Midterm 1 Review Problems

Here are 8 problems that cover most of the topics on the midterm. Try to do these problems on your own before looking at the solutions. Don't be discouraged if you don't get the full answer, remember there is lots of partial credit given!
At the end of the document check out the studying and test taking tips!

Problem 1. A mass-spring system has natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$ and damping constant $\gamma = \frac{b}{2m}$ (where the damping force is $F = -bv$.) After N cycles, the system loses half its energy to the damping medium. Find N in terms of ω_0 and γ.

The amplitude decays exponentially and the energy is proportional to the square of the amplitude.

$$E(t) = \frac{1}{2}kA(t)^2 \tag{1}$$

$$A(t) = A_0 e^{-\gamma t} \tag{2}$$

$$\tag{3}$$

Compare the amplitude at a time t_0 to a time t_1 which is N cycles later. Each cycle takes time equal to one period.

$$t_1 = t_0 + NT \tag{4}$$

$$E(t_1) = \frac{1}{2}E(t_0) \tag{5}$$

$$A(t_1)^2 = \frac{1}{2}A(t_0)^2 \tag{6}$$

$$A_0 e^{-\gamma t_1} = \frac{1}{\sqrt{2}}A_0 e^{-\gamma t_0} \tag{7}$$

$$e^{-\gamma(t_0+NT)} = \frac{1}{\sqrt{2}}e^{-\gamma t_0} \tag{8}$$

$$e^{-\gamma NT} = \frac{1}{\sqrt{2}} \tag{9}$$

$$N = \frac{1}{-\gamma T}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{\ln(2)}{2\gamma T} \tag{10}$$

$$\tag{11}$$

The period depends on the frequency of the damped oscillator.

$$T = \frac{2\pi}{\omega'} \tag{12}$$

$$\omega' = \sqrt{\omega_0^2 - \gamma^2} \tag{13}$$

$$N = \frac{\ln(2)}{2\gamma}\frac{\sqrt{\omega_0^2 - \gamma^2}}{2\pi} \tag{14}$$

Problem 2. To demonstrate standing waves, one end of a string is attached to a tuning fork with frequency f (treat the point attached to the tuning fork as a node, as the picture shows, even though that physically doesn't make too much sense). The other end of the string passes over a pulley and is connected to a suspended mass M as shown in the figure. The value of M is such that the standing wave pattern has four "loops." The length of the string from the tuning fork to the point where the string touches the top of the pulley is L. The linear density of the string is μ, and remains constant throughout the experiment.

a. Determine the wavelength of the standing wave.

b. Determine the speed of the transverse waves along the string.

c. The speed of waves along the string increases with increasing tension in the string. Indicate whether the value of M should be increased or decreased in order to double the number of loops in the standing wave pattern. Find an expression for the number of loops n in terms of f, L, μ, M, and g.

d. If a point on the string at an antinode had a maximum vertical displacement of a during one complete cycle, what is the amplitude of the standing wave?

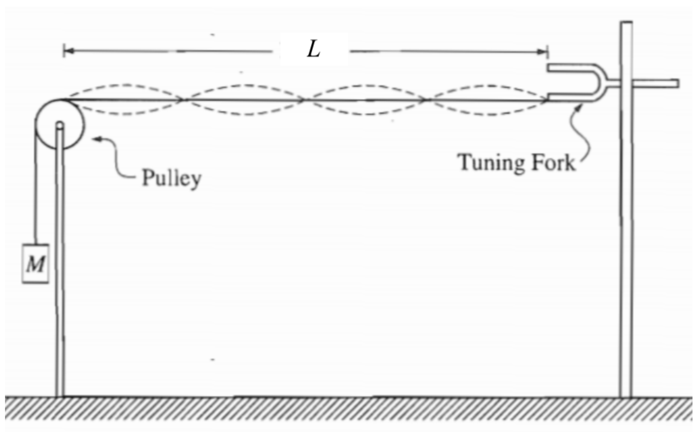

Figure 1: Problem 2

The wavelength is the total length divided by the number of waves. In this case $\lambda = L/2$. The speed of the waves is the wavelength times the frequency, so $v = fL/2$. Let n equal the number of loops, so the number of waves is $n/2$. In the figure, $n = 4$.

$$v = \sqrt{\frac{T}{\mu}} \tag{15}$$

$$T = Mg \tag{16}$$

$$f\lambda = \frac{fL}{n/2} = \sqrt{\frac{Mg}{\mu}} \tag{17}$$

$$n = 2fL\sqrt{\frac{\mu}{Mg}} \tag{18}$$

To double the number of loops we should decrease the mass. If an antinode moves a total distance of a, then the amplitude is $a/2$. Maximum vertical displacement means the length of the path traveled by the point as it moves from maximum to minimum.

Problem 3. Determine in what way and how many times will the fundamental tone frequency of a stretched wire change if it is cut to shorten its length by 35% and the tension increased by 70%.

Changing the length of the wire will not change the mass density, μ.

$$v = f\lambda = \sqrt{\frac{T}{\mu}} \tag{19}$$
$$\lambda = 2L \tag{20}$$
$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{21}$$
$$\frac{f_2}{f_1} = \frac{\sqrt{T_2}/L_2}{\sqrt{T_1}/L_1} = \sqrt{\frac{T_2}{T_1}}\frac{L_1}{L_2} \tag{22}$$
$$T_2 = 1.7T_1 \tag{23}$$
$$L_2 = 0.65L_1 \tag{24}$$
$$\frac{f_2}{f_1} = \frac{\sqrt{1.7}}{0.65} \approx 2 \tag{25}$$

The frequency doubles.

Problem 4. A physical pendulum performs small oscillations about the horizontal axis with frequency ω_1. When a small body of mass m is fixed to the pendulum at a distance l below the axis, the oscillation frequency becomes equal to ω_2. Find the moment of inertia of the pendulum relative to the oscillation axis.

Let m_0, d_0, and I denote the initial mass, initial center of mass, and initial moment of inertia respectively. The problem is asking for I so we need to eliminate m_0 and d_0. The new moment of inertia with the additional mass attached is $I + ml^2$.

First let's re-derive the frequency for a physical pendulum in case you don't have it memorized during the test.

$$\tau = I\ddot{\theta} = \vec{r} \times \vec{F} \tag{26}$$
$$\tau = I\ddot{\theta} = -rdmg\sin\theta \tag{27}$$
$$\tau = I\ddot{\theta} \approx -rdmg\theta \tag{28}$$
$$\ddot{\theta} = -\frac{rdmg}{I}\theta \tag{29}$$
$$\rightarrow \omega = \sqrt{\frac{rdmg}{I}} \tag{30}$$

Now we can write down our frequencies in terms of the physical pendulum described in the problem.

$$\omega_1 = \sqrt{\frac{m_0 g d_0}{I}} \tag{31}$$
$$m_0 g d_0 = I\omega_1^2 \tag{32}$$
$$d = \frac{m_0 d_0 + ml}{m_0 + m} \tag{33}$$
$$\omega_2 = \sqrt{\frac{g\left(m_0 d_0 + ml\right)}{I + ml^2}} \tag{34}$$
$$\omega_2^2 = \frac{\left(I\omega_1^2 + gml\right)}{I + ml^2} \tag{35}$$
$$I = ml^2\left(\frac{\omega_2^2 - g/l}{\omega_2^2 - \omega_1^2}\right) \tag{36}$$

Problem 5. A spring-loaded toy gun is used to shoot a ball of mass M straight up in the air. The ball is not attached to the spring. The ball is pushed down onto the spring so that the spring is compressed a distance S below its unstretched point. After release, the ball reaches a maximum height $3S$, measured from the unstretced position of the spring as shown in the figure.

a. Find the spring constant of the spring

b. Find the equilibrium point of the ball when it is sitting on the spring with no forces other than gravity and the spring acting on it. Clearly indicate the point you are using as the origin of your coordinate system and what direction is positive.

c. Now, the ball is glued onto the spring so that it oscillates up and down rather than flying off the spring. The spring is again compressed the same distance S below its *unstretched* point. Write an equation for the position of the ball as a function of time after it is released. Clearly indicate the point you are using as the origin of your coordinate system and what direction is positive.

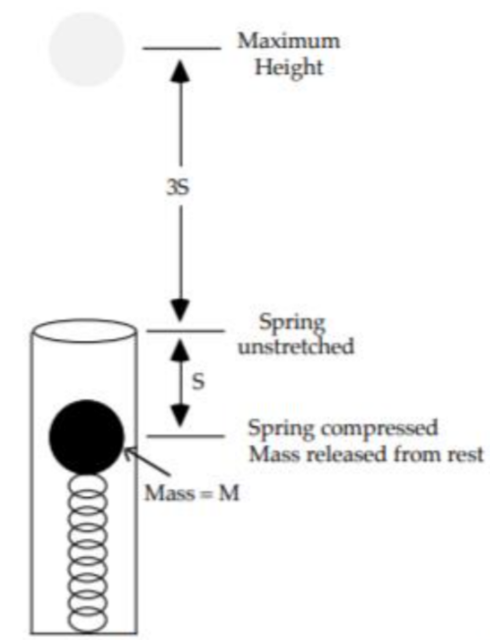

Figure 2: Problem 5

We can start by defining a coordinate system. The system is being described with respect to the unstretched length of the spring, so let's set $y = 0$ to be at that point and choose up as positive. Then conservation of energy can be used to solve for k.

$$Mg(3S) = \frac{1}{2}kS^2 - MgS \tag{37}$$
$$k = \frac{8Mg}{S} \tag{38}$$

The equilibrium position of the ball is when all forces cancel out (the ball is not accelerating).

$$-Mg - ky_{eq} = 0 \tag{39}$$
$$y_{eq} = -\frac{Mg}{k} = -\frac{S}{8} \tag{40}$$

If we redefine $x = y - y_{eq}$ so that $x = 0$ is the equilibrium point, the system follows the simple harmonic equation of motion.

$$\frac{d^2x}{dt^2} = -\omega^2 x \tag{41}$$
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8g}{S}} \tag{42}$$
$$x(t) = A\cos(\omega t + \phi) \tag{43}$$
$$x(0) = S - y_{eq} = \frac{7S}{8} \tag{44}$$
$$\frac{dx}{dt}(0) = 0 \tag{45}$$
$$\phi = 0 \tag{46}$$
$$x(t) = \frac{7S}{8}\cos(\sqrt{\frac{8g}{S}}t) \tag{47}$$

We could also write the equation of motion in terms of y.

$$y(t) = y_{eq} + x(t) \tag{48}$$
$$y(t) = -\frac{S}{8} - \frac{7S}{8}\cos(\sqrt{\frac{8g}{S}}t) \tag{49}$$

Problem 6. A siren of frequency f is moving away from the listener with speed v_1 relative to the air (speed of sound v), and the listener is moving toward the siren with a speed $v_2 < v_1$. What frequency does the listener hear?

Here are three different ways to think about the solution:

1) The positive direction is from the source to the listener. The frequency decreases.

$$v_L = -v_2 \tag{50}$$
$$v_S = -v_1 \tag{51}$$
$$f_L = \frac{v + v_2}{v + v_1}f \tag{52}$$

2) You have the formula memorized and it's $f_L = \frac{v+v_L}{v+v_S}f_S$. Now, should the velocities be positive or negative? The siren is moving away from the listener, so the wavelength is being "dragged out",

making the frequency lower. Therefore we want to increase the denominator by adding v_1, the velocity of the siren. The listener is running towards the siren, which will increase the speed at which he passes the waves, increasing the frequency. Therefore, we want to increase the numerator by adding v_2.

All together the solution is (the same)

$$f_L = \frac{v + v_2}{v + v_1} f.$$

3) We can derive what happens to the frequency from first principles. Let's start with the moving siren which is outputing frequency f. Say the siren releases the start of the wave at $t = 0$. It will release the end of the wave at $t = T = 1/f$. During that time, the beginning of the wave has moved vT to the right and the siren has moved $v_1 T$ to the left. Therefore the wavelength will be

$$\lambda = vT + v_1 T \tag{53}$$

$$= \frac{v + v_1}{f}. \tag{54}$$

Now this wave is moving towards the person. Say the first part of the wave hits the person at time $t = 0$. To find the frequency the person hears, we want to know how long until the person reaches the end of the wave. The person is moving to the left and the wave is moving to the right at the speed of sound, so we can find this time by using $\lambda = (v + v_2)t$. We find $t = \frac{\lambda}{v+v_2}$. Solving for the frequency the person hears,

$$f_L = \frac{1}{t - 0} \tag{55}$$

$$= \frac{v + v_2}{\lambda} \tag{56}$$

$$= \frac{v + v_2}{v + v_1} f \tag{57}$$

Problem 7. A turntable with diameter d rotates at angular speed ω. Two speakers, each giving off sound of frequency f, are attached to the rim of the table at opposite ends of a diameter. A listener stands in front of the turntable. Assume the turntable is small compared to how far away the person is standing. What is the greatest beat frequency the listener will receive from this system?

The maximum beat frequency occurs when one speaker is moving directly away from and the other is moving directly toward the listener. The tangential speed of a point on a rotating object is $v_T = R\omega$ where R is the distance to the axis of rotation. The emitter moving towards the speaker (1) has an increased frequency, and the emitter moving away from the speaker (2) has a decreased frequency.

$$f_{L1} = \frac{v}{v - d\omega/2} f \tag{58}$$

$$f_{L2} = \frac{v}{v + d\omega/2} f \tag{59}$$

$$f_{beat} = |f_{L1} - f_{L2}| \tag{60}$$

$$f_{beat} = vf \left(\frac{d\omega}{v^2 - (d\omega/2)^2} \right) \tag{61}$$

$$\tag{62}$$

See Solution 6 (part 3) for a derivation of the Doppler shift frequency from an emitter.

Problem 8. A ball is suspended by a thread of length l at the point O on the wall, forming a small angle α with the vertical as shown in the figure. Then the thread with the ball was deviated through a small angle β, where $\beta > \alpha$, and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

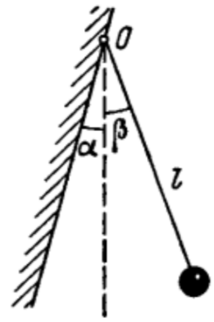

Figure 3: Problem 8

Define $\theta = 0$ to be the vertical position and θ positive to the right so the ball's initial position is $\theta = \beta$. For small β this is a simple pendulum whenever $\theta > -\alpha$. In a perfectly elastic collision with the wall, the ball reflects with the same velocity with which it hit the wall. Energy is conserved and it will swing back to its maximum, $\theta = \beta$. The period of this pendulum, T, will be twice the time a simple pendulum takes to go from $\theta = \beta$ to $\theta = -\alpha$ since a collision with the wall takes no time and the system behaves symmetrically before and after a collision. Use the equation of motion for a simple pendulum assuming small oscillations. We are solving for the period, T, such that $\theta(T/2) = -\alpha$.

$$\theta(t) = A\cos(\omega t + \phi) \tag{63}$$

$$\omega = \sqrt{\frac{g}{l}} \tag{64}$$

$$\theta(0) = \beta \tag{65}$$

$$\frac{d\theta}{dt}(0) = 0 \tag{66}$$

$$\phi = 0 \tag{67}$$

$$\theta(t) = \beta\cos(\sqrt{\frac{g}{l}}t) \tag{68}$$

$$\theta\left(\frac{T}{2}\right) = -\alpha \tag{69}$$

$$-\alpha = \beta\cos\left(\sqrt{\frac{g}{l}}\frac{T}{2}\right) \tag{70}$$

$$T = 2\sqrt{\frac{l}{g}}\arccos\left(\frac{-\alpha}{\beta}\right) \tag{71}$$

$$T = 2\sqrt{\frac{l}{g}}\left(\pi - \arccos\left(\frac{\alpha}{\beta}\right)\right) \tag{72}$$

Studying Tips

- Do all of the homework. :p
- Don't memorize formulas. Understand the derivations for each formula you use. Go back and re-derive formulas you need when practicing problems.
- Try problems first without looking at the solutions. Once you've given it full effort, check the solution. Then try again without looking at the solution. Repeat until solved.
- After finishing a problem, go back and think about the process of how you did it. What formulas did you use? What principles did you use?

- Come to OH and practice there!

Test Taking Tips

- Get a full night sleep before the test. Research shows this helps with memory recall. Research also shows that students who get a full night sleep do better than students who stay up all night studying.
- Tell yourself you are going to do well before the test. In your head, say things like "I will remember how to do these problems" and "I am confident in my ability to problem solve". Don't complain with your friends right before about how you are not ready. This has also been scientifically shown to increase your score even though it sounds silly.
- Don't cheat. We have caught people cheating in the past, and we will catch you!
- Write down something for every problem. We are almost always willing to give a point or two for effort.
- Check units! If your answer has the wrong units then you've made an algebra mistake most likely. Even just writing that you have the wrong units shows the grader that you are aware of what you are doing.
- Physically prepare yourself for the test. Eat breakfast and use the bathroom before hand!