Home_Work_1

Should modify this

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\ 1.7

$$\mathbb{N}(x|m,\sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}}\exp(-\frac{1}{2\sigma^2}(x-m)^2)$$

show

def

$$I = \int_{-\infty}^{\infty}\exp(-frac12\sigma^2x^2dx)$$
$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp(-frac12\sigma^2x^2 - -frac12\sigma^2y^2)dxdy$$
$$x^2+y^2 = r^2\frac{\partial(x,y)}{\partial(r,\theta)}$$

$$\begin{aligned} I^2 &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp(-\frac{r^2}{2\sigma^2})rdrd\theta..(r^2=u) \\ &= 2\pi\int_0^{\infty}\exp(-\frac{u}{2\sigma^2})\frac{1}{2}du \\ &= \pi[\exp(-\frac{u}{2\sigma^2})(-2\sigma^2)]_0^{\infty} \\ &= 2\pi\sigma^2 \Rightarrow I = (2\pi\sigma^2)^{\frac{1}{2}} \end{aligned}$$

$$\rightarrow \int_{-\infty}^{\infty}\mathbb{N}(x|m,\sigma^2)dx = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}}\int_{-infty}^{\infty}\exp(-\frac{y^2}{2\sigma^2})dy = \frac{I}{2\pi\sigma^2}^f rac12 = 1$$

\ 1.10

$$E[x+z] = E[x] + E[z]$$

show

def

$$\begin{aligned} E[x+z] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x+z)f(x+z)dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf(x+z)dzdx + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} zf(x+z)dxdz \\ &= \int_{-\infty}^{\infty} xf(x)dx + \int_{-\infty}^{\infty} zf(z)dz \end{aligned}$$

\ 1.10-2 $$ Var[x+z] = Var[x] + Var[z]

def

$$\begin{aligned} Var[x+z] &= E[(x+z)^2] - (E[(x+z)])^2 \\ &= E[x^2] - (E[x])^2 + E[z^2] - (E[z])^2 + 2E[xz] - 2E[x]E[z]..Cov(x,z) = 2E[xz] = 2E[x]E[z] \\ &= Var[x] + Var[z] \end{aligned}$$

\ 1.12

$$E[m_{ML}] = \frac{1}{N}\sum_{n=1}^{N} E[x_n] = m$$

$$\begin{aligned}
E(\sigma^2_{ML}) &= E\left[\frac{1}{N}\sum_{n=1}^{N}(x_n - \frac{1}{N}\sum_{m=1}^{N} x_m)^2]\right] \\
&= \frac{1}{N}\sum_{n=1}^{N} E[x_n^2 - \frac{2}{N}x_m \sum_{m=1}^{N} x_m + \frac{1}{N^2}\sum_{m=1}^{2}\sum_{l=1}^{N} x_m x_l] \\
&= m^2 + \sigma^2 - 2(m^2 + \frac{1}{N}\sigma^2) + m^2 + \frac{1}{N}\sigma^2 \\
&= (\frac{N-1}{N})\sigma^2
\end{aligned}$$