Discussion week 5

Introduction to finance and marketing for engineers

Week 5

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Time value of money

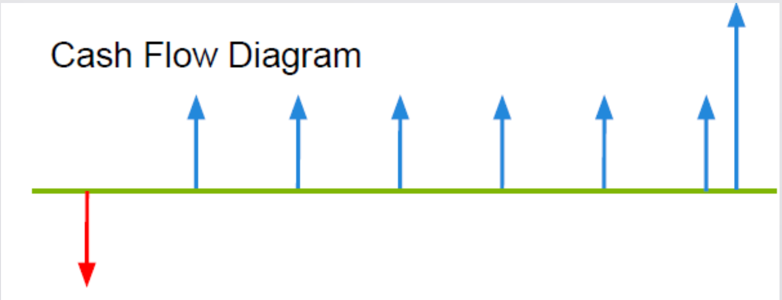
Is 10 dollars today the same as 10 dollars tomorrow?

The answer is no because for any given cash flow we need to look at the time frame.

Cash Flow Diagram: shows the incoming (blue) and outgoing (red) cashflows.

Suppose we put money in the bank. We are making an investment.

Banks have an annual interest rate of around 2%. After one year, you will have $102 (future value). After two years, that 100 dollars becomes $104.04 (future value).



Present and future value

Future Value (FV) = Present Value (PV) x ((1 + Interest Rate)nth time frame)

Present Value (PV) = Future Value (FV) / ((1 + Interest Rate)nth time frame)

We can write this in our notation:

$$FV = P(FV/PV, i, n) = PV(1+i)^n$$

$$PV = P(PV/FV, i, n) = \frac{FV}{(1+i)^n}$$

P is denoted as the principal or our investment

i is the return rate

n is the time frame (usually in years)

Frequent compounding

Given an annual percentage rate (APR), we are often to pay in monthly or quarterly periods.

To get the percentage for each period, we simply divide by the period.

$$FV = PV(1 + (\frac{i}{p}))^{np}$$

Suppose we put \$100 in the bank compounded monthly for two years. How much will we have?

Recall that if we compounded annually we get \$104.04

FV = 100 $(1 + \frac{0.02}{12})^{2x12}$ = \$104.07

Effective rate

We can also get the APR or the effective rate if given a rate with a different time period.

Effective Rate (Ia):

$$I_a = (1 + (\frac{i}{p}))^p - 1$$

Suppose we invest $100 today and the following occurs:

You get a 12% APR, compounded yearly. After one year:

FV = 100 (1+0.12) = $112

You get a 12% APR, compounded monthly. After one year:

FV = 100 $(1 + 0.12/12)^{12}$= $112.68

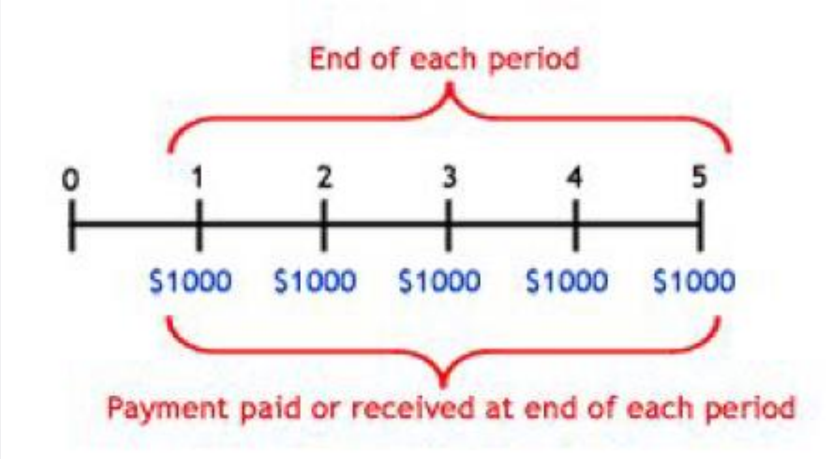
Our effective rate in this case is 12.68%, $I_a = (1 + \frac{0.12}{12})^{12} - 1 = 12.68\%$

Annuity

Annuity: a series of fixed payments required from you or paid to you at a specified frequency over the course of a fixed time period.

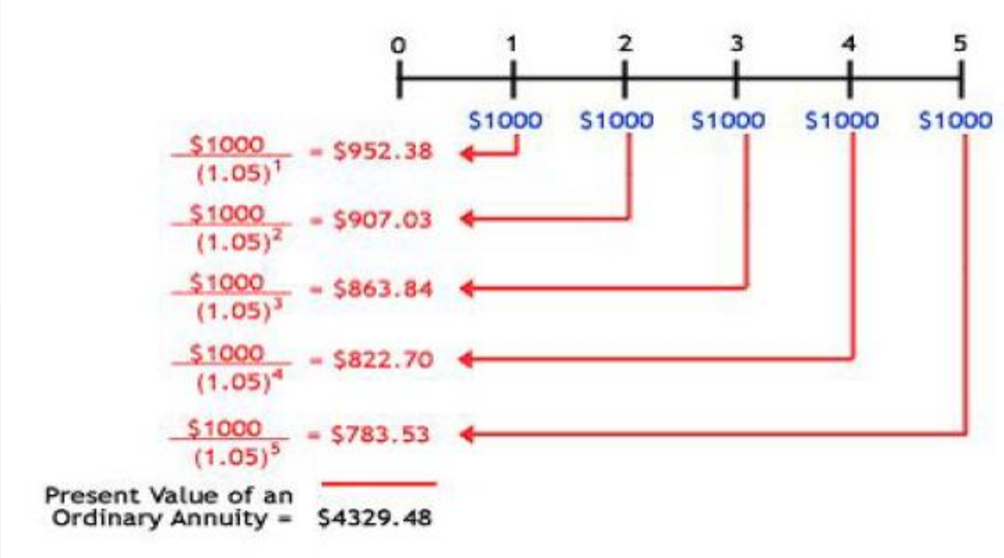
The most common payment frequencies are yearly, semi-annually (twice a year), quarterly and monthly.

If we were to pick up fixed amount of cash flow for 5 years at an interest rate of 5%, this is what it would look like:



Annuity

Sometimes it is not useful to see how much we are going to collect in the future since we are still in the present:



We can also determine the Present Value of the Annuity via the following:

$$PV = (PV/A, i, n) = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

Perpetuity

What if we have an investment that pays forever?

If there is a cash flow that comes in every year with 1000 dollars.

How much would this cash flow give you if this went on for a long period of time?

A perpetuity is a cash flow that runs for a very long time.

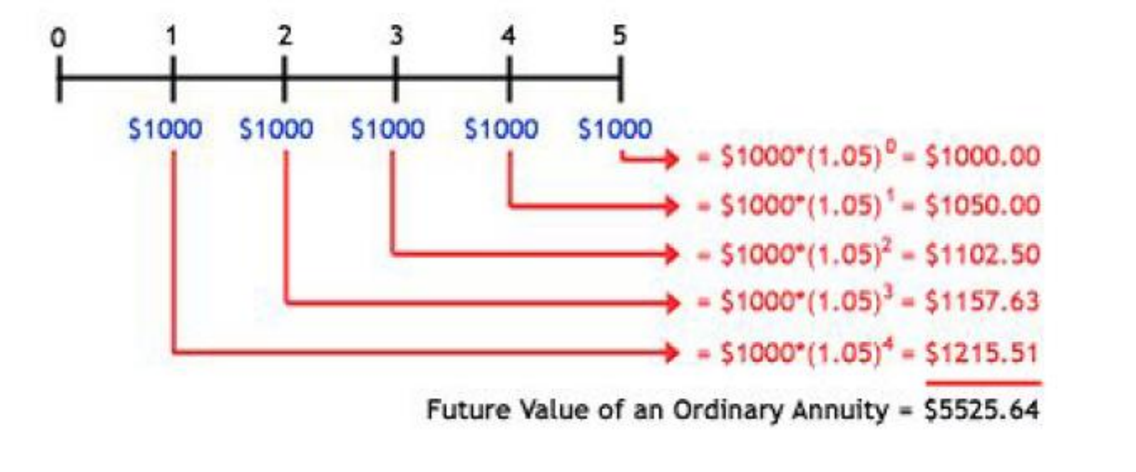
Note: because of the time value of money, eventually the money discounted to the present will be worth almost nothing. Therefore

we will converge to some value.

$$PV = \frac{A}{i}$$

Annuity

We can sum up all of these cash flows to see what kind of money we have received at the end of the 5 year period:



We define the Future Value Annuity formula to be the following:

$$FV = (FV/A, i, n) = A\frac{(1+i)^n - 1}{i}$$

Net Present Value

Net Present Value: the difference between total discounted benefits and total discounted costs brought to the present.

In other words, it is the *sum of all your cash flows discounted to the present*

$$NPV = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_n}{(1+r)^n}$$

Note: NPV is heavily reliant on the accuracy of discount rates

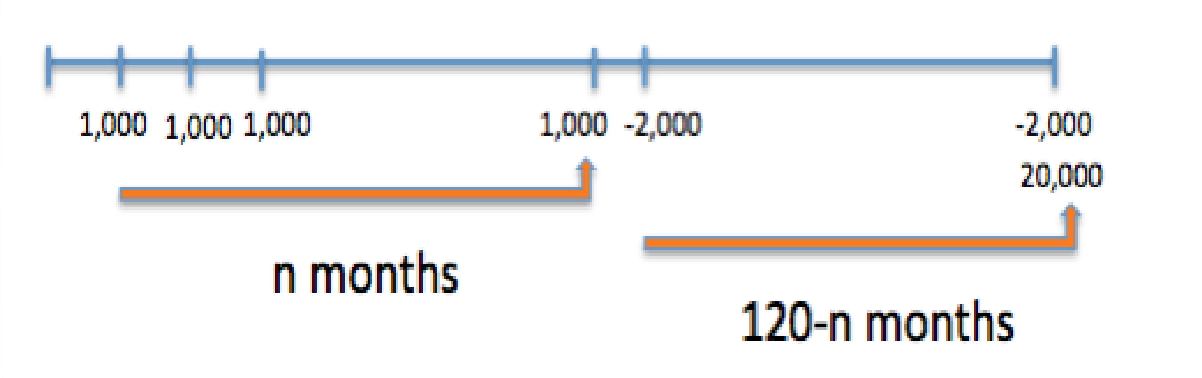
Exercise 1

Boeing would like to create a fund to fulfill future battery problem obligation. Beginning a month from today, Boeing will start to invest $1,000 each month.

At some point, it will stop investing and start withdrawing $2,000 per month and still have $20,000 in this fund10 years from now.

When can Boeing start withdrawing money if APR is 12% and compounding is done monthly?

Solution



$$i = \frac{12\%}{12} = 1\%$$

$$NPV = 1{,}000\left({}^{P}/_{A}, 1\%, n\right) - \frac{2{,}000\left({}^{P}/_{A}, 1\%, 120 - n\right)}{(1+0.01)^{n}} = \frac{20{,}000}{(1+0.01)^{120}}$$

$$n = 66.7$$

Exercise 2

Suppose you invested \$1000 compounded yearly at 12% APR for 1 year.

You are offered to make another investment but compounded monthly.

At what interest rate should it be such that you should invest?

Solution

$1000(1.12) = 1000\ (1+\frac{i}{12})^{12}$

$$\left((\frac{1120}{1000})^{\frac{1}{12}} - 1 \right) 12 = i$$

i = 0.1138 = 11.38%

Exercise 3

You are offered $20,000 in 4 years at the cost of $X today. At most how much should be X for this offer to be attractive if you know you can make 5% in the coming two years and 8% thereafter.

Solution

$$X = \frac{20000}{(1+5\%)^2(1+8\%)^2}$$

$$X = 15552.6$$

Exercise 4

You want to have \$10,000 saved ten years from now. How much less do you have to deposit today to reach this goal if you can earn 6% rather than 5% on your savings?

Solution

Let 10,000 to be the future value in year 10

$$\Delta PV = \frac{10000}{1.05^{10}} - \frac{10000}{1.06^{10}} = 555.2$$

Exercise 5

A company is expecting to get \$10,000 a year from today if it invests \$8,600 now on project A. Alternatively, project B asks for twice the cost of A now and promises twice what A provides, but two years from today. Should the company take any of these projects if the best return that can be obtained in the market is 15%?

Solution

Project A	Project B
-8600	-17200
+10000	0
0	20000

$$NPV_A = -8600 + \frac{10000}{(1+15\%)^1} = 96$$

$$NPV_B = -17200 + \frac{20000}{(1+15\%)^2} = -2077$$

Alternatively, if the company invests in the market, in one year, investment would grow into 8,600*1.15 = 9,890 which is less than what project A offers.
In two years, 17,200 would grow into $17,200*1.15^2 =$22,747 which is more than what project B offers. Take project A.

Cost

A project cash flow should be treated on a “what-if-this-existed” basis.

Important Terms:

Sunk Costs: costs that already occurred, accepting or rejecting does not change the fact these costs occurred, ie. market research done prior to the project

Opportunity Costs: the best alternative use of resources, ie. if you had a piece of land, instead of building a factory, you may want a farm

Side Effects: embarking on this current project may affect other projects

ie. it may re-direct of funds from profitable segments to new projects and areas (*erosion*)

ie. introduce new products into a market where these products are already established (*cannibalism*)

ie. investing in a new project may improve sales on another segment (*synergy*)

Project evaluation

There are several evaluation methods:

- Net Present Value (NPV)
- Payback Period (PP)
- Discounted Payback Period (DPP)
- Profitability Index (PI)
- Incremental Profitability Index (IPI)

Payback period

Payback Period: the amount of time it will take for the project to re-obtain its initial cash flow ("break-even")

We use that amount as a cut off for accepting a project

Note: Payback Period does not take into account of the time-value of money

Example

For the comparison of project A and B below:

Project A takes 2 years to pay back its initial investment, Project B takes a little more than 2 years to break even.

Therefore, we choose project A

Year	Project A	Project B
0	-100,000	-150,000
1	50,000	70,000
2	50,000	70,000
3	50,000	210,000

Exercise 6

A project will bring money for 4 years after its initial investment of \$10,000. In each of the first 3 years cash flow will be \$X, and in 4^{th} year, it will be \$6,000. If the payback period of this project is 3 years and 2 months, what is NPV given APR is 10%?

Solution

(10,000-3X) = 6000 * (2months/12months)

X = 3,000

NPV = 3,000/1.1 + 3,000/1.1^2 + 3,000/1.1^3 + 6,000/1.1^4

NPV = $1,558.64

Payback period evaluation

The Payback Period is primitive and has its flaws, but it provides a number of benefits as well!

Pros	Cons
Simple and easy to use	Does not account for time-value of money
Break-even in defined time-frame	Does not account for future cash flows

Example

It will cost $2,600 to acquire a small machine. Revenue is expected to be $1,400 a year for three years. After the three years, the machine is expected to be worthless What is the payback period of the machine?

A. 0.86 years

B. 1.46 years

C. 1.86 years

D. 2.46 years

E. 2.86 years

Payback period=1+$\frac{\$2600-\$1400}{\$1400}$ =1.86 years

Discounted payback period

Discounted Payback Period: a hybrid between Net Present Value and Payback Period

Takes into account of time-value of money and select the option that meets the cut off period

1. Discount all cash flows
2. Compute the payback period given discounted cash flows
3. Select the option that meets the time requirement

Note: Discounted Payback Period is not the same as combining NPV and PP!

Example

For the comparison of project A and B below:

Given a discount rate of 10%

Project A takes a little more than 2 years to pay back its initial investment

Project B takes a approximately 3 years to break even

Therefore, we choose project A

Year	Project A	Discounted A	Project B	Discounted B
0	-100,000	-100,000	-150,000	-150,000
1	50,000	45,454	30,000	27,272
2	50,000	41,322	30,000	24,793
3	50,000	37,565	300,000	225,394