

# CS 181 HW8 2021 CS181

YIQIAO JIN

TOTAL POINTS

**12 / 12**

## QUESTION 1

1 Two left-most reductions in CFG **4 / 4**

✓ - **0 pts** Perfectly correct

- **1.5 pts** One of your reductions is wrong

- **3 pts** Both of your reductions are wrong

- **1 pts** Reduction is correct, but one of your reductions is not left-most

- **2 pts** Reduction is correct, but both of your reductions are not left-most

- **4 pts** Did not answer this question

## QUESTION 2

TM (mixed) Construction **8 pts**

2.1 a. Procedure **6 / 6**

✓ + **6 pts** Correct

+ **4 pts** Partially correct

+ **2 pts** Attempted

+ **5 pts** Almost Correct

+ **0 pts** Not attempted

2.2 b. Brief explanation **2 / 2**

✓ + **2 pts** Correct

+ **1 pts** Attempted, partially correct

+ **0 pts** Not attempted or wrong

# Homework 8

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Name: Yiqiao Jin

UID: 305107551

## 1

The grammar is ambiguous. The string  $baabba$  can be reduced in two different ways using this grammar:

$$\underline{b}aabba \mapsto B\underline{a}abba \mapsto BA\underline{a}bba \mapsto B\underline{A}Abba \mapsto \underline{B}C\underline{b}ba \mapsto A\underline{b}ba \mapsto AB\underline{b}a \mapsto ABB\underline{a} \mapsto AB\underline{B}A \mapsto \underline{A}BS \mapsto \underline{A}B \mapsto S$$
$$\underline{b}aabba \mapsto B\underline{a}abba \mapsto BA\underline{a}bba \mapsto BAA\underline{b}ba \mapsto BAA\underline{B}ba \mapsto B\underline{A}Sba \mapsto BA\underline{b}a \mapsto BAB\underline{a} \mapsto \underline{BABA} \mapsto \underline{BAS} \mapsto \underline{BA} \mapsto S$$

There are more than 2 possible reductions. For example, a slightly modified version of 1 gives:

$$\underline{b}aabba \mapsto B\underline{a}abba \mapsto BA\underline{a}bba \mapsto B\underline{A}Abba \mapsto \underline{B}C\underline{b}ba \mapsto A\underline{b}ba \mapsto AB\underline{b}a \mapsto \underline{A}BBa \mapsto \underline{A}Da \mapsto \underline{B}\underline{a} \mapsto \underline{BA} \mapsto S$$

## 2

a

Let  $M_A$  be a TM decider for  $L_A$  (Recursive language). Let  $M_P$  be a TM recognizer for  $L_P$  (R.E. language).

We can construct a **Universal Turing Machine**  $M$  for the union of languages  $L_P \cup L_A$ .

We use the  $M$  to simulate  $M_A$  on a given input string  $w$  for  $L_A$ :

- If  $M_A$  halts and accepts,  $M$  halts and accepts
- If  $M_A$  halts and rejects, we use  $M$  to simulate  $M_P$  on  $w$

Note that, since  $M_A$  is a decider, it must always halt on its input (instead of entering infinite loops), and either accept or reject.

Similarly, we can use  $M$  to simulate  $M_P$  on a given input string  $w$  for  $L_P$ :

- If  $M_P$  halts and accepts,  $M$  halts and accepts

## 1 Two left-most reductions in CFG 4 / 4

✓ - 0 pts Perfectly correct

- 1.5 pts One of your reductions is wrong
- 3 pts Both of your reductions are wrong
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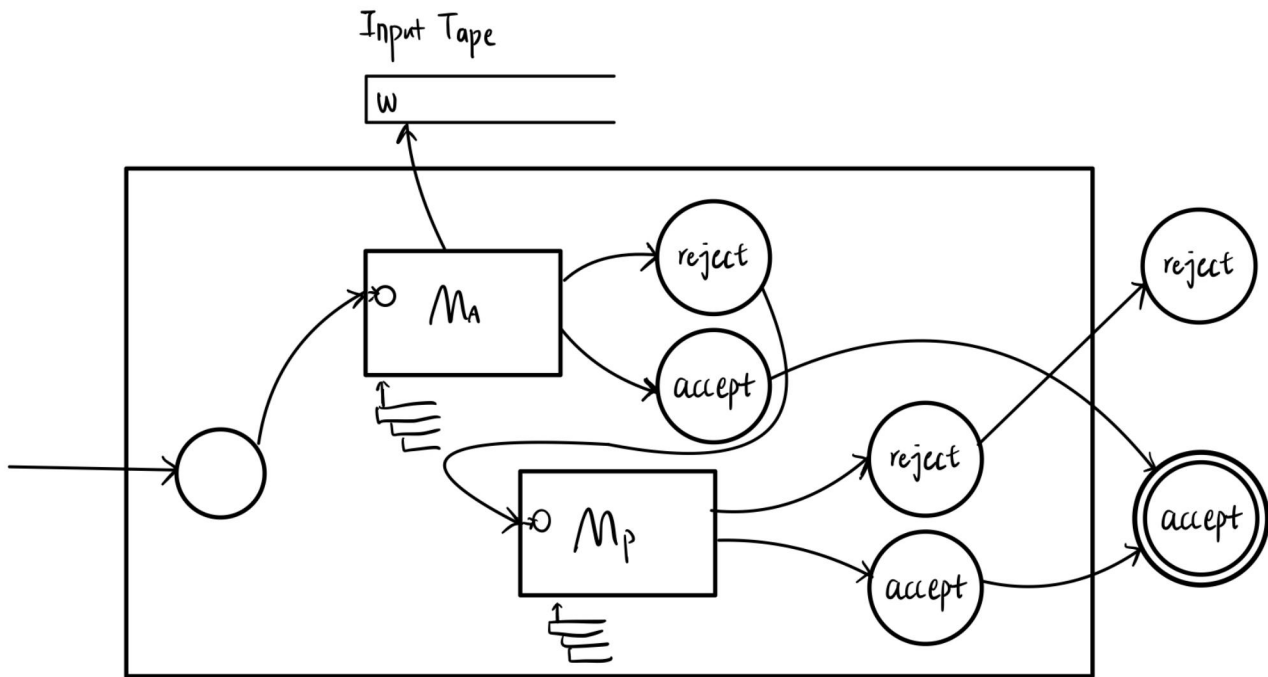
Similarly, we can use  $M$  to simulate  $M_P$  on a given input string  $w$  for  $L_P$ :

- If  $M_P$  halts and accepts,  $M$  halts and accepts

- If  $M_P$  halts and rejects,  $M$  halts and rejects
- If  $M_P$  does not halt, this means the input string cannot be accepted. So  $M$  does not halt, either. It continuously runs as  $M_P$  does.

This way,  $M$  may fail to halt on some inputs in  $L_P \cup L_A$ . This means  $L_P \cup L_A$  is Recursively Enumerable.

A picture for this **Universal Turing Machine**:



**b**

In order to decide whether  $w$  is in  $L_P \cup L_A$ , we need to let  $M$  simulate  $M_P$  in cases that  $w$  is not in  $L_A$  in our first step, which means that it can receive any strings as  $M_P$  receives. However,  $L_P$  is Recursively Enumerable but not necessarily Recursive. This means  $M_P$  does not always halt on its inputs and can only **recognize** (instead of **decide**) strings that belong to  $L_P$ . Similarly,  $M$  may not halt on its inputs. So  $M$  is a Turing Machine Procedure (but not an Algorithm) that recognizes the union  $L_P \cup L_A$ .

## 2.1 a. Procedure 6 / 6

✓ + **6 pts** Correct

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## 2.2 b. Brief explanation 2 / 2

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