1.解: ① $(\frac{dy}{dx})^2-2\frac{dy}{dx}-3=0$

② $a_{12}^2-a_{11}a_{22}=1+3=4>0$ 双曲型

$\begin{cases}C_1=y+x\\ C_2=y-3x\end{cases}$ 令 $\begin{cases}\xi=y+x\\ \eta=y-3x\end{cases}$ $Q=\begin{pmatrix}1&1\\-3&1\end{pmatrix}$ 则 $\begin{pmatrix}\bar{a}_{11}&\bar{a}_{12}\\ \bar{a}_{12}&\bar{a}_{22}\end{pmatrix}=\begin{pmatrix}1&1\\-3&1\end{pmatrix}\begin{pmatrix}1&1\\1&-3\end{pmatrix}\begin{pmatrix}1&-3\\1&1\end{pmatrix}$

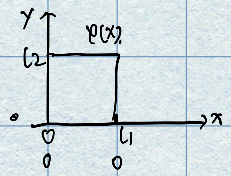
$=\begin{pmatrix}0&-8\\-8&0\end{pmatrix}$ $\bar{b}_1=L\xi-c\xi=0$ $\bar{c}=c=0$

$\bar{b}_2=L\eta-c\eta=0$ $\bar{f}=f=0$

③ $u_{\xi\eta}=0 \Rightarrow u=f(y+x)+g(y-3x)$

2.解: $\lambda_n=[\frac{(n+\frac{1}{2})\pi}{L}]^2$. $X_n(x)=\sin\frac{(n+\frac{1}{2})\pi}{L}x$

3.解:



$\begin{cases}u_{xx}+u_{yy}=0 & 0<x<l_1,\ 0<y<l_2\\ u|_{x=0}=0,\ u|_{x=l_1}=0\\ u|_{y=0}=0\ \ u|_{y=l_2}=\varphi(x). & \varphi(0)=0.\end{cases}$

令 $u(x,y)=X(x)Y(y) \Rightarrow X''Y+XY''=0 \Rightarrow \frac{X''}{X}+\frac{Y''}{Y}=0 \Rightarrow \frac{X''}{X}=\frac{-Y''}{Y}=-\lambda$

$\Rightarrow \begin{cases}X''+\lambda X=0\\ Y''-\lambda Y=0.\end{cases} \Rightarrow \begin{cases}X''+\lambda X=0\\ X(0)=X(l_1)=0.\end{cases}$ $\lambda_n=(\frac{n\pi}{l_1})^2. \Rightarrow X_n(x)=\sin\frac{n\pi}{l_1}x.$

又 $Y_n(y)=C\cdot e^{\sqrt{\lambda}y}+De^{-\sqrt{\lambda}y}=C\cdot e^{\frac{n\pi}{l_1}y}+D\cdot e^{-\frac{n\pi}{l_1}y}$

$\Rightarrow u(x,y)=\sum_{n=1}^{\infty}(C\cdot e^{\frac{n\pi}{l_1}y}+D\cdot e^{-\frac{n\pi}{l_1}y})\cdot\sin\frac{n\pi}{l_1}x$

$\Rightarrow \begin{cases}0=\sum_{n=1}^{\infty}(C+D)\sin\frac{n\pi}{l_1}x \Rightarrow 0=C+D \Rightarrow -C=D\\ \varphi(x)=\sum_{n=1}^{\infty}(C\cdot e^{\frac{n\pi}{l_1}l_2}+De^{-\frac{n\pi}{l_1}l_2})\sin\frac{n\pi}{l_1}x\end{cases}$

$\Rightarrow \varphi(x)=\sum_{n=1}^{\infty}C(e^{\frac{n\pi}{l_1}l_2}-e^{-\frac{n\pi}{l_1}l_2})\sin\frac{n\pi}{l_1}x$

$\Rightarrow C=\frac{2}{l_1}\int_0^{l_1}\frac{1}{e^{\frac{n\pi}{l_1}l_2}-e^{-\frac{n\pi}{l_1}l_2}}\cdot\varphi(x)\sin\frac{n\pi}{l_1}x\,dx$

$=\frac{2}{l_1(e^{\sim}-e^{\sim})}\int_0^{l_1}\varphi(x)\sin\frac{n\pi}{l_1}x\,dx$

$\Rightarrow u(x,y)=\sum_{n=1}^{\infty}C(e^{\frac{n\pi}{l_1}y}-e^{-\frac{n\pi}{l_1}y})\sin\frac{n\pi}{l_1}x$

4.解: 令 $u(x,t)=v(x,t)+w(x,t)$

$\begin{cases}v_{tt}=v_{xx}\\ v|_{t=0}=0.\ v_t|_{t=0}=\sin x\end{cases} \Rightarrow v(x,t)=\frac{1}{2}\int_{x-t}^{x+t}\sin\xi\,d\xi=\frac{1}{2}\sin x\sin t$

$$\begin{cases} w_{tt} = w_{xx} + t\sin x \\ w|_{t=0} = w_t|_{t=0} = 0 \end{cases} \Rightarrow w(x,t) = \int_0^t \frac{1}{2} \int_{x-(t-\tau)}^{x+(t-\tau)} 2\cdot\sin\xi \, d\xi \, d\tau = \frac{1}{2}\sin x(\cos t - 1)$$

$$\Rightarrow u(x,t) = \frac{1}{2}\sin x(\sin t + \cos t - 1)$$

6. 解. $\frac{s}{s^2-2s+5} = \frac{s}{(s-1)^2+2^2}$

$$\Rightarrow L^{-1}\left(\frac{1}{s^2-2s+5}\right) = \frac{1}{2}\sin 2x e^x$$

$$L^{-1}\left(\frac{s}{s^2-2s+5}\right) = \left(\frac{1}{2}\sin 2x e^x\right)' = \cos 2x e^x + \frac{1}{2}\sin 2x e^x = e^x\left(\frac{1}{2}\sin 2x + \cos 2x\right)$$

7. 解, $\begin{cases} \Delta G = -\delta(M-M_0) & x>0, y>0 \\ G|_{x=y=0} = 0 \end{cases}$

$$G(M,M_0) = \frac{1}{2\pi}\left(\ln\frac{1}{r_{MM_0}} - \ln\frac{1}{r_{MM_1}}\right) - \frac{1}{2\pi}\left(\ln\frac{1}{r_{MM_2}} - \ln\frac{1}{r_{MM_3}}\right)$$

8. 解: $\int x^4 J_1(x)dx$ $\qquad [x^n J_n(x)]' = x^n J_{n-1}(x)$

$$= \int x^2 x^2 J_1(x) dx$$

$$= \int x^2 d x^2 J_2(x) = x^4 J_2(x) - 2\int x^3 J_2(x) dx = x^4 J_2(x) - 2x^3 J_3(x) + C$$