1.) $\rho = \rho_0(1 - r/a) \quad r<a$

$\rho = 0 \quad r>a.$

a.) $Q = 4\pi\rho_0 \int_0^a dr\,(1 - r/a) r^2 = 4\pi\rho_0\left(\frac{a^3}{3} - \frac{a^3}{4}\right)$

$Q = \int \rho\, dV = \int \rho\,(4\pi r^2 dr).$

$= \frac{4\pi\rho_0 a^3}{12} = \boxed{\frac{\pi\rho_0 a^3}{3} \Rightarrow Q.}$

b.) $r>a$

$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}.$

$E(4\cancel{\pi}r^2) = \frac{\cancel{\pi}\rho_0 a^3}{3\varepsilon_0}. \Rightarrow \boxed{|\vec{E}| = \frac{\rho_0 a^3}{12\varepsilon_0 r^2}, \text{ radially outward.}}$

$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{\cancel{\pi}\rho_0 a^3}{12\cancel{\pi}\varepsilon_0 r}. \Rightarrow \boxed{V = \frac{\rho_0 a^3}{12\varepsilon_0 r}.}$

c.) $r<a.$

$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}.$

$E(4\pi r^2) = \frac{1}{\varepsilon_0}\int_0^r \rho\, dV = \frac{1}{\varepsilon_0}\rho_0 \int_0^r \left(1 - \frac{r'}{a}\right) 4\pi r'^2 dr'.$

$= \frac{4\pi}{\varepsilon_0}\rho_0 \int_0^r \left(1 - \frac{r'}{a}\right) r'^2 dr'$

$E(\cancel{4\pi}r^2) = \frac{\cancel{4\pi}\rho_0}{\varepsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{4a}\right).$

$\Rightarrow \boxed{|\vec{E}| = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3} - \frac{r^2}{4a}\right). \text{ radially outward.}}$

$$V = V(a) + \int_r^a E(r)\,dr = V(a) + \frac{\rho_0}{\varepsilon_0}\int_r^a \left(\frac{r}{3} - \frac{r^2}{4a}\right)dr.$$

$$= V(a) + \frac{\rho_0}{\varepsilon_0}\left(\frac{r^2}{6} - \frac{r^3}{12a}\right)\Bigg|_r^a$$

$$V = \left(\rho_0\frac{a^2}{12\varepsilon_0} + \frac{\rho_0}{\varepsilon_0}\frac{a^2}{6} - \frac{\rho_0}{\varepsilon_0}\frac{r^2}{6} - \frac{\rho_0 a^3}{\varepsilon_0 12a} + \frac{\rho_0}{\varepsilon_0}\frac{r^3}{12a}\right).$$

$$\boxed{V = \frac{\rho_0}{\varepsilon_0}\left(\frac{a^2}{6} - \frac{r^2}{6} + \frac{r^3}{12a}\right)}$$

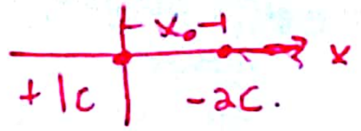
d.) $U = \dfrac{\varepsilon_0}{2}\displaystyle\int_{\substack{\text{all}\\\text{space}}} E^2\,dV.$

$$U = \frac{\rho_0^2}{2\varepsilon_0}\int_0^a \left(\frac{r^2}{9} - \frac{r^3}{6a} + \frac{r^4}{16a^2}\right)4\pi r^2\,dr.$$

$$+ \frac{\rho_0^2 a^6}{288\,\varepsilon_0}\int_a^\infty \left(\frac{1}{r^4}\right)4\pi r^2\,dr.$$

$$\boxed{U = \frac{\rho_0^2 a^5}{\varepsilon_0} * 0.0648.}$$

(2).



a.) $\vec{E}$ field can only be zero to the left of the charge at the origin. $(x<0)$.

$$E = E_1 + E_2 = \frac{kq_1}{x^2} + \frac{kq_2}{(x+x_0)^2}$$ (q_2 is negative).

this is ok since q_2 is $(-)$.

$$E = 0 \Rightarrow \sqrt{\frac{kq_1}{x^2}} = \sqrt{\frac{-kq_2}{(x+x_0)^2}}.$$

$$\frac{x}{\sqrt{q_1}} = \frac{x+x_0}{\sqrt{-q_2}}.$$

$$\frac{x_0}{\sqrt{-q_2}} = x\left(\frac{1}{\sqrt{q_1}} - \frac{1}{\sqrt{-q_2}}\right).$$

$$x = \frac{x_0}{\frac{\sqrt{-q_2}}{\sqrt{q_1}} - 1}$$

$$\boxed{x = \frac{1}{\sqrt{2}-1} \simeq -2.4\text{m}.}$$

b.) We need y components from both charges to balance.

$$E_{1y} = \frac{kq_1}{y^2}$$ if we are a distance along the y-axis.

$$E_{2y} = \frac{kq_2}{x_0^2+y^2}\frac{x_0}{\sqrt{x_0^2+y^2}} = \frac{kq_2 y}{(x_0^2+y^2)^{3/2}}.$$

We need $E_{1y} = -E_{2y}$.

$$\Rightarrow \frac{kq_1}{y^2} = \frac{-kq_2 y}{(x_0^2+y^2)^{3/2}}.$$

$$q_1(x_0^2+y^2)^{3/2} = -q_2 y^3.$$

$$q_1^{2/3}(x_0^2+y^2) = (-q_2)^{2/3} y^2.$$

$$y^2\left(q_1^{2/3} - (-q_2)^{2/3}\right) = q_1^{2/3} x_0^2$$

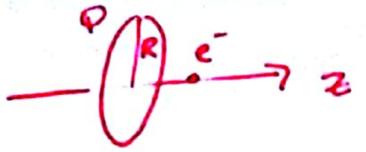
$$\boxed{y = \frac{\pm q_1^{1/3} x_0}{\sqrt{(-q_2)^{2/3} - q_1^{2/3}}} \approx \pm 1.30\,\text{m}.}$$

3. a.) Let each charge sit at each numeral of a clock.

· Net force on a charge at the center is $\boxed{\text{zero}}$. The forces cancel out in pairs as we go around the clock.

b.) Now, we have a 13-sided polygon. Due to the symmetry, the force at the center is still $\boxed{\text{zero.}}$

(Think of a circular ring with uniform charge. The force at the center is zero no matter if I break it up into an even or odd number of parts).

4.) 

$$|\vec{E}| = \frac{q|z|}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$ for ring.

points to $+z$ direction when $z>0$.
" " $-z$ direction when $z<0$.

$$F_z = (e)E_z = \frac{-eqz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}.$$

For small oscillations, $|z| << R$

$$(z^2+R^2)^{3/2} \simeq R^3$$

$$F_z \simeq \frac{-eqz}{4\pi\varepsilon_0 R^3} = \frac{eq}{4\pi\varepsilon_0 R^3}(z).$$

$$\Rightarrow k = \frac{eq}{4\pi\varepsilon_0 R^3}.$$

$$\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 R^3 m}}}$$

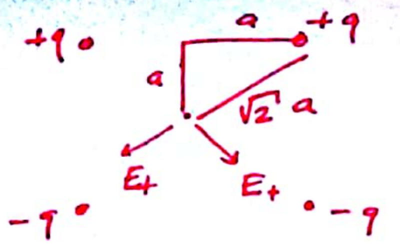
5.) Construct an 8-faced closed surface (w/ 2 pyramids). with charge at the center.

total flux $= \frac{q}{\varepsilon_0}$

→ Each face is identical $\Rightarrow$ $\boxed{\frac{q}{8\varepsilon_0} \text{ flux through each face}}$.

$\boxed{\frac{q}{2\varepsilon_0}}$ is flux emerging from upper triangles.

(6.)



- E field from upper charges points downward.
- The same argument holds from the bottom two charges.

(these two add).

a.) $E_{total} = 4\,|\vec{E}_+|\sin\theta\,(-\hat{j}) = 4k\frac{q}{2a^2}\left(\frac{1}{\sqrt{2}}\right)(-\hat{j})$.

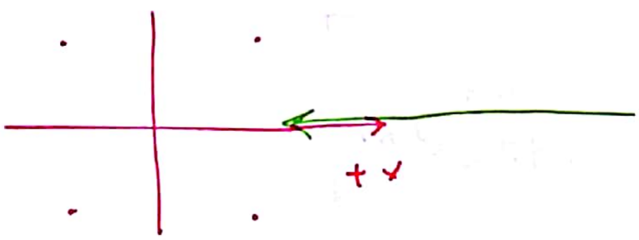
$$E_{total} = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}\,q}{a^2}(-\hat{j}).$$

b.) $\underset{\text{origin}}{V(0)} - V(\infty) = V(0)$

$$= k\frac{q}{(2a^2)^{1/2}} + k\frac{q}{(2a^2)^{1/2}} + \frac{k(-q)}{(2a^2)^{1/2}} + \frac{k(-q)}{(2a^2)^{1/2}}.$$

$$\boxed{V(0) = 0.}$$

c.)

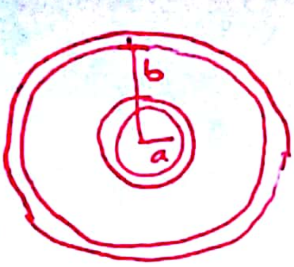


path is along the x-axis from ∞ to the origin.

This is a trivial path because the $\vec{E}$ field points in the $(-\hat{j})$ direction so $\int \vec{E}\cdot d\vec{l} = 0$.

$$-\int \vec{E}\cdot d\vec{l} = \Delta V = 0.$$ ✓

(7.)



a.) • $(r<a)$ $\vec{E}=0$ (no $Q_{enclosed}$).

• $(a<r<b)$ ⟹ choose spherical Gaussian surface.

$$\oint \vec{E}\cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}.$$

$$\vec{E} = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}\,\hat{r}$$

• $(r>a)$ $\vec{E}=0$ ($Q_{enclosed}=0$).

b.) $$C = \frac{Q}{|\Delta V|} = \frac{Q}{\left|-\int_{r=a}^{r=b}\vec{E}\cdot d\vec{l}\right|} = \frac{Q}{\left|-\int_a^b \frac{Q\,dr}{4\pi\varepsilon_0 r^2}\right|} = \frac{Q}{\left|\frac{+Q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)\right|}.$$

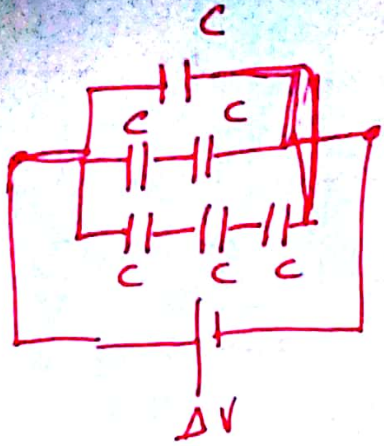
$$\boxed{C = \frac{4\pi\varepsilon_0 ab}{(b-a)}} \quad (b>a).$$

c.) $U = \dfrac{Q^2}{2C}.$ $b \to 2b.$: Q stays the same.
: C changes.

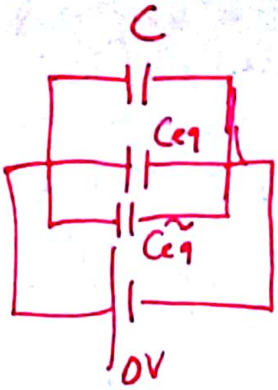
$$\Delta U = \frac{Q^2}{2}\left(\frac{(2b-a)}{4\pi\varepsilon_0 2ba} - \frac{(b-a)}{4\pi\varepsilon_0 ba}\right).$$

$$\boxed{\Delta U = \frac{Q^2}{16\pi\varepsilon_0 b}}$$

8.



$\Rightarrow$



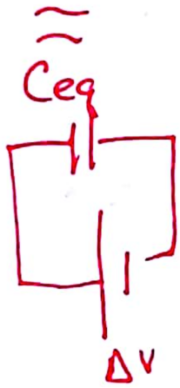
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}.$$

$$C_{eq} = \frac{C}{2}.$$

$$\frac{1}{\tilde{C}_{eq}} = \frac{3}{C} =$$

$$\tilde{C}_{eq} = \frac{C}{3}$$

$\Rightarrow$



$$\rightarrow \tilde{\tilde{C}}_{eq} = \tilde{C}_{eq} + C_{eq} + C$$

$$= \frac{C \cdot 2}{3 \cdot 2} + \frac{C \cdot 3}{2 \cdot 3} + \frac{6C}{6}.$$

$$\boxed{\tilde{\tilde{C}}_{eq} = \frac{11C}{6}.}$$