Midterm 2 Review Problems

Problem 1. Figure 1 shows a finite line of charge positioned along the y-axis between $y = -a$ and $y = +a$. The line of charge has a uniform positive linear charge density λ.

a. Derive integral expressions for the x and y components of the electric field at point P, which is located along the x-axis at a distance x from the line of charge. Then, evaluate the integrals to obtain the magnitude and direction of the electric field at point P. Express your answer in terms of the given parameters and fundamental constants.

b. Now, determine the magnitude and direction of the electric field at point P due to an infinitely long line charge by evaluating the expression from Part A in the limit $a \gg x$.

Figure 2 shows two identical infinite line charges of positive linear charge density λ that are placed in the xy plane and aligned along the y-axis. The two line charges are separated by a distance d, with each line charge positioned at a distance $d/2$ from the y-axis.

c. Determine the magnitude and direction of the electric field as a function of x in the range $d/2 < x < +d/2$ in the xy plane (i.e., between the two line charges).

d. A positive point charge q_0 that has a mass m_0 is placed between the two line charges in the xy plane at a very small displacement from the origin along the x-axis, such that $|x| \ll d$. Show that the resulting motion of the charge will be approximately simple harmonic motion, and derive an expression for the frequency of oscillations. Express your answer in terms of the given parameters and fundamental constants. You will need the following Taylor series approximations which are valid for $c \ll 1$:

$$\frac{1}{1+c} \approx 1 - c \tag{1}$$

$$\frac{1}{1-c} \approx 1 + c \tag{2}$$

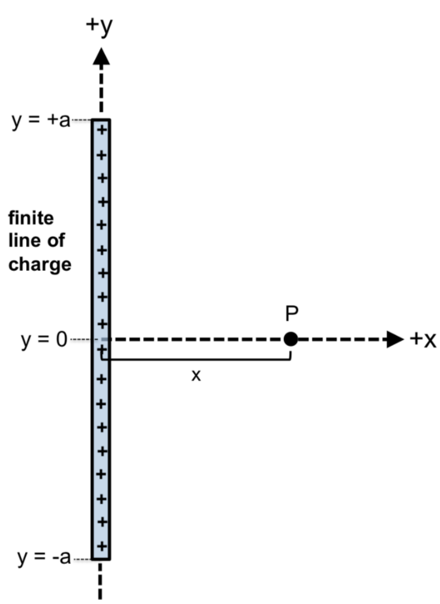

Figure 1: Problem 1

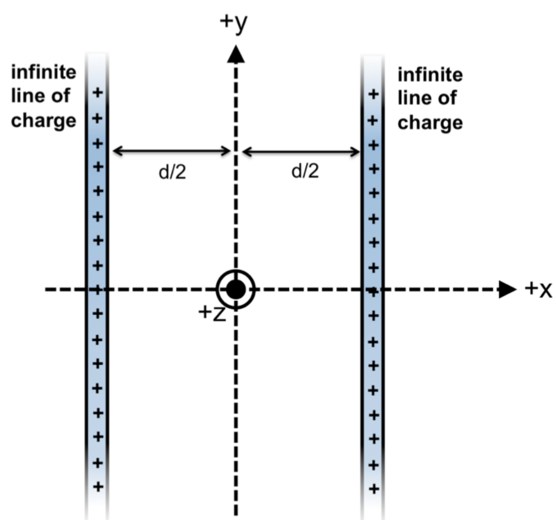

Figure 2: Problem 1

Problem 2. As shown in Figure 3, a spherical conductor of radius R holds a net positive charge $+q$. The conductor also contains two internal cavities. One of the cavities contains a positive point charge $+2q$, and the other cavity contains a positive point charge $+3q$. For this problem, assume the space in the cavities and outside of the spherical conductor is vacuum.

a. In the figure, sketch the distribution of charge on the conductor and the electric field lines outside of the conductor ($r > R$). Use "+" signs for positive charge and "-" signs for negative charge. The spacing between the signs should represent the relative charge density.

b. Determine the magnitude and direction of the electric field as a function of radial distance r from the conductor center outside of the conductor ($r > R$) and inside of the material of the conductor. Express your answer in terms of the given parameters and fundamental constants.

c. Taking the electrostatic potential to be zero infinitely far away, determine the potential as a function of radial distance r from the conductor center outside of the conductor ($r > R$) and inside of the material of the conductor. Express your answer in terms of the given parameters and fundamental constants.

Problem 3. Figure 4 shows the cross-section of an infinitely long insulating cylindrical shell of inner radius r_a and outer radius r_b that has a uniform positive volume charge density ρ_0. The insulating shell is centered within an infinitely long uncharged conducting cylindrical shell of inner radius r_c and outer radius r_d.

a. Determine the surface charge density on the inner surface of the conducting shell and the outer surface of the conducting shell. Express your answers in terms of the given parameters and fundamental constants.

b. Determine the magnitude and direction of the electric field as a function of radial distance r from the center inside the insulating shell ($r_a < r < r_b$) and in the vacuum region between the insulating shell and the conducting shell ($r_b < r < r_c$). Express your answers in terms of the given parameters and fundamental constants.

c. Calculate the voltage (i.e., the magnitude of the potential difference) between the outer surface of the insulating shell ($r = r_b$) and the inner surface of the conducting shell ($r = r_c$). Express your answer in terms of the given parameters and fundamental constants.

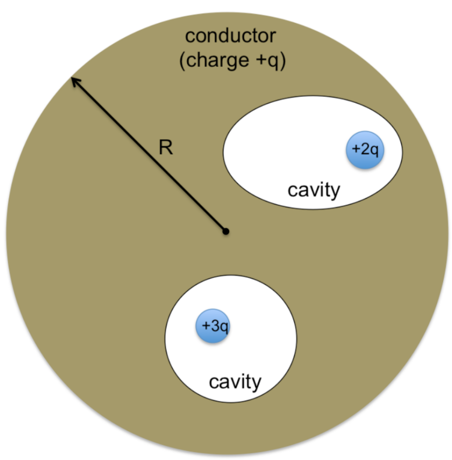

Figure 3: Problem 2

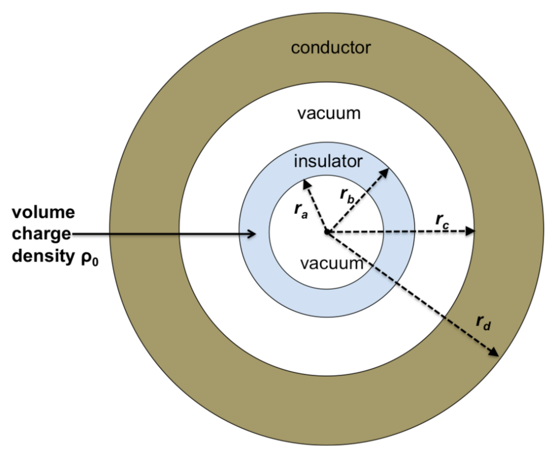

Figure 4: Problem 3

Problem 4. The small sphere A in Figure 5 has a positive charge Q_A. The large sphere B_1 is a thin shell of nonconducting material with a net charge that is uniformly distributed over its surface. Sphere B_1 has a mass M_1, a radius R_1, and is suspended from an uncharged nonconducting thread. Sphere B_1 is in equilibrium when the thread makes an angle θ_1 with the vertical. The centers of the spheres are at the same vertical height and are a horizontal distance d apart, as shown.

a. Calculate the charge on sphere B_1.

b. Suppose that sphere B_1 is replaced by a second suspended sphere B_2 that has the same mass, radius, and charge, but that is conducting. Equilibrium is again established when sphere A is a distance d from sphere B_2 and their centers are at the same vertical height. State whether the equilibrium angle θ_2 will be less than, equal to, or greater than θ_1. Justify your answer.

Now suppose there is a very long, horizontal, nonconducting tube, as shown in the top view in Figure 6. The tube is hollow with thin walls of radius R and uniform positive charge per unit length λ. To the left of the tube is the same small sphere A with charge Q_A a distance d away.

c. Use Gauss' Law to show that the electric field at a perpendicular distance r from the tube is given by the expression $E = C/r$ where $r > R$. Find the constant C.

d. The small sphere A is now brought into the vicinity of the tube and is held at a distance of d from the center of the tube. Calculate the repulsive force that the tube exerts on the sphere.

e. Calculate the work done against electrostatic repulsion to move sphere A toward the tube from a distance d to a distance $d/5$ from the tube.

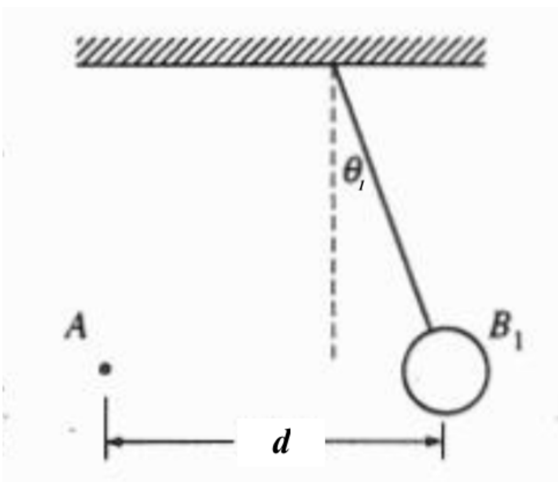

Figure 5: Problem 4

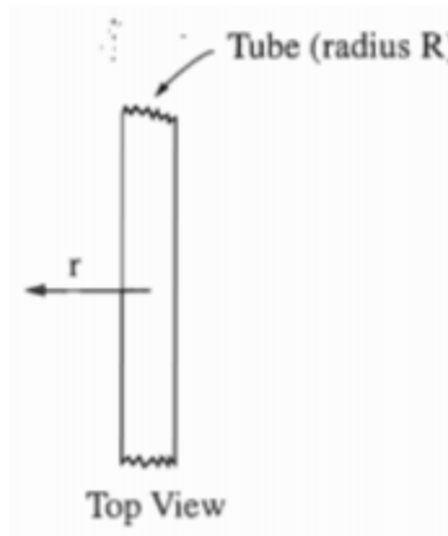

Figure 6: Problem 4

Problem 5. The field potential in a certain region of space depends only on the x coordinate as Equation 3, where a and b are constants. Find the electric field $\vec{E}$.

$$\phi = -ax^3 + b \tag{3}$$

Problem 6. A point charge q is located at a distance r from the center of an uncharged conducting spherical layer whose inside and outside radii are equal to R_1 and R_2 respectively. Find the potential at the center if $r < R_1$.

Problem 7. A charge $+Q$ is deposited on a conductive wire (line charge) of length L. An electrically neutral, conductive cylindrical shell of inner radius R_1, outer radius R_2, and length L is symmetrically positioned around the wire as in Figure 7. In this problem, $R_2 \ll L$. Calculate the following:

a. The line charge density on the wire, λ.

b. The surface charge density on the inner (σ_{in}) and outer (σ_{out}) shell of the cylinder and the volume charge density (ρ) inside the cylindrical shell, for $R_1 < r < R_2$. Always specify the sign of the charge.

c. The electric field $\vec{E}$ everywhere in space.

d. The flux of the electric field (Φ_E) through a cube of side $2R_2$ centered on the center of the wire as in the figure.

Now, deposit a charge $-Q$ on the cylindrical shell. Calculate the following:

e. The surface charge density on the inner (σ'_{in}) and outer (σ'_{out}) shell of the cylinder and the volume charge density (ρ') inside the cylindrical shell, for $R_1 < r < R_2$.

f. The electric field $\vec{E}'$ everywhere in space.

g. The difference in potential V between the cylinder and the wire: $V = \phi_{cylinder} - \phi_{wire}$.

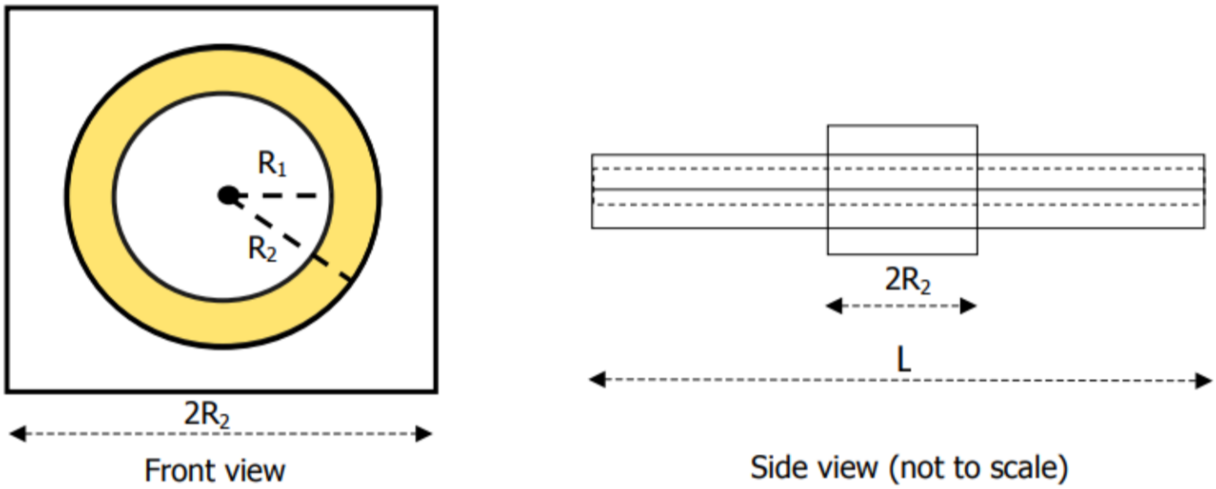

Figure 7