CDF:

$$\begin{aligned} F_x(x) = P(x \leq x) &= P(\ln y \leq x) \\ &= P\left(y \leq e^x\right) \\ &= F_y\left(e^x\right) \end{aligned}$$

PDF $f_x(x) = \frac{d}{\partial x} F_y\left(e^x\right) = \frac{d}{\partial x} F_y\left(e^x\right) = e^x f_y\left(e^x\right) = e^x e^{-e^x} = \begin{cases} e^{x-e^x} \\ 0 \end{cases}, x \in \mathbb{R}.$