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T ex 22 QF

Let $S(T) = e^{x(T)}$ by pg 49

fair price of option at $t=T$

$$V(T) = \frac{e^{-r(T)}}{\pi} \int_0^\infty Re\left(\tilde{f}(R+iu) X_t(u-iR)\right) du$$

with $X_t(u) = E_Q\left(e^{iuX(T)} \mid F_t\right)$ eq 4.5

given $f(x) = \cancel{(S(T)-K)^+ \cdot S(T)}$
$(e^x - K)^+ e^x$
as $S(T) = e^x$

we have:

bilateral laplace transform eq 4.6

$$\tilde{f}(z) = \int_{-\infty}^{\infty} (e^x - K)^+ e^x \cdot e^{-zx} \, dx$$

$$= \int_{\log K}^{\infty} (e^x - K) e^{x(1-z)} \, dx$$

$$= \int_{\log K}^{\infty} e^{x(1+1-z)} - K e^{x(1-z)} \, dx$$

$$= \int_{\log K}^{\infty} e^{x(2-z)} - K e^{x(1-z)} \, dx$$

$$\left[\frac{e^{x(2-z)}}{(2-z)} - \frac{K e^{x(1-z)}}{(1-z)} \right]_{x=\log K}^{x=\infty}$$

plugging in $\log K$ lower limit, we get:

$$\left[\frac{e^{\log K \cdot (2-z)}}{(2-z)} - \frac{K\, e^{\log K \cdot (1-z)}}{(1-z)} \right]_{\cancel{x=\log K}}^{\cancel{x=\infty}}$$

$$\left[\cancel{K^{(2-z)}}\, \frac{K^{(2-z)}}{(2-z)} - \frac{K^{(2-z)}}{(1-z)} \right]_{x=\log K}^{x=\infty}$$

plugging in ∞ gets us 0.

$$\left[0 - \frac{K^{(2-z)}}{(2-z)} + \frac{K^{(2-z)}}{(1-z)} \right]$$

$$= \frac{K^{(2-z)}}{(1-z)} - \frac{K^{(2-z)}}{(2-z)} \rightarrow \tilde{f}(z)$$

$Re(z) > 2$ else $e^{(2-z)x}$ does not tend to 0 as $x \to \infty$

Simply from pg 50, we have

$$\cancel{\phi_T(u) = \exp(iu(\log S(0) + r(T)) - \lambda(iu - u^2))}$$

$$X_T(\overset{z}{u - iR}) = \exp\left(i\cancel{z}(\log S(T) + rT - (iz + z^2)\frac{\sigma^2}{2}(T)\right)$$

plugging both back in we get

$$V(\cancel{t}) = \frac{e^{-r(T)}}{\pi}\int_0^\infty Re\left(\frac{-K^{(2-(R+iu))}}{(2-(R+iu))} + \frac{K^{(2-(R+iu))}}{(1-(R+iu))}\right)$$

$$\cdot \exp\left[i(u - iR)\left[e^x + rT - (i \cdot \underbrace{(u - iR)}_{iu - R} + (u - iR)^2 \cdot \frac{\sigma^2}{2}(T)\right)\right.$$

$$\exp\left[\left[iu - \underset{R}{iR}\right)\right.$$

$$= \frac{e^{-r(T)}}{\pi}\int_0^\infty Re\left(\frac{-K^{(2-(R+iu))}}{(2-(R+iu))} + \frac{K^{(2-(R+iu))}}{(1-(R+iu))}\right) \times$$

$$\times \exp\left[\left(iu - R\right)\left(\log S(T) + \left(r - \frac{\sigma^2}{2}\right)T - \frac{(u - iR)^2\sigma^2}{2}T\right)\right]$$