Final Exam Review Problems

Problem 1. As shown in Figure 1, two masses M and m are connected by a very light rigid bar and attached to an ideal massless spring of spring constant k. Assume the mass of the rigid bar is negligible.

a. Using Newton's Second Law, write the differential equation for $x(t)$, the masses' displacement from equilibrium as a function of time, in terms of m, M, and k.

b. Assume that at $t = 0$ the masses are at a positive position $x_0 > 0$ and have a positive velocity $v_{x0} > 0$. Moreover, assume that at $t = 0$ the total kinetic energy of the masses is equal to the potential energy of the spring. Write the soution for $x(t)$ in terms of m, M, k, and x_0.

c. Now consider a physical pendulum consisting of a solid, uniform sphere of radius R suspended on a wire of negligible mass and also of length R, as shown in Figure 2. What must the distance R be so that the period of the pendulum for small oscillations matches the period of the mass-spring system from Figure 1? Give an expression for R in terms of m, M, k, and gravitational acceleration g. (First, find the moment of inertia of a solid, uniform sphere of mass m_s and radius R about an axis through its center.)

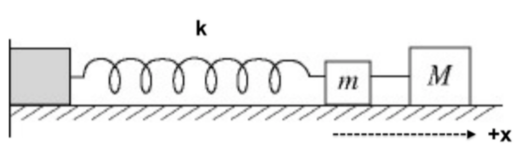

Figure 1: Problem 1

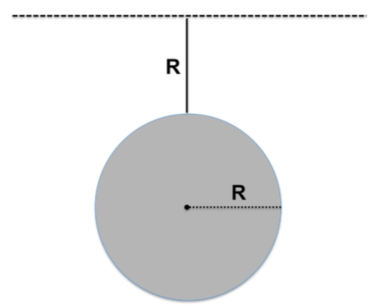

Figure 2: Problem 1

Problem 2. Consider a musical pipe of length L_P that is open at both ends. Air is blown into the pipe, exciting a longitudinal normal mode. Measurements indicate that there are four pressure fluctuation nodes associated with this normal mode.

a. In the space in Figure 3, draw a representation of this normal mode in terms of the pressure fluctuation, labeling all nodes and anti-nodes. You do not need to label the amplitudes.

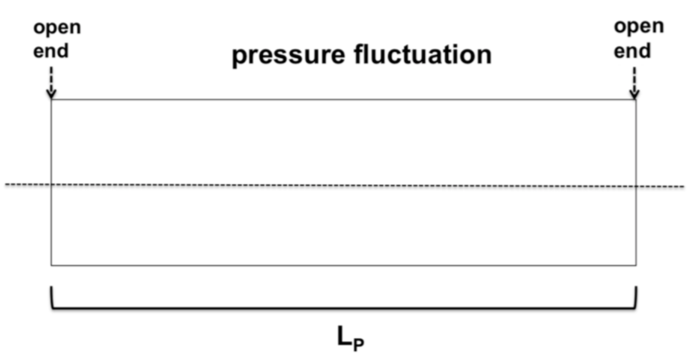

Figure 3: Problem 2

b. Measurements indicate that the pressure fluctuation amplitude is p_{max} and the speed of sound in air is v_s. Determine the expressions for the pressure fluctuation $P_W(x,t)$ as a function of x-position and time t in terms of p_{max}, v_s, and L_P. Assume the ends of the pipe are at $x = 0$ and $x = L_P$, respectively, and that the particles at $x = 0$ have their most positive (i.e. rightmost) displacement at $t = 0$. (Therefore, the pressure at $(x = 0, t = 0)$ is zero.)

c. In a second identical pipe of length L_P, a normal mode corresponding to the next allowed harmonic above the one in Parts (a) and (b) is excited. Both pipes are played simultaneously. At what speed would the original pipe need ot be moved towards a stationary listener such that the listener hears the same frequency from both pipes? Express your answer in terms of v_s.

Problem 3. Figure 4 shows a thin ring of radius a that contains a uniformly distributed negative charge Q (such that $Q < 0$). The ring is placed in the xy-plane, such that the z-axis defines the central axis of the ring.

a. Determine the magnitude and direction of the electric field at point P, which is located at coordinate z along the z-axis, due to the ring of charge. Derive and then solve the necessary integral expression(s) and/or make arguments from symmetry. Express your answer in terms of Q, the given parameters, and fundamental constants.

Figure 5 shows a thin, uniformly charged disk of radius R and negative surface charge density σ (such that $\sigma < 0$). The disk is placed in the xy-plane such that the z-axis defines the central axis of the disk.

b. Determine the magnitude and direction of the electric field at point P, which is located at coordinate z along the z-axis, due to the disk of charge. Derive and then solve the necessary integral expression(s) and/or make arguments from symmetry. Express your answer in terms of σ, the given parameters, and fundamental constants. (Make use of the result to Part (a).)

Figure 6 shows a thin, uniformly charged, infinite sheet of negative surface charge density σ (such that $\sigma < 0$). The sheet is placed in the xy-plane.

c. Using the result from Part (b), take the appropriate limit to determine the magnitude and direction of the electric field at point P due to an infinite sheet of uniform negative surface charge density σ. Express your answer in terms of the given parameters and fundamental constants.

d. Using Gausss Law, determine the magnitude and direction of the electric field at point P due to an infinite sheet of uniform negative surface charge density σ. Demonstrate the application of Gausss Law and clearly state arguments from symmetry. Your answer should match the answer to Part (c).

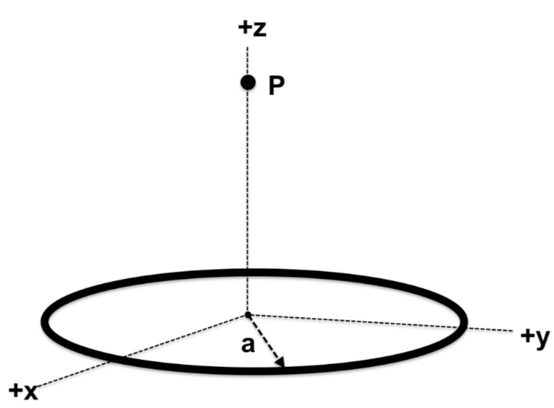

Figure 4: Problem 3

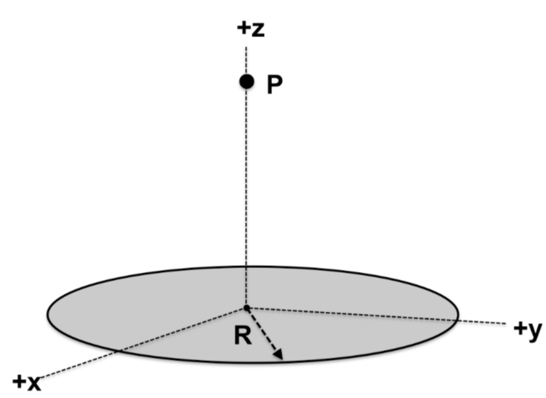

Figure 5: Problem 3

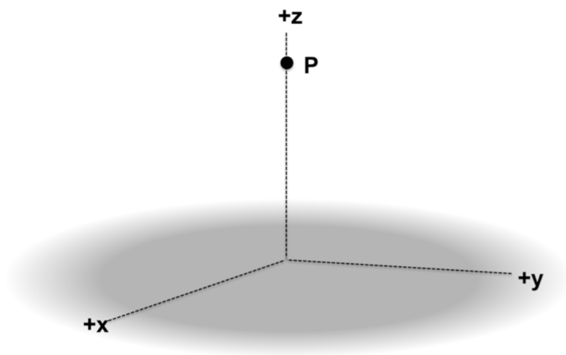

Figure 6: Problem 3

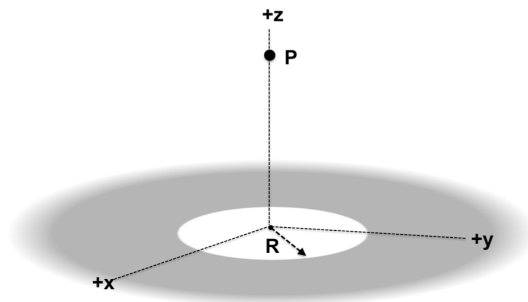

Figure 7: Problem 3

Figure 7 shows a thin, uniformly charged infinite sheet of negative surface charge density σ (such that $\sigma < 0$). The sheet is placed in the xy-plane. A circular section of radius R is removed from the center of the sheet such that the z-axis runs through the center of the resulting hole.

e. Determine the magnitude and direction of the electric field at point P, which is located at coordinate z along the z-axis, due to the infinite sheet with a hole. Express your answer in terms of the given parameters and fundamental constants. (Use the principle of superposition and the results to the previous parts of this problem.)

f. A positive point charge q_0 that has a mass m_0 is now placed at point P. Assuming that point P is very close to the origin such that $|z| \ll R$, (1) show that the resulting motion of the point charge will be approximately simple harmonic motion and (2) derive an expression for the period of oscillation. Express your answer in terms of the given parameters and fundamental constants.

Problem 4. Figure 8 shows a diagram for a circuit containing two identical batteries, each of unknown EMF $\mathcal{E}$ and an unknown internal resistance r. The circuit also contains two resistors of resistances R and $3R$. There are two switches S1 and S2, each of which can either be in the open or closed position. An ideal ammeter and an ideal voltmeter are also connected to the circuit as shown. When both switch S1 and switch S2 are open, the voltmeter reads a potential difference V_0. When switch S1 is closed but switch S2 is open, the voltmeter reads a potential difference V_1, where $V_1 < V_0$. Assume that the voltmeter leads are connected in such a way that the measured potential differences are positive ($V_0, V_1 > 0$).

a. Solve for (1) the battery EMF $\mathcal{E}$ and (2) the internal resistance r in terms of R, V_0, and V_1.

b. What is the reading of the ammeter and the direction of the current through switch S1 when (1) both switches are open, (2) switch S1 is closed but switch S2 is open, and (3) both switches are closed? Give the answers in terms of R, V_0, and V_1. State the directions as either left, right, or none.

c. What is the reading of the voltmeter when both switches are closed?

Problem 5. Three parallel-plate capacitors of capacitances C, $2C$, and $3C$ are connected to an ideal battery of EMF $\mathcal{E}$ and a switch S as shown in Figure 9.

a. With the switch S in the closed position, you perform the following sequence of steps:
   (1) You measure the charge magnitude on the capacitor with capacitance C and record the value as $Q_{initial}$.
   (2) You insert a dielectric material of dielectric constant $K = 2$ between the plates of the capacitor with capacitance $3C$. The dielectric fully fills the volume between the plates.
   (3) You measure the charge magnitude on the capacitor with capacitance C again and record the value as Q_{final}.

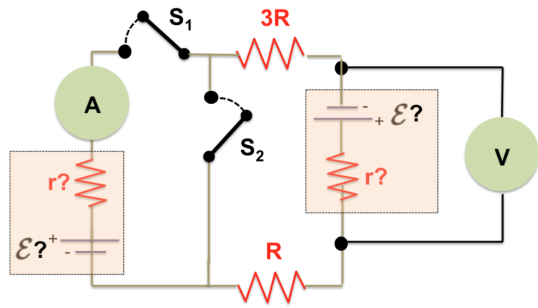

Figure 8: Problem 4

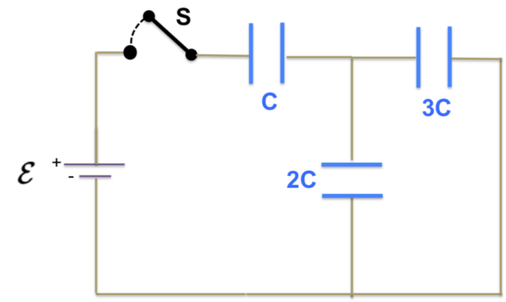

Figure 9: Problem 5

What is the ratio $Q_{final}/Q_{initial}$? Express your answer as a numerical fraction.

b. With the dielectric of dielectric constant $K = 2$ still inserted into the capacitor of capacitance $3C$, you now perform the following sequence of steps:
   (1) You flip switch S into the open position.
   (2) You remove the dielectric.
   (3) You measure the voltage across the capacitor of capacitance $3C$.

   What is the voltage across the capacitor of capacitance $3C$? Express your answer in terms of $\mathcal{E}$.

Problem 6. Two batteries are connected to three conducting wires. Battery 1, which has EMF $2\mathcal{E}$ and negligible internal resistance, has its positive terminal connected to wire 1 and wire 3 and its negative terminal connected to wire 2 and wire 3. Battery 2, which has EMF $3\mathcal{E}$ and negligible internal resistance, has its positive terminal connected to wire 1 and its negative terminal connected to wire 2. Wire 1 and wire 2 have the same resistance, R. Wire 3 has a different resistance R_3.

a. Draw a circuit diagram for the above circuit, clearly labeling the EMF's and resistances. (Model each *real* wire as a resistor connected to *ideal* wires.)

b. Determine the amount of current passing through (1) wire 1, (2) wire 3, and (3) battery 1 in terms of $\mathcal{E}$, R, and R_3. You do not need to state the direction of the current.

c. If you want each wire to dissipate an equal amount of power, what should be the resistance R_3 of wire 3 in terms of R?

Problem 7. The circuit in Figure 10 has been in position a for a long ime. At time $t = 0$ the switch is thrown to position b.

a. What is the current through the resistor just before and just after the switch is thrown?

b. What is the charge on the capacitor just before and just after the switch is thrown?

c. What is the charge on the capacitor at time $t > 0$?

d. After having been in position b for a long time, the switch is thrown back to a. What is the charge on the capacitor at time t after this change? What is the charge on the capacitor just after the switch is thrown?

Problem 8. A source of sonic oscillations with frequency f_0 and a receiver are located on the same line normal to a wall. Both the source and the receiver are stationary, and the wall recedes from the source with velocity u. Find the beat frequency registered by the receiver. Denote the velocity of sound as v_{snd}.

Problem 9. For the circuit in Figure 11, the switch is closed at $t = 0$ after having been open for a long time. Find the charge on the capacitor as a function of time. Determine the time constant and the maximum charge on the capacitor in this circuit.

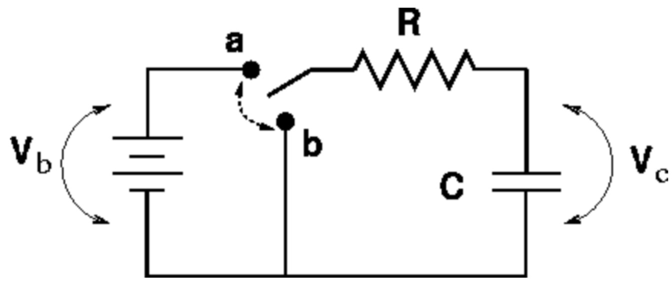

Figure 10: Problem 7

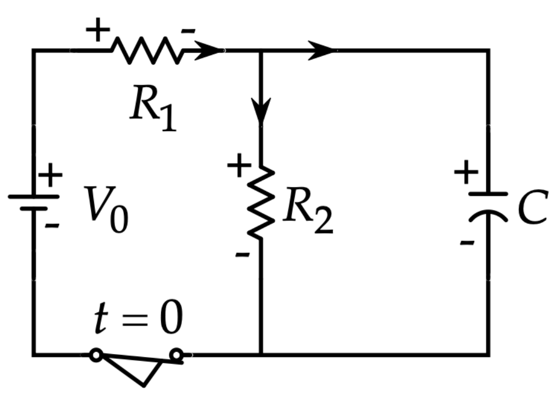

Figure 11: Problem 9