3

Vectors

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

SOLUTIONS TO THINK-PAIR-SHARE AND ACTIVITIES

***TP3.1** **Conceptualize** Imagine your situation, staring at a radar screen with five ships indicated. You want to choose which ship to send, but how do you do it, especially when they all have different speeds? Should you simply send the ship that has the highest speed?

Categorize Based on the assumption in the problem statement, we can use the particle under constant velocity model for each ship.

Analyze First, find the x and y components of the four ships, as well as the sinking ship, using east as the $+x$ direction and north as the $+y$ direction:

Ship #	**x (km)**	**y (km)**
1	–24.2	26.8
2	–32.6	18.1
3	–6.00	8.25
4	–50.3	9.77
Sinking	–30.1	41.4

Now find the differences in x components and the differences in y components between the sinking ship and each of the other ships:

Ship #	**Δx (km)**	**Δy (km)**
1	5.94	–14.6
2	–2.53	–23.3
3	24.1	–33.2
4	–20.2	–31.7

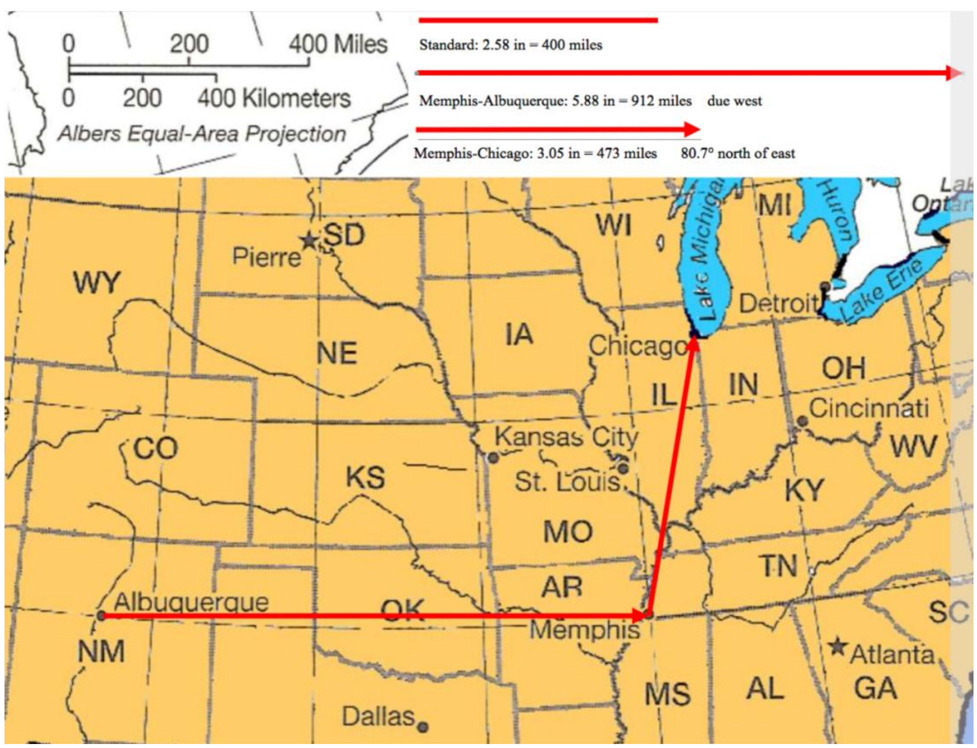
Finally, use the Pythagorean theorem to find the distance from each ship to the sinking ship and use the speed, assumed constant, to find the time interval required for the ship to reach the sinking ship:

Ship #	Distance from Sinking Ship (km)	Maximum Speed (km/h)	Time Interval (h)
1	15.8	30.0	0.53
2	23.5	38.0	0.62
3	10.2	32.0	1.28
4	51.2	45.0	0.83

Finalize Well, the fastest ship would not get there first! In fact, you need to alert the *slowest* ship, because it is the closest and, despite its lower speed, it will arrive at the sinking ship first.

Answer: ship 1

***TP3.2** A typical drawing will look like that below.



Albuquerque (35°06′39″N, 106°36′36″W) and Memphis (35°07′03″N 89°58′16″W) are almost on the exact same latitude, so we can say that the vector from Albuquerque to Memphis is pointed due east. The comparison to the scale in the diagram shows a distance of 912 miles, so we can write the Albuquerque–Memphis vector as

$$\vec{\mathbf{r}}_{\text{A–M}} = 912\hat{\mathbf{i}} \text{ mi}$$

The comparison to the scale for the Memphis–Chicago vector shows a distance of 473 miles. A protractor shows that the vector makes an angle of 80.7° with the latitude line running through Memphis. Therefore, the Memphis–Chicago vector is

$$\begin{aligned}\vec{\mathbf{r}}_{\text{M–C}} &= \left(473\cos 80.7°\hat{\mathbf{i}} + 473\sin 80.7°\hat{\mathbf{j}}\right) \text{mi} \\ &= \left(76.4\hat{\mathbf{i}} + 467°\hat{\mathbf{j}}\right) \text{mi}\end{aligned}$$

A vector drawn from Albuquerque to Chicago would be the sum of these two vectors:

$$\vec{\mathbf{r}}_{\text{A–C}} = \vec{\mathbf{r}}_{\text{A–M}} + \vec{\mathbf{r}}_{\text{M–C}} = 912\hat{\mathbf{i}} \text{ mi} + \left(76.4\hat{\mathbf{i}} + 467°\hat{\mathbf{j}}\right) \text{mi} = \left(988\hat{\mathbf{i}} + 467°\hat{\mathbf{j}}\right) \text{mi}$$

Use the Pythagorean theorem to find the distance between Albuquerque and Chicago:

$$\left|\vec{\mathbf{r}}_{\text{A–C}}\right| = \sqrt{(988)^2 + (467)^2} \text{ mi} = 1.10 \times 10^3 \text{ mi}$$

The direction of Chicago from Albuquerque is

$$\theta = \tan^{-1}\left(\frac{467\text{ mi}}{988\text{ mi}}\right) = 25.3^\circ$$

Answer: Answers will vary.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 3.1 Coordinate Systems

P3.1 (a) The distance between the points is given by

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2} \end{aligned}$$

$$d = \sqrt{25.0 + 49.0} = \boxed{8.60\text{ m}}$$

(b) To find the polar coordinates of each point, we measure the radial distance to that point and the angle it makes with the $+x$ axis:

$$r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47\text{ m}}$$

$$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4^\circ}$$

$$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24\text{ m}}$$

$\theta_2 = \boxed{135^\circ}$ measured from the $+x$ axis.

P3.2 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore,

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$$(x_1, y_1) = \boxed{(2.17, 1.25)\text{ m}}$$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$$(x_2,\ y_2) = \boxed{(-1.90, 3.29)\text{ m}}$$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2}\text{ m} = \boxed{4.55\text{ m}}$

P3.3 For polar coordinates (r, θ), the Cartesian coordinates are $(x = r\cos\theta, y = r\sin\theta)$, if the angle is measured relative to the $+x$ axis.

(a) $\boxed{(-3.56\text{ cm},\ -2.40\text{ cm})}$

(b) $(+3.56\text{ cm}, -2.40\text{ cm}) \rightarrow \boxed{(4.30\text{ cm}, -34.0°)}$

(c) $(7.12\text{ cm}, 4.80\text{ cm}) \rightarrow \boxed{(8.60\text{ cm}, 34.0°)}$

(d) $(-10.7\text{ cm}, 7.21\text{ cm}) \rightarrow \boxed{(12.9\text{ cm}, 146°)}$

P3.4 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

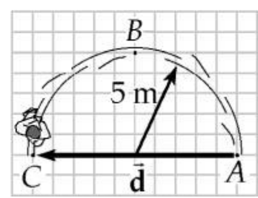
$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}$$

(b) $\sqrt{(-2x)^2+(-2y)^2} = \boxed{2r}$. This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180°+\theta}$.

(c) $\sqrt{(3x)^2+(-3y)^2} = \boxed{3r}$. This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta \text{ or } 360-\theta}$.

Section 3.2 Vector and Scalar Quantities

P3.5 In solving this problem we must contrast displacement with distance traveled. We draw a diagram of the skater's path in ANS. FIG. P3.9, which is the view from a hovering helicopter so that we can see the circular path as circular in shape. To start with a concrete example, we have chosen to draw motion *ABC* around one half of a circle of radius 5 m.



ANS. FIG. P3.5

The displacement, shown as $\vec{\mathbf{d}}$ in the diagram, is the straight-line change in position from starting point *A* to finish *C*. In the specific case we have chosen to draw, it lies along a diameter of the circle. Its magnitude is $\left|\vec{\mathbf{d}}\right| = \left|-10.0\hat{\mathbf{i}}\right| = 10.0$ m.

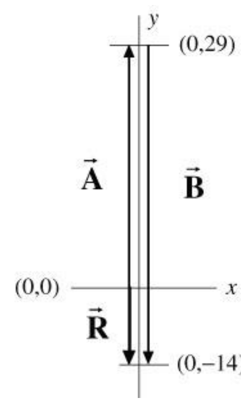
The distance skated is greater than the straight-line displacement. The distance follows the curved path of the semicircle (ABC). Its length is half of the circumference: $s = \frac{1}{2}(2\pi r) = 5.00\pi \text{ m} = 15.7 \text{ m}$.

A straight line is the shortest distance between two points. For any nonzero displacement, less or more than across a semicircle, the distance along the path will be greater than the displacement magnitude. Therefore:

The situation can never be true because the distance is an arc of a circle between two points, whereas the magnitude of the displacement vector is a straight-line cord of the circle between the same points.

Section 3.3 Basic Vector Arithmetic

P3.6 We are given $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative y direction to the point (0, −14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point

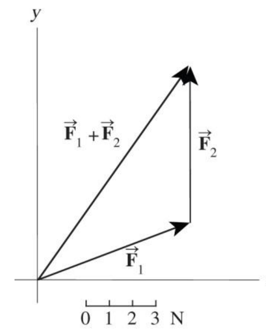


ANS. FIG. P3.6

(0, −14) as shown in the sketch at the right. From this, it is seen that

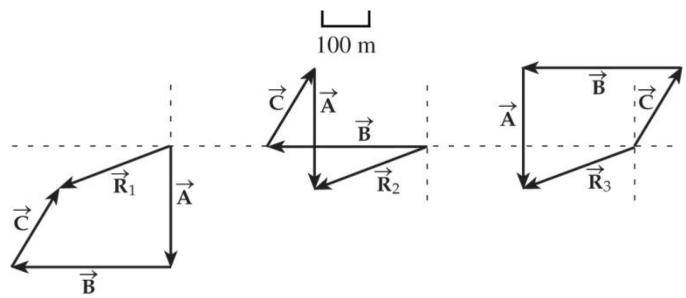
$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y \text{ direction}}$

P3.7 We find the resultant $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$. The resultant force vector $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x \text{ axis}}$.



ANS. FIG. P3.7

P3.8 (a) The three diagrams are shown in ANS. FIG. P3.8 below.



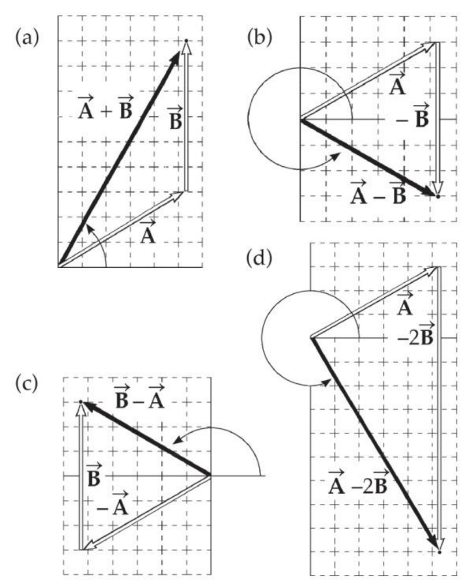
ANS. FIG. P3.8

(b) The diagrams in ANS. FIG. P3.8 represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$.

P3.9 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{5.2 \text{ m at } 60°}$

(b) $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{3.0 \text{ m at } 330°}$

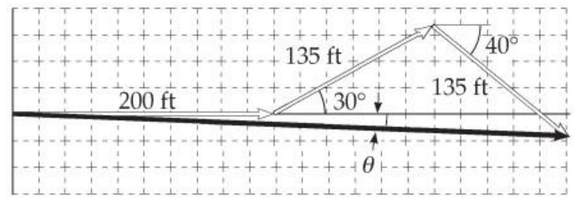


ANS. FIG. P3.9

(c) $\vec{\mathbf{B}} - \vec{\mathbf{A}} = \boxed{3.0 \text{ m at } 150^\circ}$

(d) $\vec{\mathbf{A}} - 2\mathbf{B} = \boxed{5.2 \text{ m at } 300^\circ}$

P3.10 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be $\boxed{d = 420 \text{ ft and } \theta = -3^\circ}$.



(Scale: 1unit = 20 ft)

ANS. FIG. P3.10

Section 3.4 Components of a Vector and Unit Vectors

P3.11 (a) $\boxed{\text{Yes}}$.

(b) Let v represent the speed of the camper. The northward component of its velocity is $v \cos 8.50°$. To avoid crowding the minivan we require $v \cos 8.50° \geq 28$ m/s.

$$\boxed{\begin{array}{l}\text{We can satisfy this requirement simply by taking} \\ v \geq (28.0 \text{ m/s})/\cos 8.50^\circ = 28.3 \text{ m/s.}\end{array}}$$

P3.12 The person would have to walk

$$(3.10 \text{ km})\sin 25.0^\circ = \boxed{1.31 \text{ km north}}$$

and $(3.10\text{ km})\cos 25.0° = \boxed{2.81\text{ km east}}$

P3.13 We use the unit-vector addition method. It is just as easy to add three displacements as to add two. We take the direction east to be along $+\hat{\mathbf{i}}$. The three displacements can be written as:

$$\begin{aligned}\vec{\mathbf{d}}_1 &= (-3.50\text{ m})\hat{\mathbf{j}}\\ \vec{\mathbf{d}}_2 &= (8.20\text{ m})\cos 45.0°\hat{\mathbf{i}} + (8.20\text{ m})\sin 45.0°\hat{\mathbf{j}}\\ &= (5.80\text{ m})\hat{\mathbf{i}} + (5.80\text{ m})\hat{\mathbf{j}}\\ \vec{\mathbf{d}}_3 &= (-15.0\text{ m})\hat{\mathbf{i}}\end{aligned}$$

The resultant is

$$\begin{aligned}\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 &= (-15.0\text{ m} + 5.80\text{ m})\hat{\mathbf{i}} + (5.80\text{ m} - 3.50\text{ m})\hat{\mathbf{j}}\\ &= (-9.20\text{ m})\hat{\mathbf{i}} + (2.30\text{ m})\hat{\mathbf{j}}\end{aligned}$$

(or 9.20 m west and 2.30 m north).

The magnitude of the resultant displacement is

$$\left|\vec{\mathbf{R}}\right| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20\text{ m})^2 + (2.30\text{ m})^2} = \boxed{9.48\text{ m}}$$

The direction of the resultant vector is given by

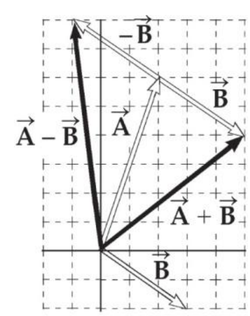
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.30\text{ m}}{-9.20\text{ m}}\right) = \boxed{166°}$$

P3.14 (a) See figure to the right.

$$\begin{aligned}\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} &= 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\\ &= \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}\end{aligned}$$

(b)

$$\begin{aligned}\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} &= 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}\\ &= \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}\end{aligned}$$



ANS. FIG. P3.14

$$\vec{\mathbf{C}} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$$

(c) $$\vec{\mathbf{D}} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$$

$$\vec{\mathbf{D}} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$$

P3.15 (a) The single force is obtained by summing the two forces:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

$$\vec{\mathbf{F}} = 120 \cos (60.0°)\hat{\mathbf{i}} + 120 \sin (60.0°)\hat{\mathbf{j}}$$
$$- 80.0 \cos (75.0°)\hat{\mathbf{i}} + 80.0 \sin (75.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

We can also express this force in terms of its magnitude and direction:

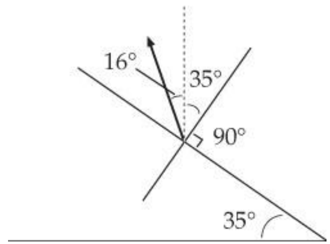
$$\left|\vec{\mathbf{F}}\right| = \sqrt{39.3^2 + 181^2} \text{ N} = \boxed{185 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) A force equal and opposite the resultant force from part (a) is required for the total force to equal zero:

$$\vec{\mathbf{F}}_3 = -\vec{\mathbf{F}} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$$

P3.16 We take the x axis along the slope downhill. (Students, get used to this choice!) The y axis is perpendicular to the slope, at 35.0° to the vertical. Then the displacement of the snow makes an angle of 90.0° + 35.0° + 16.0° = 141° with the x axis.



ANS. FIG. P3.16

(a) $\boxed{\begin{array}{l}\text{Its component parallel to the surface is}\\ (1.50\text{ m})\cos 141° = -1.17\text{ m},\\ \text{or 1.17 m toward the top of the hill.}\end{array}}$

(b) $\boxed{\begin{array}{l}\text{Its component perpendicular to the surface is}\\ (1.50\text{ m})\sin 141° = 0.944\text{ m, or 0.944 m away}\\ \text{from the snow.}\end{array}}$

P3.17 (a) We add the components of the three vectors:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 6\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{D}}\right| = \sqrt{6^2 + 2^2} = \boxed{6.32\text{ m at }\theta = 342°}$$

(b) Again, using the components of the three vectors,

$$\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}} = -2\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{E}}\right| = \sqrt{2^2 + 12^2} = \boxed{12.2\text{ m at }\theta = 99.5°}$$

P3.18 We are given $\vec{\mathbf{A}} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$, and $\vec{\mathbf{B}} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$, and $\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$. Solving for $\vec{\mathbf{C}}$ gives

$$3\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\vec{\mathbf{C}} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}} \quad\text{or}\quad C_x = \boxed{7.30\text{ cm}};\ C_y = \boxed{-7.20\text{ cm}}$$

P3.19 Hold your fingertip at the center of the front edge of your study desk, defined as point *O*. Move your finger 8 cm to the right, then 12 cm vertically up, and then 4 cm horizontally away from you. Its location relative to the starting point represents position vector $\vec{\mathbf{A}}$. Move three-fourths of the way straight back toward *O*. Now your fingertip is at the location of $\vec{\mathbf{B}}$. Now move your finger 50 cm straight through *O*, through your left thigh, and down toward the floor. Its position vector now is $\vec{\mathbf{C}}$.

We use unit-vector notation throughout. There is no adding to do here, but just multiplication of a vector by two different scalars.

(a) $\vec{\mathbf{A}} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{A}}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\vec{\mathbf{C}} = -3\vec{\mathbf{A}} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.20 We carry out the prescribed mathematical operations using unit vectors.

(a)
$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\ \text{m}}$$
$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00\ \text{m})^2 + (1.00\ \text{m})^2 + (3.00\ \text{m})^2} = \boxed{5.92\ \text{m}}$$

(b)
$$\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{m}}$$
$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00\ \text{m})^2 + (11.0\ \text{m})^2 + (15.0\ \text{m})^2} = \boxed{19.0\ \text{m}}$$

P3.21 The component description of $\vec{\mathbf{A}}$ is just restated to constitute the answer to part (a): $A_x = -3.00$, $A_y = 2.00$.

(a) $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{2.00}{-3.00}\right) = -33.7°$$

θ is in the second quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

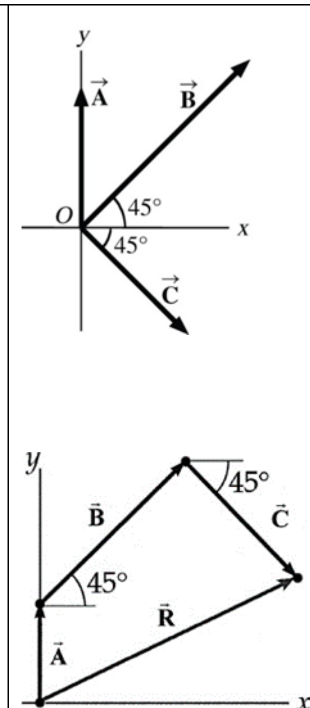
(c) $R_x = 0$, $R_y = -4.00$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, thus $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$.

Therefore, $\vec{\mathbf{B}} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.22

The given diagram shows the vectors individually, but not their addition. The second diagram represents a map view of the motion of the ball. According to the definition of a displacement, we ignore any departure from straightness of the actual path of the ball. We model each of the three motions as straight. The simplified problem is solved by straightforward application of the component method of vector addition. It works for adding two, three, or any number of vectors.



ANS. FIG. P3.22

(a) We find the two components of each of the three vectors

$$A_x = (20.0 \text{ units})\cos 90° = 0$$

and $A_y = (20.0\,\text{units})\sin 90° = 20.0 \text{ units}$

$$B_x = (40.0\,\text{units})\cos 45° = 28.3 \text{ units}$$

and $B_y = (40.0\,\text{units})\sin 45° = 28.3 \text{ units}$

$$C_x = (30.0\,\text{units})\cos 315° = 21.2 \text{ units}$$

and $C_y = (30.0\,\text{units})\sin 315° = -21.2 \text{ units}$

Now adding,

$$R_x = A_x + B_x + C_x = (0 + 28.3 + 21.2) \text{ units} = 49.5 \text{ units}$$

and $R_y = A_y + B_y + C_y = (20 + 28.3 - 21.2) \text{ units} = 27.1 \text{ units}$

so $\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b)

$$\left|\vec{\mathbf{R}}\right| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$$

P3.23 (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0$$

Substituting $a = 1.33b - 4.33$ into the second equation, we find

$$-8(1.33b - 4.33) + 3b + 19 = 0 \rightarrow 7.67b = 53.67 \rightarrow b = 7.00$$

and so a = 1.33(7.00) – 4.33 = 5.00.

Thus $\boxed{a = 5.00,\ b = 7.00}$. Therefore, $5.00\vec{\mathbf{A}} + 7.00\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$.

(b) $\boxed{\text{In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation, as each component gives us one equation.}}$

P3.24 We are given $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$. The magnitude of the vector is therefore

$$\left|\vec{\mathbf{B}}\right| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

And the angle of the vector with the three coordinate axes is

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2^\circ} \text{ is the angle with the } x \text{ axis}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8^\circ} \text{ is the angle with the } y \text{ axis}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4^\circ} \text{ is the angle with the } z \text{ axis}$$

P3.25 We use the numbers given in Problem 3.11:

$$\vec{\mathbf{A}} = 3.00 \text{ m},\ \theta_A = 30.0^\circ$$

$$A_x = A\cos\theta_A = 3.00\cos 30.0^\circ = 2.60 \text{ m},$$

$$A_y = A\sin\theta_A = 3.00\sin 30.0^\circ = 1.50 \text{ m}$$

So $$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = (2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}) \text{ m}$$

$$\vec{\mathbf{B}} = 3.00 \text{ m},\ \theta_B = 90.0^\circ$$

$$B_x = 0,\ B_y = 3.00\text{ m} \rightarrow \vec{\mathbf{B}} = 3.00\hat{\mathbf{j}}\text{ m}$$

then
$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)\text{m}}$$

P3.26(a) Her net x (east-west) displacement is –3.00 + 0 + 6.00 = +3.00 blocks, while her net y (north-south) displacement is 0 + 4.00 + 0 = +4.00 blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the x axis (eastward direction) is

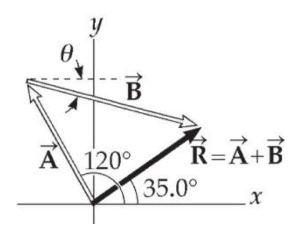
$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{\text{5.00 blocks at 53.1° N of E}}$.

(b) The total distance traveled is 3.00 + 4.00 + 6.00 = $\boxed{\text{13.00 blocks}}$.

P3.27 We will use the component method for a precise answer. We already know the total displacement, so the algebra of solving a vector equation will guide us to do a subtraction.

We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:



ANS. FIG. P3.27

$$A_x = 150\cos 120° = -75.0\text{ cm}$$

$$A_y = 150\sin 120° = 130\text{ cm}$$

$$R_x = 140\cos 35.0° = 115\text{ cm}$$

$$R_y = 140\sin 35.0° = 80.3\text{ cm}$$

Therefore,

$$\vec{\mathbf{B}} = [115-(-75)]\hat{\mathbf{i}} + [80.3-130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right) \text{ cm}$$
$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

P3.28 First, we sum the components of the two vectors for the male:

$$d_{3mx} = d_{1mx} + d_{2mx} = 0 + (100 \text{ cm})\cos 23.0° = 92.1 \text{ cm}$$
$$d_{3my} = d_{1my} + d_{2my} = 104 \text{ cm} + (100 \text{ cm})\sin 23.0° = 143.1 \text{ cm}$$
$$\boxed{\begin{array}{l}\text{magnitude: } d_{3m} = \sqrt{(92.1 \text{ cm})^2 + (143.1 \text{ cm})^2} = 170.1 \text{ cm} \\ \text{direction: } \tan^{-1}(143.1/92.1) = 57.2° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

followed by the components of the two vectors for the female:

$$d_{3fx} = d_{1fx} + d_{2fx} = 0 + (86.0 \text{ cm})\cos 28.0° = 75.9 \text{ cm}$$
$$d_{3fy} = d_{1fy} + d_{2fy} = 84.0 \text{ cm} + (86.0 \text{ cm})\sin 28.0° = 124.4 \text{ cm}$$
$$\boxed{\begin{array}{l}\text{magnitude: } d_{3f} = \sqrt{(75.9 \text{ cm})^2 + (124.4 \text{ cm})^2} = 145.7 \text{ cm} \\ \text{direction: } \tan^{-1}(124.4/75.9) = 58.6° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

P3.29 The hurricane's first displacement is

$$(41.0 \text{ km/h})(3.00 \text{ h}) \text{ at } 60.0° \text{ N of W}$$

and its second displacement is

$$(25.0 \text{ km/h})(1.50 \text{ h}) \text{ due North}$$

With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\begin{aligned}
&[(41.0\text{ km/h})\cos 60.0°](3.00\text{ h})(-\hat{\mathbf{i}}) \\
&\qquad +[(41.0\text{ km/h})\sin 60.0°](3.00\text{ h})\hat{\mathbf{j}} \\
&\qquad +(25.0\text{ km/h})(1.50\text{ h})\hat{\mathbf{j}} \\
&= 61.5\text{ km}(-\hat{\mathbf{i}}) + 144\text{ km }\hat{\mathbf{j}}
\end{aligned}$$

with magnitude $\sqrt{(61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km}}$.

P3.30 Let the positive x direction be eastward, the positive y direction be vertically upward, and the positive z direction be southward. The total displacement is then

$$\begin{aligned}
\vec{\mathbf{d}} &= (4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}})\text{ cm} + (3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}})\text{ cm} \\
&= (4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}})\text{ cm}
\end{aligned}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\text{ cm} = \boxed{10.4\text{ cm}}$.

(b) Its angle with the y axis follows from

$\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

P3.31 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$, where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 8\,040\text{ m}/30\text{ s} = 268\text{ m/s}$. The position vector as a function of time is

$$\vec{\mathbf{P}} = (268\text{ m/s})t\hat{\mathbf{i}} + (7.60 \times 10^3\text{ m})\hat{\mathbf{j}}$$

At $t = 45.0$ s, $\vec{\mathbf{P}} = \left[1.21 \times 10^4\hat{\mathbf{i}} + 7.60 \times 10^3\hat{\mathbf{j}}\right]$ m. The magnitude is

$$\vec{\mathbf{P}} = \sqrt{\left(1.21\times10^4\right)^2 + \left(7.60\times10^3\right)^2}\ \text{m} = \boxed{1.43\times10^4\ \text{m}}$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{7.60\times10^3}{1.21\times10^4}\right) = \boxed{32.2^\circ \text{ above the horizontal}}$$

P3.32 Note that each shopper must make a choice whether to turn 90° to the left or right, each time he or she makes a turn. One set of such choices, following the rules in the problem, results in the shopper heading in the positive y direction and then again in the positive x direction.

Find the magnitude of the sum of the displacements:

$$\vec{\mathbf{d}} = (8.00\text{ m})\hat{\mathbf{i}} + (3.00\text{ m})\hat{\mathbf{j}} + (4.00\text{ m})\hat{\mathbf{i}} = (12.00\text{ m})\hat{\mathbf{i}} + (3.00\text{ m})\hat{\mathbf{j}}$$

$$\text{magnitude: d} = \sqrt{(12.00\text{ m})^2 + (3.00\text{ m})^2} = 12.4\text{ m}$$

$$\boxed{\text{Impossible because 12.4 m is greater than 5.00 m.}}$$

P3.33 The displacement from the start to the finish is

$$16\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (11\hat{\mathbf{i}} + 9\hat{\mathbf{j}})$$

The displacement from the starting point to A is $f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right)$ meters.

(a) The position vector of point A is

$$5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right) = \boxed{\left[(5+11f)\hat{\mathbf{i}} + (3+9f)\hat{\mathbf{j}}\right]\ \text{m}}$$

(b) For $f = 0$ we have the position vector $\boxed{(5+0)\hat{\mathbf{i}} + (3+0)\hat{\mathbf{j}} \text{ meters.}}$

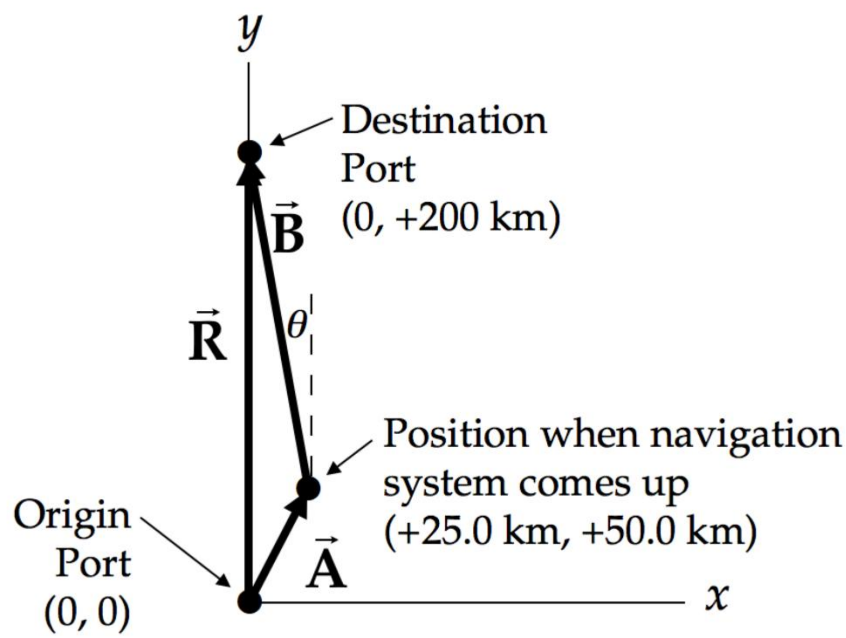
(c) $\boxed{\text{This is reasonable because it is the location of the starting point, } 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} \text{ meters.}}$

(d) For $f = 1 = 100\%$, we have position vector

$$(5+11)\hat{\mathbf{i}} + (3+9)\hat{\mathbf{j}} \text{ meters} = \boxed{16\hat{\mathbf{i}} + 12\hat{\mathbf{j}} \text{ meters}}.$$

(e) $\boxed{\text{This is reasonable because we have completed the trip, and this is the position vector of the endpoint.}}$

***P3.34** **Conceptualize** A drawing is always a good way to help understand a vector problem. Below is a sketch of the situation.



The desired displacement from the position when the system comes up is vector $\vec{\mathbf{B}}$ in the diagram. The desired heading angle is shown as the angle θ.

Categorize This is a relatively simple problem in vector addition in two dimensions.

Analyze As noted in the diagram, we denote the early part of the displacement as vector $\vec{\mathbf{A}}$, the desired vector from the point at which the navigation system comes up as $\vec{\mathbf{B}}$, and the resultant, which is the originally desired displacement, $\vec{\mathbf{R}}$. Vector addition of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ gives us

$$\vec{\mathbf{A}}+\vec{\mathbf{B}}=\vec{\mathbf{R}} \quad \rightarrow \quad \left(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}\right)+\left(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}\right)=\left(R_x\hat{\mathbf{i}}+R_y\hat{\mathbf{j}}\right) \qquad (1)$$

Separate the x and y components:

$$\begin{aligned} A_x\hat{\mathbf{i}}+B_x\hat{\mathbf{i}}=R_x\hat{\mathbf{i}} \quad &\rightarrow \quad A_x+B_x=R_x \\ A_y\hat{\mathbf{j}}+B_y\hat{\mathbf{j}}=R_y\hat{\mathbf{j}} \quad &\rightarrow \quad A_y+B_y=R_y \end{aligned} \qquad (2)$$

Substitute numerical values:

$$\begin{aligned} 25.0\text{ km}+B_x=0 \quad &\rightarrow \quad B_x=-25.0\text{ km} \\ 50.0\text{ km}+B_y=200\text{ km} \quad &\rightarrow \quad B_y=150\text{ km} \end{aligned}$$

Therefore, expressing the heading with respect to due north,

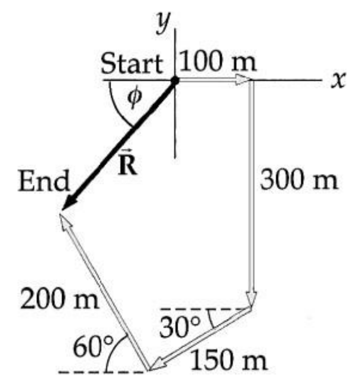
$$\tan\theta=\frac{B_x}{B_y}=\frac{25.0\text{ km}}{150\text{ km}}=0.167 \quad \rightarrow \quad \theta=9.46^\circ$$

Finalize You should advise the captain to set a heading of 9.46° west of north and maintain that heading to arrive at the destination port. This is the vector $\vec{\mathbf{B}}$ in the figure in the Conceptualize step.

Answer: 9.46° west of north

Additional Problems

P3.35 On our version of the diagram we have drawn in the resultant from the tail of the first arrow to the head of the last arrow. The resultant displacement $\vec{\mathbf{R}}$ is equal to the sum of the four individual displacements, $\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4$. We translate from the pictorial representation to a mathematical representation by writing the individual displacements in unit-vector notation:



ANS. FIG. P3.35

$$\vec{\mathbf{d}}_1 = 100\hat{\mathbf{i}} \text{ m}$$

$$\vec{\mathbf{d}}_2 = -300\hat{\mathbf{j}} \text{ m}$$

$$\vec{\mathbf{d}}_3 = (-150 \cos 30°)\hat{\mathbf{i}} \text{ m} + (-150 \sin 30°)\hat{\mathbf{j}} \text{ m} = -130\hat{\mathbf{i}} \text{ m} - 75\hat{\mathbf{j}} \text{ m}$$

$$\vec{\mathbf{d}}_4 = (-200 \cos 60°)\hat{\mathbf{i}} \text{ m} + (200 \sin 60°)\hat{\mathbf{j}} \text{ m} = -100\hat{\mathbf{i}} \text{ m} + 173\hat{\mathbf{j}} \text{ m}$$

Summing the components together, we find

$$R_x = d_{1x} + d_{2x} + d_{3x} + d_{4x} = (100 + 0 - 130 - 100) \text{ m} = -130 \text{ m}$$

$$R_y = d_{1y} + d_{2y} + d_{3y} + d_{4y} = (0 - 300 - 75 + 173) \text{ m} = -202 \text{ m}$$

so altogether

$$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{m}}$$

Its magnitude is

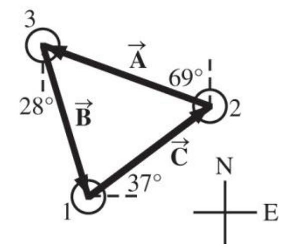
$$\left|\vec{\mathbf{R}}\right| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240 \text{ m}}$$

We calculate the angle $\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-202}{-130}\right) = 57.2°$.

The resultant points into the third quadrant instead of the first quadrant. The angle counter-clockwise from the $+x$ axis is

$$\theta = 180 + \phi = \boxed{237°}$$

P3.36 Let A represent the distance from island 2 to island 3. The displacement is $\vec{\mathbf{A}} = A$ at 159°. Represent the displacement from 3 to 1 as $\vec{\mathbf{B}} = B$ at 298°. We have 4.76 km at $37° + \vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$.



ANS. FIG. P3.36

For the x components:

$$\begin{aligned}(4.76\text{ km})\cos 37° + A\cos 159°& \\ +B\cos 298° &= 0 \\ 3.80\text{ km} - 0.934A + 0.470B &= 0\end{aligned}$$

$$B = -8.10\text{ km} + 1.99A$$

For the y components:

$$\begin{aligned}(4.76\text{ km})\sin 37° + A\sin 159° + B\sin 298° &= 0 \\ 2.86\text{ km} + 0.358A - 0.883B &= 0\end{aligned}$$

(a) We solve by eliminating B by substitution:

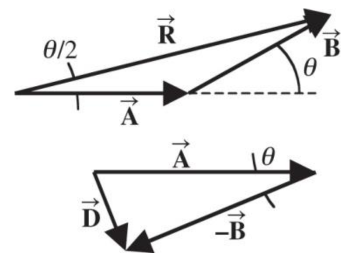
$$\begin{aligned}2.86\text{ km} + 0.358A - 0.883(-8.10\text{ km} + 1.99A) &= 0 \\ 2.86\text{ km} + 0.358A + 7.15\text{ km} - 1.76A &= 0\end{aligned}$$

$$10.0\text{ km} = 1.40A$$

$$A = \boxed{7.17\text{ km}}$$

(b) $B = -8.10\text{ km} + 1.99(7.17\text{ km}) = \boxed{6.15\text{ km}}$

P3.37 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$. This can be seen from



ANS. FIG. P3.37

applying the law of cosines to the isosceles triangle and using the fact that $B = A$.

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

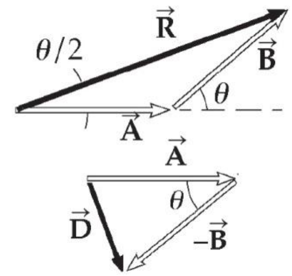
gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\dfrac{\theta}{2}\right) = 200A\sin\left(\dfrac{\theta}{2}\right)$. This gives

$$\tan\left(\frac{\theta}{2}\right) = 0.010 \text{ and } \boxed{\theta = 1.15°}$$

P3.38 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then



ANS. FIG. P3.38

$R = 2A\cos\left(\frac{\theta}{2}\right)$. This can be seen by applying the law of cosines to the isosceles triangle and using the fact that $B = A$. Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or

$$\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right) \text{ giving } \boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}.$$

The larger R is to be compared to D, the smaller the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ becomes.

P3.39 (a) Take the x axis along the tail section of the snake. The displacement from tail to head is

$$(240\text{ m})\hat{\mathbf{i}} + \left[(420-240)\text{ m}\right]\cos(180° - 105°)\hat{\mathbf{i}}$$
$$-(180\text{ m})\sin 75°\hat{\mathbf{j}} = 287\text{ m}\hat{\mathbf{i}} - 174\text{ m}\hat{\mathbf{j}}$$

Its magnitude is $\sqrt{(287)^2 + (174)^2}$ m = 335 m.

From $v = \dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335 \text{ m}(\text{h})(1 \text{ km})(3600 \text{ s})}{(12 \text{ km})(1000 \text{ m})(1 \text{ h})} = 101 \text{ s}$$

$$\text{Olaf: } \Delta t = \frac{420 \text{ m} \cdot \text{s}}{3.33 \text{ m}} = 126 \text{ s}$$

Inge wins by $126 - 101 = \boxed{25.4 \text{ s}}$.

(b) Olaf must run the race in the same time:

$$v = \frac{d}{\Delta t} = \frac{420 \text{ m}}{101 \text{ s}}\left(\frac{3600 \text{ s}}{1 \text{ h}}\right)\left(\frac{\text{km}}{10^3 \text{ m}}\right) = \boxed{15.0 \text{ km/h}}$$

P3.40 (a) The very small differences between the angles suggests we may consider this region of Earth to be small enough so that we may consider it to be flat (a plane); therefore, we may consider the lines of latitude and longitude to be parallel and perpendicular, so that we can use them as an *xy* coordinate system. Values of latitude, θ, increase as we travel north, so differences between latitudes can give the y coordinate. Values of longitude, ϕ, increase as we travel west, so differences between longitudes can give the x coordinate. Therefore, our coordinate system will have $+y$ to the north and $+x$ to the west.

Since we are near the equator, each line of latitude and longitude may be considered to form a circle with a radius equal to the radius of Earth, $R = 6.36 \times 10^6$ m. Recall the length s of an arc of a circle of radius R that subtends an angle (in radians) $\Delta\theta$ (or $\Delta\phi$) is given by $s = R\Delta\theta$ (or $s = R\Delta\phi$). We can use this equation to find the components of the displacement from the starting point to the tree—these are parallel to the x and y coordinates axes. Therefore, we can regard the origin to be the starting point and the displacements as the x and y coordinates of the tree.

The angular difference $\Delta\phi$ for longitude values is (west being positive)

$$\begin{aligned}\Delta\phi &= [75.64426° - 75.64238°] \\ &= (0.00188°)(\pi \text{ rad} / 180°) \\ &= 3.28 \times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the x coordinate (displacement west)

$$x = R\Delta\phi = \left(6.36 \times 10^{6} \text{ m}\right)\left(3.28 \times 10^{-5} \text{ rad}\right) = 209 \text{ m}$$

The angular difference $\Delta\theta$ for latitude values is (north being positive)

$$\begin{aligned}\Delta\theta &= \left[0.00162° - (-0.00243°)\right] \\ &= (0.00405°)(\pi \text{ rad} / 180°) \\ &= 7.07 \times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the y coordinate (displacement north)

$$y = R\Delta\theta = \left(6.36 \times 10^{6} \text{ m}\right)\left(7.07 \times 10^{-5} \text{ rad}\right) = 450 \text{ m}$$

The distance to the tree is

$$d = \sqrt{x^2 + y^2} = \sqrt{(209 \text{ m})^2 + (450 \text{ m})^2} = \boxed{496 \text{ m}}$$

The direction to the tree is

$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{450\text{ m}}{209\text{ m}}\right) = 65.1° = \boxed{65.1°\text{ N of W}}$$

(b) $\boxed{\text{Refer to the arguments above. They are justified because the distances involved are small relative to the radius of Earth.}}$

P3.41 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\vec{\mathbf{R}}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c)

$$\cos\theta_x = \frac{R_x}{|\vec{\mathbf{R}}|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{|\vec{\mathbf{R}}|}\right) = \boxed{57.7°\text{ from } +x}$$

$$\cos\theta_y = \frac{R_y}{|\vec{\mathbf{R}}|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{|\vec{\mathbf{R}}|}\right) = \boxed{74.5°\text{ from } +y}$$

$$\cos\theta_z = \frac{R_z}{|\vec{\mathbf{R}}|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{|\vec{\mathbf{R}}|}\right) = \boxed{36.7°\text{ from } +z}$$

***P3.42**

Solution

Conceptualize The aircraft are flying in three-dimensional space, so the distance between them is difficult to visualize easily or draw on paper based on the numbers provided. Your goal is to determine a straight-line distance between them to make sure they are far enough apart to be safe.

Categorize Despite the difficulty in visualizing the positions of the airplanes, this is a relatively simple substitution problem.

Write a vector expression for each airplane using the suggested axis orientations:

$$\vec{\mathbf{r}}_1 = \left(19.2\cos 25.0°\,\hat{\mathbf{i}} + 19.2\sin 25.0°\,\hat{\mathbf{j}} + 0.800\hat{\mathbf{k}}\right)\text{ km}$$
$$= \left(17.40\hat{\mathbf{i}} + 8.114\hat{\mathbf{j}} + 0.800\hat{\mathbf{k}}\right)\text{ km}$$
$$\vec{\mathbf{r}}_2 = \left(17.6\cos 20.0°\,\hat{\mathbf{i}} + 17.6\sin 20.0°\,\hat{\mathbf{j}} + 1.100\hat{\mathbf{k}}\right)\text{ km}$$
$$= \left(16.54\hat{\mathbf{i}} + 6.020\hat{\mathbf{j}} + 1.100\hat{\mathbf{k}}\right)\text{ km}$$

Find the difference vector between these two positions:

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left(-0.86\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.300\hat{\mathbf{k}}\right)\text{ km}$$

Use the Pythagorean theorem to find the distance between the planes:

$$\left|\Delta\vec{\mathbf{r}}\right| = \sqrt{(-0.86)^2 + (-2.09)^2 + (0.300)^2}\text{ km} = 2.28\text{ km}$$

The two airplanes are just slightly farther apart than the minimum separation distance.]

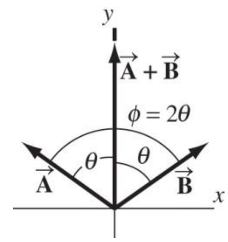
Answer: 2.28 km

P3.43 (a) $\dfrac{d\vec{\mathbf{r}}}{dt} = \dfrac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{k}}\right)}{dt} = -2\hat{\mathbf{k}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{k}}}$

(b) The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, and so on. The object is moving straight downward at 2 m/s, so $\frac{d\vec{\mathbf{r}}}{dt}$ represents $\boxed{\text{its velocity vector}}$.

P3.44 Since

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.00\hat{\mathbf{j}},$$



ANS. FIG. P3.44

we have

$$(A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$$

giving $\quad A_x + B_x = 0 \rightarrow A_x = -B_x.$

Because the vectors have the same magnitude and x components of equal magnitude but of opposite sign, the vectors are reflections of each other in the y axis, as shown in the diagram. Therefore, the two vectors have the same y components:

$$A_y = B_y = (1/2)(6.00) = 3.00$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

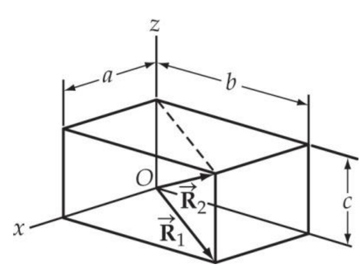
$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600 \rightarrow \theta = 53.1°$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then

$\boxed{\phi = 2\theta = 106°}$.

P3.45 (a) From the picture, $\boxed{\vec{\mathbf{R}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}.}$

(b) $\boxed{R_1 = \sqrt{a^2 + b^2}}$



ANS. FIG. P3.45

(c) $\boxed{\vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_1 + c\hat{\mathbf{k}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}}$

Challenge Problems

P3.46 (a) You start at point A: $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$, to move to

$$\vec{\mathbf{r}}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$$

Now the displacement from your current position to C is

$$\vec{\mathbf{r}}_C - \vec{\mathbf{r}}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}$$

You cover one-third, moving to

$$\vec{\mathbf{r}}_3 = \vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}$$

The displacement from where you are to D is

$$\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}$$

You traverse one-quarter of it, moving to

$$\begin{aligned}\vec{\mathbf{r}}_4 &= \vec{\mathbf{r}}_3 + \frac{1}{4}\left(\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right)\\ &= 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\end{aligned}$$

The displacement from your new location to E is

$$\vec{\mathbf{r}}_E - \vec{\mathbf{r}}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\vec{\mathbf{r}}_4 + \Delta\vec{\mathbf{r}}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}$$

The treasure is at $\boxed{(10.0\text{ m}, 16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\vec{\mathbf{r}}_A + \frac{1}{2}(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A) = \left(\frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B}{2}\right)$$

then to

$$\frac{(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B)}{2} + \frac{\vec{\mathbf{r}}_C - (\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B)/2}{3} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C}{3}$$

then to

$$\frac{(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C)}{3} + \frac{\vec{\mathbf{r}}_D - (\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C)/3}{4} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D}{4}$$

and last to

$$\frac{(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D)}{4} + \frac{\vec{\mathbf{r}}_E - (\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D)/4}{5}$$
$$= \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D + \vec{\mathbf{r}}_E}{5}$$

This center of mass of the tree distribution is the same location whatever order we take the trees in.

ANSWERS TO QUICK-QUIZZES

1. vectors: (b), (c); scalars: (a), (d), (e)

2. (c)

3. (b) and (c)

4. (b)

5. (c)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P3.2	(a) (2.17, 1.25) m, (−1.90, 3.29) m; (b) 4.55m
P3.4	(a) r, 180° – θ; (b) 180° + θ; (c) $-\theta$
P3.6	$\vec{\mathbf{B}}$ is 43 units in the negative y direction
P3.8	(a) See ANS. FIG. P3.8; (b) The sum of a set of vectors is not affected by the order in which the vectors are added.
P3.10	approximately 420 ft at 23°
P3.12	1.31 km north and 2.81 km east

P3.14 (a) See ANS. FIG. P3.14; (b) $5.00\hat{\mathbf{i}}+4.00\hat{\mathbf{j}}, -1.00\hat{\mathbf{i}}+8.00\hat{\mathbf{j}}$; (c) 6.40 at 38.7°, 8.06 at 97.2°

P3.16 (a) Its component parallel to the surface is (1.50 m) cos 141° = −1.17 m, or 1.17 m toward the top of the hill; (b) Its component perpendicular to the surface is (1.50 m) sin 141° = 0.944 m, or 0.944 m away from the snow.

P3.18 $C_x = 7.30$ cm; $C_y = -7.20$ cm

P3.20 (a) $5.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}$, 5.92 m;(b) $(4.00\hat{\mathbf{i}}-11.0\hat{\mathbf{j}}+15.0\hat{\mathbf{k}})$ m, 19.0) m

P3.22 (a) $49.5\hat{\mathbf{i}}+27.1\hat{\mathbf{j}}$;(b) 56.4, 28.7°

P3.24 59.2° with the x axis, 39.8° with the y axis, 67.4° with the z axis

P3.26 (a) 5.00 blocks at 53.1° N of E; (b) 13.00 blocks

P3.28 magnitude: 170.1 cm, direction: 57.2° above $+x$ axis (first quadrant); magnitude: 145.7 cm, direction: 58.6° above $+x$ axis (first quadrant)

P3.30 (a) 10.4 cm (b) u 5 35.5°

P3.32 Impossible because 12.4 m is greater than 5.00 m

P3.34 9.46° west of north

P3.36 (a) 7.17 km; (b) 6.15 km

P3.38 $\theta = 2\tan^{-1}\left(\frac{1}{n}\right)$

P3.40 (a) 496 m, 65.1° N of W; (b) The arguments are justified because the distances involved are small relative to the radius of the Earth.

P3.42 2.29 km

P3.44 $\phi = 2\theta = 106°$

P3.46 (a) (10.0 m, 16.0 m) (b) This center of mass of the tree distribution is the same location whatever order we take the trees in.