CS 181 HW3 2021 CS181

YIQIAO JIN

TOTAL POINTS

26 / 28

QUESTION 1

1 Pumping Lemma 5 / 7

- + 7 pts Correct or nearly correct
- √ + 1 pts Appropriate "s"
 - + 2 pts Effective use of constraints on "xyz"
- √ + 2 pts Show coverage of all cases of "xyz"
- **0.5 pts** should give more complete justification that your proof covers all cases of xyz

√ + 2 pts Sound logic in every case

- **0.5 pts** Should clearly state that |xy| \$\leq\$ p means xy is all a's, not just mention that |xy| \$\leq\$ p.
 - 0.5 pts minor error in logic
 - 1 pts should give more complete justification logic
- **2 pts** You cannot assign/assume any particular value for x, y, or z.
 - 1 pts Cannot assume p is even
 - + 0 pts No answer
 - -2 for use of constraints because the three cases are not necessary.

QUESTION 2

2 Regular Expression 4/4

- √ 0 pts Correct
 - 1 pts Almost correct
 - 3 pts Not correct
 - 4 pts Not Attempted

QUESTION 3

3 NFA 5/5

√ - 0 pts Correct

- 1 pts Minor mistakes, your NFA cannot accept some of the strings in the language. We use following strings to test your NFA.
- ababccbcccc

- ababcccc
- ababacccc
- ababbcccc
- ababbacccc
- babaacccc
- babaabcccc
- babaaccccabba
- ababbaccccbaab
- 1 pts Minor mistakes, your NFA will accept some strings that are not in the language, for example,
- ababccc
- babaabab
- ababbaba
- ccccbaba
- ccccabab
- ccccaababa
- ccccbbabab
- 1 pts Does not effectively use the nondeterminism in the NFA
- 1 pts Invalid NFA, or your NFA does not satisfy the definition of NFA, or do not explicitly specify the final states, or forget to specify the transition for some states.
- **5 pts** Totally wrong, or you did not answer this problem at all.

QUESTION 4

Parse Trees & Derivations 4 pts

4.1 a Parse Tree 1/1

- √ 0 pts Correct
 - 0.5 pts Small mistake
 - 1 pts Incorrect

4.2 b Left-most Derivation 1/1

√ - 0 pts Correct

- 0.5 pts Missing Steps
- 0.5 pts Not Left-most
- 1 pts Incorrect

4.3 c Parse Tree 1/1

√ - 0 pts Correct

- 0.5 pts Small Mistake
- 1 pts Incorrect

4.4 d Left-most Derivation 1/1

√ - 0 pts Correct

- **0.5 pts** Missing Steps
- 0.5 pts Not Left-most
- 1 pts Incorrect

QUESTION 5

5 CFG; 4/4

√ - 0 pts Correct

- 1 pts Can not express arbitrary number of begin end blocks beside each other, e.g. bbs;e;bs;e;bs;e;e;
- **2 pts** Begin and end blocks don't have to line up. e.g. bbs;e; or bs;e;e; is generated even though it shouldn't be.
 - 1 pts No specified start variable
- 1 pts Extra semicolons generated (e.g. bs;;bs;e;e; or b;s;e;)
- 1 pts Can not express multiple statements beside each other (e.g. bs;s;s;e;)
- **1 pts** Missing semicolons on s (e.g. bse; is generated)
- 1 pts Can not express single statement e.g. bs;e;
- **1 pts** Can't do some orders of statements and blocks e.g. bbs;e;s;e; or bs;bs;e;e;
- 1 pts Can't have arbitrary nestings next to each other (like bbbs;e;e;bbs;e;e;e;
- 1 pts Doesn't necessarily have an outside begin end pair (e.g. generates s; or b or nothing at all, or bs;e;bs;e;)
- 1 pts There isn't necessarily an outer be pair (e.g. bs;e;bs;e; can be generated)

- **0 pts** Click here to replace this description.

QUESTION 6

6 CFG, 4/4

√ - 0 pts Correct

- 1 pts Adjacent begin statements either can't exist (can't generate bbse,bsee) or can be missing commas between (e.g. bbsebsee)
- 1 pts Can't generate arbitrarily many adjacent (or nested) begin/ends (e.g. bbse,bse,bsee or bbsee or bs,bse,bsee)
- 2 pts zBegin/end not guaranteed to match (e.g. could generate bsee or bssee)
 - 1 pts No specified start variable
 - 1 pts Unnecessary semicolons
- **1 pts** Doesn't necessarily generate outside begin end pair (e.g. bsebse or bse,bse or s or s, or epsilon)
- 1 pts Can't generate certain orders of statements, for example bbse,se or bs,bsee
- 1 pts begin/end statements can be empty (e.g. generates be)
- 1 pts Could be missing commas (e.g. this can generate bsse or bss,se or bbsese)
- **1 pts** Can't generate arbitrarily many s's (e.g. bs,s,se)
- 1 pts Can generate extra commas (e.g. bse, or bs,e or b,,,,se)
- 1 pts Can't do arbitrary nesting of begin/end statements (e.g. bbbseee

QUESTION 7

7 Postponed to next weeK: GNFA o / o

√ - 0 pts Correct

Homework 3

Name: Yiqiao Jin UID: 305107551

1

We prove that $L = \{a^{2n}b^n|n \ge 0\}$ is not regular by contradiction.

Suppose L is an FSL. Let p be the pumping length. So we can choose $s=a^{2n}b^n\in L$. Assume L is regular. Here, s can be written as s=xyz, the concatenation of some substrings x,y,z, where:

- 1. for each $i \geq 0$, $xy^iz \in A$
- 2. |y| = m > 0
- 3. $|xy| \le p$

We consider 3 cases for the formation of y:

1a

The string y consists only of a's. In this case, the number of a's in the string xyyz is more than 2n, but the number of b's remains the same (n). So xyyz is not a member of L, which violates condition 1 of the Pumping Lemma. This case is a contradiction.

1b

The string y consists only of b's. In this case, the number of a's in the string xyyz remains 2n. However, the number of b's > n. So xyyz is still not a member of L, which violates condition 1 of the Pumping Lemma.

1c

The string y consists of both a's and b's. In this case, it is possible that within the string xyyz, the number of a's is twice the number of b's, specifically, when $y=a^{2m}b^m$ for some m>0). But they will be out of order with some b's before a's. Hence xyyz is still not a member of L, which is a contradiction.

From 1a-c, we cannot avoid the contradiction if we assume that L is regular, so L is not regular.

1 Pumping Lemma 5/7

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- √ + 1 pts Appropriate "s"
 - + 2 pts Effective use of constraints on "xyz"
- √ + 2 pts Show coverage of all cases of "xyz"
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Let
$$\Sigma = \{a, b, c\}$$
.

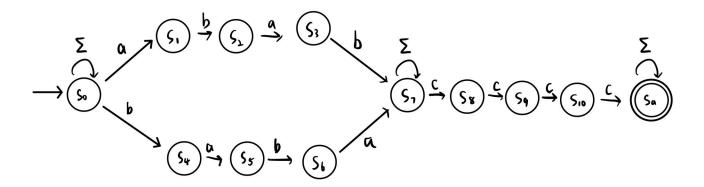
$$L_2 = (a(\Sigma)^*(\Sigma \setminus \{a\})) \cup (b(\Sigma)^*(\Sigma \setminus \{b\})) \cup (c(\Sigma)^*(\Sigma \setminus \{c\}))$$

The $(\Sigma)^*$ in the middle requires that the arbitrary symbols between the start symbol and end symbol can appear any times in $[0, \infty)$.

The a at the beginning of the string and $(\Sigma \setminus \{a\})$ at the end of the string require that the start and end symbols are different. The same is true for b and c

3

The following NFA recognizes L_3



The above diagram shows that the NFA recognizes strings with the following pattern:

$$\Sigma^*(abab \cup baba) \Sigma^* cccc \Sigma^*$$
 , where $\Sigma = (a,b,c)$.

At the beginning of the string, we non-deterministically loop on Σ before we detect the start of substring abab (which is a) and baba (which is b). This means any characters in (a,b,c) are acceptable before we recognize the substring.

We then move onto either of branches representing abab and baba by transition into either S_1 or S_4 . After we continuously read the 4 symbols in the substring and before we read the cccc, we non-deterministically loop on Σ at S_7 . Note that substring like babab is acceptable for both branches, and we can transition into either S_1 or S_4 non-deterministically.

Then, in S_7 to S_{10} , we try to detect cccc. Finally, we transition into the final state S_a and non-deterministically loop on Σ since the string has already satisfied all of its requirements.

2 Regular Expression 4/4

- √ 0 pts Correct
 - 1 pts Almost correct
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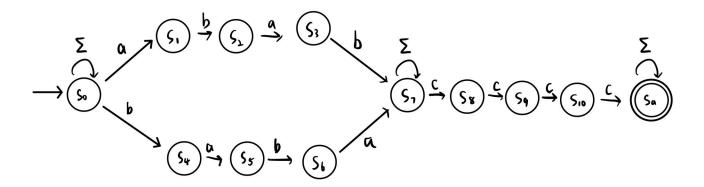
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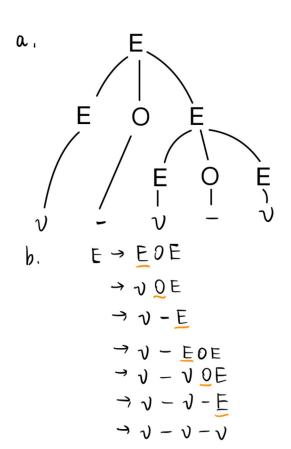
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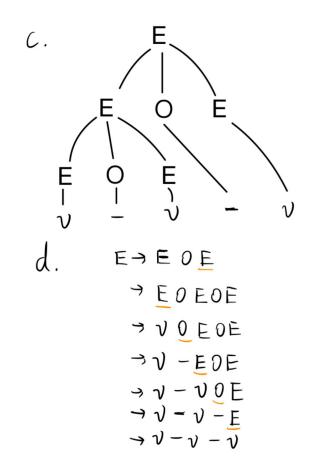
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3 NFA 5/5

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 - 1 pts Does not effectively use the nondeterminism in the NFA
- 1 pts Invalid NFA, or your NFA does not satisfy the definition of NFA, or do not explicitly specify the final states, or forget to specify the transition for some states.
 - 5 pts Totally wrong, or you did not answer this problem at all.





Let $\Sigma = \{b, e, s, ; \}$. Then L_5 is specified by the grammar G:

 $\mathbf{S}
ightarrow b\mathbf{A}e;$ (Rule 1)

 $\mathbf{A}
ightarrow b\mathbf{A}e;|s;|\mathbf{A}\mathbf{A}$ (Rule 2)

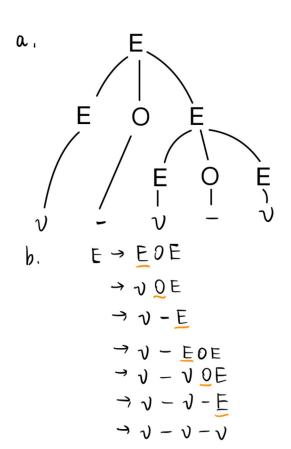
We use **bold** capital letters to represent nonterminal symbols, and lowercase letters to represent terminal symbols.

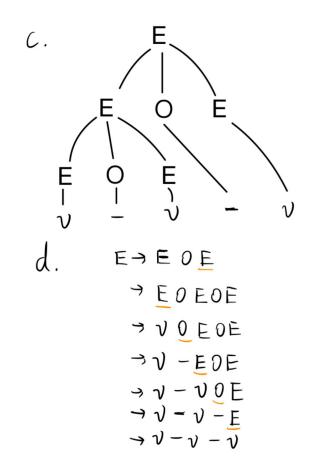
Rule 1 specifies that every string is generated from the start variable S. It must begin with b and end with e;

- Spawn a new begin-end statement pair, followed by ';'
- Generate a single statement s; (this is a terminal)
- ullet Generate two statements f A (variables, or nonterminals), separated by ';'

4.1a Parse Tree 1/1

- √ 0 pts Correct
 - 0.5 pts Small mistake
 - 1 pts Incorrect





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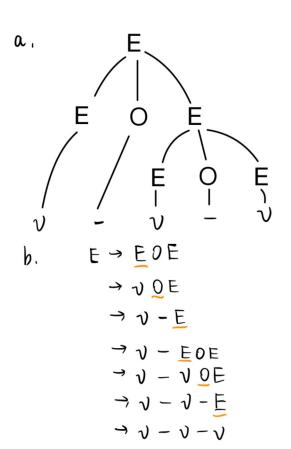
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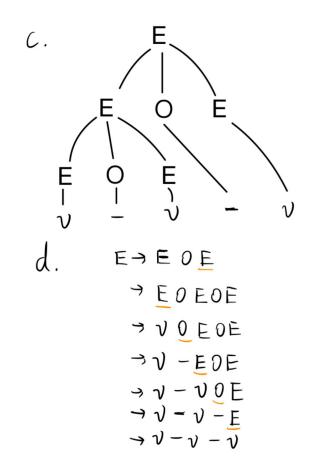
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4.2 b Left-most Derivation 1/1

- √ 0 pts Correct
 - **0.5 pts** Missing Steps
 - 0.5 pts Not Left-most
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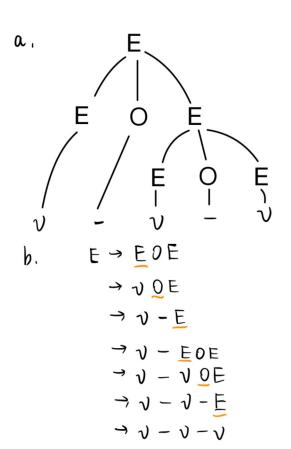
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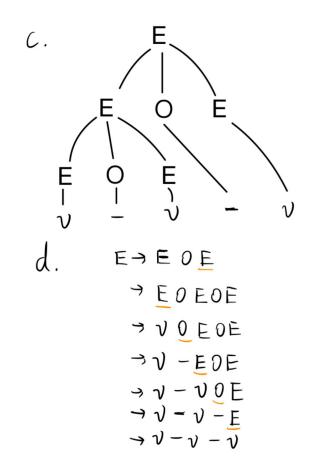
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4.3 c Parse Tree 1/1

- √ 0 pts Correct
 - 0.5 pts Small Mistake
 - 1 pts Incorrect





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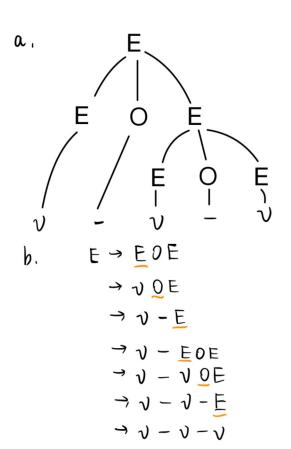
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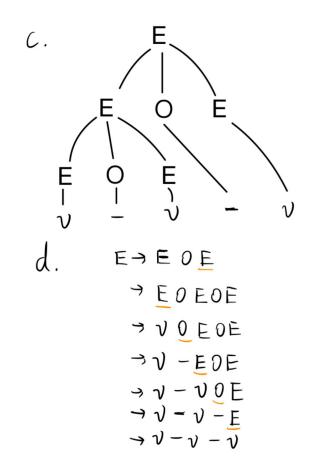
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4.4 d Left-most Derivation 1/1

- √ 0 pts Correct
 - **0.5 pts** Missing Steps
 - 0.5 pts Not Left-most
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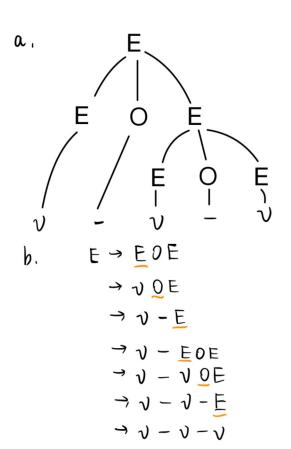
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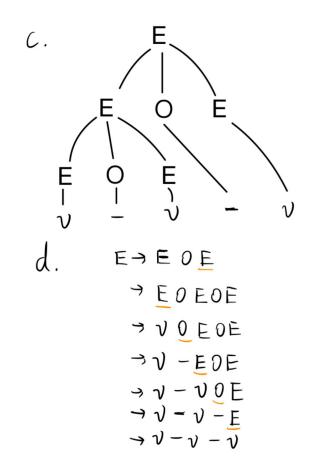
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5 CFG; 4/4

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- 1 pts Can not express arbitrary number of begin end blocks beside each other, e.g. bbs;e;bs;e;bs;e;e;
- 2 pts Begin and end blocks don't have to line up. e.g. bbs;e; or bs;e;e; is generated even though it shouldn't be.
 - 1 pts No specified start variable
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Rule 1 specifies that every string is generated from the start variable S. It must begin with b and end with e;

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G is a Context-Free Grammar for the language L_6

$$\mathbf{S} o b\mathbf{A}e$$
 (Rule 1)

$$\mathbf{A}
ightarrow b\mathbf{A}e|s|\mathbf{A},\mathbf{A}$$
 (Rule 2)

In Rule 1, the symbol S is the start variable. This guarantee that all strings generated are enclosed in a pair of beginning and ending symbol b and e.

The second rule specifies that every new string spawned by ${\bf A}$ can be one of

- Some string generated by A, enclosed in a begin-end block
- A single statement s
- Two new strings generated by **A**, separated by ','.

7

The string aaab can be accepted by the following ways:

$$q_{start} \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{\varepsilon} q_1 \xrightarrow{ab} q_{accept}$$

$$q_{start} \xrightarrow{\varepsilon} q_2 \xrightarrow{aa} q_1 \xrightarrow{ab} q_{accept}$$

(Note: there is a 3rd way):

$$q_{start} \stackrel{\varepsilon}{\longrightarrow} q_2 \stackrel{\varepsilon}{\longrightarrow} q_1 \stackrel{aa}{\longrightarrow} q_1 \stackrel{ab}{\longrightarrow} q_{accept}$$

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