University Physics Volume I
Unit 2: Waves and Acoustics
Chapter 15: Oscillations

Conceptual Questions

1. What conditions must be met to produce SHM?

Solution

The restoring force must be proportional to the displacement and act opposite to the direction of motion with no drag forces or friction. The frequency of oscillation does not depend on the amplitude.

3. Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

Solution

Examples: Mass attached to a spring on a frictionless table, a mass hanging from a string, a simple pendulum with a small amplitude of motion. All of these examples have frequencies of oscillation that are independent of amplitude.

5. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

Solution

Since the frequency is proportional to the square root of the force constant and inversely proportional to the square root of the mass, it is likely that the truck is heavily loaded, since the force constant would be the same whether the truck is empty or heavily loaded.

7. Describe a system in which elastic potential energy is stored.

Solution

In a car, elastic potential energy is stored when the shock is extended or compressed. In some running shoes elastic potential energy is stored in the compression of the material of the soles of the running shoes. In pole vaulting, elastic potential energy is stored in the bending of the pole

9. The temperature of the atmosphere oscillates from a maximum near noontime and a minimum near sunrise. Would you consider the atmosphere to be in stable or unstable equilibrium?

Solution

The overall system is stable. There may be times when the stability is interrupted by a storm, but the driving force provided by the sun bring the atmosphere back into a stable pattern.

11. If the maximum speed of the mass attached to a spring, oscillating on a frictionless table, was increased, what characteristics of the rotating disk would need to be changed?

Solution

The maximum speed is equal to $v_{\max} = A\omega$ and the angular frequency is independent of the amplitude, so the amplitude would be affected. The radius of the circle represents the amplitude of the circle, so make the amplitude larger.

13. A pendulum clock works by measuring the period of a pendulum. In the springtime the clock runs with perfect time, but in the summer and winter the length of the pendulum changes. When most materials are heated, they expand. Does the clock run too fast or too slow in the summer? What about the winter?

Solution

The period of the pendulum is $T = 2p\sqrt{L/g}$. In summer, the length increases, and the period increases. If the period should be one second, but period is longer than one second in the summer, it will oscillate fewer than 60 times a minute and clock will run slow. In the winter it will run fast.

15. Give an example of a damped harmonic oscillator. (They are more common than undamped or simple harmonic oscillators.)

Solution
A car shock absorber.

17. Most harmonic oscillators are damped and, if undriven, eventually come to a stop. Why?

Solution
The second law of thermodynamics states that perpetual motion machines are impossible. Eventually the ordered motion of the system decreases and returns to equilibrium.

19. Do you think there is any harmonic motion in the physical world that is not damped harmonic motion? Try to make a list of five examples of undamped harmonic motion and damped harmonic motion. Which list was easier to make?

Solution
All harmonic motion is damped harmonic motion, but the damping may be negligible. This is due to friction and drag forces. It is easy to come up with five examples of damped motion: (1) A mass oscillating on a hanging on a spring (it eventually comes to rest). (2) Shock absorbers in a car (thankfully they also come to rest). (3) A pendulum is a grandfather clock (weights are added to add energy to the oscillations). (4) A child on a swing (eventually comes to rest unless energy is added by pushing the child). (5) A marble rolling in a bowl (eventually comes to rest). As for the undamped motion, even a mass on a spring in a vacuum will eventually come to rest due to internal forces in the spring. Damping may be negligible, but cannot be eliminated.

Problems

21. Prove that using $x(t) = A\sin(\omega t + \phi)$ will produce the same results for the period for the oscillations of a mass and a spring. Why do you think the cosine function was chosen?

Solution
Proof

23. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

Solution
0.400 s/beat

25. A stroboscope is set to flash every 8.00×10^{-5} s. What is the frequency of the flashes?

Solution
12,500 Hz

27. Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

Solution
a. 340 km/hr; b. 11.3×10^3 rev/min

29. A mass m_0 is attached to a spring and hung vertically. The mass is raised a short distance in the vertical direction and released. The mass oscillates with a frequency f_0. If the mass is

replaced with a mass nine times as large, and the experiment was repeated, what would be the frequency of the oscillations in terms of f_0?

Solution

$f = \frac{1}{3} f_0$

31. By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?

Solution

0.009 kg; 2%

33. It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective force constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85-kg team?

Solution

a. 1.57×10^5 N/m; b. 77 kg, yes, he is eligible to play

35. When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m. (a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

Solution

a. 6.53×10^3 N/m; b. yes, when the man is at his lowest point in his hopping the spring will be compressed the most

37. The length of nylon rope from which a mountain climber is suspended has an effective force constant of 1.40×10^4 N/m. (a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg? (b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? *Hint:* Use conservation of energy. (c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

Solution

a. 1.99 Hz; b. 50.2 cm; c. 0.710 m

39. (a) A novelty clock has a 0.0100-kg-mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

Solution

a. 0.335 m/s; b. 5.61×10^{-4} J

41. A student stands on the edge of a merry-go-round which rotates five times a minute and has a radius of two meters one evening as the sun is setting. The student produces a shadow on the nearby building. (a) Write an equation for the position of the shadow. (b) Write an equation for the velocity of the shadow.

Solution

a. $x(t) = 2\text{m}\cos\left(0.52\text{s}^{-1}t\right)$; b. $v(t) = (-1.05 \text{ m/s})\sin\left(0.52\text{s}^{-1}t\right)$

43. Some people think a pendulum with a period of 1.00 s can be driven with “mental energy” or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

Solution
24.8 cm

45. How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

Solution
4.01 s

47. Two parakeets sit on a swing with their combined CMs 10.0 cm below the pivot. At what frequency do they swing?

Solution
1.58 s

49. A pendulum with a period of 2.00000 s in one location ($g = 9.80\,\text{m/s}^2$) is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

Solution
$9.82002\ \text{m/s}^2$

51. The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Solution
9%

53. If a car has a suspension system with a force constant of $5.00 \times 10^4\ \text{N/m}$, how much energy must the car’s shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

Solution
141 J

55. Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_\text{s} = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_\text{k} = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

Solution
a. $4.90 \times 10^{-3}\text{m}$; b. $1.15 \times 10^{-2}\text{m}$

Additional Problems

57. A diver on a diving board is undergoing SHM. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

Solution
94.7 kg

59. The device pictured in the following figure entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring. (a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its force constant? (b) What is

the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m?

(credit: Lisa Doehnert)

Solution
a. 314 N/m; b. 1.00 s; c. 1.25 m/s

61. Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is $1.63\,\text{m/s}^2$.

Solution
ratio of 2.45

63. If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

Solution
The length must increase by 0.0116%.

65. A 2.00-kg object hangs, at rest, on a 1.00-m-long string attached to the ceiling. A 100-g object is fired with a speed of 20 m/s at the 2.00-kg object, and the two objects collide and stick together in a totally inelastic collision. Write an equation for the motion of the system after the collision. Assume air resistance is negligible.

Solution
$\theta = (0.31\,\text{rad})\sin\left(3.13\,\text{s}^{-1}t\right)$

67. A 2.00-kg block lies at rest on a frictionless table. A spring, with a spring constant of 100 N/m is attached to the wall and to the block. A second block of 0.50 kg is placed on top of the first block. The 2.00-kg block is gently pulled to a position $x = +A$ and released from rest. There is a coefficient of friction of 0.45 between the two blocks. (a) What is the period of the oscillations? (b) What is the largest amplitude of motion that will allow the blocks to oscillate without the 0.50-kg block sliding off?

Solution
a. 0.99 s; b. 0.11 m

Challenge Problems

69. Near the top of the Citigroup Center building in New York City, there is an object with mass of $4.00\times10^5\,\text{kg}$ on springs that have adjustable force constants. Its function is to dampen wind-

driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement from equilibrium?

Solution

a. 3.95×10^{6} N/m; b. 7.90×10^{6} J

71. Consider the van der Waals potential $U(r)=U_o\left[\left(\frac{R_o}{r}\right)^{12}-2\left(\frac{R_o}{r}\right)^{6}\right]$, used to model the potential energy function of two molecules, where the minimum potential is at $r=R_o$. Find the force as a function of *r*. Consider a small displacement $r=R_o+r'$ and use the binomial theorem:

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots,$$

to show that the force does approximate a Hooke's law force.

Solution

$F\approx -\text{constant } r'$

73. (a) The springs of a pickup truck act like a single spring with a force constant of 1.30×10^{5} N/m. By how much will the truck be depressed by its maximum load of 1000 kg? (b) If the pickup truck has four identical springs, what is the force constant of each?

Solution

a. 7.54 cm; b. 3.25×10^{4} N/m