10.1 Recursion: Introduction

An **algorithm** is a sequence of steps for solving a problem. For example, an algorithm for making lemonade is:



Figure 10.1.1: Algorithms are like recipes.



- Make lemonade:
- Add sugar to pitcher
- Add lemon juice
- Add water
- Stir

Some problems can be solved using a recursive algorithm. A **recursive algorithm** solves a problem by breaking that problem into smaller subproblems, solving these subproblems, and combining the solutions.

Figure 10.1.2: Mowing the lawn can be broken down into a recursive process.



- Mow the lawn
 - Mow the frontyard
 - Mow the left front
 - Mow the right front
 - Mow the backyard
 - Mow the left back
 - Mow the right back

AhramKim@u.boisestate.edu

An algorithm that is defined by repeated applications of the same algorithm on smaller problems is a **recursive** algorithm. The mowing algorithm consists of applying the mowing algorithm on smaller pieces of the yard.

At some point, a recursive algorithm must describe how to actually do something, known as the **base case**. The mowing algorithm could thus be written as:

- Mow the lawn
 - If lawn is less than 100 square meters
 - Push the lawnmower left-to-right in adjacent rows

- Else
 - Mow one half of the lawn
 - Mow the other half of the lawn

PARTICIPATION ACTIVITY	10.1.1: Recursion.	
1) Helping N p	eople: Our boisestate edu p that person. ECS253Fall2017 ne first N/2 people, then help N/2 people.	
O False 2) Driving to the Go 1 mile. Turn left on Go 1/2 mile O True	ne store: Main Street.	
If N is 1, dor Else, find the zipcodes les on the left, a	e middle zipcode. Put all ss than the middle zipcode all greater ones on the right. he left, then sort the right. Ahram Kim Qu. boisestate.eo	

10.2 Recursive functions

A function may call other functions, including calling itself. A function that calls itself is a **recursive function**.

Aug. 27th, 2017 18:15

PARTICIPATION ACTIVITY

10.2.1: A recursive function example.

```
#include <stdio.h>

void CountDown(int countInt) {
    if (countInt = 0) {
        printf("GO!\n");
    }

else {
        printf("%d\n", countInt);
        CountDown(countInt = 1);
    }

int main(void) {
        CountDown(2);
        return 0;
}
```

```
int main (void)
   CountDown (2);
   return 0;
void CountDown(int countInt) {
                                    countInt: 2
   if (countInt == 0) {
      printf("GO!\n");
      printf("%d\n", countInt);
      CountDown (countInt - 1);
   return;
                                    countInt: 1
void CountDown(int countInt)
   if (countInt == 0) {
    printf("GO!\n");
      printf("%d\n", countInt);
      CountDown (countInt - 1);
   return:
                                   countInt: 0
void CountDown(int countInt) {
   if (countInt == 0) {
      printf("GO!\n");
   else |
      printf("%d\n", countInt);
      CountDown (countInt - 1);
   return;
```

Ahram Kim

Each call to CountDown() effectively creates a new "copy" of the executing function, as shown on the right. Returning deletes that copy.

The example is for demonstrating recursion; counting down is otherwise better implemented with a loop.

Recursion may be direct, such as f() itself calling f(), or indirect, such as f() calling g() and g() calling f().

PARTICIPATION ACTIVITY

10.2.2: Thinking about recursion. Kim Qu. boisestate.ed

Refer to the above CountDown example for the following.

1) How many times is CountDown() called if main() calls CountDown(5)?

Check

Show answer

2) How many times is CountDown() called if main() calls CountDown(0)?

Check Show answer

3) Is there a difference in how we define the parameters of a recursive versus non-recursive function? Answer yes or no.

CHALLENGE ACTIVITY

10.2.1: Calling a recursive function.

Write a statement that calls the recursive function BackwardsAlphabet() with parameter startingLetter.

```
1 #include <stdio.h>
  void BackwardsAlphabet(char currLetter){
     if (currLetter == 'a') {
       printf("%c\n", currLetter);
 5
 6
 7
     else{
       printf("%c ", currLetter);
 8
 9
       BackwardsAlphabet(currLetter - 1);
10
11
     return;
12 }
13
14 int main(void) {
                                          Ahram Kim
     char startingLetter = '-';
15
16
                           AhramKim@u.boisestate.edu
     startingLetter = 'z';
17
18
     /* Your solution goes here
19
                            BOISESTATECS253Fall2017
20
21
     return 0:
                                Aug. 27th, 2017 18:15
Run
```

10.3 Recursive algorithm: Search

Consider a guessing game program where a friend thinks of a number from 0 to 100 and you try to guess the number, with the friend telling you to guess higher or lower until you guess correctly. What algorithm would you use to minimize the number of guesses?

A first try might implement an algorithm that simply guesses in increments of 1:

- Is it 0? Higher
- n@u.boisestate.edu Is it 1? Higher
- CS253Fall2017 Is it 2? Higher

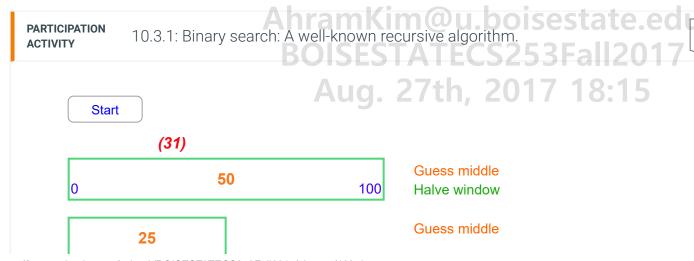
This algorithm requires too many guesses (50 on average). A second try might implement an algorithm that guesses by 10s and then by 1s:

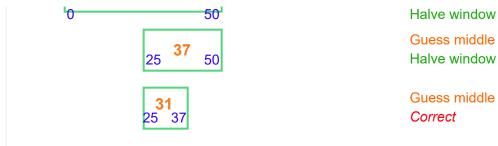
- Is it 10? Higher
- Is it 20? Higher
- Is it 30? Lower
- Is it 21? Higher
- Is it 22? Higher
- Is it 23? Higher

This algorithm does better but still requires about 10 guesses on average: 5 to find the correct tens digit and 5 to guess the correct ones digit. An even better algorithm uses a binary search. A **binary search** algorithm begins at the midpoint of the range and halves the range after each guess. For example:

- Is it 50 (the middle of 0-100)? Lower
- Is it 25 (the middle of 0-50)? Higher
- Is it 37 (the middle of 25-50)? Lower
- Is it 31 (the middle of 25-37).

After each guess, the binary search algorithm is applied again, but on a smaller range, i.e., the algorithm is recursive.





A recursive function is a natural match for the recursive binary search algorithm. A function GuessNumber(lowVal, highVal) has parameters that indicate the low and high sides of the guessing range. The function guesses at the midpoint of the range. If the user says lower, the function calls GuessNumber(lowVal, midVal). If the user says higher, the function calls GuessNumber(midVal + 1, highVal)^{mid}.

Figure 10.3.1: A recursive function Find() carrying out a binary search algorithm.

```
#include <stdio.h>
void GuessNumber(int lowVal, int highVal) {
   int midVal = 0;
                     // Midpoint of low and high value
  char userAnswer = '-'; // User response
  midVal = (highVal + lowVal) / 2;
  // Prompt user for input
  printf("Is it %d? (I/h/y): ",midVal);
  scanf(" %c", &userAnswer);
  if( (userAnswer != 'I') && (userAnswer != 'h') ) { // Base case: Found number
     printf("Thank you!\n");
  else {
                                                  // Recursive case: split into lower OR upper half
     if (userAnswer == 'l') {
                                                  // Guess in lower half
        GuessNumber(lowVal, midVal);
                                                  // Recursive call
                                                  // Guess in upper half
     else {
        GuessNumber(midVal + 1, highVal);
                                                 // Recursive call
                                                         Ahram Kim
  return;
                                    AhramKim@u.boisestate.edu
int main(void) {
                                                                     CS253Fall2017
  // Print game objective, user input commands
  printf("Choose a number from 0 to 100.\formun");
                                            Aug. 27th, 2017 18:15
  printf("Answer with:\n");
printf(" | I (your num is
            l (your num is lower)\n");
  printf("
            h (your num is higher)₩n");
  printf("
            any other key (guess is right).₩n");
   // Call recursive function to guess number
  GuessNumber(0, 100);
  return 0;
```

```
Choose a number from 0 to 100.

Answer with:

I (your num is lower)

h (your num is higher)

any other key (guess is right).

Is it 50? (I/h/y): I

Is it 25? (I/h/y): h

Is it 38? (I/h/y): I

Is it 32? (I/h/y): I

Is it 29? (I/h/y): h

Is it 31? (I/h/y): y

Thank you!
```

AhramKim@u.boisestate.edu BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15

PARTICIPATION ACTIVITY

10.3.2: Binary search tree tool.

The following program guesses the hidden number known by the user.

```
#include <stdio.h>
void Find(int low, int high) {
  int mid=0; // Midpoint of low..high
char answer='\0';
  mid = (high + low)/2;
   printf("Is it %d? (1/h/y): ", mid);
   scanf(" %c", &answer);
   if( (answer != '1')
        && (answer != 'h') ) { // Base case:
      printf("Thank you!\n"); // Found number!
   else { // Recursive case: Guess in
      // Recursive case. Guess in
// lower or upper half of range
if (answer == 'l') { // Guess in lower half
    Find(low, mid); // Recursive call
         else {
                                                     Ahram Kim
   }
   return;
                                 AhramKim@u.boisestate.edu
int main() {
                                                                CS253Fall2017
  printf("Choose a number from 0 to 100.\n");
   printf("Answer with:\n");
  printf(" 1 (your num is lower)\n");
printf(" h (your num is higher)\n");
             h (your num is higher) \n"); any other key (guess is right).\n");
  printf("
  printf("
   Find(0, 100);
   return:
}
```

```
int main() {
   printf("Choose a number from 0 to 100.\n");
            printf("Answer with:\n");
                       1 (your num is lower)\n");
h (your num is higher)\n");
             printf("
                        any other key (guess is right).\n");
                Find(0, 100);
Aug. 27th, 2017 18:15
```

The recursive function has an if-else statement. The if branch ends the recursion, known as the **base case**. The else branch has recursive calls. Such an if-else pattern is common in recursive functions.

Search is commonly performed to quickly find an item in a sorted list stored in an array or vector. Consider a list of attendees at a conference, whose names have been stored in alphabetical order in an array or vector. The following quickly determines whether a particular person is in attendance.

Figure 10.3.2: Recursively searching a sorted list u.boisestate.edu
BOISESTATECS253Fall2017
Aug. 27th, 2017 18:15

Ahram Kim

```
#include <stdio.h>
#include <string.h>
/* Finds index of string in array of strings, else -1.
   Searches only with index range low to high
  Note: Upper/lower case characters matter
const int NUM_ATTENDEES = 6;
                                    // Number of total attendees
const int CHAR_LIMIT_PER_NAME = 50; // Limit size of attendee name
int FindMatch(char stringsList[NUM_ATTENDEES][CHAR_LIMIT_PER_NAME], char itemMatch[],
              int lowVal, int highVal) {
 int midVal = 0;  // Midpoint of low and high values
int itemPos = -1; // Position where item found, -1 if not found
   int rangeSize = 0; // Remaining range of values to search for match
   rangeSize = (highVal - lowVal) + 1;
   midVal = (highVal + lowVal) / 2;
   if (strcmp(stringsList[midVal], itemMatch) == 0) {
                                                           Base case 1: item found at midVal position
      itemPos = midVal;
   else if (rangeSize == 1) {
                                                         // Base case 2: match not found
      itemPos = -1;
   else {
                                                         // Recursive case: search lower or upper half
      if (strcmp(itemMatch, stringsList[midVal]) < 0) { // Search lower half, recursive call</pre>
         itemPos = FindMatch(stringsList, itemMatch, lowVal, midVal);
      }
      else {
                                                         // Search upper half, recursive call
         itemPos = FindMatch(stringsList, itemMatch, midVal + 1, highVal);
   return itemPos;
int main(void) {
   char attendeesList[NUM_ATTENDEES][CHAR_LIMIT_PER_NAME]; // List of attendees
   char attendeeName[50] = "";
                                                            // Name of attendee to match. 50 is CHAR_LIMIT_PER_NAME
   int matchPos = 0;
                                                            // Matched position in attendee list
   // Omitting part of program that adds attendees
   // Instead, we insert some sample attendees in sorted order
   strcpy(attendeesList[0], "Adams, Mary");
   strcpy(attendeesList[1], "Carver, Michael");
   strcpy(attendeesList[2], "Domer, Hugo");
   strcpy(attendeesList[3], "Fredericks, Carlos");
   strcpy(attendeesList[4], "Li, Jie");
                                                              Ahram Kim
   strcpy(attendeesList[5], "Zuckerberg, AI");
                                                       nKim@u.boisestate.edu
   // Prompt user to enter a name to find
   printf("Enter person's name: Last, First: ");
   fgets(attendeeName, CHAR_LIMIT_PER_NAME, stdin);
   attendeeName[strlen(attendeeName) - 1] = '\0'; // remove newline
   // Call function to match name, output results matchPos = FindMatch(attendeesList, attendeeName, 0, NUM_ATTENDEES - 1);
   // Call function to match name, output results
   if (matchPos >= 0) {
     printf("Found at position %d.\m", matchPos);
   else {
     printf("Not found. \n");
   return 0;
}
```

Enter person's name: Last, First: Meeks, Stan Not found.
...
Enter person's name: Last, First: Adams, Mary Found at position 0.
...
Enter person's name: Last, First: Li, Jie Found at position 4.

AhramKim@u.boisestate.edu BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15

FindMatch() restricts its search to elements within the range lowVal to highVal. main() initially passes a range of the entire list: 0 to (list size - 1). FindMatch() compares to the middle element, returning that element's position if matching. If not matching, FindMatch() checks if the window's size is just one element, returning -1 in that case to indicate the item was not found. If neither of those two base cases are satisfied, then FindMatch() recursively searches either the lower or upper half of the range as appropriate.

•		
PARTICIPATION ACTIVITY	10.3.3: Recursive search algorithm.	
Consider the a	above FindMatch() function for finding an item in a sorted list.	
the item be element 6,	list has elements 0 to 50 and eing searched for is at how many times will n() be called?	
Check	Show answer Ahram Kim	
elements 0 element 0 i recursive c	betically ascending list has ramkim@u.boisestate.ed to 50, and the item at BOISESTATECS253Fall2017 is "Bananas", how many calls to FindMatch() will be Aug. 27th, 2017 18:15 ing the failed search for	-
Check	Show answer	

PARTICIPATION ACTIVITY 10.3.4: Recursive calls.	
A list has 5 elements numbered 0 to 4, with these letter values: 0: A, 1: B, 2: D, 3: E, 4: F.	
1) To search for item C, the first call is FindMatch(0, 4). What is the second call to FindMatch()? O FindMatch(0, 0) O FindMatch(0, 2) O FindMatch(3, 4)	
 2) In searching for item C, FindMatch(0, 2) is called. What happens next? O Base case 1: item found at midVal. O Base case 2: rangeSize == 1, so 	
no match.	
O Recursive call: FindMatch(2, 2)	

Exploring further:

 Binary search from wikipedia.com

(*mid) Because midVal has already been checked, it need not be part of the new window, so midVal + 1 rather than midVal is used for the window's new low side, or midVal - 1 for the window's new high side. But the midVal - 1 can have the drawback of a non-intuitive base case (i.e., midVal < lowVal, because if the current window is say 4..5, midVal is 4, so the new window would be 4..4-1, or 4..3). rangeSize == 1 is likely more intuitive, and thus the algorithm uses midVal rather than midVal - 1. However, the algorithm uses midVal + 1 when searching higher, due to integer rounding. In particular, for window 99..100, midVal is 99 ((99 + 100) / 2 = 99.5, rounded to 99 due to truncation of the fraction in integer division). So the next window would again be 99..100, and the algorithm would repeat with this window forever. midVal + 1 prevents the problem, and doesn't miss any numbers because midVal was checked and thus need not be part of the window.

10.4 Adding output statements for debugging

Recursive functions can be particularly challenging to debug. Adding output statements can be helpful. Furthermore, an additional trick is to indent the print statements to show the current depth of recursion. The following program adds a parameter indent to a FindMatch() function that searches a sorted list for an item. All of FindMatch()'s print statements start with printf("% ...", indentAmt, ...); Indent is typically some number of spaces. main() sets indent to three spaces. Each recursive call adds three more spaces. Note how the output now clearly shows the recursion depth.

Ahram Kim

Figure 10.4.1: Output statements can help debug recursive functions, especially if indented based on recursion depth.

```
#include <stdio.h>
#include <string.h>
/* Finds index of string in array of strings, else -1.
      Searches only with index range low to high
      Note: Upper/lower case characters matter
const int NUM_ATTENDEES = 6;
                                                                            // Number of total attendees
const int CHAR_LIMIT_PER_NAME = 50; // Limit size of attendee name
int FindMatch(char stringsList[NUM_ATTENDEES][CHAR_LIMIT_PER_NAME], char itemMatch[],
                               int lowVal, int highVal, char indentAmt[CHAR_LIMIT_PER_NAME]) { // indentAmt used for print debug
       int midVal = 0;
                                                               // Midpoint of low and high values
       int itemPos = -1;
                                                              // Position where item found, -1 if not found
      int rangeSize = 0;
                                                              // Remaining range of values to search for match
       char indentNext[50] = ""; // Stores next recursion level indentation amount
       // Update indent amount for next level of recursion
      strcpy(indentNext, indentAmt);
      strcat(indentNext, '
      printf("%sFind() range %d %d\n", indentAmt, lowVal, highVal);
      rangeSize = (highVal - lowVal) + 1;
      midVal = (highVal + lowVal) / 2;
       if (strcmp(stringsList[midVal], itemMatch) == 0) { // Base case 1: Found at mid
             printf("%sFound person.\n", indentAmt);
             itemPos = midVal;
      else if (rangeSize == 1) {
            printf("%sPerson not found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\found.\foun
             itemPos = -1;
                                                                                                                             // Recursive case: search lower or upper half
      else {
             if (strcmp(itemMatch, stringsList[midVal]) < 0) { // Search lower half, recursive call</pre>
                   printf("%sSearching lower half.\footbolden", indentAmt);
                    itemPos = FindMatch(stringsList, itemMatch, lowVal, midVal, indentNext);
                                                                                                                            // Search upper half, recursive ca
                   printf("%sSearching upper half.\footnote{\text{Wn}}", indentAmt);
                    itemPos = FindMatch(stringsList, itemMatch, midVal + 1, highVal, indentNext);
      printf("%sReturning pos = %d.\n", indentAmt, itemPos);
      return itemPos;
int main(void) {
```

```
char attendeesList[NUM_ATTENDEES][CHAR_LIMIT_PER_NAME]; // List of attendees
                                                      // Name of attendee to match. 50 is CHAR_LIMIT_PER_NAME
  char attendeeName[50] = "";
  int matchPos = 0;
                                                      // Matched position in attendee list
  char indentLevel[50] = "";
                                                      // Stores indentation (3 spaces)
  // Omitting part of program that adds attendees
  // Instead, we insert some sample attendees in sorted order
  strcpy(attendeesList[0], "Adams, Mary");
  strcpy(attendeesList[1], "Carver, Michael");
  strcpy(attendeesList[2], "Domer, Hugo");
  strcpy(attendeesList[3], "Fredericks, Carlos");
  strcpy(attendeesList[4], "Li, Jie");
  strcpy(attendeesList[5], "Zuckerberg, AI");
 fgets(attendeeName, CHAR_LIMIT_PER_NAME, stdin);
  attendeeName[strlen(attendeeName) - 1] = '₩0'; // remove newline
  strcpy(indentLevel,
  // Call function to match name, output results
  matchPos = FindMatch(attendeesList, attendeeName, 0, NUM_ATTENDEES - 1, indentLevel);
  if (matchPos >= 0) {
     printf("Found at position %d.\footbooks, matchPos);
  else {
     printf("Not found. ₩n");
  return 0;
Enter person's name: Last, First: Meeks, Stan
  Find() range 0 5
  Searching upper half.
     Find() range 3 5
     Searching upper half.
        Find() range 5 5
        Person not found.
        Returning pos = -1.
     Returning pos = -1.
  Returning pos = -1.
Not found.
Enter person's name: Last, First: Adams, Mary
  Find() range 0 5
  Searching lower half.
     Find() range 0 2
     Searching lower half.
                                                       Ahram Kim
        Find() range 0 1
        Found person.
                                   AhramKim@u.boisestate.edu
        Returning pos = 0.
     Returning pos = 0.
  Returning pos = 0.
                                                                    CS253Fall2017
Found at position 0.
                                           Aug. 27th, 2017
```

Some programmers like to leave the output statements in the code, commenting them out with "//" when not in use. The statements actually serve as a form of comment as well.

More advanced techniques for handling debug output exist too, such as **conditional compilation** (beyond this section's scope).

PARTICIPATION ACTIVITY

Refer to the above code using indented output statements. 1) The above debug approach requires an extra parameter be passed to indicate the amount of indentation. True Ahram Kim O False 2) Each recursive call should add a few spaces to the indent parameter. \$253Fall 2017 ı. 27th, 2017 18:15 C False 3) The function should remove a few spaces from the indent parameter before returning. O True C False

PARTICIPATION ACTIVITY

10.4.2: Output statements in a recursive function.

- Run the recursive program, and observe the output statements for debugging, and that the person is correctly not found.
- Introduce an error by changing itemPos = -1 to itemPos = 0 in the range size == 1 base case.
- Run the program, notice how the indented print statements help isolate the error of the person incorrectly being found.

```
Load default template...

Anramkim & boisestate.edu

#include <stdio.h>
#include <string.h>

/* Finds index of string in array of strings, else -1. th,
Searches only with index range low to high
Note: Upper/lower case characters matter

*/

const int NUM_ATTENDEES = 6;  // Number of total atte
const int CHAR_LIMIT_PER_NAME = 50; // Limit size of attend

int FindMatch(char stringsList[NUM_ATTENDEES][CHAR_LIMIT_PE
int lowVal, int highVal, char indentAmt[CHAR_
```

```
int midVal = 0; // Midpoint of low and high va
int itemPos = -1; // Position where item found,
int rangeSize = 0; // Remaining range of values t
```

10.5 Creating a recursive function

Creating a recursive function can be accomplished in two steps.

- **Write the base case** Every recursive function must have a case that returns a value without performing a recursive call. That case is called the **base case**. A programmer may write that part of the function first, and then test. There may be multiple base cases.
- Write the recursive case -- The programmer then adds the recursive case to the function.

The following illustrates for a simple function that computes the factorial of N (i.e. N!). The base case is N = 1 or 1! which evaluates to 1. The base case is written as if $(N \le 1)$ { fact = 1; }. The recursive case is used for N > 1, and written as else { fact = N * NFact(N - 1); }.

PARTICIPATION ACTIVITY

10.5.1: Writing a recursive function for factorial: First write the base case, then add the recursive case.

Start

Write and test base case (non-recursive case)

Add and test recursive case

```
int NFact(int N) {
   int factResult;
   if (N <= 1) { // Base case
      factResult = 1;
   }
   // FIXME: Finish
   return factResult;
}
// main(): Get N, print NFact(N)</pre>
```

```
Enter N: 1
N! is: 1
```

Enter N: 1 1 1 2 2 0 U N! is: 1 Enter N: 6 N! is: 720

A <u>common error</u> is to not cover all possible base cases in a recursive function. Another <u>common error</u> is to write a recursive function that doesn't always reach a base case. Both errors may lead to infinite recursion, causing the program to fail.

Typically, programmers will use two functions for recursion. An "outer" function is intended to be called from other parts of the program, like the function int CalcFactorial(int inVal). An "inner" function is intended only to be called from that outer function, for example a function int _CalcFactorial(int inVal) (note the "_"). The outer function may check for a valid input value, e.g., ensuring inVal is not negative, and then calling the inner function. Commonly, the inner function has parameters that are mainly of use as part of the recursion, and need not be part of the outer function, thus keeping the outer function more intuitive.

PARTICIPATION 10.5.2; Creatin	ng recursion. 7 18:15	
 Recursive functions can be accomplished in one step, repeated calls to itself. True 		
O False		
 2) A recursive function with p counts up from any negative 0. An appropriate base cas == 0. O True O False 	ve number to	
3) A recursive function can hat cases, such as N == 0 returning 1.		
O True	Ahram Kim	
O False	AhramKim@u.boises	tate.edu

Before writing a recursive function, a programmer should determine:

- 1. Does the problem naturally have a recursive solution?
- 2. Is a recursive solution better than a non-recursive solution?

For example, computing N! (N factorial) does have a natural recursive solution, but a recursive solution is not better than a non-recursive solution. The figure below illustrates how the factorial computation can be implemented as a loop. Conversely, binary search has a natural recursive solution, and that solution may be easier to understand than a non-recursive solution.

Figure 10.5.1: Non-recursive solution to compute N!

```
for (i = inputNum; i > 1; --i) {
   facResult = facResult * i;
}
```

PARTICIPATION ACTIVITY

10.5.3: When recursion is appropriate.

AhramKim@u.boisestate.edu

- 1) N factorial (N!) is commonly implemented as a recursive function due to being easier to understand and executing faster than a loop implementation.
 - O True
 - O False

PARTICIPATION ACTIVITY

10.5.4: Output statements in a recursive function.

Implement a recursive function to determine if a number is prime. Skeletal code is provided in the IsPrime function.

Load default template...

Run

```
#include <stdio.h>
 4 // Returns 0 if value is not prime, 1 if value is prime
  int IsPrime(int testVal, int divVal)
 6
      // Base case 1: 0 and 1 are not prime, testVal is not pr
7
      // Base case 2: testVal only divisible by 1, testVal is Kim
 8
9
10
11
      // Recursive Case
         // Check if testVal can be evenly divided by
12
13
         // Hint: use the % operator
14
         // If not, recursive call to isPrime with testVal and
15
16
      return 0;
17 }
18
19 int main(void){
20
21
```

CHALLENGE

ACTIVIT

10.5.1: Recursive function: Writing the base case.

Write code to complete DoublePennies()'s base case. Sample output for below program:

Number of pennies after 10 days: 1024

Note: These activities may test code with different test values. This activity will perform three tests, with startingPennies = 1 and userDays = 10, then with startingPennies = 1 and userDays = 40, then with startingPennies = 1 and userDays = 1. See How to Use zyBooks.

Also note: If the submitted code has an infinite loop, the system will stop running the code after a few seconds, and report "Program end never reached." The system doesn't print the test case that caused the reported message.

```
1 #include <stdio.h>
 3 // Returns number of pennies if pennies are doubled numDays times
   long long DoublePennies(long long numPennies, int numDays){
      long long totalPennies = 0;
      /* Your solution goes here */
9
      else {
        totalPennies = DoublePennies((numPennies * 2), numDays - 1);
10
11
12
13
      return totalPennies;
14 }
15
16 // Program computes pennies if you have 1 penny today,
17 // 2 pennies after one day, 4 after two days, and so on
18 int main(void) |{|
      long long startingPennies = 0;
20
      int userDays = 0;
21
```

CHALLENGE ACTIVITY

Run

10.5.2: Recursive function: Writing the recursive case.

Write code to complete PrintFactorial()'s recursive case. Sample output if userVal is 5:

Ahram Kim
AhramKim@u.boisestate.edu

$$5! = 5 * 4 * 3 * 2 * 1 = 120$$

1 #include <stdio.h>

```
void PrintFactorial(int factCounter, int factValue){
      int nextCounter = 0;
      int nextValue = 0;
 5
 6
      if (factCounter == 0) {
                                    // Base case: 0! = 1
 7
        printf("1\n");
 8
 9
      else if (factCounter == 1) {
                                     // Base case: Print 1 and result
10
         printf("%d = %d\n", factCounter, factValue);
11
12
                                      // Recursive case
      else {
13
        printf("%d * ", factCounter);
14
         nextCounter = factCounter - 1;
15
        nextValue = nextCounter * factValue;
16
17
           Your solution goes here */53Fall2017
18
19
        Aug. 27th, 2017 18:15
Run
```

10.6 Recursive math functions

Fibonacci sequence

Recursive functions can solve certain math problems, such as computing the Fibonacci sequence. The *Fibonacci sequence* is 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc.; starting with 0, 1, the pattern is to compute the next number by adding the previous two numbers.

Below is a program that outputs the Fibonacci sequence values step-by-step, for a user-entered number of steps. The base case is that the program has output the requested number of steps. The recursive case is that the program needs to compute the number in the Fibonacci sequence.

Figure 10.6.1: Fibonacci sequence step-by-step.

Ahram Kim

Ahram Kim

Ahram Kim

Ahram Kim

BOISESTATECS253Fall2017

Aug. 27th, 2017 18:15

```
#include <stdio.h>
/* Output the Fibonacci sequence step-by-step.
   Fibonacci sequence starts as:
   0 1 1 2 3 5 8 13 21 ... in which the first
   two numbers are 0 and 1 and each additional
   number is the sum of the previous two numbers
void ComputeFibonacci(int fibNum1, int fibNum2, int runCnt) {
   printf("%d + %d = %d\n", fibNum1, fibNum2, fibNum1 + fibNum2);
   // Recursive case: compute next value
   else {
      ComputeFibonacci(fibNum2, fibNum1 + fibNum2, runCnt -
int main(void) {
   int runFor = 0; // User specified number of values computed
   // Output program description
  printf("This program outputs theWn");
printf("Fibonacci sequence step-by-step,Wn");
   printf("starting after the first 0 and 1.\forall n\forall n\forall n\forall n\forall n
   // Prompt user for number of values to compute
   printf("How many steps would you like? ");
   scanf("%d", &runFor);
   // Output first two Fibonacci values, call recursive function
   printf("O\mun1\mun");
   ComputeFibonacci(0, 1, runFor);
   return 0;
```

Ahram Kim AhramKim@u.boisestate.edu BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15

PARTICIPATION ACTIVITY

10.6.1: Recursive Fibonacci.

Complete ComputeFibonacci() to return F_N , where F_0 is 0, F_1 is 1, F_2 is 1, F_3 is 2, F_4 is 3, and continuing: F_N is $F_{N-1} + F_{N-2}$. Hint: Base cases are N == 0 and N == 1.

```
#include <stdio.h>
 4 int ComputeFibonacci(int N) {
      printf("FIXME: Complete this function.\n");
 6
      printf("Currently just returns 0.\n");
 7
 8
 9
10
      return 0;
                 Ahram Kim
11 }
12
13 int main(void) {
14 int N = 4; // F_N, starts at 0 | Sestate ecu
15
      printf("F_%d is %d\n", N, ComputeFibonacci(N));
16
17
                 27th, 2017 18:15
18
19 }
20
Run
```

Greatest common divisor (GCD)

Recursion can solve the greatest common divisor problem. The **greatest common divisor** (GCD) is the largest number that divides evenly into two numbers, e.g. GCD(12, 8) = 4. One GCD algorithm (described by Euclid around 300 BC) subtracts the smaller number from the larger number until both numbers are equal. Ex:

- GCD(12, 8): Subtract 8 from 12, yielding 4.
- GCD(4, 8): Subtract 4 from 8, yielding 4.
- GCD(4, 4): Numbers are equal, return 4

The following recursively computes the GCD of two numbers. The base case is that the two numbers are equal, so that number is returned. The recursive case subtracts the smaller number from the larger number and then calls GCD with the new pair of numbers.

Figure 10.6.2: Calculate greatest common divisor of two numbers.

```
#include <stdio.h>
/* Determine the greatest common divisor
  of two numbers, e.g. GCD(8, 12) = 4
int GCDCalculator(int inNum1, int inNum2) {
   int gcdVal = 0; // Holds GCD results
   if(inNum1 == inNum2) {
                            // Base case: Numbers are equal
      gcdVal = inNum1;
                            // return value
  else {
                           // Recursive case: subtract smaller from larger
      if (inNum1 > inNum2) { // call fucntion with new values
     gcdVal = GCDCalculator(inNum1 - inNum2, inNum2);
}
     else {
    gcdVal = GCDCalculator(inNum1, inNum2 - inNum1);
               ig. 27th, 2017 18:15
  return gcdVal
}
int main(void) {
   int gcdInput1 = 0; // First input to GCD calc
   int gcdInput2 = 0; // Second input to GCD calc
   int gcdOutput = 0; // Result of GCD
   // Print program function
  printf("This program outputs the greatest \wn");
  printf("common divisor of two numbers.\wn");
   // Prompt user for input
  printf("Enter first number: ");
   scanf("%d", &gcdInput1);
  printf("Enter second number: ");
  scanf("%d", &gcdInput2);
   // Check user values are > 1, call recursive GCD function
   if ((gcdInput1 < 1) || (gcdInput2 < 1)) {</pre>
     printf("Note: Neither value can be below 1.\wn");
   }
  else {
     gcdOutput = GCDCalculator(gcdInput1, gcdInput2);
     printf("Greatest common divisor = %d\n", gcdOutput);
  return 0;
}
```

```
This program outputs the greatest common divisor of two numbers.
Enter first number: 12
Enter second number: 8
Greatest common divisor = 4
...

This program outputs the greatest common divisor of two numbers.
Enter first number: 456
Enter second number: 784
Greatest common divisor = 8
...

This program outputs the greatest common divisor of two numbers.
Enter first number: 0
Enter second number: 10
Note: Neither value can be below 1.
```

Ahram Kim AhramKim@u.boisestate.edu BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15

10.6.2: Recursive GCD example.

1) How many calls are made to GCDCalculator() function for input values 12 and 8?

1) 1

2 Ahram Kim
A 3

2) What is the base case for the GCD 5 3 F 112 0 17 algorithm?

1) When both inputs to the function are equal.

2) When both inputs are greater than
1.

Exploring further:

 Fibonacci sequence from wikipedia.com

O When inNum1 > inNum2.

• GCD algorithm from wikipedia.com

CHALLENGE ACTIVITY

10.6.1: Writing a recursive math function.

Write code to complete RaiseToPower(). Sample output if userBase is 4 and userExponent is 2 is shown below. Note: This example is for practicing recursion; a non-recursive function, or using the built-in function pow(), would be more common.

 $4^2 = 16$

BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15

```
1 #include <stdio.h>
2
3 int RaiseToPower(int baseVal, int exponentVal){
4   int resultVal = 0;
5
6   if (exponentVal == 0) {
7   resultVal = 1;
```

```
10
       resultVal = baseVal * /* Your solution goes here */;
11
12
     return resultVal;
13
14 }
15
  int main(void) {
16
17
     int userBase = 0;
     int userExponent = 0;
     userBase = 4; Ahram Kim
                   @u.boisestate.edu
                     ECS253Fall2017
```

Aug. 27th, 2017 18:15

10.7 Recursive exploration of all possibilities

Recursion is a powerful technique for exploring all possibilities, such as all possible reorderings of a word's letters, all possible subsets of items, all possible paths between cities, etc. This section provides several examples.

Word scramble

Consider printing all possible combinations (or "scramblings") of a word's letters. The letters of abc can be scrambled in 6 ways: abc, acb, bac, bca, cab, cba. Those possibilities can be listed by making three choices: Choose the first letter (a, b, or c), then choose the second letter, then choose the third letter. The choices can be depicted as a tree. Each level represents a choice. Each node in the tree shows the unchosen letters on the left, and the chosen letters on the right.

PARTICIPATION 10.7.1: Exploring all possibilities viewed as a tree of choices. **ACTIVITY** Ahram Kim AhramKim@u.boisestate.edu BOISESTATECS253Fall2017 abc/ □ bc/<u>a</u> ac/b ab/c Choose 2nd letter □ □ □ c/<u>ba</u> a/<u>bc</u> □ □ □ b/<u>ca</u> a/<u>cb</u> c/<u>ab</u> b/ac Choose 3rd letter /abc /<u>acb</u> /bac /bca /cab /cba

The tree guides creation of a recursive exploration function to print all possible combinations of a string's letters. The function takes two parameters: unchosen letters, and already chosen letters. The base case is no unchosen letters, causing printing of the chosen letters. The recursive case calls the function once for each letter in the unchosen letters. The above animation depicts how the recursive algorithm traverses the tree. The tree's leaves (the bottom nodes) are the base cases.

The following program prints all possible ordering of the letters of a user-entered word.

```
Aug. 27th, 2017 18:15
Figure 10.7.1: Scramble a word's letters in every possible way.
#include <stdio.h>
#include <string.h>
const int MAX_ARR_SIZE = 50;
                                               // Word size limit
void RemoveFromIndex(char* origString, int remLoc); // Remove letter at location i from string c
void InsertAtIndex(char* origString, char* addChar,
                 int addLoc);
                                               // Add letter n to location i of string c
/* Output every possible combination of a word.
  Each recursive call moves a letter from
  remainLetters" to scramLetters".
void ScrambleLetters(char* remainLetters, // Remaining letters
                   char* scramLetters) { // Scrambled letters
   char tmpString[2] = ""; // Using c string for access to strcat
   int i = 0;
                        // Loop index
   if(strlen(remainLetters) == 0) {
                                          // Base case: All letters used
     printf("%s\mun", scramLetters);
                                          // Recursive case: move a letter from
  else {
                                          // remaining to scrambled letters
     for (i = 0; i < strlen(remainLetters); ++i) {</pre>
                                                       Ahram Kim
        // Move letter to scrambled letters
        tmpString[0] = remainLetters[i];
        strcat(scramLetters, tmpString);
RemoveFromIndex(remainLetters, i);
                                                                   CS253Fall201
        ScrambleLetters(remainLetters, scramLetters);
        // Put letter back in remaining letters
        scramLetters[strlen(scramLetters)-1]='W0'; UO 27th, 2017 18:15
        InsertAtIndex(remainLetters, tmpString, i);
  return;
int main(void) {
  char wordScramble[50] = ""; // User defined word to scramble. 50 is MAX_ARR_SIZE
   char finishScramble[50] = ""; // Temp string already scrambled. 50 is MAX_ARR_SIZE
```

// Init strings

```
strcpy(wordScramble, "");
   strcpy(finishScramble, "");
   // Prompt user for input
   printf("Enter a word to be scrambled: ");
   scanf("%s", wordScramble);
   // Call recursive function
   ScrambleLetters(wordScramble, finishScramble);
   return 0;
                         Ahram Kir
// Remove letter at location remLoc from string origString
void RemoveFromIndex(char* origString, int remLoc) {
   char tmpString[50] = ""; // Temp string to extract char. 50 is MAX_ARR_SIZE
   strcpy(tmpString, "");
   strncat(tmpString, origString, remLoc); // Copy before location remLoc
   strncat(tmpString, origString + remLoc + 1,
strlen(origString) - remLoc); // Copy after location remLoc
   strcpy(origString, tmpString);
                                             // Copy back to orignal string
// Add letter addChar to location addLoc of string origString
void InsertAtIndex(char* origString, char* addChar, int addLoc) {
   char tmpString[50] = "";
                                 // Temp string to add char. 50 is MAX_ARR_SIZE
   strcpy(tmpString,"");
                                             // Init string
   strncat(tmpString, origString, addLoc); // Copy before location addLoc
   strncat(tmpString, addChar, 1);
                                             // Copy letter addChar to location addLoc
   strncat(tmpString, origString + addLoc,
           strlen(origString) - addLoc); // Copy after location addLoc
   strcpy(origString, tmpString);
Enter a word to be scrambled: cat
cta
act
atc
tca
tac
```

PARTICIPATION ACTIVITY

10.7.2: Letter scramble.

Ahram Kim 1) What is the output of ScrambleLetters("xy", "")? Determine ramKim@u.boisestate.edu vour answer by manually tracing the ESTATECS253Fall201 code, not by running the program. Aug. 27th, 2017 18:15

O yx xy

O xx yy xy yx

O xy yx

Shopping spree

Recursion can find all possible subsets of a set of items. Consider a shopping spree in which a person can select any 3-item subset from a larger set of items. The following program prints all possible 3-item subsets of a given larger set. The program also prints the total price of each subset.

ShoppingBagCombinations() has a parameter for the current bag contents, and a parameter for the remaining items from which to choose. The base case is that the current bag already has 3 items, which prints the items. The recursive case moves one of the remaining items to the bag, recursively calling the function, then moving the item back from the bag to the remaining items.

Figure 10.7.2: Shopping spree in which a user can fit 3 items in a shopping bag.

```
#include <stdio.h>
                                                                                      Milk
                                                                                                        = $45
                                                                                            Belt
                                                                                                  Toys
#include <stdlib.h>
                                                                                      Milk
                                                                                            Belt
                                                                                                        = $38
                                                                                                  Cups
#include <string.h>
                                                                                                        = $45
                                                                                      Milk
                                                                                            Toys
                                                                                                  Belt
#include <stdbool.h>
                                                                                      Milk
                                                                                            Toys
                                                                                                   Cups
                                                                                                        = $33
                                                                                      Milk
                                                                                            Cups
                                                                                                  Belt
                                                                                                        = $38
typedef struct | tem_struct {
                                                                                      Milk
                                                                                            Cups
                                                                                                   Toys
                                                                                                        = $33
   char itemName [50]; // Name of item
                                                                                                   Toys
                                                                                                        = $45
                                                                                      Belt
                                                                                            Milk
   int priceDollars; // Price of item
                                                                                                   Cups
                                                                                                        = $38
                                                                                      Belt
                                                                                            Milk
} Item;
                                                                                      Belt
                                                                                                   Milk
                                                                                                        = $45
                                                                                             Toys
                                                                                      Belt
                                                                                            Toys
                                                                                                        = $55
                                                                                                   Cups
const int TOTAL_ITEMS = 4;
                                              // Total number of items available
                                                                                      Belt
                                                                                            Cups
                                                                                                   Milk
                                                                                                        = $38
const unsigned int MAX_SHOPPING_BAG_SIZE = 3; // Max number of items in shopping bag
                                                                                      Belt
                                                                                            Cups
                                                                                                   Toys
                                                                                                        = $55
                                                                                      Toys
                                                                                            Milk
                                                                                                   Belt
                                                                                                        = $45
/* Output every possible combination of items that
                                                                                       Toys
                                                                                            Milk
                                                                                                   Cups
                                                                                                        = $33
   fit in a shopping bag. Each recursive call moves
                                                                                       Toys
                                                                                            Belt
                                                                                                   Milk
                                                                                                        = $45
   one item into the bag.
                                                                                       Toys
                                                                                            Belt
                                                                                                   Cups
                                                                                                        = $55
                                                                                                        = $33
                                                                                       Toys
                                                                                            Cups
                                                                                                   Milk
void ShoppingBagCombinations(Item* currBag.
                                                   // Bag contents
                                                                                                        = $55
                                                                                       Toys
                                                                                            Cups
                                                                                                   Belt
                             Item* remainingItems, // Available items
                                                                                                        = $38
                                                                                       Cups
                                                                                            Milk
                                                                                                   Belt
                             bool* beenAdded,
                                                   // Items already in shopping bag
                                                                                                        = $33
                                                                                       Cups
                                                                                            Milk
                                                                                                   Toys
                                                   // Current shopping bag size
                             int bagCnt) {
                                                                                       Cups
                                                                                            Belt
                                                                                                   Milk
                                                                                                        = $38
   int bagValue = 0; // Cost of items in shopping bag
                                                                                                  Toys
                                                                                                        = $55
                                                                                       Cups
                                                                                            Belt
                    // Loop index
   int i = 0;
                                                                                                  Milk
                                                                                                        = $33
                                                                                       Cups
                                                                                            Toys
                                                                                            Toys
                                                                                                  Belt
                                                                                                        = $55
                                                                                       Cups
   if (bagCnt = MAX_SHOPPING_BAG_SIZE) { // Base case: Shopping bag full
     bagValue = 0;
      for(i = 0; i < bagCnt; ++i) {</pre>
        printf("%s ", currBag[i].itemName);
        bagValue += currBag[i].priceDollars;
     printf("= $%d\m", bagValue);
                                          // Recursive case: move one
   else {
      for (i = 0; i < TOTAL_ITEMS; ++i) { // item to bag</pre>
         if (!beenAdded[i]) {
                                              amKim@u.boisestate.edu
            // Move item to bag
           beenAdded[i] = true;
           currBag[bagCnt] = remainingItems[i];
           ShoppingBagCombinations(currBag, remainingItems,
                                   beenAdded, bagCnt + 1);
            // Take item out of bag
           beenAdded[i] = false;
   return;
int main(void) {
```

```
item* possibleitems = NULL, // Possible snopping items
Item* shoppingBag = NULL; // Current shopping bag
bool* itemBeenAdded = NULL; // Track if item already in bag
                           // Temp item
Item tmpGroceryItem;
possibleItems = (Item*)malloc(sizeof(Item) * TOTAL_ITEMS);
shoppingBag = (Item*)malloc(sizeof(Item) * TOTAL_ITEMS);
itemBeenAdded = (bool*)malloc(sizeof(bool) * TOTAL_ITEMS);
// No items added yet
itemBeenAdded[0] = false;
itemBeenAdded[1] = false;
itemBeenAdded[2] = false:
itemBeenAdded[3] = false;
// Populate grocery with different items sestate.edu
strcpy(tmpGroceryItem.itemName, "Milk");
tmpGroceryItem.priceDollars = 2;
possibleItems[0] = tmpGroceryItem;
strcpy(tmpGroceryItem.itemName, "Belt");
tmpGroceryItem.priceDollars = 24;
possibleItems[1] = tmpGroceryItem;
strcpy(tmpGroceryItem.itemName, "Toys");
tmpGroceryItem.priceDollars = 19;
possibleItems[2] = tmpGroceryItem;
strcpy(tmpGroceryItem.itemName, "Cups");
tmpGroceryItem.priceDollars = 12;
possibleItems[3] = tmpGroceryItem;
// Try different combinations of three items
ShoppingBagCombinations(shoppingBag, possibleItems,
                        itemBeenAdded, 0);
return 0;
```

PARTICIPATION

10.7.3. All letter combinations

ACTIVITY	10.7.0.7 th letter combinations.	_
	agCombinations(), how are in the remainingItems Ahram Kim	
O None	AhramKim@u.boisestate.edu	
O 3	BOISESTATECS253Fall2017	
O 4	Aug. 27th, 2017 18:15	
	() calls agCombinations(), how are in currBag list?	

4		
3) After main() calls ShoppingBagCombinations(), what happens first?		
O The base case prints Milk, Belt, Toys.		
O The function bags one item,		
An makes recursive call. Looisesta	ate.edu	
O The function bags 3 items, makes recursive call.	12017	
4) After ShoppingBagCombinations()	3:15	
returns back to main(), how many items are in the remainingItems list?		
O None		
O 4		
5) How many recursive calls occur before the first combination is printed?		
O None		
O 1		
O 3		
6) What happens if main() only put 2, rather than 4, items in the possibleItems list?		
O Base case never executes; nothing printed.		
O Infinite recursion occurs.	Ahram Kim	

Traveling salesman

AhramKim@u.boisestate.edu BOISESTATECS253Fall2017

Recursion is useful for finding all possible paths. Suppose a salesman must travel to 3 cities: Boston, Chicago, and Los Angeles. The salesman wants to know all possible paths among those three cities, starting from any city. A recursive exploration of all travel paths can be used. The base case is that the salesman has traveled to all cities. The recursive case is to travel to a new city, explore possibilities, then return to the previous city.

Figure 10.7.3: Find distance of traveling to 3 cities.

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <stdbool.h>
const int NUM_CITIES = 3; // Number of cities
int cityDistances[3][3]; // Distance between cities
char cityNames[3][50];
                         // City names
/* Output every possible travel path.
   Each recursive call moves to a new city.
void TravelPaths(int* currPath, int* toVisit,
  bool* haveBeen, int cityCnt) { estate edu
   int totalDist = 0; // Total distance given current path
int i = 0; // Loop index
   if (cityCnt == NUM_CITIES) { // Base case: Visited all cities
      totalDist = 0; // return total path distance
      for (i = 0; i < cityCnt; ++i) {
                    ", cityNames[currPath[i]]);
         printf("%s
         if (i > 0) {
            totalDist += cityDistances[currPath[i - 1]][currPath[i]];
      printf("= %d\n", totalDist);
   else {
                                // Recursive case: pick next city
      for (i = 0; i < NUM\_CITIES; ++i) {
         if (!haveBeen[i]) {
            // Add city to travel path
            haveBeen[i] = true;
            currPath[cityCnt] = toVisit[i];
            TravelPaths(currPath, toVisit, haveBeen, cityCnt+1);
            // Remove city from travel path
                                                                                                         = 2971
                                                                        Boston
                                                                                 Chicago
                                                                                          Los Angeles
           haveBeen[i] = false;
                                                                        Boston
                                                                                Los Angeles
                                                                                              Chicago
                                                                                                         = 4971
                                                                        Chicago
                                                                                                         = 3920
                                                                                 Boston
                                                                                         Los Angeles
                                                                                                         = 4971
                                                                        Chicago
                                                                                 Los Angeles
                                                                                                Boston
                                                                        Los Angeles
                                                                                                         = 3920
                                                                                      Boston
                                                                                               Chicago
                                                                        Los Angeles
                                                                                                Boston
                                                                                                         = 2971
                                                                                      Chicago
   return;
int main(void) {
   int* currPath = NULL; // Current path traveled
                                                            Ahram Kim
   int* toVisit = NULL; // Cities left to visit
bool* haveBeen = NULL; // City already visited
                                                               n@u.boisestate.edu
   // Initialize distances array
   cityDistances[0][0] = 0;
                                                                              253Fall2017
   cityDistances[0][1] = 960; // Boston-Chicago
   cityDistances[0][2] = 2960; // Boston-Los Angeles
   cityDistances[1][0] = 960; // Chicago-Boston
                                                                    h. 2017 18:15
   cityDistances[1][1] = 0;
   cityDistances[1][2] = 2011; // Chicago-Los Angeles
   cityDistances[2][0] = 2960; // Los Angeles-Boston
   cityDistances[2][1] = 2011; // Los Angeles-Chicago
   cityDistances[2][2] = 0;
   strcpy(cityNames[0], "Boston");
strcpy(cityNames[1], "Chicago");
strcpy(cityNames[2], "Los Angeles");
   currPath = (int*)malloc(sizeof(int) * NUM_CITIES);
   talliait - (int+)mallag(aizaaf(int) + NUM CITICO)
```

```
haveBeen = (bool*)malloc(sizeof(bool) * NUM_CITIES);

toVisit[0] = 0;
toVisit[1] = 1;
toVisit[2] = 2;

haveBeen[0] = false;
haveBeen[1] = false;
haveBeen[2] = false;

// Explore different paths
TravelPaths(currPath, toVisit, haveBeen, 0);

return 0;

BOISESTATECS253Fall2017
```

PARTICIPATION ACTIVITY

10.7.4: Recursive exploration.

- 1) You wish to generate all possible 3letter subsets from the letters in an Nletter word (N>3). Which of the above recursive functions is the closest?
 - O ShoppingBagCombinations
 - O ScrambleLetters
 - O main()

Exploring further:

 Recursion trees from wikipedia.org

Ahram Kim

10.8 Stack overflowhramKim@u.boisestate.edu BOISESTATECS253Fall2017

Recursion enables an elegant solution to some problems. But, for large problems, deep recursion can cause memory problems. Part of a program's memory is reserved to support function calls. Each function call places a new **stack frame** on the stack, for local parameters, local variables, and more function items. Upon return, the frame is deleted.

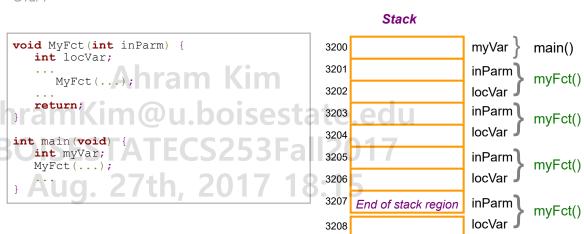
Deep recursion could fill the stack region and cause a **stack overflow**, meaning a stack frame extends beyond the memory region allocated for stack, Stack overflow usually causes the program to crash and report an error like: segmentation fault, access violation, or bad access.

PARTICIPATION ACTIVITY

Start

void in ...

10.8.1: Recursion causing stack overflow.



Deep recursion may cause stack overflow, causing program to crash

The animation showed a tiny stack region for easy illustration of stack overflow.

The size of parameters and local variables results in a larger stack frame. Large vectors, arrays, or strings declared as local variables, or passed by copy, can lead to faster stack overflow.

A programmer can estimate recursion depth and stack size to determine whether stack overflow might occur. Sometime a non-recursive algorithm must be developed to avoid stack overflow.

PARTICIPATION ACTIVITY	10.8.2: Stack overflow.
most one s	s stack region can store at tack frame. Ahram Kim
O True O False	AhramKim@u.boisestate.edu
2) The size of O True O False	the stack is unlimited. BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15
frame for a	rflow occurs when the stack function call extends past he stack's memory.
O True	

O False

4) The following recursive function will result in a stack overflow.

int RecAdder(int inValue) {
 return RecAdder(inValue + 1);
}

O True
O False
Ahram Kim @u.boisestate.edu
BOISESTATECS253Fall2017

10.9 C example: Recursively output permutations

PARTICIPATION ACTIVITY

10.9.1: Recursively output permutations.

The below program prints all permutations of an input string of letters, one permutation per line. Ex: The six permutations of "cab" are:

cab

cba

acb

abc

bca

bac

Below, the PermuteString function works recursively by starting with the first character and permuting the remainder of the string. The function then moves to the second character and permutes the string consisting of the first character and the third through the end of the string, and so on.

- 1. Run the program and input the string "cab" (without quotes) to see that the above output is produced.
- 2. Modify the program to print the permutations in the opposite order, and also to output a permutation count on each line.
- 3. Run the program again and input the string cab. Check that the output is reversed.
- 4. Run the program again with an input string of abcdefg. Why did the program take longer to produce the results?
- 1 #include <stdio.h>

```
2 #include <string.h>
   4 // FIXME: Use a static variable to count permutations. Why must it be static?
   6 void RemoveFromIndex(char* origString, int remLoc);
     void InsertAtIndex(char* origString, char* addChar, int addLoc);
   7
     void PermuteString(char* remainLetters, char* permutedLetters) {
   9
        char tmpString[2] = "";
  10
        int i = 0;
  11
  12
        tmpString[1] = '\0';
  13
        if (strlen(remainLetters) == 0) {
                                                    // Base case: All letters used
  14
  15
           // FIXME: add count for each permutation
         printf("%s\n", permutedLetters);
 16
  17
                                                    // Recursive case: move a letter from
  18
                                                    // remaining to permuted letters
  19
              FIXME: Change loop to output permutations in reverse order
  20
                    <del>Z/U1, ZU1/ 18:1</del>:
cab
```

Run

PARTICIPATION ACTIVITY

10.9.2: Recursively output permutations (solution).

Below is the solution to the above problem.

```
1 #include <stdio.h>
   #include <string.h>
                                                           // For counting permutations
   static int permutationCount = 0;
   void RemoveFromIndex(char* origString, int remLoc);
void InsertAtIndex(char* origString, char* addChar, int addLoc);
7
8
9 void PermuteString(char* remainLetters,
                                                 // Remaining letters
                        char* permutedLetters) { // Permuted letters
10
       char tmpString[2] = "";
11
12
       int i = 0;
13
       tmpString[1] = '\0';
14
       if (strlen(remainLetters) == 0) {
                                                       // Base case: All letters used
15
                                                       // Counting permutations
16
          ++permutationCount;
17
          printf("%d) %s\n", permutationCount, permutedLetters);
18
       }
19
       else {
                                                       // Recursive case: move a letter from
20
                                                       // remaining to permuted letters
          for (i = (strlen(remainLetters) - 1); i >= 0; --i) {
```

	- ,	
	cab abcdefg	
	Run	
	Ahram Kim	
	AhramKim@u.boisestate.edu	l
4	BOISESTATECS253Fall2017	>
	Aug. 27th, 2017 18:15	

Ahram Kim AhramKim@u.boisestate.edu BOISESTATECS253Fall2017 Aug. 27th, 2017 18:15