Name: _____

Midterm 2

Math 253

Fall 2022

You have 50 minutes to complete this exam and turn it in. You may use a scientific calculator, but not a graphing one, and you may not consult the internet or other people. If you have a question, don't hesitate to ask — I just may not be able to answer it. **Enough work should be shown that there is no question about the mathematical process used to obtain your answers.**

1. (16 points) Multiple choice. You don't need to show your work.

a) (4 points) Suppose $a_n \geq 0$ for all n. Which of the following circumstances is **not** possible?

A) $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

B) $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} (-1)^n a_n$ diverges.

C) $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

D) $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} (-1)^n a_n$ diverges.

b) (4 points) Consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$. What is its interval of convergence?

A) $(0, 3)$.

B) $(-1, 1)$.

C) $(-3, 3)$.

D) $(-\infty, \infty)$

c) (4 points) What are the first three terms of $\left(\sum_{n=0}^{\infty} nx^n\right)\left(\sum_{n=0}^{\infty} 2^n x^n\right)$?

A) $0 + x + 4x^2$.

B) $1 + x + 2x^2$.

C) $0 + 2x + 8x^2$.

D) $1 + 4x + 4x^2$.

d) (4 points) The series $\sum_{n=1}^{\infty} \frac{1}{(-n)^n}$

A) converges absolutely.

B) converges conditionally.

C) diverges.

2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) Does the series $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^{2/3}}$ converge or diverge?

b) (8 points) Does the series $\sum_{k=0}^{\infty} \frac{2^{k+1}}{k!} k^2$ converge or diverge?

c) (8 points) Does $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ converge or diverge?

d) (8 points) Let $f(x) = \sum_{n=0}^{\infty} x^n$ and $g(x) = \sum_{n=2}^{\infty} \left(x - \frac{1}{2}\right)^n$. What is the interval of convergence of $f + g$?

3. (32 points) Consider the series $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$.

a) (8 points) Does $f(-1)$ converge absolutely, converge conditionally, or diverge?

b) (12 points) Estimate $f\left(-\frac{1}{2}\right)$ to within .02 of its actual value.

c) (12 points) Using your answer to part a), determine the interval and radius of convergence of f.

d) (2 points extra credit) Using your answer to part b), what do suspect the exact value of $f\left(-\frac{1}{2}\right)$ is? Hint: it involves ln.