

CDF:

$$\begin{aligned}F_x(x) &= P(x \leq x) = P(\ln y \leq x) \\&= P(y \leq e^x) \\&= F_y(e^x)\end{aligned}$$

PDF

$$\begin{aligned}f_x(x) &= \frac{\partial}{\partial x} F_y(e^x) \\&= e^x f_y(e^x) = e^x e^{-e^x} = \begin{cases} e^{x-e^x}, & x \in \mathbb{R}. \\ 0 \end{cases}\end{aligned}$$