15.6 •• A fisherman notices that his boat is moving up and down periodically, owing to waves on the surface of the water. It takes 2.5 s for the boat to travel from its highest point to its lowest, a total distance of 0.53 m. The fisherman sees that the wave crests are spaced 4.8 m apart. (a) How fast are the waves traveling? (b) What is the amplitude of each wave? (c) If the total vertical distance traveled by the boat were 0.30 m but the other data remained the same, how would the answers to parts (a) and (b) change?

EVALUATE: For a given v, a larger f corresponds to smaller λ. For the same f, λ increases when v increases.

15.6. **IDENTIFY:** The fisherman observes the amplitude, wavelength, and period of the waves.

SET UP: The time from the highest displacement to lowest displacement is $T/2$. The distance from highest displacement to lowest displacement is $2A$. The distance between wave crests is λ, and the speed of the waves is $v = f\lambda = \lambda/T$.

EXECUTE: **(a)** $T = 2(2.5\text{ s}) = 5.0\text{ s}$. $\lambda = 4.8\text{ m}$. $v = \dfrac{4.8\text{ m}}{5.0\text{ s}} = 0.96\text{ m/s}$.

(b) $A = (0.53\text{ m})/2 = 0.265\text{ m}$ which rounds to 0.27 m.

(c) The amplitude becomes 0.15 m but the wavelength, period and wave speed are unchanged.

EVALUATE: The wavelength, period and wave speed are independent of the amplitude of the wave.

15.18 •• A 1.50-m string of weight 0.0125 N is tied to the ceiling at its upper end, and the lower end supports a weight W. Ignore the very small variation in tension along the length of the string that is produced by the weight of the string. When you pluck the string slightly, the waves traveling up the string obey the equation

$$y(x, t) = (8.50\text{ mm})\cos(172\text{ rad/m}\ x - 4830\text{ rad/s}\ t)$$

Assume that the tension of the string is constant and equal to W. (a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight W? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling *down* the string?

15.18. IDENTIFY: For transverse waves on a string, $v = \sqrt{F/\mu}$. The general form of the equation for waves traveling in the $+x$-direction is $y(x,t) = A\cos(kx - \omega t)$. For waves traveling in the $-x$-direction it is $y(x,t) = A\cos(kx + \omega t)$. $v = \omega/k$.

SET UP: Comparison to the general equation gives $A = 8.50\text{ mm}$, $k = 172\text{ rad/m}$ and $\omega = 4830\text{ rad/s}$. The string has mass 0.00128 kg and $\mu = m/L = 0.000850\text{ kg/m}$.

EXECUTE: **(a)** $v = \dfrac{\omega}{k} = \dfrac{4830\text{ rad/s}}{172\text{ rad/m}} = 28.08\text{ m/s}$. $t = \dfrac{d}{v} = \dfrac{1.50\text{ m}}{28.08\text{ m/s}} = 0.0534\text{ s} = 53.4\text{ ms}$.

(b) $W = F = \mu v^2 = (0.000850\text{ kg/m})(28.08\text{ m/s})^2 = 0.670\text{ N}$.

(c) $\lambda = \dfrac{2\pi\text{ rad}}{k} = \dfrac{2\pi\text{ rad}}{172\text{ rad/m}} = 0.0365\text{ m}$. The number of wavelengths along the length of the string is $\dfrac{1.50\text{ m}}{0.0365\text{ m}} = 41.1$.

(d) For a wave traveling in the opposite direction, $y(x,t) = (8.50\text{ mm})\cos([172\text{ rad/m}]x + [4830\text{ rad/s}]t)$.

EVALUATE: We have assumed that the tension in the string is constant and equal to W. This is reasonable since $W \gg 0.0125\text{ N}$, so the weight of the string has a negligible effect on the tension.

15.20 •• A heavy rope 6.00 m long and weighing 29.4 N is attached at one end to a ceiling and hangs vertically. A 0.500-kg mass is suspended from the lower end of the rope. What is the speed of transverse waves on the rope at the (a) bottom of the rope, (b) middle of the rope, and (c) top of the rope? (d) Is the tension in the middle of the rope the average of the tensions at the top and bottom of the rope? Is the wave speed at the middle of the rope the average of the wave speeds at the top and bottom? Explain.

15.20. IDENTIFY: The rope is heavy, so the tension at any point in it must support not only the weight attached but the weight of the rope below that point. Assume that the rope is uniform.

SET UP: $v = \sqrt{F/\mu}$ and $\mu = m/L$ = [(29.4 N)/(9.80 m/s^2)]/(6.00 m) = 0.500 kg/m.

EXECUTE: **(a)** At the bottom, the rope supports only the 0.500-kg object, so $T = mg$ = (0.500 kg)(9.80 m/s^2) = 4.90 N. Now use $v = \sqrt{F/\mu}$ find v.
v = [(4.90 N)/(0.500 kg/m)]$^{1/2}$ = 3.13 m/s.

(b) At the middle, the tension supports the 0.500-kg object plus half the weight of the rope, so T = (29.4 N)/2 + 4.90 N = 19.6 N. Therefore v = [(19.6 N)/(0.500 kg/m)]$^{1/2}$ = 6.26 m/s.

(c) At the top, the tension supports the entire rope plus the object, so T = 29.4 N + 4.90 N = 34.3 N. Therefore v = [(34.3 N)/(0.500 kg/m)]$^{1/2}$ = 8.28 m/s.

(d) T_{middle}= 19.6 N. $T_{\text{av}} = (T_{\text{top}} + T_{\text{bot}})/2$ = (34.3 N + 4.90 N)/2 = 19.6 N, which is equal to T_{middle}.
v_{middle} = 6.26 m/s. v_{av} = (8.28 m/s + 3.13 m/s)/2 = 5.71 m/s, which is not equal to v_{middle}.

EVALUATE: The average speed is not equal to the speed at the middle because the speed depends on the square root of the tension. So even though the tension at the middle is the average of the top and bottom tensions, that is not true of the wave speed.

15.25 •• A jet plane at takeoff can produce sound of intensity 10.0 W/m^2 at 30.0 m away. But you prefer the tranquil sound of normal conversation, which is 1.0 μW/m^2. Assume that the plane behaves like a point source of sound. (a) What is the closest distance you should live from the airport runway to preserve your peace of mind? (b) What intensity from the jet does your friend experience if she lives twice as far from the runway as you do? (c) What power of sound does the jet produce at takeoff?

15.25. IDENTIFY: For a point source, $I = \dfrac{P}{4\pi r^2}$ and $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$.

SET UP: $1\,\mu\text{W} = 10^{-6}$ W

EXECUTE: **(a)** $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (30.0\text{ m})\sqrt{\dfrac{10.0\text{ W/m}^2}{1\times 10^{-6}\text{ W/m}^2}} = 95\text{ km}$

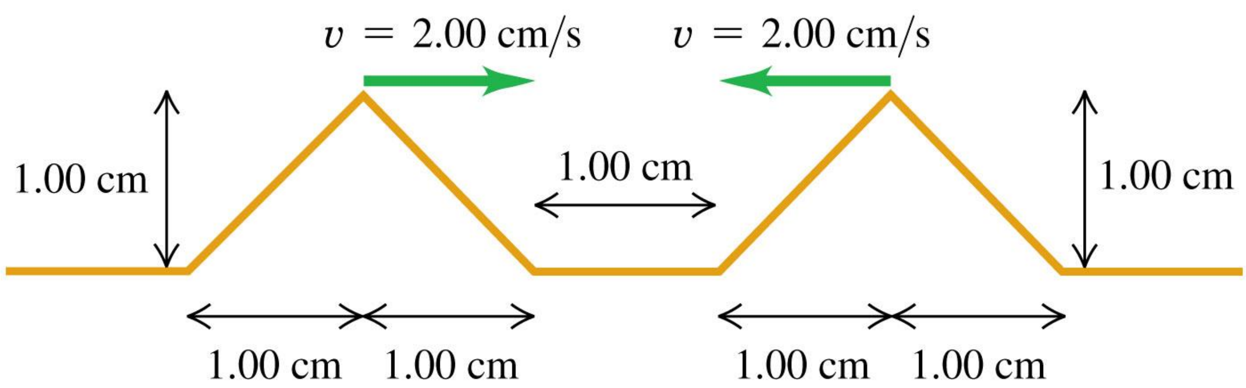
(b) $\dfrac{I_2}{I_3} = \dfrac{r_3^2}{r_2^2}$, with $I_2 = 1.0\,\mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\dfrac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\,\mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\text{ W/m}^2)(4\pi)(30.0\text{ m})^2 = 1.1\times 10^5\text{ W}$

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.

15.32 • **Interference of Triangular Pulses.** Two triangular wave pulses are traveling toward each other on a stretched string as shown in Fig. E15.32. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at $t = 0$. Sketch the shape of the string at $t = 0.250$ s, $t = 0.500$ s, $t = 0.750$ s, $t = 1.000$ s, and $t = 1.250$ s.

Figure **E15.32**



15.32. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.32.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

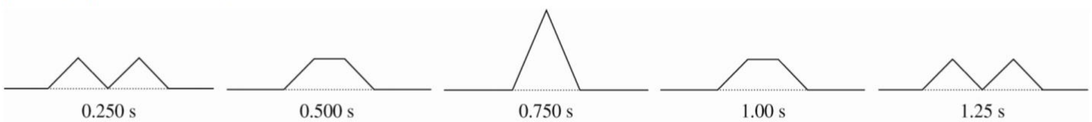
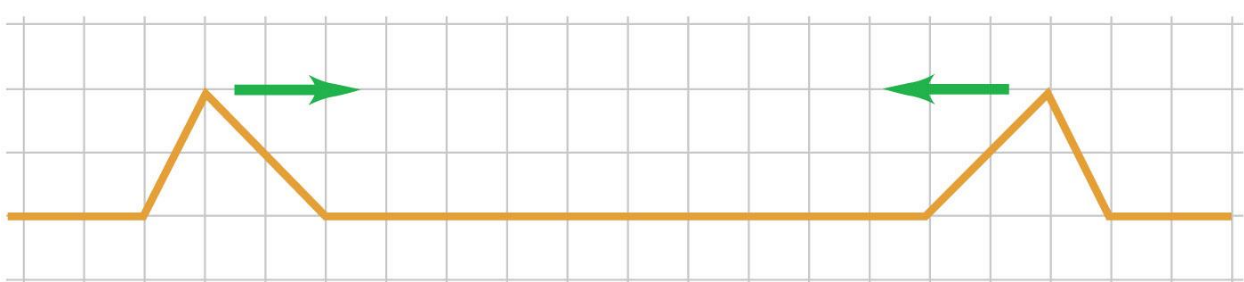


Figure 15.32

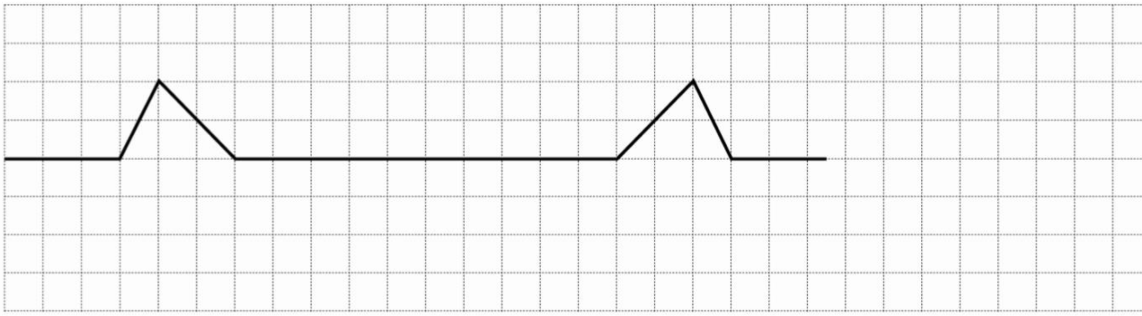
15.34 •• Two pulses are moving in opposite directions at 1.0 cm/s on a taut string, as shown in Fig. E15.34. Each square is 1.0 cm. Sketch the shape of the string at the end of (a) 6.0 s; (b) 7.0 s; (c) 8.0 s.

Figure **E15.34**

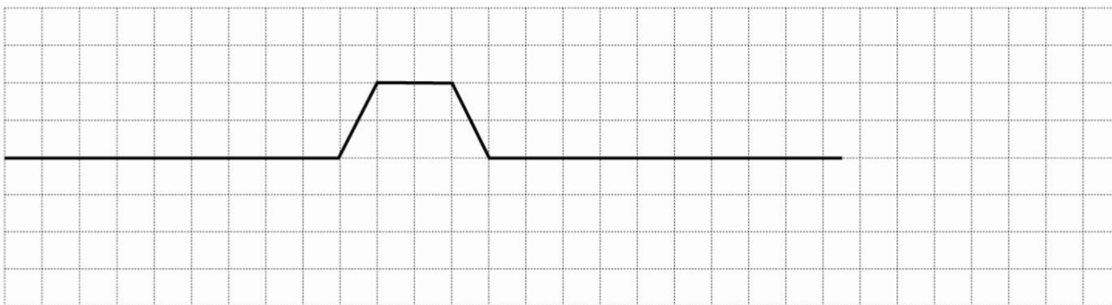


15.34. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.34.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

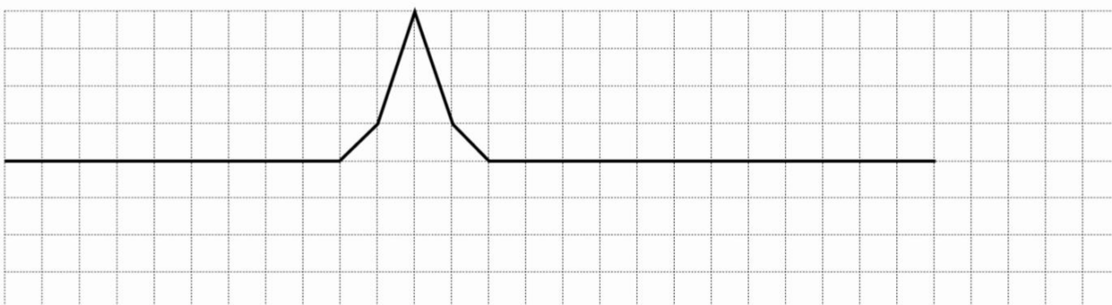
$t = 0$



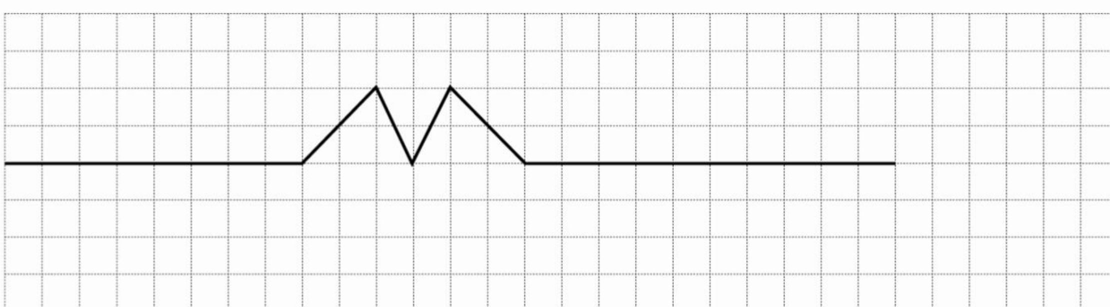
$t = 6.0$ s



$t = 7.0$ s

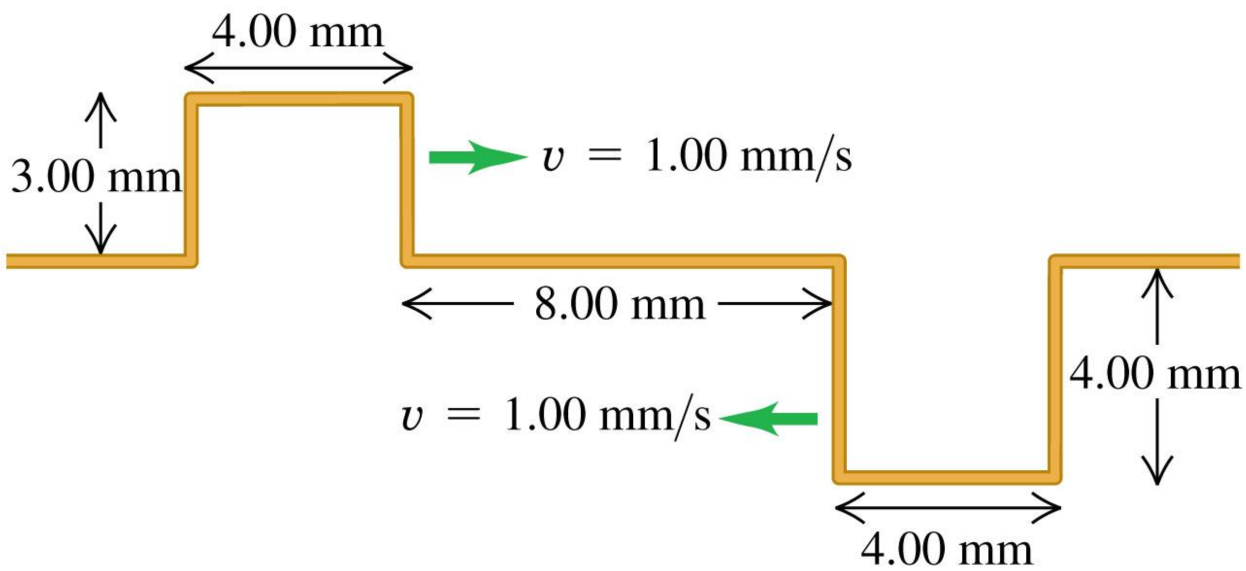


$t = 8.0$ s



15.35 •• **Interference of Rectangular Pulses.** Figure E15.35 shows two rectangular wave pulses on a stretched string traveling toward each other. Each pulse is traveling with a speed of 1.00 mm/s and has the height and width shown in the figure. If the leading edges of the pulses are 8.00 mm apart at $t = 0$, sketch the shape of the string at $t = 4.00$ s, $t = 6.00$ s, and $t = 10.0$ s.

Figure E15.35

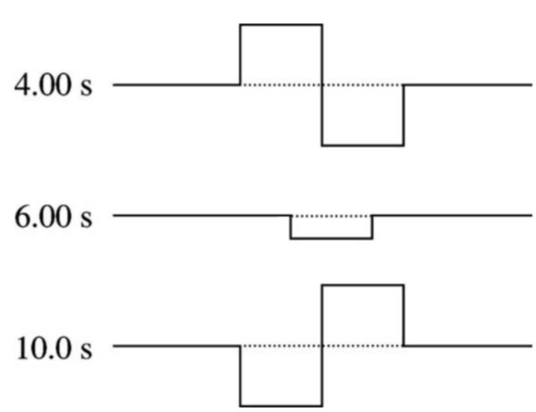


15.35. IDENTIFY: Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.35.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.



15.44 •• One string of a certain musical instrument is 75.0 cm long and has a mass of 8.75 g. It is being played in a room where the speed of sound is 344 m/s. (a) To what tension must you adjust the string so that, when vibrating in its second overtone, it produces sound of wavelength 0.765 m? (Assume that the breaking stress of the wire is very large and isn't exceeded.) (b) What frequency sound does this string produce in its fundamental mode of vibration?

15.44. IDENTIFY: $v = \sqrt{F/\mu}$. $v = f\lambda$. The standing waves have wavelengths $\lambda_n = \dfrac{2L}{n}$ and frequencies $f_n = nf_1$. The standing wave on the string and the sound wave it produces have the same frequency.

SET UP: For the fundamental $n = 1$ and for the second overtone $n = 3$. The string has $\mu = m/L = (8.75\times10^{-3}\ \text{kg})/(0.750\ \text{m}) = 1.17\times10^{-2}\ \text{kg/m}$.

EXECUTE: **(a)** $\lambda = 2L/3 = 2(0.750\ \text{m})/3 = 0.500\ \text{m}$. The sound wave has frequency

$f = \dfrac{v}{\lambda} = \dfrac{344\ \text{m/s}}{0.765\ \text{m}} = 449.7\ \text{Hz}$. For waves on the string,

$v = f\lambda = (449.7\ \text{Hz})(0.500\ \text{m}) = 224.8\ \text{m/s}$. The tension in the string is

$F = \mu v^2 = (1.17\times10^{-2}\ \text{kg/m})(224.8\ \text{m/s})^2 = 591\ \text{N}$.

(b) $f_1 = f_3/3 = (449.7\ \text{Hz})/3 = 150\ \text{Hz}$.

EVALUATE: The waves on the string have a much longer wavelength than the sound waves in the air because the speed of the waves on the string is much greater than the speed of sound in air.

15.50 •• CP A 1750-N irregular beam is hanging horizontally by its ends from the ceiling by two vertical wires (A and B), each 1.25 m long and weighing 0.290 N. The center of gravity of this beam is one-third of the way along the beam from the end where wire A is attached. If you pluck both strings at the same time at the beam, what is the time delay between the arrival of the two pulses at the ceiling? Which pulse arrives first? (Ignore the effect of the weight of the wires on the tension in the wires.)

15.50. IDENTIFY: Apply $\Sigma\tau_z = 0$ to find the tension in each wire. Use $v = \sqrt{F/\mu}$ to calculate the wave speed for each wire and then $t = L/v$ is the time for each pulse to reach the ceiling, where $L = 1.25$ m.

SET UP: The wires have $\mu = \dfrac{m}{L} = \dfrac{0.290\text{ N}}{(9.80\text{ m/s}^2)(1.25\text{ m})} = 0.02367$ kg/m. The free-body diagram for the beam is given in Figure 15.50. Take the axis to be at the end of the beam where wire A is attached.

EXECUTE: $\Sigma\tau_z = 0$ gives $T_B L = w(L/3)$ and $T_B = w/3 = 583$ N. $T_A + T_B = 1750$ N, so $T_A = 1167$ N.

$$v_A = \sqrt{\frac{T_A}{\mu}} = \sqrt{\frac{1167\text{ N}}{0.02367\text{ kg/m}}} = 222\text{ m/s}.\quad t_A = \frac{1.25\text{ m}}{222\text{ m/s}} = 0.00563\text{ s} = 5.63\text{ ms}.$$

$$v_B = \sqrt{\frac{583\text{ N}}{0.02367\text{ kg/m}}} = 156.9\text{ m/s}.\quad t_B = \frac{1.25\text{ m}}{156.9\text{ m/s}} = 0.007965\text{ s} = 7.965\text{ ms}.$$

$\Delta t = t_B - t_A = 7.965\text{ ms} - 5.63\text{ ms} = 2.34\text{ ms}.$

EVALUATE: The wave pulse travels faster in wire A, since that wire has the greater tension, so the pulse in wire A arrives first.