

X be amount bidden, W be profit. Let y be the minimum bid of the rest.

$$\text{For a fixed } x, \omega = \begin{cases} x - y, & f_y(y) = \frac{1}{70} \\ 0, & f_y(y) = \frac{y-70}{70} \end{cases}$$

$$\begin{aligned} P(\text{Win bid}) &= P(x < Y) \\ &= 1 - P(Y < x) \\ &= 1 - P(Y \leq x) \\ &= 1 - \frac{x-70}{70} \\ &= 2 - \frac{x}{70} \end{aligned}$$

$$\begin{aligned} E(W) &= (x-100) \left(2 - \frac{x}{70} \right) \\ &= 2x - 200 - \frac{x^2}{70} + \frac{10x}{7} \end{aligned}$$

$$\frac{d}{dx} E(W) = 2 - \frac{2x}{70} + \frac{10}{7} \rightarrow 2 - \frac{2x}{70} + \frac{10}{7} = 0$$

$$x = 120$$

Check whether this is max: $\left(2 - \frac{2x}{70} + \frac{10}{7} \right)' = -2^{-\frac{2}{70}} < 0$ (max)

$$\therefore x = \$120,000$$