CS 421 Algorithms (Summer 2018)

Homework #1 (70 points), Due Date: 5/22/2018 (Tuesday)

- **Q1(10 points): Asymptotic Notations**

  (a)(3 points) Which one of the following is first wrong statement?

  1. $\Theta(n) + O(n) = \Theta(n)$
  2. $\Theta(n) + O(n) = O(n)$
  3. $\Theta(n) + \Omega(n) = \Theta(n)$
  4. $f(n) = o(g(n))$ implies $g(n) = \Omega(f(n))$

  (b)(7 points) Try to use the basic definition of Θ-notation to show $n^2 - 10\, n \log_2 n = \Theta(n^2)$.

- **Q2(12 points): Divide-and-Conquer**

Suppose that a computer does not know how to apply dynamic programming techniques to compute a function $f(n)$, but it knows how to use the `divide and Conquer` approach to compute $f(n)$ as follows. The computer takes only constant time for scalar arithmetic operations.

$$f(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ f(n-1) + f(n-2) + n \log n & \text{if } n > 1 \end{cases}$$

(a)(8 points) Please write down the three steps of `Divide`, `Conquer` and `Combine` to describe how the computer calculates $f(n)$.

`Divide: Do nothing.`

`Conquer:`

`Combine:`

(b)(4 points) Please write down the running time recurrence if $f(n)$ is computed using the above approach.

- **Q3(24 points): Recurrences**

  (a)(8 points) Given a recurrence $T(n) = 3T(n-1) + 1$, please draw the recursion tree and derive a tight bound of $T(n)$.

(b)(8 points) Given a recurrence $T(n) = 2T(n-1) + n$, please use the substitution method to verify $T(n) = O(2^n)$.
`Hint: use the hypothesis` $T(n) \leq c(2^n - n)$ `for some` $c > 0$.

(c)(8 points) Please solve the recurrence $T(n) = 2T(n-1) + n^2$ using the Master Method.
`Hint: try to transfer the equation to another form and then solve it.`

- **Q4(24 points): Dynamic programming**

(a)(9 points) For a `Matrix-Chain` problem with 4 matrices A_1, A_2, A_3 and A_4, please construct and draw the two tables as in the book if the dimension vector for these four matrices is $< 3, 1, 5, 4, 2 >$.

(b)(3 points) Based on the tables in (a), what is the optimal parenthesization for the product $A_1 A_2 A_3 A_4$?

(c)(9 points) For a LCS (longest common subsequence) problem with two input sequences $X =< C, A, A, B, B, D, C >$ and $Y =< C, B, A, D, B, B, C >$, please draw the table(s) as in the book.

(d)(3 points) Based on the table(s) in (c), what is the longest common subsequence for X and Y ?