Chapter 6

Modeling Random Events:
The Normal and Binomial Models

Probability Model and Distribution

- A probability model is a description of how a statistician thinks data are produced
  - Uniform
  - Linear
  - Normal
- A probability distribution or probability distribution function (pdf) is a tool that helps us keep track of all the outcomes of a random experiment and their probabilities

Discrete vs Continuous

- A random variable is called discrete if the outcomes are values that can be listed or counted.
  - Number of classes taken
  - The roll of a die
- A random variable is called continuous if the outcomes cannot be listed because they occur over a range.
  - Time to finish the exam
  - Exact weight

Discrete vs Continuous?

Clicker!

Length of your left index finger

A. Discrete

B. Continuous

Discrete vs Continuous?

Length of your left index finger

A. Discrete

B. Continuous

Discrete vs Continuous?

Clicker!

Number of children in a family

A. Discrete

B. Continuous

Discrete vs Continuous?

Number of children in a family

A. Discrete

B. Continuous

Discrete vs Continuous?

Clicker!

Number of devices in the house that connect to the Internet

A. Discrete

B. Continuous

Discrete vs Continuous?

Number of devices in the house that connect to the Internet

A. Discrete

B. Continuous

Discrete vs Continuous?

Clicker!

Sodium concentration in the bloodstream

A. Discrete

B. Continuous

Discrete vs Continuous?

Sodium concentration in the bloodstream

A. Discrete

B. Continuous

Discrete Probability Distributions

- The most common way to display a probability distribution function (pdf) for discrete data is with a table.
- The probability distribution table always has two columns (or rows)
  - The first, x, displays all the possible outcomes.
  - The second, $P(x)$, displays the probabilities for these outcomes

Examples of Probability Distributions

Important: The sum of all the probabilities must equal 1

Die Roll

x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

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x	P(x)
1	0.290
2	0.146
3	0.163
4	0.195
5	0.206

Example

Roll a fair six-sided die. You will win \$4 if you roll a 5 or a 6. You will lose \$5 if you roll a 1. You will lose \$1 if you roll a 2. Any other outcome, you will win or lose \$0.

- What is the probability distribution table for the amount of money you will win?

Clicker! When you're done, click in choice A!

Example

Roll a fair six-sided die. You will win $4 if you roll a 5 or a 6. You will lose $5 if you roll a 1. You will lose $1 if you roll a 2. Any other outcome, you will win or lose $0.

- What is the probability distribution table for the amount of money you will win?

Die Roll

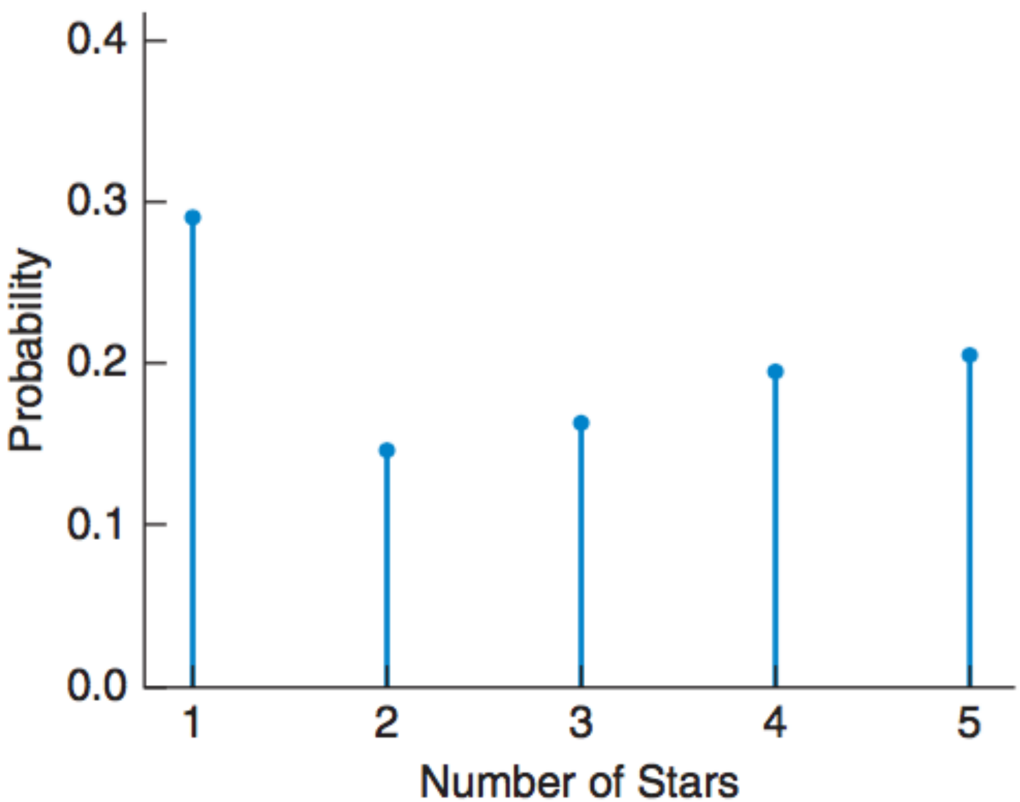
x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Winnings

Winnings	P(x)
-$5	1/6
-$1	1/6
$0	2/6 or 1/3
$4	2/6 or 1/3

Examples of Probability Distributions

The pdf can also be displayed as a graph.



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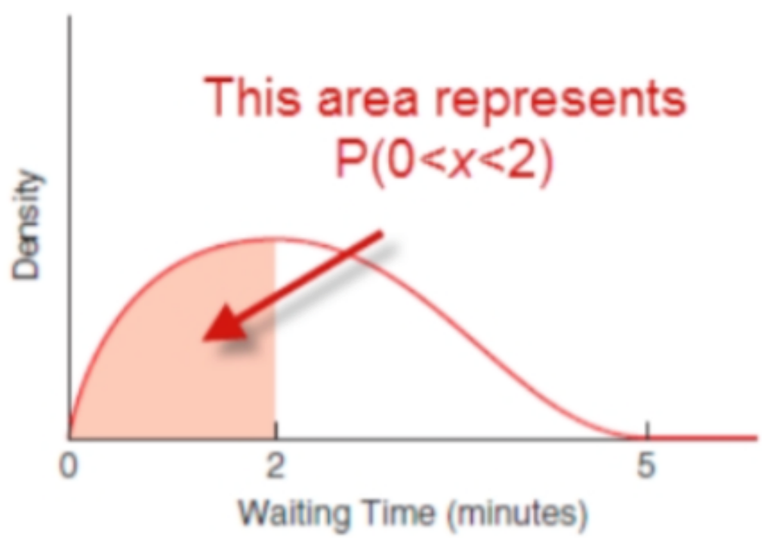
x	P(x)
1	0.290
2	0.146
3	0.163
4	0.195
5	0.206

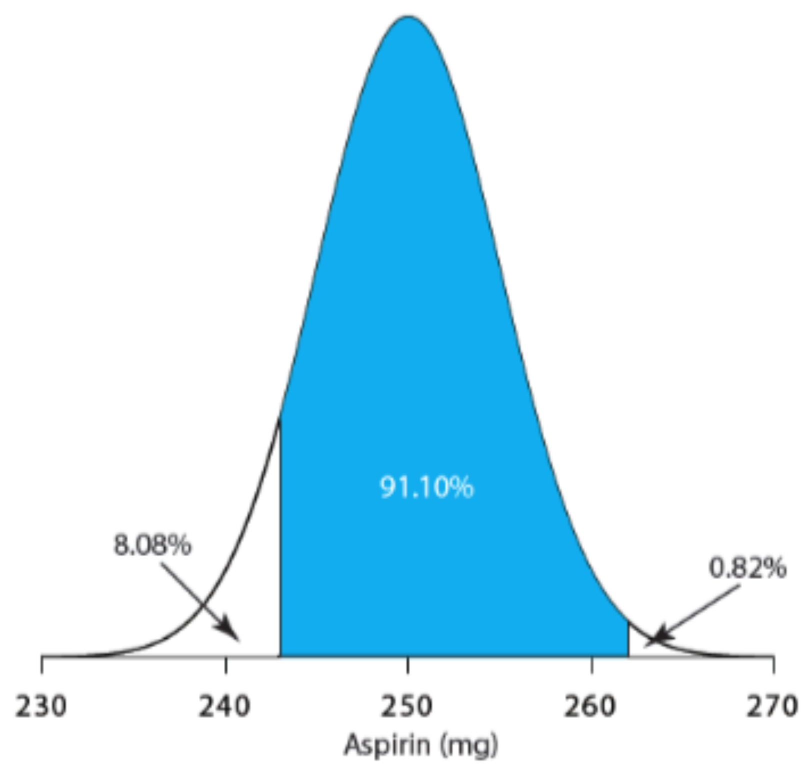
Continuous Probability Distribution Function

- Often represented as a curve
  - The area under the curve between two values of x represents the probability of x being between the two values.
  - The total area under the curve must equal 1.
  - The curve can't lie below the x-axis.



The Normal Model

(Gaussian distribution or Bell Curve)



The Normal Model

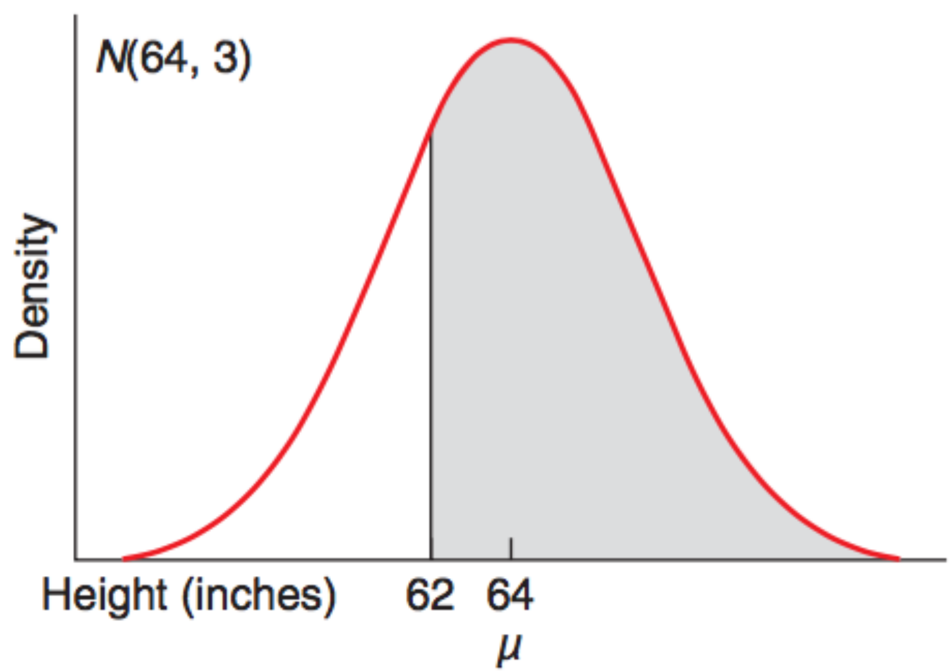
- The Normal Model is a good fit if:
  - The distribution is unimodal.
  - The distribution is approximately symmetric.
  - The distribution is approximately bell shaped.
- A Normal distribution is defined by the mean μ and standard deviation σ. Shorthand for a normal distribution is

$$N(\mu,\sigma)$$

Finding Probabilities

Let's assume N(64,3) gives a good approximation of adult women's heights in the US (in inches).

- Draw the normal curve and shade the area representing the probability of finding a woman taller than 62 inches.



Finding Probabilities

Now want to know the probability of a randomly chosen woman is between 62 inches and 67 inches tall.

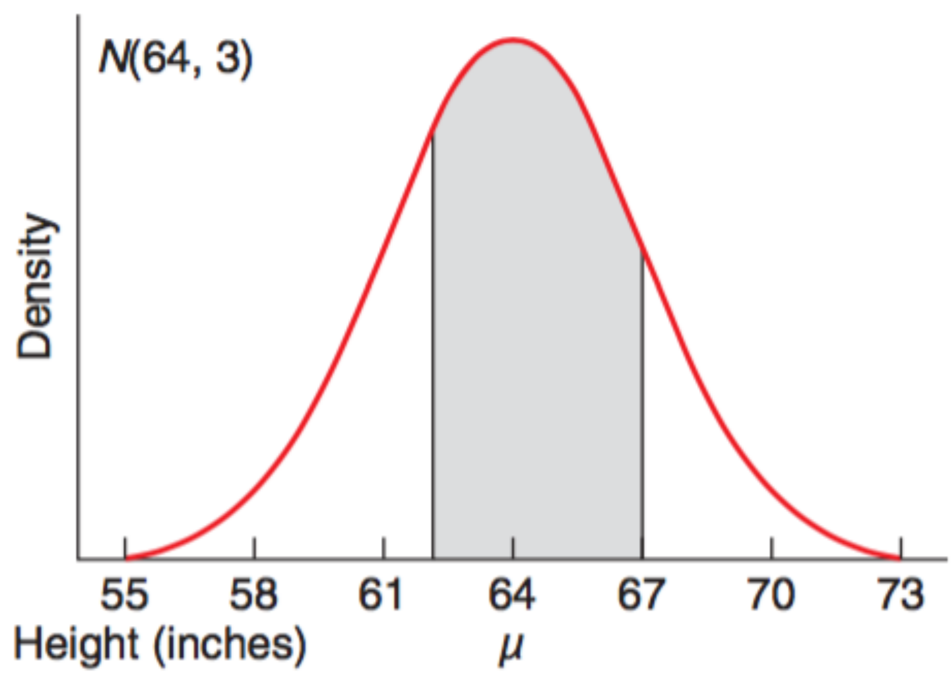
- Draw the normal curve and shade the area.

Clicker! When you’re done, click in choice B!

Finding Probabilities

Now want to know the probability of a randomly chosen woman is between 62 inches and 67 inches tall.

- Draw the normal curve and shade the area.



Finding Probabilities

What is the probability the randomly chosen woman is less than 62 inches tall?

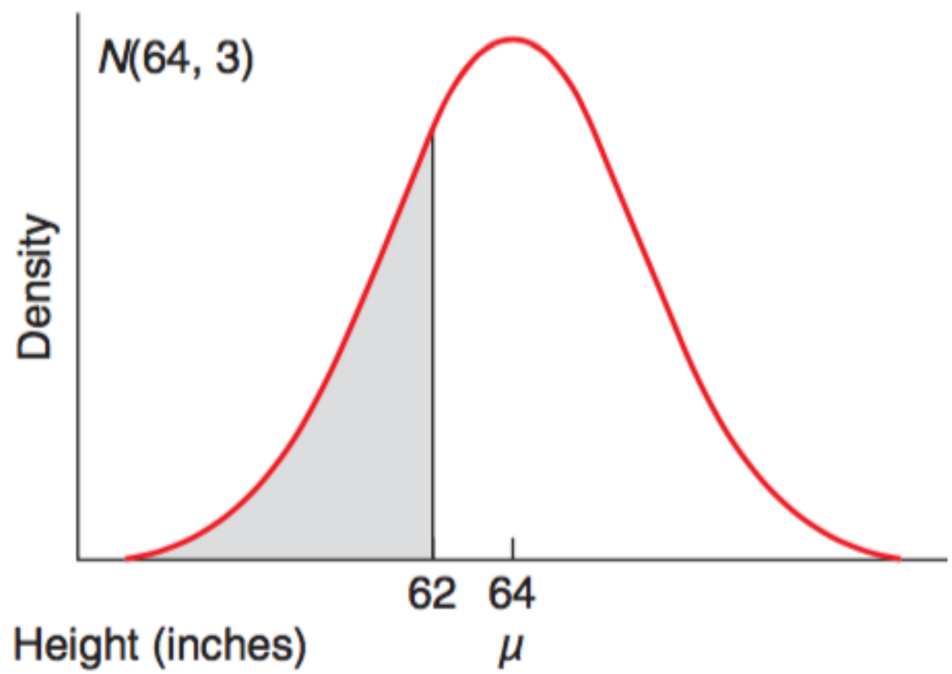
- Draw the normal curve and shade the area.

Clicker! When you’re done, click in choice C!

Finding Probabilities

What is the probability the randomly chosen woman is less than 62 inches tall?

- Draw the normal curve and shade the area.



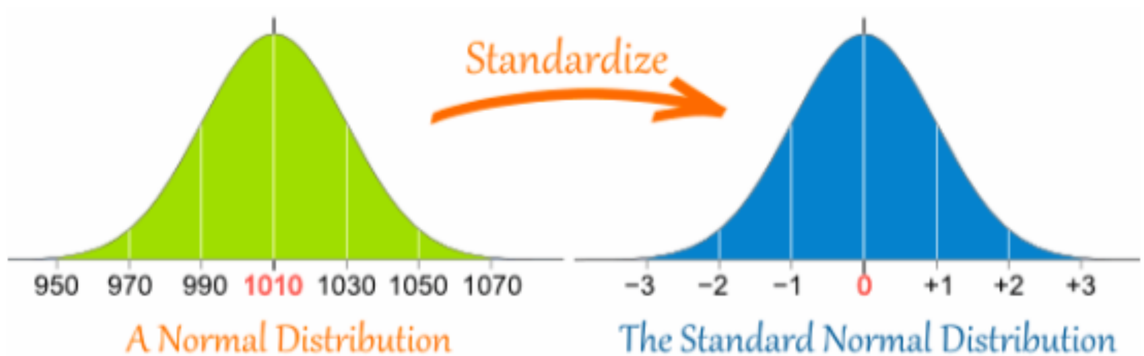
Standardizing with z-scores

- Reminder: z-scores are standardized scores.
- Z-scores are used to compare individual data values to their mean relative to their standard deviation.
- The formula for calculating the z-score of a data values is:

$$z = \frac{\text{observation - mean}}{SD}$$

z-scores

- Standardizing data into z-scores shifts the data by subtracting the mean and rescales the values by dividing by their standard deviation.
- Standardizing into z-scores does not change the shape of the distribution.
- It shifts the mean to 0 and the standard deviation to 1.



Shape, Center and Spread of z-scores

- Z-scores for normally distributed variables are also normally distributed, but with mean 0 and standard deviation 1.

$$z \sim N(0, 1)$$

- Z-scores for a variable with some other distribution (right skewed, uniform, etc.) will follow the same shape as the original distribution, but with mean 0 and standard deviation 1.

Calculating Percentiles and Probabilities with Normal Models

- Since z-scores tell us whether or not an observation is unusual, they can also tell us how unusual the observation is (i.e. how likely it is to observe such a value)
- So far we have only been able to tell how unusual an observation is if it was exactly 1, 2, or 3 standard deviations from the mean (using the Empirical Rule).
- What happens if we have a z-score of 2.5 or -1.3?

Calculating percentiles using the z-table

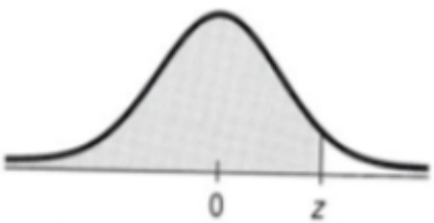
- ACT scores are distributed normally with mean 21 and standard deviation 5. If Adam got a 27 on his ACT, what is his percentile score?
- Note: percentile score means what percent is below the observed value.
- First we compute our z-score:

$$z = \frac{\text{obs - mean}}{SD} = \frac{27-21}{5} = 1.20$$

- Now we go to the z-table.

Using the z-table

Table Z (cont.)
Areas under the standard Normal curve



z	Second decimal place in z: 0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265

- We have z=1.20.
- Z-scores occur on the outer edges of the z-table, probabilities are in the middle.
- The probabilities refer to the area to the left of the z-score.
- Note: It’s best to round the z-score to 2 decimal places.

Calculating percentiles using the z-table

- ACT scores are distributed normally with mean 21 and standard deviation 5. If Adam got a 27 on his ACT, what is his percentile score?
- With a z-score of 1.20, we found the value 0.8849 on the z-table.
- Adam's score is the 88.49th percentile, i.e. he scored higher than 88.49% of the test takers.

Percentiles to Probabilities

- If a score of 27 is higher than about 88.49% of all scores on this test, this means that the probability of scoring lower than 27 is 0.8849.

$$P(\text{ACT score} < 27) = 0.8849$$

- Similarly, the probability of scoring higher than 27 is the complement of this probability:

$$P(\text{ACT score} > 27) = 1 - 0.8849 = 0.1151$$

- Note: Complement probabilities complete each other to 1; the area under the normal curve is equal to 1, so when we know the probability of one side, we can simply subtract it from 1 to get the other side.

Practice Drawing the Normal Curve

Draw the following normal curves:

- What percent of standard normal is found where $z < -1.1$?

- What percent of standard normal is found where $z > -2.09$?

- What percent of standard normal is found where $-1 < z < 2.5$?

Review

- Normal Model - distribution is unimodal, symmetric, and bell shaped.
- To find a probability, first calculate the z-score, then draw the normal model and use the z-table to find the corresponding probability to the z-score.
- The z-tables in the book and on CCLE display the probabilities which refer to the area to the left of the z-score.
- Shorthand for a normal distribution is

$$N(\mu, \sigma)$$

Example

Small newborn seal pups have a lower chance of survival than larger newborn pups. Suppose the length of a newborn seal pup follows a Normal distribution with a mean length of 29.5 inches and a standard deviation of 1.2 inches.

- What is the probability that a newborn pup selected at random is shorter than 28.0 inches?
- To answer the question let's first find the z-score.
- What is the corresponding z-score?

Clicker!

A. 1.25

B. -1.25

C. -1.5

D. 1.5

Example

Small newborn seal pups have a lower chance of survival than larger newborn pups. Suppose the length of a newborn seal pup follows a Normal distribution with a mean length of 29.5 inches and a standard deviation of 1.2 inches.

- What is the probability that a newborn pup selected at random is <u>shorter</u> than 28.0 inches?
- What is the corresponding z-score?

$$z = \frac{28\text{-}29.5}{1.2} = \frac{-1.5}{1.2} = -1.25$$

Hint: Now that you have the z-score draw the normal curve. Next, look for the z-score on the z-table to find the probability.

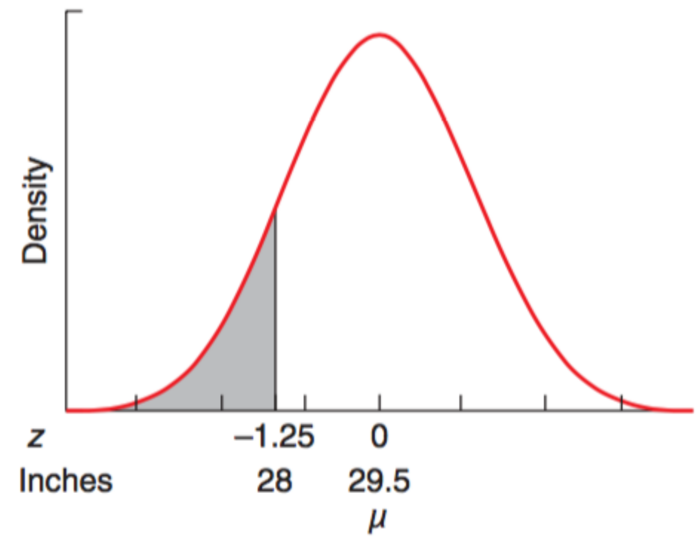
Example

Suppose the length of a newborn seal pup follows a Normal distribution with a mean length of 29.5 inches and a standard deviation of 1.2 inches.

- What is the probability that a newborn pup selected at random is shorter than 28.0 inches?

$$z = \frac{28\text{-}29.5}{1.2} = \frac{-1.5}{1.2} = -1.25$$

z	.00	.01	.02	.03	.04	.05
−1.3	.0968	.0951	.0934	.0918	.0901	.0885
−1.2	.1151	.1131	.1112	.1093	.1075	**.1056**



The probability is 10.56% or approximately 11%.

Example

Suppose the length of a newborn seal pup follows a Normal distribution with a mean length of 29.5 inches and a standard deviation of 1.2 inches.

- What is the probability that a newborn pup selected at random is <u>longer</u> than 28.0 inches?

- To answer the question which area under the curve are we shading?

Clicker!

  A. Area to the right of 28

  B. Area to the left of 28

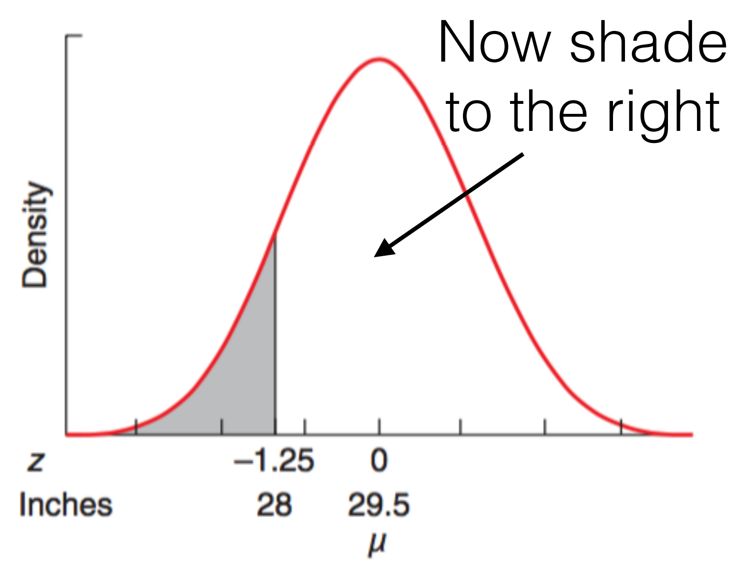
Example

Suppose the length of a newborn seal pup follows a Normal distribution with a mean length of 29.5 inches and a standard deviation of 1.2 inches.

- What is the probability that a newborn pup selected at random is longer than 28.0 inches?

$$z = \frac{28\text{-}29.5}{1.2} = \frac{-1.5}{1.2} = -1.25$$

z	.00	.01	.02	.03	.04	.05
−1.3	.0968	.0951	.0934	.0918	.0901	.0885
−1.2	.1151	.1131	.1112	.1093	.1075	**.1056**



Since we are looking the area to the right of the z-score, the probability is 1-.1056 = .8944 or 89.44%.

Example

Small newborn seal pups have a lower chance of survival than larger newborn pups. Suppose the length of a newborn seal pup follows a Normal distribution with a mean length of 29.5 inches and a standard deviation of 1.2 inches.

- What is the probability that a newborn pup selected at random is between 27 and 31 inches long?

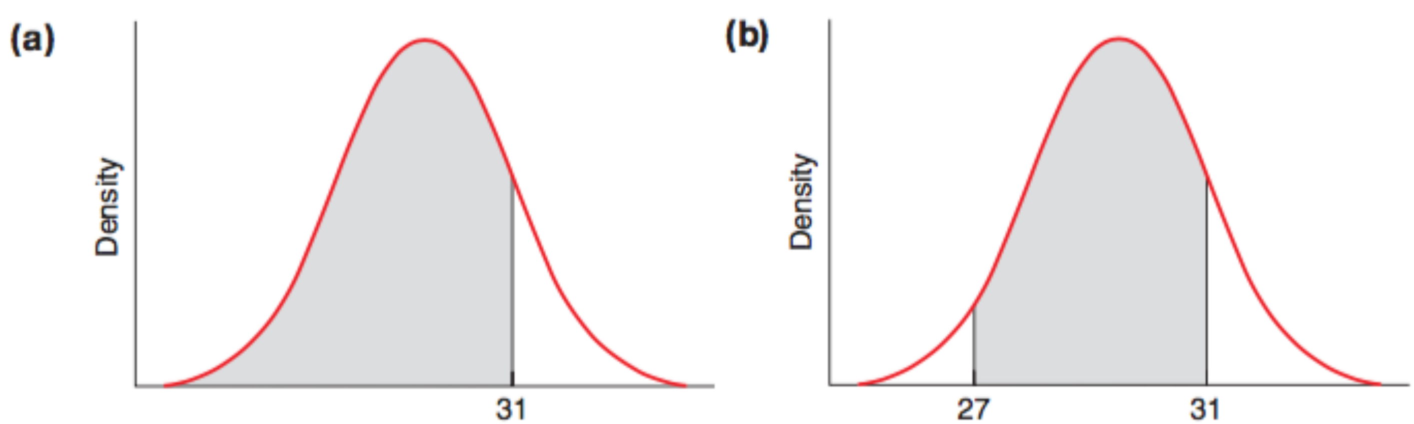
Example

What is the probability that a newborn pup selected at random is between 27 and 31 inches long?

- First, we find the area less than 31 inches long.
- Next, we "chop off" the area less than 27 inches long.

$$z = \frac{31\text{-}29.5}{1.2} = \frac{1.5}{1.2} = 1.25$$

$$z = \frac{27\text{-}29.5}{1.2} = \frac{-2.5}{1.2} = -2.08$$



Example

What is the probability that a newborn pup selected at random is between 27 and 31 inches long?

- First, we find the area less than 31 inches long.

$$z = \frac{31\text{-}29.5}{1.2} = \frac{1.5}{1.2} = 1.25$$

- Using the z-table we find the probability to be 0.8944.
- Next, we find the area less than 27 inches long.

$$z = \frac{27\text{-}29.5}{1.2} = \frac{-2.5}{1.2} = -2.08$$

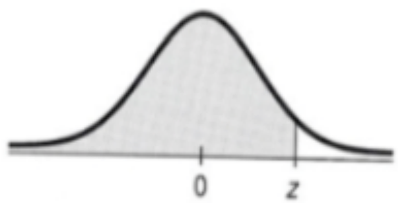
- Using the z-table we find the probability to be 0.0188.
- Finally, we subtract the smaller area from the bigger area:

0.8944 - 0.0188 = 0.8756

Finding Measurements from Percentiles

Let’s assume SAT scores follow a N(1500, 300). If Sophie scored at the 76th percentile, what was her actual score?

Table Z (cont.)
Areas under the standard Normal curve



	Second decimal place i					
z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
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1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265

Finding Measurements from Percentiles

Let's assume SAT scores follow a N(1500, 300). If Sophie scored at the 76th percentile, what was her actual score?

- From the table, we found the corresponding z-score of 0.71 for the 76th percentile, so:

$$z = 0.71 = \frac{score - 1500}{300}$$

$$score - 1500 = 0.71 * 300$$

$$score - 1500 = 213$$

$$score = 213 + 1500$$

$$score = 1713$$

Example

Clicker!

Let's assume SAT scores follow a N(1500, 300). If Michael scored at the 3rd percentile, what is the corresponding z-score?

A. -3.40
B. -2.75
C. -1.88
D. -3.39

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

Example

Let's assume SAT scores follow a N(1500, 300). If Michael scored at the 3rd percentile, what is the corresponding z-score? -1.88

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

Example

Let's assume SAT scores follow a N(1500, 300). If Michael scored at the 3rd percentile, what was his actual score?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

Example

Let's assume SAT scores follow a N(1500, 300). If Michael scored at the 3rd percentile, what was his actual score?

- From the table, we found the corresponding z-score of -1.88 for the 3rd percentile, so:

$$z = -1.88 = \frac{score - 1500}{300}$$

$$score - 1500 = -1.88 \times 300$$

$$score - 1500 = -564$$

$$score = -564 + 1500$$

$$score = 936$$

Example

Let's assume SAT scores follow a N(1500, 300). Between what two scores do the middle 50% of SAT test takers score?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549

Example

Let's assume SAT scores follow a N(1500, 300). Between what two scores do the middle 50% of SAT test takers score?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

The middle 50% is the 25th percentile and 75th percentile.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549

Remember Q1 and Q3!

Example

Let's assume SAT scores follow a N(1500, 300). Between what two scores do the middle 50% of SAT test takers score?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Let's start with the 25th percentile. The corresponding z-score is between -0.67 and -0.68 so we can estimate it to be -0.675.

Example

Let's assume SAT scores follow a N(1500, 300). Between what two scores do the middle 50% of SAT test takers score?

We use the z-score of -0.675 to find the corresponding test score.

$$z = -0.675 = \frac{score - 1500}{300}$$

$$score - 1500 = -0.675 \times 300$$

$$score - 1500 = -202.5$$

$$score = -202.5 + 1500$$

$$score = 1297.5 \approx 1298$$

Example

Let's assume SAT scores follow a N(1500, 300). Between what two scores do the middle 50% of SAT test takers score?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549

Next we look for the 75th percentile. The corresponding z-score is between 0.67 and 0.68 so we can estimate it to be 0.675.

Example

Let's assume SAT scores follow a N(1500, 300).
Between what two scores do the middle 50% of SAT test takers score?

We use the z-score of 0.675 to find the corresponding test score.

$$z = 0.675 = \frac{score - 1500}{300}$$

$$score - 1500 = 0.675 \times 300$$

$$score - 1500 = 202.5$$

$$score = 202.5 + 1500$$

$$score = 1702.5 \approx 1703$$

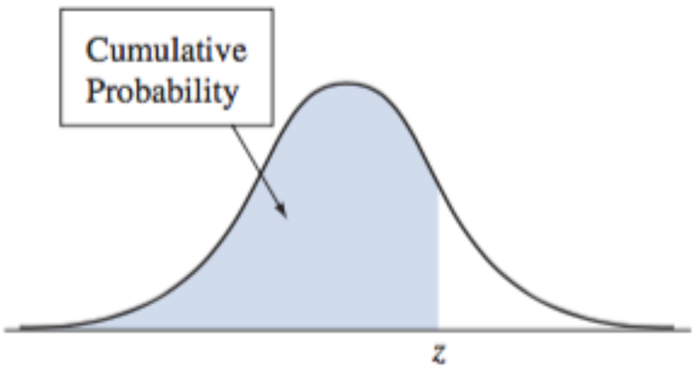
So the middle 50% will be between scores 1298 and 1703.

Example

Clicker!

Find the z-score that gives a left area of 0.6950.

A. 0.69
B. 0.51
C. 0.72



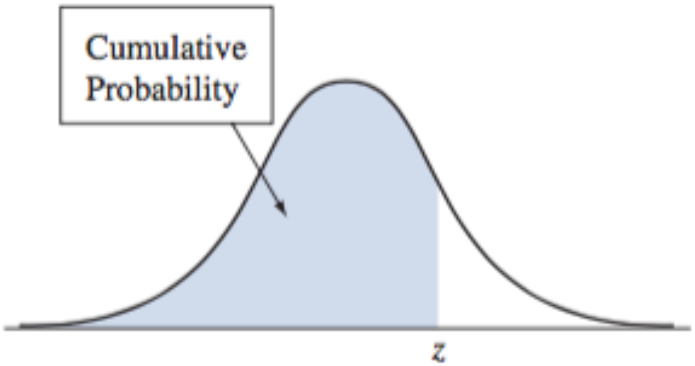
Cumulative probability for z is the area under the standard normal curve to the left of z

Standard Normal Cumulative Probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

Example

Find the z-score that gives a left area of 0.6950.



z-score is 0.51

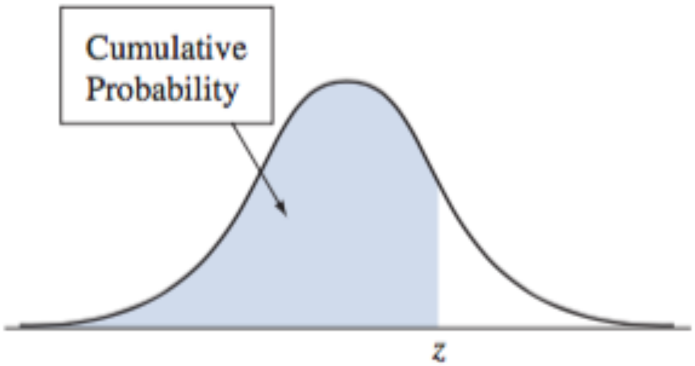
Cumulative probability for z is the area under the standard normal curve to the left of z

Standard Normal Cumulative Probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

Example

Find the z-score that gives a right area of 0.3936.



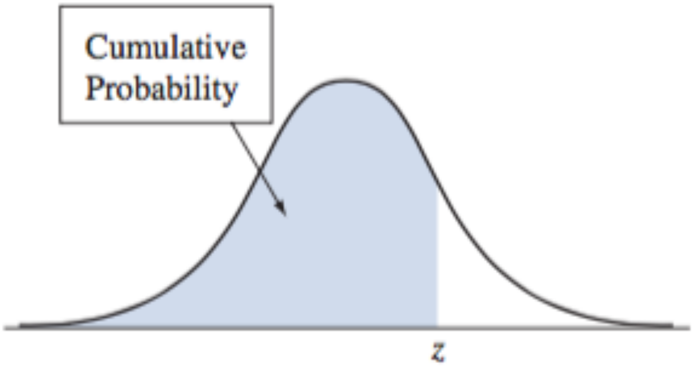
Cumulative probability for z is the area under the standard normal curve to the left of z

Standard Normal Cumulative Probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

Example

Find the z-score that gives a right area of 0.3936.



left area = 1 - 0.3936 = 0.6064

z-score is 0.27

Cumulative probability for *z* is the area under the standard normal curve to the left of *z*

Standard Normal Cumulative Probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

The Binomial Model

- Discrete probability distribution
    - the outcome variable is discrete (counts)
- All 4 characteristics must be present:
    - A fixed number of trials.
    - Only two outcomes are possible at each trial, “success and failure”. Ex. heads/tails, male/female.
    - The probability of success is the same at each trial.
    - The trials are independent.

The Binomial Model

Suppose you flip a fair coin 10 times and want to know the total number of heads. Can the binomial model be applied to this situation?

- Are all 4 characteristics present?
  - A fixed number of trials.
  - Only two outcomes are possible at each trial, “success and failure”.
  - The probability of success is the same at each trial.
  - The trials are independent.

The Binomial Model

Suppose you flip a fair coin 10 times and want to know the total number of heads. Can the binomial model be applied to this situation?

- Are all 4 characteristics present?

1. A fixed number of trials.

Clicker!

A. Yes

B. No

The Binomial Model

Suppose you flip a fair coin 10 times and want to know the total number of heads. Can the binomial model be applied to this situation?

- Are all 4 characteristics present?

Clicker!

2. Only two outcomes are possible at each trial, “success and failure”.

A. Yes

B. No

The Binomial Model

Suppose you flip a fair coin 10 times and want to know the total number of heads. Can the binomial model be applied to this situation?

- Are all 4 characteristics present?

Clicker!

3. The probability of success is the same at each trial.

    A. Yes

    B. No

The Binomial Model

Suppose you flip a fair coin 10 times and want to know the total number of heads. Can the binomial model be applied to this situation?

Clicker!

- Are all 4 characteristics present?

   4. The trials are independent.

      A. Yes

      B. No

The Binomial Model

Suppose you flip a fair coin 10 times and want to know the total number of heads. Can the binomial model be applied to this situation?

- Are all 4 characteristics present?
  - A fixed number of trials. **Yes, 10 trials.**
  - Only two outcomes are possible at each trial, “success and failure”. **Yes, heads and tails.**
  - The probability of success is the same at each trial. **Yes, probability is 0.50.**
  - The trials are independent. **Yes, the coin flips are independent.**

Binomial or Not?

Clicker!

- Ask a group of 50 randomly selected people what their favorite food is and record it.

  A. Binomial

  B. Not Binomial

Binomial or Not?

- Ask a group of 50 randomly selected people what their favorite food is and record it.
  - This is not a binomial experiment. There are more than 2 outcomes at each trial. Instead, we can ask the group of people if they like pizza.

Binomial or Not?

Clicker!

- A student guesses on every question of a test that has 20 multiple choice questions (4 choices each). Record the number of questions the student gets right.

    A. Binomial

    B. Not Binomial

Binomial or Not?

- A student guesses on every question of a test that has 20 multiple choice questions (4 choices each). Record the number of questions the student gets right.
  - Yes this is a binomial experiment. Set number of trials, 2 outcomes, probability of success is the same at each trial, and the trials are independent.

Binomial or Not?

Clicker!

A married couple plans to have three children, and they are wondering how many boys they should expect to have. Assume none of the children will be twins or multiples. Also assume the probability that a child will be a boy is 0.50.

- Is this a binomial experiment?

    A. Yes

    B. No

Hint: Remember all 4 characteristics!

Binomial or Not?

Check assumptions:

- A fixed number of trials: 3 children
- Only two outcomes are possible at each trial: boy or girl
- The probability of success is the same at each trial: 0.50 probability of a boy
- The trials are independent. The gender of each child is independent of the gender of the others.

Binomial or Not?

Clicker!

A married couple plans to have three children, and they are wondering how many boys they should expect to have. Assume none of the children will be twins or multiples. Also assume the probability that a child will be a boy is 0.50.

- What if two of the children turned out to be a set of identical twins?
- Would this be a binomial experiment?

A. Yes

B. No

Binomial or Not?

- What if two of the children turned out to be a set of identical twins?
  - The genders of identical twins are not independent. If one is a boy, the other must be a boy.

Binomial Properties

- Parameters:
  - n: fixed number of trials
  - p: probability of success (stays constant)
  - x: number of successes we're interested in

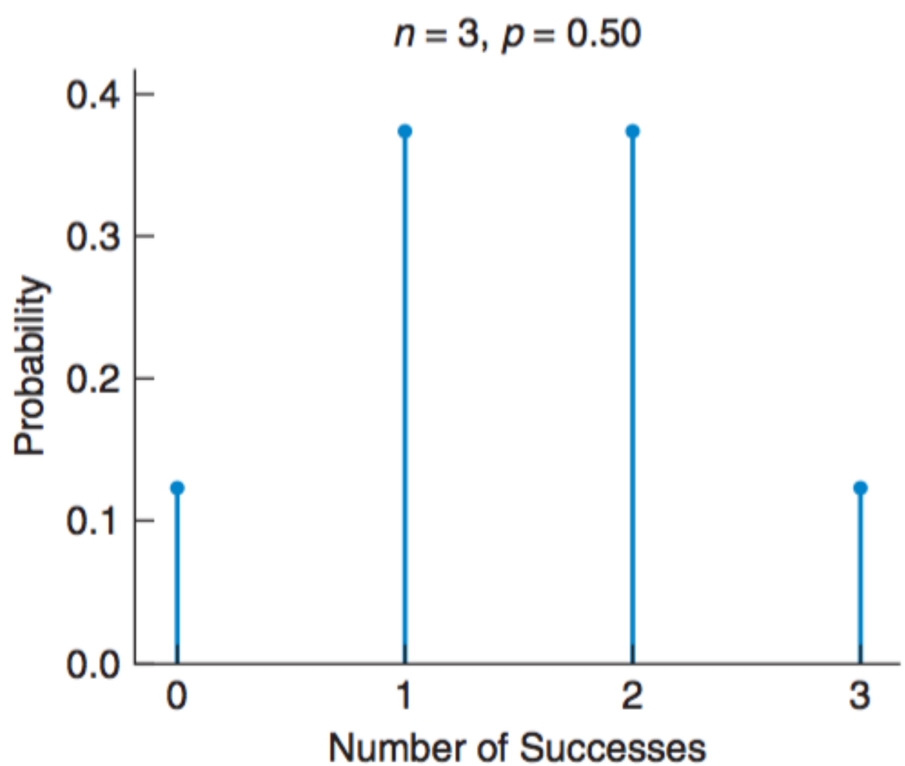
$$b(n, p, x)$$

Visualizing the Binomial Distribution

- Let's assume we flip a coin 10 times and we record the number of heads.
- The binomial model allows for flexibility in n and p. If we flip a coin 6 times instead of 10, it is still a binomial experiment. The probability of success can also change from 0.5 to 0.6 and we will still have a binomial experiment.
- However, for different values of n and p, the binomial distribution looks different.

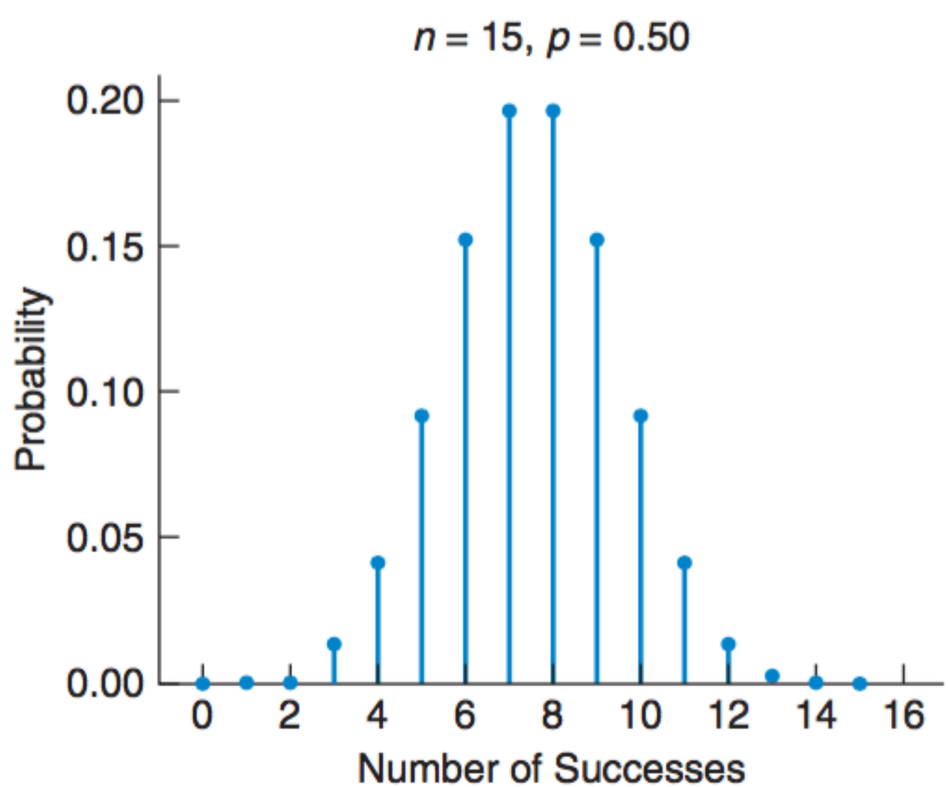
Visualizing the Binomial Distribution

- Here we have n=3 and p=0.5. The distribution looks symmetric.
- The probability of getting exactly 2 heads (successes) in 3 coin flips is almost 0.40. The probability of 0 heads (no successes) is the same as the probability of 3 heads (all successes).



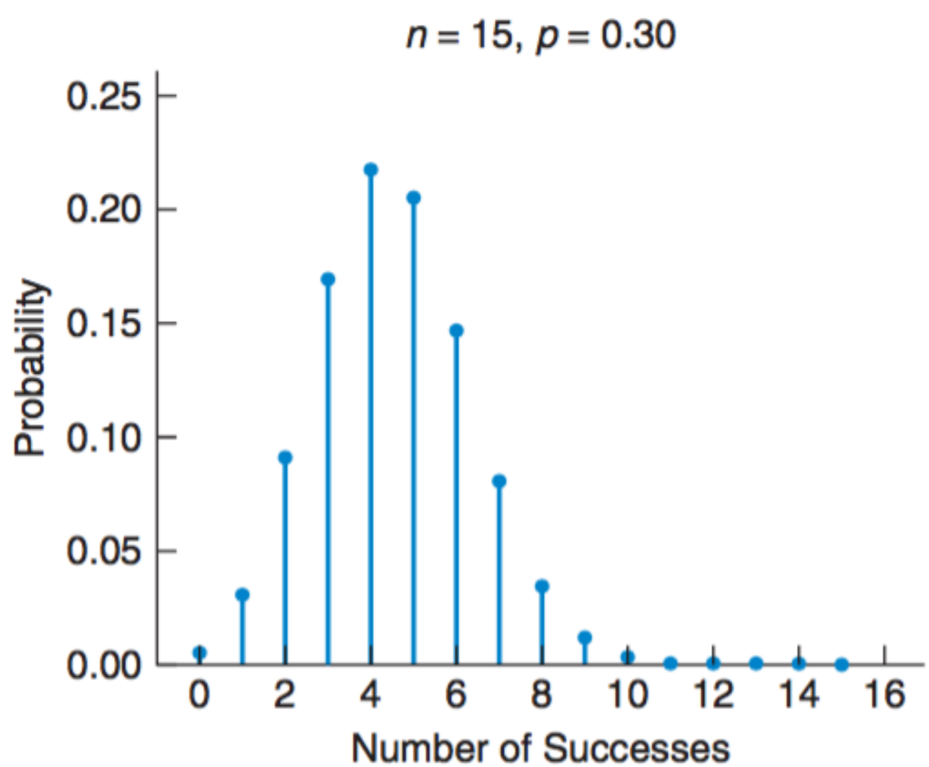
Visualizing the Binomial Distribution

- Now we have n=15 and p=0.5.
- The distribution still looks symmetric because the chance of success is the same as the chance of failure (50/50).



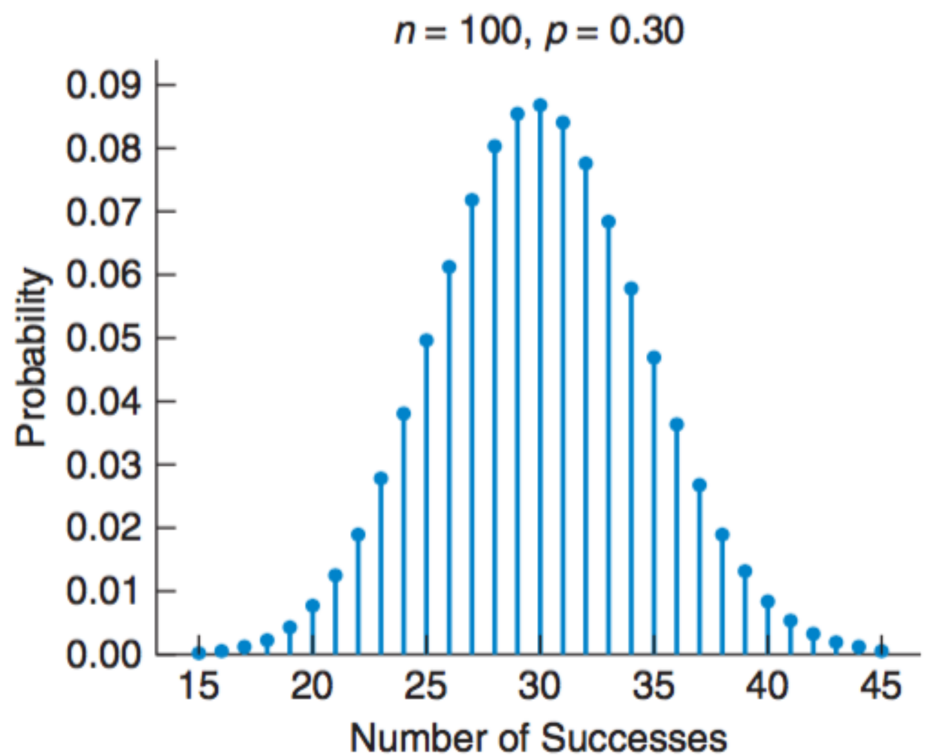
Visualizing the Binomial Distribution

- What happens when we change the probability of success? n=15, p=0.3. Now the chances of success (heads) are lower hence it is more likely to get a lower number of successes.
- The distribution is right-skewed.



Visualizing the Binomial Distribution

- However, if we increased the number of trials to 100, the distribution would start to look more symmetric.
- Binomial distributions have an interesting property that if the number of trials is large enough, the distributions are symmetric, even if p is closer to 0 or 1.



The Binomial Model $b(n, p, x)$

- Let's say you toss a coin 10 times. If each side is equally likely, what is the probability of getting 4 heads?
- All the properties for the binomial distributions are met so we would represent this with $b(10, 0.50, 4)$
- The easiest and most accurate way to find binomial probabilities is to use technology such as calculators or software.

The Binomial Model $b(n, p, x)$

- We can also use a table, such as the one available for you in Appendix A of the textbook.
- This table lists binomial probabilities for values of n between 2 and 15 and for several different values of p.

Table 3: Binomial Probabilities

		p													
n	*x*	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	*x*
2	0	.980	.902	.810	.640	.490	.360	.250	.160	.090	.040	.010	.002	0+	0
	1	.020	.095	.180	.320	.420	.480	.500	.480	.420	.320	.180	.095	.020	1.
	2	0+	.002	.010	.040	.090	.160	.250	.360	.490	.640	.810	.902	.980	2
3	0	.970	.857	.729	.512	.343	.216	.125	.064	.027	.008	.001	0+	0+	0
	1	.029	.135	.243	.384	.441	.432	.375	.288	.189	.096	.027	.007	0+	1
	2	0+	.007	.027	.096	.189	.288	.375	.432	.441	.384	.243	.135	.029	2
	3	0+	0+	.001	.008	.027	.064	.125	.216	.343	.512	.729	.857	.970	3
4	0	.961	.815	.656	.410	.240	.130	.062	.026	.008	.002	0+	0+	0+	0
	1	.039	.171	.292	.410	.412	.346	.250	.154	.076	.026	.004	0+	0+	1
	2	.001	.014	.049	.154	.265	.346	.375	.346	.265	.154	.049	.014	.001	2
	3	0+	0+	.004	.026	.076	.154	.250	.346	.412	.410	.292	.171	.039	3
	4	0+	0+	0+	.002	.008	.026	.062	.130	.240	.410	.656	.815	.961	4

Example

30% of released prisoners return to prison within three years of their release. Suppose a prison in Texas released 15 prisoners.

- What is the probability that exactly 8 out of 15 will end up back in prison within three years?

Binomial Probabilities (*continued*)

		p										
n	*x*	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90
15	0	.860	.463	.206	.035	.005	0+	0+	0+	0+	0+	0+
	1	.130	.366	.343	.132	.031	.005	0+	0+	0+	0+	0+
	2	.009	.135	.267	.231	.092	.022	.003	0+	0+	0+	0+
	3	0+	.031	.129	.250	.170	.063	.014	.002	0+	0+	0+
	4	0+	.005	.043	.188	.219	.127	.042	.007	.001	0+	0+
	5	0+	.001	.010	.103	.206	.186	.092	.024	.003	0+	0+
	6	0+	0+	.002	.043	.147	.207	.153	.061	.012	.001	0+
	7	0+	0+	0+	.014	.081	.177	.196	.118	.035	.003	0+
	8	0+	0+	0+	.003	.035	.118	.196	.177	.081	.014	0+
	9	0+	0+	0+	.001	.012	.061	.153	.207	.147	.043	.002

Example

30% of released prisoners return to prison within three years of their release. Suppose a prison in Texas released 15 prisoners.

- What is the probability that exactly 8 out of 15 will end up back in prison within three years? The probability is 3.5%.

$$b(15, 0.30, 8) = .035$$

Binomial Probabilities (*continued*)

		p										
n	*x*	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90
15	0	.860	.463	.206	.035	.005	0+	0+	0+	0+	0+	0+
	1	.130	.366	.343	.132	.031	.005	0+	0+	0+	0+	0+
	2	.009	.135	.267	.231	.092	.022	.003	0+	0+	0+	0+
	3	0+	.031	.129	.250	.170	.063	.014	.002	0+	0+	0+
	4	0+	.005	.043	.188	.219	.127	.042	.007	.001	0+	0+
	5	0+	.001	.010	.103	.206	.186	.092	.024	.003	0+	0+
	6	0+	0+	.002	.043	.147	.207	.153	.061	.012	.001	0+
	7	0+	0+	0+	.014	.081	.177	.196	.118	.035	.003	0+
	8	0+	0+	0+	.003	.035	.118	.196	.177	.081	.014	0+
	9	0+	0+	0+	.001	.012	.061	.153	.207	.147	.043	.002

Example

Clicker!

We draw 10 cards at random from a large deck of cards which contains an equal number of 5 unique shapes. We ask a person to guess which card has been drawn.

- What is the probability of getting exactly 5 correct answers if the person is simply guessing?

  A. 0

  B. 0.026

  C. 0.246

		p													
n	*x*	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	*x*
10	0	.904	.599	.349	.107	.028	.006	.001	0+	0+	0+	0+	0+	0+	0
	1	.091	.315	.387	.268	.121	.040	.010	.002	0+	0+	0+	0+	0+	1
	2	.004	.075	.194	.302	.233	.121	.044	.011	.001	0+	0+	0+	0+	2
	3	0+	.010	.057	.201	.267	.215	.117	.042	.009	.001	0+	0+	0+	3
	4	0+	.001	.011	.088	.200	.251	.205	.111	.037	.006	0+	0+	0+	4
	5	0+	0+	.001	.026	.103	.201	.246	.201	.103	.026	.001	0+	0+	5
	6	0+	0+	0+	.006	.037	.111	.205	.251	.200	.088	.011	.001	0+	6
	7	0+	0+	0+	.001	.009	.042	.117	.215	.267	.201	.057	.010	0+	7
	8	0+	0+	0+	0+	.001	.011	.044	.121	.233	.302	.194	.075	.004	8
	9	0+	0+	0+	0+	0+	.002	.010	.040	.121	.268	.387	.315	.091	9
	10	0+	0+	0+	0+	0+	0+	.001	.006	.028	.107	.349	.599	.904	10

Example

- What is the probability of getting exactly 5 correct answers if the person is simply guessing? The probability is 0.026 or 2.6%.

$$b(10, 0.20, 5)$$

		p													
n	**x**	**.01**	**.05**	**.10**	**.20**	**.30**	**.40**	**.50**	**.60**	**.70**	**.80**	**.90**	**.95**	**.99**	**x**
10	0	.904	.599	.349	.107	.028	.006	.001	0+	0+	0+	0+	0+	0+	0
	1	.091	.315	.387	.268	.121	.040	.010	.002	0+	0+	0+	0+	0+	1
	2	.004	.075	.194	.302	.233	.121	.044	.011	.001	0+	0+	0+	0+	2
	3	0+	.010	.057	.201	.267	.215	.117	.042	.009	.001	0+	0+	0+	3
	4	0+	.001	.011	.088	.200	.251	.205	.111	.037	.006	0+	0+	0+	4
	5	0+	0+	.001	.026	.103	.201	.246	.201	.103	.026	.001	0+	0+	5
	6	0+	0+	0+	.006	.037	.111	.205	.251	.200	.088	.011	.001	0+	6
	7	0+	0+	0+	.001	.009	.042	.117	.215	.267	.201	.057	.010	0+	7
	8	0+	0+	0+	0+	.001	.011	.044	.121	.233	.302	.194	.075	.004	8
	9	0+	0+	0+	0+	0+	.002	.010	.040	.121	.268	.387	.315	.091	9
	10	0+	0+	0+	0+	0+	0+	.001	.006	.028	.107	.349	.599	.904	10

Example

We draw 10 cards at random from a large deck of cards which contains an equal number of 5 unique shapes. We ask a person to guess which card has been drawn.

- What is the probability of getting 5 or more of the cards correct out of 10 trials?

		p													
n	x	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	x
10	0	.904	.599	.349	.107	.028	.006	.001	0+	0+	0+	0+	0+	0+	0
	1	.091	.315	.387	.268	.121	.040	.010	.002	0+	0+	0+	0+	0+	1
	2	.004	.075	.194	.302	.233	.121	.044	.011	.001	0+	0+	0+	0+	2
	3	0+	.010	.057	.201	.267	.215	.117	.042	.009	.001	0+	0+	0+	3
	4	0+	.001	.011	.088	.200	.251	.205	.111	.037	.006	0+	0+	0+	4
	5	0+	0+	.001	.026	.103	.201	.246	.201	.103	.026	.001	0+	0+	5
	6	0+	0+	0+	.006	.037	.111	.205	.251	.200	.088	.011	.001	0+	6
	7	0+	0+	0+	.001	.009	.042	.117	.215	.267	.201	.057	.010	0+	7
	8	0+	0+	0+	0+	.001	.011	.044	.121	.233	.302	.194	.075	.004	8
	9	0+	0+	0+	0+	0+	.002	.010	.040	.121	.268	.387	.315	.091	9
	10	0+	0+	0+	0+	0+	0+	.001	.006	.028	.107	.349	.599	.904	10

Example

- What is the probability of getting 5 or more of the cards correct out of 10 trials? The probability is 0.033 or 3.3%.

$$b(10,0.20,5)+b(10,0.20,6)+b(10,0.20,7)+b(10,0.20,8)+b(10,0.20,9)+b(10,0.20,10)$$
$$=0.026+.006+.001+0+0+0=0.033$$

		p													
n	*x*	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	*x*
10	0	.904	.599	.349	.107	.028	.006	.001	0+	0+	0+	0+	0+	0+	0
	1	.091	.315	.387	.268	.121	.040	.010	.002	0+	0+	0+	0+	0+	1
	2	.004	.075	.194	.302	.233	.121	.044	.011	.001	0+	0+	0+	0+	2
	3	0+	.010	.057	.201	.267	.215	.117	.042	.009	.001	0+	0+	0+	3
	4	0+	.001	.011	.088	.200	.251	.205	.111	.037	.006	0+	0+	0+	4
	5	0+	0+	.001	.026	.103	.201	.246	.201	.103	.026	.001	0+	0+	5
	6	0+	0+	0+	.006	.037	.111	.205	.251	.200	.088	.011	.001	0+	6
	7	0+	0+	0+	.001	.009	.042	.117	.215	.267	.201	.057	.010	0+	7
	8	0+	0+	0+	0+	.001	.011	.044	.121	.233	.302	.194	.075	.004	8
	9	0+	0+	0+	0+	0+	.002	.010	.040	.121	.268	.387	.315	.091	9
	10	0+	0+	0+	0+	0+	0+	.001	.006	.028	.107	.349	.599	.904	10

Example

Clicker!

We draw 10 cards at random from a large deck of cards which contains an equal number of 5 unique shapes. We ask a person to guess which card has been drawn.

- What is the probability of getting fewer than 5 of the cards correct out of 10 trials?

    A. 0.033

    B. 0.967

    C. 0.026

		p													
n	x	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	x
10	0	.904	.599	.349	.107	.028	.006	.001	0+	0+	0+	0+	0+	0+	0
	1	.091	.315	.387	.268	.121	.040	.010	.002	0+	0+	0+	0+	0+	1
	2	.004	.075	.194	.302	.233	.121	.044	.011	.001	0+	0+	0+	0+	2
	3	0+	.010	.057	.201	.267	.215	.117	.042	.009	.001	0+	0+	0+	3
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	8	0+	0+	0+	0+	.001	.011	.044	.121	.233	.302	.194	.075	.004	8
	9	0+	0+	0+	0+	0+	.002	.010	.040	.121	.268	.387	.315	.091	9
	10	0+	0+	0+	0+	0+	0+	.001	.006	.028	.107	.349	.599	.904	10

Example

- What is the probability of getting fewer than 5 of the cards correct out of 10 trials? The probability is 1-0.033=0.967 or 96.7%.

$$b(10,0.20,0)+b(10,0.20,1)+b(10,0.20,2)+b(10,0.20,3)+b(10,0.20,4)$$
$$=0.107+.268+.302+.201+0.088$$

		p													
n	x	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	x
10	0	.904	.599	.349	.107	.028	.006	.001	0+	0+	0+	0+	0+	0+	0
	1	.091	.315	.387	.268	.121	.040	.010	.002	0+	0+	0+	0+	0+	1
	2	.004	.075	.194	.302	.233	.121	.044	.011	.001	0+	0+	0+	0+	2
	3	0+	.010	.057	.201	.267	.215	.117	.042	.009	.001	0+	0+	0+	3
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	8	0+	0+	0+	0+	.001	.011	.044	.121	.233	.302	.194	.075	.004	8
	9	0+	0+	0+	0+	0+	.002	.010	.040	.121	.268	.387	.315	.091	9
	10	0+	0+	0+	0+	0+	0+	.001	.006	.028	.107	.349	.599	.904	10

Center and Spread of Binomial

- The mean of the binomial probability distribution can be found with the following formula:

$$\mu = np$$

- The mean number of successes in a binomial experiment is the number of trials times the probability of a success. This is sometimes referred to as the expected value.
- The standard deviation of the binomial probability distribution which measures the spread is:

$$\sigma = \sqrt{np\left(1-p\right)}$$

- We would interpret the standard deviation by saying “give or take”.

Example

During his last season with the Cleveland Cavaliers, LeBron James had a free throw percentage of 65%. Assume the free throw shots are independent.

- If LeBron James has 600 free throws in an upcoming season, how many would you expect him to make, give or take how many?

Example

During his last season with the Cleveland Cavaliers, LeBron James had a free throw percentage of 65%. Assume the free throw shots are independent.

- If LeBron James has 600 free throws in an upcoming season, how many would you expect him to make, give or take how many?

Find the mean. $\mu = np = 600 \times 0.65 = 390$

Find the standard deviation. $\sigma = \sqrt{np(1-p)} = \sqrt{600 \times 0.65(1-0.65)} = 11.68$

We would expect him to make about 390 free throws, give or take about 12.

$$390 + 12 = 402$$

$$390 - 12 = 378$$

Hence, we would expect him to make between 378 and 402 free throws.