LINMA2171 (2025–2026) — Numerical Analysis: Approximation, Interpolation, Integration

Homework 1

Deadline: Tuesday 7 October 2025 10:30am

Hermite interpolation Let $x_0, \dots, x_n$ be distinct interpolation points and let $\mathcal{P}_n$ denote the set of real polynomials with degree at most n. Given a function $f : \mathbb{R} \to \mathbb{R}$ (m times differentiable), m order Hermite interpolation aims to find a polynomial $p : \mathbb{R} \to \mathbb{R}$ that interpolates the data points $(x_i, f(x_i), f'(x_i), \dots, f^{(m)}(x_i))_{i=0}^n$, namely

$$p^{(l)}(x_i) = f^{(l)}(x_i), \quad i = 0, \dots, n, \quad l = 0, \dots, m.$$

Firstly, we consider $q_{2n+1} \in \mathcal{P}_{2n+1}$ such that

$$q_{2n+1}(x) = \sum_{i=0}^{n} f(x_i) H_i(x) + \sum_{i=0}^{n} f'(x_i) K_i(x), \tag{1}$$

where

$$\begin{aligned} H_i(x) &= (1 - 2L_i'(x_i)(x - x_i))L_i(x)^2, \\ K_i(x) &= (x - x_i)L_i(x)^2 \end{aligned}$$

and $L_i(x)$ is the Lagrange polynomial associated to x_i defined by

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}.$$

1. Show that the polynomial given by (1) interpolates $(x_i, f(x_i), f'(x_i))_{i=0}^n$.

2. Propose a polynomial $p_{3n+2} \in \mathcal{P}_{3n+2}$ of the form

$$p_{3n+2}(x) = \sum_{i=0}^{n} f(x_i)\alpha_i(x) + \sum_{i=0}^{n} f'(x_i)\beta_i(x) + \sum_{i=0}^{n} f''(x_i)\gamma_i(x) \tag{2}$$

   that interpolates $(x_i, f(x_i), f'(x_i), f''(x_i))_{i=0}^n$. Give the explicit formulas for the functions $\alpha_i(x)$, $\beta_i(x)$ and $\gamma_i(x)$ in (2) with full details of your computations.

3. Implement the second order Hermite interpolation (2) with $f(x) = e^{-\frac{x^2}{2}}$ on the interval $[-5, 5]$. Display the polynomial $p_{3n+2}(x)$ for several values of n and comment. *Hint : If your implementation is correct, you should recover $f(x)$ when it is a polynomial with degree at most $3n + 2$.*

You should provide clear graphs and discuss all your numerical results.

Practical information.

Meetings with the TA: by appointment
Writing : You must do all the writing (report and code) ***individually***. Never share your production, but you are allowed, and even encouraged, to exchange ideas on how to address the homework. If you use an LLM, mention it in the report and explain for which purpose it was used. However, we believe that this assignment will be more useful to you—and the report more pleasant to read for us—if you use LLMs sparingly, if at all.
Questions: Feel free to contact me at timothe.taminiau@uclouvain.be to ask questions or to set up a meeting.

Submission: Using Moodle. Keep in mind that the deadline is automatically enforced, using the clock of Moodle!
Python/Octave/Matlab codes: Please don't copy your code to your report and add the `.py/.ipynb/.m` files as attachments. Add a script named `run` that reproduces all your results.
Language: Reports in French are accepted without penalty. However, English is strongly encouraged. The quality of the English will not impact the grade, provided that the text is intelligible.

Deadline: 7 October, 2025 at 10:30