Given that $X = \ln Y$, it is strictly monotone, it is increasing on $(0, \infty)$ Thus we can directly apply Monotone Transformation Theorem

$$f_x(x) = \begin{cases} f_y\left(g^{-1}(x)\right) \cdot \left|\frac{d}{dx} g^{-1}(x)\right| & \text{if } y = g(x) \quad x = \ln y = g(y) \\ 0 & \text{if } y \neq g(x) \quad y = e^x = g^{-1}(x) \end{cases}$$

$$= \begin{cases} e^{-e^x} \cdot |e^x| = e^{-e^x} \cdot e^x & (e^x > 0) \\ 0 & \text{omitted, because } y > 0 \text{ and the pdf of x is always not zero.} \end{cases}$$