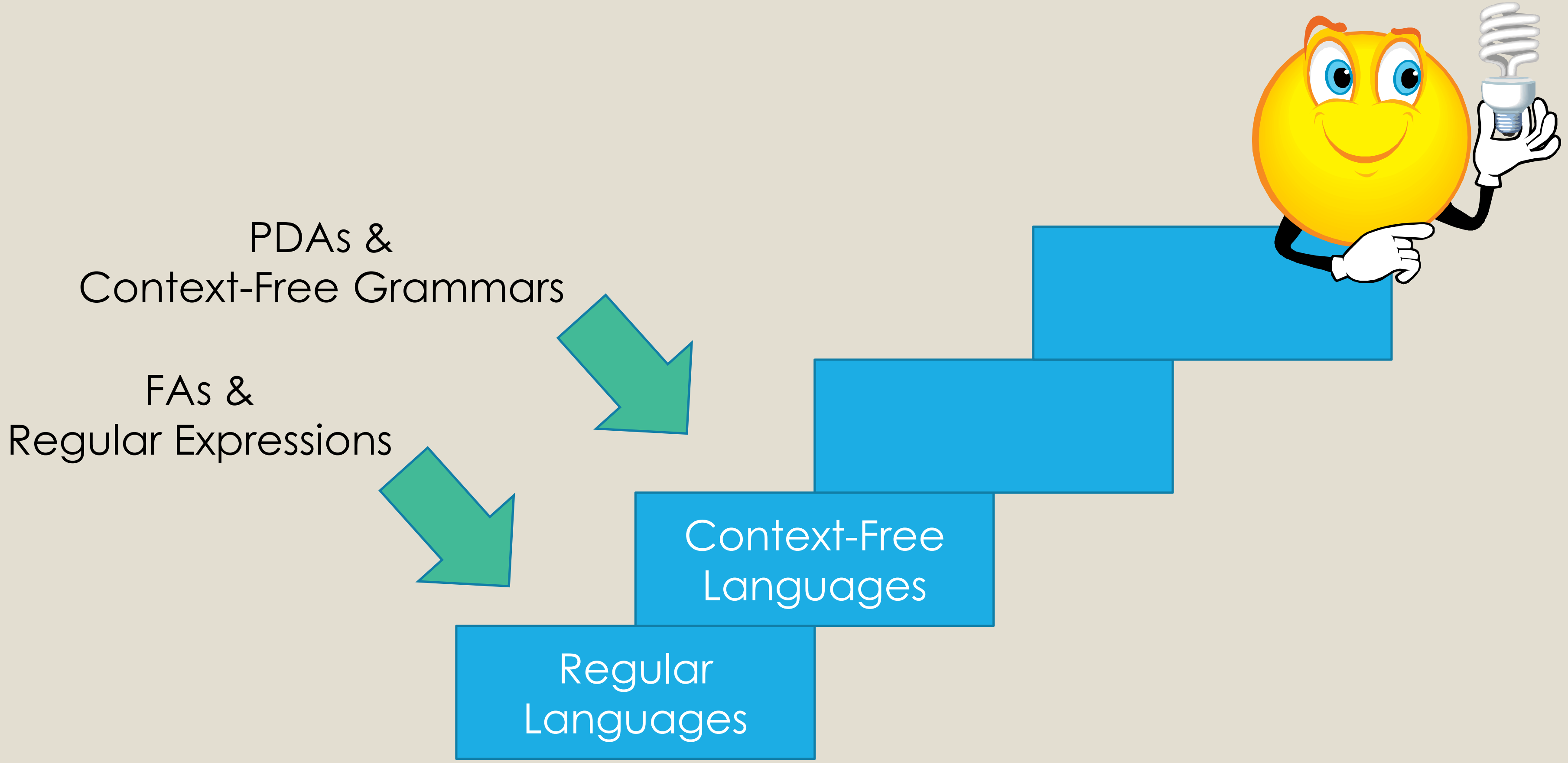
MIDTERM 2 REVIEW

Chapter 2


PDAs &
Context-Free Grammars
FAs &
Regular Expressions
Context-Free
Languages
Regular
Languages

Topics

- **Context-Free Languages**
  - Context-Free Grammar
  - PDA
  - Equivalence CFG and PDA
  - Pumping lemma for context free languages (Theorem 2.34 and lecture notes)
- **Vocabulary**
  - CFG: definition, terminal, variables, production rules, derivation, ambiguity
  - PDA: definition, tracing strings, acceptance conditions
  - PDAs & State Machines
  - Regular vs Context Free Languages

Recognizing Languges

- Regular but not CF (R) , Context Free (C) , Non context free (N)

No.	Language	Type
1	$\{a^i b^j c^k d^l \mid (i < j) \text{ or } (k < l)\}$	C
2	$\{a^i b^j c^k d^l \mid (i < j) \text{ and } (k < l)\}$	C
3	$\{a^i b^j c^k d^l \mid (i \neq j) \text{ and } (k \neq l)\}$	C
4	$\{a^i b^j c^k d^l \mid (i = k) \text{ and } (j = l) \}$	N
5	$\{a^i b^j c^k d^l \mid (i = j) < 4 \text{ and } (k = l) < 2 \}$	R
6	$\{a^i b^j c^k d^l \mid (i + j) < (k + l)\}$	C

CFG

- Provide a CFG for the following languages
  - $A=\{0^a1^a0^b \mid a,b > 0\}$

    $S \rightarrow AB$

    $A \rightarrow 0A1 \mid 01$

    $B \rightarrow 0B \mid 0$

  - $B=\{0^a1^b0^b \mid a > 0,\ b \geq 0\}$

    $S \rightarrow 0AB$

    $A \rightarrow 0A \mid \epsilon$

    $B \rightarrow 0B1 \mid \epsilon$

CFG

- Describe the language of the following grammar G, using set notation

S-> aSb | aaXb

X->YZ | cXdd

Y->aY | a

Z->bZ | ϵ

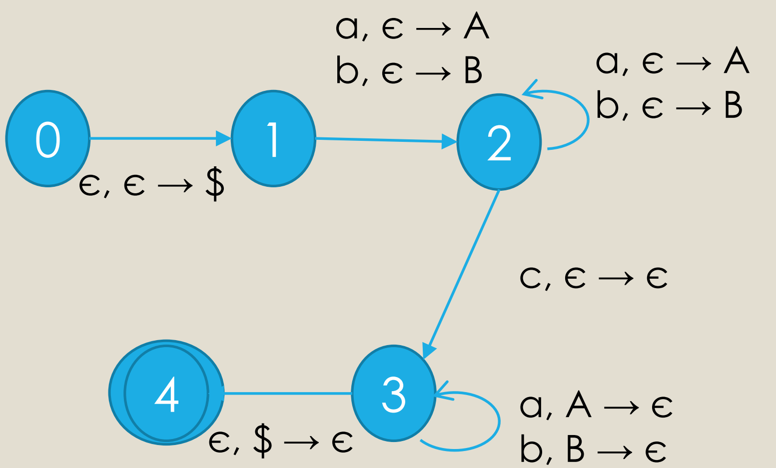
$$L(G) = \{a^{n+1}\, c^k a^m b^p\, d^{2k} b^n \mid n, m \geq 1; k, p \geq 0\}$$

PDAs & Pumping Lemma

- Consider the following languages. If content free, provide the PDA, if not use the pumping lemma to prove it
  - A={wcw^R| w is a string over {a, b} and |w| >0}
  - B={ww^Rw| w is a string over {0, 1} }

PDA

- Consider the PDA for A={wcw^R | w is a string over {a, b} and |w| >0}
  - Do any of the following strings are accepted by the PDA? (Trace, i.e., derive, each string to show accept/reject)
    - abcab
    - bacab
    - c



[0,abcab, ϵ]
[1,abcab, $]
[2,bcab, A$]
[2,cab, BA$]
[3,ab, BA$]
Reject

[0, bacab, ϵ]
[1, bacab, $]
[2, acab, B$]
[2, cab, AB$]
[3,ab, AB$]
[3,b, B$]
[3, ϵ, $]
[4, ϵ, ϵ]
Accept

[0,c, ϵ]
[1,c, $]
Reject

Pumping Lemma

Proof: For the sake of contradiction, assume that the language $L = \{ww^Rw \mid w \in \{0, 1\}^*\}$ is context-free. The Pumping Lemma must then apply; let **k** be the pumping length. Consider the string $s= 0^k1^k1^k0^k0^k1^k$

Since $|s| \geq k$, it must be possible to split s into five pieces **uvxyz** satisfying the conditions of the Pumping Lemma. The substrings v and y must collectively contain some symbols since $|vy| > 0$. We consider the following exhaustive cases.

Case 1: The substrings v and/or y contain some symbols from the first block of k 0s. Since $|vxy| \leq k$, v and y cannot contain any 0s from the second block of 2k 0s. Consider the string $uv^0xy^0z = uxz$. The string uxz must be of the form $0^i\, 1^j\, 0^{2k}1^k$ where $i < k$ and $j \leq 2k$. If uxz $\in L$, it must be of the form ww^Rw. Since uxz is of the form $0^i\, 1^j\, 0^{2k}1^k$ and of length at least 5k, the first w must begin with the block of $i < k$ 0s followed by some number of 1s. Thus, w^Rw must contain a block of at most $2i < 2k$ 0s. But uxz contains a block of 2^k 0s, a contradiction.

Pumping Lemma

Case 2: The substrings v and/or y contain some symbols from the first block of 2k 1s. Since $|vxy| \le k$, v and y cannot contain any 1s from the second block of k 1s. Consider the string $uv^0xy^0z = uxz$. The string uxz must be of the form $0^i\ 1^j\ 0^l\ 1^k$ where $i \le k$, $j < 2k$, and $l \le 2k$. If $uxz \in L$, it must be of the form ww^Rw. Since uxz is of the form $0^i\ 1^j\ 0^l\ 1^k$ and of length at least 5k, the last w must end with the block of k 1s preceded by some number of 0s. Thus, ww^R must contain a block of 2k 1s. But uxz contains a block of $j < 2k$ 1s, a contradiction.

Case 3:The substrings v and/or y contain some symbols from the second block of 2k 0s. Since $|vxy| \le k$, v and y cannot contain any 0s from the first block of k 0s. Consider the string uv0xy0 z = uxz. The string uxz must be of the form $0^k\ 1^i\ 0^j\ 1^l$ where $i \le 2k$, $j < 2k$, and $l \le k$. If $uxz \in L$, it must be of the form ww^Rw. Since uxz is of the form $0^k\ 1^i\ 0^j\ 1^l$ and of length at least 5k, the first w must begin with the block of k 0s followed by some number of 1s. Thus, w^Rw must contain a block of 2k 0s. But uxz contains a block of $j < 2k$ 0s, a contradiction.

Pumping Lemma

Case 4: The substrings v and/or y contain some symbols from the second block of k 1s. Since $|vxy| \le k$, v and y cannot contain any 1s from the first block of 2k 1s. Consider the string $uv^0xy^0z = uxz$. The string uxz must be of the form $0^k 1^{2k} 0^i 1^j$ where $i \le 2k$ and $j < k$. If $uxz \in L$, it must be of the form ww^Rw. Since uxz is of the form $0^k 1^{2k} 0^i 1^j$ and of length at least 5k, the second w must end with the block of $j < k$ 1s preceded by some number of 0s. Thus, ww^R must contain a block of at most $2j < 2k$ 1s. But uxz contains a block of 2k 1s, a contradiction.

Thus, the Pumping Lemma is violated under all circumstances, and the language in question cannot be context-free.

Pumping Lemma

Remember the **choice of a particular string s is critical** to the proof.

One might think that any string of the form ww^Rw would suffice, but this is not correct.

Consider the trivial string $0^k0^k0^k=0^{3k}$ which is of the form ww^Rw. Letting $v = 0$, $x = \varepsilon$, and $y = 00$, we have $uv^ixy^iz=0^{3(k+i-1)}$ which is an element of L since it is a string consisting of a multiple of three 0s.

Another seemingly “good” strings is $s= 0^k110^k0^k1=0^k110^{2k}1$. However, this is also not a good choice. Let $v = 0$, $x = 11$, and $y = 00$ (i.e., v consists of the 0 immediately preceding the 11, x is the 11, and y consists of the two 0s immediately following the 11). We then have $uv^ixy^iz = {}^{0k+i-1}110^{2(k+i-1)}1$ which is an element of L for all i since $uv^ixy^iz = ww^Rw$ where $w = 0^{k+i-11}$.

So once again, remember that the choice of the string to be pumped is critical!