习题二

X	2	3	4	5	6	7	8	9	10	11	12
P	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

习题三

1个盒子有球的概率 $C_3^1/3^3=3/27$

2个盒子有球的概率 $C_3^2\cdot C_2^1 C_3^2/3^3=18/27$

3个球选2个放入一个盒子

3个盒子选2个

2个盒子选一个

3个盒子有球的概率 $C_3^1\cdot C_2^1/3^3=6/27$

X	1	2	3
P	$3/27$	$18/27$	$6/27$

$$E(X)=1\times\frac{3}{27}+2\times\frac{18}{27}+3\times\frac{6}{27}=\frac{19}{9}$$

X^2	1	4	9
P	$3/27$	$18/27$	$6/27$

$$E(X^2)=1\times\frac{3}{27}+4\times\frac{18}{27}+9\times\frac{6}{27}=\frac{43}{9}$$

$$D(X)=E(X^2)-[E(X)]^2=\frac{43}{9}-\frac{19}{9}\times\frac{19}{9}=\frac{26}{81}$$

习题四

X	0	1	2
P	$1/5$	$3/5$	$1/5$
	$\frac{C_4^3}{C_6^3}$	$\frac{C_2^1C_4^2}{C_6^3}$	$\frac{C_4^1}{C_6^3}$

Y \ X	0	1	2
0	$\frac{1}{3}$	$\frac{2}{9}$	0
1	$\frac{2}{3}$	$\frac{5}{9}$	$\frac{2}{3}$
2	0	$\frac{2}{9}$	$\frac{1}{3}$

Y	0	1	2
P	$3/15$	$9/15$	$3/15$

$3/5\times 2/9+1/5\times 1/3$

$1/5\times 1/3+3/5\times 2/9$

$1/5\times 2/3+3/5\times 5/9+1/5\times 2/3$

习题五

X \ Y	0	1	合计
0	$1/4$	$1/8$	$3/8$
1	$3/8$	$1/4$	$5/8$
合计	$5/8$	$3/8$	

$\therefore X,Y$不独立

X	0	1
P	$3/8$	$5/8$

$E(X) = 5/8$

$D(X) = \frac{15}{64}$

同理

Y	0	1
P	$5/8$	$3/8$

$E(Y) = 3/8$

$D(Y) = \frac{15}{64}$

Q $E(XY) = \frac{1}{4}$ $\therefore \text{cov}(X,Y) = E(XY) - E(X)E(Y) = 1/4 - 15/64 = 1/64$

$$\therefore r_{XY} = \frac{\text{cov}(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{1/64}{15/64} = \frac{1}{15}$$

$$F(x,y) = \begin{cases} 0 & x < 0 \text{或} y < 0 \\ 1/4 & 0 \le x < 1, 0 \le y < 1 \\ 3/8 & 0 \le x < 1, y \ge 1 \\ 5/8 & x \ge 1, 0 \le y < 1 \\ 1 & x \ge 1, y \ge 1 \end{cases}$$

习题六

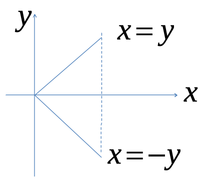
全概率公式

Y	0	1
P	$(1-p)p_0 + pq_1$	$pq_1 + (1-p)(1-p_0)$

发生误码的概率 $P = pq_1 + (1-p)(1-p_0)$

习题十六

$$f(x,y) = \begin{cases} 1 & |y| < x < 1 \\ 0 & \text{其他} \end{cases}$$



$f_X(x) = \int_{-x}^{x} dy = 2x$

$f_y(y) = \int_{|y|}^{1} |dx = 1 - |y|$

由于 $f(x,y) \ne f_X(x) f_y y$ $\therefore$ 不成立

$$E(x)=\int_0^1 2x^2dx=\tfrac{2}{3}\qquad E(y)=\int_0^1(y-y^2)dy+\int_{-1}^0(y+y^2)dy=\tfrac{1}{2}$$

$$E(XY)=\iint xyf(x,y)dxdy=\int_0^1 xdx\int_{-x}^{x}ydy=0$$

$$\mathrm{cov}(X,Y)=E(XY)-E(X)E(Y)=-\frac{1}{3}\quad \text{相关}$$

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{1}{1-|y|}\qquad f_{y|x}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{1}{2x}$$

$$E[X|Y=y]=\int_{|y|}^1 xf_{X|Y}(x|y)dx=\frac{|y|+1}{2}\qquad |y|<1$$

$$E[X|Y]=\frac{1+|Y|}{2}\qquad |Y|<1$$

$$E[Y|X=x]=\int_{-x}^{x}yf_{Y|X}(y|x)dy=0\qquad 0<x<1$$

$$E[Y|X]=0\qquad 0<x<1$$

习题十七

$$f_Y(y)=\int_0^{+\infty}\tfrac{1}{y}e^{-\frac{x}{y}}e^{-y}dx=e^{-y}\qquad 0<y<+\infty$$

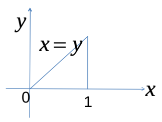
$$f_{X|Y}(x,y)=\frac{f(x,y)}{f_Y(y)}=\tfrac{1}{y}e^{-\frac{x}{y}}e^{-y}/e^{-y}=\tfrac{1}{y}e^{-\frac{x}{y}}\qquad \begin{matrix}0<x<+\infty\\0<y<+\infty\end{matrix}$$

$$E[X|Y=y]=\int_0^{+\infty}x\cdot\tfrac{1}{y}e^{-\frac{x}{y}}dx\quad \text{令}u=\tfrac{x}{y}\Big|_0^{+\infty}$$

$$\text{原式}=\int_0^{+\infty}ue^{-u}d(uy)=y\int_0^{+\infty}ue^{-u}du=y\qquad 0<y<+\infty$$

$$E[X|Y]=Y\qquad 0<Y<+\infty$$

习题十八



$$f_X(x)=\int_0^x e^{-x}dy=xe^{-x}\qquad 0<x<1$$

$$f_Y(y)=\int_y^1 e^{-x}dx=e^{-y}-1\qquad 0<y<1$$

由于$f(x,y)\neq f_X(x)f_Y(y)$　x,y不独立

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{e^{-x}}{xe^{-x}}=\frac{1}{x}\qquad 0<x<1$$

$$E[Y|X=x]=\int_0^x yf_Y(y|x)dy=\int_0^x y\cdot\tfrac{1}{x}dy=\frac{x}{2}\qquad 0<x<1$$

$$E[Y|X]=\frac{x}{2}\qquad 0<x<1$$

习题二十二

由于正态分布的可加性

$Y_1,Y_2,Y_3\sim N(0,1)$

$E(Y_1Y_2)$

$$=E\left(X_1^2+X_1X_2-2X_1X_3-X_1X_1-X_2^2+2X_2X_3\right)\frac{1}{2\sqrt{3}}=0$$

同理$E(Y_2Y_3)=E(Y_1Y_3)=0$

$\therefore \mathrm{cov}(Y_1,Y_2)=\mathrm{cov}(Y_2,Y_3)=\mathrm{cov}(Y_1,Y_3)=0$

利用课本$p.45$例14的方法还可证明 $E[Y_1Y_2Y_2]=0$

$=E[Y_1]E[Y_2]E[Y_3]$

$\Rightarrow Y_1,Y_2,Y_3$相互独立

得Y_1,Y_2,Y_3相互独立，且服从$N(0,1)$的正态分布

习题二十五

题目有错误，应为 $X \sim P(l_1), Y \sim P(l_2)$，求证 $(X+Y) \sim P(l_1+l_2)$

$$j_{X+Y}(u) = j_X(u) j_Y(u) = E\left[e^{iuX}\right] E\left[e^{iuY}\right] = e^{l_1 t(e^{iu}-1)} e^{l_2 t(e^{iu}-1)}$$

$$= E\left[e^{iuZ}\right], Z \sim P(l_1+l_2)$$

$$\Rightarrow (X+Y) \sim P(l_1+l_2)$$

$$P\{X=k|X+Y=n\} = \frac{P\{X=k, x+y=n\}}{P\{x+y=n\}} = \frac{P\{x=k\}P\{y=n-k\}}{P\{X+Y=n\}}$$

$$= \frac{\frac{l_1^k}{k!} e^{l_1} \cdot \frac{l_2^{(n-k)}}{(n-k)!} e^{l_2}}{\frac{(l_1+l_2)^2}{n!} e^{l_1+l_2}} = C_n^k \left(\frac{l_1}{l_1+l_2}\right)^k \left(\frac{l_2}{l_1+l_2}\right)^{n-k}$$

$$E[X|X+Y=n] = \sum_{k=0}^{+\infty} x_k C_n^k \left(\frac{l_1}{l_1+l_2}\right)^k \left(\frac{l_2}{l_1+l_2}\right)^{n-k} \qquad k \le n$$

$$= \sum_{k=0}^{+\infty} x_k C_n^k \left(\frac{l_1}{l_1+l_2}\right)^k \left(\frac{l_2}{l_1+l_2}\right)^{n-k}$$

习题二十七

X	2	$3+E(X)$	$5+E(X)$
P	$1/3$	$1/3$	$1/3$

$$E(X) = 2 \times 1/3 + [3+E(X)] \times 1/3 + [5+E(X)] \times 1/3$$

$$= \frac{2}{3} E(X) + \frac{10}{3} \qquad \therefore E(X) = 10$$

习题二十九

记作 $x_i = \begin{cases} 1 & 第\, i+1 \,层停止 \\ 0 & 不停止 \end{cases}$

$Y = \sum_{i=1}^{N} x_i$ $\quad$ Y 表示电梯的停止次数 $\qquad E(Y) = \sum_{i=1}^{N} E(x_i)$

设 M 为电梯内的人数 $\quad M \sim P(10)$

$$E[M_i] = E[E[x_i|M]] = \sum_{k=1}^{\infty} E[x_i|M=k] \cdot P\{m=k\}$$

$$= \sum_{k=1}^{\infty} \left(1 - \left(\tfrac{N-1}{N}\right)^k\right) P\{M=k\} = \sum_{k=1}^{\infty} P\{M=k\} + \sum_{k=1}^{\infty} \left(\tfrac{N-1}{N}\right)^k P\{M=k\}$$

$$= (1-e^{-l}) + \sum_{k=1}^{\infty} \left(\tfrac{N-1}{N}\right)^k \frac{(l)^k}{k!} e^{-l} = (1-e^{-l})$$

$$+ \sum_{k=0}^{\infty} \left(\tfrac{N-1}{N}\right)^k \frac{(l)^k}{k!} e^{-l} - e^{-l}$$

$$= 1 - 2e^{-l} + e^{-l} e^{\left(\frac{N-1}{N}\right)l} = 1 - 2e^{-l} + e^{\frac{-l}{N}}$$

习题三十

$$E[Y] = E[E(Y|X)] = \sum_{i=1}^{n} E(Y|x=i) \cdot P\{x=i\}$$

$$= \sum_{i=1}^{n} \frac{i+1}{2} \frac{1}{n} = \frac{1}{2n} \frac{(2+n+1)n}{2} = \frac{(2+n+1)}{4}$$

习题三十六

$$Y=\sum_{k=1}^{N}X_k$$

$$E(Y)=E\{E(Y|N)\}=\sum_{k=0}^{+\infty}E[Y|N=k]P\{N=k\}$$ N 与 Y 独立

$$=\sum_{k=0}^{+\infty}E\left[\sum_{k=1}^{N}X_i\right]P\{N=k\}=\sum_{K=0}^{+\infty}KE(X)\cdot P\{N=k\}=E(X)E(N)$$

$$D(X)=E[D(X|Y)]+D[E(X|Y)]$$

p.24(9)全方差公式

$$\therefore\ D(Y)=E[D(Y|N)]+D[E(Y|N)]\quad Y=\sum_{k=1}^{N}X_k$$

$$=E\left[D\left(\sum_{k=1}^{N}X_k|N\right)\right]+D\left[E\left(\sum_{k=1}^{N}X_k|N\right)\right]$$

$$=E[D(X)N]+D[E(X)N]$$

$$D(Y)=E(N)D(X)+D(N)E^2(X)$$

$$j_Y(u)=E[e^{iYu}]=E[E(e^{iYu}|N)]$$

$$=\sum_{k=0}^{+\infty}E(e^{iYu}|N=k)P\{N=k\}$$

$$=\sum_{k=0}^{+\infty}E\left[e^{iu\sum_{k=1}^{N}X_i}\right]P\{N=k\}=\sum_{k=0}^{+\infty}[j_X(u)]^kP\{N=k\}$$