Computational Methods in Astronomy, IUCAA, 2020

Module D1 (NS): Gas Dynamics

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1. Solve the 1D Burgers' equation using the PENCIL CODE:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \nu\frac{\partial^2 v}{\partial x^2}\,.$$

(a) Here $v(x,t)$ is the velocity field, and ν is the viscosity. Consider a domain $x \in [x_1, x_2]$ with $x_1 = -20$ and $x_2 = +20$. Set the flow boundary conditions on the leftmost and rightmost points as, $v_1 = +1.0$ and $v_2 = -1.0$, respectively, at $t = 0$.

(b) Come up with a criteria to define the thickness (Δ) of the shock. Empirically determine Δ for, at least, four different choices of ν; show the ν-dependence of Δ in a plot; and make a qualitative comparison with the theory of Burgers' shock.

(c) What would happen if we slightly change the flow boundary condition such that $v_1 = +1.05$ and $v_2 = -1.0$ at $t = 0$, at a fixed value of ν, say, 0.4, while keeping everything else as same.
(Note: Try to guess the solution before simulating this case.)

(d) Perform a new simulation with parameters as in (c) above, and show the solution (v) as a contour plot in a space-time (x-t) diagram.
(Note: Run this for a relatively longer time to see what happens at late time, and make a comment about this.)