1.解: ① $\left(\frac{dy}{dx}\right)^2-3\frac{dy}{dx}+2=0$

② $a_{12}^2-a_{11}a_{22}=\frac{9}{4}-2>0$. 则为双曲型.

$\begin{cases} C_1=y-x \\ C_2=y-2x \end{cases}$ 则令 $\begin{cases} \xi=y-x \\ \eta=y-2x \end{cases}$ $\Rightarrow Q=\begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}$

③. $\begin{pmatrix} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{pmatrix}=Q\begin{pmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & 2 \end{pmatrix}Q^T=\begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3/2 \\ 3/2 & 2 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$

$\bar{b}_1=L\xi-C\xi=0$ $\quad \bar{b}_2=L\eta-C\eta=0$ $\quad \bar{c}=C=0$ $\quad \bar{f}=f=0$

$\Rightarrow U_{\xi\eta}=0$ $\Rightarrow U=f(y-x)+g(y-2x)$

2.解: 令 $u(x,t)=V(x,t)+W(x)$

$\begin{cases} 0=a^2W_{xx}+f(x) \\ W(0)=A \quad W(L)=B \end{cases} \Rightarrow W(x)$

$\begin{cases} V_{tt}=a^2V_{xx} \\ V|_{x=0}=V|_{x=L}=0 \\ V|_{t=0}=\varphi(x)-W(x) \quad V_t|_{t=0}=\psi(x). \end{cases}$ 此时可利用分离变量法求解

3.解: $dQ=C\rho\rho\, dx\, U_t\, dt$ $\quad dQ_1=-KAU_x(x,t)dt$ $\quad dQ_2=-KAU_x(x+dx,t)dt$

又 $dQ'=dQ_1-dQ_2$. $\Rightarrow U_t=\frac{k}{C\rho}U_{xx}=a^2U_{xx}$

$\Rightarrow \begin{cases} U_t=a^2U_{xx} \\ U|_{x=0}=0, \quad U|_{x=L}=0 \\ U|_{t=0}=\varphi(x) \end{cases}$ 令 $U(x,t)=X(x)T(t)$

$\begin{cases} X''+\lambda X=0 \\ T'+\lambda a^2T=0 \end{cases} \Rightarrow \begin{cases} X''+\lambda X=0 \\ X(0)=X(L)=0 \end{cases}$ $\quad \lambda_n=\left(\frac{n\pi}{L}\right)^2$ $\quad X_n(x)=\sin\frac{n\pi}{L}x$

$T_n(t)=C\cdot e^{-\lambda a^2t}=C\cdot e^{-\frac{n\pi a^2}{L}t}$

$\Rightarrow U(x,t)=\sum_{n=1}^{\infty}C\cdot e^{-\frac{n\pi a^2}{L}t}\cdot\sin\frac{n\pi}{L}x$

$\Rightarrow \varphi(x)=\sum_{n=1}^{\infty}C\cdot\sin\frac{n\pi}{L}x.$ $\quad$ 则 $C=\frac{2}{L}\int_0^L\varphi(x)\sin\frac{n\pi}{L}x\,dx$

4.解: 令 $U(x,t)=V(x,t)+W(x,t)$

$\begin{cases} V_{tt}=9V_{xx} \\ V|_{t=0}=x^2+x+18 \\ V_t|_{t=0}=\sin x+18 \end{cases} \Rightarrow V(x,t)=\frac{1}{2}\left[(x+3t)^2+(x+3t)+18-(x-3t)^2-(x-3t)-18\right]$

$+\frac{1}{6}\int_{x-3t}^{x+3t}(\sin\xi+18)d\xi$

$\begin{cases} W_{tt}=9W_{xx}+x^2e^t \\ W|_{t=0}=0 \end{cases} \Rightarrow W(x,t)=\int_0^t\frac{1}{6}\int_{x-3(t-\tau)}^{x+3(t-\tau)}(\xi^2e^{\tau})d\xi d\tau$

$\left\lfloor W|_{t=0}=0 \right.$

$\Rightarrow u(x,t)=v(x,t)+w(x,t)$

5. 解: $= \int_0^{+\infty} e^{-a^2 2\omega^2} \frac{e^{j\omega x}+e^{-j\omega x}}{2} d\omega \quad = \frac{1}{2}\int_0^{+\infty} e^{-a^2 2\omega^2 + j\omega x} d\omega + \frac{1}{2}\int_0^{+\infty} e^{-a^2 2\omega^2 - j\omega x} d\omega$

$= \frac{1}{2}\cdot\frac{1}{-2a^2 2\omega + jx} e^{-a^2 2\omega^2 + j\omega x}\Big|_0^{+\infty}$

$+ \frac{1}{2}\cdot\frac{1}{-2a^2 2\omega - jx} e^{-a^2 2\omega^2 - j\omega x}\Big|_0^{+\infty}$

$= -\frac{1}{2}\frac{1}{jx} - \frac{1}{2}\frac{1}{-jx}$

$= \frac{1}{2}\left(\frac{1}{jx} - \frac{1}{jx}\right) = 0$

6. 解:

$F(s) = \frac{\frac{1}{3}}{s^2+2s+2} + \frac{\frac{2}{3}}{s^2+2s+5}$

$= \frac{1}{3}\frac{1}{(s+1)^2+1} + \frac{2}{3}\frac{2^2}{(s+1)^2+4}\cdot\frac{1}{4}$

根据 $\widehat{f(\sin ax)} = \frac{a}{s^2+a^2}$

$\Rightarrow L^{-1}\{F(s)\} = \frac{1}{3}\sin x\cdot e^{-x} + \frac{1}{6}\sin 2x e^{-x}$

7. 解: (1) $\begin{cases} \Delta G = -\delta(M-M_0) \\ G|_S = 0. \end{cases}$ $\quad x^2+y^2+z^2 \leq r^2$

$G(M,M_0) = \frac{1}{4\pi}\left(\frac{1}{r_{MM_0}} - \frac{R}{r_0}\frac{1}{r_{MM_1}}\right)$

(2). $\frac{\partial G(M,M_0)}{\partial \vec{n}} = -\frac{1}{4\pi R}\frac{R^2-r_0^2}{[R^2+r_0^2-2Rr_0\cos\gamma]^{3/2}}$

9. 解: (1). $P_n(x) = \sum_{n=1}^{M} (-1)^m \frac{(2n-2m)!}{2^n m!(n-m)!(n-2m)!} x^{n-2m}$ $\quad m = \begin{cases} \frac{n}{2} & n=2k \\ \frac{n-1}{2} & n=2k+1 \end{cases}$

$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$

$\Rightarrow 2+3x+x^2 = C_0P_0(x)+C_1P_1(x)+C_2P_2(x)$

$= C_0 + C_1 x + C_2\cdot\frac{1}{2}(3x^2-1)$ $\Rightarrow \begin{cases} C_0-\frac{1}{2}C_2=2 \\ C_1=3 \\ \frac{3}{2}C_2=1 \end{cases} \Rightarrow \begin{cases} C_0=\frac{7}{3} \\ C_1=3 \\ C_2=\frac{2}{3} \end{cases}$

$= C_0 - \frac{1}{2}C_2 + C_1 x + \frac{3}{2}C_2 x^2$

$\Rightarrow f(x) = 2+3x+x^2 = \frac{7}{3} + 3x + \frac{2}{3}\cdot\frac{1}{2}(3x^2-1)$