Discussion 1 - Intro and Math Review

Problem 1. Name 3 systems that undergo periodic motion.

Problem 2. Taylor expand the following functions about the given point up to second order.

$$f(x - x_0) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!}f''(x_0)(x - x_0)^2 + \frac{1}{3!}f'''(x_0)(x - x_0)^3 + ... \tag{1}$$

a. $f(x) = \cos(4x)$ about $x = 0$

b. $f(x) = \ln(3 + 4x)$ about $x = 0$

c. $f(x) = \frac{1}{x^4}$ about $x = 1$

d. $f(x) = (1 + x)^n$ about $x = 0$ for any n

Problem 3. Calculate the total mass of a sphere with radius R and non-uniform mass density $\rho = Cr$, where C is a constant and r is the distance from the center. [In Eq. 2, dV is the volume of an infinitesimal piece of the object.]

$$M = \int \rho dV \tag{2}$$

Problem 4. Calculate the moment of inertia of a rod (thin cylinder) with total mass M and length L rotating about an axis perpendicular to the rod, passing through the center of the rod. [In Eq. 3, y is the distance from the axis to a infinitesimal piece of the object with mass dm.]

$$I = \int y^2 dm \tag{3}$$

Problem 5. The following functions represent the displacement of an object as a function of time. Calculate the velocity, $\frac{dx}{dt}$, and acceleration, $\frac{d^2x}{dt^2}$, for each of the functions.

a. $x(t) = \cos(t)$

b. $x(t) = A\cos(\omega t)$

c. $x(t) = A\cos(t - \frac{\pi}{2})$

d. $x(t) = A\cos(t + \frac{\pi}{4})$

Problem 6. Derive the formula for the oscillation frequency for (a) an ideal horizontal spring (spring constant k) with a block of mass m attached to the end (no gravity, no friction), (b) an ideal pendulum with a block of mass m hanging from the end of a string of length l, undergoing small oscillations.