



Cryptography Information Security (CSC-407)

Fall 2024 (BSE-7A & 7B)



Basic Terminologies

- Plaintext: the original intelligible message.
- Ciphertext: the coded unintelligible message.
- Enciphering\Encryption: the process of converting plaintext to ciphertext.
- Deciphering\Decryption: the process of restoring plaintext from ciphertext.



Basic Terminologies (Cont.)

- Cryptography: the study of encryption.
- Cryptanalysis: techniques used for deciphering a message without any knowledge of the enciphering details, such as the keys used to perform encryption.
- Cryptology: The field of science that encompasses cryptography and cryptanalysis together.



Cryptographic Systems

 Cryptographic systems are characterized by three dimensions:

Type of operations used for converting plaintext to ciphertext

Substitution

Transposition

Number of keys used

Symmetric, singlekey, secret-key, conventional encryption

Asymmetric, twokey, or public-key encryption The way in which plaintext is processed

Block cipher

Stream cipher



Cryptographic Systems (Cont.)

Type of operations used for transforming plaintext to ciphertext:

- All encryption algorithms are based on two general principles:
 - a. Substitution, in which each element in the plaintext (bit, letter, group of bits or letters) is mapped into another element.
 - b. Transposition, in which elements in the plaintext are rearranged.



Cryptographic Systems (Cont.)

The way in which the plaintext is processed:

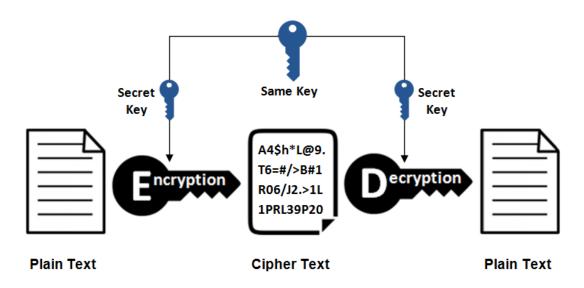
- Block cipher processes the input one block of elements at a time, producing an output block for each input block.
- Stream cipher processes the input elements continuously, producing output one element at a time, as it goes along.



Cryptographic Algorithms

- Cryptographic algorithms can be grouped into:
 - 1. Symmetric-key Algorithms: cryptography algorithms that use the same cryptographic keys for both encryption and decryption.

 Symmetric Encryption

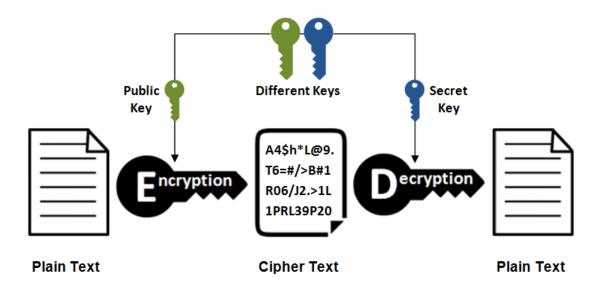




Cryptographic Algorithms (Cont.)

- Cryptographic algorithms can be grouped into:
 - 2. Asymmetric-key Algorithms: cryptography algorithms that uses pairs of keys, i.e. public keys and private keys, to encrypt and decrypt data.

 Asymmetric Encryption





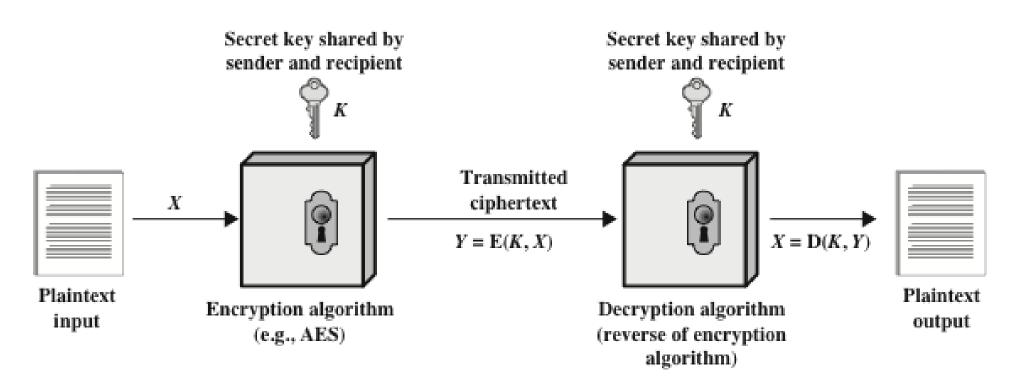
Cryptographic Algorithms (Cont.)

- Symmetric Encryption is used to conceal the contents of blocks or streams of data of any size, including messages, files, encryption keys and passwords.
- Asymmetric Encryption is used to conceal small blocks of data, such as encryption keys and hash function values, which are used in digital signatures.

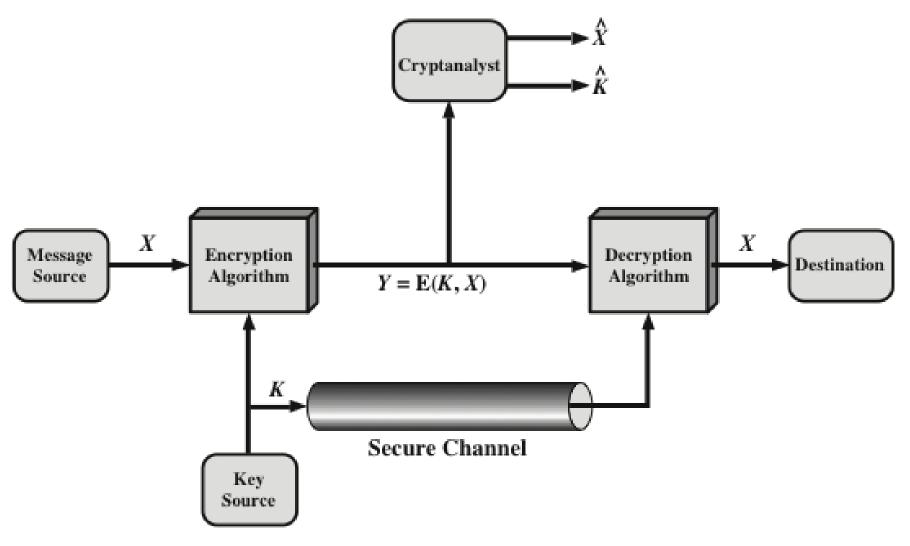
Symmetric Ciphers (Substitution, Transposition)



Symmetric Encryption



A general model for the symmetric encryption.





Properties of secret key in symmetric encryption:

- The **key** is input to **encryption algorithm** along with **plaintext**.
- The **key** is a value independent of the **plaintext** and the **algorithm**.
- The algorithm will produce a different output depending on the specific key being used. Hence, for a given message, two different keys will produce two different ciphertexts.
- The **encryption algorithm** performs various substitutions and transpositions on the **plaintext**, where the exact substitutions and transpositions depends on the **key**.



• Kerckhoff's principle: one should always assume that the adversary knows the encryption/decryption algorithm. The resistance of the cipher to attack must be based only on the

secrecy of the key.





- It is "impractical" to decrypt a message on the basis of ciphertext plus knowledge of encryption/decryption algorithm.
- There is no need to keep the algorithm secret; but only keep the key secret. This feature makes symmetric key feasible for widespread use. Hence, manufacturers can and have developed low-cost chip implementations of data encryption algorithms.
- With the use of symmetric encryption, the principal security problem is maintaining the secrecy of the key.



Cryptanalysis and Brute-Force

- There are two general approaches for attacking a conventional encryption scheme:
 - Cryptanalysis, rely on nature of the algorithm plus some general characteristics of plaintext or plaintext-ciphertext pairs. This attack attempts to deduce a specific plaintext or the key being used.
 - Brute-force attack, the attacker tries every possible key on a piece of ciphertext until an intelligible translation is obtained. *On average, half of all possible keys must be tried to achieve success!*









Cryptanalysis and Brute-Force (Cont.)

• In brute-force attack, the attack is proportional to key size.

Key Size (bits)	Number of Alternative Keys	Time required at 1 decryption/µs	Time required at 10 ⁶ decryptions/µs	
32	$2^{32} = 4.3 \times 10^9$	35.8 minutes	2.15 milliseconds	
56	$2^{56} = 7.2 \times 10^{16}$	1142 years	10.01 hours	
128	$2^{128} = 3.4 \times 10^{38}$	$5.4 \times 10^{24} \text{ years}$	$5.4 \times 10^{18} \text{ years}$	
168	$2^{168} = 3.7 \times 10^{50}$	$5.9 \times 10^{36} \text{ years}$	$5.9 \times 10^{30} \text{years}$	

Substitution Technique



Substitution Technique

- Techniques in which the letters of plaintext are replaced by other letters, numbers or symbols.
- If the plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns.
- Substitution ciphers can be categorized as either monoalphabetic ciphers or polyalphabetic ciphers.



Monoalphabetic Ciphers

- In **monoalphabetic** substitution, the relationship between a symbol in the plaintext to a symbol in the ciphertext is always **one-to-one**.
- Example: the following shows a plaintext and its corresponding ciphertext. The cipher is monoalphabetic because both *l*'s are encrypted as *O*'s.

Plaintext: hello

Ciphertext: KHOOR



Caesar Cipher

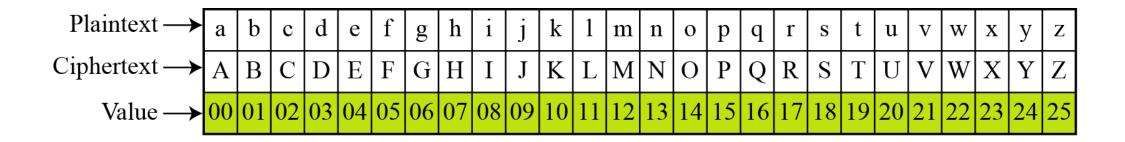
• Caesar cipher is a monoalphabetic cipher that involves replacing each letter of alphabet with that standing 3 places further down the alphabet, where alphabet is wrapped around so the letter following Z is A.

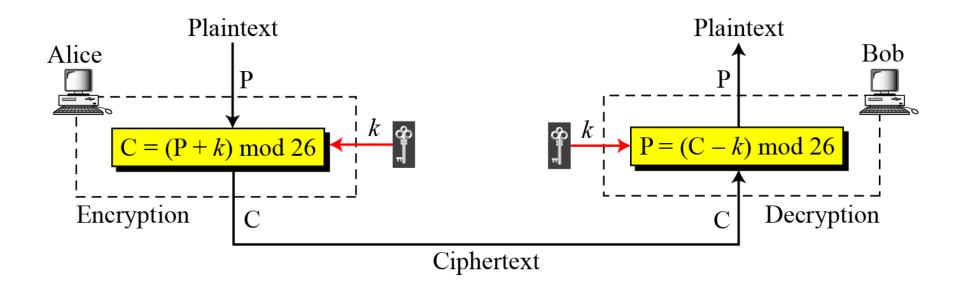
```
plain: meet me after the toga party cipher: PHHW PH DIWHU WKH WRJD SDUWB
```

• The shift may be of any amount, so the general Caesar cipher is called an additive cipher.



Additive Cipher





- The plaintext, ciphertext and key are integers in \mathbb{Z}_{26} .
- The general algorithm is:

$$C = E(k, P) = (P + k) \mod 26$$

• The decryption algorithm is simply:

$$P = D(k, C) = (C - k) \mod 26$$

- Where, k takes on a value in the range 1 to 25).
- With k = 0, ciphertext is same as plaintext. Hence, only 25 keys are useful.



- Example 01: use the additive cipher with k = 15 to encrypt the message "hello".
- Solution:

Plaintext: $h \rightarrow 07$	Encryption: $(07 + 15) \mod 26$	Ciphertext: $22 \rightarrow W$
Plaintext: $e \rightarrow 04$	Encryption: $(04 + 15) \mod 26$	Ciphertext: $19 \rightarrow T$
Plaintext: $1 \rightarrow 11$	Encryption: $(11 + 15) \mod 26$	Ciphertext: $00 \rightarrow A$
Plaintext: $1 \rightarrow 11$	Encryption: $(11 + 15) \mod 26$	Ciphertext: $00 \rightarrow A$
Plaintext: $o \rightarrow 14$	Encryption: $(14 + 15) \mod 26$	Ciphertext: $03 \rightarrow D$



- Example 02: use the additive cipher with k = 15 to decrypt the message "WTAAD".
- Solution:

Ciphertext: $W \rightarrow 22$	Decryption: $(22 - 15) \mod 26$	Plaintext: $07 \rightarrow h$
Ciphertext: $T \rightarrow 19$	Decryption: $(19 - 15) \mod 26$	Plaintext: $04 \rightarrow e$
Ciphertext: A \rightarrow 00	Decryption: $(00-15) \mod 26$	Plaintext: $11 \rightarrow 1$
Ciphertext: A \rightarrow 00	Decryption: $(00 - 15) \mod 26$	Plaintext: $11 \rightarrow 1$
Ciphertext: D \rightarrow 03	Decryption: $(03 - 15) \mod 26$	Plaintext: $14 \rightarrow 0$



- If it is known that a given ciphertext is an Additive cipher, then bruteforce cryptanalysis is easily performed.
- Simply try all the 25 possible keys.
- In this example, the plaintext leaps out as occupying the third line.
- *Note:* with only 25 possible keys, the Caesar cipher is far from secure.

	PHHW	PH	DIWHU	WKH	WRJD	SDUWB
KEY						
1			chvgt		_	
2	nffu	nf	bgufs	uif	uphb	qbsuz
3	meet	me	after	the	toga	party
4	ldds	ld	zesdq	sgd	snfz	ozqsx
5	kccr	kc	ydrcp	rfc	rmey	nyprw
6	jbbq	jb	xcqbo	qeb	qldx	mxoqv
7	iaap	ia	wbpan	pda	pkcw	lwnpu
8	hzzo	hz	vaozm	ocz	ojbv	kvmot
9	gyyn	gу	uznyl	nby	niau	julns
10	fxxm	fx	tymxk	max	mhzt	itkmr
11	ewwl	ew	sxlwj	lzw	lgys	hsjlq
12	dvvk	dv	rwkvi	kyv	kfxr	grikp
13	cuuj	cu	qvjuh	jxu	jewq	fqhjo
14	btti	bt	puitg	iwt	idvp	epgin
15	assh	as	othsf	hvs	hcuo	dofhm
16	zrrg	zr	nsgre	gur	gbtn	cnegl
17	yqqf	уq	mrfqd	ftq	fasm	bmdfk
18	хрре	хр	lqepc	esp	ezrl	alcej
19	wood	wo	kpdob	dro	dyqk	zkbdi
20	vnnc	vn	jocna	cqn	схрј	yjach
21	ummb	um	inbmz	bpm	bwoi	xizbg
22	tlla	tl	hmaly	aol	avnh	whyaf
23	skkz	sk	glzkx	znk	zumg	vgxze
24	rjjy	rj	fkyjw	ymj	ytlf	ufwyd
25		_	ejxiv		_	
						E 11 /



Exploiting Monoalphabetic Ciphers

• Cryptanalysis with frequency of characters in English:

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
Е	12.7	Н	6.1	W	2.3	K	0.08
Т	9.1	R	6.0	F	2.2	J	0.02
A	8.2	D	4.3	G	2.0	Q	0.01
О	7.5	L	4.0	Y	2.0	X	0.01
I	7.0	С	2.8	P	1.9	Z	0.01
N	6.7	U	2.8	В	1.5		
S	6.3	M	2.4	V	1.0		



Exploiting Monoalphabetic Ciphers (Cont.)

• Example: the attacker has intercepted the following ciphertext. Using a statistical attack, find the plaintext.

XLILSYWIMWRSAJSVWEPIJSVJSYVQMPPMSRHSPPEVWMXMWASVX-LQSVILY-VVCFIJSVIXLIWIPPIVVIGIMZIWQSVISJJIVW

• Solution: after tabulating the frequency of letters in this ciphertext, we get: I = 14, V = 13, S = 12, and so on. The most common character is I with 14 occurrences. This means key = 4.

the house is now for sale for four million dollars it is worth more hurry before the seller receives more offers



Exploiting Monoalphabetic Ciphers

- Monoaphabetic ciphers are easy to break because they reflect **frequency** of the original alphabet.
- However, a countermeasure is to provide multiple substitutes for a single letter.





Polyalphabetic Ciphers

- In polyalphabetic substitution, each occurrence of a character may have a different substitute.
- The relationship between a character in plaintext to a character in ciphertext is **one-to-many**.
- E.g. "a" could be enciphered as "D" at beginning of text, but as "N" at the middle.
- Benefit of polyalphabetic ciphers:
 - Hides letter frequency of the language.
 - Cannot use frequency statistics to break the ciphertext.



Polyalphabetic Ciphers (Cont.)

- We need to make each ciphertext character dependent on both corresponding plaintext character(s) and position of plaintext character(s) in the message.
- The key should be a stream of subkeys, i.e. $k = \{k_1, k_2, k_3, ...\}$.
- k_i is used to encipher the *i*th character in plaintext to create the *i*th character in ciphertext.

Hill Cipher

- The plainttext is divided into **equal-size blocks** that are encrypted one at a time, where *m* is the size of the block.
- Each character in a block contributes to the encryption of other characters in the block.
- Hill cipher algorithm takes *m* successive plaintext letters and substitutes by *m* ciphertext letters.
- The key is a square matrix of size $m \times m$.



• Key in Hill cipher:

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1m} \\ k_{21} & k_{22} & \dots & k_{2m} \\ \vdots & \vdots & & \vdots \\ k_{m1} & k_{m2} & \dots & k_{mm} \end{bmatrix}$$

• The key matrix needs to have a multiplicative inverse, where not all square matrices do in \mathbb{Z}_{26} .

- Substitution is determined by m linear equations.
- If we call m characters in plaintext block P_1 , P_2 , ..., P_m , the corresponding characters in ciphertext block are C_1 , C_2 , ..., C_m .
- Each ciphertext character depends on all plaintext characters.

$$C_{1} = P_{1} k_{11} + P_{2} k_{21} + \dots + P_{m} k_{m1}$$

$$C_{2} = P_{1} k_{12} + P_{2} k_{22} + \dots + P_{m} k_{m2}$$

$$\dots$$

$$C_{m} = P_{1} k_{1m} + P_{2} k_{2m} + \dots + P_{m} k_{mm}$$

• E.g. for m = 3, the system can be described as:

$$c_1 = (k_{11}p_1 + k_{21}p_2 + k_{31}p_3) \mod 26$$

 $c_2 = (k_{12}p_1 + k_{22}p_2 + k_{32}p_3) \mod 26$
 $c_3 = (k_{13}p_1 + k_{23}p_2 + k_{33}p_3) \mod 26$

• This can be expressed in terms of row vectors and matrices as:

$$(c_1 c_2 c_3) = (p_1 p_2 p_3) \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \bmod 26$$



• Example 01: consider the plaintext "paymoremoney" and use the encryption key for encryption purpose through Hill cipher, where m = 3.

$$\mathbf{K} = \begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}$$

Solution:

- The first three letters of the plaintext are represented by the vector (15 0 24).
- Then $(15\ 0\ 24)$ **K** = $(303\ 303\ 531)$ mod $26 = (17\ 17\ 11) = RRL$.

Hill Cipher (Cont.)

Solution (Cont.):

- Continuing this way, the ciphertext for the entire plaintext is RRLMWBKASPDH.
- Decryption requires using the inverse of the matrix **K**.

$$\mathbf{K}^{-1} = \begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix}$$

• It can be seen that if the matrix K^{-1} is applied to the ciphertext, then the plaintext is recovered.



Hill Cipher (Cont.)

- Example 02: the plaintext "code is ready" can make a 3×4 matrix, where m = 4, when adding extra bogus character "z" to the last block along with removing spaces.
- Ciphertext is:

"OHKNIHGLLISS"

• Note: (mod 26) is applied.

$$\begin{bmatrix} C \\ 14 & 07 & 10 & 13 \\ 08 & 07 & 06 & 7 \\ 5 & 08 & 18 & 18 \end{bmatrix} = \begin{bmatrix} 02 & 14 & 03 & 04 \\ 08 & 18 & 17 & 04 \\ 00 & 03 & 24 & 25 \end{bmatrix} \begin{bmatrix} 09 & 07 & 11 & 13 \\ 04 & 07 & 05 & 06 \\ 02 & 21 & 14 & 09 \\ 03 & 23 & 21 & 08 \end{bmatrix}$$

a. Encryption

$$\begin{bmatrix} P & C & K^{-1} \\ 02 & 14 & 03 & 04 \\ 08 & 18 & 17 & 04 \\ 00 & 03 & 24 & 25 \end{bmatrix} = \begin{bmatrix} 14 & 07 & 10 & 13 \\ 08 & 07 & 06 & 7 \\ 5 & 08 & 18 & 18 \end{bmatrix} \begin{bmatrix} 02 & 15 & 22 & 03 \\ 15 & 00 & 19 & 03 \\ 09 & 09 & 03 & 11 \\ 17 & 00 & 04 & 07 \end{bmatrix}$$

b. Decryption



Hill Cipher (Cont.)

Hill Cipher Benefits:

- It completely hides single-letter frequencies.
- The use of a larger matrix hides more frequency information.
- A Hill cipher hides not only single-letter but also two-letter frequency information.
- Hill cipher is strong against a ciphertext-only attack.

Transposition Technique



Transposition Ciphers

- Transposition Ciphers does not substitute one symbol for another, rather it changes location of the symbols.
- Transposition Ciphers performs some sort of *permutation* on the plaintext letters i.e. reorders (transposes) the symbols.
- Has two types of ciphering:
 - 1. Keyless transposition ciphers
 - 2. Keyed transposition ciphers



Keyless Transposition Ciphers

- Simple transposition ciphers, which were used in the past, are keyless. **E.g.**:
 - Text is written into table column by column and then transmitted row by row.
 - Text is written into table row by row and then transmitted column by column.



Keyless Transposition Ciphers (Cont.)

• Example: sender and receiver can agree on the number of columns. Sender writes the same plaintext, row by row, in a table of four columns.

m	e	e	t
m	e	a	t
t	h	e	p
a	r	k	

• The ciphertext is "MMTAEEHREAEKTTP".



Keyed Transposition Ciphers

- In keyed ciphers, the plaintext is divided into groups of predetermined size, **called blocks**, and then a **key** is used to permute the characters in each block separately.
- Example: plaintext message "Enemy attacks tonight".
- Solution: arrange the text in blocks of size 5 characters each. Followed by sending each character, within a block, in the sequence defined by key.

enemy attac kston ightz



Keyed Transposition Ciphers (Cont.)

- Example (Cont.):
- The use key is

Encryption ↓

3	1	4	5	2
1	2	3	4	5

↑ Decryption

• The permutation yields

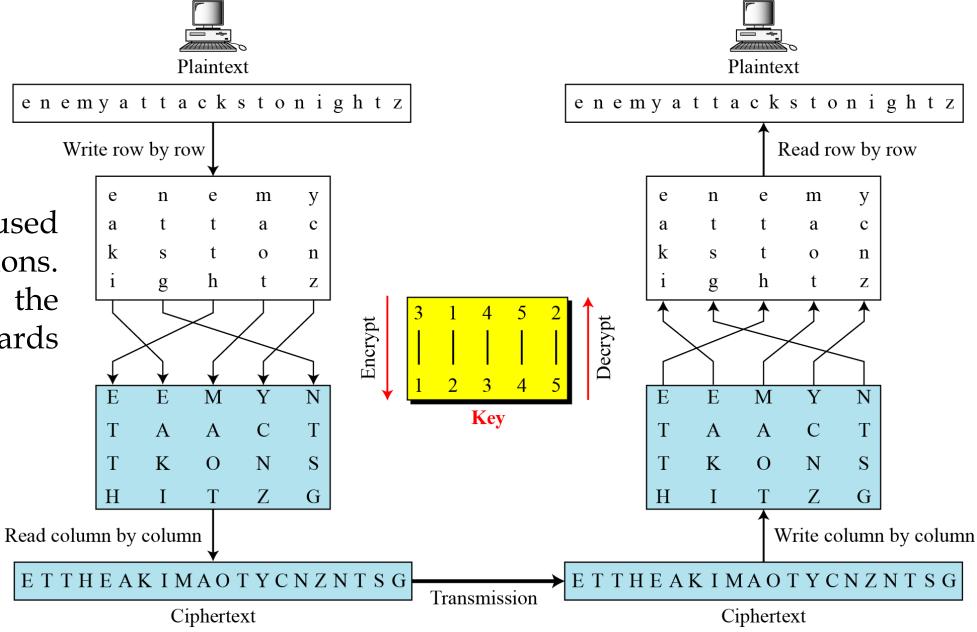
E E M Y N T A A C T T K O N S H I T Z G



Columnar Transposition Ciphers

- To achieve better scrambling, we can combine the two approaches (Keyless & Keyed).
- Below is an *example* in which the following steps are followed:
 - 1. Text is written into table row by row.
 - 2. Permutation is done by reordering the columns, i.e. according to the provided key.
 - 3. A new table is read column by column.
- *Note*: the 1st and 3rd steps provide a keyless reordering, while the second step provides a keyed reordering.

A single key is used directions. two Downwards for the encryption; upwards for decryption.

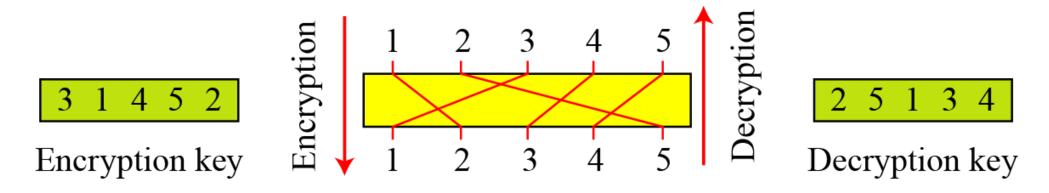


Alice



Columnar Transposition Ciphers (Cont.)

• It is customary to create two keys:

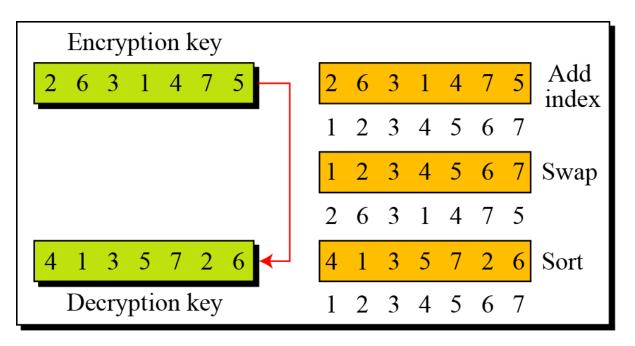


- In encryption key (3 1 4 5 2), the first entry shows that column 3 in source becomes column 1 in destination.
- In decryption key (2 5 1 3 4), the first entry shows that column 2 in source becomes column 1 in destination.



Columnar Transposition Ciphers (Cont.)

• It is possible to create the decryption key if the encryption key is provided as given below:

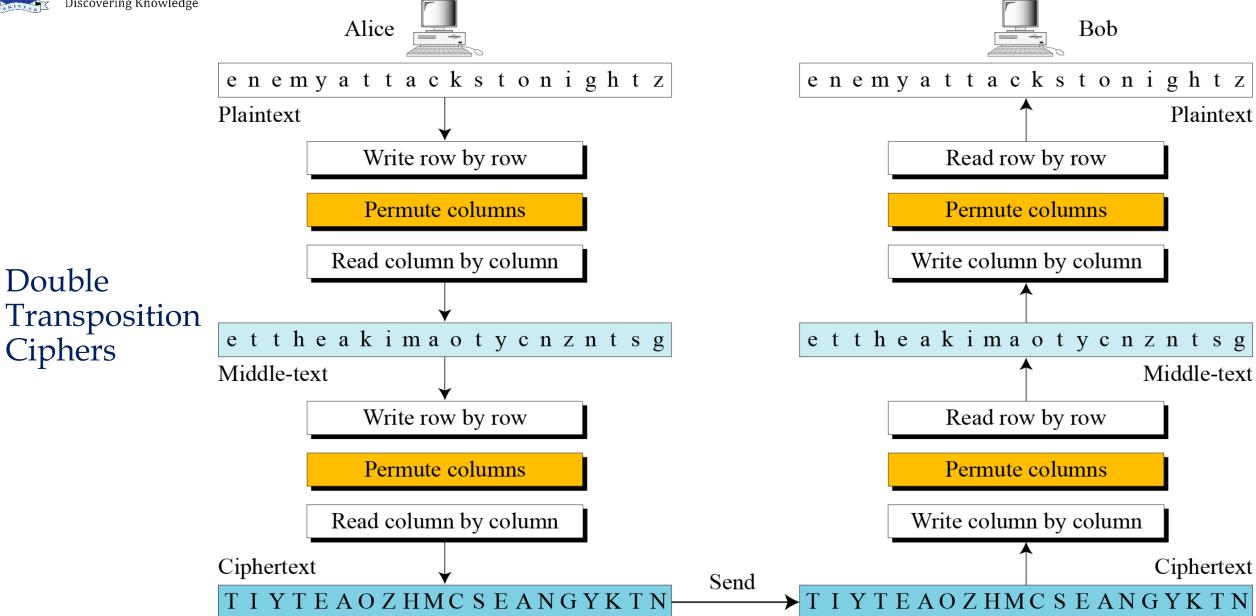


```
Given: EncKey [index]
index ← 1
while (index ≤ Column)
{
    DecKey[EncKey[index]] ← index
    index ← index + 1
}
Return: DecKey [index]
```

a. Manual process

b. Algorithm





Double

Ciphers

Asymmetric Ciphers (Public-Key Cryptography)

Overview



- Public-key cryptography is **asymmetric**, i.e. involving use of *two separate keys*.
- Asymmetric Keys: two "related" keys, a public and private key, used to perform complementary operations.
- There is nothing about **symmetric** or **asymmetric** encryption that makes one superior to another w.r.t. *cryptanalysis*.
- Computational overhead of public-key encryption exists w.r.t. key management.



Public-Key Cryptosystems



• The concept of public-key cryptography evolved from an attempt to solve two of the most difficult problems in symmetric encryption:

Key distribution

• How to have secure communications in general without having to trust a Key Distribution Center with your key

Digital signatures

• How to verify that a message comes from the claimed sender

Delic Private

Public-Key Cryptosystems (Cont.)

Mechanism in public-key cryptosystem:

- Each user generates a **pair of keys** to be used for encryption and decryption of messages.
- Each user places one of the two keys in a public register (*this is the public key*), while the companion key is kept private.
- All participants have access to public keys, hence each user has a collection of public keys obtained from others.

Public-Key Cryptosystems (Cont.)

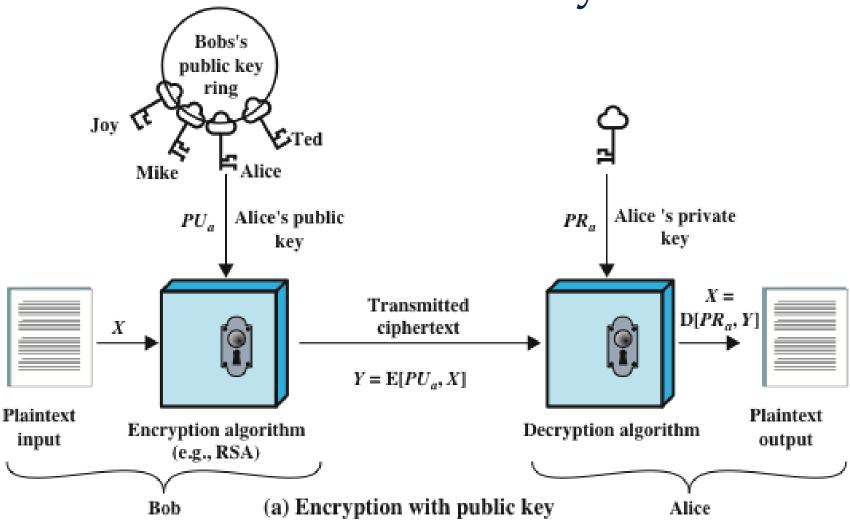
Mechanism in public-key cryptosystem (Cont.):

- Private keys are generated locally by each participant and therefore need never be distributed.
- At any time, a system can change its private key and publish the companion public key to replace its old public key.



Public-Key Cryptosystems: Confidentiality

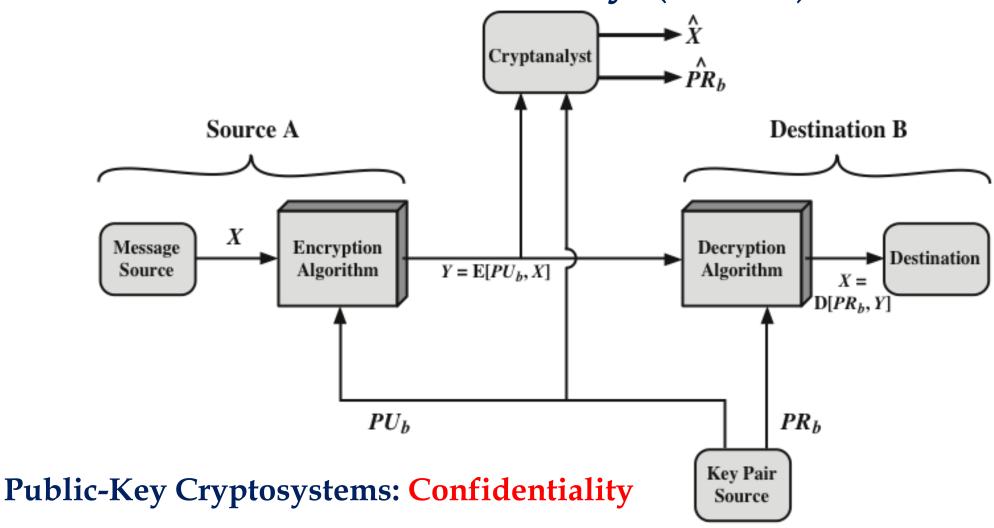






Public-Key Cryptosystems: Confidentiality (Cont.)







Public-Key Cryptosystems: Confidentiality (Cont.)



Example:

- If **Bob** wishes to send a confidential message to **Alice**, **Bob** encrypts the message using **Alice's public key**.
- When Alice receives the message, she decrypts it using her private key, where no other recipient can decrypt the message because only Alice knows Alice's private key.

Public-Key Cryptosystems: Authentication

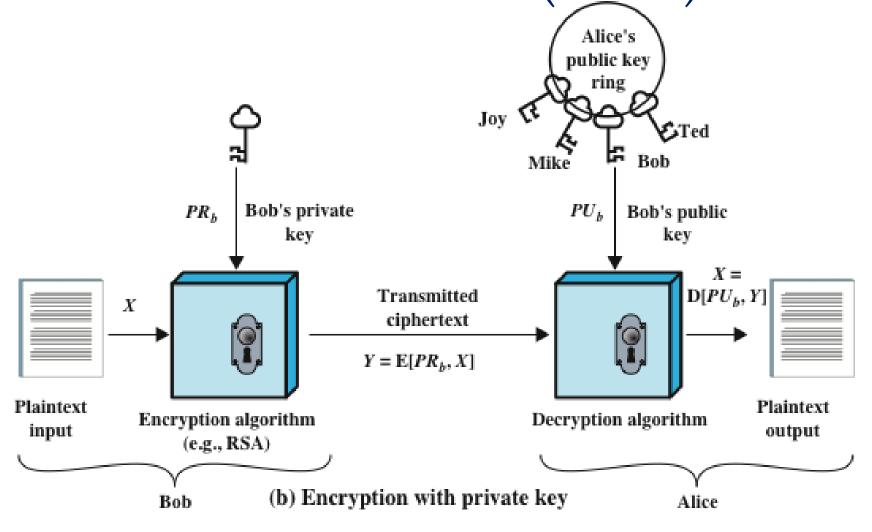


• Since either of the **two related keys** can be used for encryption with the other used for decryption, the public-key encryption can also be used to provide **authentication**.

Example:

- A prepares a message to \mathbf{B} and encrypts it using $\mathbf{A's}$ private key.
- B can decrypt the message using A's public key.
- Because the message was encrypted using **A's** private key, *only A* could have prepared the message not anyone else.
- Hence, entire encrypted message serves as a digital signature.

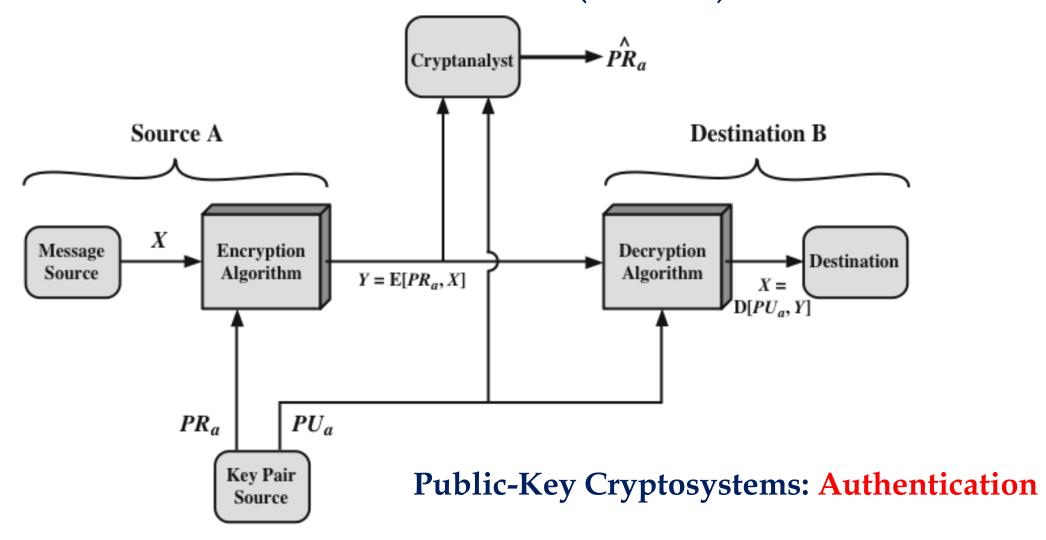
Public-Key Cryptosystems: Authentication (Cont.)





Public-Key Cryptosystems: Authentication (Cont.)







Public-Key Cryptosystems: Confidentiality and Authentication

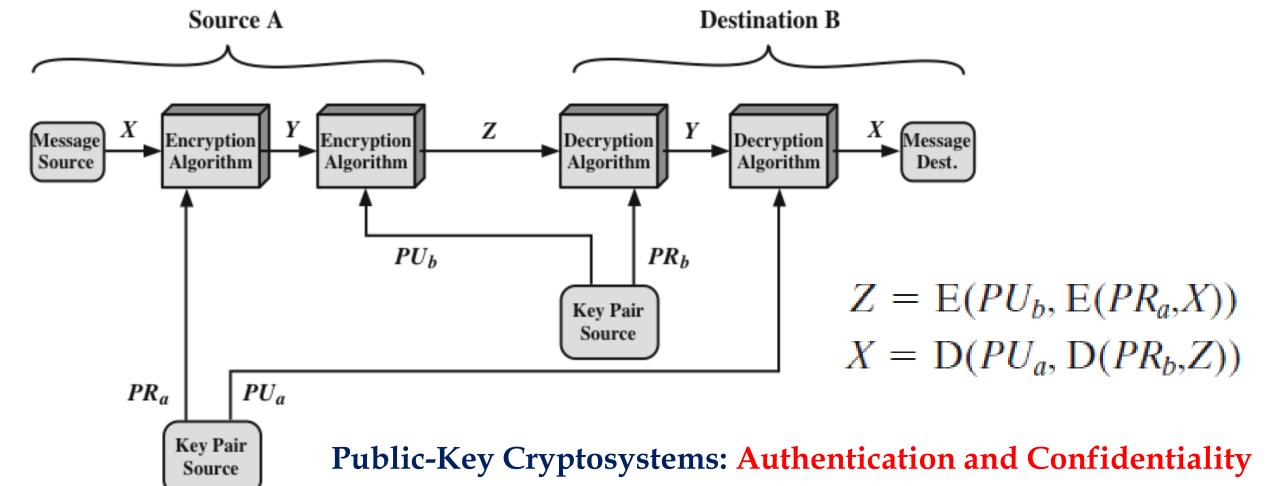


- The encryption process, using the private key for encryption, does not provide confidentiality.
- The message being sent is safe from **authentication** issues but not from **eavesdropping**.
- There is no protection of **confidentiality** because any observer can decrypt the message by using the sender's public key.
- It is possible to provide both **authentication** and **confidentiality** by a **double use of the public-key scheme**.



Public-Key Cryptosystems:





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Public-Key Cryptosystems: Confidentiality and Authentication (Cont.)

Working mechanism:

- We begin by encrypting a message using sender's private key. Hence, providing digital signature.
- Next, we encrypt again using the receiver's public key. Hence, generating the final ciphertext. This final ciphertext can be decrypted only by the receiver who has the matching private key. Thus, confidentiality is achieved.
- The receiver decrypts the received **ciphertext** first by its **own private key**. Followed by decrypting the result with the **sender's public key**. By that, the **plaintext** is obtained.



Application of Public-Key Cryptosystems



- Depending on the application, sender uses either the sender's private key or receiver's public key.
- Broadly, we can classify the use of public-key cryptosystems into three categories:

Encryption / Decryption

• The sender encrypts a message with the recipient's public key

Digital signature

• The sender "signs" a message with its private key

Key exchange

 Two sides cooperate to exchange a session key, which is a secret key for symmetric encryption



Application of Public-Key Cryptosystems (Cont.)



- Some algorithms are suitable for all three applications, whereas others can be used only for one or two of these applications.
- Table below indicates the applications supported by the algorithms.

Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Elliptic Curve	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes
DSS	No	Yes	No

Thank You!