CS 181: Formal Languages and Automata Theory	**Spring 2021**
Homework 7	
Assigned: 11 May	*Due: 17 May 9:00pm PDT*

Note Submission deadline is 9:00pm.

Problem 1

Let $\Sigma = \{a, b, c, d\}$. Consider the following CFG. The start symbol is S:

$$\begin{aligned} S &\longrightarrow SS \mid P \mid B \mid \epsilon \\ P &\longrightarrow aSc \mid ac \\ B &\longrightarrow bSd \mid bd \end{aligned}$$

a. Prove that this grammar is ambiguous.

b. Briefly explain how the grammar works in a manner that causes it to be ambiguous.

c. Define a new grammar that is unambiguous or "fix" the given grammar to make it unambiguous. As always, *briefly* explain how your grammar is designed to be correct and unambiguous.

Problem 2

Let Σ be some alphabet appropriate for representing digraphs. Assume that the encoding represents the graph as a list of edges in the form of ordered pairs of nodes represented in some appropriate manner. (Assume the set of all nodes of the graph can be determined from the list of edges.) The pseudo-code below defines a Deterministic TM, $C0$, which is an attempt to program a TM to recognize the following language, L, over Σ: L equals all strings which encode a graph, G, such that G has at least one directed cycle containing the first node in the encoding of the graph as a string. The first node in the graph encoding is defined as the source node in the first ordered pair in the list of edges - we call this node "$NODE0$". Answer the following questions about the pseudo-code for $C0$:

a. Does $C0$ correctly recognize the language? Briefly explain why or why not.

b. Regardless of the correctness of the pseudo-code, Is $C0$ an algorithm? or is it a procedure?

Pseudo-code for $C0$:
$C0$ uses a work tape to store a value, "CurrentNode", used during the computation (along with other values used to implement the details of the steps which we are assuming can be done based on Church's Thesis).

Initialization:
Test whether the input tape contains a valid encoding of a digraph.
If not, halt and reject.
If there is at least one edge out-going from "NODE0", then choose the first outgoing edge from "NODE0", and call that edge (NODE0, NextNode),
else halt and reject.
Write the value "NextNode" to the location "CurrentNode" on the work tape.

Begin While the value of "CurrentNode" on the work tape does not equal $NODE0$:

1. If there is at least one edge out-going from "CurrentNode", then choose the first outgoing edge from "CurrentNode", and call that edge (CurrentNode,NextNode). Write the value NextNode to the location $CurrentNode$ on the work tape

   else halt and reject.

End while loop.
Halt and accept.

Problem 3

Let $\Sigma = \{a, b\}$, and let L be the language over Σ:

$$L = \left\{ w \in \Sigma^+ \;\middle|\; |w| \text{ is even, and } w \text{ contains at least one } a \text{ in the first half and exactly one } b \text{ in the second half} \right\}$$

Give a context-free grammar (CFG) for L, and briefly explain how it works to correctly represent the language. There are no additional requirements for your solution other than what is stated here, except the usual guidance to make sure your answer is clear and understandable and avoids too much unnecessary complexity. In particular, there is no credit for the grammar being unambiguous (if possible.)