Midterm 2 Review Problems

Problem 1. Figure 1 shows a finite line of charge positioned along the y-axis between $y = -a$ and $y = +a$. The line of charge has a uniform positive linear charge density λ.

a. Derive integral expressions for the x and y components of the electric field at point P, which is located along the x-axis at a distance x from the line of charge. Then, evaluate the integrals to obtain the magnitude and direction of the electric field at point P. Express your answer in terms of the given parameters and fundamental constants.

b. Now, determine the magnitude and direction of the electric field at point P due to an infinitely long line charge by evaluating the expression from Part A in the limit $a \gg x$.

Figure 2 shows two identical infinite line charges of positive linear charge density λ that are placed in the xy plane and aligned along the y-axis. The two line charges are separated by a distance d, with each line charge positioned at a distance $d/2$ from the y-axis.

c. Determine the magnitude and direction of the electric field as a function of x in the range $d/2 < x < +d/2$ in the xy plane (i.e., between the two line charges).

d. A positive point charge q_0 that has a mass m_0 is placed between the two line charges in the xy plane at a very small displacement from the origin along the x-axis, such that $|x| \ll d$. Show that the resulting motion of the charge will be approximately simple harmonic motion, and derive an expression for the frequency of oscillations. Express your answer in terms of the given parameters and fundamental constants. You will need the following Taylor series approximations which are valid for $c \ll 1$:

$$\frac{1}{1+c} \approx 1 - c \tag{1}$$

$$\frac{1}{1-c} \approx 1 + c \tag{2}$$

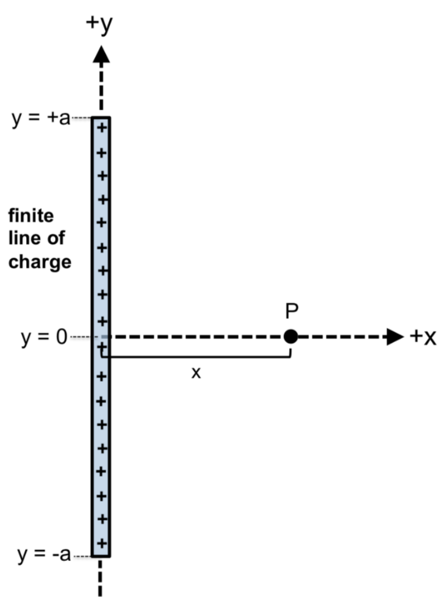


Figure 1: Problem 1

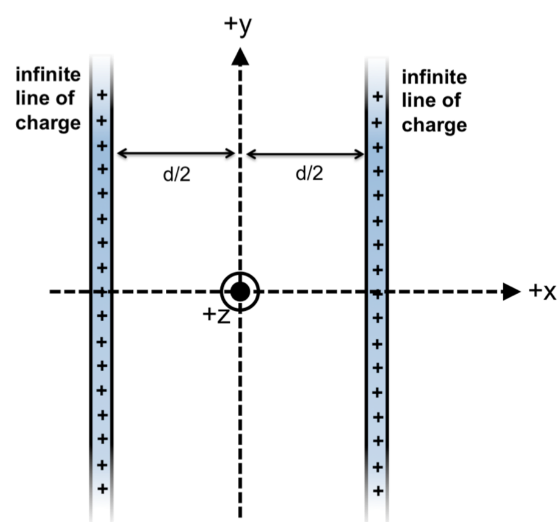
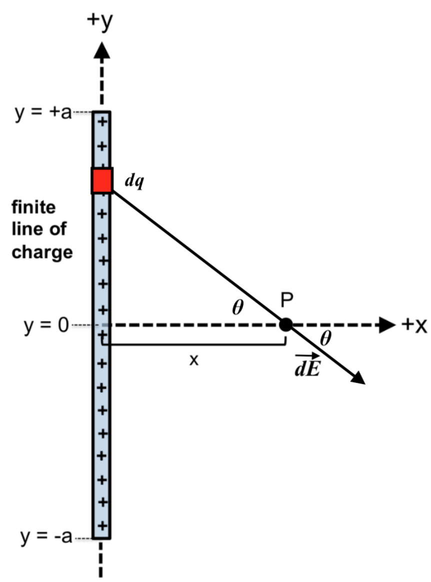


Figure 2: Problem 1

a. Consider a piece of the line with charge $dq = \lambda dy$. The x-component of the field due to this charge is $dE_x = |d\vec{E}| \cos\theta$ and the y-component is $dE_y = -|d\vec{E}| \sin\theta$ where θ is shown in the figure.



$$d\vec{E} = k\frac{dq}{r^2}\hat{r} \tag{3}$$

$$r^2 = x^2 + y^2 \tag{4}$$

$$\hat{r} = \cos\theta\hat{x} - \sin\theta\hat{y} \tag{5}$$

$$\cos\theta = \frac{x}{r} \tag{6}$$

$$\sin\theta = \frac{y}{r} \tag{7}$$

$$\hat{r} = \frac{x\hat{x} - y\hat{y}}{r} \tag{8}$$

$$E_x = \int_{-a}^{a} k\frac{\lambda dy}{r^2}\frac{x}{r} \tag{9}$$

$$E_x = k\lambda x\int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}} \tag{10}$$

$$E_y = -\int_{-a}^{a} k\frac{\lambda dy}{r^2}\frac{y}{r} \tag{11}$$

$$E_y = -k\lambda\int_{-a}^{a} \frac{ydy}{(x^2+y^2)^{3/2}} \tag{12}$$

The integral for E_y can be done by u-substitution.

$$u = x^2 + y^2 \tag{13}$$

$$du = 2ydy \tag{14}$$

$$E_y = -k\lambda\int_{y=-a}^{y=a} \frac{du}{2u^{3/2}} \tag{15}$$

$$E_y = k\lambda\left(\sqrt{x^2+a^2} - \sqrt{x^2+a^2}\right) = 0 \tag{16}$$

The integral for E_x can be solved with the substitution $y = x\tan(\theta)$.

$$dy = x\sec^2\theta d\theta \tag{17}$$

$$E_x = k\lambda x\int_{y=-a}^{y=a}\frac{x\sec^2\theta d\theta}{\left(x^2+(x\tan\theta)^2\right)^{3/2}} \tag{18}$$

$$E_x = k\lambda x\int_{y=-a}^{y=a}\frac{x\sec^2\theta d\theta}{x^3\sec^3\theta} \tag{19}$$

$$E_x = \frac{k\lambda}{x}\int_{y=-a}^{y=a}\cos\theta d\theta = \frac{k\lambda}{x}\sin\theta\Big|_{y=-a}^{y=a} \tag{20}$$

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}} \tag{21}$$

$$E_x = \frac{k\lambda}{x}\frac{2a}{\sqrt{x^2+a^2}} \tag{22}$$

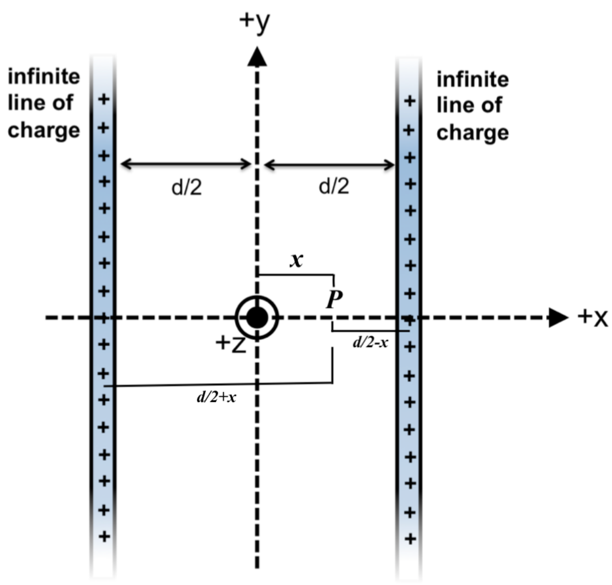
Thus, the electric field is

$$\vec{E} = \frac{2k\lambda}{x\sqrt{\left(\frac{x}{a}\right)^2+1}}\hat{x} \tag{23}$$

b. In the limit $a \gg x$, $\frac{x^2}{a^2} \ll 1$. The expression from part a becomes

$$\vec{E} \approx \frac{2k\lambda}{x}\hat{x} \tag{24}$$

c. Any point P in the xy plane between the two line charges will be a distance $d/2 + x$ from the left line and a distance $d/2 - x$ from the right line, where x is the displacement from the y-axis as shown in the figure. For any point P with $-d/2 < x < d/2$, the line charge on the left contributes



an electric field in the $\hat{x}$ direction and the line charge on the right contributes an electric field in the $-\hat{x}$ direction. The electric field at point P is

$$\vec{E} = 2k\lambda\left(\frac{1}{\frac{d}{2}+x} - \frac{1}{\frac{d}{2}-x}\right)\hat{x} \tag{25}$$

d. The acceleration of charge q_0 can be found from the electric force.

$$\vec{F_E} = q_0\vec{E} = m_0\vec{a} \tag{26}$$

$$\vec{a} = \frac{q_0}{m_0}\vec{E} = \frac{q_0 2k\lambda}{m_0}\left(\frac{1}{\frac{d}{2}+x} - \frac{1}{\frac{d}{2}-x}\right)\hat{x} \tag{27}$$

$$\vec{a} = \frac{q_0 2k\lambda}{m_0}\frac{2}{d}\left(\frac{1}{1+\frac{2x}{d}} - \frac{1}{1-\frac{2x}{d}}\right)\hat{x} \tag{28}$$

For small displacements ($|x| \ll d$), $2x/d \ll 1$ and we can use the given Taylor series approximations. The equation of motion for simple harmonic motion is $\vec{a} = -\omega^2\vec{x}$ where the frequency of small oscillations is $f = \omega/2\pi$.

$$\vec{a} \approx \frac{q_0 4k\lambda}{m_0 d}\left(1 - \frac{2x}{d} - \left(1 + \frac{2x}{d}\right)\right)\hat{x} \tag{29}$$

$$\vec{a} \approx -\frac{q_0 16k\lambda}{m_0 d^2}\vec{x} \tag{30}$$

$$\omega^2 = \frac{q_0 16k\lambda}{m_0 d^2} \tag{31}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{q_0 16k\lambda}{m_0 d^2}} = \sqrt{\frac{q_0\lambda}{\pi^3\epsilon_0 m_0 d^2}} \tag{32}$$

Problem 2. As shown in Figure 3, a spherical conductor of radius R holds a net positive charge $+q$. The conductor also contains two internal cavities. One of the cavities contains a positive point charge $+2q$, and the other cavity contains a positive point charge $+3q$. For this problem, assume the space in the cavities and outside of the spherical conductor is vacuum.

a. In the figure, sketch the distribution of charge on the conductor and the electric field lines outside of the conductor ($r > R$). Use "+" signs for positive charge and "-" signs for negative charge. The spacing between the signs should represent the relative charge density.

b. Determine the magnitude and direction of the electric field as a function of radial distance r from the conductor center outside of the conductor ($r > R$) and inside of the material of the conductor. Express your answer in terms of the given parameters and fundamental constants.

c. Taking the electrostatic potential to be zero infinitely far away, determine the potential as a function of radial distance r from the conductor center outside of the conductor ($r > R$) and inside of the material of the conductor. Express your answer in terms of the given parameters and fundamental constants.

a. All of the charge resides on the surfces of the conductor. The total charge on the cavity surfaces is equal and opposite to the enclosed point charges, from Gauss' Law. Where the point charges are closer to the cavity wall, more charge is induced on the conductor. The induced negative charge on the cavity surfaces results in an individual positive charge elsewhere on the conductor. From Gauss' Law, this induced positive charge, in addition to the net positive charge on the conductor, must reside on the outer surface. The outer surface charge is uniformly distributed and the electric field outside the conductor $r > R$ is uniformly radial.

b. The total induced charge on the cavity walls is $-2q - 3q = -5q$ so the total induced charge on the outer surface is $+5q$. The net charge on the conductor, $+q$ also resides on the outer surface so the total charge on the outer surface is $+5q + q = +6q$. Since the induced charge on the cavity walls perfectly cancels the point charges to create zero electric field within the conductor, the outer surface charge distributes uniformly. We can obtain the electric field outside the conductor by applying Gauss' Law to the spherically symmetric charge distribution on the outer surface, pretending the cavities and point charges do not exist.

$$\oint \vec{E} \cdot \hat{n} dA = \frac{Q_{encl}}{\epsilon_0} \tag{33}$$

$$\vec{E} = E_r \hat{r} \tag{34}$$

$$E_r 4\pi r^2 = \frac{6q}{\epsilon_0} \tag{35}$$

$$\vec{E}_{out} = \frac{3}{2\pi\epsilon_0} \frac{q}{r^2} \hat{r} \tag{36}$$

Inside the material of a conductor the electric field is zero.

c. Define the potential at $r = \infty$ to be zero.

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} \tag{37}$$

$$V_a = \int_a^\infty \vec{E} \cdot d\vec{l} \tag{38}$$

For $r > R$, we take a radially outward path such that $d\vec{l} = dr\hat{r}$.

$$V = \int_r^\infty \left(\frac{3}{2\pi\epsilon_0} \frac{q}{r^2} \hat{r} \right) \cdot (dr\hat{r}) \tag{39}$$

$$V = \frac{3q}{2\pi\epsilon_0} \int_r^\infty \frac{dr}{r^2} = \frac{3q}{2\pi\epsilon_0} \frac{-1}{r} \Big|_r^\infty \tag{40}$$

$$V = \frac{3q}{2\pi\epsilon_0} \frac{1}{r} \tag{41}$$

For $r < R$ at a point inside the material of the conductor, we take a radially outward path, $d\vec{l} = dr\hat{r}$ and break our path integral into two parts.

$$V = \int_r^R \vec{E} \cdot d\vec{l} + \int_R^\infty \vec{E} \cdot d\vec{l} \tag{42}$$

$$V = 0 + \frac{3q}{2\pi\epsilon_0} \frac{1}{R} \tag{43}$$

The first term in the sum is zero because $\vec{E} = 0$ inside the conductor. So the potential is constant inside the material of the conductor and is equal to $3q/(2\pi\epsilon_0 R)$ (the potential at the surface.)

Problem 3. Figure 4 shows the cross-section of an infinitely long insulating cylindrical shell of inner radius r_a and outer radius r_b that has a uniform positive volume charge density ρ_0. The insulating shell is centered within an infinitely long uncharged conducting cylindrical shell of inner radius r_c and outer radius r_d.

a. Determine the surface charge density on the inner surface of the conducting shell and the outer surface of the conducting shell. Express your answers in terms of the given parameters and fundamental constants.

b. Determine the magnitude and direction of the electric field as a function of radial distance r from the center inside the insulating shell ($r_a < r < r_b$) and in the vacuum region between the insulating shell and the conducting shell ($r_b < r < r_c$). Express your answers in terms of the given parameters and fundamental constants.

c. Calculate the voltage (i.e., the magnitude of the potential difference) between the outer surface of the insulating shell ($r = r_b$) and the inner surface of the conducting shell ($r = r_c$). Express your answer in terms of the given parameters and fundamental constants.

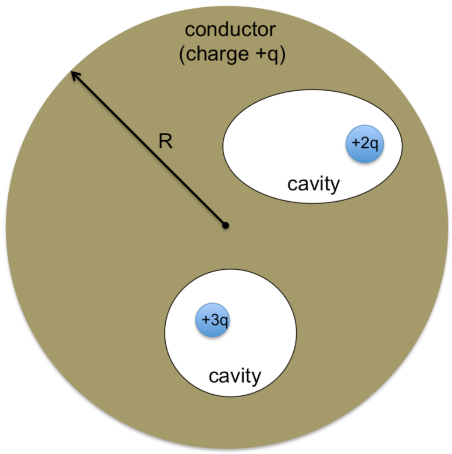


Figure 3: Problem 2

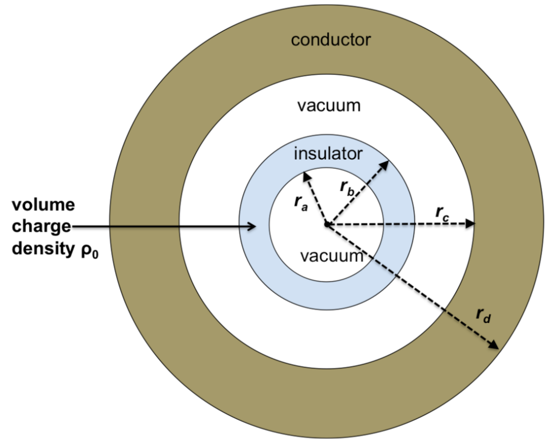


Figure 4: Problem 3

a. Consider a Gaussian cylinder with radius r such that $r > r_c$ and $r < r_d$.

$$\oint \vec{E} \cdot \hat{n} dA = \frac{Q_{enc}}{\epsilon_0} = 0 \tag{44}$$

The electric field in the material of a conductor is zero. So the charge enclosed by this Gaussian cylinder is zero. The enclosed charge consists of the postively charged insulating shell and the negative induced charge on the conductor's inner surface.

$$Q_{enc} = q_{insulator} + q_{inner} = 0 \tag{45}$$

$$q_{inner} = -q_{insulator} \tag{46}$$

Assuming the Gaussian cylinder has length L

$$q_{insulator} = \rho_0 V_{insulator} = \rho_0 \left(\pi r_b^2 - \pi r_a^2\right) L \tag{47}$$

Since $-q_{insulator}$ is uniformly distributed over a cylindrical surface of length L and radius r_c, the inner surface charge density is

$$\sigma_{in} = \frac{q_{inner}}{2\pi r_c L} = \frac{-\rho_0 \left(\pi r_b^2 - \pi r_a^2\right) L}{2\pi r_c L} = -\rho_0 \frac{r_b^2 - r_a^2}{2r_c} \tag{48}$$

Since the conductor is uncharged, $q_{outer} + q_{inner} = 0$. The charge on the outer surface is uniformly distributed over a cylindrical surface of length L and radius r_d so the outer surface charge density is

$$\sigma_{out} = \frac{-q_{inner}}{2\pi r_d L} = \rho_0 \frac{r_b^2 - r_a^2}{2r_d} \tag{49}$$

b. Drawing a cylindrical Gaussian surface and applying Gauss' law assuming cylindrical symmetry ($\vec{E} = E_r \hat{r}$) gives

$$\oint \vec{E} \cdot \hat{n} dA = E_r 2\pi r L = \frac{Q_{enc}}{\epsilon_0} \tag{50}$$

Inside the insulator ($r_a < r < r_b$), charge enclosed depends on volume enclosed by the Gaussian cylinder.

$$Q_{enc} = \rho_0 V_{enc} = \rho_0 \left(\pi r^2 - \pi r_a^2\right) L \tag{51}$$

$$\vec{E} = \frac{\rho_0}{2\epsilon_0} \left(\frac{r^2 - r_a^2}{r}\right) \hat{r} \tag{52}$$

Outside the insulator ($r_b < r < r_c$), charge enclosed is constant.

$$Q_{enc} = \rho_0 V_{insulator} = \rho_0 \left(\pi r_b^2 - \pi r_a^2\right) L \tag{53}$$

$$\vec{E} = \frac{\rho_0}{2\epsilon_0}\left(\frac{r_b^2 - r_a^2}{r}\right)\hat{r} \tag{54}$$

c. Take a radially outward path from r_b to r_c, $\vec{dl} = dr\hat{r}$.

$$V_a - V_b = \int_a^b \vec{E} \cdot \vec{dl} \tag{55}$$

$$V_{r_b} - V_{r_c} = \int_{r_b}^{r_c} \frac{\rho_0}{2\epsilon_0}\left(\frac{r_b^2 - r_a^2}{r}\right)\hat{r} \cdot dr\hat{r} \tag{56}$$

$$V_{bc} = \frac{\rho_0\left(r_b^2 - r_a^2\right)}{2\epsilon_0}\int_{r_b}^{r_c}\frac{dr}{r} \tag{57}$$

$$V_{bc} = \frac{\rho_0\left(r_b^2 - r_a^2\right)}{2\epsilon_0}\ln\left(\frac{r_c}{r_b}\right) \tag{58}$$

Since $r_c > r_b$, $r_b > r_a$, and $\rho_0 > 0$, this quantity is positive and thus gives the potential difference magnitude between r_b and r_c.

$$\left|\Delta V_{bc}\right| = \frac{\rho_0\left(r_b^2 - r_a^2\right)}{2\epsilon_0}\ln\left(\frac{r_c}{r_b}\right) \tag{59}$$

Problem 4. The small sphere A in Figure 5 has a positive charge Q_A. The large sphere B_1 is a thin shell of nonconducting material with a net charge that is uniformly distributed over its surface. Sphere B_1 has a mass M_1, a radius R_1, and is suspended from an uncharged nonconducting thread. Sphere B_1 is in equilibrium when the thread makes an angle θ_1 with the vertical. The centers of the spheres are at the same vertical height and are a horizontal distance d apart, as shown.

a. Calculate the charge on sphere B_1.

b. Suppose that sphere B_1 is replaced by a second suspended sphere B_2 that has the same mass, radius, and charge, but that is conducting. Equilibrium is again established when sphere A is a distance d from sphere B_2 and their centers are at the same vertical height. State whether the equilibrium angle θ_2 will be less than, equal to, or greater than θ_1. Justify your answer.

Now suppose there is a very long, horizontal, nonconducting tube, as shown in the top view in Figure 6. The tube is hollow with thin walls of radius R and uniform positive charge per unit length λ. To the left of the tube is the same small sphere A with charge Q_A a distance d away.

c. Use Gauss' Law to show that the electric field at a perpendicular distance r from the tube is given by the expression $E = C/r$ where $r > R$. Find the constant C.

d. The small sphere A is now brought into the vicinity of the tube and is held at a distance of d from the center of the tube. Calculate the repulsive force that the tube exerts on the sphere.

e. Calculate the work done against electrostatic repulsion to move sphere A toward the tube from a distance d to a distance $d/5$ from the tube.

a. The forces on B_1 in the horizontal direction are the Coulomb force from sphere A and the horizontal component of the tension, $T\sin\theta_1$. The vertical forces are gravity and the vertical component of the tension, $T\cos\theta_1$. The net force is zero because the system is in static equilibrium. Eliminate tension and solve for the charge on sphere B_1. The positive y direction is up and the positive

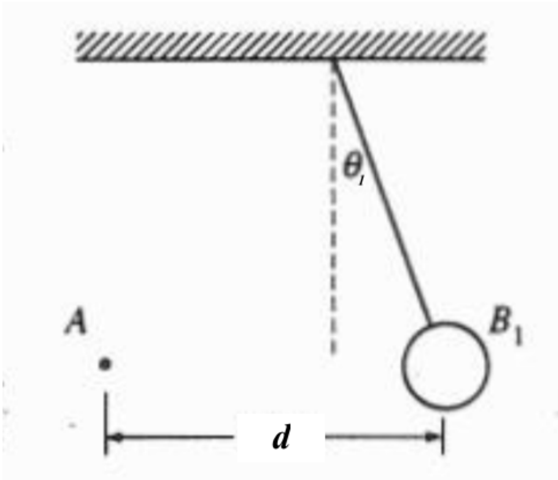


Figure 5: Problem 4

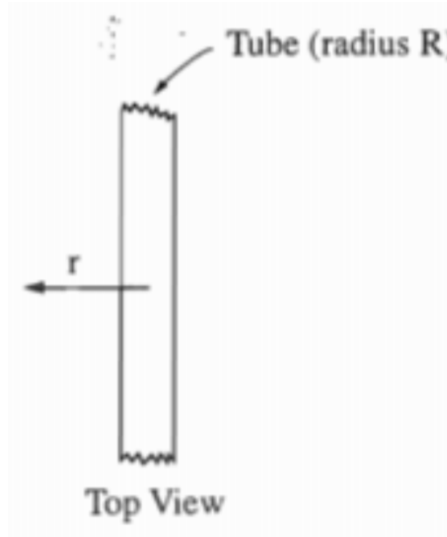


Figure 6: Problem 4

x direction is to the right.

$$\Sigma F_y = 0 = -M_1 g + T\cos\theta_1 \tag{60}$$
$$\Sigma F_x = 0 = -k\frac{Q_A Q_B}{d^2} + T\sin\theta_1 \tag{61}$$
$$T = \frac{M_1 g}{\cos\theta_1} \tag{62}$$
$$Q_B = \frac{d^2}{kQ_A} M_1 g \tan\theta_1 \tag{63}$$

b. The conductor allows charges to move. Positive charge will move to the far side of B_2 and so the charge will be farther from sphere A. Then the strength of the Coulomb force will be smaller because distance increased, so the angle should decrease to change distance in a way that compensates, restoring equilibrium.

$$\theta_2 < \theta_1 \tag{64}$$

c. For a very long positive tube, $\vec{E}$ points radially away. Consider a Gaussian cylinder of radius $r > R$ and length L.

$$\oint E_r \hat{r}\cdot\hat{r}\,dA = \frac{Q_{enc}}{\epsilon_0} \tag{65}$$
$$E_r\left(2\pi r L\right) = \frac{\lambda L}{\epsilon_0} \tag{66}$$
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}\hat{r} \tag{67}$$
$$C = \frac{\lambda}{2\pi\epsilon_0} \tag{68}$$

d. The force on a charge in an electric field is $\vec{F} = q\vec{E}$. The force points away from the charged tube.

$$F = \frac{\lambda}{2\pi\epsilon_0}\frac{Q_A}{d} \tag{69}$$

e. Work is charge times the change in electric potential.

$$W = Q_A\left(V_2 - V_1\right) \tag{70}$$

$$V_2 - V_1 = \int_{d/5}^{d} \vec{E}\cdot\vec{dl} \tag{71}$$

$$\vec{dl} = dr\hat{r} \tag{72}$$

$$W = Q_A \int_{d/5}^{d} \frac{\lambda}{2\pi\epsilon_0}\frac{dr}{r} \tag{73}$$

$$W = \frac{Q_A\lambda}{2\pi\epsilon_0}\ln\left(\frac{d}{d/5}\right) \tag{74}$$

$$W = \frac{Q_A\lambda}{2\pi\epsilon_0}\ln\left(5\right) \tag{75}$$

Problem 5. The field potential in a certain region of space depends only on the x coordinate as Equation 76, where a and b are constants. Find the electric field $\vec{E}$.

$$\phi = -ax^3 + b \tag{76}$$

$$\vec{E} = -\nabla\phi \tag{77}$$

$$\vec{E} = 3ax^2\hat{x} \tag{78}$$

$$\tag{79}$$

Problem 6. A point charge q is located at a distance r from the center of an uncharged conducting spherical layer whose inside and outside radii are equal to R_1 and R_2 respectively. Find the potential at the center if $r < R_1$.

The point charge induces a charge $-q$ on the inner surface of the conductor so that the electric field inside the conductor is zero. There is an induced charge q on the outer surface of the conductor because it is uncharged ($q_{inner} + q_{outer} = 0$.) Use superposition and think of the potential due to the point charge, the inner surface, and the outer surface independendly. The distribution of charge on the inner surface of the conductor is not uniform but the entire surface is equidistant from the center. The potential at the center is given by

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} + \frac{-q}{R_1} + \frac{q}{R_2}\right) \tag{80}$$

The surface charge on the inner layer of the conductor has some non-uniform distribution but all the charge is on the surface so it's all the same distance from the center (R_1). Let the surface charge distribution be denoted $\sigma(\theta,\phi)$. Then the potential due to these charges is

$$V = \int \frac{dq}{4\pi\epsilon_0 r} \tag{81}$$

$$dq = \sigma dA \tag{82}$$

$$-q = \int \sigma dA \tag{83}$$

$$V = \int\int \frac{\sigma(\theta,\phi)dA}{4\pi\epsilon_0 R_1} \tag{84}$$

$$V = \frac{1}{4\pi\epsilon_0 R_1}\int \sigma dA = \frac{-q}{4\pi\epsilon_0 R_1} \tag{85}$$

Problem 7. A charge $+Q$ is deposited on a conductive wire (line charge) of length L. An electrically neutral, conductive cylindrical shell of inner radius R_1, outer radius R_2, and length L is symmetrically positioned around the wire as in Figure 7. In this problem, $R_2 \ll L$. Calculate the following:

a. The line charge density on the wire, λ.

b. The surface charge density on the inner (σ_{in}) and outer (σ_{out}) shell of the cylinder and the volume charge density (ρ) inside the cylindrical shell, for $R_1 < r < R_2$. Always specify the sign of the charge.

c. The electric field $\vec{E}$ everywhere in space.

d. The flux of the electric field (Φ_E) through a cube of side $2R_2$ centered on the center of the wire as in the figure.

Now, deposit a charge $-Q$ on the cylindrical shell. Calculate the following:

e. The surface charge density on the inner (σ'_{in}) and outer (σ'_{out}) shell of the cylinder and the volume charge density (ρ') inside the cylindrical shell, for $R_1 < r < R_2$.

f. The electric field $\vec{E}'$ everywhere in space.

g. The difference in potential V between the cylinder and the wire: $V = \phi_{cylinder} - \phi_{wire}$.

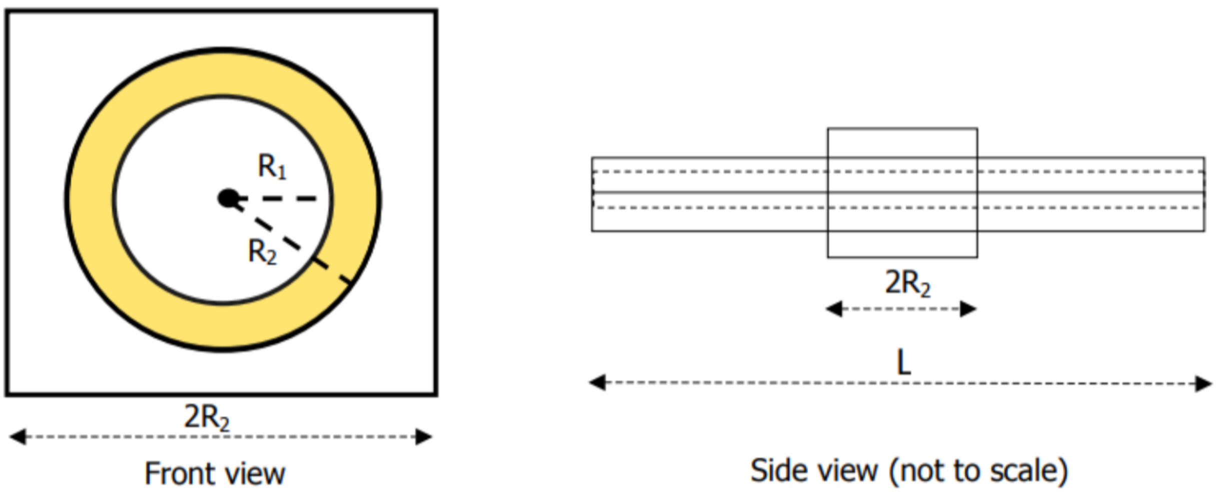


Figure 7

a. $\lambda = Q/L$

b. The charge in a conductor resides on the surface. The inner surface charge is induced by the wire and the outer surface charge adjusts to maintain charge neutrality.

$$\rho = 0 \tag{86}$$

$$0 = \sigma_{in} A_{in} + \lambda L \tag{87}$$

$$\sigma_{in} = \frac{-Q}{2\pi R_1 L} \tag{88}$$

$$0 = \sigma_{out} A_{out} + \sigma_{in} A_{in} \tag{89}$$

$$\sigma_{out} = \frac{Q}{2\pi R_2 L} \tag{90}$$

c. Use Gauss' Law. The field inside the conductive wire ($r < R_0$) is zero. For $R_0 < r < R_1$, the field is the field of a charged wire.

$$E(R_0 < r < R_1) = \frac{Q/L}{2\pi\epsilon_0 r}\hat{r} \tag{91}$$

The field in the material of a conductor is zero. Outside, the field depends on the charge enclosed by a Gaussian cylinder, but the total charge of the conductor is zero, so it is again the field of a wire.

$$E(r > R_2) = \frac{Q/L}{2\pi\epsilon_0 r}\hat{r} \tag{92}$$

d. By Gauss' law,

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0} = \frac{\lambda 2R_2}{\epsilon_0} = \frac{2QR_2}{L\epsilon_0} \tag{93}$$

e. The volume charge density inside the shell is still zero. The surface charge density on the inner shell must stay the same so the electric field inside the shell is still zero. But the surface charge density on the outer shell changes to account for the total charge, and we find that it must be zero.

$$\rho' = 0 \tag{94}$$

$$-Q = \sigma'_{in}A_{in} + \sigma'_{out}A_{out} \tag{95}$$

$$0 = \sigma'_{in}A_{in} + \lambda L \tag{96}$$

$$\sigma'_{in} = \sigma_{in} = \frac{-Q}{2\pi R_1 L} \tag{97}$$

$$\sigma'_{out} = 0 \tag{98}$$

f. The electric field for $r < R_2$ is the same as the previous case but now the electric field outside ($r > R_2$) is zero because the charge enclosed in a Gaussian cylinder is zero.

g. We find the potential by integration. The conducting cylinder is at constant potential.

$$V = \int_{r=R_0}^{R_1} \vec{E} \cdot \vec{dl} \tag{99}$$

$$\vec{dl} = dr\hat{r} \tag{100}$$

$$V = \int_{R_0}^{R_1} \frac{Q/L}{2\pi\epsilon_0 r} dr \tag{101}$$

$$V = \frac{Q}{2\pi\epsilon_0 L} \ln\left(\frac{R_1}{R_0}\right) \tag{102}$$