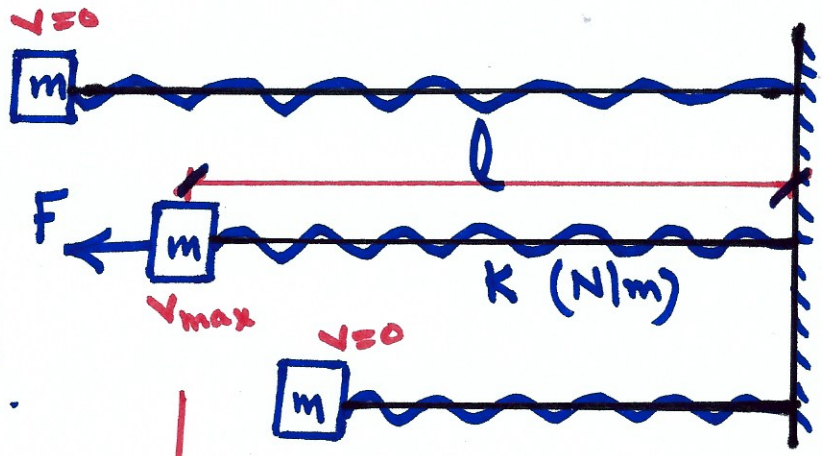
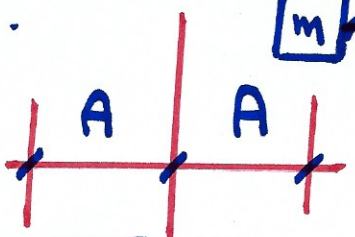
# Simple Harmonic Motion




$$F = K\,A$$




$$T = 2\pi\sqrt{\frac{m}{K}} \qquad m \to Kg,\ K \to N/m$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad f \to Hz$$

$$\frac{1}{2}\,m\,V_{max}^2 = \frac{1}{2}\,K\,A^2$$

$$\omega = 2\pi f \qquad \omega \to rad/s$$

بالاشتقاق:

$$x = A\cos(\omega t + \phi)$$

$$v = -A\omega\sin(\omega t + \phi)$$

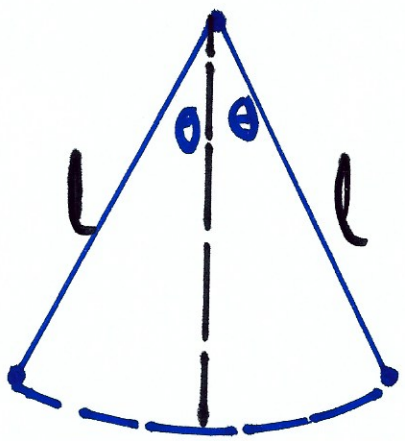
$$a = -A\omega^2\cos(\omega t + \phi)$$

$$\therefore a = -\omega^2 x$$

$$v = \omega\sqrt{A^2 - x^2}$$

## Pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$




كثير ممن حققوا النجاح
يدينون بالفضل للمشاكل التي
تعرضوا لها قبل هذا النجاح.

**14.3** • The tip of a tuning fork goes through 400 complete vibrations in 0.550 s. Find the angular frequency and the period of the motion.

$$T = \frac{t}{n} = \frac{0.550}{400}$$

$$= 1.38 * 10^{-3} \text{ s}$$

$$= \boxed{1.38} \text{ ms}$$

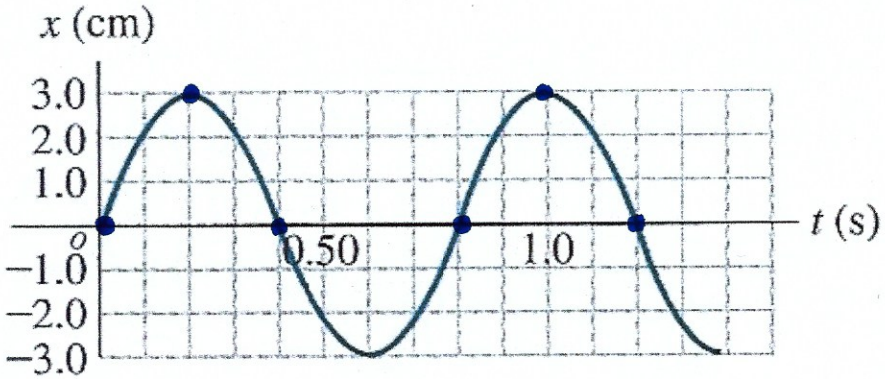
$$f = \frac{1}{T} = 727 \text{ Hz}$$

$$\omega = 2\pi f = \boxed{4570} \text{ rad/s}$$

ما رأيت ظالماً أشبه بمظلوم من الحاسد.

**14.7** • A 2.40-kg ball is attached to an unknown spring and allowed to oscillate. **Figure E14.7** shows a graph of the ball's position $x$ as a function of time $t$. What are the oscillation's (a) period, (b) frequency, (c) angular frequency, and (d) amplitude? (e) What is the force constant of the spring?

Figure **E14.7**




a] $T = 0.8$ s

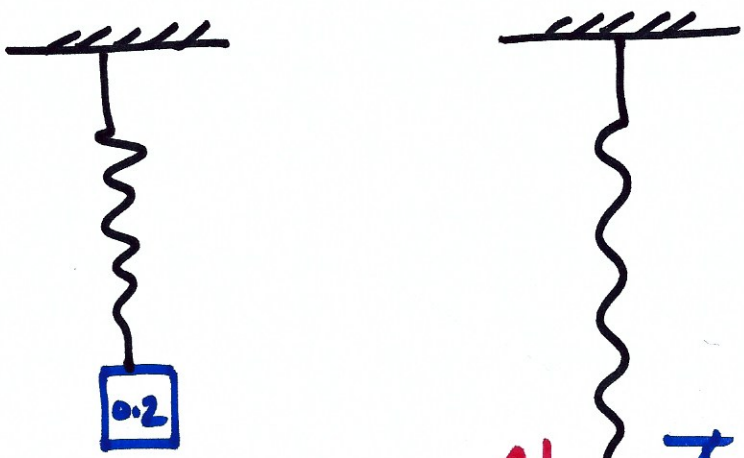
b] $f = \frac{1}{T} = 1.25$ Hz

c] $\omega = 2\pi f = 7.85$ rad/s

d] $A = 3$ cm

e] $0.8 = 2\pi\sqrt{\frac{2.4}{K}}$ $\therefore K = 148$ N/m

**14.8** •• In a physics lab, you attach a 0.200-kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 s. Find the spring's force constant.




$t = 2.6\ s$

$T = 2 * t = 5.2\ s$

$5.2 = 2\pi\sqrt{\frac{m}{K}}$ , $m = 0.2$

$\therefore K = 0.292\ N/m$

**14.12** • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the block is at $x = 0.310$ m, the acceleration of the block is $-5.96\ \text{m/s}^2$. What is the frequency of the motion?

$a$

$f$

$$a = -\omega^2 x$$

$$-5.96 = -\omega^2 (0.310)$$

$$\therefore \omega = 4.38 \text{ rad/s}$$

$$\omega = 2\pi f$$

$$\therefore f = 0.698 \text{ Hz}$$

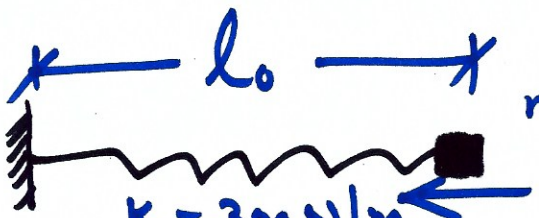
إذا لم تزد شيئا على الحياة
كنت أنت زائداً عليها.

**14.13** • A 2.00-kg, frictionless block is attached to an ideal spring with force constant 300 N/m. At $t = 0$ the spring is neither stretched nor compressed and the block is moving in the negative direction at 12.0 m/s. Find (a) the amplitude and (b) the phase angle. (c) Write an equation for the position as a function of time.

m

k

$V_{max}$




$V_{max} = 12$ m/s

a]

$$\frac{1}{2} m V_{max}^2 = \frac{1}{2} K A^2$$

$2 \quad (12)^2 \quad 300 \quad ?$

$\therefore A = 0.980$ m

b]

$x = A \cos(\omega t + \Phi)$ — Zero

$\cos\Phi = 0 \quad \therefore \Phi = 90^\circ = \frac{\pi}{2}$

c]

$x = -0.980 \cos(\omega t + \pi/2)$

$x = -0.980 \sin(\omega t)$

$\omega = \frac{V_{max}}{A} = 12.2$ rad/s

**14.15** • The point of the needle of a sewing machine moves in SHM along the $x$-axis with a frequency of 2.5 Hz. At $t = 0$ its position and velocity components are +1.1 cm and −15 cm/s, respectively. (a) Find the acceleration component of the needle at $t = 0$. (b) Write equations giving the position, velocity, and acceleration components of the point as a function of time.

a] $\omega = 2\pi f = 15.7$ rad/s

$a = -\omega^2 x = -271$ cm/s²

b] $v = \omega\sqrt{A^2 - x^2}$

$\therefore A = 1.46$ cm

$x = A\cos(\omega t + \phi)$

$\therefore \phi = 0.718$ rad

$x = A\cos(\omega t + \phi)$

$x = 1.46\cos(15.7t + 0.718)$ cm

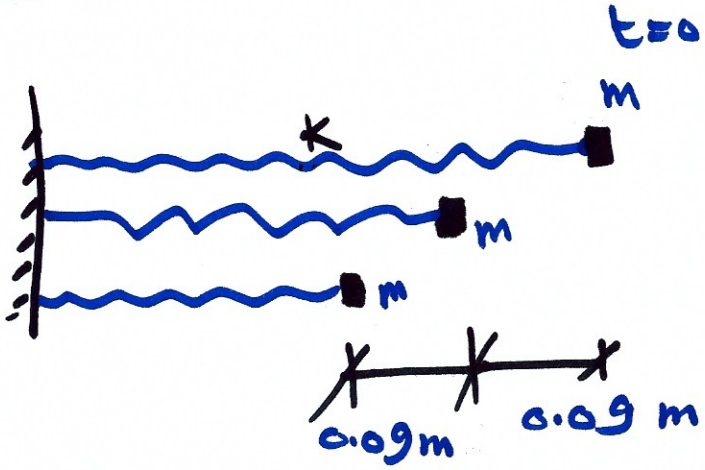
$v = -22.9\sin(15.7t + 0.718)$ cm/s

$a = -360\cos(15.7t + 0.718)$ cm/s²

حين ينجح الإنسان يقول فعلت وفعلت،
وحين يفشل يقول القدر والنصيب

**14.16** •• A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the amplitude of the motion is 0.090 m, it takes the block 2.70 s to travel from $x = 0.090$ m to $x = -0.090$ m. If the amplitude is doubled, to 0.180 m, how long does it take the block to travel (a) from $x = 0.180$ m to $x = -0.180$ m and (b) from $x = 0.090$ m to $x = -0.090$ m?

$A = 0.09$ m




$T = 2.70 \times 2 = 5.4$ s

$\omega = \frac{2\pi}{T} = 1.16$ rad/s

a]

$T = 2\pi\sqrt{\frac{m}{k}}$

T is Same, $\therefore t = \frac{T}{2} = 2.7$ s

b]

$x = A\cos(\omega t)$ Case $A = 0.09$ m

$-0.09 = 0.18\cos(1.16 t_1)$ $\therefore t_1 = 0.903$ s

$-0.09 = 0.18\cos(1.16 t_2)$ $\therefore t_2 = 1.805$ s

$\therefore t = 0.903$ s

**14.18** • A 0.450-kg object undergoing SHM has $a_x = -2.50\ \text{m/s}^2$ when $x = 0.400$ m. What is the time for one oscillation?

$$a_x = -\omega^2 r$$

$$-2.5 = -\omega^2 (0.4)$$

$$\therefore \omega = \underline{2.50}\ \text{rad/s}$$

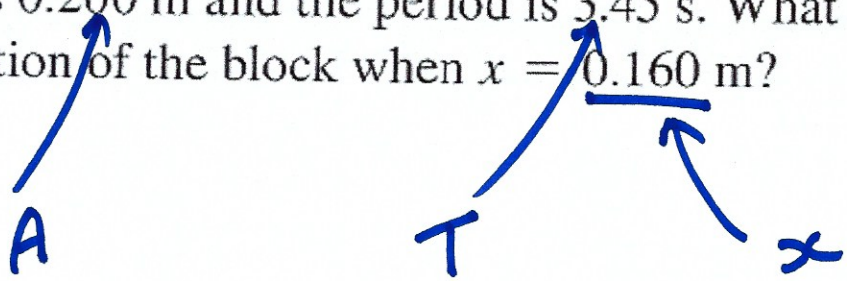
$$\omega = \frac{2\pi}{T}$$

$$2.5 \qquad ?$$

$$\therefore T = \underline{2.51}\ \text{s}$$

من المهم أن لا تُري خوفك لعدو شجاع،
لأن الشجعان يحتقرون الجبناء.

**14.26** • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.200 m and the period is 3.45 s. What are the speed and acceleration of the block when $x = 0.160$ m?




$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.45} = 1.82 \text{ rad/s}$$

$$v = \omega\sqrt{A^2 - x^2} = 1.82\sqrt{0.2^2 - 0.16^2}$$

$$= 0.218 \text{ m/s}$$

$$a = -\omega^2 x = -1.82^2 \times 0.16$$

$$= -0.530 \text{ m/s}^2$$

سيكون بإمكاننا يوما ما حساب حركة
الأجرام السماوية، ولكن ليس جنون البشر.

**14.38** • A proud deep-sea fisherman hangs a 65.0-kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.180 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?


a]

$F = K x$

$65 \times 9.8 = ? \times 0.18$

$\therefore K = 3539 \text{ N/m}$

b]

$T = 2\pi \sqrt{\frac{m}{K}}$

$= 2\pi \sqrt{\frac{65}{3539}}$

$= 0.852 \text{ s}$

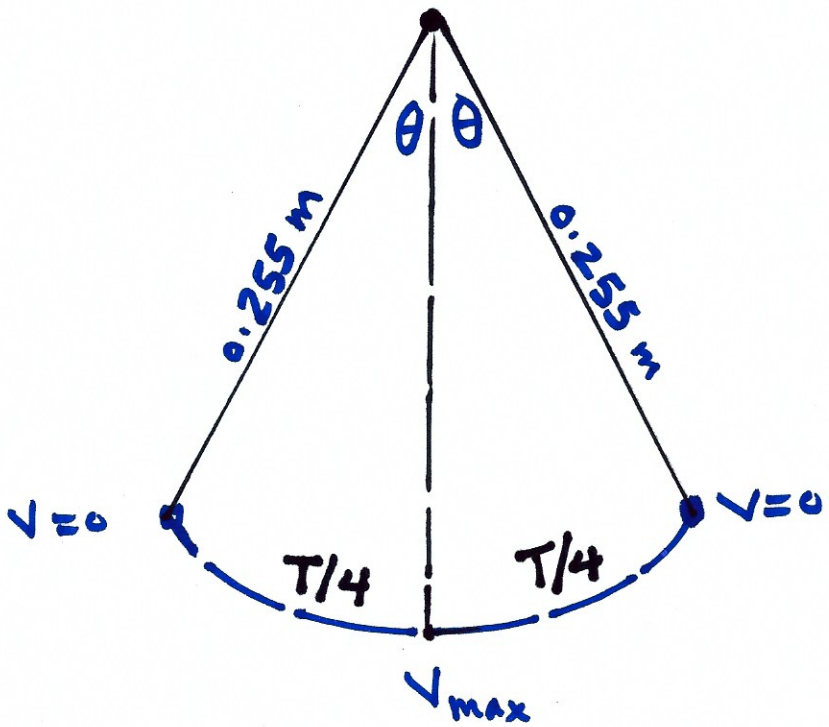
c]

$\frac{1}{2} m V_{max}^2 = \frac{1}{2} K A^2$

$\frac{1}{2} \times 65 \times V_{max}^2 = \frac{1}{2} \times 3539 \times 0.05^2$

$\therefore V_{max} = 0.369 \text{ m/s}$

**14.45** •• You pull a simple pendulum 0.255 m long to the side through an angle of 3.50° and release it. (a) How much time does it take the pendulum bob to reach its highest speed? (b) How much time does it take if the pendulum is released at an angle of 1.75° instead of 3.50°?




a]

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad L \to 0.255 \qquad g \to 9.8$$

$$= 1.01 \text{ s}$$

$$\therefore t = \frac{T}{4} = \boxed{0.253} \text{ s}$$

b]

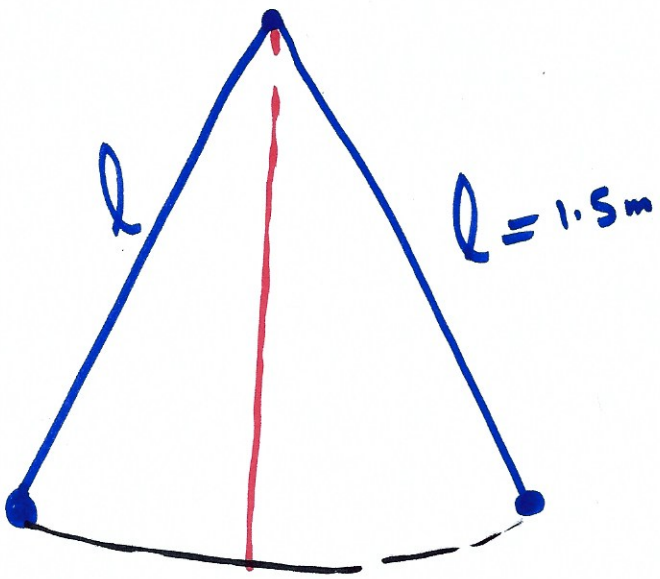
$$t = \boxed{0.253} \text{ s}$$

As it does not depend on θ

مهما أخطأنا بحقهن، مازلن يتحاملن لأجلنا، لأنهن ببساطة أمهات.

**14.47** • A building in San Francisco has light fixtures consisting of small 2.35-kg bulbs with shades hanging from the ceiling at the end of light, thin cords 1.50 m long. If a minor earthquake occurs, how many swings per second will these fixtures make?

$f$



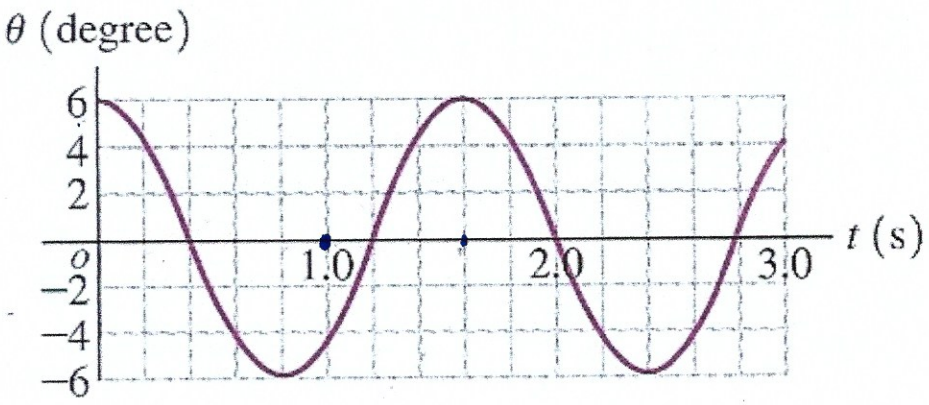

$$T = 2\pi\sqrt{\frac{l}{g}} \quad l = 1.5$$

$$= 2.46 \text{ s}$$

$$f = \frac{1}{T} = 0.407 \text{ Hz}$$

**14.50** •• In the laboratory, a student studies a pendulum by graphing the angle $\theta$ that the string makes with the vertical as a function of time $t$, obtaining the graph shown in **Fig. E14.50**. (a) What are the period, frequency, angular frequency, and amplitude of the pendulum's motion? (b) How long is the pendulum? (c) Is it possible to determine the mass of the bob?

Figure **E14.50**

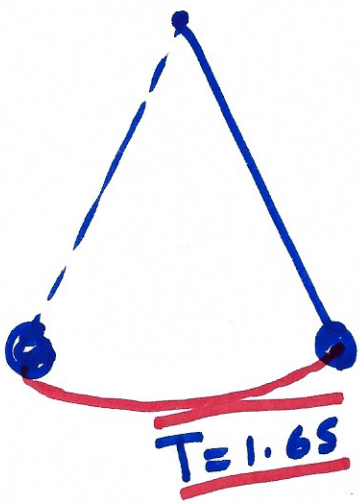



a] $T = 1.6$ s

$f = 0.625$ Hz

$\omega = 3.93$ rad/s

$A = 6°$




b] $1.6 = 2\pi\sqrt{\frac{l}{9.8}}$ ?

$\therefore l = 0.635$ m

c] It is not.

**14.66** • Four passengers with combined mass 250 kg compress the springs of a car with worn-out shock absorbers by 4.00 cm when they get in. Model the car and passengers as a single body on a single ideal spring. If the loaded car has a period of vibration of 1.92 s, what is the period of vibration of the empty car?

$F = k\,x$

$(250 * 9.8) \quad ? \quad 0.04$

$\therefore k = 61{,}250 \ N/m$

$T = 2\pi \sqrt{\frac{m + 250}{k}}$

$1.92 \qquad k \to 61{,}250$

$m = 5{,}469 \ kg$

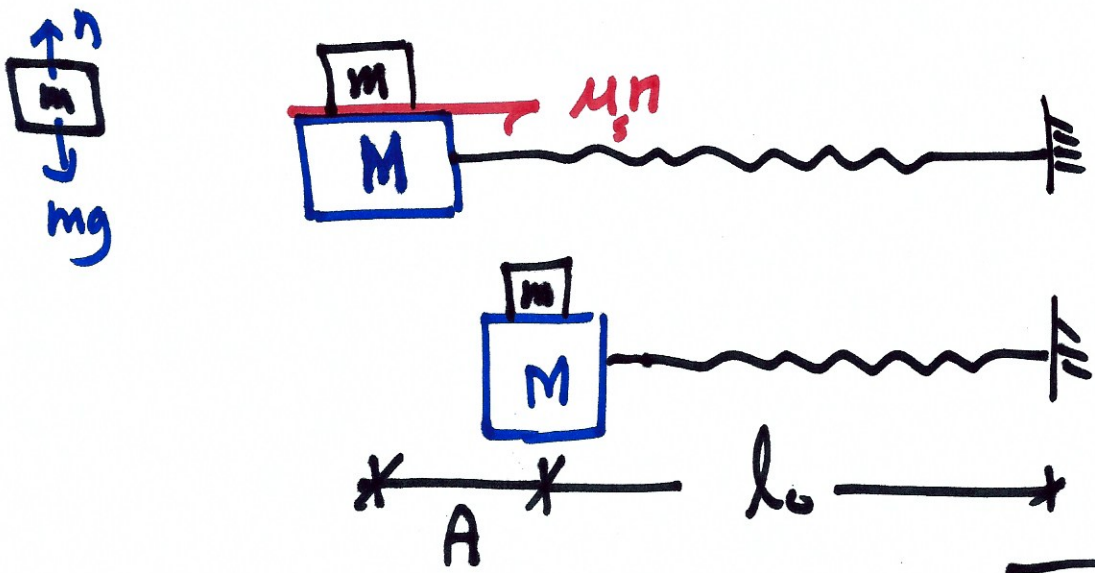
$T_{empty} = 2\pi \sqrt{\frac{m}{k}}$

$m \to 5{,}469 \qquad k \to 61{,}250$

$= 1.88 \ s$

بعض الناس لا يتغيرون،
أحيانا فقط يغيرون أزيائهم.

**14.68** •• **CP** A block with mass $M$ rests on a frictionless surface and is connected to a horizontal spring of force constant $k$. The other end of the spring is attached to a wall (**Fig. P14.68**). A second block with mass $m$ rests on top of the first block. The coefficient of static friction between the blocks is $\mu_s$. Find the *maximum* amplitude of oscillation such that the top block will not slip on the bottom block.




$$a = \omega^2 A$$

$$F = m a$$

$$\mu_s m g = m \omega^2 A$$

$$\omega^2 = \frac{K}{m}$$

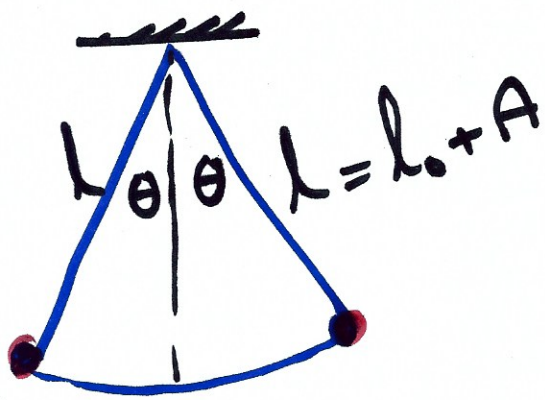
$$\mu_s g = \frac{K}{(M+m)} A$$

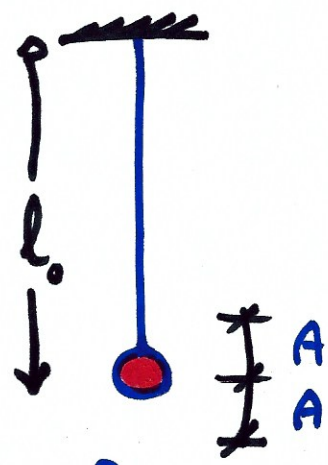
$$\therefore A = \frac{\mu_s (M+m) g}{K}$$

تعريف الحكمة: التركيز على أمر واحد بالوقت الواحد.

**14.71** ••• An apple weighs 1.00 N. When you hang it from the end of a long spring of force constant 1.48 N/m and negligible mass, it bounces up and down in SHM. If you stop the bouncing and let the apple swing from side to side through a small angle, the frequency of this simple pendulum is half the bounce frequency. (Because the angle is small, the back-and-forth swings do not cause any appreciable change in the length of the spring.) What is the unstretched length of the spring (with the apple removed)?

$$m = \frac{1}{g} = 0.102 \text{ kg}$$




$f' = 0.5 f$

$f$

$f' = 0.5\,(0.606) = 0.303$ Hz

$F = kA$ (1.48, ?)

$\therefore A = 0.676$ m

$T = \frac{1}{f'} = 3.3$ s

$T = 2\pi\sqrt{\frac{l}{g}}$ (3.3, ?, 9.8)

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (1.48, 0.102) $= 0.606$ Hz

$l = 2.70$ m $\quad \therefore l_0 = l - A = 2.03$ m

عندما يكون الله معك يصبح عندك
خيارات يستحيل عليك فهمها