

# VALUING BONDS



# BONDS

- ▶ INTEREST ONLY LOANS, PRINCIPAL OR FACE VALUE OR PAR VALUE REPAID AT END OF LOAN
- ▶ STATED INTEREST RATE CALLED COUPON
- ▶ DENOMINATIONS (OR PAR VALUES) OF CORPORATE BONDS TYPICALLY \$1,000. GOVERNMENT BONDS USUALLY HAVE GREATER PAR VALUES. BOND SELLING AT PAR IS SELLING “FLAT.”
- ▶ MATURITY SOMETIMES CASUALLY USED FOR REMAINING LIFE OF BOND OR CURRENT MATURITY
- ▶ MOST US BONDS PAY INTEREST SEMIANNUALLY
- ▶ PRICE OFTEN STATED AS PERCENTAGE OF PAR VALUE

## 6%, 5 year bonds

- ▶ CASH FLOWS AT END OF EACH YEAR (IGNORING SEMIANNUAL PAY)

1995	1996	1997	1998	1999
60	60	60	60	1,060

- ▶ SIMILAR BONDS RETURN 6.9%

$$PV = \frac{60}{(1.069)} + \frac{60}{(1.069)^2} + \frac{60}{(1.069)^3} + \frac{60}{(1.069)^4} + \frac{1,060}{(1.069)^5} = 963$$

PAR Value 1,000

Coupon = 60 > 6% of PAR Value

- ▶ BOND IS SELLING AT 96.3 (PERCENT OF PAR VALUE)

# Example

BOND IS SELLING AT 99%  
(PERCENT OF PAR VALUE)  
 $1000 \times 99\% = 990 \text{ €}$

20/04 – 20/12 = 120 jours  
1 an = 360 jours

“Coupon Couru” =  $2,5\% \times 1000 \times (120 / 360) = 8,33 \text{ €}$   
C/C (%) =  $8,33 / 1000 = 0,833 \%$

Bonds Price =  $((99 + 0,899) \times 1000) / 100 = 998,33 \text{ €}$   
 $990 + 8,33$

Obligation GAMA		Informations complémentaires	
20/04/2013	99,00 %		
Taux coupon %	2,50	Prix d'émission (euros)	1 000
Date de remboursement	20/12/2022	valeur nominale (euros)	1 000
		Date d'émission	20/12/2002
ISIN	FRXXXXX...	Date de maturité	20/12/2022
Marché	France	Emetteur	société GAMA
		Pays émetteur	France
<b>Cotation</b>			
Dernier (%)	99,00	Taux (%)	2,50
C/C (%)	0,833	Fréquence du coupon	annuel
"+" haut (%)	0,00	Coupon couru (%)	0,833
"+" bas (%)	0,00	Date du coupon précédent	20/12/2012

cours spot	998,33																					
Valeur Faciale €	1000																					
Coupon €	25																					
Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Cash €	-998,33	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	1025
YTM (IRR)	2,510329%																					

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# **AFTER BOND IS ISSUED, INTEREST RATES ON SIMILAR BONDS CHANGE BUT CASH FLOWS FROM BOND STAY SAME**

- PRICE OF BOND WILL VARY BECAUSE THE PRICE IS THE PV OF THE REMAINING CASH FLOWS
- DISCOUNT RATES CHANGE WITH CHANGES IN YIELD TO MATURITY (YTM) OR YIELD ON SIMILAR BONDS!

**PV(BOND) = PV (COUPON PAYMENTS)  
+ PV (FINAL PAYMENT)**



**PV(COUPON PAYMENTS) IS THE PV OF AN ANNUITY**

$$\begin{aligned}\text{PV(BOND)} &= 60 \left( \frac{1}{.069} - \frac{1}{.069(1.069)^5} \right) + \frac{1,000}{(1.069)^5} \\ &= 246.67 + 716.33 \\ &= \$963\end{aligned}$$

# YTM

- TURN THE QUESTION AROUND
- ASK WHAT RETURN,  $r$ , DO INVESTORS EXPECT WHEN A 5-YEAR, 6% COUPON BOND IS PRICED AT 96.3?
- WE NEED TO FIND THE VALUE OF  $r$  THAT SATISFIES THE EQUATION

$$96.3 = \frac{60}{(1+R)} + \frac{60}{(1+R)^2} + \frac{60}{(1+R)^3} + \frac{60}{(1+R)^4} + \frac{1,060}{(1+R)^5}$$

**$r$  IS THE YIELD TO MATURITY (YTM) OR YIELD  
WE ASSUME A FLAT TERM STRUCTURE  
OF INTEREST RATES**



# INTEREST RATE RISK

**WHEN MARKET INTEREST RATES RISE, BOND PRICES FALL.**

**WHEN MARKET INTEREST RATES FALL, BOND PRICES RISE.**

**BOND PRICE SENSITIVITY TO CHANGES IN INTEREST RATES  
GREATER**

- 1. LONGER CURRENT MATURITY**
- 2. LOWER THE COUPON RATE.**



# **WHY ARE LONGER MATURITY BONDS MORE SENSITIVE TO CHANGES IN MARKET INTEREST RATES?**

- MORE OF THE PRICE OF THE BOND IS DERIVED FROM CASH FLOWS (INTEREST AND PRINCIPAL) THAT OCCUR LATER IN TIME
- AND THEREFORE HAVE TO BE DISCOUNTED MORE
- MORE SENSITIVE TO CHANGES IN INTEREST RATES

## EXAMPLE:

$$\frac{1}{\left(1 + \frac{r}{2}\right)^{60}}$$

IS MORE SENSITIVE  
TO CHANGES IN  $r$  THAN

$$\frac{1}{\left(1 + \frac{r}{2}\right)^2}$$

# BONDS MAKE SEMI-ANNUAL COUPON PAYMENTS

- ANNUAL COUPON RATE IS QUOTED AS TWICE THE SEMIANNUAL COUPON RATE
  - 6% COUPON BOND PAYS \$30 TWICE A YEAR
- BOND YIELD IS QUOTED AS TWICE THE SEMIANNUAL BOND YIELD

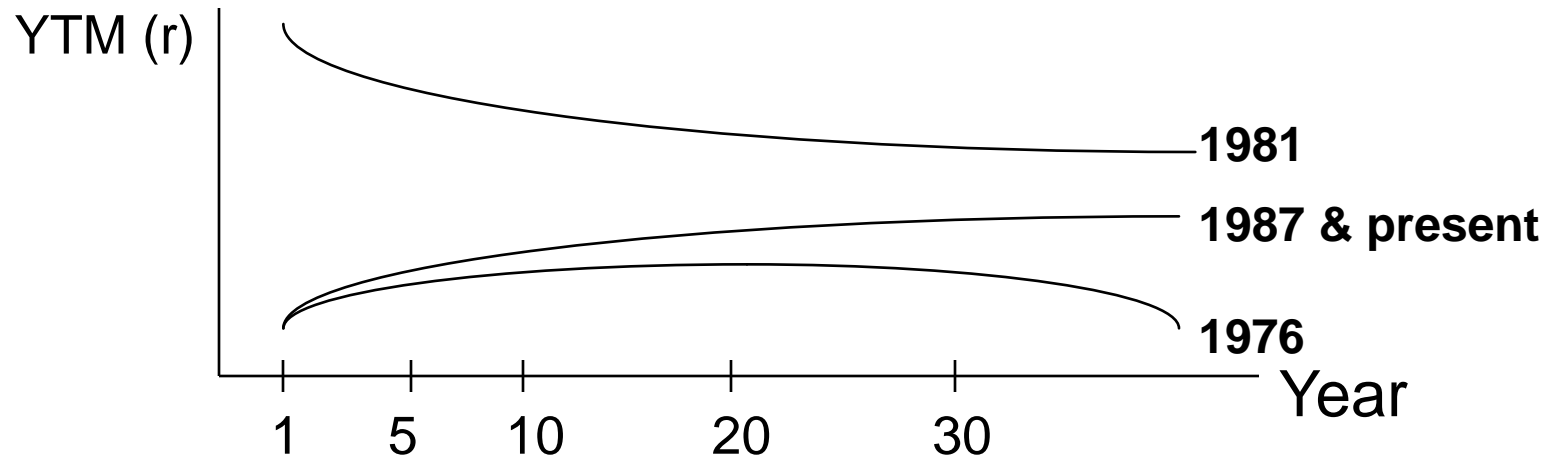
$$PV = \frac{30}{(1.0345)} + \frac{30}{(1.0345)^2} + \dots + \frac{1,030}{(1.0345)^{10}}$$

# VALUE OF A BOND

ANNUAL COUPON  $C$ , ANNUAL YIELD TO MATURITY  $r$

$$PV = \frac{\frac{C}{2}}{(1 + \frac{r}{2})} + \frac{\frac{C}{2}}{(1 + \frac{r}{2})^2} + \dots + \frac{\frac{C}{2}}{(1 + \frac{r}{2})^{2n}}$$

# TERM STRUCTURE



**Spot Rate** - The actual interest rate today ( $t=0$ )

**Forward Rate** - The interest rate, fixed today, on a loan made in the future at a fixed time.

**Future Rate** - The spot rate that is expected in the future.

**Yield To Maturity (YTM)** - The IRR on an interest bearing instrument.

# TERM STRUCTURE

$$PV = \frac{1}{(1+r_1)} + \frac{1}{(1+r_2)^2} + \dots + \frac{1}{(1+r_t)^t}$$

WE DISCOUNT CASH FLOW AT TIME 1 BY  $r_1$   
RATE APPROPRIATE FOR 1-PERIOD LOAN  
RATE FIXED TODAY, 1-PERIOD SPOT RATE

WE DISCOUNT CASH FLOW AT TIME 2 BY  $r_2$   
RATE APPROPRIATE FOR 2-PERIOD LOAN  
RATE FIXED TODAY, 2-PERIOD SPOT RATE

TERM STRUCTURE OF INTEREST RATES  
DESCRIBED BY SERIES OF INTEREST RATES  $r_1$   
 $r_2$  ETC



# YIELD TO MATURITY

- **INSTEAD OF DISCOUNTING EACH PAYMENT AT DIFFERENT RATE OF INTEREST**
  - **FIND SINGLE RATE OF INTEREST,  $r$  WHICH GIVES SAME PV**
  - **YTM**
  - **REALLY IRR**

$$PV = \frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^t}$$

**BOND TABLES SHOW BOND PRICES  
FOR DIFFERENT COUPONS AND YTM**

**BOND PRICES QUOTED AS PERCENT OF FACE VALUE**



$$D = \frac{\sum_{i=1}^n \frac{t(i) \times F_i}{(1+r)^{t(i)}}}{\sum_{i=1}^n \frac{F_i}{(1+r)^{t(i)}}}$$

# DURATION

## Example (Bond 1)

Calculate the duration of our 10.5% bond @ 8.5% YTM

Year	CF	PV@YTM	% of Total PV%	x Year	
1	105	96.77	.090	0.090	= 96,77/1078,82
2	105	89.19	.083	0.164	= 2 X 89,19/1078,82
3	105	82.21	.076	0.227	= 3 X 82,21/1078,82
4	105	75.77	.070	0.279	= 4 X 75,77/1078,82
5	1105	734.88	.681	3.406	= 5 X 734,88/1078,82
		1078.82	1.00	4.166	Duration

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$$D = \frac{\sum_{i=1}^n \frac{t(i) \times F_i}{(1+r)^{t(i)}}}{\sum_{i=1}^n \frac{F_i}{(1+r)^{t(i)}}}$$

# DURATION

## Example (Bond 2)

**Given a 5 year, 9.0%, \$1000 bond, with a 8.5% YTM, what is this bond's duration?**

Year	CF	PV@YTM	% of Total PV%	x Year
1	90	82.95	.081	0.081
2	90	76.45	.075	0.150
3	90	70.46	.069	0.207
4	90	64.94	.064	0.256
5	1090	<u>724.90</u>	<u>.711</u>	<u>3.555</u>
		<u>1019.70</u>	<u>1.00</u>	<b>4.249 Duration</b>

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# DURATION AND VOLATILITY

- DURATION MEASURES AVERAGE TIMING OF CASH FLOWS
  - $\text{DURATION} = 1 \times [\text{PV}(C_1)/V] + 2 \times [\text{PV}(C_2)/V] + \dots$
- BONDS WITH LONGER DURATION ALSO HAVE GREATER VOLATILITY

$$\text{VOLATILITY}(\%) = \text{DURATION} / (1 + \text{YIELD})$$

$$\text{VOLATILITY OF BOND(1)} (\%) = 4.166 / 1.085 = 3.84$$

$$\text{VOLATILITY OF BOND(2)} (\%) = 4.249 / 1.085 = 3.92$$



# BONDS RATING

Échelle de notation financière selon les principales  
agences de notation

Signification de la note	Moody's		Standard & Poor's		Fitch Ratings		Dagong		
	Long terme	Court terme	Long terme	Court terme	Long terme	Court terme	Long terme	Court terme	
Prime Première qualité	Aaa	P-1  Prime -1	AAA	A-1+	AAA	F1+	AAA	A-1	
High grade Haute qualité	Aa1		AA+		AA+		AA+		
	Aa2		AA		AA		AA		
	Aa3		AA-		AA-		AA-		
Upper medium grade Qualité moyenne supérieure	A1		A+	A-1	A+	F1	A+		A-2
	A2		A		A		A		
	A3	A-	A-2	A-	F2	A-			
Lower medium grade Qualité moyenne inférieure	Baa1	P-2	BBB+	A-3	BBB+	F3	BBB+	A-3	
	Baa2		BBB		BBB		BBB		
	Baa3		BBB-		BBB-		BBB-		
Non-investment grade, speculative Spéculatif	Ba1	Not prime	BB+	B	BB+	B	BB+	B	
	Ba2		BB		BB		BB		
	Ba3		BB-		BB-		BB-		
Highly speculative Très spéculatif	B1		B+		B+		B+		
	B2		B		B		B		
	B3		B-		B-		B-		
Risque élevé	Caa1	Non prime	CCC+	C	CCC	C	CCC	C	
Ultra spéculatif	Caa2		CCC						
En défaut, avec quelques espoirs de recouvrement	Caa3		CCC-						CC
	Ca		CC	C/CI/R	C	C			
	En défaut sélectif		C	SD	D	RD	D		D
D				D		D			

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# BONDS RATING

Notation des pays de la zone euro au 16 février 2015

Pays	Moody's		Standard & Poor's		Fitch Ratings		Dagong		Egan-Jones	
	Note	Persp.	Note	Persp.	Note	Persp.	Note	Persp.	Note	Persp.
 Allemagne	Aaa	▼	AAA	►	AAA	►	AA+	►	A+	▼
 Autriche	Aaa	▼	AA+	►	AAA	►	AA+	►	A	▼
 Belgique	Aa3	►	AA	►	AA	►	A+	▼		—
 Chypre	Caa3	SR	B-	►	CCC+	▼		—		—
 Espagne	Baa3	SR	BBB-	►	BBB	▼	A	▼	CC+	—
 Estonie	A1	►	AA-	▼	A+	►	A	►		—
 Finlande	Aaa	►	AA+	►	AAA	►	AAA	▼		—
 France	Aa1	▼	AA	►	AA	►	A+	▼	BBB+	▼
 Grèce	Caa1	—	B-	►	CCC	►	CC	▼		—
 Irlande	Ba1	▼	BBB+	▼	BBB+	▼	BBB	▼		—
 Italie	Baa2	▼	BBB-	►	BBB+	▼	BBB	▼	CCC+	▼
 Lettonie	?	-	?	-	?	-	?	-		—
 Luxembourg	Aaa	▼	AAA	▼	AAA	►	AAA	►		—
 Malte	A3	▼	A-	▼	A+	►	A-	▼		—
 Pays-Bas	Aaa	▼	AA+	►	AAA	►	AA+	►	A	▼
 Portugal	Ba2	►	BB	►	BB+	▼	BB+	▼	B-	—
 Slovaquie	A2	▼	A	►	A+	►		—		—
 Slovénie	Baa2	▼	A	▼	A-	▼		—		—

Légende : ► Perspective stable • ▼ Perspective négative • SR Note sous revue <sup>N1</sup> • — Perspective non communiquée

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