CS 421 Algorithm (Summer 2018)

Homework #3, final exam preparation

- **Q1(9 points): Asymptotic notations and recurrence equations**

  (a)(6 points) Let $f(n) = n^2 - 5n - 50$, please use the basic definition of Θ-notation to prove $f(n) = \Theta(n^2)$.

  (b)(3 pts)(Multiple choice) If an algorithm's running time can be expressed as a function $f(n) = O(n^2) + \Omega(n)$, then which one of the following running time bounds is not possible for the algorithm?

  1. $\Theta(\log n)$
  2. $\Theta(n)$
  3. $\Theta(n^2)$
  4. $\Theta(2^n)$

- **Q2(11 points): Divide-and-conquer and recurrence equations**

  (a)(3 pts) Assume the following are recurrence equations for four different algorithms. Which algorithm has the worst running time?

  1. $T(n) = 2T(n/2) + n$
  2. $T(n) = T(n-1) + n$
  3. $T(n) = 2T(n-1) + 1$
  4. $T(n) = T(n-1) + T(n-2) + 1$

  (b)(8 points) A ternary merge sort algorithm works the same as the ordinary two-way merge sort, except it divides an array into three (about) equal-sized subarrays and then sorts each subarray recursively. Finally, the merge-process merges three sorted subarrays back to one sorted array. What is the running time for this ternary merge sort? Please write down the recurrence equation first (3 pts), and then solve the equation (5 pts).
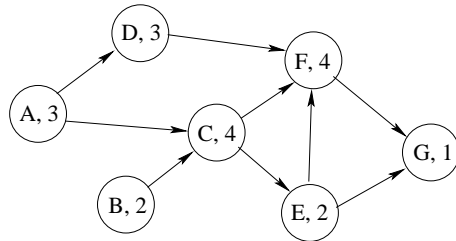
- **Q3(7 points): Greedy algorithms**

  In the activity selection problem (Chapter 16.1 of the textbook), what's the optimal solution for the following problem instance?

Activity i	Starting Time	Finishing Time
1	3	5
2	0	2
3	6	8
4	2	4
5	4	6
6	1	3
7	1	3
8	5	7
9	5	7

- **Q4(18 points): Critical path analysis**

  An activity node graph is given below.

 (a)(8 pts) Based on the discussion in class, for the given graph, what's the corresponding event node graph?

(b)(10 points) Please find the earliest completion time (EC time), latest completion time (LC time), and the slack time for each activity.

- **Q5(15 points): Single source shortest path problem**

  (a)(6 points) Given a weighted and undirected graph as below, please find a minimum spanning tree by highlighting the edges chosen.
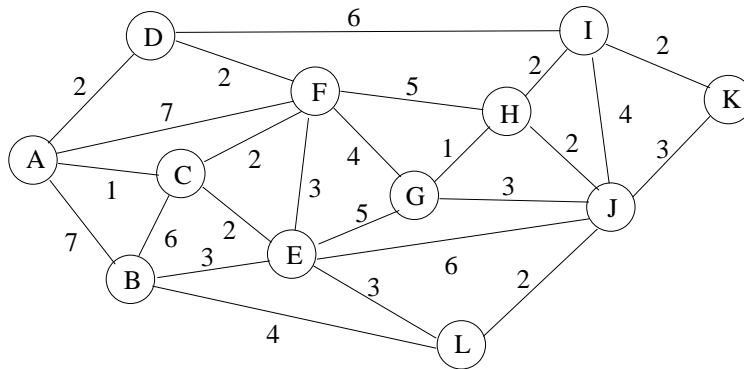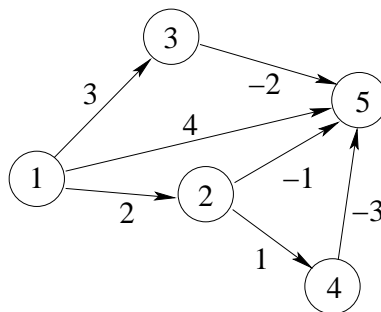

  (b)(9 points) Given a DAG as below, please find a shortest path tree (by high-lighting some edges) if A is the source vertex.
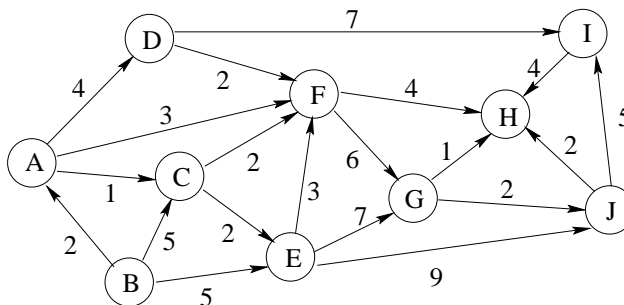
- **Q6(15 points): All pairs shortest path problem**

  A weighted and directed graph is given below.

  (a)(5 pts) For the given graph, based on the algorithm in Chapter 25.1 of the text-book, what is the initial shortest-path weight matrix $L^{(1)}$ (please see page 690 of the textbook).

  (b)(10 pts) For the given graph, based on the algorithm in Chapter 25.1, what is the next shortest-path weight matrix $L^{(2)}$?

- **Q7(15 points): Number theory**

  (a)(8 pts) Let two primes $p = 7$ and $q = 13$. Based on these two primes and let an RSA public key $e = 5$, what is the corresponding RSA private key?

  (b)(7 points) Given an integer 6, what's the RSA encrypted cipher for this integer if using the above RSA keys (encryption should use RSA public key)?