CS 181: Formal Languages and Automata Theory

Spring 2021

Homework 6

Assigned: Tuesday 4 May

Due: Monday 10 May 9:00pm PDT

Note Submission deadline is now 9:00pm.

Problem 1

Let $\Sigma = \{a, b\}$. Consider the following language over Σ :

$$\{a^ib^j \mid 0 \le i \le j\}$$

Show a CFG for this language and briefly explain how your grammar is designed to correctly represent the language. You will receive extra credit if your grammar is unambiguous and you briefly explain why you believe that is true.

Problem 2

Let $\Sigma = \{0, 1\}$, and let:

$$L = \{xx^Rx \mid x \in \Sigma^*\}$$

Show using the Pumping Lemma for CFLs (and possibly other results), that this language is not a CFL.

Problem 3

Let Σ be an alphabet with at least two symbols. Consider the following languages over Σ :

R and S are a FSLs

C is a language which can be represented by a PDA

G is a language which can be represented by an unambiguous CFG

A is a language which can be represented by an ambiguous CFG

I is an inherently ambiguous CFL

L is a language which cannot be represented by a CFG

X, given that $X \cup S = L$

Given only the information above, classify each of the following languages by stating exactly which of the following four families it could possibly be:

1 FSL | 2 CFL and Not FSL | 3 Inherently Ambiguous CFL | 4 Non-CFL

For this set of classification questions, provide a *very brief* explanation, but you do *not* need to provide any detailed justifications, proofs, examples, nor counter-examples.

Examples:

 $R \cap S$: It must be 1. The Family of FSLs is closed under intersection.

 $R \cup I$: It could be a CFL; so it could be 1, 2, or 3. I is a CFL. And since R is a FSL, it is also a CFL. The Family of CFLs is closed under union.

 $C \cup L$: It could be a non-CFL; so it could be 1, 2, 3, or 4. C is a CFL, but since L is not a CFL we cannot apply any closure properties to narrow-down the possibilities.

- a. $R \cap L$
- b. A
- c. \bar{C} (the complement of C with respect to Σ^*)
- d. G
- e. \bar{L} (the complement of L with respect to Σ^*)
- f. X