Midterm 1

Math 251

Spring 2022

You have 50 minutes to complete this exam and turn it in. **Show all your work. You may use a scientific calculator, but not a graphing one.** If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) Let f be a function that is continuous at $x = a$. Which of the following must be true?

A) f is differentiable at $x = a$.

B) $\lim_{x \to a} f(x)$ exists.

C) f is continuous.

D) $f(x)$ is defined for all x.

b) (8 points) Let $f(x) = \dfrac{x-1}{x^2-1}$. Then f has

A) a removable discontinuity at $x = 1$ and a jump discontinuity at $x = -1$.

B) a jump discontinuity at $x = 1$ and an infinite discontinuity at $x = -1$.

C) a removable discontinuity at $x = 1$ and an infinite discontinuity at $x = -1$.

D) a jump discontinuity at $x = 1$ and a jump discontinuity at $x = -1$.

c) (8 points) Which of the following functions is **not** differentiable at $x = 0$?

A) $y = |x|$.

B) $y = \dfrac{x-1}{x^2-1}$.

C) $y = x^2$.

D) $y = 0$.

d) (8 points) Which of the following is **not** equal to $f''(x)$?

A) $\dfrac{d^2}{dx^2}[f(x)]$.

B) $\lim_{x \to 0} f'(x)$.

C) $\dfrac{d}{dx}[f'(x)]$

D) $\lim_{h \to 0} \dfrac{f'(x+h) - f'(x)}{h}$.

2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

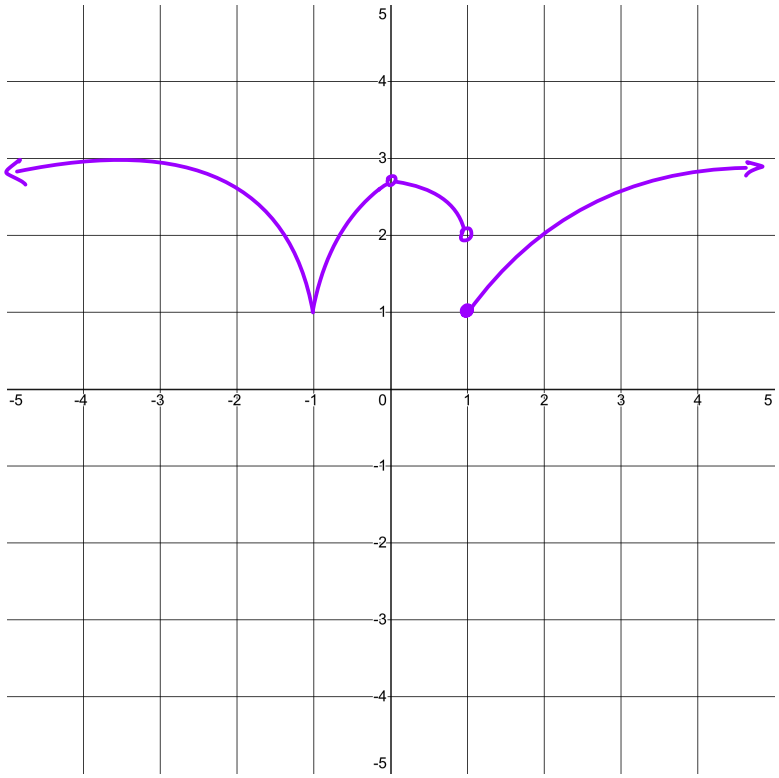
a) (8 points) Evaluate $\frac{d^2}{dy^2}[3y^4 + 2y]$.

$\frac{d}{dy}[3y^4 + 2y] = 12y^3 + 2$

$\frac{d}{dy}[12y^3 + 2] = \boxed{36y^2}$

b) (8 points) Draw a function $f(x)$ on the graph below so that:

- f is continuous but not differentiable at $x = -1$,
- f is not continuous at $x = 0$, but $\lim_{x \to 0} f(x)$ exists, and
- $\lim_{x \to 1} f(x)$ DNE.



c) (8 points) Let $g(x) = x^2 + x$. Find $g'(x)$ **from the limit definition.**

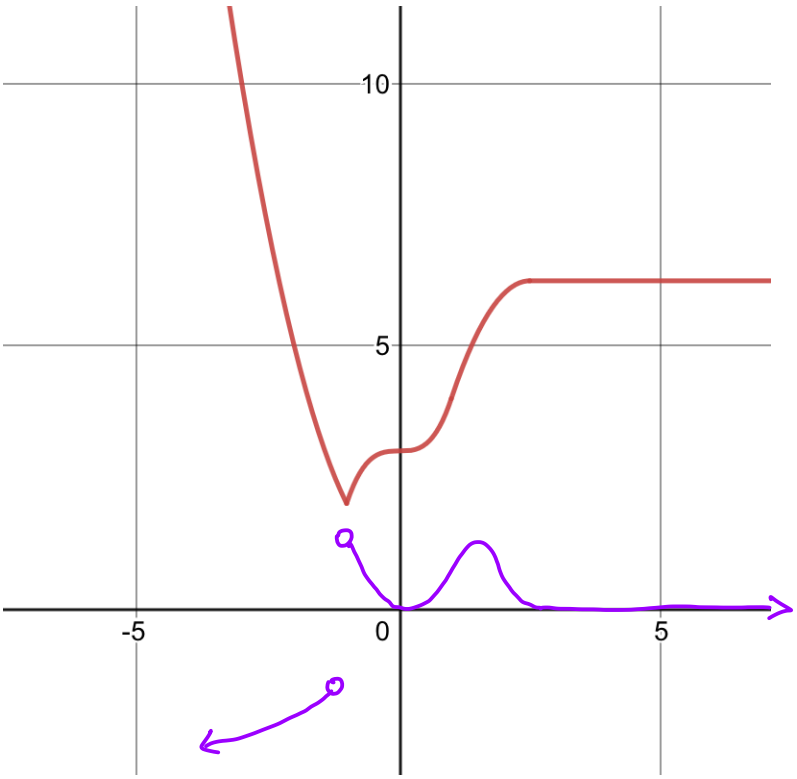
$$g'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - x^2 - x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$$

$$= \lim_{h \to 0} (2x + h + 1)$$

$$= 2x + 1.$$

d) (8 points) Given the graph of $F(x)$ below, sketch a graph of $F'(x)$.



key features:

negative and increasing before the corner
undefined at the corner
drops to zero, then back up, then back to zero after the corner — always positive

3. (32 points) You throw a ball straight up. After t seconds, its height above the ground is given by $h(t) = -4.905t^2 + 3t + 2$.

a) (12 points) Find a formula for $v(t)$, the velocity of the ball after t seconds. Include units.

$v(t) = s'(t) = -9.81t + 3 \quad \frac{m}{s}$

b) (8 points) Find a formula for $a(t)$, the acceleration of the ball after t seconds. Include units.

$a(t) = v'(t) = -9.81 \quad \frac{m}{s^2}$

c) (12 points) What is the highest the ball gets off the ground? (Hint: imagine actually throwing a ball — at the time that it reaches its maximum height, its velocity is zero).

$v(t) = 0$

$-9.81t + 3 = 0$

$t = \frac{3}{9.81} \approx .306 \text{ s}$

$h(.306) = -4.905(.306)^2 + 3(.306) + 2 \approx 2.46 \text{ m}$

d) (8 points extra credit) What is the ball's velocity when it hits the ground?

$h(t) = 0$

$-4.905t^2 + 3t + 2 = 0$

$t = \frac{-3 \pm \sqrt{9 + 4 \cdot 2 \cdot 4.905}}{-9.81}$

$t = -.402, \boxed{1.01}$

$v(1.01) = -9.81(1.01) + 3$

$= -6.91 \frac{m}{s}$

so $\boxed{6.91 \frac{m}{s}}$ down