3a_0}$, and find that the average reciprocal value is $\boxed{\text{NOT}}$ the reciprocal of the average value.

P41.42 (a) Using the same procedure that was used in the Bohr model of the hydrogen atom, we apply Newton's second law to the Earth. We simply replace the Coulomb force by the gravitational force exerted by the Sun on the Earth and find

$$G\frac{M_S M_E}{r^2} = M_E\frac{v^2}{r} \qquad \textbf{[1]}$$

where v is the orbital speed of the Earth. Next, we apply the postulate that angular momentum of the Earth is quantized in multiples of $\hbar$::

$$M_E v r = n\hbar \qquad (n = 1,\ 2,\ 3,\ \ldots)$$

Solving for v gives

$$v = \frac{n\hbar}{M_E r} \qquad \textbf{[2]}$$

Substituting [2] into [1], we find

$$r = \frac{n^2\hbar^2}{GM_S M_E^2} \qquad \textbf{[3]}$$

(b) Solving equation [3] for n gives

$$n = \sqrt{GM_S r}\,\frac{M_E}{\hbar} \qquad \textbf{[4]}$$

Taking $M_S = 1.99 \times 10^{30}$ kg, $M_E = 5.98 \times 10^{24}$ kg, $r = 1.496 \times 10^{11}$ m, $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, and $\hbar = 1.055 \times 10^{-34}\ \text{J}\cdot\text{s}$, we find

$$n = \boxed{2.53 \times 10^{74}}$$

(c) We can use equation [3] to determine the radii for the orbits corresponding to the quantum numbers n and $n + 1$:

$$r_n = \frac{n^2\hbar^2}{GM_S M_E^2} \quad \text{and} \quad r_{n+1} = \frac{(n+1)^2\hbar^2}{GM_S M_E^2}$$

Hence, the separation between these two orbits is

$$\Delta r = \frac{\hbar^2}{GM_S M_E^2}\left[(n+1)^2 - n^2\right] = \frac{\hbar^2}{GM_S M_E^2}(2n+1)$$

Since n is very large, we can neglect the number 1 in the parentheses and express the separation as

$$\begin{aligned}\Delta r &\approx \frac{\hbar^2}{GM_S M_E^2}(2n) \\ &= \frac{\left(1.054\ 6\times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times 10^{30}\ \text{kg}\right)\left(5.98\times 10^{24}\ \text{kg}\right)^2} \\ &\qquad \times\left[2\left(2.53\times 10^{74}\right)\right] \\ &= \boxed{1.18\times 10^{-63}\ \text{m}}\end{aligned}$$

(d) This number is *much smaller* than the radius of an atomic nucleus (~ 10^{-15} m), so the distance between quantized orbits of the Earth is too small to observe.

P41.43 We use Equation 41.26:

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2 - \frac{r}{a_0}\right]e^{-r/2a_0}$$

(a) By Equation 41.24,

$$P(r) = 4\pi r^2|\psi|^2 = \boxed{\frac{r^2}{8a_0^3}\left(2-\frac{r}{a_0}\right)^2 e^{-r/a_0}}$$

(b) The derivative of the radial probability is

$$\frac{dP(r)}{dr} = \frac{1}{8a_0^3}\left[2r\left(2-\frac{r}{a_0}\right)^2 - 2r^2\left(\frac{1}{a_0}\right)\left(2-\frac{r}{a_0}\right) - r^2\left(2-\frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0}$$

Simplifying the expression,

$$\frac{dP(r)}{dr} = \frac{1}{8a_0^3}\left(2-\frac{r}{a_0}\right)e^{-r/a_0}\left[4r - \frac{2r^2}{a_0} - \frac{2r^2}{a_0} - \left(\frac{2r^2}{a_0} - \frac{r^3}{a_0^2}\right)\right]$$

$$= \frac{r}{8a_0^5}\left(2-\frac{r}{a_0}\right)e^{-r/a_0}\left[4a_0^2 - 6ra_0 + r^2\right]$$

$$= \boxed{\frac{r}{8a_0^5}\left(2-\frac{r}{a_0}\right)e^{-r/a_0}\left[r^2 - 6ra_0 + 4a_0^2\right]}$$

(c) Its extremes are given by

$$\frac{dP}{dr} = \frac{r}{8a_0^5}\left(2-\frac{r}{a_0}\right)e^{-r/a_0}\left[r^2 - 6ra_0 + 4a_0^2\right] = 0$$

The roots of $\frac{dP}{dr} = 0$ at $\boxed{r = 0,\ r = 2a_0,\ \text{and}\ r = \infty}$ are minima with $P(r)$ = 0 (as shown in Figure 41.12).

(d) We require $r^2 - 6ra_0 + 4a_0^2 = 0$. The solutions are

$$r = \frac{-(-6a_0) \pm \sqrt{(-6a_0)^2 - 4(1)(4a_0^2)}}{2} = \frac{6a_0 \pm \sqrt{20a_0^2}}{2} = \boxed{\left(3 \pm \sqrt{5}\right)a_0}$$

(e) We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = \left(3-\sqrt{5}\right)a_0 = 0.764\,a_0$,

$$P(r) = \frac{(0.764\,a_0)^2}{8a_0^3}\left(2 - \frac{0.764\,a_0}{a_0}\right)^2 e^{-0.764}$$

$$= \frac{(0.764)^2}{8a_0}(2 - 0.764)^2 e^{-0.764} = \frac{0.051\,9}{a_0}$$

When $r = \left(3 + \sqrt{5}\right)a_0 = 5.236a_0$,

$$P(r) = \frac{(5.236\,a_0)^2}{8a_0^3}\left(2 - \frac{5.236\,a_0}{a_0}\right)^2 e^{-5.236}$$

$$= \frac{(5.236)^2}{8a_0}(2 - 5.236)^2 e^{-5.236} = \frac{0.191}{a_0}$$

Therefore, the most probable value of r is

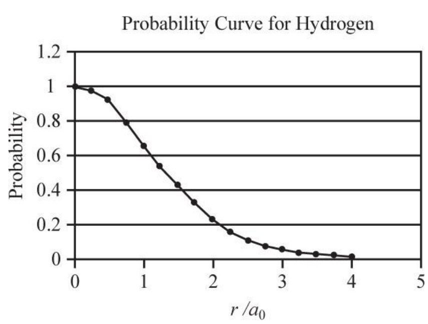
$\boxed{r = \left(3 + \sqrt{5}\right)a_0 \rightarrow P = 0.191/a_0}$.

P41.44 (a) From Equations 41.22 – 41.25,

$$P = \int_r^\infty P_{1s}(r')\,dr' = \frac{4}{a_0^3}\int_r^\infty r'^2 e^{-2r'/a_0}\,dr'$$

$$= \left[-\left(\frac{2r'^2}{a_0^2} + \frac{2r'}{a_0} + 1\right)e^{-2r'/a_0}\right]_r^\infty = \boxed{\left(\frac{2r^2}{a_0^2} + \frac{2r}{a_0} + 1\right)e^{-2r/a_0}}$$

(b) The graph is shown in ANS. FIG. P41.44.



ANS. FIG. P41.44

(c) The probability of finding the electron inside or outside the sphere of radius r is $\frac{1}{2}$:

$$\left(\frac{2r^2}{a_0^2}+\frac{2r}{a_0}+1\right)e^{-2r/a_0}=\frac{1}{2} \quad \text{or} \quad z^2+2z+2=e^z$$

where $z=\frac{2r}{a_0}$.

One can home in on a solution to this transcendental equation for r on a calculator, the result being $r=\boxed{1.34a_0}$ to three digits.

P41.45 (a) One molecule's share of volume is,

$$\text{Al: } V=\left(\frac{27.0\text{ g}}{\text{mol}}\right)\left(\frac{1\text{ mol}}{6.02\times10^{23}\text{ molecules}}\right)\left(\frac{1.00\times10^{-6}\text{ m}^3}{2.70\text{ g}}\right)$$
$$=1.66\times10^{-29}\text{ m}^3$$

$$D\approx\sqrt[3]{V}=\boxed{2.55\times10^{-10}\text{ m}\sim10^{-1}\text{ nm}}$$

$$\text{U: } V=\left(\frac{238\text{ g}}{\text{mol}}\right)\left(\frac{1\text{ mol}}{6.02\times10^{23}\text{ molecules}}\right)\left(\frac{1.00\times10^{-6}\text{ m}^3}{18.9\text{ g}}\right)$$
$$=2.09\times10^{-29}\text{ m}^3$$

$$D\approx\sqrt[3]{V}=\boxed{2.76\times10^{-10}\text{ m}\sim10^{-1}\text{ nm}}$$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge $+Ze-(Z-1)e=+e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is a_0/Z.

P41.46 An ionization energy of 4.10 eV means the ground state energy is –4.10 eV. The photon energies tell us the separation of the energy levels:

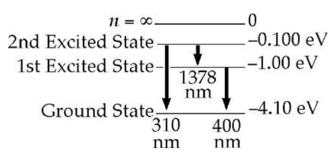
$$E = \frac{hc}{\lambda} = \frac{1\,240 \text{ eV} \cdot \text{nm}}{\lambda} = \Delta E$$

Then, $\lambda_1 = 310 \text{ nm}$, so $\Delta E_1 = 4.00 \text{ eV}$

$\lambda_2 = 400 \text{ nm}$, $\Delta E_2 = 3.10 \text{ eV}$

$\lambda_3 = 1\,378 \text{ nm}$, $\Delta E_3 = 0.900 \text{ eV}$

The energy level diagram having the fewest levels and consistent with these energies is shown in ANS. FIG. P41.46.



ANS. FIG. 41.46

***P41.47** **Conceptualize** If one of the hydrogen atoms is in its ground state, ionization requires photons of energy 13.6 eV. Therefore, the ionization is occurring from atoms in excited states.

Categorize We categorize the problem as one that requires analyzing the hydrogen atom using the Bohr model. We also model the incoming photon and the hydrogen atom as an *isolated system* for *energy*.

Analyze (a) In order to ionize the atoms with 2.28-eV photons, the energy of the atom must be higher than –2.28 eV. Consulting Figure 41.8, we see the minimum value of n for which the energy is higher than –2.28 eV is $\boxed{n = 3}$.

(b) Let us write the appropriate reduction of Equation 8.2 for the ionization process:

$$\Delta K_{\text{electron}} + \Delta U_{\text{atom}} = T_{\text{ER}} \qquad (1)$$

where T_{ER} represents the energy carried into the process by the photon. Express each of the energy changes in terms of initial and final values:

$$\left(K_f - K_i\right) + \left(0 - U_e\right) = T_{\text{ER}} \qquad (2)$$

where we have evaluated the final potential energy as zero because the electron is far from its parent atom. Now rearrange the equation:

$$K_f - \left(K_i + U_e\right) = T_{\text{ER}} \quad \rightarrow \quad K_f = T_{\text{ER}} + \left(K_i + U_e\right) \qquad (3)$$

The term on the left is the kinetic energy of the ejected electron. On the right, in the parenthesis is the sum of initial kinetic and potential energy values according to Equation 41.7, which we have ultimately evaluated in the Bohr model with Equation 41.14. Therefore, for an initial energy state corresponding to $n = 3$,

$$K_f = T_{\text{ER}} + E_3 = 2.28\text{ eV} - 1.51\text{ eV} = 0.77\text{ eV} \qquad (4)$$

Now use Equation 7.16 to find the speed of the electron:

$$K_f = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2\left(0.77\text{ eV}\right)}{9.11\times10^{-31}\text{ kg}}\left(\frac{1.602\times10^{-19}\text{ J}}{1\text{ eV}}\right)}$$
$$= 5.20\times10^{5}\text{ m/ s} = \boxed{520\text{ km/ s}}$$

Finalize The fact that this speed is only about 0.2% of the speed of light justifies our use of the classical expression for kinetic energy.]

Answers: (a) 3 (b) 520 km/s

***P41.48** **Conceptualize** Review Example 41.1. In that example, we looked at a hydrogen atom in the $n = 5$ state. Now imagine such an atom in a state of very high value of n.

Categorize We will be modeling the hydrogen atom with the Bohr theory.

Analyze From Equation 41.17, the quantum prediction of the wavelength is

$$\frac{1}{\lambda_{\text{quantum}}} = R_{\text{H}}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{k_e e^2}{2a_0 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad (1)$$

where we have substituted the definition of the Rydberg constant. From the What If? feature in Example 41.1, we see that the classical prediction of the wavelength is

$$\lambda_{\text{classical}} = \frac{c}{f_{\text{orbit}}} = \frac{2\pi rc}{v} \qquad (2)$$

Use Equations 41.8 and 41.12 to substitute for v and r in Equation (2):

$$\lambda_{\text{classical}} = \frac{2\pi\left(n_i^2 a_0\right)c}{\left(\sqrt{\dfrac{k_e e^2}{m_e\left(n_i^2 a_0\right)}}\right)} = \frac{2\pi c a_0^3}{e}\sqrt{\frac{m_e}{k_e}}\, n_i^3 \qquad (3)$$

Multiply Equations (1) and (3) to give a ratio of the classical wavelength to the quantum wavelength and clean up the constants:

$$\frac{\lambda_{\text{classical}}}{\lambda_{\text{quantum}}} = \left(\frac{k_e e^2}{2a_0 hc}\right)\left(\frac{2\pi c}{e}\sqrt{\frac{m_e a_0{}^3}{k_e}}\right) n_i^3 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{\pi e}{h}\sqrt{k_e m_e a_0}\, n_i^3 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Finally, substitute for a_0 from Equation 41.11:

$$\frac{\lambda_{\text{classical}}}{\lambda_{\text{quantum}}} = \frac{\pi e}{h}\sqrt{k_e m_e\left(\frac{\hbar^2}{m_e k_e e^2}\right)}n_i^3\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) = \tfrac{1}{2}n_i^3\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) \qquad (4)$$

It is remarkable that all of these constants reduced to just the fraction $\frac{1}{2}$! The problem statement asked for a comparison for a $\Delta n = 1$ transition, so let $n_i = n$ and $n_f = n - 1$. Substitute these values into Equation (4):

$$\frac{\lambda_{\text{classical}}}{\lambda_{\text{quantum}}} = r = \tfrac{1}{2}n^3\left[\frac{1}{(n-1)^2}-\frac{1}{n^2}\right] \qquad (5)$$

where we have used the notation r for the ratio of the classical wavelength to the quantum wavelength. Let's rearrange Equation (5) to become the following quadratic equation:

$$2(r-1)n^2 + (1-4r)n + 2r = 0 \qquad (6)$$

The solution to this quadratic equation is

$$n = \frac{(4r-1) \pm \sqrt{(1-4r)^2 - 4[2(r-1)](2r)}}{4(r-1)} = \frac{4r-1 \pm \sqrt{1+8r}}{4(r-1)}$$

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Substitute the value of $r = 1.005\,00$ for a 0.500% difference:

$$n = \frac{4(1.005\,00) - 1 \pm \sqrt{1+8(1.005\,00)}}{4(1.005\,00-1)} = 0.667 \quad \text{or} \quad 301$$

We must reject the first root, because the lowest possible value of n is 1. Therefore, the second root is the one that we want:

$$n = \boxed{301}$$

Finalize Let's combine the fractions in Equation (5):

$$r = \tfrac{1}{2}n^3\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \tfrac{1}{2}n^3\left[\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right] = \frac{n(2n-1)}{2(n-1)^2}$$

This equation clearly shows that $r \to 1$ as $n \to \infty$, so that the quantum problem reduces to the classical problem in the limits of large quantum numbers.]

Answer: 301

P41.49 (a) The length of the pulse is

$$\Delta L = c\Delta t = (3.00\times10^8\ \text{m/s})(14.0\times10^{-12}\ \text{s}) = \boxed{4.20\ \text{mm}}$$

(b) The energy of each photon is

$$E = \frac{hc}{\lambda} = 2.86\times10^{-19}\ \text{J}$$

so the number of photons in the pulse is

$$N = \frac{3.00\ \text{J}}{2.86\times10^{-19}\ \text{J/photon}} = \boxed{1.05\times10^{19}\ \text{photons}}$$

(c) The volume of the pulse is

$$V = \Delta L\pi r^2 = (4.20\ \text{mm})\left[\pi(3.00\ \text{mm})^2\right] = 119\ \text{mm}^3$$

resulting in a photon density of

$$n = \frac{1.05\times10^{19}\ \text{photons}}{119\ \text{mm}^3} = \boxed{8.82\times10^{16}\ \text{mm}^{-3}}$$

P41.50 (a) The length of the pulse is $\Delta L = \boxed{c\Delta t}$.

(b) The energy of each photon is $E = \dfrac{hc}{\lambda}$, so

$$N = \frac{T_{\text{ER}}}{E} = \boxed{\frac{\lambda T_{\text{ER}}}{hc}}$$

(c) The volume of the pulse is

$$V = \Delta L \pi \frac{d^2}{4} = c\Delta t \pi \frac{d^2}{4}$$

resulting in a photon density of

$$n = \frac{N}{V} = \frac{\lambda T_{\text{ER}}}{hc\left(c\Delta t \pi d^2/4\right)} = \boxed{\frac{4\lambda T_{\text{ER}}}{\pi h c^2 d^2 \Delta t}}$$

Challenge Problems

P41.51 (a) From Equation 41.13, the allowed energies are $E_n = \frac{k_e e^2}{2a_0}\left(\frac{1}{n^2}\right)$,

where, from Equation 41.11, the Bohr radius is

$$a_0 = \frac{\hbar^2}{m_e k_e e^2} = \frac{\left(h/2\pi\right)^2}{m_e k_e e^2} = \frac{h^2}{4\pi^2 m_e k_e e^2}$$

Combining these gives

$$E_n = \frac{k_e e^2}{2}\frac{4\pi^2 m_e k_e e^2}{h^2}\left(\frac{1}{n^2}\right) = \frac{2\pi^2 m_e k_e^2 e^4}{h^2}\left(\frac{1}{n^2}\right)$$

For a transition from state n to state $n-1$,

$$hf = \Delta E = \left(\frac{2\pi^2 m_e k_e^2 e^4}{h^2}\right)\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$

$$hf = \Delta E = \left(\frac{2\pi^2 m_e k_e^2 e^4}{h^2}\right)\frac{n^2 - \left(n^2 - 2n + 1\right)}{n^2 (n-1)^2}$$

which gives

$$f = \left(\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\right)\frac{2n-1}{n^2(n-1)^2}$$

(b) As $n \to \infty$, we find the quantum result:

$$f \to \frac{2\pi^2 m_e k_e^2 e^4}{h^3}\frac{2}{n^3} = \frac{4\pi^2 m_e k_e^2 e^4}{h^3 n^3}$$

The classical frequency is $f = \dfrac{v}{2\pi r}$, where classically, from Equation 41.8, $v^2 = \dfrac{k_e e^2}{m_e r}$. By substituting, the relation for the classical frequency becomes

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi r}\sqrt{\frac{k_e e^2}{m_e r}} = \sqrt{\frac{k_e e^2}{4\pi^2 m_e r^3}}$$

From Equation 41.10, the radius $r = r_n = \dfrac{n^2\hbar^2}{m_e k_e e^2} = \dfrac{n^2 h^2}{4\pi^2 m_e k_e e^2}$; substituting this yields

$$f = \sqrt{\frac{k_e e^2}{4\pi^2 m_e r^3}} = \sqrt{\frac{k_e e^2}{4\pi^2 m_e}\left(\frac{4\pi^2 m_e k_e e^2}{n^2 h^2}\right)^3}$$

$$= \sqrt{\frac{(4\pi^2)^2 m_e^2 k_e^4 e^8}{n^6 h^6}} = \frac{4\pi^2 m_e k_e^2 e^4}{h^3 n^3}$$

$\boxed{\text{The classical frequency is } 4\pi^2 m_e k_e^2 e^4 / h^3 n^3.}$ We see that the Bohr result for large n reduces to the classical result.

P41.52 (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\hat{\mathbf{i}} + \frac{h}{\lambda}(-\hat{\mathbf{i}}) = mv_f\hat{\mathbf{i}} \quad \text{so} \quad v_f - v_i = -\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\text{ s} = \Delta t$. Then,

$$a = \frac{v_f - v_i}{\Delta t} = -\frac{h}{m\lambda\Delta t} \sim -\frac{6.626\times 10^{-34}\text{ J}\cdot\text{s}}{\left(10^{-25}\text{ kg}\right)\left(500\times 10^{-9}\text{ m}\right)\left(10^{-8}\text{ s}\right)}$$
$$\sim \boxed{-10^{6}\text{ m/s}^2}$$

(b) With constant average acceleration,

$$v_f^2 = v_i^2 + 2a\Delta x \quad 0 \sim \left(10^3\text{ m/s}\right)^2 + 2\left(-10^6\text{ m/s}^2\right)\Delta x$$

so $\Delta x \sim \dfrac{\left(10^3\text{ m/s}\right)^2}{10^6\text{ m/s}^2}\boxed{\sim 1\text{ m}}$.

ANSWERS TO QUICK-QUIZZES

1. (c)

2. (a)

3. (b)

4. (a) five (b) nine

5. (c)

6. true

ANSWERS TO EVEN-NUMBERED PROBLEMS

P41.2 1.94 μm

P41.4 (a) See P41.4(a) for full explanation; (b) 0.846 ns

P41.6 See P41.6 for full explanation.

P41.8 (a) 97.3 nm; (b) 1.88 μm; (c) infrared; (d) Paschen; (e) 97.3 nm; (f) ultraviolet; (g) Lyman

P41.10 (a) 152 as; (b) 8.23×10^9 revolutions; (c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so we can think of it as a long time.

P41.12 (a) 1.31 μm; (b) 164 nm

P41.14 (a) $\dfrac{\hbar}{2r}$; (b) $\dfrac{\hbar^2}{2m_e r^2}$; (c) $\dfrac{\hbar^2}{2m_e r^2} - \dfrac{k_e e^2}{r}$; (d) $\dfrac{\hbar^2}{m_e k_e e^2} = a_0$; (e) –13.6 eV; (f) We find our results are in agreement with the Bohr theory.

P41.16 (a) See P41.16(a) for full explanation; (b) $E = -\dfrac{k_e e^2}{2a_0}$

P41.18 (a) See P41.18(a) for full explanation; (b) See P41.18(b) for full explanation

P41.20 (a) See P41.20(a) for a list of all sets; (b) See P41.20(b) for a list of all sets.

P41.22 0.05 T

P41.24 The electron energy is not enough to excite the hydrogen atom from its ground state to even the first excited state.

P41.26 See P41.26 for full explanation.

P41.28 See P41.28 for the complete table.

P41.30 $\dfrac{1?40\ \text{nm} \cdot \text{V}}{\Delta V}$

P41.32 (a) 1.4, 1.0; (b) See P41.32(a) for full explanation

P41.34 (a) $\dfrac{1240\ \text{V}\cdot\text{nm}}{\lambda}$; (b) The potential difference is inversely proportional to the wavelength; (c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure; (d) Yes, but it might be unlikely for a very high energy electron to stop in a single interaction to produce a high-energy gamma ray, and it might be difficult to observe the very low intensity radio waves produced as bremsstrahlung by low-energy electrons; (e) The potential difference goes to infinity as the wavelength goes to zero; (f) The potential difference goes to zero as the wavelength goes to infinity.

P41.36 (a) $\lambda_1 = 632.809\ 14$ nm, $\lambda_2 = 632.808\ 57$ nm, $\lambda_3 = 632.809\ 71$ nm, three; (b) 697 m/s (c) See P41.36(c) for full description.

P41.38 See P41.38 for full explanation.

P41.40 In order to have a maximum value of m_ℓ equal to 3, we need to have $\ell = 3$, which is inconsistent with the first result.

P41.42 (a) See P41.42(a) for full explanation; (b) 2.53×10^{74}; (c) 1.18×10^{-63} m; (d) This number is *much smaller* than the radius of an atomic nucleus ($\sim 10^{-15}$ m), so the distance between quantized orbits of the Earth is too small to observe.

P41.44 (a) $\left(\dfrac{2r^2}{a_0^2} + \dfrac{2r}{a_0} + 1\right)e^{-2r/a_0}$; (b) See ANS. FIG. P41.44; (c) 1.34 a_0

P41.46 See ANS. FIG. P41.46

P41.48 301

P42.50 (a) $c\Delta t$; (b) $\dfrac{\lambda T_{\text{ER}}}{hc}$; (c) $\dfrac{4\lambda T_{\text{ER}}}{\pi hc^2 d^2 \Delta t}$

P42.52 (a) -10^6 m/s^2; (b) ~1 m