

Fig 10

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clear; clc;

K = 1;
Omega = 50;
alpha = 2;
delta = 0.04;
P0_fixed = 0.1;
A = 0.8;
f = 0.05;

num_r = 20;
num_a = 20;
r_vec = linspace(0.1, 1.9, num_r);
a_vec = logspace(log10(0.02), log10(0.2), num_a);

grid_size = 100;
B0_vec = logspace(-2, log10(0.5), grid_size); % B0: 0.01 ~ 0.5
I0_vec = logspace(-2, log10(0.5), grid_size); % I0: 0.01 ~ 0.5
total_points = grid_size^2;

t_final = 500;
extinct_thres = 1e-8;

% ODE
options = odeset('NonNegative', [1,2,3], 'RelTol', 1e-12, 'MaxStep', 0.1);
tspan = [0, t_final];

survival_ratio_map = zeros(num_r, num_a);

fprintf('Starting 20x20 parameter scan...\n');
```

Starting 20x20 parameter scan...

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for r_idx = 1:num_r
    r = r_vec(r_idx);
    fprintf('Processing r = %.3f (%d/%d)\n', r, r_idx, num_r);

    for a_idx = 1:num_a
        a = a_vec(a_idx);

        state_matrix = zeros(grid_size, grid_size);
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parfor i = 1:grid_size
    for j = 1:grid_size
        B0 = B0_vec(j);
        I0 = I0_vec(i);
        y0 = [B0; I0; P0_fixed];

        [~, y] = ode15s(@(t,y) fluctuate_1B(t, y, A,f,r, K, a, Omega,alpha, delta), ...
            tspan, y0, options);

        n = size(y, 1);
        last_y_B = y(floor(n*0.9):end, 1);

        if min(last_y_B) >= extinct_thres
            state_matrix(i, j) = 1;
        else
            state_matrix(i, j) = 0;
        end
    end
end

survival_ratio_map(r_idx, a_idx) = sum(state_matrix(:)) / total_points;

if mod(a_idx, ceil(num_a/10)) == 0 || a_idx == num_a
    fprintf(' -> a = %.4f, survival = %.3f\n', a, survival_ratio_map(r_idx, a_idx));
end
end
end

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Processing r = 0.100 (1/20)
Starting parallel pool (parpool) using the 'local' profile ...
Connected to the parallel pool (number of workers: 8).
-> a = 0.0226, survival = 0.996
-> a = 0.0288, survival = 0.966
-> a = 0.0367, survival = 0.858
-> a = 0.0467, survival = 0.735
-> a = 0.0595, survival = 0.623
-> a = 0.0759, survival = 0.517
-> a = 0.0967, survival = 0.413
-> a = 0.1232, survival = 0.325
-> a = 0.1570, survival = 0.242
-> a = 0.2000, survival = 0.155
Processing r = 0.195 (2/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.967
-> a = 0.0367, survival = 0.878
-> a = 0.0467, survival = 0.768
-> a = 0.0595, survival = 0.643
-> a = 0.0759, survival = 0.528
-> a = 0.0967, survival = 0.417
-> a = 0.1232, survival = 0.310
-> a = 0.1570, survival = 0.220
-> a = 0.2000, survival = 0.141
Processing r = 0.289 (3/20)

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-> a = 0.0226, survival = 0.990
-> a = 0.0288, survival = 0.938
-> a = 0.0367, survival = 0.847
-> a = 0.0467, survival = 0.755
-> a = 0.0595, survival = 0.626
-> a = 0.0759, survival = 0.492
-> a = 0.0967, survival = 0.376
-> a = 0.1232, survival = 0.268
-> a = 0.1570, survival = 0.176
-> a = 0.2000, survival = 0.110
Processing r = 0.384 (4/20)
-> a = 0.0226, survival = 0.963
-> a = 0.0288, survival = 0.879
-> a = 0.0367, survival = 0.776
-> a = 0.0467, survival = 0.683
-> a = 0.0595, survival = 0.562
-> a = 0.0759, survival = 0.425
-> a = 0.0967, survival = 0.299
-> a = 0.1232, survival = 0.193
-> a = 0.1570, survival = 0.111
-> a = 0.2000, survival = 0.058
Processing r = 0.479 (5/20)
-> a = 0.0226, survival = 0.928
-> a = 0.0288, survival = 0.799
-> a = 0.0367, survival = 0.677
-> a = 0.0467, survival = 0.568
-> a = 0.0595, survival = 0.489
-> a = 0.0759, survival = 0.385
-> a = 0.0967, survival = 0.266
-> a = 0.1232, survival = 0.166
-> a = 0.1570, survival = 0.070
-> a = 0.2000, survival = 0.025
Processing r = 0.574 (6/20)
-> a = 0.0226, survival = 0.891
-> a = 0.0288, survival = 0.709
-> a = 0.0367, survival = 0.560
-> a = 0.0467, survival = 0.474
-> a = 0.0595, survival = 0.392
-> a = 0.0759, survival = 0.299
-> a = 0.0967, survival = 0.208
-> a = 0.1232, survival = 0.114
-> a = 0.1570, survival = 0.043
-> a = 0.2000, survival = 0.012
Processing r = 0.668 (7/20)
-> a = 0.0226, survival = 0.860
-> a = 0.0288, survival = 0.615
-> a = 0.0367, survival = 0.467
-> a = 0.0467, survival = 0.401
-> a = 0.0595, survival = 0.366
-> a = 0.0759, survival = 0.310
-> a = 0.0967, survival = 0.213
-> a = 0.1232, survival = 0.113
-> a = 0.1570, survival = 0.035
-> a = 0.2000, survival = 0.005
Processing r = 0.763 (8/20)
-> a = 0.0226, survival = 0.844
-> a = 0.0288, survival = 0.520
-> a = 0.0367, survival = 0.409
-> a = 0.0467, survival = 0.363
-> a = 0.0595, survival = 0.337
-> a = 0.0759, survival = 0.284
-> a = 0.0967, survival = 0.176
-> a = 0.1232, survival = 0.082

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-> a = 0.1570, survival = 0.022
-> a = 0.2000, survival = 0.004
Processing r = 0.858 (9/20)
-> a = 0.0226, survival = 0.850
-> a = 0.0288, survival = 0.448
-> a = 0.0367, survival = 0.364
-> a = 0.0467, survival = 0.323
-> a = 0.0595, survival = 0.295
-> a = 0.0759, survival = 0.233
-> a = 0.0967, survival = 0.140
-> a = 0.1232, survival = 0.059
-> a = 0.1570, survival = 0.007
-> a = 0.2000, survival = 0.002
Processing r = 0.953 (10/20)
-> a = 0.0226, survival = 0.890
-> a = 0.0288, survival = 0.410
-> a = 0.0367, survival = 0.328
-> a = 0.0467, survival = 0.289
-> a = 0.0595, survival = 0.241
-> a = 0.0759, survival = 0.204
-> a = 0.0967, survival = 0.143
-> a = 0.1232, survival = 0.061
-> a = 0.1570, survival = 0.009
-> a = 0.2000, survival = 0.000
Processing r = 1.047 (11/20)
-> a = 0.0226, survival = 0.982
-> a = 0.0288, survival = 0.381
-> a = 0.0367, survival = 0.298
-> a = 0.0467, survival = 0.260
-> a = 0.0595, survival = 0.218
-> a = 0.0759, survival = 0.208
-> a = 0.0967, survival = 0.152
-> a = 0.1232, survival = 0.071
-> a = 0.1570, survival = 0.008
-> a = 0.2000, survival = 0.001
Processing r = 1.142 (12/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.364
-> a = 0.0367, survival = 0.273
-> a = 0.0467, survival = 0.235
-> a = 0.0595, survival = 0.216
-> a = 0.0759, survival = 0.207
-> a = 0.0967, survival = 0.154
-> a = 0.1232, survival = 0.075
-> a = 0.1570, survival = 0.016
-> a = 0.2000, survival = 0.002
Processing r = 1.237 (13/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.354
-> a = 0.0367, survival = 0.252
-> a = 0.0467, survival = 0.214
-> a = 0.0595, survival = 0.200
-> a = 0.0759, survival = 0.195
-> a = 0.0967, survival = 0.152
-> a = 0.1232, survival = 0.071
-> a = 0.1570, survival = 0.019
-> a = 0.2000, survival = 0.001
Processing r = 1.332 (14/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.358
-> a = 0.0367, survival = 0.235
-> a = 0.0467, survival = 0.194
-> a = 0.0595, survival = 0.182
-> a = 0.0759, survival = 0.178

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-> a = 0.0967, survival = 0.139
-> a = 0.1232, survival = 0.065
-> a = 0.1570, survival = 0.016
-> a = 0.2000, survival = 0.003
Processing r = 1.426 (15/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.375
-> a = 0.0367, survival = 0.224
-> a = 0.0467, survival = 0.178
-> a = 0.0595, survival = 0.165
-> a = 0.0759, survival = 0.163
-> a = 0.0967, survival = 0.127
-> a = 0.1232, survival = 0.050
-> a = 0.1570, survival = 0.008
-> a = 0.2000, survival = 0.002
Processing r = 1.521 (16/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.403
-> a = 0.0367, survival = 0.215
-> a = 0.0467, survival = 0.164
-> a = 0.0595, survival = 0.151
-> a = 0.0759, survival = 0.148
-> a = 0.0967, survival = 0.109
-> a = 0.1232, survival = 0.046
-> a = 0.1570, survival = 0.005
-> a = 0.2000, survival = 0.002
Processing r = 1.616 (17/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.454
-> a = 0.0367, survival = 0.208
-> a = 0.0467, survival = 0.151
-> a = 0.0595, survival = 0.137
-> a = 0.0759, survival = 0.131
-> a = 0.0967, survival = 0.093
-> a = 0.1232, survival = 0.045
-> a = 0.1570, survival = 0.005
-> a = 0.2000, survival = 0.002
Processing r = 1.711 (18/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.557
-> a = 0.0367, survival = 0.205
-> a = 0.0467, survival = 0.140
-> a = 0.0595, survival = 0.125
-> a = 0.0759, survival = 0.111
-> a = 0.0967, survival = 0.081
-> a = 0.1232, survival = 0.046
-> a = 0.1570, survival = 0.009
-> a = 0.2000, survival = 0.001
Processing r = 1.805 (19/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 0.718
-> a = 0.0367, survival = 0.205
-> a = 0.0467, survival = 0.131
-> a = 0.0595, survival = 0.112
-> a = 0.0759, survival = 0.089
-> a = 0.0967, survival = 0.074
-> a = 0.1232, survival = 0.050
-> a = 0.1570, survival = 0.008
-> a = 0.2000, survival = 0.000
Processing r = 1.900 (20/20)
-> a = 0.0226, survival = 1.000
-> a = 0.0288, survival = 1.000
-> a = 0.0367, survival = 0.212
-> a = 0.0467, survival = 0.122

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-> a = 0.0595, survival = 0.101
-> a = 0.0759, survival = 0.073
-> a = 0.0967, survival = 0.072
-> a = 0.1232, survival = 0.055
-> a = 0.1570, survival = 0.009
-> a = 0.2000, survival = 0.001

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% ===== Visualization =====
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```

figure;
imagesc(a_vec, r_vec, survival_ratio_map);
hcb = colorbar('Location','eastoutside');
colormap("turbo");

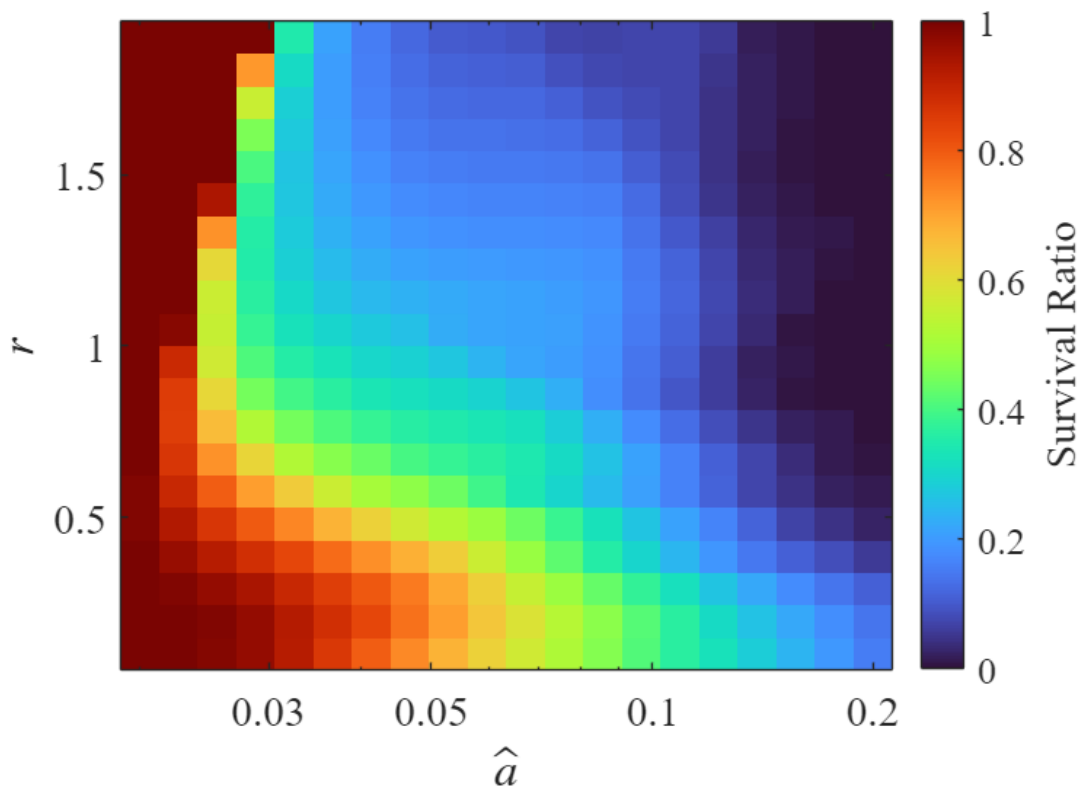
hcb.Label.String = 'Survival Ratio';
hcb.Label.FontSize = 16;
hcb.Label.FontName = 'Times New Roman';
xlabel('$\hat{a}$', 'Interpreter', 'latex', 'FontSize', 18, 'FontName', 'Times New Roman');
ylabel('$r$', 'Interpreter', 'latex', 'FontSize', 18, 'FontName', 'Times New Roman');
%title('Survival Ratio (Fraction of Initial Conditions Surviving)', 'FontSize', 20, 'FontName', 'Times New Roman');

ax = gca;
ax.XScale = 'log';
xticks([0.03, 0.05, 0.1, 0.2]);
set(ax, 'FontSize', 16, 'FontName', 'Times New Roman');

ylim([min(r_vec) max(r_vec)]);
set(gca, 'YDir', 'normal');

grid off;
box on;
axis tight;

```



```
clear;clc;
K = 1;
a_values = [0.038, 0.12];
Omega = 50;
alpha = 2;
delta = 0.04;
A = 0.8;
f = 0.05;
r_values = [0.2, 0.6, 1.0, 1.4, 1.8];
tspan = [0 500];

options = odeset('NonNegative', [1,2,3], 'RelTol', 1e-12, 'MaxStep', 0.1);

B0_1 = 0.1; I0_1 = 0.05; P0_1 = 0.1;
y0_1 = [B0_1; I0_1; P0_1];

B0_2 = 0.02; I0_2 = 0.3; P0_2 = 0.1;
y0_2 = [B0_2; I0_2; P0_2];

colors = lines(length(r_values));
```

```

figWidth_cm = 8;
figHeight_cm = 6;
dpi = 300;
figure('Position', [100, 100, figWidth_cm*dpi/2.54, figHeight_cm*dpi/2.54]);

t1 = tiledlayout(2, 2, 'TileSpacing', 'tight');

axes_handles = cell(2, 2);

y0s = {y0_1, y0_2};

for i_row = 1:2
    for j_col = 1:2
        ax = nexttile;
        axes_handles{i_row, j_col} = ax;
        hold(ax, 'on');

        y0 = y0s{i_row};
        a = a_values(i_row);

        for k = 1:length(r_values)
            r = r_values(k);
            if j_col == 1
                [t, y] = ode15s(@(t,y) ode1B(t, y, r, K, a, Omega, alpha, delta), tspan, y0, op
            else
                [t, y] = ode15s(@(t,y) fluctuate_1B(t, y, A, f, r, K, a, Omega, alpha, delta),
            end

            y_scaled = y * 1e8;
            plot(ax, t, y_scaled(:,1), '-', 'Color', colors(k,:), 'LineWidth', 1.5);
            ylim([10^-1, 10^8]);
            xlim([0, 500]);
        end

        yline(ax, 1, '--k', 'LineWidth', 1.5);

        ax.YScale = 'log';
        grid(ax, 'off');
        box(ax, 'on');
        ax.FontSize = 12;
    end
end

axes_handles{1, 1}.XTickLabel = {};
axes_handles{1, 2}.XTickLabel = {};

```



```
axes_handles{1, 2}.YTickLabel = {};
axes_handles{2, 2}.YTickLabel = {};
```

```
ax_12 = axes_handles{1, 2};
legend_strings = arrayfun(@(r) sprintf('r = %.1f', r), r_values, 'UniformOutput', false);
legend(ax_12, legend_strings, 'Location', 'southeast', 'FontSize', 10);
```

