

Name: \_\_\_\_\_

# Midterm 2

Math 256

Spring 2023

You have 50 minutes to complete this exam and turn it in. You may use a 3x5 inch two-sided handwritten index card and a scientific calculator, but not a graphing one, and you may not consult the internet or other people. If you have a question, don't hesitate to ask — I just may not be able to answer it. **Enough work should be shown that there is no question about the mathematical process used to obtain your answers.**

You should expect to spend about one minute per question per point it's worth — there are 50 points possible on the exam and 50 minutes total.



**Part I** (9 points) Multiple choice. You don't need to show your work.

1. (3 points) Only one of the following four matrices is invertible. Which one is it?

A)  $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ .

B)  $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ .

C)  $\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .

D) A  $3 \times 3$  matrix  $\mathbf{D}$  with eigenvectors  $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ , and  $\begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$ , corresponding to eigenvalues 1, 0, and  $-6$ .

2. (3 points) To solve a nonhomogeneous DE, we can use the method of undetermined coefficients or the method of variation of parameters. Which of the following DEs can **only** be solved with variation of parameters?

A)  $y'' + 2y = e^t$ .

B)  $y'' + 4y' = \sin(t) + \cos(2t)$ .

C)  $y''' - y' + y = t^2 e^{-3t}$ .

D)  $y' + y = \csc(t)$ .

3. (3 points) Matrix  $\mathbf{A}$  has 4 rows and 3 columns, and matrix  $\mathbf{C}$  has 4 rows and 2 columns. For the product  $\mathbf{AB} = \mathbf{C}$  to be defined, what must be the shape of  $\mathbf{B}$ ?

A)  $3 \times 2$ .

B)  $4 \times 4$ .

C)  $3 \times 3$ .

D) There is no shape that makes the product defined.

**Part II** (12 points) Short-answer. Explain your reasoning and show your work for each question.

1. (4 points) One of the eigenvectors of  $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$  is  $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$  with eigenvalue  $\lambda = 3$ . What is  $\mathbf{A}^8 \mathbf{v}$  (i.e. the result of multiplying  $\mathbf{A}$  by  $\mathbf{v}$  eight times)?

2. (4 points) Give an example of a differential equation whose general solution is

$$y = c_1 \cos(2t) + c_2 \sin(2t) + c_3 t \cos(2t) + c_4 t \sin(2t).$$

3. (4 points) Let  $\mathbf{B} = \begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$ . Find  $\mathbf{B}^{-1}$ .

**Part III** (29 points) More involved questions with multiple parts.

1. (14 points) Let's look at a few variations of a DE.

a) (2 points) Find the general solution to  $y'' - 4y = 0$ .

b) (6 points) Find a particular solution to  $y'' - 4y = e^{2t}$  using undetermined coefficients.

c) (6 points) Find a particular solution to  $y'' - 4y = e^{2t}$  (the same as in part b) using variation of parameters.

**2.** (15 points) Consider the system of equations

$$3x - 2y = 5$$

$$4x - 3y = 8.$$

a) (2 points) Write this system as an augmented matrix of the form  $\left[ \mathbf{A} \mid \mathbf{b} \right]$ .

b) (5 points) Solve for  $\mathbf{x}$  by row-reducing  $\mathbf{A}$ . Clearly indicate every row operation.



c) (3 points) Find the eigenvalues of  $\mathbf{A}$ .

d) (5 points) Find the corresponding eigenvectors of  $\mathbf{A}$ .