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| **CS 181: Formal Languages and Automata Theory** | **Spring 2021** |
| Homework 9 Solution | |
| *Assigned: TBD* | *Due: TBD* |

# Problem 1

Let alphabet $\Sigma = \{0, 1\}$.
Recall we defined a "run" of a symbol in a string to be a sequence of one or more of the same symbol with no other symbols in-between and no more of that symbol adjacent to it. E.g., the string 01000110 contains exactly the following five runs: 0, 1, 000, 11, and 0.
Consider the following language over $\Sigma$:

$$L = \{\ w \in \Sigma^* \mid \text{in } w \text{ none of the runs of 0 are of equal length }\}$$

E.g., $L$ contains 011000001001, and $L$ does not contain 1001000100.

Decide whether $L$ is a finite state language. If so, prove it by any method discussed in class. If not, prove that it is not finite state by any method discussed in class.
**Solution:**
$L$ is not finite state. (In fact, it is not context free, but you do not have to prove that.) Suppose $L$ is a finite state language to achieve a contradiction. Then, by pumping lemma there exists $p > 0$ the pumping length. We will choose $s = 0^{p+1}10^{p}10^{p-1}1....0^{2}10 \in L$. That is, $s$ is a run of $p+1$ 0's followed by a 1, then a run $p$ 0's followed by a 1, and so on decreasing the length of the run of 0's by one each time until we reach a run of length 1. The key here is that in $s$ after the initial block of $p+1$ zeros, it is guaranteed to have a run of zeros of every possible length between $p$ and 1, inclusive.
We observe that $|s| > p$, which means that the pumping lemma applies. In particular, there exists strings $x, y, z$ such that $s = xyz$, with $|xy| \leq p$, $|y| \geq 1$, and $xy^{i}z \in L$ for all $i \geq 0$. Since $|xy| \leq p$, $y$ must consist exclusively of 0's from the first block.
Now, consider $s' = xy^{0}z \in L$. By the FSL pumping lemma, we know that $s' = 0^{p+1-|y|}10^{p}10^{p-1}1....0^{2}10$. And we know that $1 \leq |y| \leq p$, which means that the first run of 0's in s' has length between $p$ and 1 inclusive. Runs of 0's of all of these lengths already exist in $s'$ after the first 1, so $s' \notin L$. This is a contradiction, so $L$ is not a FSL.
Many of you found this technique, and I am very glad to see that. And I am especially proud that some of you discovered that it is also possible to prove this result by considering the complement of the language, proving that it is not a FSL (using the Pumping Lemma of course), and then noting that since the complement is not a FSL, then the language itself cannot be a FSL either. Both approaches earned full credit, as long as you did it correctly. This sort of indirect approach is why in previous assignments we sometimes said "using the ... Pumping Lemma and possibly other results".