STATISTICAL LEARNING THEORY & APPLICATIONS

Assignment & Research Report

Fall, 2020

1. Submission Method & Due Day

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2. Research Report (20% of Final Grading)

1. **Scope:** Study some papers related to statistical learning theory and application for a subject or direction of your interest, with at least five papers in the top conferences and journals in the past five years, for example in Science, Nature, IEEE Transactions on Pattern Analysis and Machine Intelligence, IEEE Transactions on Image Processing , IEEE CVPR, IEEE ICCV, and etc.
2. **Format:**
   (a) Write a research report or literature review on the theory and application of statistical learning;
   (b) Written in English with no less than 2,000 words;
   (c) Electronic version in Word or PDF format;
   (d) Citing at least 5 excellent papers;
   (e) Indicating your name and student ID in the report or review.
3. **Submission Requirements:**
   (a) One research report or review;
   (b) Five electronic versions of the cited references.

3. Assignment (20% of Final Grading)

3.1. Problem #1

Determine whether the following functions are kernel functions and explain the reasons.

Assuming $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{R}^n$:

$$k(\mathbf{x}, \mathbf{y}) = (\mathbf{y}^T \mathbf{x} + 10)^7 \tag{1}$$

$$k(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2 \tag{2}$$

$$k(\mathbf{x}, \mathbf{y}) = (\mathbf{x} - 5\mathbf{z})^T (\mathbf{y} - 5\mathbf{z}) \tag{3}$$

3.2. Problem #2

The following convex optimization problem with constraints is a constrained optimization problem:

$$\min_{x} \quad f\ (x) \tag{4}$$

$$\text{s.t.} \quad c_i(x) \leq 0, \quad i = 1, 2, \ldots, k \tag{5}$$

$$h_i(x) = 0, \quad i = 1, 2, \ldots, m \tag{6}$$

where the object function $f\ (x)$ and the constraint function $c_i(x)$ are all continuous and differentiable convex functions on $\mathbf{R}^n$, $h_i(x)$ is affine function on $\mathbf{R}^n$. The generalized Lagrange function for the above convex optimization problem is:

$$L(x, \alpha, \beta) = f\ (x) + \sum_{i=1}^{k} \alpha_i c_i(x) + \sum_{j=1}^{m} \beta_j h_j(x), \quad \alpha_i \geq 0 \tag{7}$$

$$\theta_P(x) = \max_{\alpha,\ \beta;\ \alpha_i \geq 0} L(x, \alpha, \beta) \tag{8}$$

$$\theta_D(\alpha, \beta) = \min_{x} L(x, \alpha, \beta) \tag{9}$$

If x has a feasible solution, please analyze the following questions from the perspective of convex and concave functions:

(1) $\theta_P(x)$ is what kind function of x ?

(2) $\theta_D(\alpha, \beta)$ is what kind of function of α, β ?