Midterm 1

Math 252

Winter 2022

You have 50 minutes to complete this exam and scan and upload it to Canvas. **Show all your work. You may use a scientific calculator, but not a graphing one.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) What is $1 + 2 + 3 + \cdots + 499 + 500$?

A) 62500.

B) 125250.

C) 175500.

D) 250000.

$\frac{500(501)}{2} = 125250$

b) (8 points) What is $\int \frac{1}{x}\, dx$?

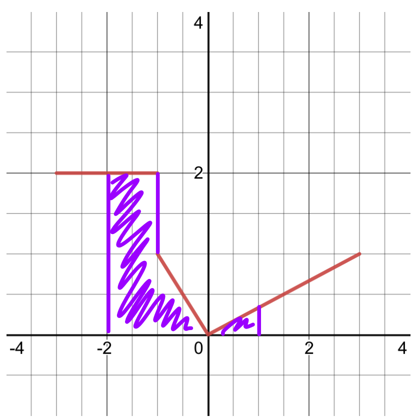
A) $\ln|x| + C$.

B) $\sin(x) + C$.

C) $-\frac{1}{x^2} + C$.

D) $x^2 + C$.

c) (8 points)



All area is above the x-axis

Let $f(t)$ be defined by the previous graph. Then $\int_{-2}^{1} f(t)\, dt$ is

A) positive.

B) negative.

C) zero.

D) undefined.

d) (8 points) With f defined from the same graph as before, let $g(x) = \int_{-3}^{x} f(t)\, dt$. Which of the following could possibly be a graph of g?

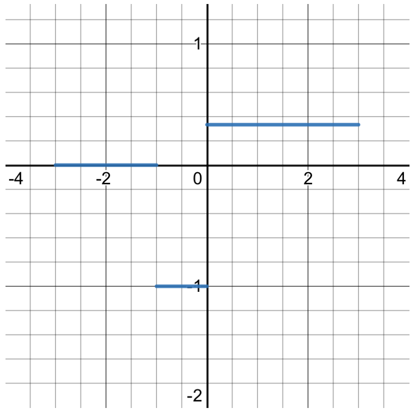
$g'(x) = f(x)$ by FTC I

$\Rightarrow$ either B or D

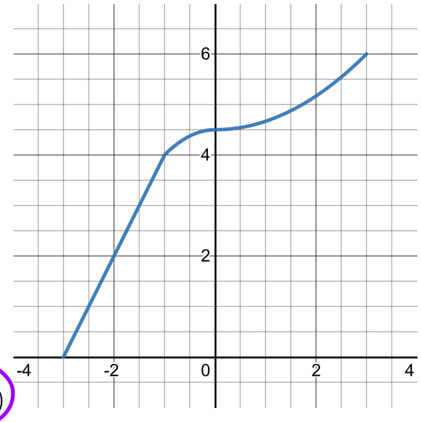
By the previous part, $g(x)$ is always positive

Alternatively, $g(-3) = 0$

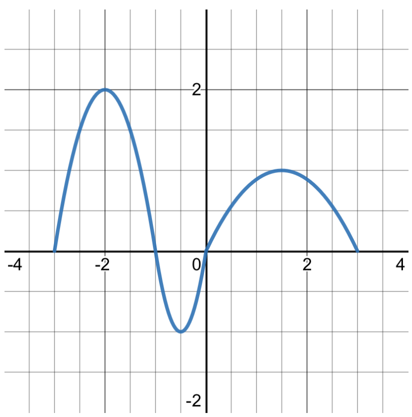
$\Rightarrow$ B



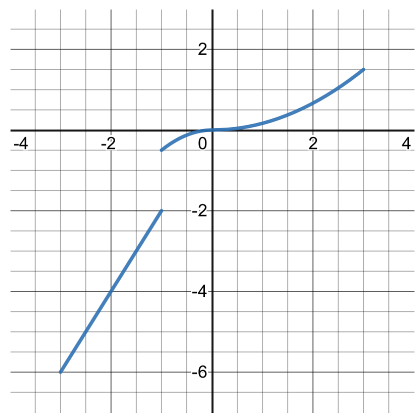
A)



B)



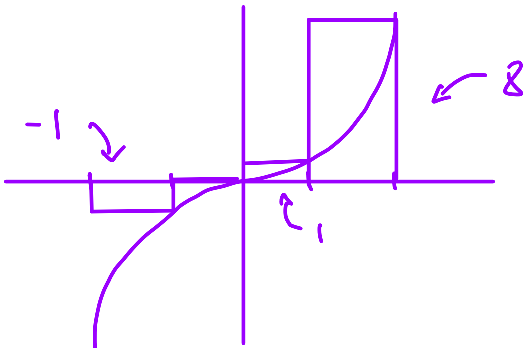
C)



D)

2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) Write and evaluate the Right Riemann sum with 4 subintervals for the function $f(x) = x^3$ on $[-2, 2]$. You don't need to simplify your answer, but it cannot contain a sum symbol.



Total : $-1 + 0 + 1 + 8 = \boxed{8}$

b) (8 points) Evaluate $\frac{d}{dx} \int_2^{\ln(x)} \frac{\sin(r)}{r}\, dr$.

Let $F(x) = \int_2^x \frac{\sin(r)}{r}\, dr$

We want $\frac{d}{dx}\left[F(\ln(x))\right] = F'(\ln(x)) \cdot \frac{d}{dx}\left[\ln(x)\right]$

$= \boxed{\frac{\sin(\ln(x))}{\ln(x)} \cdot \frac{1}{x}}$

c) (8 points) Evaluate $\int_2^4 (x^2 + x)\, dx$.

$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]\Big|_2^4$

$= \left(\frac{64}{3} + \frac{16}{2}\right) - \left(\frac{8}{3} + \frac{4}{2}\right)$

d) (8 points) Evaluate $\int 3t^3 \sin(t^4)\, dt$.

$u = t^4$

$du = 4t^3\, dt$

$\frac{1}{4} du = t^3\, dt$

$\Rightarrow \int \frac{3}{4} \sin(u)\, du$

$= -\frac{3}{4}\cos(u) + C$

$= \boxed{-\frac{3}{4}\cos(t^3) + C}$

3. (32 points) Let $v(t) = 2 - 2t$ be the velocity of a particle at time t.

a) (8 points) Find a formula for $a(t)$, the acceleration of the particle at time t.

$= v'(t) = -2$

b) (12 points) Find a formula for $s(t)$, the position of the particle at time t, given that $s(3) = 2$.

$s(t) = \int v(t)\, dt$

$= 2t - t^2 + C$

$2 = 6 - 9 + C$

$C = 5$

$s(t) = -t^2 + 2t + 5$

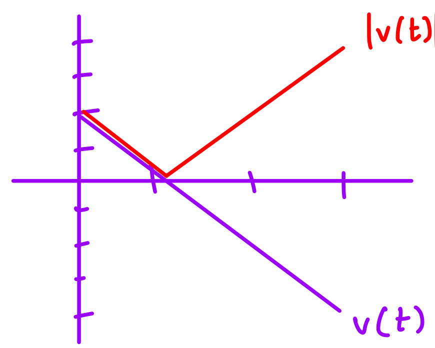
c) (12 points) Find the total distance traveled by the particle from time $t = 0$ to time $t = 3$.

$= \int_0^3 |v(t)|\, dt$

$= \int_0^1 (2-2t)\, dt + \int_1^3 -(2-2t)\, dt$

$= \left[2t - t^2\right]\Big|_0^1 - \left[2t - t^2\right]\Big|_1^3$

$= 1 - (-3 - 1) = 5$



e) (8 points extra credit) Let $e(x)$ be the average position of the particle from time 0 to time x. Find $e(x)$.

$= \frac{1}{x - 0} \int_0^x s(t)\, dt$

$= \frac{1}{x} \int_0^x (-t^2 + 2t + 5)\, dt$

$= \frac{1}{x} \left[-\frac{t^3}{3} + t^2 + 5t\right]\Big|_0^x$

$= \frac{1}{x}\left(-\frac{x^3}{3} + x^2 + 5x\right)$

$= -\frac{x^2}{3} + x + 5$