Computer Networks

Linear Block Code

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Linear Block Code

- Linear block codes are a fundamental concept in coding theory, primarily used for error detection and correction in digital communications.

- These codes allow the efficient transmission of data over noisy channels by adding redundancy to the original message.

- A linear block code is defined as an error-correcting code where any linear combination of *codewords* is also a *codeword*.
  - → This property allows for effective encoding and decoding processes.

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Parameters of LBC

- **Message (k):** The number of bits in the original message.
- **Redundancy (q):** The number of parity bits added.
- **Codeword (n):** The total number of bits in each codeword, $n = k + q$.

Matrices of LBC

- **Generator Matrix (G):** This matrix is used to generate codewords (*c*) from datawords (*d*), which is a row vector.

- **Parity Check Matrix (H):** This matrix helps in error detection and correction.

Generating the Generator Matrix

- $G = [I_k \mid P]$;
  - → I_k is the identity matrix of size k×k.
  - → P is the parity matrix that contains information about how the parity bits are derived from the data bits.
  - → **Generator Matrix:** Appending I_4 and the given P, we get:

$$I_4 = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$P = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{matrix}$$

$$G = [I_4|P] = \begin{matrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{matrix}$$

Generating the Parity Check Matrix

- $H = [P^T \mid I_q]$;
  - → **The sequence is reversed.**
  - → P^T is the transpose matrix of P.
  - → $q = 3$; I_3;

$$P^T = \begin{matrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{matrix}$$

$$I_3 = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

- **Parity Check Matrix (H):**

$$H = \begin{matrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{matrix}$$

Example: (7, 4) Hamming Code

- Let's consider a specific example of a linear block code known as the Hamming code, which is designed to detect and correct single-bit errors.
- **Define Parameters:** For a (7, 4) Hamming code:
  - → $n=7$: Total bits in the codeword.
  - → $k=4$: Number of message bits.
  - → $q=n-k=3$: Number of parity bits.

$$G = [I_4|P] = \begin{matrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{matrix}$$

- **Generator Matrix:** The generator matrix ***G*** for this code can be represented using the identity and parity matrices like this: $G=[I_k|P]$.

Example: (7, 4) Hamming Code

- **Encoding Process:** To encode a dataword, say d = [1, 0, 1, 1], and generate the codeword, we multiply it by the generator matrix.

$$c = dG = 1\ 0\ 1\ 1 \begin{matrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{matrix} = 1\ 0\ 1\ 1\ 0\ 1\ 0$$

- **Parity Check Matrix:** The parity check matrix ***H*** can be derived from $[P^T | I_q]$.

$$P^T = \begin{matrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{matrix} \qquad I_3 = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad H = \begin{matrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{matrix}$$

Example: (7, 4) Hamming Code

- **Error Detection and Correction using the Syndrome Matrix**
  - → When a transmitted codeword is received with potential errors due to noise in the channel, the received vector can be checked using the parity check matrix.
  - → **Syndrome Vector**, $s = Hc^T$ **OR** cH^T;
  - → Both are correct!
  - → We assume a correct reception if s is zero, s = 0 0 0 OR $\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

Example: (7, 4) Hamming Code

- **Error Detection and Correction using the Syndrome Matrix**
  - → **Syndrome Vector,** $s = Hc^T$ **OR** cH^T;

$$H = \begin{matrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{matrix}$$

$$c^T = \begin{matrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{matrix}$$

$$s = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

c = 1 0 1 1 0 1 0

s = 0 0 0

$$H^T = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Example: (7, 4) Hamming Code

- How do we correct if there is an error?
- Suppose we received, c = 1 0 1 1 0 0 0
- Then, syndrome vector $s = cH^T$
  - → S = 0 1 0
  - → So, the error is on the 6^{th} bit. Flip it to get the correct codeword.
  - → c = 1 0 1 1 0 1 0

c = 1 0 1 1 0 0 0

s = 0 1 0

$$H^T = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ \boxed{0 \quad 1 \quad 0} \\ 0 & 0 & 1 \end{matrix}$$

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References

- Online Website Research
- Linear Block Code - Provided Materials