Bond Valuation

Bonds Basics

Bonds: certificate that shows that a borrower owes a specified sum

the borrower has agreed to make interest and principal payments on designated dates to repay the amount

Terminology

Maturity (*t*): the date or time until the entirety of the bond is paid out or “expires”

Face Value (*F*): the payment at maturity (also known as the *Par Value*)

A bond is known to be paid at “face value” at some “maturity date”

Note: if the *Bond Value* is equal to the *Face Value* at then the bond is known as a *Par Bond*

Pure Discount Bonds

Pure Discount Bond: promises a single payment at a fixed future date

Pure Discount Bonds are also known as *Zero Coupon Bonds* due to its single payment

The holder receives no payments until maturity

ie. A&W Corp. offered a one-year pure discount bond paying $1000 in 1 year Face

Value = 1000; Maturity = 1 year

Pure Discount Bond Valuation

Consider a Pure Discount Bond that pays at face value of *F* in *t* years

where the interest rate *R* (market rate) is constant in each of the *t* years

The value of the bond (*BV*) can be computed by:

$$BV = \frac{F}{(1+R)^t}$$

BV is the amount that we are willing to pay for the bond at this very moment

Note: this is simply the *Present Value* (*PV*) of some amount paid out in the future *t* years

Pure Discount Bond Valuation

Suppose A&W Corp. offered a ten-year pure discount bond with a face value of $1000

where the market rate is 10%

Pure Discount Bond Valuation

Suppose A&W Corp. offered a ten-year pure discount bond with a face value of $1000 where the market rate is 10%

$$BV = 1000 / (1+0.10)^{10} = \$358.54$$

As the buyer, we will pay no more than $358.54 for the bond As the seller, they will sell no less than $358.54 for the bond

Level Coupon Bonds

Level Coupon Bonds: pays not only the face value at maturity, but also individual payments regularly in between

Coupon Payments (*C*): are fixed payments distributed to the holder of the bond at fixed periods (ie. every 6 months) before the bond is paid out at maturity

Coupon Rate (*r*): is coupon payment as a the percentage of the face value (generally the market rate)

$$r = \frac{C}{F}$$

Level Coupon Bond Valuation

Consider a Level Coupon Bond with *C* coupon paid every year and face value *F* maturing in *t* years where the market rate is *R*, constant over *t* years

The value of the bond (*BV*) can be computed by:

$$BV = \frac{C}{(1+R)^1} + \frac{C}{(1+R)^2} + \ldots + \frac{C}{(1+R)^t} + \frac{F}{(1+R)^t}$$

We can also represent this as an annuity of *C* over *t* years with an interest rate of *R*

$$BV = C(PV/A, R, t) + \frac{F}{(1+R)^t}$$

Level Coupon Bond Valuation

Suppose A&W Corp. offered a 20-year bond with a face value of $1000

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Level Coupon Bond Valuation

Suppose A&W Corp. offered a 20-year bond with a face value of $1000 with a coupon rate of 10%, where the market rate is 10%

We first need to find the coupon payment *C* from the coupon rate *r* $C = 1000(0.10) = 100$

We can then take the Bond Value (*BV*) as the Net Present Value of the sum of the coupon payments and the face value

$BV = 100 / (1+0.10)^1 + 100 / (1+0.10)^2 + \ldots + 100 / (1+0.10)^{20} + 1000 / (1+0.10)^{20} = \$\mathbf{1000.00}$

Or as a sum of an annuity and the face value

$BV = 100\ (PV/A,\ 10\%,\ 20) + 1000 / (1+0.10)^{20} = (100/0.10)\ [\ 1- (1/(1+0.10)^{20}\ ]+ 1000 / (1+0.10)^{20} = \$\mathbf{1000.00}$

Level Coupon Bond Valuation

Suppose A&W Corp. offered a 20-year bond with a face value of $1000 with a coupon rate of 10%, where the market rate is 10%

*BV = $**1000.00***

Notice that given a coupon rate equal to the market rate, our bond is valued at *Face Value* or *Par Value*
This happens because the coupon payments cover the discounted amount

This type of bond is also known as a *Par Bond*

Changing Market Rates

It is safe to assume that the market rates will change over time The changes to the rate also affects the value of our bonds:

If the rate drops such that $R < r$,
our bond now becomes much more valuable as the rest of the market will need to invest at a lower rate

If the rate rises such that $R > r$,
our bond's worth drops as investing in another endeavor will bring us better returns

Note:
r = coupon rate (annual payments expressed as % of bond face value)
R = market rate (discount rate)

Premium and Discount Bonds

Let's suppose the same bond from A&W: a 20-year bond was issued
with a face value of $1000 with a coupon rate of 10%, where the market rate is 10%

Suppose market rate fluctuates where $R = 8\%$, $r = 10\%$ ($R < r$)

$BV = 100 / (1+0.08)^1 + 100 / (1+0.08)^2 + \ldots + 100 / (1+0.08)^{20} + 1000 / (1+0.08)^{20} = \$\mathbf{1196.36}$

Because 1196.36 is greater than Par Value (1000.00), the bond is sold at a *premium*

This is also known as a *premium bond*

Suppose market rate fluctuates where $R = 12\%$, $r = 10\%$ ($R > r$)

$BV = 100 / (1+0.12)^1 + 100 / (1+0.12)^2 + \ldots + 100 / (1+0.12)^{20} + 1000 / (1+0.12)^{20} = \$\mathbf{850.61}$

Because 850.61 is less than Par Value (1000.00), the bond is sold at a *discount*

This is also known as a *discount bond*

Yield to Maturity

Yield to Maturity (*YTM*): is the return rate we get *as a buyer* on a bond priced at a certain value, also known as *yield*.
It is the market interest rate for bonds with similar features.

Suppose a 20-year bond is currently priced at $1196.36

with a coupon payment of $100, and the face value is $1000, what is the yield?

To get the yield, we need to find the rate, *y*, that gives the bond a value of 1196.36

Note: the yield of a bond refers to what we can get out of it if we were to buy it at a certain value

The return on the bond as a seller is the difference between the sell price and and amount paid taken to the present value

Yield to Maturity

Suppose a 20-year bond is currently priced at $1196.36

with a coupon payment of $100, and the face value is $1000, what is the yield?

$$1196.36 = 100 / (1+y)^1 + 100 / (1+y)^2 + \ldots + 100 / (1+y)^{20} + 1000 / (1+y)^{20}$$

We see that *y* is equal to **8%** when we solve for the yield

Now suppose the price drops to $850.61 due to a lack of demand for the bond

$$850.61 = 100 / (1+y)^1 + 100 / (1+y)^2 + \ldots + 100 / (1+y)^{20} + 1000 / (1+y)^{20}$$

We see that *y* is equal to **12%** when we solve for the yield

Note: We observe that the yield and the price of the bond is *inversely related*

As price goes up, yield drops down and vice versa

Exercise 1

You bought a bond with the following specifications last year on this day:

Face Value: $1,000
Time to Maturity: 2 years
Coupon Rate: 10%
On the day you bought the bond, the return you could obtain elsewhere was 10%.
Today, you can obtain 12% in the market and you just collected your $100 coupon payment.

Should you sell your bond today? At what value? Why or why not?

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Should you sell your bond today? At what value? Why or why not?

The value of the bond today is 1,100/1.12 = $982.14

If I do not sell the bond I will get $1,100 in one year's time.
If I can sell the bond at a price of $X where $X > $982.14, I should sell it because, X(1.12)>1,100.
If I can sell the bond at a lower price than $982.14, I should not sell it.
If I can sell the bond at exactly $982.14, which should be the market price of this bond when it is correctly priced, then I am indifferent between selling and not selling. Both options will give me $1,100 in one year.

Exercise 2

You bought two different bonds with the following specifications a year ago when market rate was 10%.

BOND I:
Face Value: $1,000
Time to Maturity (at the time of the issue): 3 years
Coupon Rate: 10%

BOND II:
Face Value: $1,000
Time to Maturity (at the time of the issue): 30 years
Coupon Rate: 10%

Market rate is still 10% and you just collected your first coupon payments. You expect the rates to go down to 8% next year, right after you collect your second coupon payments.

a) If you are the only person that expects this rate change, would you sell BOND I or buy more of it today? How about BOND II? You need to support your answer with calculations and concrete numbers.

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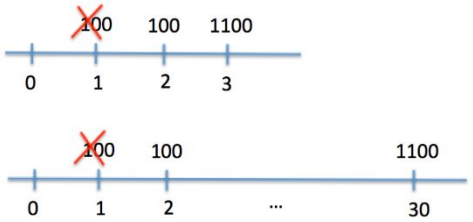

Both Bond I and Bond II will have a value of $1,000 as long as the market rate stays at 10%. If you know that the rate will go down to 8% next year, then the value of the bond next year after the collection of the second coupon is the present value of the face value discounted 1 year (28 years for Bond II) and the present value of an annuity of 100 that runs for 1 year (28 years for Bond II) calculated at 8%:

BVI = 1100/1.08 = 1,019

BVII = $100(1.08^{28} - 1)/((0.08)(1.08^{28} + 1)) + 1000/1.08^{28}$ = 1,117

Then, if I sell Bond I and buy Bond II right now, I will profit more since the value of Bond II will go up by a higher margin.

(Remember that this is the exact reasoning behind a flattening or inverted yield curve, when rates are expected to go down, people switch from short term to long term bonds)

Exercise 2

b) If the rest of the market has the same expectations about the rate change,
Would your answer to part (a) change?
How would your expected profit change compared to part (a)?
(No calculation is necessary. Explain your reasoning with a few sentences)

You would still want to switch to longer term bonds but since the rest of the market will do the same, the price of long term bonds will start to increase, erasing the higher-than-market profit opportunity you had in part (a).

Exercise 3

A bond has a price of $1,080.42. It has a face value of $1,000, a semi-annual coupon of $30, and a maturity of five years. What is its yield to maturity?

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$BV = C\,(P/A, R, t) + F / (1 + R)^t$
$1{,}080.42 = 30((1+R)^{10}-1)/(R(1+R)^{10}) + 1000 / (1+R)^{10}$
This cannot be found analytically.

Find by trial and error on excel.
R = 2.1% but this is for six months

The yield to maturity (YTM) is given per year, so, YTM = 4.2%.
$(1 + 2.1\%)^2 - 1 = 4.2\%$

Exercise – Extra Practice

This is a practice question that we may solve when we go over the default rate in week 9:

You own Corporation X bonds that have 1 year to maturity, $1,000 face value and 10% coupon rate. The promised yield on this bond is 12% today. However, there is a 5% chance that you will only get half of what is promised from now on. What is the market rate? (hint: remember that the market rate is the rate that your alternative investments offer)

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The price of the bond is P = 1,100/1.12 = $982.14
The promised cash flow is $1,100.
*However, the expected cash flow is 1,100 * 0.95 + 550 * 0.05 = $1,072.5*
Then, the expected rate this investment offers should match the market:
$982.14 = 1,072.5/(1+R)
R = 9.2%