Homework 3

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1

We prove that $L = \{a^{2n}b^n | n \geq 0\}$ is not regular by contradiction.

Suppose L is an FSL. Let p be the pumping length. So we can choose $s = a^{2n}b^n \in L$. Assume L is regular. Here, s can be written as $s = xyz$, the concatenation of some substrings x, y, z, where:

1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| = m > 0$
3. $|xy| \leq p$

We consider 3 cases for the formation of y:

1a

The string y consists only of a's. In this case, the number of a's in the string $xyyz$ is more than $2n$, but the number of b's remains the same (n). So $xyyz$ is not a member of L, which violates condition 1 of the Pumping Lemma. This case is a contradiction.

1b

The string y consists only of b's. In this case, the number of a's in the string $xyyz$ remains $2n$. However, the number of b's $> n$. So $xyyz$ is still not a member of L, which violates condition 1 of the Pumping Lemma.

1c

The string y consists of both a's and b's. In this case, it is possible that within the string $xyyz$, the number of a's is twice the number of b's, specifically, when $y = a^{2m}b^m$ for some $m > 0$). But they will be out of order with some b's before a's. Hence $xyyz$ is still not a member of L, which is a contradiction.

From 1a-c, we cannot avoid the contradiction if we assume that L is regular, so L is not regular.

2

Let $\Sigma = \{a, b, c\}$.

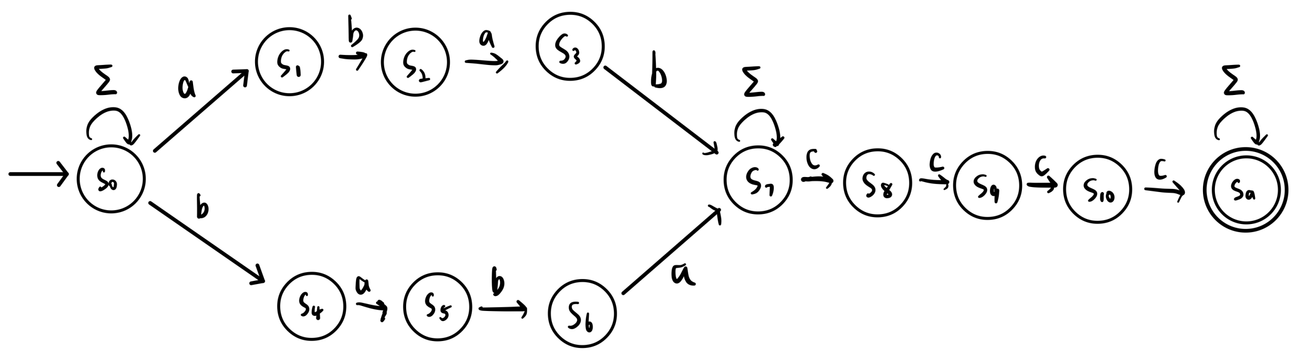
$L_2 = (a(\Sigma)^*(\Sigma \setminus \{a\})) \cup (b(\Sigma)^*(\Sigma \setminus \{b\})) \cup (c(\Sigma)^*(\Sigma \setminus \{c\}))$

The $(\Sigma)^*$ in the middle requires that the arbitrary symbols between the start symbol and end symbol can appear any times in $[0, \infty)$.

The a at the beginning of the string and $(\Sigma \setminus \{a\})$ at the end of the string require that the start and end symbols are different. The same is true for b and c

3

The following NFA recognizes L_3



The above diagram shows that the NFA recognizes strings with the following pattern:

$\Sigma^*(abab \cup baba)\Sigma^* cccc\Sigma^*$, where $\Sigma = (a, b, c)$.

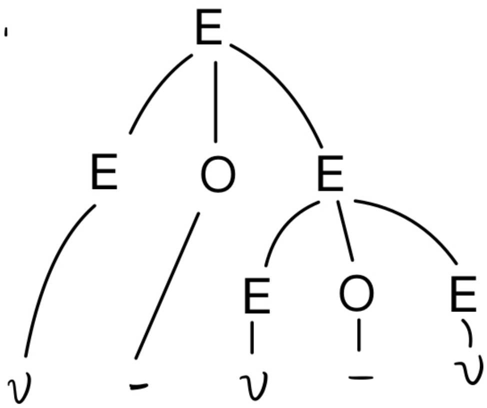
At the beginning of the string, we non-deterministically loop on Σ before we detect the start of substring $abab$ (which is a) and $baba$ (which is b). This means any characters in (a, b, c) are acceptable before we recognize the substring.

We then move onto either of branches representing $abab$ and $baba$ by transition into either S_1 or S_4. After we continuously read the 4 symbols in the substring and before we read the $cccc$, we non-deterministically loop on Σ at S_7. Note that substring like $babab$ is acceptable for both branches, and we can transition into either S_1 or S_4 non-deterministically.

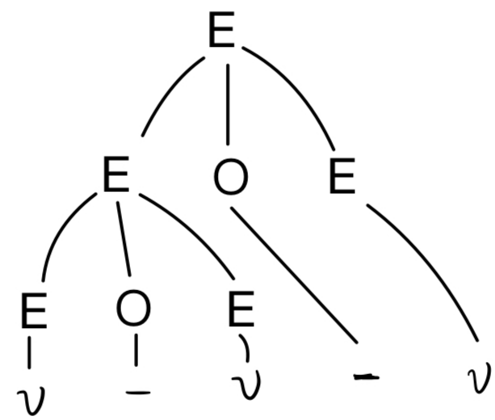
Then, in S_7 to S_{10}, we try to detect $cccc$. Finally, we transition into the final state S_a and non-deterministically loop on Σ since the string has already satisfied all of its requirements.

4

a.



c.



b.

$$
\begin{aligned}
E &\rightarrow \underline{E}OE \\
&\rightarrow v\,\underline{O}E \\
&\rightarrow v - \underline{E} \\
&\rightarrow v - \underline{E}OE \\
&\rightarrow v - v\,\underline{O}E \\
&\rightarrow v - v - \underline{E} \\
&\rightarrow v - v - v
\end{aligned}
$$

d.

$$
\begin{aligned}
E &\rightarrow EO\underline{E} \\
&\rightarrow \underline{E}OEOE \\
&\rightarrow v\,\underline{O}EOE \\
&\rightarrow v - \underline{E}OE \\
&\rightarrow v - v\,\underline{O}E \\
&\rightarrow v - v - \underline{E} \\
&\rightarrow v - v - v
\end{aligned}
$$

5

Let $\Sigma = \{b, e, s, ;\}$. Then L_5 is specified by the grammar G:

$\mathbf{S} \rightarrow b\mathbf{A}e;$ (Rule 1)

$\mathbf{A} \rightarrow b\mathbf{A}e; | s; | \mathbf{AA}$ (Rule 2)

We use $\mathbf{bold}$ capital letters to represent nonterminal symbols, and lowercase letters to represent terminal symbols.

Rule 1 specifies that every string is generated from the start variable $\mathbf{S}$. It must begin with b and end with $e;$

Rule 2 specifies the rules $\mathbf{A}$ uses to produce substrings. $\mathbf{A}$ can perform either of the following:

- Spawn a new begin-end statement pair, followed by $';'$
- Generate a single statement $s;$ (this is a terminal)
- Generate two statements $\mathbf{A}$ (variables, or nonterminals), separated by $';'$

6

G is a Context-Free Grammar for the language L_6

$\mathbf{S} \to b\mathbf{A}e$ (Rule 1)

$\mathbf{A} \to b\mathbf{A}e|s|\mathbf{A},\mathbf{A}$ (Rule 2)

In Rule 1, the symbol $\mathbf{S}$ is the start variable. This guarantee that all strings generated are enclosed in a pair of beginning and ending symbol b and e.

The second rule specifies that every new string spawned by $\mathbf{A}$ can be one of

- Some string generated by $\mathbf{A}$, enclosed in a begin-end block
- A single statement s
- Two new strings generated by $\mathbf{A}$, separated by $','$.

7

The string $aaab$ can be accepted by the following ways:

$q_{start} \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{\varepsilon} q_1 \xrightarrow{ab} q_{accept}$

$q_{start} \xrightarrow{\varepsilon} q_2 \xrightarrow{aa} q_1 \xrightarrow{ab} q_{accept}$

(Note: there is a 3rd way):

$q_{start} \xrightarrow{\varepsilon} q_2 \xrightarrow{\varepsilon} q_1 \xrightarrow{aa} q_1 \xrightarrow{ab} q_{accept}$