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Homework 1

1. $I[T; G] = H(T) - \underbrace{H(T|G)}_{\text{the target and the guess are independent of eachother, so } H(T|G) = H(T).} = H(T) - H(T) = 0$

2. $I[T; G|P] = \underbrace{H(T|P)}_{H(T) \text{ since } T \,\&\, P \text{ are independent}} - H(T|G, P)$

$= H(T) - H(T|G, P)$

$H(T) > H(T|G, P)$ because the guess and the pattern would limit the number of possible target words, giving it less entropy than knowing nothing.

So $I[T; G|P] = H(T) - H(T|G, P) > 0$.

3. $H[P|G, T] = 0$ because the pattern is completely determined by the guess and the target. So if we know the guess and target, we can find the pattern.

4. $I[T; P|G] = H(T|G) - H(T|G, P)$

$= H(T) - H(T|G, P)$.

$H(T)$ is not dependent on π. Therefore, finding π that maximizes $I[T; P|G]$ is the same as finding π that minimizes $H(T|G, P)$.

$$5.\ H(T \mid G, P) = \sum_{g \in W} \sum_{r \in P} \sum_{t \in W} P(t, g, r) \log \frac{1}{P(t \mid g, r)}$$

$$= \sum_{g \in W} \sum_{r \in P} \sum_{t \in W} P(r \mid t, g)\, P(t, g) \log \frac{1}{P(t \mid g, r)}$$

$$= \sum_{g \in W} \sum_{t \in W} P(t, g) \log \frac{1}{P(t \mid g, r)}$$

$$= \sum_{g \in W} \sum_{t \in W} P(t)\, P(g) \log \frac{1}{P(t \mid g, r)}$$

$$= \sum_{g \in W} \sum_{t \in W} \frac{1}{|W|} \pi(g) \log \frac{1}{P(t \mid g, r)}$$

$$= \frac{1}{|W|} \sum_{g \in W} \pi(g) \sum_{t \in W} \log \frac{1}{P(t \mid g, r)}$$

$$= \frac{1}{|W|} \sum_{g \in W} \pi(g) \sum_{t \in W} \log \left(\frac{1}{\frac{1}{|W'|}} \right)$$, where $W' = \{ t \in W : f(g, t) = r \}$

$$= \frac{1}{|W|} \sum_{g \in W} \pi(g) \underbrace{\sum_{t \in W} \log(|W'|)}_{\alpha_g}$$

$$= \frac{1}{|W|} \sum_{g \in W} \pi(g) \overbrace{\underbrace{\sum_{r \in P} |W'| \log(|W'|)}_{\text{which is the expression for finding } \alpha \text{ we use in our program}}}^{\alpha_g}$$

6. Let g our guess with the minimum α value. Certainly the π where $\pi(g) = 1$ is an optimal solution because if we put all the weight into g, then the weighted average would be the minimum. So $\sum_{g \in W} \pi(g) \alpha_g$ would be the minimum. Thus $H(T \mid G, P) = \frac{1}{|W|} \sum_{g \in W} \pi(g) \alpha_g$ would be the minimum. Thus the optimal solution $= \frac{1}{|W|} \sum_{g \in W} \pi(g) \alpha_g = \frac{1}{|W|} \alpha_g$.

Now, let S be the set of guesses where all $g \in S$ have the same minimum α value with some distribution π st $\sum_{g \in S} \pi(g) = 1$.

Then for all $g \in W \setminus S$, $P(g) = 0$

So $H(T \mid G, P) = \frac{1}{|W|} \sum_{g \in W} \pi(g) \alpha_g$

because

$$= \frac{1}{|W|} \sum_{g \in S} \pi(g) \alpha_g$$

$$= \frac{1}{|W|} \alpha_g \sum_{g \in S} \pi(g)$$

$= \frac{1}{|W|} \alpha_g$, which is equal to the optimal solution.

Thus, all such π lead to an optimal solution.