

# Midterm 2 Review

Math 253

## Section 4

1. Show that  $\sum_{n=1}^{\infty} \frac{1}{n^2 + 20}$  converges.

2. Does the series  $\sum_{n=1}^{\infty} \frac{1}{n^4 + n^3}$  converge or diverge?

3. Determine the behavior of  $\sum_{m=2}^{\infty} \frac{1}{\ln(m)}$ .

4. Does  $\sum_{n=1}^{\infty} \frac{1}{2^n + 3}$  converge?

5. Determine if  $\sum_{k=1}^{\infty} \frac{2^k + 3^k}{4^k}$  converges or diverges.

6. Does the series  $\sum_{n=1}^{\infty} \frac{2^n - n}{3^n}$  converge?

7. Does the series  $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n} + 10}$  converge or diverge?

8. What is the behavior of the series  $\sum_{k=1}^{\infty} \frac{k^2}{k(k+3)}$ ?

9. Determine the behavior of  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ .

## Section 5

1. Does  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$  converge absolutely, conditionally, or not at all?

2. Does  $\sum_{i=1}^{\infty} \left(-\frac{1}{i}\right)^i$  converge absolutely, conditionally, or not at all?

3. Does  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sin(n)}$  converge absolutely, converge conditionally, or diverge?

4. Show that  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$  converges, and then estimate it to within .001 of its true value.

5. Show that  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/4}}$  converges, and then estimate it to within .2 of its true value.

6. Does the series  $\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln(k)}$  converge absolutely, conditionally, or not at all?

7. Does the series  $\sum_{k=1}^{\infty} \frac{\sin(k)}{k^2}$  converge absolutely, conditionally, or not at all?

## Section 6

1. Does the series  $\sum_{n=0}^{\infty} \frac{e^n}{n!}$  converge or diverge?

2. Does the series  $\sum_{n=0}^{\infty} \frac{(-1)^n n}{2^n}$  converge absolutely, conditionally, or not at all?

3. Determine the behavior of the series  $\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$ .

4. Apply the ratio test to the following series and state what conclusion you can draw.

a)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ .

b)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ .

c)  $\sum_{n=1}^{\infty} (-1)^n$ .

5. Does  $\sum_{n=1}^{\infty} \frac{\sin^n(n)}{n^n}$  converge absolutely, conditionally, or not at all?

6. Does  $\sum_{n=1}^{\infty} \frac{n}{2^n}$  converge or diverge?

## Section 7

1. Find the interval and radius of convergence of  $\sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n(n+1)}$ .

2. Find the interval and radius of convergence of  $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ .

3. Write  $\frac{x}{1+3x^2}$  as a power series and state the interval and radius of convergence.

4. Write  $\frac{x^3}{2+x^2}$  as a power series and state the interval and radius of convergence.

## Section 8

1. Express  $f(x) = \frac{5}{(1-2x)(1+x^2)}$  as a power series in two ways: first with partial fractions and then with power series multiplication. For each, find the interval and radius of convergence, and make sure they match.

2. Express  $g(x) = \frac{6}{(1-x)(2-x)(3-x)}$  as a power series by using partial fractions and determine the interval and radius of convergence.

3. Express  $h(x) = \frac{1}{(1-x)^2}$  as a power series by using power series multiplication.