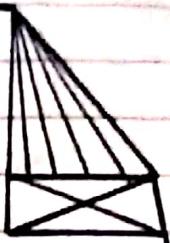


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# Assignment 01

Date \_\_\_\_\_

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**PROGRAM : BSE**  
**SECTION : 2B**  
**COURSE : DISCRETE STRUCTURE**



Ex # 1.1

Q # 01

Q Which of the following sentences are propositions?  
What are the truth values of those are propositions?

Sentences	Proposition	Truth Value
a) Boston is the capital of Massachusetts	T	T
b) Miami is the capital of Florida.	T	F
c) $2 + 3 = 5$	T	T
d) $5 + 7 = 10$	T	F
e) $2 + 2 = 11$	F	-
f) Answer the question.	F	-

Q # 03

Q What is the negation of each of these propositions?

a) Mei has an MP3 player

A Mei has not an MP3 player.

b) There is no pollution in new Jersey.

A There is pollution in new Jersey.

c)  $2 + 1 = 3$

A  $2 + 1 \neq 3$

(2)

Date \_\_\_\_\_

- d) The Summer in Maine is hot and sunny.  
 A It is not the case that the Summer in Maine is hot sunny.

### Q # 9

Let

- \*  $P$  = Swimming at the new Jersey shore is allowed.
- \*  $q$  = Sharks have been spotted near the shore.

- a)  $\neg q$   
 Sharks have not been spotted near the shore.
- b)  $P \wedge q$   
 Swimming at the new Jersey shore is allowed and sharks have been spotted near the shore.
- c)  $\neg P \vee q$   
 Swimming at the new Jersey shore is not allowed, or shark have been spotted near the shore.
- d)  $P \rightarrow \neg q$   
 If swimming at the new Jersey shore is allowed, then shark have not been spotted near the shore.
- e)  $\neg q \rightarrow P$   
 If shark have not been spotted near the shore; then swimming at the new Jersey shore is allowed.

(3)

Date \_\_\_\_\_

f)  $\neg P \rightarrow \neg q$

If swimming at the new jersey shore is not allowed, then shark have not been spotted near the shore.

g)  $P \leftrightarrow \neg q$

Swimming at the new jersey shore is allowed if and only if shark have not been spotted near the shore.

h)  $\neg P \wedge (P \vee \neg q)$

Swimming at the new jersey shore is not allowed and either swimming at the new jersey shore is allowed or shark have not been spotted near the shore.

### Q # 11

Q Let  $P$  and  $q$  be the proposition

+  $P$ : It is below freezing

+  $q$ : It is snowing.

- write the proposition using  $P$  and  $q$  and logical connective (Include negation).

a) It is below freezing and snowing.

$$P \wedge q$$

b) It is below freezing but not snowing.

$$P \wedge \neg q$$

c) It is not below freezing and it is not snowing.

$$\neg P \wedge \neg q$$

d) It is either snowing or below freezing (or both).

$$P \vee q$$

e) If it is below freezing, it is also snowing.

$$P \rightarrow q$$

(9)

Date \_\_\_\_\_

- f) Either it is below freezing or it is snowing but it is not snowing if it is below freezing.

$$(P \vee q) \wedge (P \rightarrow \neg q)$$

- g) That it is below freezing is necessary and sufficient for it to be snowing.

$$P \leftrightarrow q$$

## Q #13

Q Let  $p$  and  $q$  be the proposition

$p$ : you drove over 65 mile per hour.

$q$ : you get a speeding ticket.

\* write these proposition using  $p$  and  $q$  and logical connective (including negation)

a) you do not drive over 65 mile per hour

$$\neg p$$

b) you drive over 65 mile per hour, but you do not get a speeding ticket.

$$p \wedge \neg q$$

c) you will get a speeding ticket, if you drive over 65 miles per hour.

$$p \rightarrow q$$

d) If you do not drive over 65 miles per hour, then you will not get speeding ticket.

$$\neg p \rightarrow \neg q$$

e) Driving over 65 miles per hour is sufficient for getting speeding ticket.

$$p \rightarrow q$$

(5)

Date \_\_\_\_\_

f) you get a speeding ticket, but you do not drive over 65 miles per hour.

$$q \wedge \neg P$$

g) whenever you get a speeding ticket, you are driving over 65 miles per hour.

$$q \rightarrow P$$

$$\longleftrightarrow Q \# 15 \longleftrightarrow$$

Q15 Let  $P, q$  and  $\gamma$  be the proposition.

- $P$ : Grizzly bears have been seen in the area.

- $q$ : Hiking is safe on the trail.

- $\gamma$ : Berries are ripe along the trail.

- + write the proposition using  $P, q, \gamma$  and logical connectives (including negation).

**Answers:-**

a)  $\gamma \wedge \neg P$

b)  $\neg P \wedge q \wedge \gamma$

c)  $\gamma \rightarrow (q \leftrightarrow \neg P)$

d)  $\neg q \wedge \neg P \wedge \gamma$

e)  $(q \rightarrow (\neg \gamma \wedge \neg P)) \wedge \neg ((\neg \gamma \wedge \neg P) \rightarrow q)$

f)  $(P \wedge \gamma) \rightarrow \neg q$

(6)

## Q # 17

Date \_\_\_\_\_

- Q17 Determine whether each of these Conditional Statement is True or False

$$T \rightarrow F = F \text{ otherwise } T.$$

	conditional Statement	70)
a) If $1+1=2$ , then $2+2=5$	False	
b) If $1+1=3$ , then $2+4=4$	True	
c) If $1+1=3$ , then $2+2=5$	True	
d) If monkey can fly, then $1+1=3$	True	

## Q # 19

- Q19 For each of these Sentences, determine whether an Inclusive OR, an exclusive OR, is Intended. Explain answers.

- a) Coffee or tea comes with dinner.

Ans) Exclusive OR :-

Because only one option is req to be true i.e. only one chy is required.

- b) A password must have at least three digits or be at least eight characters long.

Ans) Inclusive OR:

Because both the situations must be true for that line

- c) The prerequisites for the course is a course in number theory or a course in Cryptography.

Inclusive OR:

Because the student will both course can be eligible.

(d) you can pay using US dollars or euros.  
Inclusive or:

Because we can pay  
either in US dollar or euros or  
by both.

**Q #25**

Q25 Write each of these propositions in the form "P if and only if q in english"

**Answers:**

a) you buy an ice cream if and only if  
it is hot outside.

b) you win the contest if and only if you  
have the winning ticket

c) you get promoted if and only if you  
have connections.

d) your mind will decay if and only if  
you watch television

e) The trainer ran late if and only if  
the days I take it.

**Q # 27**

Q27 State the converse, contrapositive and inverse  
of each of the conditional statements.

a) If it snows today, I will ski tomorrow.

- Converse:- I will ski tomorrow only if it snows today.
- Contrapositive:- If I will not ski tomorrow, then it will not have snows today.
- Inverse:- If it does not snows today, then I will not ski tomorrow.

b) I come to class whenever there is going to be a quiz.

- Converse:- I come to class only if there is going to be a quiz.
- Contrapositive:- If I do not come to class, then there will not be a quiz.
- Inverse:- If there is not going to be a quiz, then I don't come to a class.

(c) A positive integer is a prime only if it has no divisor other than 1 and itself.

- Converse:- ~~A positive integer is a prime if it has no divisor other than 1 and itself.~~
- Contrapositive ~~If a positive integer has a divisor other than 1 and itself, then it is not prime.~~
- Inverse:- ~~If a positive integer is not prime, then it has a divisor other than 1 and itself.~~

(9)

(c)

A positive integer is a prime only if it has no division other than 1 and itself

→ Converse: If it has no division other than 1 and itself then a positive integer is a prime

→ Contrapositive: If it has a division other than 1 and itself then a positive integer is not a prime

→ Inverse: If positive integer is not a prime then it has a division other than 1 and itself.

## Q # 29

Q How many rows appear in a truth table for each of these compound proposition.

a)  $P \rightarrow \neg P$

P	$\neg P$	$P \rightarrow \neg P$
T	F	F
F	T	T

Formula =  $2^n$   
where n is a number of variable.

$$\Rightarrow 2^1$$

$\Rightarrow 2$  Rows.

(b)  $(P \vee \neg q) \wedge (q \vee \neg s)$

Sol

$$2^4$$

16 rows.

(10)

Date \_\_\_\_\_

c)  $q \vee p \vee \neg s \vee \neg t \vee \neg u$ .

Sol $2^6$ 

| 64 rows |  
An

d)  $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

Sol $2^4$ 

| 16 rows |  
An

Q # 31

Q Construct a truth table for each of these compound proposition.

a)  $p \wedge \neg p$

a	P	$\neg p$	$p \wedge \neg p$
	T	F	F
	F	T	F

b)  $p \vee \neg p$

	P	$\neg p$	$p \vee \neg p$
	T	F	T
	F	T	T

11

Date \_\_\_\_\_

c)  $(P \vee \neg q) \rightarrow q$

i	P	q	$\neg q$	$P \vee \neg q$	$(P \vee \neg q) \rightarrow q$
	T	T	F	T	T
	T	F	T	T	F
	F	T	F	F	T
	F	F	T	T	F

d)  $(P \vee q) \rightarrow (P \wedge q)$

	P	q	$P \vee q$	$P \wedge q$	$(P \vee q) \rightarrow (P \wedge q)$
	T	T	T	T	T
	T	F	T	F	F
	F	T	T	F	F
	F	F	F	F	T

d)

e)  $(P \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg P)$

	P	q	$\neg P$	$\neg q$	$P \rightarrow q$	$\neg q \rightarrow \neg P$	$(P \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg P)$
	T	T	F	F	T	T	T
	T	F	F	T	F	F	T
	F	T	T	F	T	T	T
	F	F	T	T	T	T	T

f)  $(P \rightarrow q) \rightarrow (q \rightarrow P)$

	P	q	$P \rightarrow q$	$q \rightarrow P$	$(P \rightarrow q) \rightarrow (q \rightarrow P)$
	T	T	T	T	T
	T	F	F	T	T
	F	T	T	F	F
	F	F	T	T	T

# Q# 33

Date \_\_\_\_\_

Q) Construct a truth table for each of these compound proposition

(a)  $(P \vee q) \rightarrow (P \oplus q)$

	P	q	$P \vee q$	$P \oplus q$	$P \vee q \rightarrow P \oplus q$
.	T	T	T	F	F
.	T	F	T	T	T
.	F	T	T	T	T
.	F	F	F	F	T

(b)  $(P \oplus q) \rightarrow (P \wedge q)$

	P	q	$P \oplus q$	$P \wedge q$	$P \oplus q \rightarrow P \wedge q$
.	T	T	F	T	T
.	T	F	T	F	F
.	F	T	T	F	F
.	F	F	F	F	T

(c)  $(P \vee q) \oplus (P \wedge q)$

	P	q	$P \vee q$	$P \wedge q$	$(P \vee q) \oplus (P \wedge q)$
.	T	T	T	T	F
.	T	F	T	F	T
.	F	T	T	F	T
.	F	F	F	F	F

(d)  $(P \leftrightarrow q) \oplus (\neg P \leftrightarrow q)$

P	q	$\neg P$	$P \leftrightarrow q$	$\neg P \leftrightarrow q$	$(P \leftrightarrow q) \oplus (\neg P \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

(13)

Date \_\_\_\_\_

$$\textcircled{e} \quad (P \leftrightarrow q) \oplus (\neg P \leftrightarrow \neg q)$$

P	$q \vee$	$\neg q$	$\neg \neg q$	$P \leftrightarrow q$	$\neg P$	$\neg \neg P$	$(P \leftrightarrow q) \oplus (\neg P \leftrightarrow \neg q)$
T	T	F	T	T	F	T	F
T	T	F	T	T	F	F	T
T	F	T	F	F	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	T	F	T	T	T
F	F	T	F	T	T	F	T
F	F	F	T	T	T	T	F

$$\textcircled{f} \quad (P \oplus q) \rightarrow (P \oplus \neg q)$$

P	$q \vee$	$\neg q$	$P \oplus q$	$P \oplus \neg q$	$(P \oplus q) \rightarrow (P \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Q # 37

Q Construct a truth table for each of these compound propositions

$$\text{a) } P \rightarrow (\neg q \vee r)$$

P	$q \vee$	$\neg q$	$\neg \neg q$	$\neg q \vee r$	$P \rightarrow (\neg q \vee r)$
T	T	F	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

(b)  $\neg P \rightarrow (q \rightarrow \gamma)$

P	q	$\gamma$	$\neg P$	$q \rightarrow \gamma$	$\neg P \rightarrow (q \rightarrow \gamma)$
T	T	T	F	T	T
T	T	F	F	F	T
F	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

(c)  $(P \rightarrow q) \vee (\neg P \rightarrow \gamma)$

P	q	$P \rightarrow q$	$\neg P$	$\gamma$	$\neg P \rightarrow \gamma$	$(P \rightarrow q) \vee (\neg P \rightarrow \gamma)$
T	T	T	F	T	T	T
T	T	T	F	F	T	T
T	F	F	F	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	T	T	F	F	T
F	F	T	T	T	T	T
F	F	T	T	F	F	T

(d)  $(P \rightarrow q) \wedge (\neg P \rightarrow \gamma)$

P	q	$\neg \gamma$	$\neg P$	$(P \rightarrow q)$	$(\neg P \rightarrow \gamma)$	$(P \rightarrow q) \wedge (\neg P \rightarrow \gamma)$
T	F	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

(4)

Date \_\_\_\_\_

$$(P \leftrightarrow q) \vee (\neg q \leftrightarrow r)$$

P	q	$\neg q$	r	$P \leftrightarrow q$	$\neg q \leftrightarrow r$	$(P \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	F	T	T	F	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	T	F	F	F	F
F	T	F	T	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	T	F	T	F	T

$$(5) (\neg P \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r).$$

P	q	r	$\neg P$	$\neg q$	$\neg P \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg P \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	F	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Ex 1.2

Q # 02

Date \_\_\_\_\_

Q # 1

Answer

$$e \rightarrow a, \neg a \rightarrow \neg e$$

Q # 7

Q) Express the System Specification.

P: The msg is scanned for viruses.

q: The message was sent from an unknown system.

Answers:-

- a)  $q \rightarrow P$
- b)  $q \wedge \neg P$
- c)  $\neg q \rightarrow P$
- d)  $\neg q \rightarrow \neg P$

Q # 9

The system is inconsistent

Q # 11

The system is consistent.

## Q#03

Date \_\_\_\_\_

Q Show that the following propositions are logically equivalent to each other.

$$(1) P \leftrightarrow q \equiv (P \wedge q) \vee (\neg P \wedge \neg q).$$

Taking L.H.S.

$$(P \leftrightarrow q).$$

$$(P \rightarrow q) \wedge (q \rightarrow P)$$

$$(\neg P \vee q) \wedge (\neg q \vee P)$$

$$(\neg P \wedge \neg q) \vee (\neg P \wedge P) \vee (q \wedge \neg q) \wedge (q \wedge P)$$

$$(\neg P \wedge \neg q) \vee (F) \vee (F) \wedge (q \wedge P).$$

$$(\neg P \wedge \neg q) \vee (q \wedge P).$$

$$(\neg P \wedge \neg q) \vee (P \wedge q).$$

$$(P \wedge q) \vee (\neg P \wedge \neg q) = R.H.S$$

proved.

$$(2) (P \rightarrow q) \wedge (P \rightarrow r) \equiv P \rightarrow (q \wedge r).$$

Taking L.H.S.

$$(P \rightarrow q) \wedge (P \rightarrow r).$$

$$(\neg P \vee q) \wedge (\neg P \vee r).$$

$$\neg P \vee \underbrace{(q \wedge r)}_a$$

$$\neg P \vee a$$

$$P \rightarrow a$$

$$P \rightarrow (q \wedge r) = R.H.S$$

proved.