14.11 •• An object is undergoing SHM with period 0.900 s and amplitude 0.320 m. At $t = 0$ the object is at $x = 0.320$ m and is instantaneously at rest. Calculate the time it takes the object to go (a) from $x = 0.320$ m to $x = 0.160$ m and (b) from $x = 0.160$ m to $x = 0$.

14.11. IDENTIFY: For SHM the motion is sinusoidal.

SET UP: $x(t) = A\cos(\omega t)$.

EXECUTE: $x(t) = A\cos(\omega t)$, where $A = 0.320$ m and $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.900 \text{ s}} = 6.981$ rad/s.

(a) $x = 0.320$ m at $t_1 = 0$. Let t_2 be the instant when $x = 0.160$ m. Then we have

$0.160 \text{ m} = (0.320 \text{ m}) \cos(\omega t_2)$. $\cos(\omega t_2) = 0.500$. $\omega t_2 = 1.047$ rad. $t_2 = \dfrac{1.047 \text{ rad}}{6.981 \text{ rad/s}} = 0.150$ s. It takes

$t_2 - t_1 = 0.150$ s.

(b) Let t_3 be when $x = 0$. Then we have $\cos(\omega t_3) = 0$ and $\omega t_3 = 1.571$ rad. $t_3 = \dfrac{1.571 \text{ rad}}{6.981 \text{ rad/s}} = 0.225$ s.

It takes $t_3 - t_2 = 0.225 \text{ s} - 0.150 \text{ s} = 0.0750$ s.

EVALUATE: Note that it takes twice as long to go from $x = 0.320$ m to $x = 0.160$ m than to go from $x = 0.160$ m to $x = 0$, even though the two distances are the same, because the speeds are different over the two distances.

14.16 •• A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the amplitude of the motion is 0.090 m, it takes the block 2.70 s to travel from $x = 0.090$ m to $x = -0.090$ m. If the amplitude is doubled, to 0.180 m, how long does it take the block to travel (a) from $x = 0.180$ m to $x = -0.180$ m and (b) from $x = 0.090$ m to $x = -0.090$ m?

14.16. IDENTIFY: The motion is SHM, and in each case the motion described is one-half of a complete cycle.

SET UP: For SHM, $x = A\cos(\omega t)$ and $\omega = \dfrac{2\pi}{T}$.

EXECUTE: **(a)** The time is half a period. The period is independent of the amplitude, so it still takes 2.70 s.

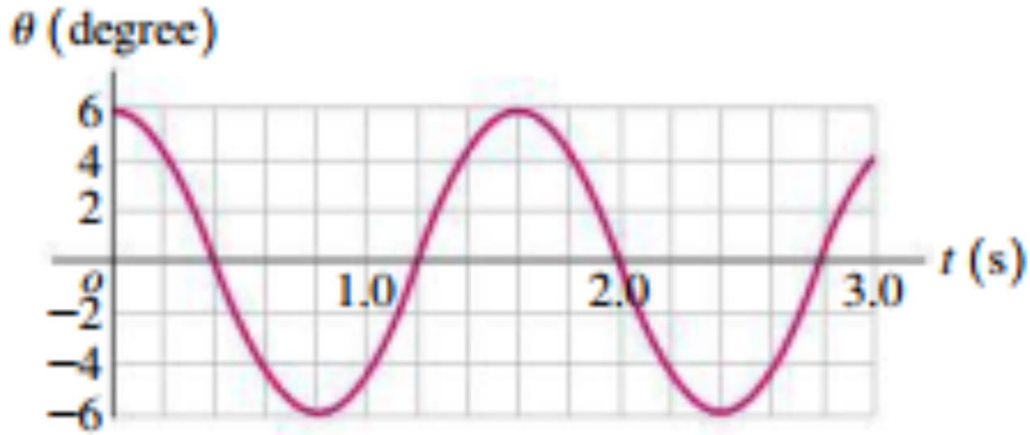
(b) $x = 0.090$ m at time t_1. $T = 5.40$ s and $\omega = \dfrac{2\pi}{T} = 1.164$ rad/s. $x_1 = A\cos(\omega t_1)$. $\cos(\omega t_1) = 0.500$.

$\omega t_1 = 1.047$ rad and $t_1 = 0.8997$ s. $x = -0.090$ m at time t_2. $\cos(\omega t_2) = -0.500$ m. $\omega t_2 = 2.094$ rad and $t_2 = 1.800$ s. The elapsed time is $t_2 - t_1 = 1.800 \text{ s} - 0.8997 \text{ s} = 0.900$ s.

EVALUATE: It takes less time to travel from ±0.090 m in (b) than it originally did because the block has larger speed at ± 0.090 m with the increased amplitude.

14.50 •• In the laboratory, a student studies a pendulum by graphing the angle θ that the string makes with the vertical as a function of time t, obtaining the graph shown in Fig. E14.50. (a) What are the period, frequency, angular frequency, and amplitude of the pendulum's motion? (b) How long is the pendulum? (c) Is it possible to determine the mass of the bob?

Figure E14.50



14.50. **IDENTIFY** and **SET UP:** The period is for the time for one cycle. The angular amplitude is the maximum value of θ.

EXECUTE: **(a)** From the graph with the problem, $T = 1.60$ s. $f = \frac{1}{T} = 0.625$ Hz. $\omega = 2\pi f = 3.93$ rad/s.

From the graph we also determine that the amplitude is 6 degrees.

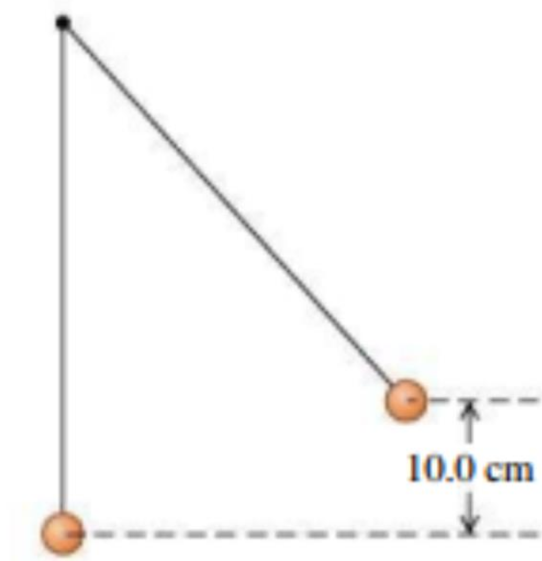
(b) $T = 2\pi\sqrt{\frac{L}{g}}$ so $L = g\left(\frac{T}{2\pi}\right)^2 = (9.80\text{ m/s}^2)\left(\frac{1.60\text{ s}}{2\pi}\right)^2 = 0.635$ m.

(c) No. The graph is unchanged if the mass of the bob is changed while the length of the pendulum and amplitude of swing are kept constant. The period is independent of the mass of the bob.

EVALUATE: The amplitude of the graph in the problem does not decrease over the time shown, so there must be little or no friction in this pendulum.

14.85 • **CP** In Fig. P14.85 the upper ball is released from rest, collides with the stationary lower ball, and sticks to it. The strings are both 50.0 cm long. The upper ball has mass 2.00 kg, and it is initially 10.0 cm higher than the lower ball, which has mass 3.00 kg. Find the frequency and maximum angular displacement of the motion after the collision.

Figure P14.85



14.85. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2gh_0 = \frac{1}{2}m_2v^2$ and

$$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s}.$$

Momentum conservation during collision: $m_2v = (m_2 + m_3)V$ and

$$V = \frac{m_2v}{m_2 + m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s}.$$

Energy conservation during upward swing: $Mgh_f = \frac{1}{2}MV^2$ and

$$h_f = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm}.$$

Figure 14.85 shows how the maximum angular displacement is calculated from h_f. $\cos\theta = \frac{48.4\text{ cm}}{50.0\text{ cm}}$ and

$\theta = 14.5°$. $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{0.500\text{ m}}} = 0.705\text{ Hz}.$

EVALUATE: $14.5° = 0.253$ rad. $\sin(0.253\text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and the equation $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ is accurate.

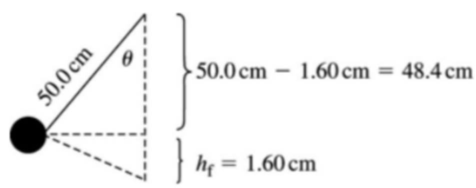


Figure 14.85

14.54 •• We want to hang a thin hoop on a horizontal nail and have the hoop make one complete small-angle oscillation each 2.0 s. What must the hoop's radius be?

14.54. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

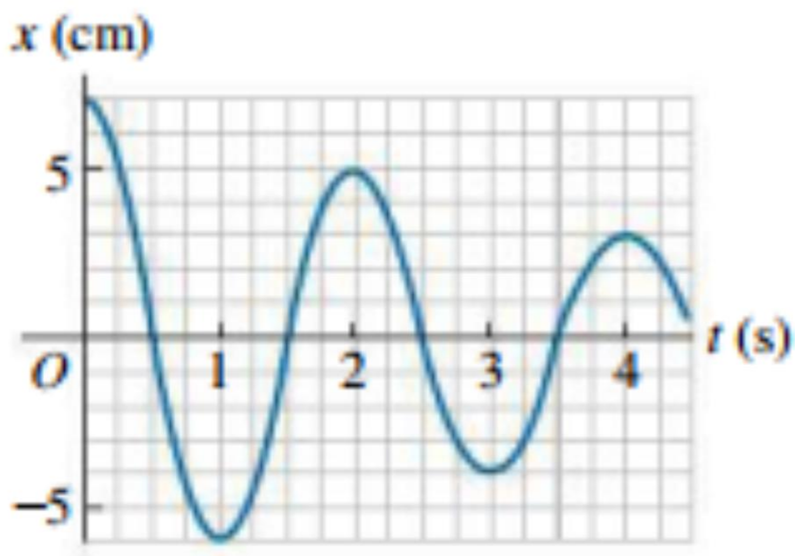
SET UP: From the parallel axis theorem, the moment of inertia of the hoop about the nail is $I = MR^2 + MR^2 = 2MR^2$. $d = R$.

EXECUTE: Solving for R, $R = gT^2/8\pi^2 = 0.496$ m.

EVALUATE: A simple pendulum of length $L = R$ has period $T = 2\pi\sqrt{R/g}$. The hoop has a period that is larger by a factor of $\sqrt{2}$.

14.62 •• A mass is vibrating at the end of a spring of force constant 225 N/m. Figure E14.62 shows a graph of its position x as a function of time t. (a) At what times is the mass not moving? (b) How much energy did this system originally contain? (c) How much energy did the system lose between $t = 1.0$ s and $t = 4.0$ s? Where did this energy go?

Figure E14.62



14.62. IDENTIFY: The graph shows that the amplitude of vibration is decreasing, so the system must be losing mechanical energy.

SET UP: The mechanical energy is $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2.$

EXECUTE: **(a)** When $|x|$ is a maximum and the tangent to the curve is horizontal the speed of the mass is zero. This occurs at $t = 0$, $t = 1.0$ s, $t = 2.0$ s, $t = 3.0$ s and $t = 4.0$ s.

(b) At $t = 0$, $v_x = 0$ and $x = 7.0$ cm so $E_0 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(0.070\text{ m})^2 = 0.55\text{ J}.$

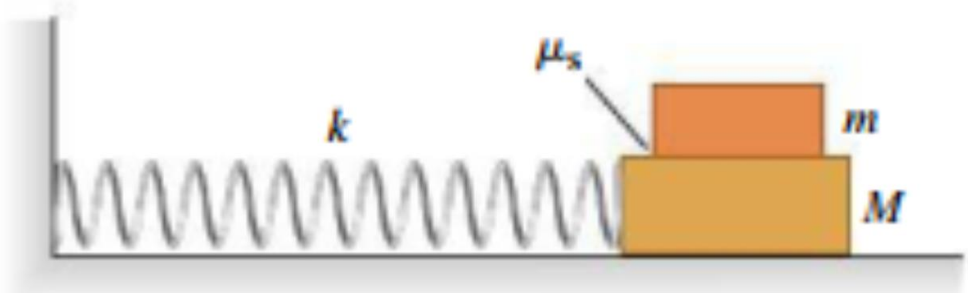
(c) At $t = 1.0$ s, $v_x = 0$ and $x = -6.0$ cm so $E_1 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(-0.060\text{ m})^2 = 0.405\text{ J}.$

At $t = 4.0$ s, $v_x = 0$ and $x = 3.0$ cm so $E_4 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(0.030\text{ m})^2 = 0.101\text{ J}.$ The mechanical energy "lost" is $E_1 - E_4 = 0.30$ J. The mechanical energy lost was converted to other forms of energy by nonconservative forces, such as friction, air resistance, and other dissipative forces.

EVALUATE: After a while the mass will come to rest and then all its initial mechanical energy will have been "lost" because it will have been converted to other forms of energy by nonconservative forces.

14.68 •• **CP** A block with mass M rests on a frictionless surface and is connected to a horizontal spring of force constant k. The other end of the spring is attached to a wall (**Fig. P14.68**). A second block with mass m rests on top of the first block. The coefficient of static friction between the blocks is μ_s. Find the *maximum* amplitude of oscillation such that the top block will not slip on the bottom block.

Figure **P14.68**



14.68. IDENTIFY: In SHM, $a_{\text{max}} = \dfrac{k}{m_{\text{tot}}}A$. Apply $\Sigma \vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

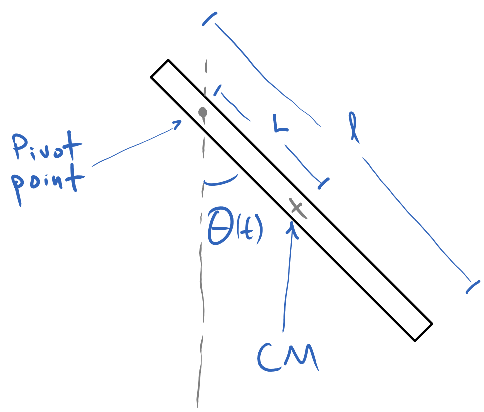
EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\Sigma F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{\text{max}} = \left(\dfrac{k}{M+m}\right)A$. Set $a_{\text{max}} = a$ and solve for A: $\mu_s g = \left(\dfrac{k}{M+m}\right)A$ and $A = \dfrac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.

Example - The Physical Pendulum

A 1.80-kg, 0.599 m long rod is pivoted 0.250 m from its center of mass and allowed to swing as a physical pendulum. The period for small-angle oscillations is 1.22 s. (a) What is the moment of inertia of the wrench about an axis through the pivot? (b) If the rod is initially displaced 0.400 rad from its equilibrium position, what is the angular speed of the rod as it passes through the equilibrium position?

(a)



Given: $M = 1.80\ kg$

$\ell = 0.599\ m$

$L = 0.250\ m$

$T = 1.22\ s$

Recall: Parallel Axis Theorem

$$\underbrace{I_p}_{\text{parallel axis through pivot point}} = \overbrace{I_{cm}}^{\text{axis through CM}} + M\underbrace{L^2}_{\text{pivot-CM distance}}$$

Recall that, for a rod that pivots about its CM:

$$I_{cm} = \frac{1}{12}M\ell^2$$

We have: $I_p = \frac{1}{12}M\ell^2 + ML^2 = \boxed{0.166\ kg\cdot m^2}$

There is another way to solve for I: we are given T!!!

We know that, by definition: $\omega = \frac{2\pi}{T} = \sqrt{\frac{MgL}{I}}$

So: $I = \left(\frac{T}{2\pi}\right)^2 MgL = \left(\frac{1.22\ s}{2\pi}\right)^2 (1.80\ kg)(9.8\ m/s^2)(0.250\ m)$

$= 0.166\ kg\cdot m^2$ ✓

(b)

We have: $\theta(t) = \Theta \cos(\omega t + \phi)$

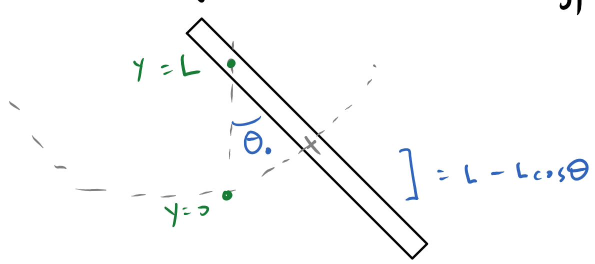
Because we are given θ_0, and are told the rod is at rest at $t=0$, we know $\phi = 0$.

This question is <u>not</u> asking for the angular <u>frequency</u> of oscillations, ω.

This question is asking for the angular <u>speed</u>,

To distinguish from ω → $\Omega(t) = \frac{d}{dt}\theta(t)$

We can use Energy:



$E_i = MgL(1-\cos\theta)$

$E_f = \frac{1}{2} I \Omega_{max}^2$

$E_i = E_f \quad \rightarrow \quad \Omega_{max}^2 = 2\underbrace{\frac{MgL}{I}}_{\omega^2 = \left(\frac{2\pi}{T}\right)^2}(1-\cos\theta)$

$$\Omega_{max} = \left(\frac{2\pi}{T}\right)\sqrt{2(1-\cos\theta_0)}$$

$$= \boxed{2.05 \text{ rad/s}}$$

There is another way to solve:

$\Omega(t) = \frac{d}{dt}\theta(t) = -\omega\underbrace{\Theta}_{0.400 \text{ rad}}\sin(\omega t)$

The rod reaches equilibrium from rest in $\frac{1}{4}T$:

$\Omega\left(\frac{1}{4}T\right) = -\omega\Theta\sin\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right) = -\omega\Theta\sin\left(\frac{\pi}{2}\right) = -\omega\Theta$

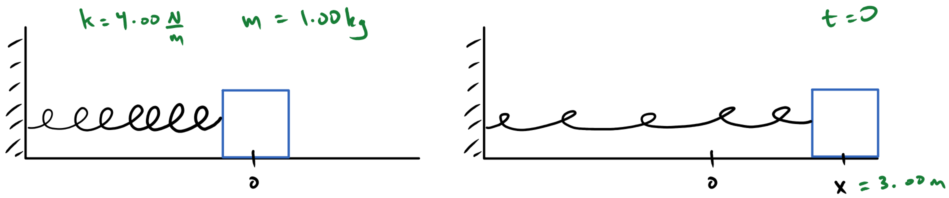
Drop. Asked for speed only, <u>not</u> direction

$$\Omega_{max} = \omega \Theta = \frac{2\pi}{1.22\ s} \times 0.400\ rad = \boxed{2.06\ rad/s}$$

It is a small approximation error. This difference is the result of the small angle approximation we made to obtain $\theta(t) = \Theta \cos(\omega t + \phi)$.

We did not use any approximations in solving with energy.

EXAMPLE Mass on a Spring 3



Consider a block of mass $m = 1.00$ kg on a spring with spring constant $k = 4.00$ N/m. The spring and block are on a horizontal table, and the spring is attached to the wall. At $t = 0$, the block is pulled 400 cm from its equilibrium position then released. Find the equation that describes its position in time, $x(t)$.

We know $x(t) = A\cos(\omega t + \phi)$. We must find A, ω, ϕ

<u>A</u>

At $t = 0$, when the block is released, it will immediately move left. The block position will never exceed $x = 400$ cm. Therefore, $A = 400$ cm.

The block will oscillate between $x = 400$ cm and $x = -400$ cm.

<u>ω</u>

We are given k and m: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.00\ \text{N/m}}{1.00\ \text{kg}}} = 2.00$ rad/s

$\underline{\phi}$

From our knowledge of cosine behavior, it may be immediately apparent that $\phi = 0$ because $x = A$ at $t = 0$.

$$\begin{aligned} x(0) &= A \quad (\text{given}) \\ &= A\cos(\omega \cdot 0 + \phi) \\ &= A\cos(\phi) \qquad \Rightarrow \qquad \phi = 0 \end{aligned}$$

Putting it all together, we have:

$$\boxed{x(t) = (4.00\,\text{cm})\cos(2.00\,t)}$$

EXAMPLE Mass on a Spring 4

Suppose we have a system similar to the example above, with $k = 4.00$ N/m and $m = 1.00$ kg. However, this time we did not observe the motion ourselves. A friend did observe the motion, but gives us limited information. When this friend begins to pay attention, the block is at position 4.00 cm, and 1.34 s later the block is at position -2.09 cm.

(a) What is the Amplitude of this oscillator?

(b) Describe the motion of the block in those 1.34 seconds.

(a) The equation of motion is: $x(t) = A\cos(\omega t + \phi)$.

From the previous example, we know $\omega = 2.00$ rad/s.

We are given two positions and one time. The simplest assignment of variables is:

$$t_0 = 0, \qquad x_0 = 400\text{ cm}$$
$$t_1 = 1.34\text{ s}, \qquad x_1 = -209\text{ cm}$$

This assignment yields the following equations:

$$x_0 = A\cos(\phi) \qquad (\omega \cdot 0 = 0)$$
$$x_1 = A\cos(\omega t_1 + \phi)$$

We have 2 equations and 2 unknowns: A, ϕ. Therefore, we can substitute the first equation into the second equation.

We will also need the trig identity:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Solve:

$$\begin{aligned} x_1 &= A\cos(\omega t_1 + \phi) \\ &= A\cos(\omega t_1)\cos(\phi) - A\sin(\omega t_1)\sin(\phi) \\ &= x_0\cos(\omega t_1) - x_0\tan(\phi)\sin(\omega t_1) \end{aligned}$$

used $A\cos(\phi) = x_0$ used $A = x_0/\cos(\phi)$

Some algebra:

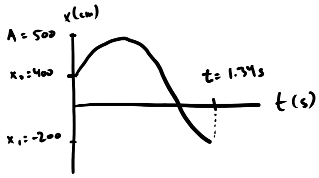
$$\sin(\omega t_1)\tan(\phi) = \cos(\omega t_1) - x_1/x_0$$
$$\tan(\phi) = \cot(\omega t_1) - (x_1/x_0)\csc(\omega t_1)$$

$$\phi = \tan^{-1}\left[\cot(\omega t_1) - (x_1/x_0)\csc(\omega t_1)\right]$$

$$= -0.697 \approx -\frac{2\pi}{9}$$

$$A = x_0 / \cos(\phi)$$
$$= 522 \text{ cm}$$

(b) Plot $x(t) = 522 \cos\left(2t - \frac{2\pi}{9}\right)$



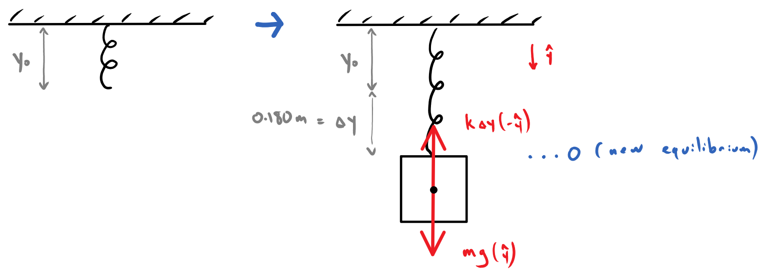
When your friend begins his observation, the block is moving to the right, toward A = 500 cm. After 1.34 s, the block is moving toward position -500 cm.

Example Vertical SHM

14.38 • A proud deep-sea fisherman hangs a 65.0-kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.180 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?

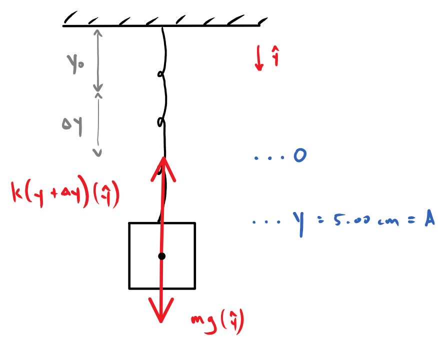
(*University Physics, 14th Ed*)

(a)



$$mg = kx \text{ so } k = \frac{mg}{x} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{0.180\text{ m}} = 3.54\times10^3\text{ N/m}$$

(b)



$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{65.0\text{ kg}}{3.54\times10^3\text{ N/m}}} = 0.8514\text{ s which rounds to } 0.851\text{ s}$$

(c) $v(t) = -\omega A\sin(\omega t + \phi)$

$$v_{max} = \omega A = \frac{2\pi}{T}A$$

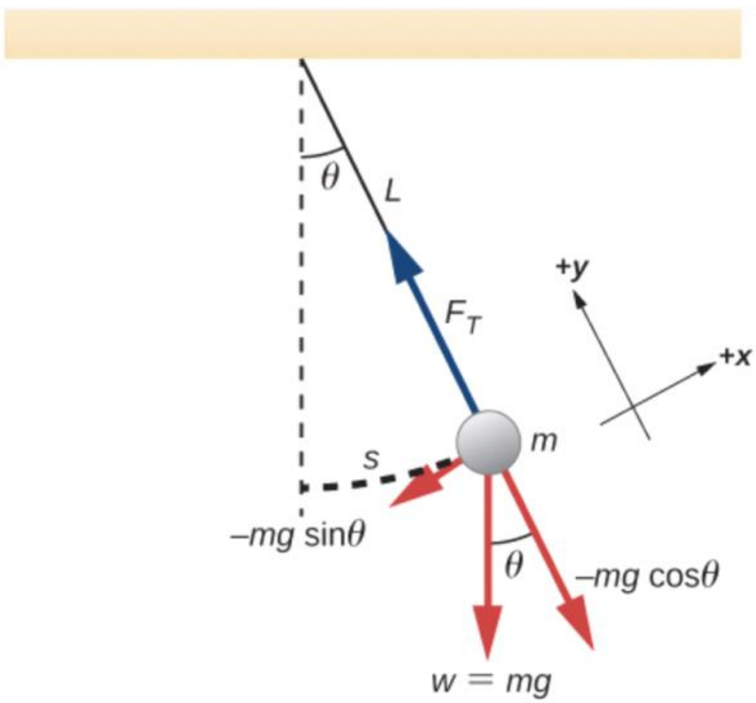
$$v_{\max} = 2\pi fA = \frac{2\pi A}{T} = \frac{2\pi(0.0500\text{ m})}{0.8514\text{ s}} = 0.369\text{ m/s.}$$

Example The Simple Pendulum

14.45 •• You pull a simple pendulum 0.240 m long to the side through an angle of 3.50° and release it. (a) How much time does it take the pendulum bob to reach its highest speed? (b) How much time does it take if the pendulum is released at an angle of 1.75° instead of 3.50°?

(University Physics, 14th Ed)

① Review : NOT part of the problem.



$F_T = mg\cos\theta$

We want to know θ(t). Therefore, we naturally go with torque:

$$\tau = -L(mg\sin\theta)$$

$$I\alpha = -L(mg\sin\theta)$$

$$I\frac{d^2\theta}{dt^2} = -L(mg\sin\theta)$$

$$mL^2\frac{d^2\theta}{dt^2} = -L(mg\sin\theta)$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$$ For small angles, $\sin\theta \approx \theta$

For small angles, $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta .$$

If we set $g/l = \omega^2$, then:

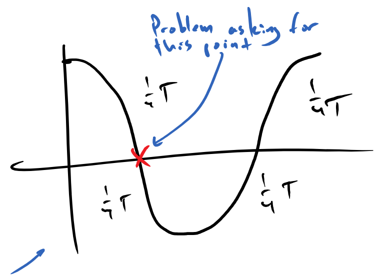
$$\theta(t) = \Theta_{\underset{=A}{}}\cos(\omega t + \phi)$$

② Solve the problem.

We are given: $l = 0.240\,m$, $\Theta_0 = 3.50° = A$

(a)

$t_{v_{max}} = \frac{1}{4}T$. How?



NOTE:

Careful !!! Only cut this way when all pieces are identical

Ex: Can also cut at $\frac{1}{2}T$, but not $\frac{1}{6}T$

In case it is not obvious, I assign a HW problem that demonstrates this.

Back to the problem:

$$\omega = \sqrt{\frac{g}{l}} = \frac{2\pi}{T}$$

$$\frac{1}{4}T = \frac{1}{4}\left(2\pi\sqrt{\frac{l}{g}}\right) = \frac{\pi}{2}\sqrt{\frac{0.240\,m}{9.8\,m/s^2}} = \boxed{0.25\,s}$$

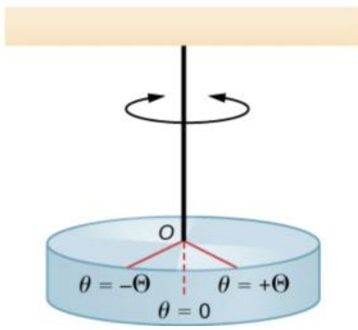
(b) Same! $\omega = \sqrt{\frac{g}{l}}$. No A-dependence.

EXAMPLE The Torsional Pendulum

14.40 •• A uniform, solid metal disk of mass 6.50 kg and diameter 24.0 cm hangs in a horizontal plane, supported at its center by a vertical metal wire. You find that it requires a horizontal force of 4.23 N tangent to the rim of the disk to turn it by 3.34°, thus twisting the wire. You now remove this force and release the disk from rest. (a) What is the torsion constant for the metal wire? (b) What are the frequency and period of the torsional oscillations of the disk? (c) Write the equation of motion for $\theta(t)$ for the disk.

(University Physics, 14th Ed)

① Review



analogous to $F = -kx$

$$\tau = -\kappa \theta$$

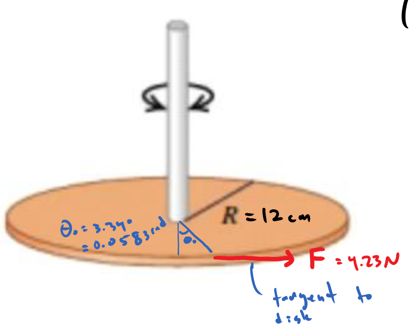
$$\Sigma \tau = I\alpha = I \frac{d^2}{dt^2}\theta = -\kappa\theta$$

If we set $\kappa / I \equiv \omega^2$, then

$$\theta(t) = \Theta \cos(\omega t + \phi)$$

② Solve the problem.

We are given:



(a) Use: $\tau = -\kappa\theta \rightarrow \kappa = -\tau/\theta$

$$= -\frac{(-4.23\ N)(0.12\ m)}{0.0583\ rad}$$

$$= \boxed{8.71\ \frac{N \cdot m}{rad}}$$

14.40. IDENTIFY: The torsion constant κ is defined by $\tau_z = -\kappa\theta$. $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ and $T = 1/f$. $\theta(t) = \Theta\cos(\omega t + \phi)$.

SET UP: For the disk, $I = \frac{1}{2}MR^2$. $\tau_z = -FR$. At $t = 0$, $\theta = \Theta = 3.34° = 0.0583$ rad, so $\phi = 0$.

EXECUTE: **(a)** $\kappa = -\frac{\tau_z}{\theta} = -\frac{-FR}{0.0583\text{ rad}} = +\frac{(4.23\text{ N})(0.120\text{ m})}{0.0583\text{ rad}} = 8.71\text{ N}\cdot\text{m/rad}$

(b) $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} = \frac{1}{2\pi}\sqrt{\frac{2\kappa}{MR^2}} = \frac{1}{2\pi}\sqrt{\frac{2(8.71\text{ N}\cdot\text{m/rad})}{(6.50\text{ kg})(0.120\text{ m})^2}} = 2.17$ Hz. $T = 1/f = 0.461$ s.

(c) $\omega = 2\pi f = 13.6$ rad/s. $\theta(t) = (3.34°)\cos([13.6\text{ rad/s}]t)$.

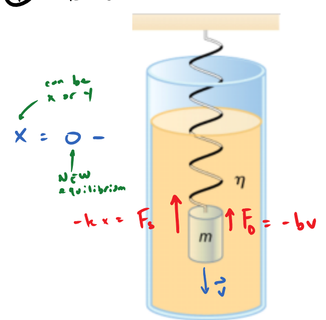
EVALUATE: The frequency and period are independent of the initial angular displacement, so long as this displacement is small.

$\boxed{\text{EXAMPLE}}$ Damped Oscillations

14.60 •• A 50.0-g hard-boiled egg moves on the end of a spring with force constant $k = 25.0$ N/m. Its initial displacement is 0.300 m. A damping force $F_x = -bv_x$ acts on the egg, and the amplitude of the motion decreases to 0.100 m in 5.00 s. Calculate the magnitude of the damping constant b.

(University Physics, 14th Ed.)

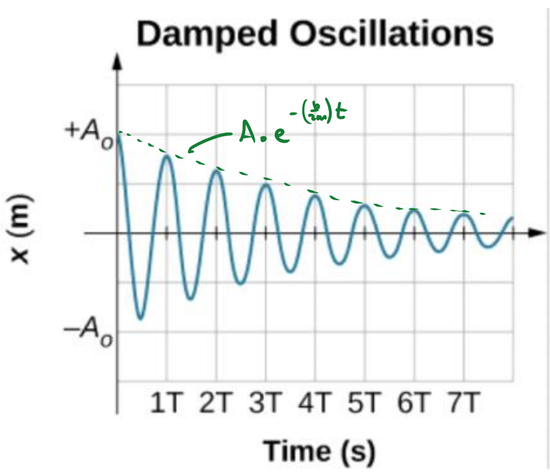
① Review



$$\Sigma F = ma = -bv - kx$$ (Fg doesn't show up at new equilibrium)

$$m\frac{d^2}{dt^2} = -b\frac{d}{dt}x - kx$$

$$x(t) = A_0 e^{-\frac{b}{2m}t}\cos(\omega t + \phi), \quad \omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$



② Solve the problem

We only need $A_0 e^{-\left(\frac{b}{2m}\right)t}$ to solve this; all damping is controlled by this term.

We are told that:

$t_0 = 0$, $t_1 = 5$s, $A_0 = 0.300$m, $A_1 = 0.100$m

$$A_0 e^{-\frac{b}{2m}t_1} = A_1$$

$$b = \frac{-2m}{t_1}\ln\left(\frac{A_1}{A_0}\right)$$

$$= \boxed{0.220\ \frac{kg}{s}}$$

Note: we did not need/use k

$\boxed{\text{EXAMPLE}}$ Forced Oscillations

14.63 • A sinusoidally varying driving force is applied to a damped harmonic oscillator of force constant k and mass m. If the damping constant has a value b_1, the amplitude is A_1 when the driving angular frequency equals $\sqrt{k/m}$. In terms of A_1, what is the amplitude for the same driving frequency and the same driving force amplitude F_{max}, if the damping constant is (a) $3b_1$ and (b) $b_1/2$?

(University Physics, 14th Ed)

① REVIEW

For the spring-in-viscous fluid above, now assume there is a driving force on the mass $F_0 \sin(\omega t)$:

$$m\frac{d^2}{dt^2}x = -kx - b\frac{d}{dt}x + F_0 \sin(\omega t)$$

driving force ang freq $\neq \sqrt{\frac{k}{m}}$

We are now calling $\sqrt{\frac{k}{m}} \equiv \omega_0$

x(t) derivation beyond scope of this course.

$$A = \frac{F_0}{\sqrt{m^2(\omega^2-\omega_0^2)^2 + b^2\omega^2}}$$

← recommend to put on index card.

when $\omega = \omega_0$, A = maximum. This is resonance.

② Solve the problem

When $\omega = \sqrt{\frac{k}{m}}$ (driving force (students: be very clear)), then

$$A_1 = \frac{F_0}{\sqrt{m^2\left(\frac{k}{m} - \omega_0^2\right)^2 + b_1^2\frac{k}{m}}} = \sqrt{\frac{m}{k}}\,\frac{F_0}{b_1}$$

ω_0^2: also $= \frac{k}{m}$

F_0 unknown

If $b = 3b_1$, then

$$A = \frac{F_0}{\sqrt{0 + (3b_1)^2\frac{k}{m}}} = \sqrt{\frac{m}{k}}\,\frac{F_0}{3b_1} = \boxed{\frac{1}{3}A_1}$$

Can do same for $b = b_1/2$, will get $\boxed{2A_1}$