

$$\begin{aligned}
F_X(x) &= P(X \leq x) = P(\ln Y \leq x) \\
&= P(Y \leq e^x) \\
&= \int_{e^-}^{e^x} e^{-y} dy \\
&= [-e^{-y}]_0^{e^x} \\
&= 1 - e^{-e^x} \\
f_x(x) &= F'_{X(x)} = e^{-e^x} \cdot e^x \\
&= e^{x-e^x} \quad (\text{for } x \in \mathbb{R})
\end{aligned}$$