Physics 1B Midterm 2 Supplementary Practice Solutions

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for Dr. Andrea Chaney's class

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Note: This is the first time that I am writing practice problems for a class; as such, the difficulty is somewhat variable and may not be reflective of the actual exam. In addition, some of the questions were written in a theoretical sense and may actually be an impossible scenario in real life. However, I believe these problems are very useful to help you understand the concepts that will be tested. Any feedback or comments are always appreciated.

Problem 1

a) To find the flux, we use $\Phi = \int \mathbf{E} \cdot d\mathbf{A}$. However, what we notice is that the electric field is constant at $r = e$, since for radii immediately smaller, the electric field is 0. Thus, the surface integral turns into $E \cdot A$:

$$\begin{aligned}\Phi &= \mathbf{E}(e) \cdot 4\pi e^2 \\ &= E_0 \left(\frac{e}{2e}\right)^2 4\pi e^2 \\ &= E_0 \left(\frac{1}{4}\right) 4\pi e^2 \\ &= \boxed{E_0 \pi e^2}\end{aligned}$$

b) For $r \geq e$:

$$\begin{aligned}\phi &= -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{r} \\ &= -\int_{\infty}^{r} E_0 \left(\frac{e}{2r}\right)^2 dr = -E_0 \left(\frac{-e^2}{4r}\bigg|_{\infty}^{r}\right) = \boxed{\frac{E_0 e^2}{4r}}\end{aligned}$$

For $d \leq r < e$:

$$\phi = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{r} = -\int_{\infty}^{e} \mathbf{E} \cdot d\mathbf{r} - \int_{e}^{r} \mathbf{E} \cdot d\mathbf{r}$$

$$= \frac{E_0 e^2}{4e} - \int_{e}^{r} 0 \; dr = \boxed{\frac{E_0 e}{4}}$$

For $c \leq r < d$:

$$\phi = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{r} = -\int_{\infty}^{d} \mathbf{E} \cdot d\mathbf{r} - \int_{d}^{r} \mathbf{E} \cdot d\mathbf{r}$$

$$= \frac{E_0 e}{4} - \int_{d}^{r} \frac{E_0 c}{\sqrt{r-c}} dr$$

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Note: Use u-substitution where $u = r - c$.

$$= \frac{E_0 e}{4} - \int_{d-c}^{r-c} \frac{E_0 c}{\sqrt{u}} du$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{u}\Big|_{d-c}^{r-c}$$

$$= \boxed{\frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} + 2E_0 c\sqrt{r-c}}$$

For $b \leq r < c$:

$$\phi = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{r} = -\int_{\infty}^{c} \mathbf{E} \cdot d\mathbf{r} - \int_{c}^{r} \mathbf{E} \cdot d\mathbf{r}$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - \int_{c}^{r} E_0 (r-b)(c-r) dr$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - \int_{c}^{r} E_0 (-r^2 + br + cr - bc) dr$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(-\frac{r^3}{3} + \frac{br^2 + cr^2}{2} - bcr \right) \Big|_{c}^{r}$$

$$= \boxed{\frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(-\frac{r^3}{3} + \frac{br^2 + cr^2}{2} - bcr + \frac{c^3}{3} - \frac{bc^2 + c^3}{2} + bc^2 \right)}$$

For $a \leq r < b$:

$$\phi = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{r} = -\int_{\infty}^{b} \mathbf{E} \cdot d\mathbf{r} - \int_{b}^{r} \mathbf{E} \cdot d\mathbf{r}$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(\frac{c^3 - b^3}{3} + \frac{b^3 + cb^2 - bc^2 - c^3}{2} - bc(b-c) \right) - \int_{b}^{r} 0 \, dr$$

$$= \boxed{\frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(\frac{c^3 - b^3}{3} + \frac{(b+c)(b^2 - c^2)}{2} - bc(b-c) \right)}$$

For $0 \leq r < a$:

$$\phi = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{r} = -\int_{\infty}^{a} \mathbf{E} \cdot d\mathbf{r} - \int_{a}^{r} \mathbf{E} \cdot d\mathbf{r}$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(\frac{c^3 - b^3}{3} + \frac{(b+c)(b^2 - c^2)}{2} - bc(b-c) \right) - \int_{a}^{r} E_0 \left(\frac{r}{a}\right)^2 dr$$

$$= \frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(\frac{c^3 - b^3}{3} + \frac{(b+c)(b^2 - c^2)}{2} - bc(b-c) \right) - E_0 \left(\frac{r^3}{3a^2}\right)\Bigg|_{a}^{r}$$

$$= \boxed{\frac{E_0 e}{4} - 2E_0 c\sqrt{d-c} - E_0 \left(\frac{c^3 - b^3}{3} + \frac{(b+c)(b^2 - c^2)}{2} - bc(b-c) \right) - \frac{E_0 r^3}{3a^2} + \frac{E_0 a}{3}}$$

Problem 2

a) In a system of two conducting spheres, charges can freely move around, so the electric potential will be the same amongst both spheres. The net charge quantity is positive, so both spheres have positive charge. Given that the electric potential is constant throughout the system, there is less charge in the smaller sphere.

b) (Note: An earlier version of this problem asked about the electric flux through the sides of cubes; this version failed to account for the varying electric field at various points throughout the sides of the cubes. The problem has since been revised to reflect the intended difficulty.)
The electric flux through a region of the smaller sphere bounded by a circle with radius r will flow evenly throughout a similar region (bounded by a circle with radius $3r$ on the larger sphere that is coaxial with the first region). Thus, we can simply divide πr^2 by $9\pi r^2$ (the area of the circle bounding the region of the larger sphere). The result is

that $\boxed{\dfrac{1}{9}}$ of the flux from a region of the smaller sphere bounded by a circle with radius r flows through a region (coaxial to the first) of the larger sphere bounded by a circle with radius r.