Discussion 5 - Coulomb's Law

Problem 1. State Coulomb's Law and explain what every variable means.

$$\vec{F}_E = k\frac{q_1 q_2}{r^2}\hat{r} \tag{1}$$

One of the charges is the "source" and the other is the "test". The force acts *on* the test charge *due to* the source. The vector $\hat{r}$ points from the source charge to the test charge. If $q_1 q_2 < 0$, the force should be attractive and in the $-\hat{r}$-direction.

$$\vec{F}_E = -k\frac{|q_1||q_2|}{r^2}\hat{r} \tag{2}$$

Coulomb's law states that the force on point charge 2 from point charge 1 is

$$\vec{F}_{1on2} = k\frac{q_1 q_2}{r^2}\hat{r}$$

q_1 and q_2 are the charges of the two point charges, and have units of coulombs (C). r is the distance between the two charges. $\hat{r}$ points from the source charge to the test charge (source = 1, test = 2 in this case).

The coefficient out front gets the units right. Knowing the units of force, we can see that $[k] = \mathrm{N{\cdot}m^2/C^2}$. In SI units, k is written as

$$k = \frac{1}{4\pi\epsilon_0}.$$

ϵ_0 is called the electric constant, and has the opposite units of k, $[\epsilon_0] = \mathrm{C^2/(N \cdot m^2)}$.

Problem 2. Two point charges, $q_1 > 0$ and $q_2 < 0$, are separated by a distance r. Find the magnitude and direction of the electric force that

a. q_1 exerts on q_2

b. q_2 exerts on q_1.

a. Use Coulomb's Law. The vector $\hat{r}$ points from the source charge to the test charge.

$$\vec{F}_{1on2} = k\frac{q_1 q_2}{r^2}\hat{r} \tag{3}$$

The charges are opposite sign so they attract each other.

b. By Newton's 3rd law, the forces are equal and opposite. Now q_2 is the source and q_1 is the test.

$$\vec{F}_{2on1} = -\vec{F}_{1on2} \tag{4}$$

Problem 3. Three point charges are arranged on a line. Charge q_3 is at the origin. Charge q_2 is at x_2. Charge q_1 is at x_1. What is q_1 if the net force on q_3 is zero and $x_1 x_2 < 0$?

Use Coulomb's Law and Superposition of Electric Forces

$$F_{2on3} = k\frac{q_2 q_3}{x_2^2} \tag{5}$$

$$F_{1on3} = k\frac{q_1 q_3}{x_1^2} \tag{6}$$

To cancel out the force of q_2 on q_3, we need the force of q_1 on q_3 to be in the opposite direction. The net force on q_3 is zero. Thus,

$$F_{2on3} + F_{1on3} = 0 \tag{7}$$
$$k\frac{q_2 q_3}{x_2^2} = k\frac{q_1 q_3}{x_1^2} \tag{8}$$
$$q_1 = -q_2\frac{x_1^2}{x_2^2} \tag{9}$$
$$\tag{10}$$

Problem 4. Positive charge Q is distributed uniformly along the x-axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x-axis at $x = a + b$, a distance b to the right of the end of Q as shown in the figure.

a. Calculate the x- and y-components of the electric field produced by the charge distribution Q on the points on the positive x-axis where $x > a$.

b. Calculate the force (magnitude and direction) that the charge distribution Q exerts on q.

c. Show that if $b \gg a$, the magnitude of the force in part (b) is approximately kQq/b^2. Explain why this result is obtained.

a. The y-component of the electric field is zero.

$$dQ = \frac{Q}{a}dx' \tag{11}$$
$$dE_x = k\frac{dQ}{(x - x')^2} \tag{12}$$
$$E_x = \int_0^a k\frac{Q}{a}\frac{dx'}{(x - x')^2} \tag{13}$$
$$u = x - x' \tag{14}$$
$$E_x = k\frac{Q}{a}\int_{x-a}^{x}\frac{du}{u^2} \tag{15}$$
$$E_x = k\frac{Q}{a}(\frac{1}{x - a} - \frac{1}{x}) = \frac{kQ}{x(x - a)} \tag{16}$$

b.

$$\vec{F} = q\vec{E} \tag{17}$$
$$F_x = k\frac{qQ}{b(a + b)} \tag{18}$$

c. If $b \gg a$, then $a + b \approx b$. The field of the rod looks like the field of a point charge far away.

$$F_x = k\frac{qQ}{b^2} \tag{19}$$

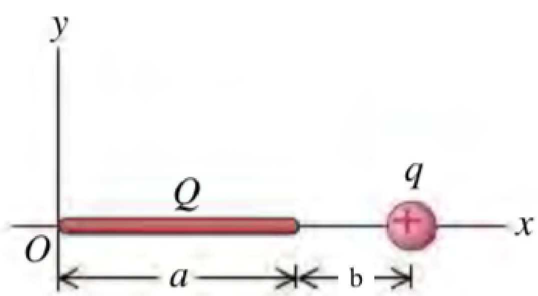

Figure 1: Problem 4