



$t=0$

$t=1$

$t=2$

$t=3$



(1) calculating values at the leaf node

$$\text{Node } G = \max(1.2 - 6.0395, 0) = 0$$

$$\text{Node } H = \max(1.2 - 1.8210, 0) = 0$$

$$\text{Node } I = \max(1.2 - 0.5490, 0) = 0.651$$

$$\text{Node } J = \max(1.2 - 0.1656, 0) = 1.0344$$

(2) Values at child nodes

$$\text{Node } D = e^{-0.05} [0.3948 \cdot 0 + 0.6052 \cdot 0] = 0$$

$$\text{Node } E = e^{-0.05} [0.3948 \cdot 0 + 0.6054 \cdot 0.651] = 0.3749$$

$$\max(0.3749, 1.2 - 0.9999) = 0.3749$$

we don't exercise!

$$\text{Node } F = e^{-0.05} [0.3948 \cdot 0.651 + 0.6054 \cdot 1.0344] = 0.8402$$

$$\max(0.8402, 1.2 - 0.3015)$$

$$\max(0.8402, 0.8985) = 0.8985$$

we exercise.



$$\text{Node B} = e^{-0.05} [0.3948 \cdot 0 + 0.6054 \cdot 0.3749]$$

$$= 0.2159$$

$$\max(0.2159, 1.2 - 1.8211) = 0.2159$$

we don't exercise.

$$\text{Node C} = e^{-0.05} [0.3948 \cdot 0.3749 + 0.6054 \cdot 0.8985]$$

$$= 0.6582$$

$$\max(0.6582, 1.2 - 0.5491)$$

$$\max(0.6582, 0.6509) = 0.6582$$

we don't exercise.

(iii) Value of root node

$$A = e^{-0.05} [0.3948 \cdot 0.2159 + 0.6054 \cdot 0.6582]$$

$$= 0.4601$$

$$\max(0.4601, 1.2 - 0.1) = 0.4601$$

we don't exercise.



⑥ Replicating portfolio : using eq 1.11

$P = \begin{pmatrix} P_0 \\ P_1 \end{pmatrix}$  where  $P_0$  is risk free <sup>bond</sup> ~~asset~~  
 $P_1$  is risky asset.

portfolio to replicate payoffs at nodes  
~~1.8211~~ B and C

$$= 1.8211 \Delta + B e^{0.05} = 0.2159 \quad \text{eq ①}$$

$$0.5491 \Delta + B e^{0.05} = 0.6582$$

value of  
replicating  
portfolio  
= value of  
put option

$$1.272 \Delta = -0.4423$$

$$\boxed{\Delta = -0.3477} \rightarrow \text{sub ①}$$

$$1.8211(-0.3477) + B e^{0.05} = 0.2159$$

$$\boxed{B = 0.8077}$$

$$P_A = \begin{pmatrix} 0.8077 \\ -0.3477 \end{pmatrix}$$



At node B

$$3.364 \Delta + Be^{0.05} = 0$$

$$0.9999 \Delta + Be^{0.05} = 0.3749$$

$$2.3165 \Delta = -0.3749$$

$$\Delta = -0.1618 \rightarrow \text{sub back}$$

$$3.3164 (-0.1618) + Be^{0.05} = 0$$

$$B = 0.5104$$

$$\begin{pmatrix} 0.5104 \\ -0.1618 \end{pmatrix}$$

At node C

$$0.9999 \Delta + \overline{0.3015} B = 0.3749$$

$$0.3015 \Delta + Be^{0.05} = 0.8985$$

$$\Delta = -0.7497 \rightarrow \text{sub back}$$

$$0.9999 (-0.7497) + Be^{0.05} = 0.3749$$

$$B = 0.4757$$

$$\begin{pmatrix} 0.4757 \\ -0.7497 \end{pmatrix}$$



At node D

$$6.0395 \Delta + Be^{0.05} = 0$$

$$-1.8210 \Delta + Be^{0.05} = 0$$

$$\boxed{\Delta = 0}$$

$$\boxed{B = 0}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

At node E

$$1.8210 \Delta + Be^{0.05} = 0$$

$$-0.5490 \Delta + Be^{0.05} = 0.651$$

$$\boxed{\Delta = -0.5118}$$

$$\boxed{B = 0.8866}$$

$$1.8210(-0.5118) + Be^{0.05} = 0$$

$$\begin{pmatrix} 0.8866 \\ -0.5118 \end{pmatrix}$$

At node F

$$0.5490 \Delta + Be^{0.05} = 0.651$$

$$-0.1656 \Delta + Be^{0.05} = 1.0344$$

$$\boxed{\Delta = -0.19928} - 1$$

$$0.5490(-0.19928) + Be^{0.05} = 0.651$$

$$\boxed{B = 1.1415}$$

$$\begin{pmatrix} 1.1415 \\ -1 \\ -0.19928 \end{pmatrix}$$