

$$\begin{aligned}
F_{x(x)} &= P(x \leq x) = P(\ln Y \leq x) = P(Y \leq e^x) = P(0 < y \leq e^x). \\
&= \int_0^{e^x} \cdot e^{-y} dy \\
&= -[e^{-y}]_0^{e^x} \\
&= -[e^{-e^x} - 1]. \\
&= 1 - e^{-e^x}
\end{aligned}$$

$$F_x(x) = \begin{cases} 1 - e^{-e^x}, & \text{for } x > 0. \\ 0, & \text{for } x \leq 0. \end{cases}$$

$$f_x(x) = \begin{cases} e^x e^{-e^x}, & \text{for } x > 0 \\ 0, & \text{otherwise.} \end{cases}$$