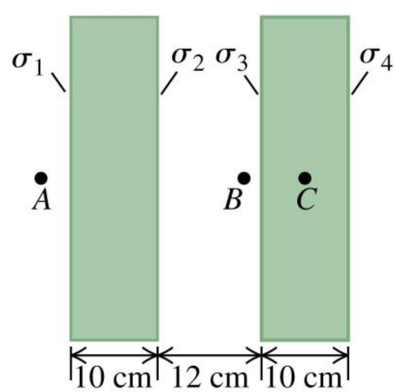
22.30 • Two very large, nonconducting plastic sheets, each 10.0 cm thick, carry uniform charge densities σ_1, σ_2, σ_3, and σ_4 on their surfaces (**Fig. E22.30**). These surface charge densities have the values $\sigma_1 = -6.00\ \mu\text{C/m}^2$, $\sigma_2 = +5.00\ \mu\text{C/m}^2$, $\sigma_3 = +2.00\ \mu\text{C/m}^2$, and $\sigma_4 = +4.00\ \mu\text{C/m}^2$. Use Gauss's law to find the magnitude and direction of the electric field at the following points, far from the edges of these sheets: (a) point A, 5.00 cm from the left face of the left-hand sheet; (b) point B, 1.25 cm from the inner surface of the right-hand sheet; (c) point C, in the middle of the right-hand sheet.

Figure E22.30



22.30. IDENTIFY: The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \dfrac{|\sigma_2|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_1|}{2\epsilon_0} = \dfrac{|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|}{2\epsilon_0}$.

$$E_A = \frac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82 \times 10^5\ \text{N/C to the left.}$$

(b) $E_B = \dfrac{|\sigma_1|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} = \dfrac{|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|}{2\epsilon_0}$.

$$E_B = \frac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95 \times 10^5\ \text{N/C to the left.}$$

(c) $E_C = \dfrac{|\sigma_4|}{2\epsilon_0} + \dfrac{|\sigma_1|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} - \dfrac{|\sigma_3|}{2\epsilon_0} = \dfrac{|\sigma_4| + |\sigma_1| - |\sigma_2| - |\sigma_3|}{2\epsilon_0}$.

$$E_C = \frac{1}{2\epsilon_0}(4\mu\text{C/m}^2 + 6\mu\text{C/m}^2 - 5\mu\text{C/m}^2 - 2\mu\text{C/m}^2) = 1.69 \times 10^5\ \text{N/C to the left.}$$

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.39 • The Coaxial Cable. A long coaxial cable consists of an inner cylindrical conductor with radius a and an outer coaxial cylinder with inner radius b and outer radius c. The outer cylinder is mounted on insulating supports and has no net charge. The inner cylinder has a uniform positive charge per unit length λ. Calculate the electric field (a) at any point between the cylinders a distance r from the axis and (b) at any point outside the outer cylinder. (c) Graph the magnitude of the electric field as a function of the distance r from the axis of the cable, from $r = 0$ to $r = 2c$. (d) Find the charge per unit length on the inner surface and on the outer surface of the outer cylinder.

22.39. **(a) IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.39a.

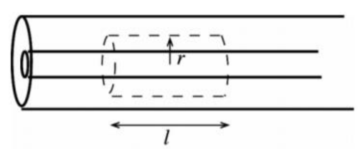


Figure 22.39a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\varepsilon_0}$.

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) IDENTIFY and SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.39b.

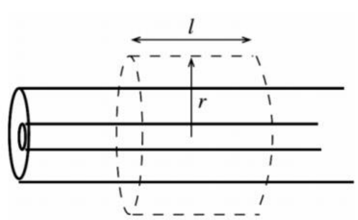


Figure 22.39b

EXECUTE: $\Phi_E = E(2\pi rl)$.

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\varepsilon_0}$.

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(c) IDENTIFY and EXECUTE: $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ for $a < r < b$; $E = 0$ for $b < r < c$;

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.39c.

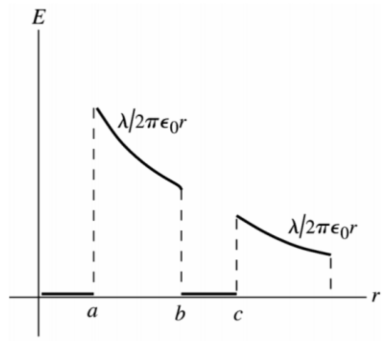


Figure 22.39c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

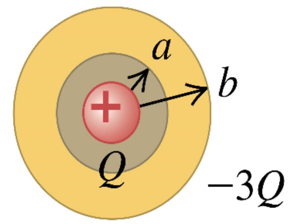
(d) IDENTIFY and SET UP: <u>inner surface:</u> Apply Gauss's law to a Gaussian cylinder with radius r, where $b < r < c$. We know E on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

<u>outer surface:</u> The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per unit length $+\lambda$ on the outer surface.

22.44 • A conducting spherical shell with inner radius a and outer radius b has a positive point charge Q located at its center. The total charge on the shell is $-3Q$, and it is insulated from its surroundings (**Fig. P22.44**). (a) Derive expressions for the electric-field magnitude E in terms of the distance r from the center for the regions $r < a$, $a < r < b$, and $r > b$. What is the surface charge density (b) on the inner surface of the conducting shell; (c) on the outer surface of the conducting shell? (d) Sketch the electric field lines and the location of all charges. (e) Graph E as a function of r.

Figure **P22.44**



22.44. IDENTIFY: Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: **(a)** For $r < a$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge because $E = 0$ inside the conductor, the total charge on the inner surface is $-Q$ and the surface charge density on the inner surface is $\sigma = -\frac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\frac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.44a.

(e) The graph of E versus r is sketched in Figure 22.44b.

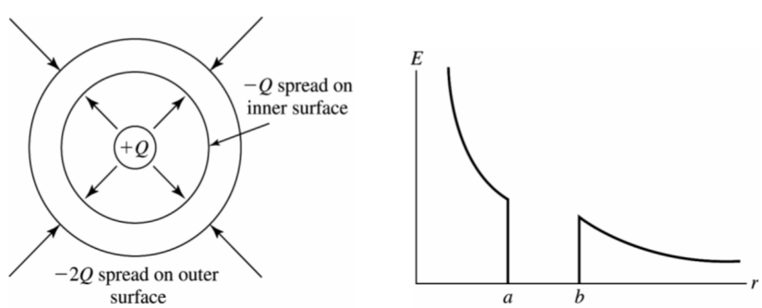
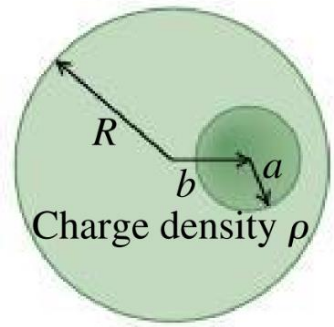


Figure 22.44

EVALUATE: For $r < a$ the electric field is due solely to the point charge Q. For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

22.57 • (a) An insulating sphere with radius a has a uniform charge density ρ. The sphere is not centered at the origin but at $\vec{r} = \vec{b}$. Show that the electric field inside the sphere is given by $\vec{E} = \rho(\vec{r} - \vec{b})/3\epsilon_0$. (b) An insulating sphere of radius R has a spherical hole of radius a located within its volume and centered a distance b from the center of the sphere, where $a < b < R$ (a cross section of the sphere is shown in **Fig. P22.57**). The solid part of the sphere has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole. [*Hint:* Use the principle of superposition and the result of part (a).]

Figure **P22.57**

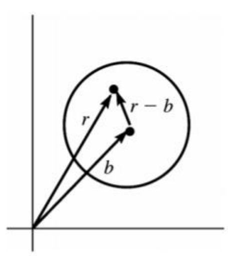


22.57. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\varepsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\varepsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E} = \rho\vec{r}'/3\varepsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.57.



EXECUTE: Thus $\vec{E} = \dfrac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0}$.

Figure 22.57

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r}=\vec{b}$.

SET UP: $\vec{E}=\vec{E}_{\text{uniform}}+\vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho-\rho=0$.)

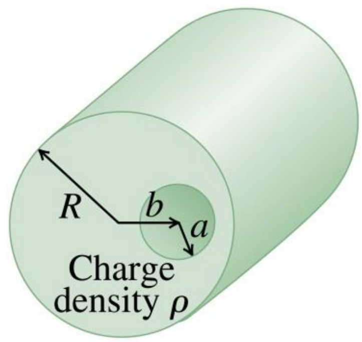
EXECUTE: $\vec{E}_{\text{uniform}}=\dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}}=\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}$, at points inside the hole. Then $\vec{E}=\dfrac{\rho\vec{r}}{3\epsilon_0}+\left(\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}\right)=\dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.58 • A very long, solid insulating cylinder has radius R; bored along its entire length is a cylindrical hole with radius a. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$ (**Fig. P22.58**). The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole. (*Hint:* See Problem 22.57.)

Figure P22.58



22.58. **IDENTIFY:** We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.58, $\vec{r}' = \vec{r} - \vec{b}$. Problem 22.41 shows that at points within a long insulating cylinder, $\vec{E} = \dfrac{\rho\vec{r}}{2\varepsilon_0}$.

EXECUTE: $\vec{E}_{\text{off-axis}} = \dfrac{\rho\vec{r}'}{2\varepsilon_0} = \dfrac{\rho(\vec{r} - \vec{b})}{2\varepsilon_0}$. $\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{off-axis}} = \dfrac{\rho\vec{r}}{2\varepsilon_0} - \dfrac{\rho(\vec{r} - \vec{b})}{2\varepsilon_0} = \dfrac{\rho\vec{b}}{2\varepsilon_0}$.

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b = 0$ and $E_{\text{hole}} = 0$.

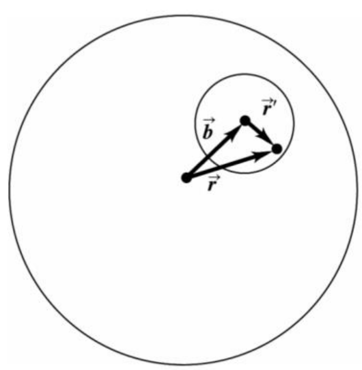


Figure 22.58