CASH FLOW VALUATION

Single Period:

Do you prefer to get \$1,000 now or in a year? Your answer is most probably "now".

Do you prefer \$1,000 now or \$1,100 in a year? The answer depends on the interest rate. If interest rate is greater than 10%, you prefer \$1,000 now, if less than 10%, you prefer \$1,100 in a year. If the interest rate is exactly 10%, you are indifferent.

\$1000 now, will be \$1,080 in 1 year if r=8%

FV = $C_0\ (1+r)$ PV = $\frac{C_1}{1+r}$

Consider an investment which costs \$1,000 now and promises to pay \$1,200 in one year. What is the value of this investment? The interest rate on a comparable investment is 8%.

NPV: PV of future cash flows minus the PV of cost, NPV=1,200/(1.08) - 1,000 = 111.11

Multi-period:

Invest \$1,000 at 10% for 2 years.

1,000(1+.1) = 1,000+100 end of year 1

1,000(1+.1)(1+.1)=1,000+100+100+10 = 1,210 end of year 2

Compound interest refers to the accumulating interest over the principal as well as over the interests of previous terms.

FV = $C_0(1+r)^T$ PV = $\frac{C_1}{(1+r)^T}$

You can use the tables provided at the end of your book to arrive at compounding factors.

TABLE 1 Future Value of \$1
FV = \$1 $(1+i)^n$

n/i	1.0%	1.5%	2.0%	2.5%
1	1.01000	1.01500	1.02000	1.02500
2	1.02010	1.03022	1.04040	1.05063
3	1.03030	1.04568	1.06121	1.07689
4	1.04060	1.06136	1.08243	1.10381
5	1.05101	1.07728	1.10408	1.13141

TABLE 2 Present Value of \$1
PV = $\frac{\$1}{(1+i)^n}$

n/i	1.0%	1.5%	2.0%	2.5%
1	0.99010	0.98522	0.98039	0.97561
2	0.98030	0.97066	0.96117	0.95181
3	0.97059	0.95632	0.94232	0.92860
4	0.96098	0.94218	0.92385	0.90595
5	0.95147	0.92826	0.90573	0.88385

Example: To finance your company's operations you borrow a long-term loan of \$10,000 today. You will payback \$10,612 in 3 years. How much r the bank charges?

PV=10,000

FV = $10{,}000(1+r)^3$ =10,612, solve for r.

Example: You have \$5,000 today. r=8%. How long would it take to double your money?

PV=5,000

FV = 2x5,000=10,000=$5{,}000(1+0.08)^n$, solve for n.

Net Present Value of a Cash Flow: NPV = $-C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} \dots + \frac{C_T}{(1+r)^T}$

Compounding and Discounting may occur more frequently than yearly:

FV = $C_0(1 + r/m)^{mT}$

Effective Annual Interest Rate (EAR) = $(1 + r/m)^{m}$-1

Continuous Compounding: FV= $C_0\ e^{rT}$

Perpetuity: A constant stream of cash flows without end.

PV = C/r

Growing Perpetuity: Same as perpetuity, except, the cash stream is not constant but growing at a certain rate, g.

PV = C/(r-g)   provided that g<r

Annuity: A constant stream of cash flows for T periods.

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Growing Annuity:

$$PV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{(1+r)}\right)^T\right]$$

By using these basic formulas you can solve different kinds of problems such as:

1. Delayed Annuity
2. Annuity beginning now instead of at period 1
3. Infrequent annuity
4. Equating present value of two annuities.

Here are some examples with solutions:

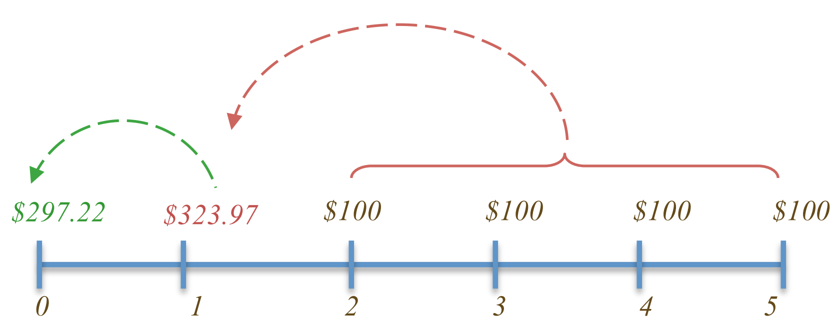
Perpetuity and Annuity Examples:

1. What is the present value of a four-year annuity of \$100 per year that makes its first payment two years from today if the discount rate is 9%?

$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$

$$PV_0 = \frac{\$327.97}{1.09} = \$297.22$$

Exercise: Solve the same problem using the annuity formula



2. Effective Rate: If your company invests on increasing sales today at a cost of \$100,000, for the coming 4 years sales are projected to go up by \$6,000 in the first three quarters and \$15,000 in the 4^{th} quarter. What is the effective annual rate of this investment?

Is this correct? $100,000(1+r)^4 = 4*(3*6,000+15,000)$

No! This does not take into account the time value of money!!!

Cash, that you will be obtaining or paying at different times, cannot be simply added or subtracted!

Rather, all the cash need to be brought in to the same period. Let r be the annual rate that you would obtain annually if you were to invest your money on an alternative project. Then:

$$100,000 = 6,000/(1+r/4)+6,000/(1+r/4)^2+6,000/(1+r/4)^3+15,000/(1+r/4)^4+6,000/(1+r/4)^5+\ldots$$

$$+15,000/(1+r/4)^{16}$$

This can be simplified as

$$100,000 = 6,000\ A^{16}{}_{r/4\%} + 9,000[1/(1+r/4)4+1/(1+r/4)^8+1/(1+r/4)12+1/(1+r/4)^{16}$$

You can solve for r by trial and error, using excel etc…

3. Assume your company has \$50K in its cash account. According to the general economic forecasts, your best investment opportunity in the market seems to be returning 2% in the coming two years and 4% thereafter.

Instead, a young engineer employee proposes to invest the money on overhauling the production system which will leave the sales the same but is projected to improve the profit margin from 13% to 14%. What should be your current sales for overhauling to be the preferred alternative?

Let the Sales be X. Is this correct? :

$$50,000 = 0.01X/1.02 +0.01X/1.02^2 +0.01X/1.04^3 +0.01X/1.04^4 +\ldots.$$

No!!! Each term in the above equation finds the present value of the extra profit obtained through years 1,2,3…

For example, the fourth term is the answer to the following question: “how much do I need to invest now to have 0.01X dollars 4 years from today?” If we discount by 4% for all of those 4 years, the answer to this question will be incorrect!

Instead, all the cash flow should be discounted at 4% beyond the second year and only at 2% for the first two years. Then:

$$50,000 = 0.01X/1.02 +0.01X/1.02^2 +(1/1.02^2) * [0.01X/1.04 +0.01X/1.04^2 +\ldots.]$$

By using the Perpetuity formula for the last term:

$$50,000 = 0.01X/1.02 +0.01X/1.02^2 +(1/1.02^2) * (0.01X/0.04)$$

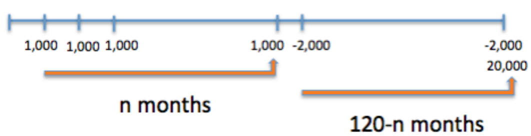
X = \$192,524

4. Consider a 4 year loan with fixed annual payments. The interest rate is 8% ,and the principal amount is $5,000. Following table shows the amortization of this loan.

Exercise: Calculate the fixed payment (i.e. $1,509.60) using the annuity formula.

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	5,000.00	1,509.60	400.00	1,109.60	3,890.40
2	3,890.40	1,509.60	311.23	1,198.37	2,692.03
3	2,692.03	1,509.60	215.36	1,294.24	1,397.79
4	1,397.79	1,509.60	111.82	1,397.78	0.01
Totals		6,038.40	1,038.41	4,999.99	

5. Boeing would like to create a fund to fulfill future battery problem obligations. Beginning a month from today, Boeing will start to invest $1,000 each month. At some point, it will stop investing and start withdrawing $2,000 per month and still have $20,000 in this fund right after its withdrawal 10 years from now. When can Boeing start withdrawing money if APR is 12.06% and compounding is done monthly?



The future value of the cash stream of $1,000 from 1st through nth months should be equal to the present value of the cash stream of $2,000 from (n+1)st through 120th month plus the present value of $20,000:

$$1000\frac{(1+r)^n-1}{r}=2000\frac{(1+r)^{(120-n)}-1}{(1+r)^{(120-n)}r}+\frac{20{,}000}{(1+r)^{(120-n)}}$$

n=42.45, so Boeing can approximately stop investing and start withdrawing after 3 and a half years.

Following are left as an exercise:

Alternative Solution: The above method carries all cash flow to month n. Solve the same problem by carrying everything to month 0.

Variation on the same problem: Assume everything stays the same except that the compounding is done every 15 days. What would be n now?