19/07/20

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Parcial ANDO 2019-2

Exercice 2 :

$$X = \begin{array}{c} \begin{array}{ccc} \overline{X^1} & \overline{X^2} & \overline{X^3} \end{array} \\ \begin{pmatrix} -6 & 2 & -5 \\ -1 & -2 & 0 \\ 1 & 0 & 2 \\ 1 & 4 & -2 \\ -1 & 2 & -4 \\ 0 & -6 & -3 \end{pmatrix} \end{array}$$

$\boxed{1.}$

$\overline{X^1} = \frac{1}{6}(-6 \cancel{-1+1+1-1}) \boxed{=-1}$ ✓

$\overline{X^2} = \frac{1}{6}(2-2+4+2-6) \boxed{=0}$ ✓

$\overline{X^3} = \frac{1}{6}(-5 \cancel{+2-2} -4-3) \cancel{\frac{-12}{6}}$ $\boxed{=-2}$ ✓

$D = \frac{1}{6}$ $g = (-1, 0, -2)$

$\boxed{2.}$ $Y = \begin{pmatrix} -5 & 2 & -3 \\ 0 & -2 & 2 \\ 2 & 0 & 4 \\ 2 & 4 & 0 \\ 0 & 2 & -2 \\ 1 & -6 & -1 \end{pmatrix}$

$\boxed{3.}$ $V = {}^tYDY$

$${}^tYY = \begin{pmatrix} 34 & -8 & 22 \\ -8 & 64 & -8 \\ 22 & -8 & 34 \end{pmatrix}$$

$$V = \begin{array}{cc} \begin{array}{ccc} x & y & z \end{array} & \\ \begin{pmatrix} \frac{17}{3} & \frac{-4}{3} & \frac{11}{3} \\ \frac{-4}{3} & \frac{32}{3} & \frac{-4}{3} \\ \frac{11}{3} & \frac{-4}{3} & \frac{17}{3} \end{pmatrix} & \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

$\boxed{4.}$

$\cancel{\frac{17}{6} \quad \frac{-4}{3} \quad \frac{11}{3}}$

$$\det \begin{vmatrix} \cancel{\frac{21}{3}} 8-\lambda & 8-\lambda & 8-\lambda \\ \frac{-4}{3} & \frac{32}{3}-\lambda & \frac{-4}{3} \\ \frac{11}{3} & \frac{-4}{3} & \frac{17}{3}-\lambda \end{vmatrix} = \begin{vmatrix} 0 & 0 & 8-\lambda \\ -12+\lambda & 12-\lambda & \frac{-4}{3} \\ 5 & -7+\lambda & \frac{17}{3}-\lambda \end{vmatrix} = (8-\lambda)(-12+\lambda)(12-\lambda) - 5\times(-7+\lambda)$$

$= (8-\lambda)$

②

$$\begin{vmatrix} \frac{17}{3}-\lambda & -\frac{4}{3} & \frac{11}{3} \\ -\frac{4}{3} & \frac{32}{3}-\lambda & -\frac{4}{3} \\ \frac{11}{3} & -\frac{4}{3} & \frac{17}{3}-\lambda \end{vmatrix} = \begin{vmatrix} 8-\lambda & 8-\lambda & 8-\lambda \\ -\frac{4}{3} & \frac{32}{3}-\lambda & -\frac{4}{3} \\ \frac{11}{3} & -\frac{4}{3} & \frac{17}{3}-\lambda \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 8-\lambda \\ -12+\lambda & 12-\lambda & -\frac{4}{3} \\ 5 & -7+\lambda & \frac{17}{3}-\lambda \end{vmatrix}$$

$\lambda_1 = 8 \quad , \lambda_2 = 12 \quad , \lambda_3 = 2$

% inertie : $\dfrac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\lambda_3} \boxed{= 0{,}91}$

5. On isole 2 valeurs propres (les plus grandes). λ_1 et λ_2.

• $E_{12} = \mathrm{Ker}(V - 12I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$$E(12) \begin{pmatrix} \frac{17}{3}-12 & -\frac{4}{3} & \frac{11}{3} \\ -\frac{4}{3} & \frac{32}{3}-12 & -\frac{4}{3} \\ \frac{11}{3} & -\frac{4}{3} & \frac{17}{3}-12 \end{pmatrix}$$

③

$$\cancel{\left(\frac{17}{3}-12\right)} x \begin{cases} \frac{-19}{3}x + \frac{-4}{3}y + \frac{11}{3}z = 0 \\ \frac{-4}{3}x + \frac{-4}{3}y + \frac{-4}{3}z = 0 \\ \frac{11}{3}x + \frac{-4}{3}y + \frac{-19}{3}z = 0 \end{cases}$$

$$\begin{cases} -10x + 10z = 0 \\ \frac{-8}{3}x + \frac{-4}{3}y = 0 \\ \frac{11}{3}x + \frac{-4}{3}y + \frac{-19}{3}z = 0 \end{cases}$$

$$\begin{cases} x = z \\ x = -\frac{1}{2}y \end{cases} \Rightarrow \begin{cases} x = z \\ -2x = y \end{cases}$$

$$E_{12} = \text{Vect}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

- $E_8 = \text{Ker}(V - 8I) \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$$\begin{pmatrix} \frac{17}{3}-8 & \frac{-4}{3} & \frac{11}{3} \\ \frac{-4}{3} & \frac{32}{3}-8 & \frac{-4}{3} \\ \frac{11}{3} & \frac{-4}{3} & \frac{17}{3}-8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \begin{cases} \frac{-7}{3}x + \frac{-4}{3}y + \frac{11}{3}z = 0 \\ \frac{-4}{3}x + \frac{8}{3}y + \frac{-4}{3}z = 0 \\ \frac{11}{3}x + \frac{-4}{3}y + \frac{-7}{3}z = 0 \end{cases}$$

$$\cancel{\frac{8}{3}x + \frac{8}{3}z = 0} \quad \cancel{x = -z}$$

$$\Rightarrow \cancel{y = 0}$$

$$\begin{cases} x = z \\ y = x \end{cases} \quad E_8 = \text{Vect}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

④ $E_{12} = Vect\begin{pmatrix}1\\-2\\1\end{pmatrix} \quad E_8 = Vect\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$U_1 = \frac{1}{\sqrt{(1,-2,1)^T(1,-2,1)}} = \frac{1}{\sqrt{1+4+1}} = \frac{1}{\sqrt{6}} \times \begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt6}\\ \frac{-2}{\sqrt6}\\ \frac{1}{\sqrt6}\end{pmatrix}$$

$$U_2 = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt3} \times \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt3}\\ \frac{1}{\sqrt3}\\ \frac{1}{\sqrt3}\end{pmatrix}$$

$\boxed{6.}$ $C^{(i)} = Y \times U_i$

$\cancel{\frac{-5}{\sqrt6} + \frac{-4}{\sqrt6} + \frac{-3}{\sqrt6}}$

$\cancel{\frac{4}{\sqrt6} +}$

$$C^{(1)} = \begin{pmatrix}-12\\6\\6\\-6\\-6\\12\end{pmatrix} \times \frac{1}{\sqrt6}$$

$$C^{(2)} = \begin{pmatrix}-6\\0\\6\\6\\0\\-6\end{pmatrix} \times \frac{1}{\sqrt3}$$

$\sigma(X) = \sqrt{\frac{17}{3}}, \quad \sigma(Y) = \sqrt{\frac{32}{3}}, \quad \sigma(Z) = \sqrt{\frac{17}{3}}$

$$\sigma_{C^{(1)}} = \sqrt{\frac{1}{6} \times \left(\frac{1}{\sqrt6}\right)^2 \times \left((-12)^2 + 6^2 + 6^2 + (-6)^2 + (-6)^2 + 12^2\right)} = 2\sqrt3$$

$$\sigma_{C^{(2)}} = \sqrt{\frac{1}{6} \times \left(\frac{1}{\sqrt3}\right)^2 \times \left((-6)^2 + 6^2 + 6^2 + (-6)^2\right)} = 2\sqrt2$$

5 $X^1 C^{(1)} D$, $X^1 C^{(2)} D$, $Y^1 C^{(1)} D$, $Y^2 C^{(1)}$

$$Y^{(1)} C^{(1)} D = 72 \times \frac{1}{\sqrt{6}} \times \frac{1}{6} = \frac{72}{6\sqrt{6}} = 16\sqrt{6}$$