

Chapter 1 Exercises

Processor	Clock Rate (GHz)	CPI
P1	3	1.5
P2	2.5	1.0
P3	4.0	2.2

Problem 1.1

1.5 <§1.6> Consider three different processors P1, P2, and P3 executing the same instruction set. P1 has a 3 GHz clock rate and a CPI of 1.5. P2 has a 2.5 GHz clock rate and a CPI of 1.0. P3 has a 4.0 GHz clock rate and has a CPI of 2.2.

a. Which processor has the highest performance expressed in instructions per second?

$$\hookrightarrow \left(\frac{\text{Clock Rate}}{\text{CPI}} \right)$$

$$P1: 3 \text{ GHz} / 1.5 = 2 \times 10^9 \text{ (Instructions/sec)}$$

$$P2: 2.5 \text{ GHz} / 1.0 = 2.5 \times 10^9 \text{ (Instructions/sec)}$$

$$P3: 4.0 \text{ GHz} / 2.2 = 1.82 \times 10^9 \text{ (Instructions/sec)}$$

b. If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.

* Number of cycles

$$P1: 3 \text{ GHz} \times 10 = 3 \times 10^{10}$$

$$P2: 2.5 \text{ GHz} \times 10 = 2.5 \times 10^{10}$$

$$P3: 4 \text{ GHz} \times 10 = 4 \times 10^{10}$$

* Number of instructions

$$P1: \frac{(3 \text{ GHz} \times 10)}{1.5} = 2 \times 10^{10}$$

$$P2: \frac{(2.5 \text{ GHz} \times 10)}{1.0} = 2.5 \times 10^{10}$$

$$P3: \frac{(4 \text{ GHz} \times 10)}{2.2} = 1.82 \times 10^{10}$$

$$\left(\frac{\text{number of instructions} \times \text{CPI}}{\text{Clock Rate}} \right)$$

c. We are trying to reduce the execution time by 30%, but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

* Reduce 30%, increase CPI 20%

$$\text{Execution time} \times 0.7 = (\text{number of instruction} \times \text{CPI} \times 1.2) / (\text{new clock rate})$$

$$\text{New Clock Rate} = (\text{clock rate} \times 1.2) / 0.7 = 1.71 \times \text{clock rate}$$

$$P1: 3 \text{ GHz} \times 1.71 = 5.13 \text{ GHz}$$

$$P2: 2.5 \text{ GHz} \times 1.71 = 4.27 \text{ GHz}$$

$$P3: 4 \text{ GHz} \times 1.71 = 6.84 \text{ GHz}$$

Problem 1.2

1.7 [15] <§1.6> Compilers can have a profound impact on the performance of an application. Assume that for a program, compiler A results in a dynamic instruction count of $1.0E9$ and has an execution time of 1.1 s, while compiler B results in a dynamic instruction count of $1.2E9$ and an execution time of 1.5 s.

a. Find the average CPI for each program given that the processor has a clock cycle time of 1 ns.

$$\text{CPI} = \frac{\text{CPU Time}}{(\text{Instruction count} \times \text{Clock cycle time})}$$

$$\text{CPI of compiler A} = \frac{1.1}{1.0E9 \times 1.0E-9} = 1.1$$

$$\text{CPI of compiler B} = \frac{1.5}{1.2E9 \times 1.0E-9} = 1.25$$

b. Assume the compiled programs run on two different processors. If the execution times on the two processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?

$$\begin{aligned} \text{Clock Rate (freq)} &= \frac{1}{\text{Clock cycle Time}} \\ &= \frac{1}{\frac{\text{Instruction count} \times \text{CPI}}{\text{CPU Time}}} \end{aligned}$$

$$\frac{f_B}{f_A} = \frac{(\text{Instr count}(B) \times \text{CPI}(B))}{(\text{Instr count}(A) \times \text{CPI}(A))} = \frac{(1.25 \times 1.2 \times 10^9)}{(1.1 \times 1.0 \times 10^9)} = 1.36$$

$$\text{cycle time compiler A} = 1.36 \times \text{cycle time compiler B}$$

c. A new compiler is developed that uses only $6.0E8$ instructions and has an average CPI of 1.1 . What is the speedup of using this new compiler versus using compiler A or B on the original processor?

$$\text{CPU Time} = (\text{Instruction count} \times \text{CPI} \times \text{Clock cycle time})$$

$$\frac{(\text{CPU Time})_A}{(\text{CPU Time})_{\text{new}}} = \frac{(1.0E9 \times 1.1 \times 1.0E-9)}{(6.0E8 \times 1.1 \times 1.0E-9)} = 1.67$$

$$\frac{(\text{CPU Time})_B}{(\text{CPU Time})_{\text{new}}} = \frac{(1.2E9 \times 1.25 \times 1.0E-9)}{(6.0E8 \times 1.1 \times 1.0E-9)} = 2.27$$

Problem 1.3 • Die Area = $\frac{\text{wafer area } (\pi r^2)}{\text{dies per wafer}}$, • Yield = $\frac{1}{(1 + (\text{defect per area} \times \frac{\text{die area}}{2}))^2}$

1.10 Assume a 15 cm diameter wafer has a cost of 12, contains 84 dies, and has 0.020 defects/cm². Assume a 20 cm diameter wafer has a cost of 15, contains 100 dies, and has 0.031 defects/cm².

1.10.1 [10] <§1.5> Find the yield for both wafers.

* A wafer

$$\text{Die Area} = \frac{3.14 \times (\frac{15}{2})^2}{84} = 2.1$$

$$\text{Yield} = \frac{1}{(1 + (0.020 \times \frac{2.1}{2}))^2} = \frac{1}{1.04244} = 0.96$$

* B wafer

$$\text{Die Area} = \frac{3.14 \times (\frac{20}{2})^2}{100} = 3.14$$

$$\text{Yield} = \frac{1}{(1 + (0.031 \times \frac{3.14}{2}))^2} = \frac{1}{1.0997} = 0.909$$

1.10.2 [5] <§1.5> Find the cost per die for both wafers.

• Cost per die = $\frac{\text{cost per wafer}}{\text{Dies per wafer} \times \text{yield}}$, Dies per wafer $\approx \frac{\text{Wafer area}}{\text{Die area}}$

$$\text{A cost per die} = \frac{12}{84 \times 0.96} = 0.1485 \approx 0.15$$

$$\text{B cost per die} = \frac{15}{100 \times 0.909} = 0.1650 \approx 0.17$$

1.10.3 [5] <§1.5> If the number of dies per wafer is increased by 10% and the defects per area unit increases by 15%, find the die area and yield.

	A wafer	B wafer
Dies per wafer	$84 + (84 \times \frac{10}{100}) = 92.4$	$100 + (100 \times \frac{10}{100}) = 110$
Wafer area	$\pi \times (\frac{15}{2})^2 = 176.625$	$\pi \times (\frac{20}{2})^2 = 314.159$
Dies area	$176.625 / 92.4 = 1.912$	$314.159 / 110 = 2.856$
Defects per area	$0.02 + (0.02 \times \frac{15}{100}) = 0.023$	$0.031 + (0.031 \times \frac{15}{100}) = 0.0356$
Yield	$\frac{1}{(1 + (0.023 \times \frac{1.912}{2}))^2} = 0.9574$	$\frac{1}{(1 + (0.0356 \times \frac{2.856}{2}))^2} = 0.9056$

1.10.4 [5] <§1.5> Assume a fabrication process improves the yield from 0.92 to 0.95. Find the defects per area unit for each version of the technology given a die area of 200 mm².

$$\hookrightarrow \left(\sqrt{\frac{1}{\text{Yield}}} - 1 \right) \times \frac{2}{\text{die area}}$$

$$\text{Yield } 0.92 = \left(\sqrt{\frac{1}{0.92}} - 1 \right) \times \frac{2}{200} = 0.00042 = 0.042 \text{ defects/cm}^2$$

$$\text{Yield } 0.95 = \left(\sqrt{\frac{1}{0.95}} - 1 \right) \times \frac{2}{200} = 0.00025 = 0.025 \text{ defects/cm}^2$$

Problem 1.4

1.14 Assume a program requires the execution of 50×10^6 FP instructions, 110×10^6 INT instructions, 80×10^6 L/S instructions, and 16×10^6 branch instructions. The CPI for each type of instruction is 1, 1, 4, and 2, respectively. Assume that the processor has a 2 GHz clock rate.

1.14.1 [10] <§1.10> By how much must we improve the CPI of FP instructions if we want the program to run two times faster?

$$\begin{aligned} \text{Clock cycle} &= (\text{CPI}_{\text{FP}} \times \text{InstCount}_{\text{FP}}) + \dots + (\text{CPI}_{\text{branch}} \times \text{InstCount}_{\text{branch}}) \\ &= (50 \times 10^6 \times 1) + (110 \times 10^6 \times 1) + (80 \times 10^6 \times 4) + (16 \times 10^6 \times 2) \\ &= 512 \times 10^6 \end{aligned}$$

$$\text{Execution time} = \Sigma (\text{clock cycle}) / (\text{clock rate})$$

$$\bullet \text{ Initial execution time for FP} = (512 \times 10^6) / (2 \times 10^9) = 256 \times 10^{-3} \text{ sec}$$

$$\begin{aligned} \text{CPI}_{\text{FP}} &= \frac{512 \times 10^6}{2} - [(110 \times 10^6 \times 1) + (80 \times 10^6 \times 4) + (16 \times 10^6 \times 2)] \\ &= \frac{(256 \times 10^6) - (462 \times 10^6)}{50 \times 10^6} = \frac{-206 \times 10^6}{50 \times 10^6} = -4.12 \quad \text{cannot improve CPI of FP instructions} \end{aligned}$$

1.14.2 [10] <§1.10> By how much must we improve the CPI of L/S instructions if we want the program to run two times faster?

$$\begin{aligned} \text{CPI}_{\text{L/S}} &= \frac{512 \times 10^6}{2} - [(50 \times 10^6 \times 1) + (110 \times 10^6 \times 1) + (16 \times 10^6 \times 2)] \\ &= \frac{(256 \times 10^6) - (192 \times 10^6)}{80 \times 10^6} = \frac{64 \times 10^6}{80 \times 10^6} = 0.8 \end{aligned}$$

$$\therefore \text{We must improve the CPI of L/S instruction} = \frac{4}{0.8} = 5$$

CPI of L/S instructions must improve by 5 times

1.14.3 [5] <§1.10> By how much is the execution time of the program improved if the CPI of INT and FP instructions is reduced by 40% and the CPI of L/S and Branch is reduced by 30%?

$$\begin{aligned} \text{CPI}_{\text{FP}} &= 1 - 1 \times 0.4 = 0.6 \\ \text{CPI}_{\text{INT}} &= 1 - 1 \times 0.4 = 0.6 \\ \text{CPI}_{\text{L/S}} &= 4 - 4 \times 0.3 = 2.8 \\ \text{CPI}_{\text{branch}} &= 2 - 2 \times 0.3 = 1.4 \end{aligned} \quad \left. \begin{array}{l} \bullet \text{ Clock cycle} = (50 \times 10^6 \times 0.6) + (110 \times 10^6 \times 0.6) \\ \quad \quad \quad + (80 \times 10^6 \times 2.8) + (16 \times 10^6 \times 1.4) \\ \quad \quad \quad = 342.4 \times 10^6 \\ \bullet \text{ Execution Time} = \frac{342.4 \times 10^6}{2 \times 10^9} = 171.2 \times 10^{-3} \text{ sec} \end{array} \right\}$$