DC Circuits Kudu Graded Homework Solutions

Question 1

All resistors in parallel will yield the smallest equivalent resistance.

Series: $R_{eq} = R_1 + R_2 + R_3 \ldots$

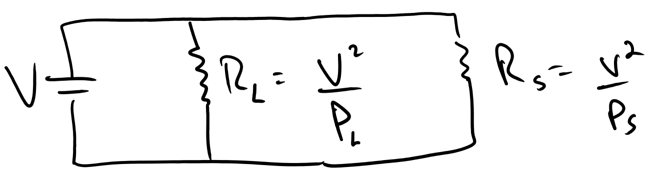
Adding resistors in series always increases equivalent resistance.

Parallel: $R_{eq} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

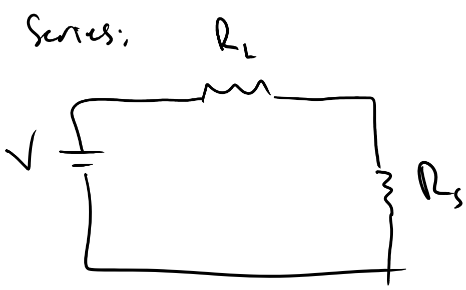
Adding resistors in parallel increases the denominator, thus decreasing the resistance.

Question 2

Given P_L, P_s when connected in parallel. Use: $P = \dfrac{V^2}{R} \rightarrow R_i = \dfrac{V^2}{P_i}$



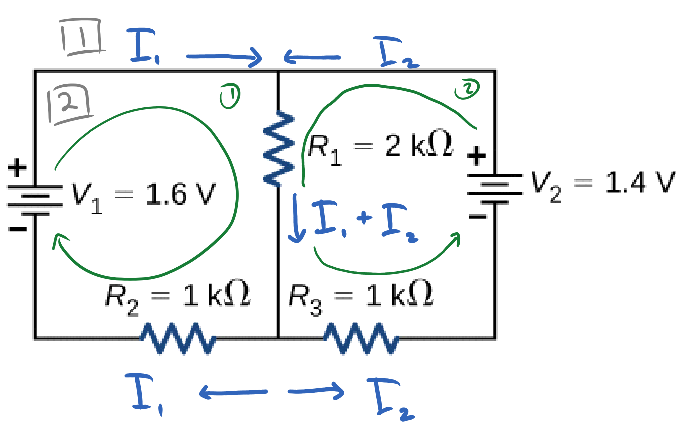
In Series:



$$P = \frac{V^2}{R_{eq}} = \frac{V^2}{R_L + R_s} = \frac{V^2}{\frac{V^2}{P_s} + \frac{V^2}{P_L}}$$

$$P = \left(P_s^{-1} + P_L^{-1}\right)^{-1}$$

Question 3



1. We don't know the direction of *I*'s. We guess. If solving produces negative *I*, then we know direction is flipped. Guessing:

2. Choose loops. Have 2 unknowns, need 2 equations. Will use 2 loops:

Loop 1: $V_1 - (I_1 + I_2)R_1 - I_1 R_2 = 0 \quad \rightarrow \quad I_1 = \frac{V_1 - I_2 R_1}{R_1 + R_2}$

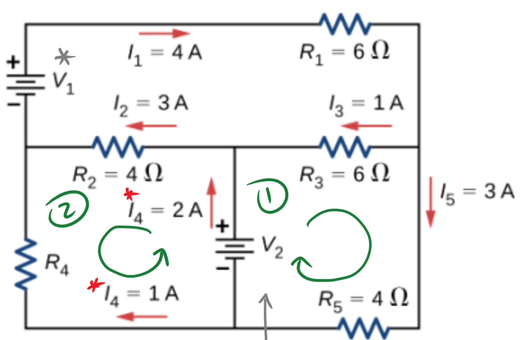
Loop 2: $V_2 - (\underbrace{I_1}_{\text{Use above sln.}} + I_2)R_1 - I_2 \underbrace{R_3}_{= R_2} = 0$

Algebra ...

$$I_2 = \frac{V_2(R_1 + R_2) - R_1 V_1}{R_2(2R_1 + R_2)} = 0.2 \text{ mA} \rightarrow I_1 = \frac{V_1 - I_2 R_1}{R_1 + R_2} = 0.4 \text{ mA}$$

$$I_1 + I_2 = \boxed{0.6 \text{ mA}}$$

Question 4



Using this loop to find V_2

V_1 unknown. Avoiding.

① $V_2 + \underbrace{I_3 R_3 - I_5 R_5}_{\text{All known}} = 0 \rightarrow V_2 = 6V$

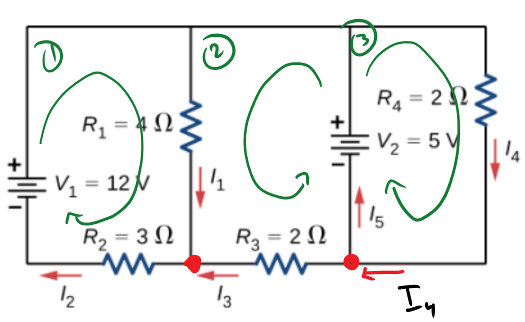
$V_2 = I_5 R_5 - I_3 R_3 = 6V$

② $\underbrace{V_2 - I_2 R_2 + \overset{= 1A^*}{I_4} R_4}_{\text{All known}} = 0$ Careful!

$R_4 = (I_2 R_2 - V_2) / I_4 = \boxed{6\,\Omega}$

Question 5

Use: $P = I^2 R$



Junction Rule

- $I_1 + I_3 = I_2$
- $I_3 + I_5 = I_4$

3 unknowns left. Need 3 more equations.
Use loop rule

Loop Rule

1. $V_1 - I_1 R_1 - \overbrace{(I_1 + I_3)}^{=I_2} R_2 = 0 \rightarrow I_1 = \dfrac{V_1 - I_3 R_2}{R_1 + R_2}$

2. $V_2 - I_1 R_1 + I_3 R_3 = 0$

$$\downarrow$$

$$I_3 = \frac{V_2(R_1 + R_2) - V_1 R_1}{R_1(R_2 + R_3) + R_2 R_3} = 0.5\ A$$

$$\downarrow$$

$$I_1 = \frac{V_1 R_3 + V_2 R_2}{R_1(R_2 + R_3) + R_2 R_3} = 1.5\ A$$

$$I_2 = I_1 + I_3 = 2\ A$$

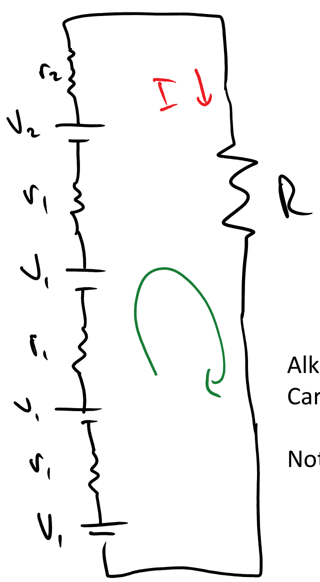
3. $V_2 - I_4 R_4 = 0 \rightarrow I_4 = \dfrac{V_2}{R_4} = 2.5\ A$

$$I_5 = I_4 - I_3 = 2\ A$$

$$P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4$$

$$= \boxed{34\ W}$$

Question 6



Alkaline cells: V_1
Carbon-zinc: V_2

Note: the battery makeup is irrelevant.

$$3(V_1 - I r_1) + V_2 - I r_2 - I R = 0$$

$$3V_1 + V_2 = 3 I r_1 + I r_2 + I R$$

$$= I(3r_1 + r_2 + R)$$

$$\boxed{I = \frac{3V_1 + V_2}{3r_1 + r_2 + R}}$$

Question 7

$\tau = RC$

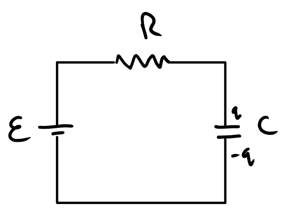
Range: $C\,(R_1, R_2) = (2, 15)\,s$

$(R_1, R_2) = \frac{1}{0.5\,\mu F}(2, 15)\,s$

$= \boxed{(4, 30)\,M\Omega}$

Question 8

This problem doesn't state whether the capacitor is initially charged or discharged. But it does not matter, because the current equation for both cases is the same.



$I = I_0 e^{-t/RC} = \frac{\varepsilon}{R} e^{-t/RC}$ $\qquad \tau = RC$

$\boxed{I(\tau) = \frac{\varepsilon}{R} e^{-1}}$

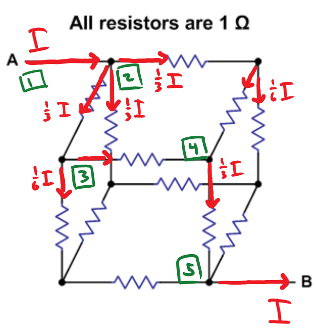
Question 9

Need: $\tau < 100\,\mu s$ $\qquad \tau = RC$ $\qquad$ Given: $R = 1\,k\Omega$

$C < \frac{100\,\mu s}{R} \rightarrow$ Max $C = \frac{100\,\mu s}{1\,k\Omega} = \boxed{100\,nF}$

Question 10

This requires the Kirchhoff's Junction Rule and Ohm's Law to solve.



1 If points A and B were connected to the terminals of a battery, then the current I out of the battery at point A MUST be the current into the battery at point B. This allows us to write:

$$V_A - V_B = I R_{eq}$$

2 Because all resistors are the same, the current will split evenly at the first junction.

3 Because all resistors are the same, the current will split evenly at this junction.

4 $\frac{1}{6}I$ and $\frac{1}{6}I$ combine and produce current out $\frac{1}{3}I$.

5 $\frac{1}{3}I + \frac{1}{3}I + \frac{1}{3}I$ combine to produce current out I.

Any path we follow from A to B will yield: $V_A - V_B = \frac{1}{3}IR + \frac{1}{6}IR + \frac{1}{3}IR = \frac{5}{6}IR = IR_{eq}$

$\Rightarrow \boxed{R_{eq} = \frac{5}{6}R}$