Bond Valuation: Default Rates

Bonds Basics

Bonds: certificate that shows that a borrower owes a specified sum

the borrower has agreed to make interest and principal payments on designated dates to repay the amount

Terminology

Maturity (*t*): the date or time until the entirety of the bond is paid out or “expires”

Face Value (*F*): the payment at maturity (also known as the *Par Value*)

A bond is known to be paid at “face value” at some “maturity date”

Note: if the *Bond Value* is equal to the *Face Value* at then the bond is known as a *Par Bond*

Pure Discount Bond Valuation

Consider a Pure Discount Bond that pays at face value of *F* in *t* years

where the interest rate *R* (market rate) is constant in each of the *t* years

The value of the bond (*BV*) can be computed by:

$$BV = \frac{F}{(1+R)^t}$$

BV is the amount that we are willing to pay for the bond at this very moment

Note: this is simply the *Present Value* (*PV*) of some amount paid out in the future *t* years

Level Coupon Bonds

Level Coupon Bonds: pays not only the face value at maturity, but also individual payments regularly in between

Coupon Payments (*C*): are fixed payments distributed to the holder of the bond at fixed periods (ie. every 6 months) before the bond is paid out at maturity

Coupon Rate (*r*): is coupon payment as a the percentage of the face value (generally the market rate)

$$r = \frac{C}{F}$$

Level Coupon Bond Valuation

Consider a Level Coupon Bond with *C* coupon paid every year and face value *F* maturing in *t* years where the market rate is *R*, constant over *t* years

The value of the bond (*BV*) can be computed by:

$$BV = \frac{C}{(1+R)^1} + \frac{C}{(1+R)^2} + \ldots + \frac{C}{(1+R)^t} + \frac{F}{(1+R)^t}$$

We can also represent this as an annuity of *C* over *t* years with an interest rate of *R*

$$BV = C(PV/A, R, t) + \frac{F}{(1+R)^t}$$

Yield to Maturity

Yield to Maturity (*YTM*): is the return rate we get *as a buyer* on a bond priced at a certain value, also known as *yield*.
It is the market interest rate for bonds with similar features.

Suppose a 20-year bond is currently priced at $1196.36

with a coupon payment of $100, and the face value is $1000, what is the yield?

To get the yield, we need to find the rate, *y*, that gives the bond a value of 1196.36

Note: the yield of a bond refers to what we can get out of it if we were to buy it at a certain value

The return on the bond as a seller is the difference between the sell price and and amount paid taken to the present value

Yield to Maturity

Suppose a 20-year bond is currently priced at $1196.36

with a coupon payment of $100, and the face value is $1000, what is the yield?

$$1196.36 = 100 / (1+y)^1 + 100 / (1+y)^2 + \ldots + 100 / (1+y)^{20} + 1000 / (1+y)^{20}$$

We see that *y* is equal to **8%** when we solve for the yield

Now suppose the price drops to $850.61 due to a lack of demand for the bond

$$850.61 = 100 / (1+y)^1 + 100 / (1+y)^2 + \ldots + 100 / (1+y)^{20} + 1000 / (1+y)^{20}$$

We see that *y* is equal to **12%** when we solve for the yield

Note: We observe that the yield and the price of the bond is *inversely related*

As price goes up, yield drops down and vice versa

Yield to Maturity

Let us denote the yield to maturity (YTM) as y:

$$y = y_{curr} + y_{cap}$$

where y_{curr} denotes the current yield and y_{cap} denotes the capital gains yield

$$y_{curr} = \frac{C}{BV} \qquad y_{cap} = \frac{P_1 - P_0}{P_0}$$

Note:

discount bonds: $y > y_{curr} >$ coupon rate

premium bonds: $y < y_{curr} <$ coupon rate

P_0 = Purchase Price

P_1 = Selling Price

Yield to Maturity

Note:

Conceptually, the current yield represents the return an investor would expect if the owner purchased the bond and held it for a year (not until maturity). It represents the annual payout as a percentage of the current market price you'll actually pay.

A capital gains yield is the rise in the price of a security (e.g. stock, bond etc) between when you purchase it and when you sell it.

Probability of Default

Up until now, we've only examined cases where there is no chance of default. U.S. government bonds are considered the safest investments as the government has never been bankrupt (default). Corporate bonds, however, always have a chance of defaulting or have some default risk.
In the event of a default, you will receive less than the promised amount of coupon payments or principal payments (or both).

Promised Yield: the promised rate of return of a bond
Expected Yield: the expected rate of return considering the chance of default
Note: promised yield is always greater than or equal to expected yield

"Hair cut" refers to the % amount of previously agreed upon coupon payments or principal payments that is not paid in the event of a default. A 30% hair cut would result in the payments being 70% of the original amount.

Probability of Default

If there is a chance of default, we can calculate the expected bond value and expected yield by using the expected value of coupon payments and principal payments in the Bond Value equation:

$$BV = \frac{\hat{C}}{(1+R)^1} + \frac{\hat{C}}{(1+R)^2} + \ldots + \frac{\hat{C}}{(1+R)^t} + \frac{\hat{F}}{(1+R)^t}$$

The *expected value* of the coupon payments and principal payments are:

$$\hat{C} = (1-d)(C) + (d)(1-h)(C)$$
$$\hat{F} = (1-d)(F) + (d)(1-h)(F)$$

d = default rate (% chance of default)

h = hair cut rate (% less original payment in event of default)

Exercise 1

You bought a 4-year bond with 10% coupon rate and $1,000 face value is offering a 9% current yield today, exactly two years after its issue date. If the market rate is expected to stay the same for the foreseeable future,

a) What is the yield on this bond today?

b) What would be the capital gains rate from this year to next year?

Exercise 1

You bought a 4-year bond with 10% coupon rate and $1,000 face value is offering a 9% current yield today, exactly two years after its issue date. If the market rate is expected to stay the same for the foreseeable future,

a) What is the yield on this bond today?

Current Yield = Coupon Payment / Current Price of the Bond
9% = 100 / Current Price of the Bond
Current Price of the Bond = $1,111.11

The yield that corresponds to this bond is
$1{,}111.11 = 100/(1+r) + 1100/(1+r)^2$
r = 4.1%

Exercise 1

You bought a 4-year bond with 10% coupon rate and $1,000 face value is offering a 9% current yield today, exactly two years after its issue date. If the market rate is expected to stay the same for the foreseeable future,

b) What would be the capital gains rate from this year to next year?

Solution Alternative 1:
Price next year is 1100/1.041 = 1,056.67
Capital Gains is (1,056.67-1,111.11) /1,111.11 = - 4.9%

Solution Alternative 2:
Yield = Current Yield + Capital Gains Rate
Capital Gains Rate = 9%-4.1% = -4.9%

Exercise 2

Suppose you are thinking of investing on Greek government bonds. Each Greek bond will pay $80.00 at the end of every year for 4 years. At the end of 4th year, each bond will also pay back the principal of $1000. Today is the 1st day of year 1.

a) What is the coupon rate of this bond?

b) What is the price of such a bond if the promised yield is 15%?

Exercise 2

Suppose you are thinking of investing on Greek government bonds. Each Greek bond will pay $80.00 at the end of every year for 4 years. At the end of 4th year, each bond will also pay back the principal of $1000. Today is the 1st day of year 1.

a) What is the coupon rate of this bond?

80/1000=8%

b) What is the price of such a bond if the promised yield is 15%?

Exercise 2

Suppose you are thinking of investing on Greek government bonds. Each Greek bond will pay $80.00 at the end of every year for 4 years. At the end of 4th year, each bond will also pay back the principal of $1000. Today is the 1st day of year 1.

b) What is the price of such a bond if the promised yield is 15%?

$80/1.15 + 80/1.15^2 + 80/1.15^3 + 80/1.15^4 + 1000/1.15^4 = 800.15$

Note that the first four terms of the left hand side is the present value calculation of an annuity. We could have also used the annuity formula, or the table to find the present value of a cash stream, 80, that runs for 4 years.

Exercise 2

c) Greece has some probability of going bankrupt within 1 year. Suppose that the probability of Greek default is 30% and if Greece defaults, assume that it will only pay 50% (this is called a “hair cut”) of the agreed amount of coupon and principal payments in that and the subsequent years.

Alternatively, you can invest on a 4-year US bond with 4% coupon and $1,000 face value that sell for $964.54 today. At most how much would you pay for a Greek bond given the information provided in parts (a), (b), and (c)?

Exercise 2

c) Greece has some probability of going bankrupt within 1 year. Suppose that the probability of Greek default is 30% and if Greece defaults, assume that it will only pay 50% (this is called a “hair cut”) of the agreed amount of coupon and principal payments in that and the subsequent years.

Alternatively, you can invest on a 4-year US bond with 4% coupon and $1,000 face value that sell for $964.54 today. At most how much would you pay for a Greek bond given the information provided in parts (a), (b), and (c)?

The yield offered by the US Bond is

$$964.54 = 40/(1+r) + 40/(1+r)^2 + 40/(1+r)^3 + 40/(1+r)^4 + 1000/(1+r)^4$$

Solving this equation by trial and error on excel, r = 5%

Market rate is r (best alternative investment)

Exercise 2

c) Greece has some probability of going bankrupt within 1 year. Suppose that the probability of Greek default is 30% and if Greece defaults, assume that it will only pay 50% (this is called a “hair cut”) of the agreed amount of coupon and principal payments in that and the subsequent years.

Alternatively, you can invest on a 4-year US bond with 4% coupon and $1,000 face value that sell for $964.54 today. At most how much would you pay for a Greek bond given the information provided in parts (a), (b), and (c)?

We can either calculate the value of the bond under default and no default cases and then take the expected value or, calculate the expected value that corresponds to each period and use the new cash flow in calculating the value of the bond:
Expected Coupon payments: 0.3*40 + 0.7*80 = 68,
Expected face value payment: 0.3*500 + 0.7*1,000 = 850
$68/1.05 + 68/1.05^2 + 68/1.05^3 + 68/1.05^4 + 850/1.05^4 = 940.42$

Exercise 3

You own Corporation X bonds that have 1 year to maturity, $1,000 face value and 10% coupon rate. The promised yield on this bond is 12% today. However, there is a 5% chance that you will only get half of what is promised from now on.

What is the market rate? (hint: remember that the market rate is the rate that your alternative investments offer)

Exercise 3

You own Corporation X bonds that have 1 year to maturity, $1,000 face value and 10% coupon rate. The promised yield on this bond is 12% today. However, there is a 5% chance that you will only get half of what is promised from now on.

What is the market rate? (hint: remember that the market rate is the rate that your alternative investments offer)

The price of the bond is P = 1,100/1.12 = $982.14

The promised cash flow is $1,100.
However, the expected cash flow is 1,100 * 0.95 + 550 * 0.05 = $1,072.5
Then, the expected rate this investment offers should match the market:
$982.14 = 1,072.5/(1+r)
r = 9.2%