BRAC University
CSE230 : Discrete Mathematics

Final Examination
5th September, 2022
Duration : 2 hours (4:30 pm - 6:30 pm)
Total Marks : 60

[Answer any 6 out of 10 questions. Answer all the sub-parts of a question together. The parts of a question are ordered according to the difficulty level. The hardest part in some questions weighs the least.]

Q01: [CO2] [Set]
A tourist fair was held last December where 300 people attended a survey. As per the survey, 30% visited England, 25% visited the United States, 20% visited Japan and 5% visited both England and the United States. Tourists that visited Japan, none of them visited England and the United States.

a) **Draw** the Venn diagram representing England as 'E', the United States as 'S' and Japan as 'J'. **[4 marks]**
b) **Find** the cardinality of the set of individuals who participated in the survey went to neither of the countries. **[3 marks]**
c) **Find** the cardinality of the complement of the E set. **[2 marks]**
d) 10% of the people who visited Japan also visited Bangkok. **Find** the cardinality of the set of people who visited Bangkok. **[1 mark]**

Q02: [CO1] [Conditional Probability]
The probability of a male being corona infected is 0.7 and if a male is infected, then the probability of his wife being corona infected is 0.9. But if the person is not infected, even then there is a chance for his wife being infected with probability 0.8.

a) Find the probability of the wife being infected **[2 marks]**
b) Find the probability of a male person not being infected if his wife is found to be infected? **[4 marks]**
c) Find the probability of the person being infected if his wife is found to be not infected? **[4 marks]**

Q04: [CO1] [Basic Probability]
A four sided die has 1,2,3, and 4 written on each of its sides. Suppose, you are repeatedly rolling two four-sided dice together. One of them is blue and the other is brown. If the blue die shows 2 and the brown die shows 4, we express the result as (2,4).

a) What is the probability that the first result will be (1,1)? **[1 mark]**
b) What is the probability that the first (2,2) will appear on the 5th roll? **[2 marks]**

c) What is the probability that (3,3) will appear exactly twice in 7 trials? **[3 marks]**
d) What is the probability that (4,4) will appear **at most** once in 5 trials? **[4 marks]**

Q06: [CO4] [Combinatorics]
BracU is arranging a table tennis tournament. 10 men and 5 women have shown their interest to participate in the tournament. However, the selection committee has decided to choose 8 players. Find the number of different ways this can be achieved if

a) There are no restrictions. **[2 mark]**
b) There is at least one woman in the tournament. **[3 marks]**
c) There are more men than women. **[3 marks]**
d) A particular man 'X' and a particular woman 'Y' can not participate in the tournament together. **[2 marks]**

Q07: [CO4] [Binomial]

a) $(a + by)^3$
b) $(1 + \frac{3}{q^2} + p + 2r)^{8a+3b}$

Answer the following questions considering the given expressions:

1) If the coefficient of y^2 is 81 and the coefficient of y^3 is 27, what are the values of a and b where $a, b > 0$ and $a, b \in R$? **[4 marks]**
2) What is the value of y in the expression $(a + by)^3$ if the 3rd term is equal to 121? **[3 marks]**
3) Using the value of a and b from the previous question, find the coefficient of the term $p^5 r^2 q^{-4}$. **[3 marks]**

Q08: [CO4] [Combinatorics]

1) How many ways are there for the letters of the word "DIAPHANOUS" to be arranged so that all the vowels stay together? **[2 marks]**
2) Consider the set of digits: {1,2,4,5,6,7}. How many odd numbers of 4 digits can be made taking each digit from the set only once? **[3 marks]**
3) How many different ways are there for 5 midfielders, 3 strikers and 6 defenders to seat in a row, where
    a) No two defenders sit side by side. **[3 marks]**
    b) Exactly four defenders sit together. (i.e. they sit side by side and there are always other type of players, if any, on both ends) **[2 marks]**

Q10: [CO2] [Function]

The function f is defined by $f(x) = 2 - \sqrt{x + 5}$ for $-5 \leq x \leq 0$

a) Find $f^{-1}(x)$ and find its range. **[4 marks]**

b) Write down the range of f. **[2 marks]**

c) Another function g is defined by $g(x) = \frac{4}{x}$. Solve $fg(x) = 0$ **[4 marks]**