Discussion 8 - Electric Potential

Problem 1. Two spherical shells have a common center. The inner shell has radius R_1 and charge q_1 and the outer shell has radius $R_2 = 3R_1$ and charge q_2. Both charges are spread uniformly over the shell surface. What is the electric potential due to the two shells at the following distances from their common center? Take $V = 0$ at a large distance from the shells.

a. $r = R_1/2$

b. $r = 2R_1$

c. $r = 4R_1$

Problem 2. A disk with radius R has uniform surface charge density σ.

a. By regarding the disk as a series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity.

b. Calculate $-\frac{\partial V}{\partial x}$. Show that the result agrees with E_x on axis of a disk, given below. Show that $\vec{E} = -\nabla V$.

$$E_x = \frac{\sigma}{2\epsilon_0}\Big[1 - \frac{1}{\sqrt{(\frac{R}{x})^2 + 1}}\Big] \tag{1}$$

Problem 3. A solid sphere of radius R contains a total charge Q distributed uniformly throughout its volume. Find the energy needed to assemble this charge by bringing infinitesimal charges from far away. This energy is called the "self-energy" of the charge distribution. Take $R \to 0$ to determine what the self energy of a point charge is. (Hint: After you have assembled a charge q in a sphere of radius r, how much energy would it take to add a spherical shell of thickness dr having charge dq? Then integrate to get the total energy.)

Problem 4. Find the potential on the axis of a ring with radius a and total charge Q. Expand the potential far away from the ring to identify the monopole, dipole, and quadrupole terms. What is the dipole moment?

Problem 5. A very small sphere with positive charge q is released from rest at a distance d from a very long line of uniform linear charge density λ. What is the kinetic energy of the sphere when it is a distance $3d$ from the line of charge if the only force on it is the force exerted by the line of charge?