

Name: _____

Midterm 2

Math 252

Winter 2022

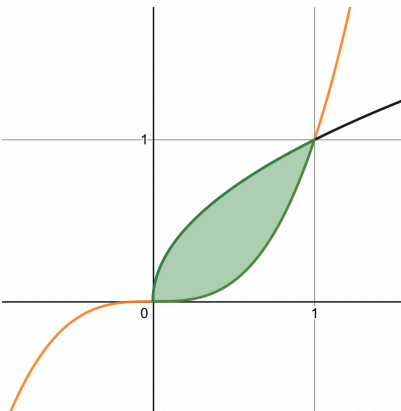
You have 50 minutes to complete this exam and turn it in. You may use a scientific calculator and a handwritten 3×5 inch index card of notes, but no other resources. When you're finished, first check your work if there is time remaining, then turn it in. If you have a question, don't hesitate to ask — I just may not be able to answer it.

Part I (24 points) Multiple choice. You don't need to show any work.

1. (8 points) Suppose $y = f(x)$, and that the graph of f is rotated about the x -axis. Then
 - A) the shell method integrates with respect to y and the disk method with respect to x .
 - B) the shell method integrates with respect to x and the disk method also with respect to x .
 - C) the shell method integrates with respect to x and the disk method with respect to y .
 - D) the shell method integrates with respect to y and the disk method also with respect to y .

2. (8 points) It takes $3 J$ of work to stretch a spring a total of 1 meter from rest. How much work does it take to compress it 2 meters from rest?
 - A) $3 J$.
 - B) $6 J$.
 - C) $9 J$.
 - D) $12 J$.

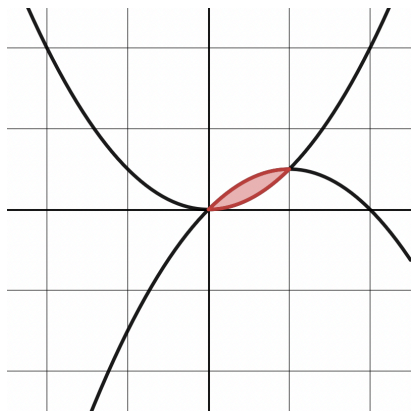
3. (8 points) Which of the following integrals calculates the area bounded by $f(x) = \sqrt{x}$ and $g(x) = x^3$?



- A) $\int_0^1 (x^3 - \sqrt{x}) \, dx$.
- B) $\int_0^1 (y^2 - \sqrt[3]{y}) \, dy$.
- C) $\int_0^1 (\sqrt{x} + x^3) \, dx$.
- D) $\int_0^1 (\sqrt[3]{y} - y^2) \, dy$.

Part II (32 points) Short answer. Show all your work.

1. (8 points) Find the area between $f(x) = x^2$ and $g(x) = x - x^2$.



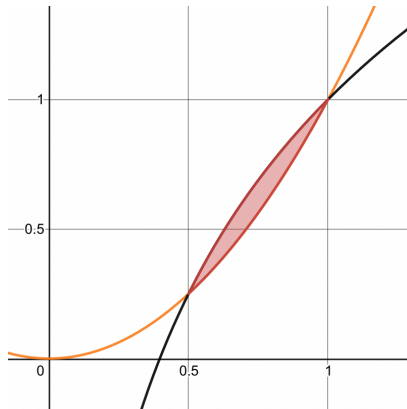
2. (8 points) Let $f(x) = 3x^2$. Set up the integrals to find the volume of the solid given by rotating the graph of f on $[0, 3]$ about the x -axis, using **both** the disk and shell methods. Don't solve either of the integrals.

3. (8 points) The density of a bar is given by $\rho(x) = \ln(x)$ for $x = e$ to $x = e^2$. Find the mass of the bar.

4. (8 points) Find the surface area of the solid created by revolving the graph of $y = x^3$ on $[0, 2]$ about the x -axis.

Part III (32 points) Longer problems that require setting up and solving integrals. Half the credit is for the set-up and half for the solving.

1. (16 points) The functions $f(x) = x^2$ and $g(x) = \frac{3}{\ln(16)} \ln(x) + 1$ intersect at $(\frac{1}{2}, \frac{1}{4})$ and $(1, 1)$ and bound a region, as shown below.



Find the volume of the solid of revolution given by rotating the region about the y -axis. You may use any method you like. **You may leave your answer in evaluation notation: e.g. $[x^2]_0^1$. No integrals should be present in your final answer.**

2. (16 points) A tank in the shape of a square pyramid has height 9 meters and a base with side length 2 meters. It's filled up to 5 meters with a liquid that has weight density $2000 \frac{N}{m^3}$. Find the work done by pumping the liquid out.

