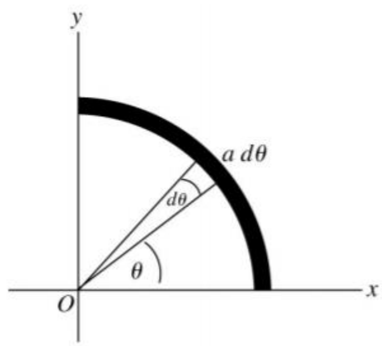
21.85 •• **CALC** Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies in the first quadrant, with the center of curvature at the origin. Find the x- and y-components of the net electric field at the origin.

21.85. IDENTIFY: Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x- and y-components of the total field.

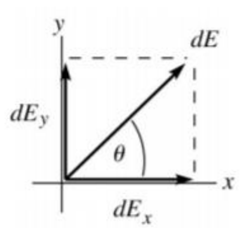
SET UP: Consider the small segment shown in Figure 21.85a.



EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\dfrac{a d\theta}{\frac{1}{2}\pi a}\right)Q = \dfrac{2Q}{\pi}d\theta.$ ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

Figure 21.85a

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.85b, along with its components.



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{|dQ|}{a^2}.$$

$$dE = \frac{Q}{2\pi^2\varepsilon_0 a^2}d\theta.$$

Figure 21.85b

$dE_x = dE\cos\theta = (Q/2\pi^2\varepsilon_0 a^2)\cos\theta d\theta.$

$$E_x = \int dE_x = \frac{Q}{2\pi^2\varepsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \frac{Q}{2\pi^2\varepsilon_0 a^2}\left(\sin\theta\Big|_0^{\pi/2}\right) = \frac{Q}{2\pi^2\varepsilon_0 a^2}.$$

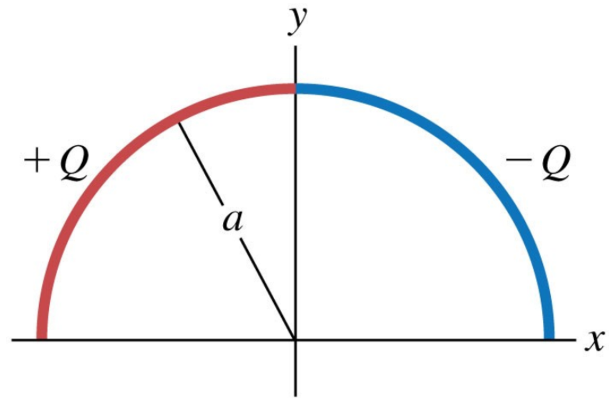
$dE_y = dE\sin\theta = (Q/2\pi^2\varepsilon_0 a^2)\sin\theta d\theta.$

$$E_y = \int dE_y = \frac{Q}{2\pi^2\varepsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \frac{Q}{2\pi^2\varepsilon_0 a^2}\left(-\cos\theta\Big|_0^{\pi/2}\right) = \frac{Q}{2\pi^2\varepsilon_0 a^2}.$$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.86 •• **CALC** A semicircle of radius a is in the first and second quadrants, with the center of curvature at the origin. Positive charge $+Q$ is distributed uniformly around the left half of the semicircle, and negative charge $-Q$ is distributed uniformly around the right half of the semicircle (**Fig. P21.86**). What are the magnitude and direction of the net electric field at the origin produced by this distribution of charge?

Figure **P21.86**



21.86. **IDENTIFY:** We must add the electric field components of the positive half and the negative half.

SET UP: From Problem 21.85, the electric field due to the quarter-circle section of positive charge has components $E_x = +\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$, $E_y = -\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$. The field due to the quarter-circle section of negative charge has components $E_x = +\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$, $E_y = +\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$.

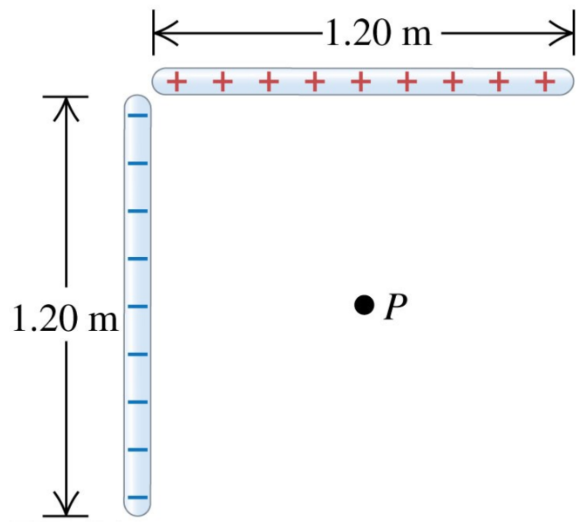
EXECUTE: The components of the resultant field is the sum of the x- and y-components of the fields due to each half of the semicircle. The y-components cancel, but the x-components add, giving

$E_x = +\dfrac{Q}{\pi^2\varepsilon_0 a^2}$, in the $+x$-direction.

EVALUATE: Even though the net charge on the semicircle is zero, the field it produces is *not* zero because of the way the charge is arranged.

21.87 •• Two 1.20-m nonconducting rods meet at a right angle. One rod carries $+2.50\ \mu\text{C}$ of charge distributed uniformly along its length, and the other carries $-2.50\ \mu\text{C}$ distributed uniformly along it (**Fig. P21.87**). (a) Find the magnitude and direction of the electric field these rods produce at point P, which is 60.0 cm from each rod. (b) If an electron is released at P, what are the magnitude and direction of the net force that these rods exert on it?

Figure **P21.87**



21.87. IDENTIFY: Each wire produces an electric field at P due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is

$E_{\text{net}} = 2E_1 \cos 45° = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at P is eE.

EXECUTE: **(a)** The net field is $E_{\text{net}} = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$.

$$E_{\text{net}} = \frac{2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\ \text{C})\cos 45°}{(0.600\ \text{m})\sqrt{(0.600\ \text{m})^2+(0.600\ \text{m})^2}} = 6.25\times10^4\ \text{N/C}.$$

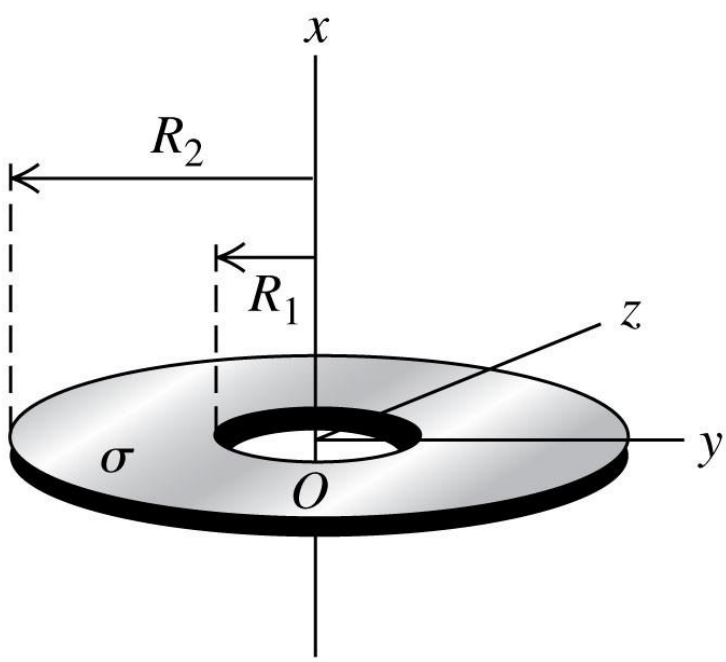
The direction is 225° counterclockwise from an axis pointing to the right at point P.

(b) $F = eE = (1.60\times10^{-19}\ \text{C})(6.25\times10^4\ \text{N/C}) = 1.00\times10^{-14}\ \text{N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.91 •• **CP** A thin disk with a circular hole at its center, called an *annulus*, has inner radius R_1 and outer radius R_2 (**Fig. P21.91**). The disk has a uniform positive surface charge density σ on its surface. (a) Determine the total electric charge on the annulus. (b) The annulus lies in the yz-plane, with its center at the origin. For an arbitrary point on the x-axis (the axis of the annulus), find the magnitude and direction of the electric field $\vec{E}$. Consider points both above and below the annulus. (c) Show that at points on the x-axis that are sufficiently close to the origin, the magnitude of the electric field is approximately proportional to the distance between the center of the annulus and the point. How close is "sufficiently close"?

Fig. P21.91



21.91. IDENTIFY: Apply the formula for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - 1/\sqrt{(R/x)^2 + 1}\right].$$

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma.$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2 + 1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2 + 1}\right]\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$

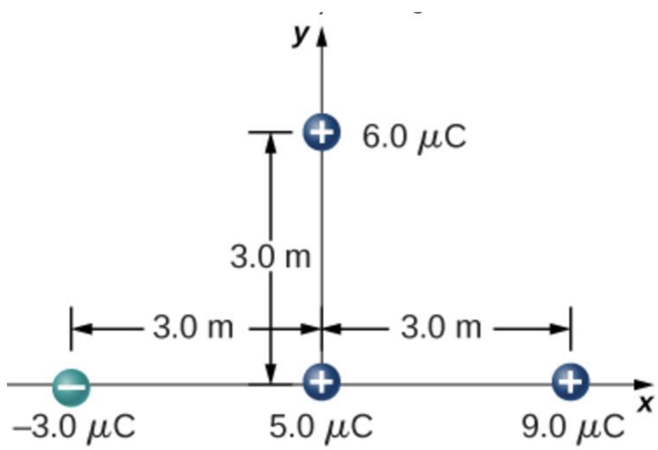
$\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(1/\sqrt{(R_1/x)^2 + 1} - 1/\sqrt{(R_2/x)^2 + 1}\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$ The electric field is in the $+x$-direction at points above the disk and in the $-x$-direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{\boldsymbol{i}}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2 + 1} = \dfrac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives

$\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\dfrac{|x|^2}{x}\hat{\boldsymbol{i}} = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{\boldsymbol{i}}.$ Sufficiently close means that $(x/R_1)^2 \ll 1$.

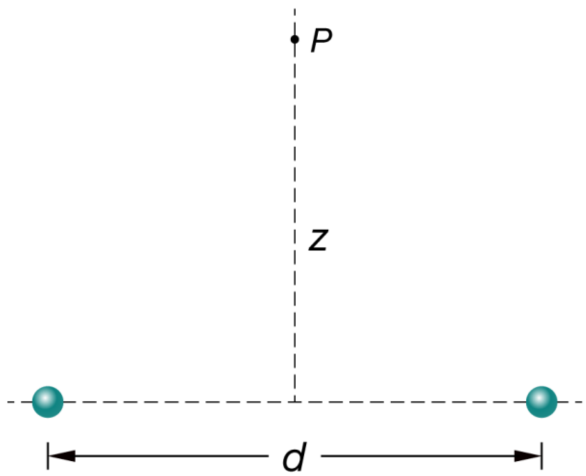
Kudu Example: Question 1 in Electric Charges HW

What is the force on the 5.0 μC charge shown below?



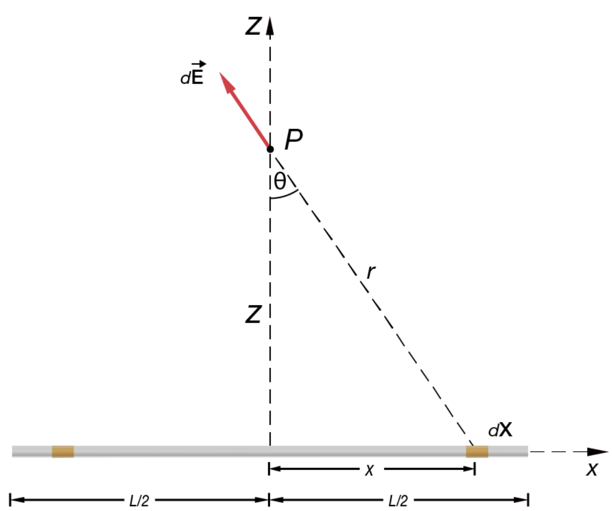
Kudu Example: The Electric Field above Two Equal Charges

(a) Find the electric field (magnitude and direction) a distance z above the midpoint between two equal charges +q that are a distance d apart (See figure). Check that your result is consistent with what you'd expect when $z \gg d$.
(b) The same as part (a), only this time make the right-hand charge -q instead of +q.



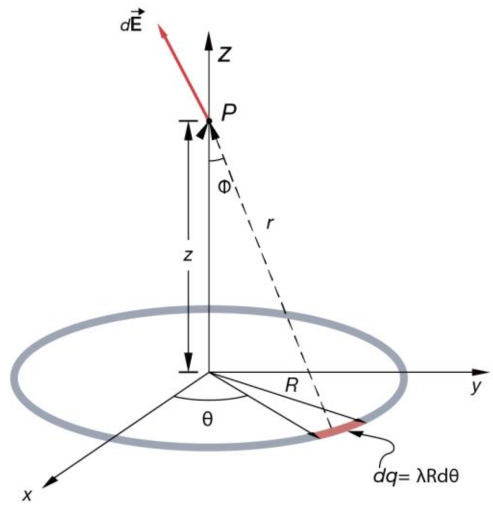
Kudu Example: Electric Field of a Line Segment

Find the electric field a distance z above the midpoint of a straight line segment of length L that carries a uniform line charge density λ.



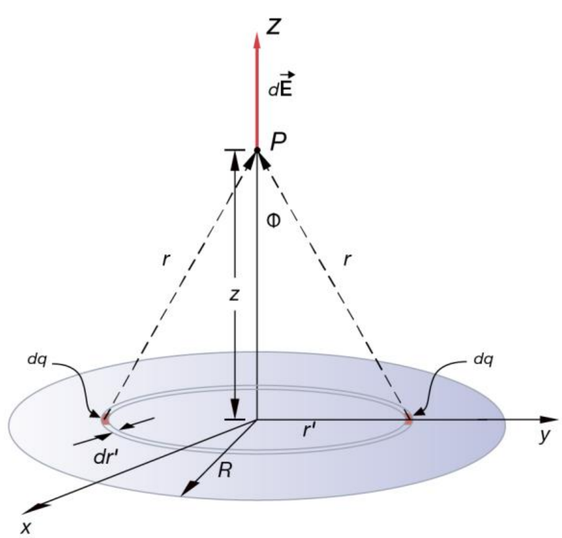
Kudu Example: Electric Field due to a Ring of Charge

A ring with radius R has a uniform charge density λ, with units of coulomb per unit meter of arc. Find the electric field at a point on the axis passing through the center of the ring.



Kudu Example: Electric Field of a Disk

Find the electric field of a circular thin disk of radius R and uniform charge density σ at a distance zzabove the center of the disk.



Kudu Example: The Field of Two Infinite Planes

Find the electric field everywhere resulting from two infinite planes with equal but opposite charge densities.