# Discussion 8 - Capacitance and Dielectrics

**Problem 1.** Two long, coaxial cylindrical conductors are separated by vacuum. The inner cylinder has radius $r_a$ and linear charge density $+\lambda$. The outer cylinder has inner radius $r_b$ and linear charge density $-\lambda$ as shown in Figure 1.

a. Find the capacitance per unit length for this capacitor. The potential outside a charged conducting cylinder relative to some finite reference point, $r_0$, is

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r} \tag{1}$$

b. What is the electric field energy density in the region between the conductors at a distance $r$ from the axis?

c. Integrate the energy density calculated in part (b) over the volume between the conductors in a length $L$ of the capacitor to obtain the total electric-field energy per unit length.

d. Use Eq. 2 and the capcitance per unit length calculated in part (a) to calculate $U/L$. Does your result agree with that obtained in part (c)?

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \tag{2}$$

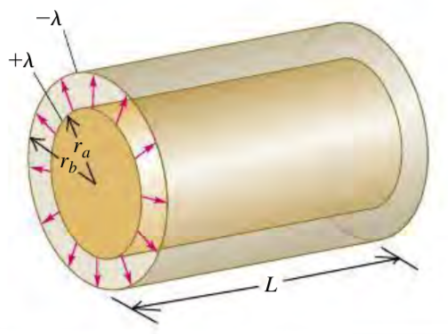


Figure 1: Problem 1

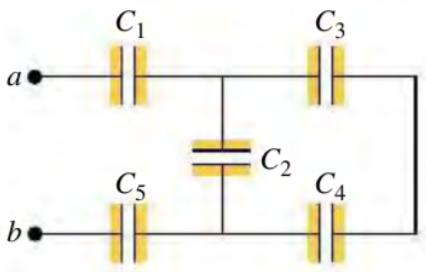


Figure 2: Problem 2

**Problem 2.** In Figure 2, $C_1 = C_5$ and $C_2 = C_3 = C_4$. The applied potential is $V_{ab}$.

a. What is the equivalent capacitance of the network between points $a$ and $b$ in terms of $C_1$ and $C_2$?

b. Calculate the charge on each capacitor and the potential difference across each capacitor.

**Problem 3.** A parallel-plate capacitor has capacitance $C$ when the volume between the plates is filled with air. The plates are circular, with radius $R$. The radius is much bigger than the separation between the plates. The capacitor is connected to a battery, and a charge of magnitude $Q$ goes onto each plate. With the capacitor still connected to the battery, a slab of dielectric is inserted between the plates, completely filling the space between the plates. After the dielectric has been inserted, the charge on each plate has magnitude $1.8Q$.

a. What is the dielectric constant $K$ of the dielectric?

b. What is the potential difference between the plates before and after the dielectric has been inserted?

c. What is the electric field at a point midway between the plates before and after the dielectric has been inserted?

d. With the dielectric inserted, the capacitor is disconnected from the battery. What is the electric field at a point midway between the plates if the dielectric is then removed?