Name: _____

Midterm 1

Math 253

Fall 2022

You have 50 minutes to complete this exam and turn it in. You may use a scientific calculator, but not a graphing one, and you may not consult the internet or other people. If you have a question, don't hesitate to ask — I just may not be able to answer it. **Enough work should be shown that there is no question about the mathematical process used to obtain your answers.**

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) In which circumstance is a sequence (a_n) guaranteed to converge?

A) (a_n) is bounded above and decreasing.

B) (a_n) is bounded above and increasing.

C) (a_n) is bounded below and increasing.

D) (a_n) is bounded above and below.

b) (8 points) Let b_n be a sequence defined by $b_1 = 3$ and $b_n = 4 - \dfrac{4}{b_{n-1}}$. What is $\lim(b_n)$?

A) 1.

B) 2.

C) 4.

D) The limit does not exist.

c) (8 points) Which of the following series **diverges**?

A) $\displaystyle\sum_{n=43}^{\infty} \frac{13}{\sqrt{n}}$.

B) $\displaystyle\sum_{k=170}^{\infty} \left(\frac{4}{7}\right)^k$.

C) $\displaystyle\sum_{j=82}^{\infty} 5j^{-6}$.

D) None of the above.

d) (8 points) The Harmonic series

A) converges by the divergence test.

B) diverges by the divergence test.

C) converges by the integral test.

D) diverges by the integral test.

2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) Let (b_n) be a sequence defined by $b_1 = 7$ and $b_n = 2nb_{n-1}$. Find an explicit formula for b_n and verify that it satisfies the recurrence.

b) (8 points) Evaluate $\sum_{k=0}^{\infty} \frac{2}{5^k}$ or show it diverges.

c) (8 points) Evaluate $\sum_{n=1}^{\infty} \left(2^n - 2^{n-1}\right)$ or show it diverges.

d) (8 points) Does the series $\sum_{i=1}^{\infty} \frac{i^2}{i^2 + 2i + 3}$ converge? Why or why not?

4. (24 points) Consider the series $\sum_{n=1}^{\infty} \frac{90}{n^4}$.

a) (4 points) Find the first three partial sums.

b) (4 points) Show that the series converges.

c) (16 points) Estimate the value of the series to within .25 of its actual value.

d) (2 points extra credit) Using your answer to part c), what do you suspect the actual value of the series is? (Hint: it's something raised to the 4th power.)