Discussion 3 - Mechanical Waves

Problem 1. A block with mass m moves in a viscous fluid that provides a frictional force, $F = -bv$. The block is attached on the left to a spring with spring constant k. The system is driven from the right with an oscillating force $F = F_0 \cos(\omega_d t)$.

a. Write a differential equation that describes the motion of the block.

b. Find the amplitude of the motion.

c. What driving frequency, ω_d, will maximize the amplitude?

Problem 2. Show that a standing wave is the superposition of an incident wave moving to the right and a reflected wave moving to the left with a π phase shift. A wave reflected from a fixed end of a string is inverted. How does the amplitude of the standing wave compare to the amplitude of the incident wave? You may need to use the identity:

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta \tag{1}$$

Problem 3. A horizontal string tied at both ends is vibrating in its fundamental mode. The *traveling* waves have speed v, frequency f, amplitude A, and wavelength λ. Consider the points located at $x = \lambda/2$, $x = \lambda/4$, and $x = \lambda/8$ from the left-hand end of the string.

a. Calculate the maximum transverse velocity and maximum transverse acceleration at these points.

b. What is the amplitude of the motion at these points?

c. How much time does it take the string to go from its largest upward displacement to its largest downward displacement at these points?

Problem 4. A vibrating string of length L is under a tension of F_T. The results from five successive stroboscopic pictures are shown in the figure. The strobe rate is set at N flashes per second, and observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between.

a. Find the period, frequency, and wavelength for the traveling waves on this string.

b. In what normal mode (harmonic) is the string vibrating?

c. What is the speed of the traveling waves on the string?

d. How fast is point P moving when the string is in

   i. position 1

   ii. position 3

e. What is the mass of this string?

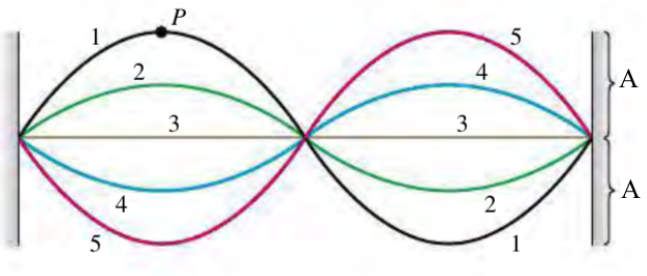

Figure 1: Problem 4