Helical Dynamo

Reference: A. Brandenburg and K. Subramanian, Physics Reports, 417, 2005

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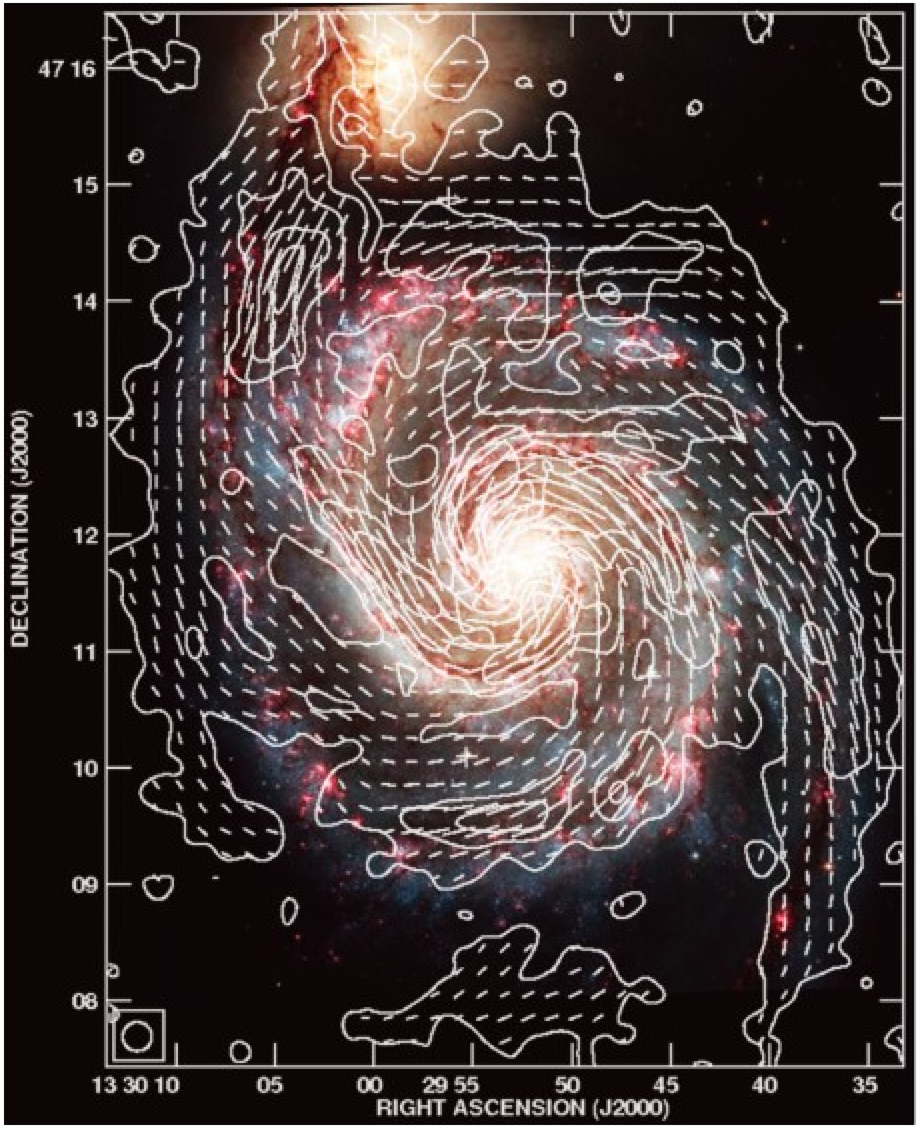
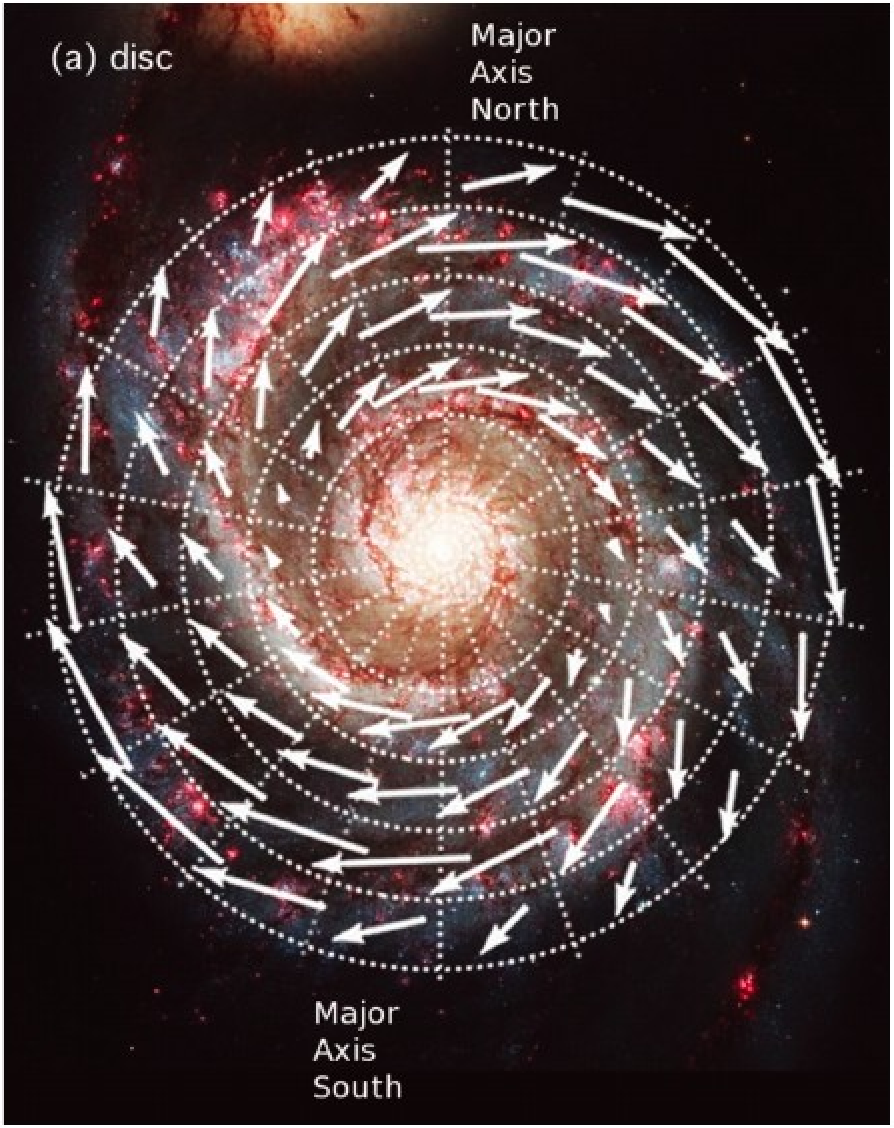
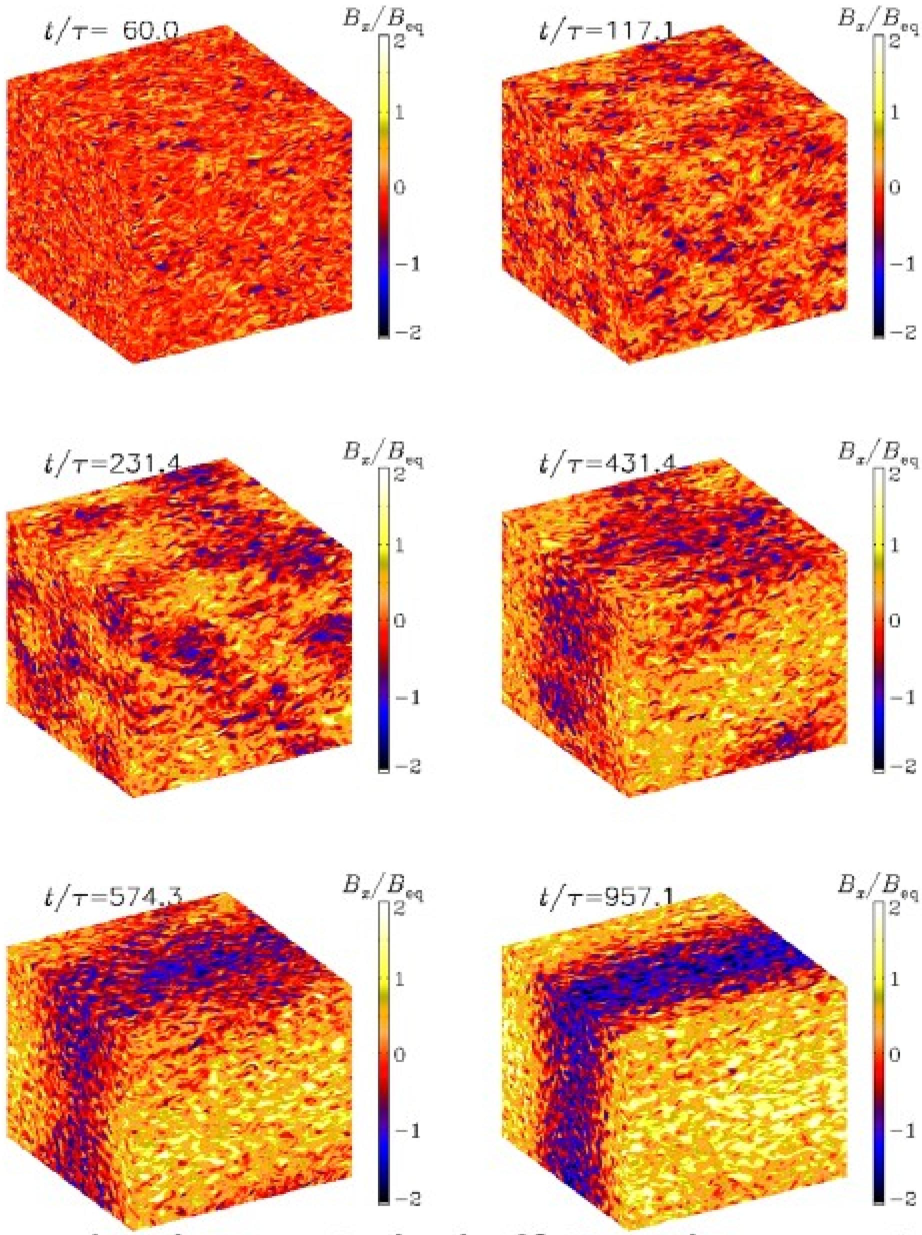
Large-scale magnetic fields: observations





Fletcher, Beck, Shukurov, Berkhuijsen & Horellou 2011

Dynamo generated LS fields: simulations



- e-folding time of SS field ~ *l/u* of smallest eddies (<< 10 Myr)
- e-folding time of LS dynamo > $1/\Omega$
- => expect SS field to saturate while LS field is still very small and growing exponentially

Brandenburg, Sokoloff & Subramanian 2012

Basics of Magnetohydrodynamics (MHD)

Maxwell's equations (cgs units):

$$\frac{1}{c}\frac{\partial \boldsymbol{B}'}{\partial t} = -\boldsymbol{\nabla}\times\boldsymbol{E}' \; ; \qquad \boldsymbol{\nabla}\cdot\boldsymbol{B}' = 0$$

$$\frac{1}{c}\frac{\partial \boldsymbol{E}'}{\partial t} = \boldsymbol{\nabla}\times\boldsymbol{B}' - \frac{4\pi}{c}\boldsymbol{J}' \; ; \qquad \boldsymbol{\nabla}\cdot\boldsymbol{E}' = 4\pi\rho_e$$

Standard Ohm's law in a fixed frame of reference:

$$\boldsymbol{J}' = \sigma\left(\boldsymbol{E}' + \frac{\boldsymbol{v}'\times\boldsymbol{B}'}{c}\right)$$

Eliminating $\boldsymbol{J}'$ and defining resistivity as $\eta = c^2/4\pi\sigma$, we write

$$\frac{\eta}{c^2}\frac{\partial \boldsymbol{E}'}{\partial t} + \boldsymbol{E}' = \frac{\eta}{c}\boldsymbol{\nabla}\times\boldsymbol{B}' - \frac{\boldsymbol{v}'\times\boldsymbol{B}'}{c}$$

Basics of MHD: the induction equation

Faraday time–scale: $\tau_f = \eta / c^2 \sim 10^{-14}\, T_4^{-3/2}\, s$ for ionized plasma

Thus the displacement current term may be safely ignored, and we get

$$\boldsymbol{E}' = \frac{\eta}{c} \boldsymbol{\nabla} \times \boldsymbol{B}' - \frac{\boldsymbol{v}' \times \boldsymbol{B}'}{c}$$

Using this in the Maxwell equation for evolution of magnetic field

$$\frac{\partial \boldsymbol{B}'}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v}' \times \boldsymbol{B}' - \eta \boldsymbol{\nabla} \times \boldsymbol{B}')$$

which is known as the *induction equation*. For homogeneous η, we may write

$$\frac{\partial \boldsymbol{B}'}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v}' \times \boldsymbol{B}') + \eta \nabla^2 \boldsymbol{B}'$$

MHD: some comments

Non-relativistic limit of Maxwell's equations

Plasma is usually modelled as a single fluid which is highly conducting

Lorentz force: $\boldsymbol{F}'_L = \rho_e \boldsymbol{E}' + \boldsymbol{J}' \times \boldsymbol{B}'/c$; $\rho_e = (en_p - en_e)$; $\boldsymbol{J}' = (en_p \boldsymbol{v}'_p - en_e \boldsymbol{v}'_e)$

Electric part of the Lorentz force is neglected as compared to the magnetic part

$\eta \to 0$ in induction equation $\Rightarrow$ Flux freezing

$\boldsymbol{v}' = 0$ in induction equation $\Rightarrow$ Decay due to diffusion

$\boldsymbol{B}' = \mathbf{0}$ is a valid solution of the induction equation

Dynamo action: Conversion of kinetic energy into magnetic energy *"without any electric current at infinity"*

The flow $\boldsymbol{v}'$ may act as a dynamo

Basics of mean field theory

Reynolds averaging: $\boldsymbol{B}' = \boldsymbol{B} + \boldsymbol{b}$; $\boldsymbol{v}' = \boldsymbol{V} + \boldsymbol{v}$; *(Total = Mean + Fluctuation)*

$$\langle \boldsymbol{B}' \rangle = \boldsymbol{B} \; ; \quad \langle \boldsymbol{b} \rangle = 0 \; ; \quad \langle\langle \boldsymbol{B}' \rangle\rangle = \boldsymbol{B} \; ; \quad \langle \boldsymbol{B}'_1 + \boldsymbol{B}'_2 \rangle = \langle \boldsymbol{B}'_1 \rangle + \langle \boldsymbol{B}'_2 \rangle$$

$$\langle \langle \boldsymbol{B}'_1 \rangle \langle \boldsymbol{B}'_2 \rangle \rangle = \boldsymbol{B}_1 \boldsymbol{B}_2 \; ; \quad \langle \boldsymbol{B} \boldsymbol{v} \rangle = 0$$

$$\left\langle \frac{\partial \boldsymbol{B}'}{\partial t} \right\rangle = \frac{\partial \boldsymbol{B}}{\partial t} \; ; \quad \left\langle \frac{\partial \boldsymbol{B}'}{\partial X_i} \right\rangle = \frac{\partial \boldsymbol{B}}{\partial X_i}$$

Apply these averaging techniques to induction equation and write *two* equations

for the mean and the fluctuating fields

Mean field theory: Turbulent dynamo

Work out:

$$\langle \boldsymbol{v}' \times \boldsymbol{B}' \rangle = \langle (\boldsymbol{V} + \boldsymbol{v}) \times (\boldsymbol{B} + \boldsymbol{b}) \rangle = \boldsymbol{V} \times \boldsymbol{B} + \langle \boldsymbol{v} \times \boldsymbol{b} \rangle$$

Mean field (large–scale) equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{V} \times \boldsymbol{B} + \boldsymbol{\mathcal{E}} - \eta \boldsymbol{\nabla} \times \boldsymbol{B})$$

Mean EMF $\qquad \boldsymbol{\mathcal{E}} = \langle \boldsymbol{v} \times \boldsymbol{b} \rangle$

Fluctuating field (small–scale) equation

$$\frac{\partial \boldsymbol{b}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{V} \times \boldsymbol{b} + \boldsymbol{v} \times \boldsymbol{B} - \eta \boldsymbol{\nabla} \times \boldsymbol{b}) + \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{b} - \boldsymbol{\mathcal{E}})$$

The last term is Nonlinear in fluctuations !

First order smoothing approximation (FOSA)

As we saw, finding $\boldsymbol{\mathcal{E}}$ poses a *closure problem*

Neglect the nonlinear term in the equation for $\boldsymbol{b}$–field *(valid for low* Rm *or small correlation time)*

This approximation is called FOSA !

Mean EMF in kinematic limit: $\boldsymbol{\mathcal{E}} = \alpha\, \boldsymbol{B} \; - \; \eta_t \, \boldsymbol{\nabla} \times \boldsymbol{B}$

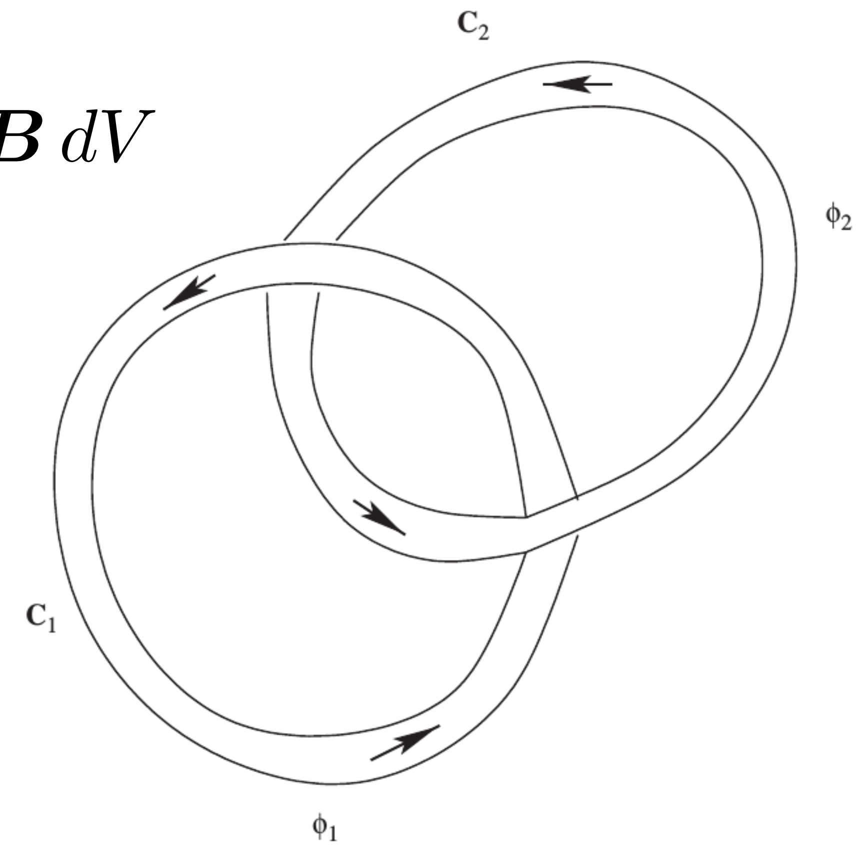
$$\alpha \approx -\frac{1}{3}\,\tau_c \langle \boldsymbol{v} \cdot (\boldsymbol{\nabla} \times \boldsymbol{v}) \rangle$$ *Depends on kinetic helicity!*

$$\eta_t \approx \frac{1}{3}\,\tau_c \langle \boldsymbol{v} \cdot \boldsymbol{v} \rangle$$ *Depends on energy density of turbulence!*

Magnetic Helicity: A Topological Invariant

- Definition: $$H = \int_V \boldsymbol{A} \cdot \boldsymbol{B} \, dV$$
- Evolution: $$\frac{dH}{dt} = -2\eta \int_V \boldsymbol{J} \cdot \boldsymbol{B} \, dV$$

$$H = +2\phi_1 \phi_2$$



- Nearly conserved for high Rm systems
- Even with magnetic reconnection events
- Recently tested in solar context
- Effective tracer of origin of magnetic field
- To see, e.g., if B-fields are *bihelical*

- Seehafer 1990
- Brandenburg & Subramanian 2005
- Pariat et al. 2015