Q3 (a)

Date:

starting with:

$$\exp((2r+\sigma^2)\Delta t) = q u^2 + (1-q) d^2$$

let $\boxed{q = 0.5}$

$$\exp((2r+\sigma^2)\Delta t) = \frac{u^2 + d^2}{2}$$

let $\alpha = \exp(r\Delta t)$

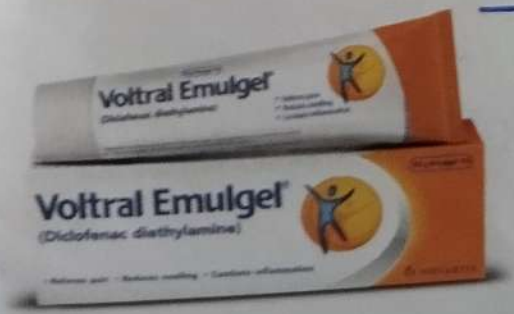
$$\alpha^2 \cdot \exp(\sigma^2 \Delta t) = \frac{u^2 + d^2}{2} \quad -(1)$$

From eq 1o1, we have

$$q = \frac{\exp(r\Delta t) - d}{u - d}$$

$$\frac{u-d}{2} = \alpha - d$$

$$\boxed{d = 2\alpha - u} \rightarrow \text{subs in } (1)$$



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we have: $\alpha^2 \exp(\sigma^2 \Delta t) = \frac{u^2 + (2\alpha - u)^2}{2}$

$\alpha^2 \exp(\sigma^2 \Delta t) = \frac{2u^2 + 4\alpha^2 - 4\alpha u}{2}$

Solving and rearranging gives:

$u^2 - 2\alpha u + (2\alpha^2 - \alpha^2 \exp(\sigma^2 \Delta t)) = 0$

Solving the quadratic formulae for u:

$u = \frac{2\alpha \pm \sqrt{4\alpha^2 - 8\alpha^2 + 4\alpha^2 \exp(\sigma^2 \Delta t)}}{2}$

$u = \alpha \pm \sqrt{\alpha^2 (1 - \exp(\sigma^2 \Delta t))}$

given $d = 2\alpha - u$

$d = 2\alpha - (\alpha \pm \sqrt{\alpha^2 (1 - \exp(\sigma^2 \Delta t)}$

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$$d = \alpha \pm \sqrt{\alpha^2(1 - \exp(\sigma^2 \Delta t))}$$

To ensure u stands for up move and d stands for down move, we have:

$$u = \alpha + \sqrt{\alpha^2(1 - \exp(\sigma^2 \Delta t))}$$
$$d = \alpha - \sqrt{\alpha^2(1 - \exp(\sigma^2 \Delta t))}$$
$$q = 0.5$$