Experiment 0: Sensor Calibration and Linear Regression

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(2.)
We are given that:

$$\Delta v = \Delta v_{best} \pm \partial \Delta v$$

$$\Delta t = \Delta t_{best} \pm \partial \Delta t$$

$$a = \frac{\Delta v}{\Delta t}$$

To solve for the equation of the uncertainty of the acceleration, we follow the process outlined in Section (ii.2) in the lab manual. We start with Equation (ii.14) from the lab manual which states that for a function $f = f(x, \dots, z)$, the uncertainties $\partial x, \dots, \partial z$ can be used to determine the uncertainty ∂f of the function f.

$$\partial f = \left.\sqrt{\left(\frac{\partial f}{\partial x}\partial x\right)^2 + \cdots + \left(\frac{\partial f}{\partial z}\partial z\right)^2}\,\right|_{x_{best}, \dots, z_{best}}$$

Now we can substitute the values Δv and Δt into this equation in order to solve for the uncertainty ∂a of $a = a(\Delta v, \Delta t)$:

$$\partial a = \sqrt{\left(\frac{\partial a}{\partial \Delta v}\partial \Delta v\right)^2 + \left(\frac{\partial a}{\partial \Delta t}\partial \Delta t\right)^2}$$

We recognize that the first term in each set of parentheses is the partial derivative of a with respect to Δv and Δt respectively. Solving each partial derivative, we get the following identities:

$$\frac{\partial a}{\partial \Delta v} = \frac{\partial}{\partial \Delta v}\left(\frac{\Delta v}{\Delta t}\right) = \frac{1}{\Delta t}$$

$$\frac{\partial a}{\partial \Delta t} = \frac{\partial}{\partial \Delta t}\left(\frac{\Delta v}{\Delta t}\right) = \frac{-\Delta v}{(\Delta t)^2}$$

We can double check our identities against equations (ii.15) and (ii.16) in the lab manual. They match, so we can move on. Plugging these new identities back in, we can solve for ∂a.

$$\partial a = \sqrt{\left(\frac{1}{\Delta t}\partial \Delta v\right)^2 + \left(\frac{-\Delta v}{(\Delta t)^2}\partial \Delta t\right)^2}$$

If we multiply the first term under the radical by a factor of 1 = $\Delta v^2/\Delta v^2$, then we can simplify the equation and plug in the values Δv_{best} and Δt_{best} for Δv and Δt respectively. We arrive at a slightly simplified version of Equation (ii.17) from the lab manual.

$$\partial a = \left|\frac{\Delta v}{\Delta t}\right| \sqrt{\frac{\partial \Delta v^2}{\Delta v^2} + \frac{\partial \Delta t^2}{\Delta t^2}}\Bigg|_{\Delta v_{best},\ \Delta t_{best}}$$

The values that were pulled out of the radical are positive because no real result that comes out of a radical can be negative (at least for values we are taking the square root of). As our final result, we have the following equation as the uncertainty for acceleration:

$$\partial a = \left|\frac{\Delta v_{best}}{\Delta t_{best}}\right| \sqrt{\frac{\partial \Delta v_{best}^2}{\Delta v_{best}^2} + \frac{\partial \Delta t_{best}^2}{\Delta t_{best}^2}}$$

(3.)
The Capstone can display up to 16 digits. The actual measurement precision depends entirely on the sensor precision. In this example, since the precision of the sensor is 4 digits, that means that the 5th through 10th decimal place are imprecise and should not be included in measurements. Also, the values will fluctuate heavily beyond the 4th decimal because the sensor is doing more guesswork than actual measurement for these values since it is not precise past the 4th decimal. However, in the alternate case presented in the question where we eliminate fluctuation entirely, we might be ignoring values that are being read precisely. These values may fluctuate despite being precise because of environmental factors. It would be a poor choice to eliminate precision that the sensor can provide.

(4.)

Force Sensor Data

Mass (g)	Force (N)	Sensor Voltage (V)
0.0	0.00	0.0014
49.9	0.49	-0.0773
100.0	0.98	-0.1556
150.0	1.47	-0.2333
199.5	1.96	-0.3112
250.1	2.45	-0.3888
300.2	2.94	-0.4668
349.9	3.43	-0.5441
400.1	3.92	-0.6220

Figure 1: This table shows the data that was collected from the force sensor during this experiment. The mass values were calculated by placing combinations of various masses on a scale. To convert to force, we multiplied by a factor of 9.80 (acceleration due to gravity) as well as a factor of 0.001 which comes from converting grams into kilograms (according to the lab manual). Finally, the sensor voltage were the readings we took from the force sensor.

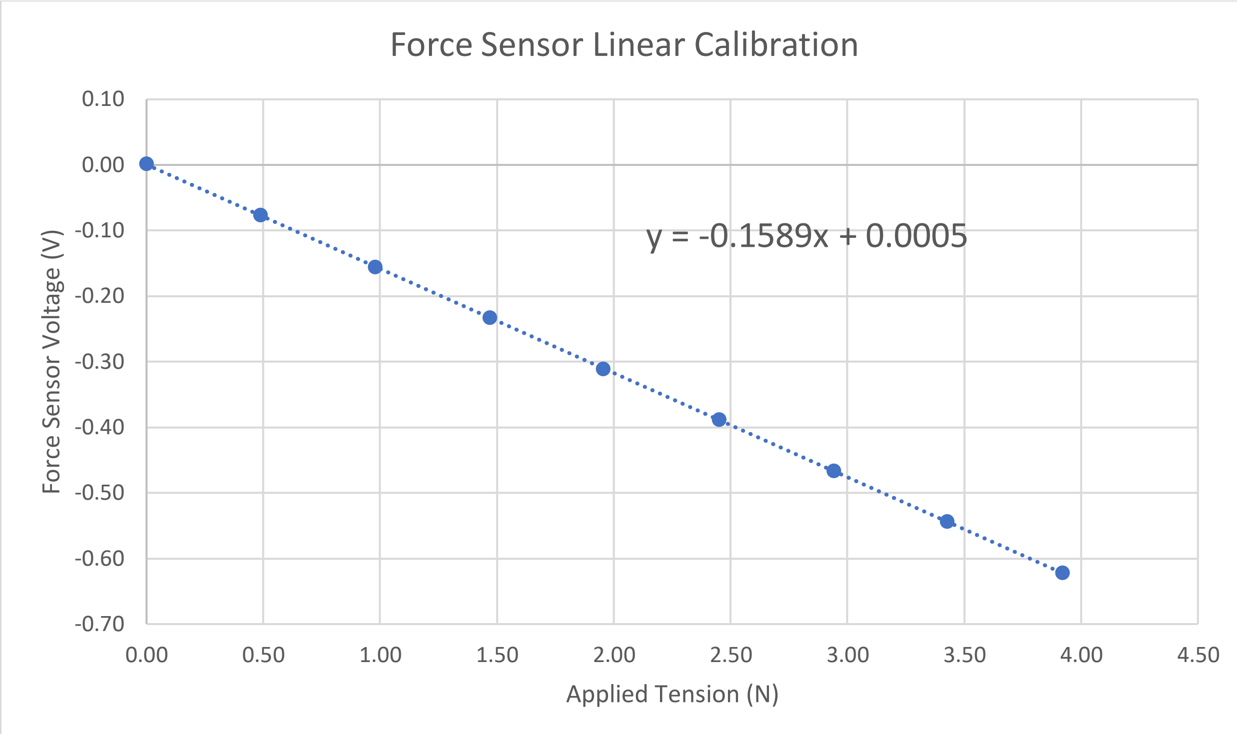

Figure 2: This is the plot of the data presented in Figure 1. The trendline and equation were generated by Microsoft Excel. The trend shows that the force sensor outputs values that are more and more negative as the applied tension increases.

(5.)
I performed a linear regression data analysis as well as generated a trendline with Microsoft Excel on the data that I recorded. The equation generated was $y = -0.1589x + 0.0005$ which is of the expected form $V = aF + b$. The value of a has units V/N and is the slope of the curve. The value of b is the intercept of the curve and indicates a skew of the data. The data analysis tool also produced the standard error for each of the values of the coefficients a and b:

Variables	*Coefficients*	*Standard Error*
Intercept	-0.0001128	0.00032523
X Variable: Force	-0.1586544	0.00013144

According to the values from the data analysis, the values of the two coefficients (in mV/N) are:

$$a = -158.7 \pm 0.1\, mV/N$$

$$b = -0.1 \pm 0.3\, mV$$

It is odd that these values are different from the coefficients in the trendline equation because I expected the trendline to be generated using the values from the regression data analysis. Nonetheless, our intercept from the plot and the data analysis tool are both nonzero. This indicates that the taring procedure did not entirely zero out the scale. Our data was skewed to be less negative than it would be if the scale was completely zeroed. Also, I noticed that as time went on, the output value of the sensor would slowly increase even with no weight attached to it. This may have also skewed the data depending on how long it took before we hung the masses on the sensor, and it could also explain the discrepancy between the values of the coefficients for the trendline and the data analysis tool results.

(6.)
We start with the equation for voltage and move the variables around to solve for force:

$$V = aF + b$$

$$V - b = aF$$

$$F = \frac{V - b}{a}$$

$$F = \frac{V}{a} - \frac{b}{a}$$

Our new coefficients are therefore $\frac{1}{a}$ for V and $\frac{-b}{a}$ for the intercept. We will label these new values as c and d, which correspond to a and b in the original equation respectively. I will use the values from the regression data analysis result for the calculation. The values of c and d are:

$$c = \frac{1}{-0.1587} = -6.301\ N/V$$

$$d = \frac{-0.0001}{-0.1587} = 0.0006\ N$$

We now use Equation (ii.23) from the lab manual to determine the uncertainties for c and d because they are both ratios of different values. We attain the following equations:

$$\partial r = \frac{1}{|a_{best}|} \sqrt{\left(\frac{\partial a}{|a_{best}|}\right)^2}$$

$$\partial s = \left|\frac{-b_{best}}{a_{best}}\right| \sqrt{\left(\frac{\partial a}{|a_{best}|}\right)^2 + \left(\frac{\partial b}{|b_{best}|}\right)^2}$$

When we plug in the values that we have for $a, b, \partial a,$ and ∂b from part (5), we get the following result:

$$\partial c = 0.0040\ N/V$$

$$\partial d = 0.0018\ N$$

Combining these values together, we find that:

$$c = -6.301 \pm 0.0040 \ N/V$$
$$d = 0.0006 \pm 0.0018 \ N$$

Finally, we put these values for c and d back into the form $F = cV + d$. Note that the units of each coefficient are excluded for readability.

$$F = (-6.301 \pm 0.0040)V + (0.0006 \pm 0.0018)$$

(7.)
It is possible for Frankie Fivefingers and Avril Armstrong to have received different final letter grades with the same average numerical score because of the way that Physics 4AL is curved within each section. A curve assigns grades to students based on their relative performance against other students. Since Frankie received a B+ with the numerical score of 84, this means that the average numerical score across all students in his section is lower than the numerical average in Avril's section. Since the average is lower in Frankie's section, he performed better relative to most students in his class, and thus he received a better grade than Avril did.