Homework 8

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1

The grammar is ambiguous. The string $baabba$ can be reduced in two different ways using this grammar:

$\underline{b}aabba \mapsto B\underline{a}abba \mapsto BA\underline{a}bba \mapsto B\underline{AA}bba \mapsto \underline{BC}bba \mapsto A\underline{b}ba \mapsto AB\underline{b}a \mapsto ABB\underline{a} \mapsto AB\underline{BA} \mapsto A\underline{BS} \mapsto \underline{AB} \mapsto S$

$\underline{b}aabba \mapsto B\underline{a}abba \mapsto BA\underline{a}bba \mapsto BAA\underline{b}ba \mapsto BA\underline{AB}ba \mapsto B\underline{AS}ba \mapsto BA\underline{b}a \mapsto BAB\underline{a} \mapsto BA\underline{BA} \mapsto B\underline{AS} \mapsto \underline{BA} \mapsto S$

There are more than 2 possible reductions. For example, a slightly modified version of 1 gives:

$\underline{b}aabba \mapsto B\underline{a}abba \mapsto BA\underline{a}bba \mapsto B\underline{AA}bba \mapsto \underline{BC}bba \mapsto A\underline{b}ba \mapsto AB\underline{b}a \mapsto A\underline{BB}a \mapsto \underline{AD}a \mapsto B\underline{a} \mapsto \underline{BA} \mapsto S$

2

a

Let M_A be a TM decider for L_A (Recursive language). Let M_P be a TM recognizer for L_P (R.E. language).

We can construct a **Universal Turing Machine** M for the union of languages $L_P \cup L_A$.

We use the M to simulate M_A on a given input string w for L_A:

- If M_A halts and accepts, M halts and accepts
- If M_A halts and rejects, we use M to simulate M_P on w

Note that, since M_A is a decider, it must always halt on its input (instead of entering infinite loops), and either accept or reject.

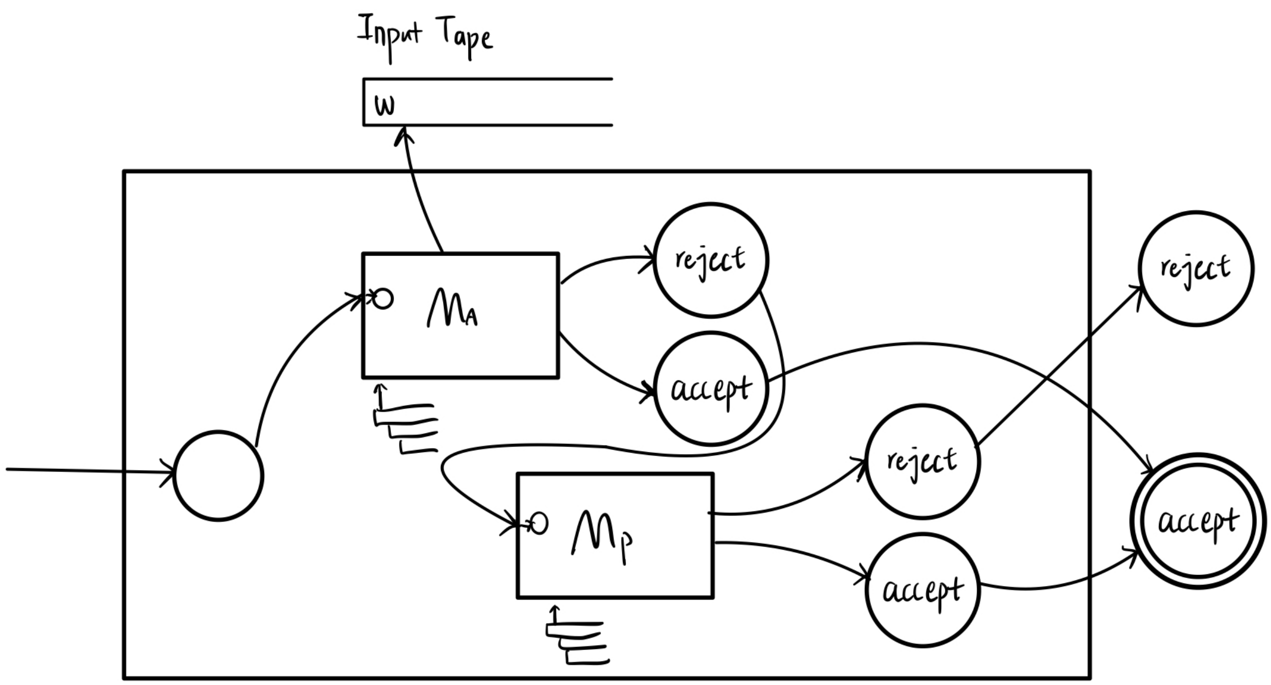
Similarly, we can use M to simulate M_P on a given input string w for L_P:

- If M_P halts and accepts, M halts and accepts

- If M_P halts and rejects, M halts and rejects
- If M_P does not halt, this means the input string cannot be accepted. So M does not halt, either. It continuously runs as M_P does.

This way, M may fail to halt on some inputs in $L_P \cup L_A$. This means $L_P \cup L_A$ is Recursively Enumerable.

A picture for this **Universal Turing Machine**:



b

In order to decide whether w is in $L_P \cup L_A$, we need to let M simulate M_P in cases that w is not in L_A in our first step, which means that it can receive any strings as M_P receives. However, L_P is Recursively Enumerable but not necessarily Recursive. This means M_P does not always halt on its inputs and can only **recognize** (instead of **decide**) strings that belong to L_P. Similarly, M may not halt on its inputs. So M is a Turing Machine Procedure (but not an Algorithm) that recognizes the union $L_P \cup L_A$.