Portfolio Analysis:

Two stocks : A & B

r_i^A : return of stock A at period i.

r_i^B : " " " B " " ".

$$\bar{r}_A = \frac{\sum_{i=1}^{N} r_i^A}{N} \rightarrow \text{Average return}$$

expected return

$$E(r_A) = \bar{r}_A$$

$$E(r_B) = \bar{r}_B$$

Expected Return is the same as average return for our purposes. (They are not exactly the same statistically!)

Covariance:

$$\sigma_{A,B} = \sum_{i=1}^{N} \frac{(r_i^A - \bar{r}_A)(r_i^B - \bar{r}_B)}{N-1}$$

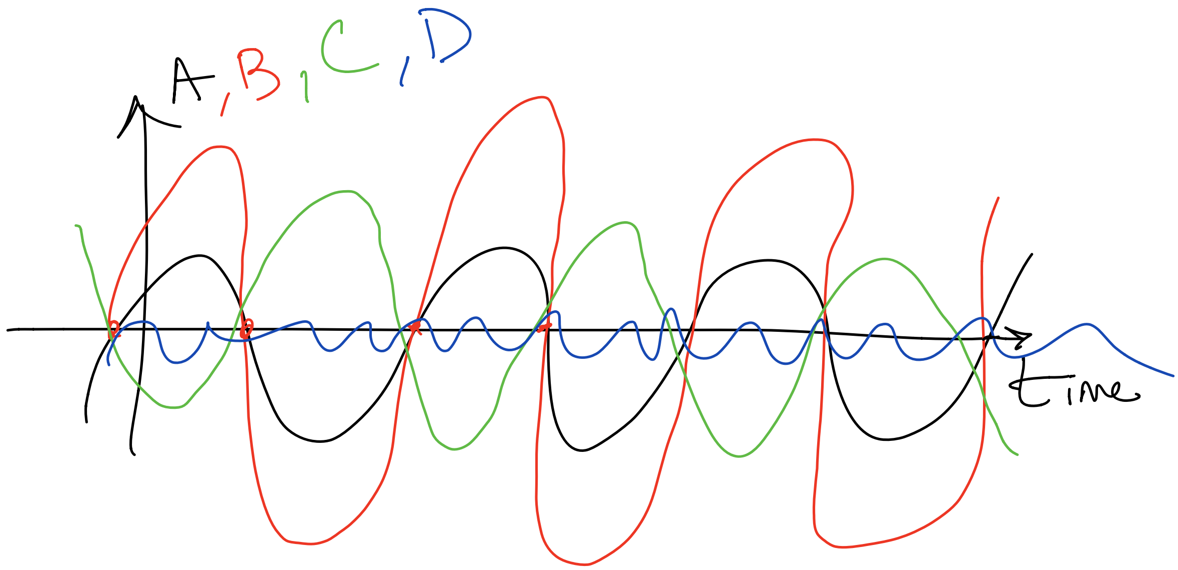
Cov(A,B)

$$\rho_{A,B} = \mathrm{Corr}(A,B) = \frac{\sigma_{A,B}}{\sigma_A \sigma_B}$$

Standard deviation of A

SD of B

$$-1 \leq \rho_{A,B} \leq 1$$



$\rho_{A,B} = +1$ A, B $\rightarrow$ Perfect positive correlation

$\rho_{A,C} = -1$ A, C $\rightarrow$ Perfect negative correlation

$\rho_{A,D} = 0$ (B, C) No correlation bw any variable and D.

Let's form a portfolio:

A, B

objective: min risk, max expected return

Budget: \$100

We can calculate and find numeric values from data for: $\bar{r}_A$, $\bar{r}_B$, σ_A, σ_B, $\sigma_{A,B}$, $\rho_{A,B}$

w_A: weight of A in the portfolio

w_B: " " B " " "

$$w_A + w_B = 1$$

Note that $w_A > 0$, $w_B > 0$ is NOT a requirement.

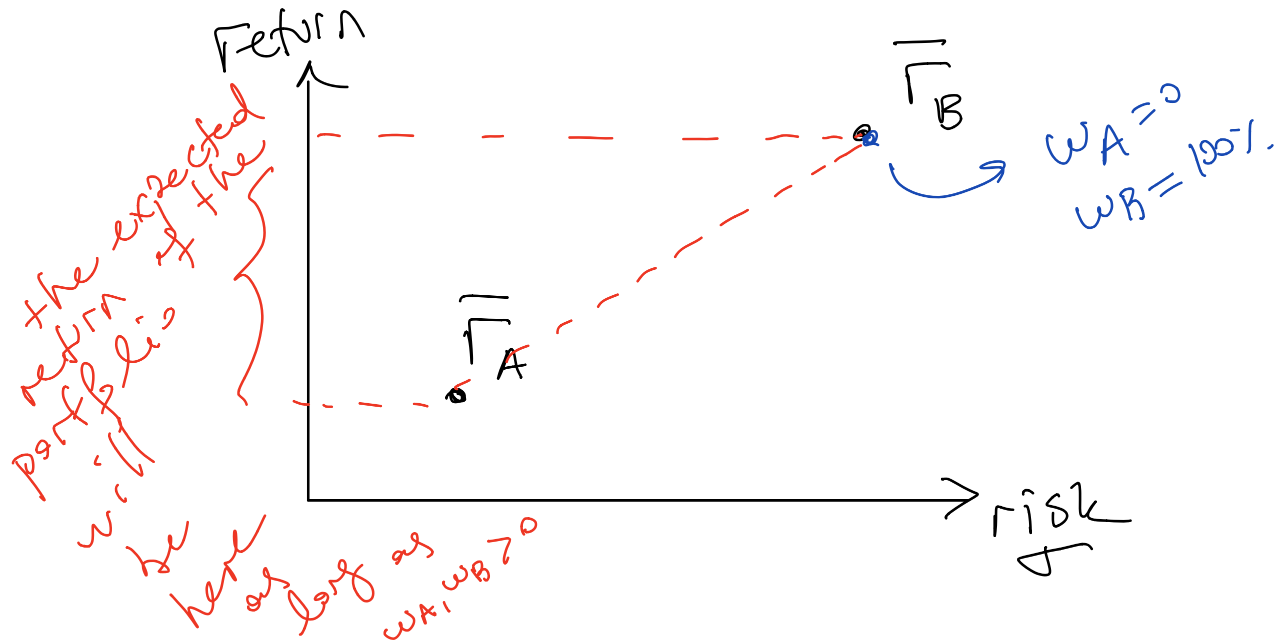
If $w_A = 0.2$
$w_B = 0.8$
then \$20 invested on A and \$80 invested on B.

P: portfolio

w_A of \$100 invested on A

w_B of \$100 " " B.

$$E(r_P) = \bar{r}_p = w_A \bar{r}_A + w_B \bar{r}_B.$$



Assume $\rho_{A,B} = +1$

then $Var(P) = \sigma_p^2$

$$= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2$$

For any $-1 \leq \rho_{A,B} \leq 1$

$$Var(P) = \sigma_p^2 =$$

$$w_A^2 \sigma_A^2 + 2 w_A w_B \sigma_{A,B} + w_B^2 \sigma_B^2$$

Because $\rho_{A,B} = \dfrac{\sigma_{A,B}}{\sigma_A \cdot \sigma_B}$

$$\Rightarrow \sigma_{A,B} = \rho_{A,B} \sigma_A \sigma_B$$

By substituting

Also equal to

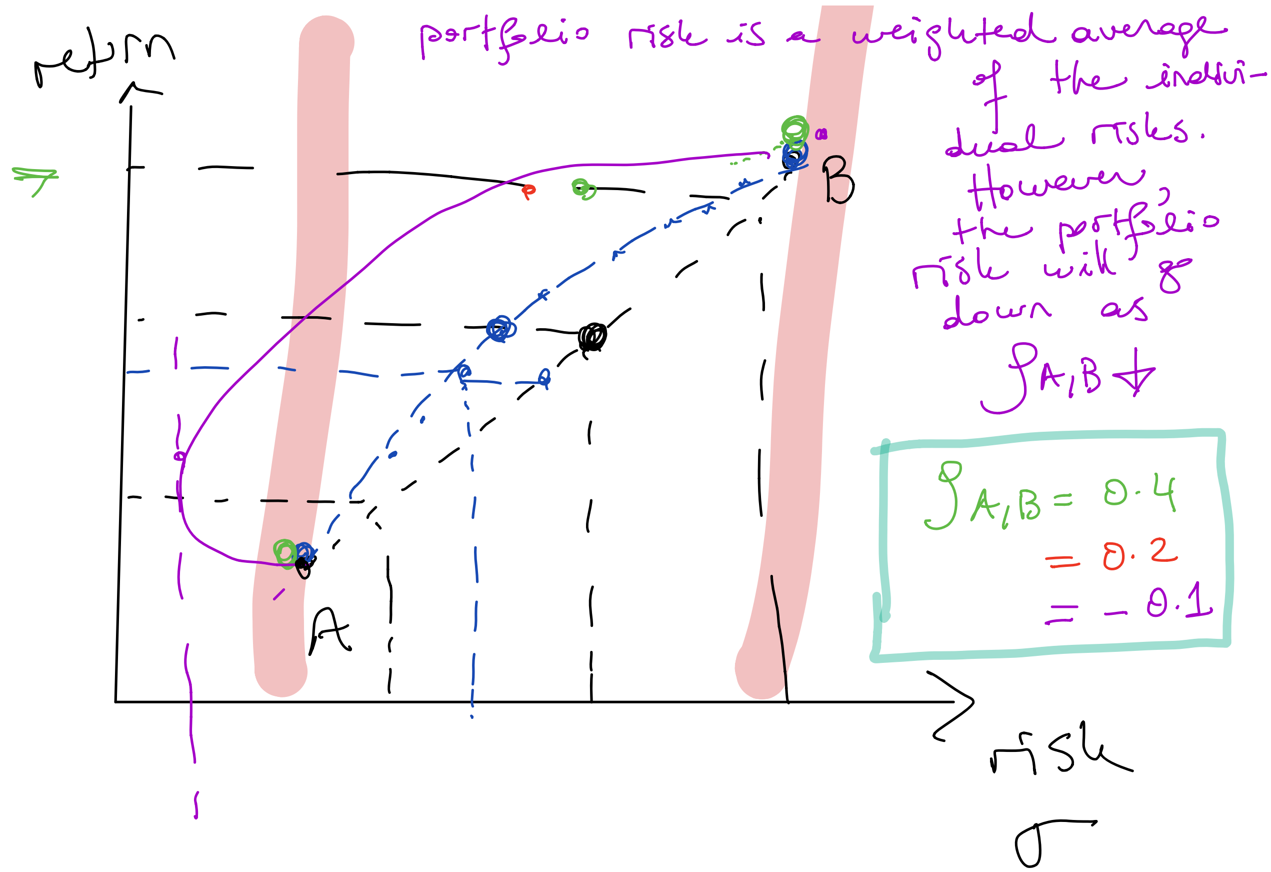
$$\sigma_p^2 = w_A^2 \sigma_A^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{A,B} + w_B^2 \sigma_B^2$$

If $\rho_{A,B} = +1$ then $\sigma_p^2 = w_A^2 \sigma_A^2 + 2 w_A w_B \sigma_A \sigma_B + w_B^2 \sigma_B^2$

This is a perfect square

$$= (w_A \sigma_A + w_B \sigma_B)^2$$

Interpretation: If there is perfect positive correlation between two assets, the

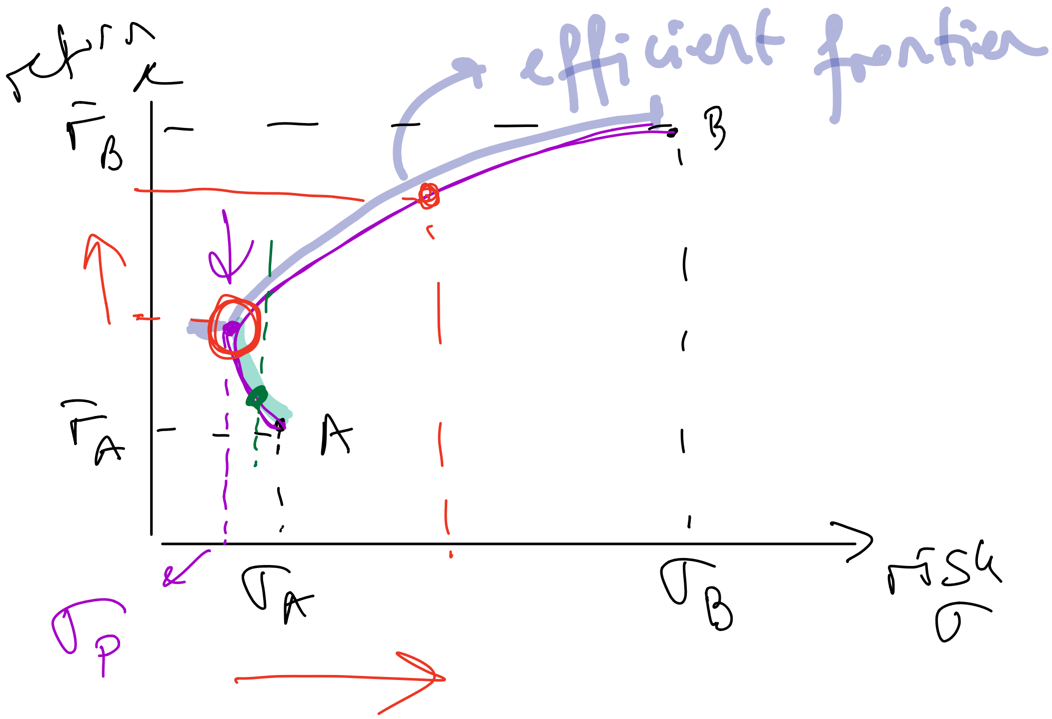


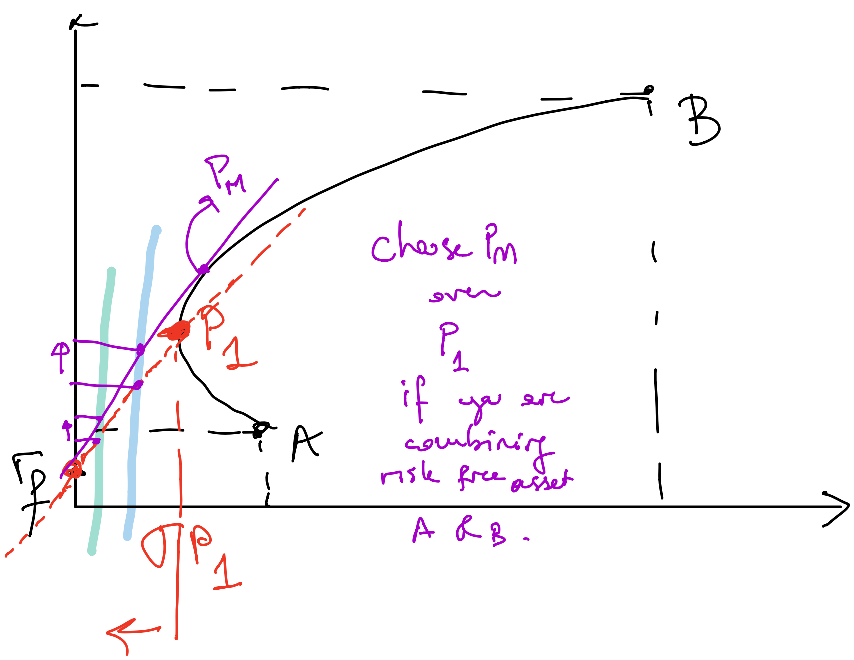
$$\min_{w_A} \sigma_P^2 = w_A^2 \sigma_A^2 + 2\sigma_A \sigma_B w_A (1-w_A) \cdot \rho_{A,B} + (1-w_A)^2 \sigma_B^2$$

$w_B = 1 - w_A$

$$\partial \sigma_P^2 / \partial w_A = 0$$

will get us the minimum variance portfolio MVP!

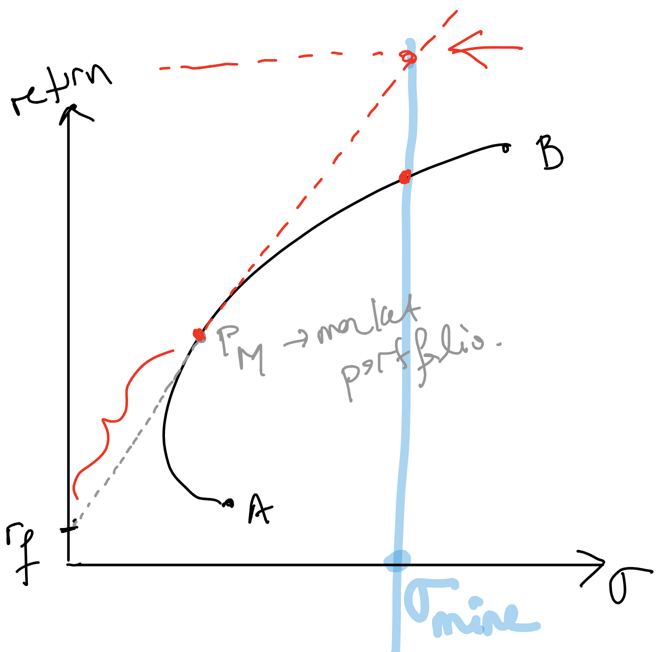




If I form a portfolio with risk free asset (A, B) → P_1

new portfolio P_2 with P_1 and risk free asset

$$\sigma_{P_2}^2 = w_{P_1}^2 \sigma_{P_1}^2 + \underbrace{\cancel{\cdots}}_{\text{zero}} + \underbrace{\cancel{\cdots}}_{\text{zero}}$$



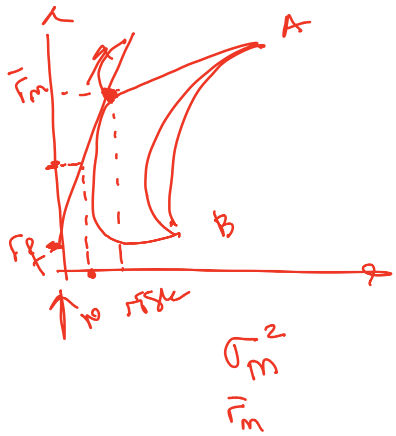
standard deviation of the risk free asset is zero! As well as its covariance to any risky asset!

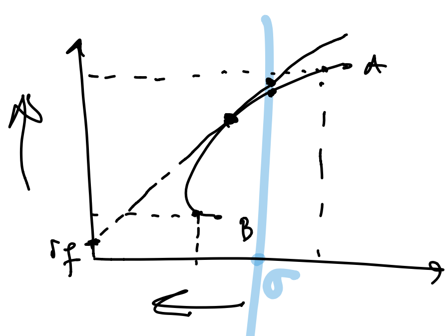
More assets, A, B, C, D, E ...

	A	B	C	D
A	$\sigma_A^2 w_A^2$	$w_A w_B \sigma_{A,B}$	$w_A w_C \sigma_{A,C}$	$w_A w_D \sigma_{A,D}$
B		$\sigma_B^2 w_B^2$		
C			$\sigma_C^2 w_C^2$	
D				...
E				
⋮				

$$\sigma_P^2 = \sum \text{all the terms}$$

opportunity set with many assets not only two, looks like this with a smooth left boundary





Numeric examples of positive & negative weights next lecture!