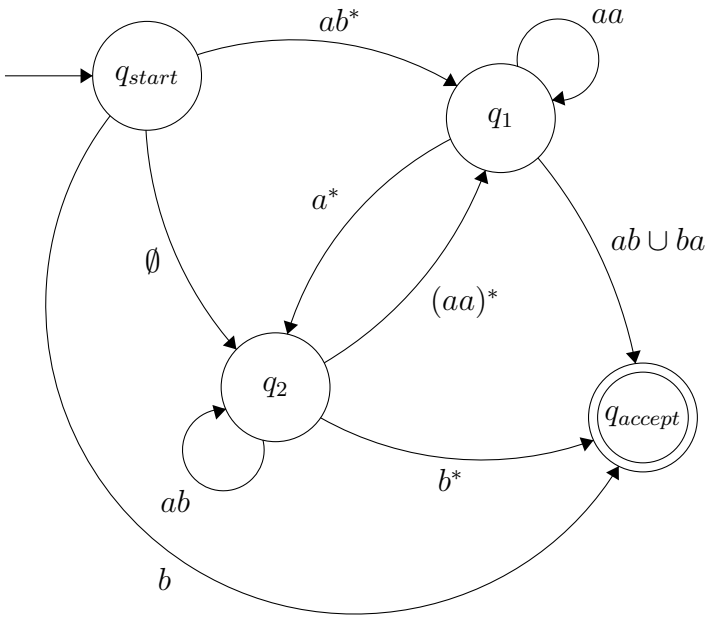
CS 181: Formal Languages and Automata Theory	**Spring 2021**
Homework 4	
Assigned: Tuesday, 20 April	*Due: Monday, 26 April 9:00pm PDT*

Note Submission deadline is now 9:00pm.

Problem 1

Refer to the GNFA diagram in Figure 1.61 on page 70 of the Sipser textbook, presented here for convenience with names given to all four of the states to make it easier to write your answers:



Show two different ways that the GNFA can accept the string "aaab". For each way, list the sequence of states and transitions and show the portion of the input string matched for each transition.

Problem 2

Consider the following Context Free Grammar (CFG), $G_2 = (V, \Sigma, R, S)$, with rule set R given by:

$$S \longrightarrow TT$$
$$T \longrightarrow 0T \mid T0 \mid \#$$

where S is the start variable; the variable set, V, is $\{S, T\}$; and the terminal set is $\Sigma = \{0, \#\}$. Describe the language represented by this grammar in clear and simple terms.

Problem 3

Recall: For any string $\omega = \omega_1\omega_2...\omega_n$, the **reverse** of ω, written as ω^R, is the string ω in reverse order, $\omega_n...\omega_2\omega_1$; and $\epsilon^R = \epsilon$. For any language, A, we have defined $A^R = \{\omega^R | \omega \in A\}$. Show that if A is a FSL (aka regular language), then so is A^R.
You may use any approach we have discussed in lecture to show that A^R will always be a FSL. Except of course you may not use the fact that the professor said the family of FSLs was closed under Reversal as your answer! If you use a model, such as a DFA or NFA, you do not need to formally prove that the model is correct. However, as always in this class, your response must include a brief explanation of how your answer correctly represents the solution.

Problem 4

Let $\Sigma = \{a, b, \#\}$. Consider following language:

$$L_4 = \{\ w = x\#y\#z \mid x, y, z \in \{a, b\}^* \text{ and } z = x^R \text{ or } |y| = 2|x|\ \}$$

Design a Context Free Grammar that generates this language. Be sure to clearly indicate your variable set, start variable, and rule set. And as always, include a *brief* description of how your grammar is designed to correctly represent the language.
There are no additional requirements on your answer other than what is stated here and the usual guidance that your answer should be correct, complete, and understandable.

Problem 5

Use the Pumping Lemma for the FSLs (and possibly other results) to show that the following language over alphabet $\Sigma = \{0, 1\}$ is not a FSL:

$$L_5 = \{\ w \mid w = xy,\ x, y \in \Sigma^*,\ |x| = |y|, \text{ and } \#(0, x) = \#(0, y)\ \}$$