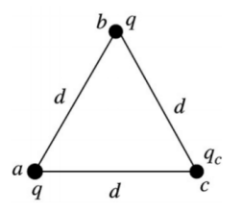
23.11 •• Three point charges, which initially are infinitely far apart, are placed at the corners of an equilateral triangle with sides d. Two of the point charges are identical and have charge q. If zero net work is required to place the three charges at the corners of the triangle, what must the value of the third charge be?

23.11. **IDENTIFY:** Apply $W_{a\to b} = U_a - U_b$. The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

SET UP: Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.



Let q_c be the third, unknown charge.

Figure 23.11

EXECUTE: $W = -\Delta U = -(U_2 - U_1)$, where W is the work done by the Coulomb force.

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c).$$

We want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$.

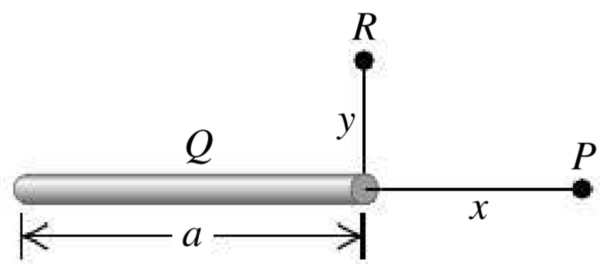
$$0 = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c).$$

$q^2 + 2qq_c = 0$ and $q_c = -q/2$.

EVALUATE: The potential energy for the two charges q is positive and for each q with q_c it is negative. There are two of the q, q_c terms so must have $q_c < q$.

23.73 • **CALC** Electric charge is distributed uniformly along a thin rod of length a, with total charge Q. Take the potential to be zero at infinity. Find the potential at the following points (**Fig. P23.73**): (a) point P, a distance x to the right of the rod, and (b) point R, a distance y above the right-hand end of the rod. (c) In parts (a) and (b), what does your result reduce to as x or y becomes much larger than a?

Figure **P23.73**



23.73. IDENTIFY: Slice the rod into thin slices and use $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$ to calculate the potential due to each slice.

Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point P are shown in Figure 23.73a.

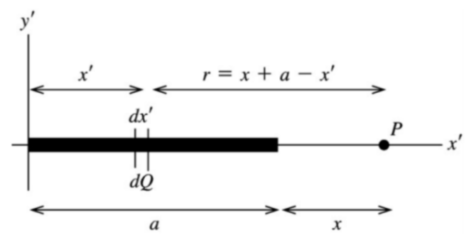


Figure 23.73a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance x given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx'. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point P. The potential at P due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total V at P due to the entire rod by integrating dV over the length of the rod $(x' = 0$ to $x' = a)$:

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\epsilon_0 a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \to \infty$, $V \to \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(\dfrac{x}{x}\right) = 0.$

(b) SET UP: An infinitesimal slice of the rod and its distance from point R are shown in Figure 23.73b.

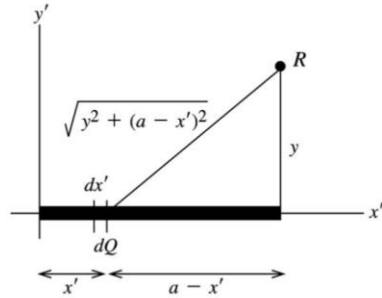


Figure 23.73b

$dQ = (Q/a)dx'$ as in part (a).

Each slice dQ is a distance $r = \sqrt{y^2 + (a - x')^2}$ from point R.

EXECUTE: The potential dV at R due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

In the integral make the change of variable $u = a - x'; du = -dx'$

$$V = -\frac{Q}{4\pi\epsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2 + u^2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(u + \sqrt{y^2 + u^2}\right)\right]_a^0.$$

$$V = -\frac{Q}{4\pi\epsilon_0 a}\left[\ln y - \ln(a + \sqrt{y^2 + a^2})\right] = \frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right)\right].$$

(The expression for the integral was found in Appendix B.)

EVALUATE: As $y \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{y}{y}\right) = 0.$

(c) SET UP: *part (a):* $V = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x + a}{x}\right) = \frac{Q}{4\pi\epsilon_0 a}\ln\left(1 + \frac{a}{x}\right).$

From Appendix B, $\ln(1 + u) = u - u^2/2\ldots$, so $\ln(1 + a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \to \frac{Q}{4\pi\epsilon_0 a}\left(\frac{a}{x}\right) = \frac{Q}{4\pi\epsilon_0 x}$. For large x, V becomes the potential of a point charge.

part (b): $V = \frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right)\right] = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{a}{y} + \sqrt{1 + \frac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1 + a^2/y^2} = (1 + a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \ldots$

Thus $a/y + \sqrt{1 + a^2/y^2} \to 1 + a/y + a^2/2y^2 + \ldots \to 1 + a/y$. And then using $\ln(1 + u) \approx u$ gives

$$V \to \frac{Q}{4\pi\epsilon_0 a}\ln(1 + a/y) \to \frac{Q}{4\pi\epsilon_0 a}\left(\frac{a}{y}\right) = \frac{Q}{4\pi\epsilon_0 y}.$$

EVALUATE: For large y, V becomes the potential of a point charge.

23.27 •• A uniformly charged, thin ring has radius 15.0 cm and total charge +24.0 nC. An electron is placed on the ring's axis a distance 30.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. (a) Describe the subsequent motion of the electron. (b) Find the speed of the electron when it reaches the center of the ring.

23.27. **(a) IDENTIFY** and **SET UP:** The electric field on the ring's axis is given by $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. The magnitude of the force on the electron exerted by this field is given by $F = eE$.

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring.

From Example 23.11, $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\sqrt{x^2+a^2}}$, where a is the radius of the ring.

EXECUTE: $x_a = 30.0$ cm, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$.

Thus $\frac{1}{2}mv^2 = U_a - U_b$.

And $U = qV = -eV$ so $v = \sqrt{\dfrac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_a^2+a^2}} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{24.0\times10^{-9}\ \mathrm{C}}{\sqrt{(0.300\ \mathrm{m})^2+(0.150\ \mathrm{m})^2}}.$$

$$V_a = 643\ \mathrm{V}.$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_b^2+a^2}} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{24.0\times10^{-9}\ \mathrm{C}}{0.150\ \mathrm{m}} = 1438\ \mathrm{V}.$$

$$v = \sqrt{\frac{2e(V_b-V_a)}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\ \mathrm{C})(1438\ \mathrm{V} - 643\ \mathrm{V})}{9.109\times10^{-31}\ \mathrm{kg}}} = 1.67\times10^7\ \mathrm{m/s}.$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.33 •• A very long insulating cylindrical shell of radius 6.00 cm carries charge of linear density 8.50 μC/m spread uniformly over its outer surface. What would a voltmeter read if it were connected between (a) the surface of the cylinder and a point 4.00 cm above the surface, and (b) the surface and a point 1.00 cm from the central axis of the cylinder?

23.33. **IDENTIFY:** For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0} \ln (r_b/r_a)$.

EXECUTE: **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0} \ln (r_b/r_a) = (8.50\times10^{-6} \text{ C/m})(2\times9.00\times10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2) \ln\left(\frac{10.0 \text{ cm}}{6.00 \text{ cm}}\right).$$

$$\Delta V = 7.82\times10^{4} \text{ V} = 78{,}200 \text{ V} = 78.2 \text{ kV}.$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

23.35 •• A very small sphere with positive charge $q = +8.00\ \mu\text{C}$ is released from rest at a point 1.50 cm from a very long line of uniform linear charge density $\lambda = +3.00\ \mu\text{C/m}$. What is the kinetic energy of the sphere when it is 4.50 cm from the line of charge if the only force on it is the force exerted by the line of charge?

23.35. IDENTIFY: The electric field of the line of charge does work on the sphere, increasing its kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$ and $K_1 = 0$. $U = qV$ so $qV_1 = K_2 + qV_2$. $V = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_0}{r}\right)$.

EXECUTE: $V_1 = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_0}{r_1}\right)$. $V_2 = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_0}{r_2}\right)$.

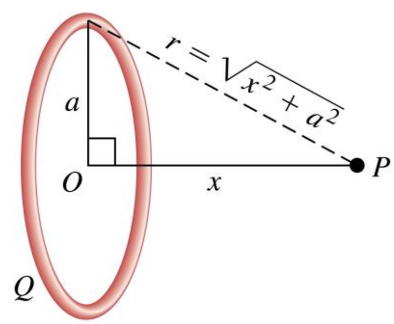
$$K_2 = q(V_1 - V_2) = \frac{q\lambda}{2\pi\epsilon_0}\left(\ln\left(\frac{r_0}{r_1}\right) - \ln\left(\frac{r_0}{r_2}\right)\right) = \frac{\lambda q}{2\pi\epsilon_0}(\ln r_2 - \ln r_1) = \frac{\lambda q}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right).$$

$$K_2 = \frac{(3.00\times10^{-6}\ \text{C/m})(8.00\times10^{-6}\ \text{C})}{2\pi(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))}\ln\left(\frac{4.50}{1.50}\right) = 0.474\ \text{J}.$$

EVALUATE: The potential due to the line of charge does *not* go to zero at infinity but is defined to be zero at an arbitrary distance r_0 from the line.

23.66 •• **CALC** A disk with radius R has uniform surface charge density σ. (a) By regarding the disk as a series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity. (*Hint:* Use the result of Example 23.11 in Section 23.3.) (b) Calculate $-\partial V/\partial x$. Show that the result agrees with the expression for E_x calculated in Example 21.11 (Section 21.5).

Hint (a) Note: the result of Example 23.11 in University Physics (14th Ed) is the potential at a point P on the ring axis at a distance x from the center of the ring.



Hint (b) Note: Example 21.11 in University Physics (14th Ed) is for the electric field of a uniformly charged disk. It is the same result found in the Kudu Example: *Electric Field of a Disk*, in the *Electric Fields* chapter.

23.66. (a) IDENTIFY: Calculate the potential due to each thin ring and integrate over the disk to find the potential. V is a scalar so no components are involved.

SET UP: Consider a thin ring of radius y and width dy. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance x from the ring is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}.$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left(\sqrt{x^2+R^2}-x\right).$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \text{ so } \sqrt{x^2+R^2}-x \approx R^2/2x.$$

Then $V \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \dfrac{Q}{4\pi\epsilon_0 x}$, as expected.

(b) IDENTIFY and SET UP: Use $E_x = -\dfrac{\partial V}{\partial x}$ to calculate E_x.

EXECUTE: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\sigma}{2\epsilon_0}\left(\dfrac{x}{\sqrt{x^2+R^2}}-1\right) = \dfrac{\sigma x}{2\epsilon_0}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x^2+R^2}}\right).$