Homework 2

Math 253

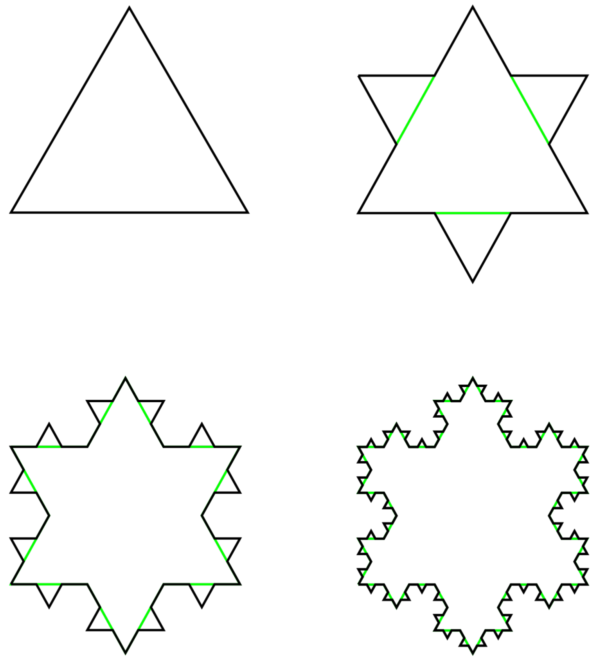
Due October 12th at 11:59 PM

Work through the following problems and write your solutions on separate sheets of paper, showing all your work. There are 32 points possible: 8 are from attempting all the problems thoroughly, and three problems will be selected to be graded for correctness for 8 points each.

5.2: 68, 73, 75, 81, 83, 87, 89, 97, 99, 101, 103

An additional problem (also required):

The Koch snowflake is a fractal, an object that is infinitely detailed no matter how far you zoom in. Begin with an equilateral triangle with side lengths equal to 1, and remove the middle third of each side, replacing it with two additional sides of equal length. Then repeat this for all 12 sides of the new figure, and so on. The limit of this sequence of figures is the completed Koch snowflake. A diagram of the first four steps is on the next page — the green lines are what has been most recently removed and are not part of the snowflake.



1. Let c_n be the number of edges in the snowflake at step n, and let l_n be their lengths. For example, $c_0 = 3$, since we start with a triangle, and $l_0 = 1$, since the sides of the triangle all have length 1. In the next step, $c_1 = 12$ and $l_1 = \frac{1}{3}$. Find explicit formulas for c_n and l_n.

2. Let p_n be the perimeter of the snowflake at step n. Using your answer to part 1, find an explicit formula for p_n. Then find the perimeter of the completed snowflake by taking the limit of the sequence (p_n).

3. Let A be the area of the completed Koch snowflake and let a_n be the amount the area increases at step n, where $n \geq 1$. For example, step 1 is going from the original triangle to the six-pointed star. That adds three triangles, each of area $\frac{\sqrt{3}}{36}$. Therefore, $a_1 = \frac{\sqrt{3}}{12}$. Using your answer to part 1, find an explicit formula for a_n. Since the area of the original triangle is $\frac{\sqrt{3}}{4}$, the total area A of the completed snowflake is

$$A = \frac{\sqrt{3}}{4} + \sum_{n=1}^{\infty} a_n.$$

Find A.

If all went well, you should have shown that the Koch snowflake is a shape with finite area but infinite perimeter. It's counterintuitive that something like it could exist!