Lecture Note

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Contents

List of Figures

Chapter 1

Vector Multiplications

1.1 Scalar Product

Let $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are two vectors. We denote the angle θ to be the angle between the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Now, we can define a scalar multiplication between those two vectors, as

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = |\vec{\mathbf{A}}||\vec{\mathbf{B}}| \cos(\theta) \tag{1.1}$$

Note that, this quantity is a scalar quantity, therefore it has no direction.

1.1.1 Scalar product as Projection

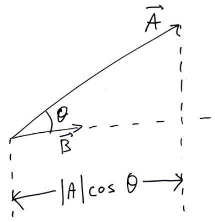

Figure 1.1: Scalar product as Projection

From the definition of scalar product, We can rewrite as,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = (|\vec{\mathbf{A}}| \cos(\theta))|\vec{\mathbf{B}}| = (|\vec{\mathbf{B}}| \cos(\theta))|\vec{\mathbf{A}}|$$

Where we can consider, $(|\vec{\mathbf{A}}| \cos(\theta))$ as a projection of $\vec{\mathbf{A}}$ on vector $\vec{\mathbf{B}}$.In the same way, we can think $(|\vec{\mathbf{B}}| \cos(\theta))$ as a projection of $\vec{\mathbf{B}}$ on vector $\vec{\mathbf{A}}$.

1.2 Some special cases

- If any of $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ or both are null vectors ($|\vec{\mathbf{A}}| = 0$ or $|\vec{\mathbf{B}}| = 0$) then,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0 \tag{1.2}$$

- If The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular, $\theta = 90°$, Then,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = |\vec{\mathbf{A}}||\vec{\mathbf{B}}| \cos(90°) = 0 \tag{1.3}$$

- Scalar product of a vector with itself:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = |\vec{\mathbf{A}}||\vec{\mathbf{A}}| \cos(0°) = |\vec{\mathbf{A}}|^2 \tag{1.4}$$

1.3 Some properties of Scalar product

- Commutative law for scalar product:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} \tag{1.5}$$

- Distributive Law:

$$\vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}} \tag{1.6}$$

- For any scalar quantity c, scalar product fulfill,

$$c(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}) = (c\vec{\mathbf{A}}) \cdot \vec{\mathbf{B}} = \vec{\mathbf{A}} \cdot (c\vec{\mathbf{B}}) = (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})c \tag{1.7}$$

1.4 Scalar product of vectors resolved in Cartesian Coordinate

In Cartesian coordinate system, we can define the unit vector $(\hat{\imath}, \hat{\jmath}, \hat{k})$ along the x, y and z-axis as,

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1 \tag{1.8}$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0 \tag{1.9}$$

As we have seen earlier in Cartesian coordinate system, we can decompose any vectors as $\vec{\mathbf{A}} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$. We can define the scalar product between two vector $\vec{\mathbf{A}} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{\mathbf{B}} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$ as,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \tag{1.10}$$

$$= A_xB_x + A_yB_y + A_zB_z \tag{1.11}$$

1.5 Exercise

- Justify the the relation in equation 2.8, 2.9 ,2.11.

- Prove that when two vector $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular, they satisfy,

$$A_x B_x + A_y B_y + A_z B_z = 0$$

- Find the value or norm of a vector $\vec{\mathbf{A}}$ by using scalar product.