电子科技大学二零 12 至二零 13 学年第 1 学期期 终 考试

数字信号处理 课程考试题 A 卷（ 120 分钟） 考试形式：开卷 考试日期 2012 年___月___日

课程成绩构成：平时 15 分， 期中 0 分， 实验 15 分， 期末 70 分

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一、 计算题（共 20 分，共 4 小题）

$h[n]$ and $x[n]$ are two finite-length sequences as given below:

$$h[n]=\{\underset{-}{1}\quad 1\quad 1\},\quad x[n]=\{\underset{-}{1}\quad 2\quad 2\quad 1\}$$

(1)（5 分） Determine the linear convolution $y_L[n]=h[n]*x[n]$ by using **the tabular method**

(2)（5 分） Determine the 4-point circular convolution $y_C[n]=h_e[n]④x[n]$ with **the tabular method** after zero-padding $h[n]$ to a length-4 sequence $h_e[n]$.

(3) （5 分）Determine the 4-point DFTs $H_e[k]$ and $X[k]$ of two length-4 sequences $h_e[n]=\{1,1,1,0\}$ and $x[n]$ by using **the matrix form of the DFT**, respectively. Notice that the element of the N-point DFT transform matrix $\mathbf{D}_N$ at the m-th row and the n-th column is calculated by $d_{kn}=W_N^{kn}=e^{j\frac{-2\pi kn}{N}}$, $0\le n,k\le N-1$, where $W_N=e^{j\frac{-2\pi}{N}}$.

(4)（5 分)Determine the IDFT of the product $Y[k]=H_e[k]X[k]$ by using **the matrix form of the IDFT** and compare the obtained result with the results $y_L[n]$ in (1) and the result $y_O[n]$ in (2). Notice that the element d'_{kn} of the IDFT transform matrix $\mathbf{D}_N{}^{-1}$ can be computed by $d'_{kn}=\frac{1}{N}(d_{kn})^{-1}$, where d_{kn} is the element of the DFT transform matrix $\mathbf{D}_N$.

Solution: （1）By using the tabular method, the linear convolution can be calculated as follows ……………………………………3

$h[n]$	1	1	1			
$x[n]$	1	2	2	1		
	1	1	1			
		2	2	2		
			2	2	2	
			1	1	1	
	1	3	5	5	3	1

The result is $y_L[n]=h[n]*x[n]$=$\{1,3,5,5,3,1\}$ ………………………………2

(2) After zero padding, the extended sequences can be written as

$$h_e[n]=\{\underline{1}\quad 1\quad 1\quad 0\},\quad x[n]=\{\underline{1}\quad 2\quad 2\quad 1\}$$ ……………1

By the tabular method, the 4-point circular convolution is calculated by ……………2

$$\begin{array}{lccccccc}
h_e[n] & \underline{1} & 1 & 1 & 0 & & & \\
x[n] & \underline{1} & 2 & 2 & 1 & & & \\
 & & & & & & & \\
 & 1 & 1 & 1 & 0 & & & \\
 & & 2 & 2 & 2 & 0 & & \\
 & & & 2 & 2 & 2 & 0 & \\
 & & & & 1 & 1 & 1 & 0 \\
\hline
 & 1 & 3 & 5 & 5 & 3 & 1 & 0 \\
 & & & & & & & \\
 & 3 & 1 & 0 & & & & \\
\hline
 & 4 & 4 & 5 & 5 & & &
\end{array}$$

The result is $y_C[n]=h_e[n]④x[n]=\{4,4,5,5\}$ …………………………2

(3) For a length-4 sequence, the DFT transform matrix is

$$\mathbf{D}_4=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}$$ ……………………………………………1

By the matrix form of the DFT， $X[k]$ can be calculated by

$$\mathbf{X} = \mathbf{D}_4\mathbf{x}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -1-j \\ 0 \\ -1+j \end{bmatrix}$$ 2

By the matrix form of the DFT, $H_e[k]$ can be calculated by

$$\mathbf{H}_e = \mathbf{D}_4\mathbf{h}_e$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$ 2

(4) The product of $\mathbf{H}_e$ and $\mathbf{X}$ can be calculated as

$$Y[k] = H_e[k]\, X[k]$$

$$= \begin{bmatrix} 6 \\ -1-j \\ 0 \\ -1+j \end{bmatrix} \odot \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$ 1

$$= \begin{bmatrix} 18 \\ -1+j \\ 0 \\ -j-1 \end{bmatrix}$$

For a length-4 sequence, the IDFT transform matrix is

$$\mathbf{D}_4^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$ ……………………………………….1

By the matrix form of the IDFT, we can compute the IDFT of the product $Y[k] = H_e[k]X[k]$ as

$$\mathbf{y} = \mathbf{D}_4^{-1}\mathbf{Y}$$
$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 18 \\ -1+j \\ 0 \\ -j-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$ ………………………………..2

Comparing the result obtained in (2), we find that

$$y[n] \neq y_L[n]$$
$$y[n] = y_C[n]$$ …………………………..1

It means that the circular convolution of two length-N sequences can be computed by the DFT method.

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二、计算题（共 25 分， 共 3 小题）

(1) （10 分）Determine the DTFT of each of the following sequences with **the commonly used DTFT pairs** and **the properties of the DTFT**,instead of the definition of DTFT:

(a) $x_1[n] = a^n u[n+1], |a| < 1$;

(b) $x_2[n] = na^n u[n+2], |a| < 1$

Some of the commonly used DTFT pairs are listed as follows

$$x[n] = a^n u[n], |a| < 1 \xleftrightarrow{DTFT} X(e^{-j\omega}) = \frac{1}{1-\alpha e^{-j\omega}}$$

$$y[n] = (n+1)a^n u[n], |a| < 1 \xleftrightarrow{DTFT} Y(e^{-j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})^2}$$

$$\delta[n-n_0] \xleftrightarrow{DTFT} e^{-j\omega n_0}$$

(2)（10 分）Find the inverse z-transform of a right-sided sequence h[n] with **the residue theorem**, whose z-transform is given by

$$H(z) = \frac{z+1}{(z-0.5)(z+0.3)}$$

(3) （5 分）Develop a parallel form realization of the above transfer function.

Solution:

（a） （5 分） We can rewrite $x_1[n] = a^n u[n+1], |a| < 1$ as follows

$$\begin{aligned} x_1[n] &= a^{-1}[a^{n+1}u[n+1]] \\ &= a^{-1}x[n+1] \end{aligned}$$ ……………………………………2

By the time shifting property of the DTFT,

$$x[n-n_0] \xleftrightarrow{DTFT} e^{-j\omega n_0} X[k]$$

and the given DTFT transform pair

$$x[n]=a^n u[n], |a|<1 \xrightleftharpoons{DTFT} X(e^{-j\omega})=\frac{1}{1-\alpha e^{-j\omega}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

we have

$$X_1[k]=a^{-1}e^{j\omega}X[k]=\frac{a^{-1}e^{j\omega}}{1-\alpha e^{-j\omega}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

(b) (5 分)

Method1: We can rewrite $x_2[n]=na^n u[n+2], |a|<1$ as follows

$$\begin{aligned} x_2[n] &= na^n u[n]+a^{-1}\delta[n+1]-2a^{-2}\delta[n+2] \\ &= (n+1)a^n u[n]-a^n u[n]+a^{-1}\delta[n+1]-2a^{-2}\delta[n+2] \quad \ldots\ldots\ldots\ldots\ldots\ldots 2 \\ &= y[n]-x[n]+a^{-1}\delta[n+1]-2a^{-2}\delta[n+2] \end{aligned}$$

where

$$x[n]=a^n u[n], |a|<1$$

$$y[n]=(n+1)a^n u[n], |a|<1$$

Based on the commonly used DTFT transform pairs,

$$\begin{aligned} x[n]=a^n u[n], |a|<1 &\xrightleftharpoons{DTFT} X(e^{-j\omega})=\frac{1}{1-ae^{-j\omega}} \\ y[n]=(n+1)a^n u[n], |a|<1 &\xrightleftharpoons{DTFT} Y(e^{-j\omega})=\frac{1}{(1-ae^{-j\omega})^2} \end{aligned} \quad \ldots\ldots\ldots\ldots 1$$

and the time-shifting property of the DTFT, we obtain

$$X_2(e^{-j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})^2} - \frac{1}{1-\alpha e^{-j\omega}} + a^{-1}e^{j\omega} - 2a^{-2}e^{j2\omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.2$$

Method2:

We can rewrite $x_2[n] = na^n u[n+2], |a| < 1$ as follows

$$\begin{aligned} x_2[n] &= a^{-2}(n+3)a^{n+2}u[n+2] - 3a^{-2}a^{n+2}u[n+2] \\ &= a^{-2}y[n+2] - 3a^{-2}x[n+2] \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.2$$

where

$$x[n] = a^n u[n], |a| < 1$$

$$y[n] = (n+1)a^n u[n], |a| < 1$$

Based on the commonly used DTFT transform pairs,

$$\begin{aligned} x[n] &= a^n u[n], |a| < 1 \xleftrightarrow{DTFT} X(e^{-j\omega}) = \frac{1}{1-ae^{-j\omega}} \\ y[n] &= (n+1)a^n u[n], |a| < 1 \xleftrightarrow{DTFT} Y(e^{-j\omega}) = \frac{1}{(1-ae^{-j\omega})^2} \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.1$$

and the time-shifting property of the DTFT, we obtain

$$X_2(e^{-j\omega}) = a^{-2}\frac{e^{j2\omega}}{(1-ae^{-j\omega})^2} - 3a^{-2}\frac{e^{j2\omega}}{1-ae^{-j\omega}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.2$$

(2) (10 分)

Method1: Based on the partial-fraction expansion method, H(z) can be written as

$$H(z) = \frac{\rho_1}{1-0.5z^{-1}} + \frac{\rho_2}{1+0.3z^{-1}} + A \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

where the coefficients are given by

$$\rho_1 = (1-0.5z^{-1})\frac{(z+1)z^{-2}}{(1-0.5z^{-1})(1+0.3z^{-1})}\Big|_{\lambda_1=0.5} = 3.75$$

$$\rho_2 = (1+0.3z^{-1})\frac{(z+1)z^{-2}}{(1-0.5z^{-1})(1+0.3z^{-1})}\Big|_{\lambda_1=-0.3} = \frac{35}{12} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

(They have been verified by Matlab code: [r,p,k]=residuez([0 1 1],[1 -0.2 -0.15]))

$$A = H(z=0) = \frac{1}{-0.5*0.3} = -\frac{20}{3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Since h[n] is a right-sided sequence, we finally obtain

$$h[n] = 3.75(0.5)^n u[n] + \frac{35}{12}(-0.3)^n u[n] - \frac{20}{3}\delta[n] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Method2:

Based on the partial-fraction expansion method, H(z) can be written as

$$H(z) = \frac{a}{z-0.5} + \frac{b}{z+0.3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Where the coefficients can be calculated by the residue theorem

$$a=(z-0.5)\frac{z+1}{(z-0.5)(z+0.3)}\Big|_{\lambda_1=0.5}=\frac{15}{8}$$

$$b=(z+0.3)\frac{z+1}{(z-0.5)(z+0.3)}\Big|_{\lambda_1=-0.3}=-\frac{7}{8}$$

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Thus, we can rewrite the transfer function as follows

$$\begin{aligned}H(z)&=\frac{\frac{15}{8}}{z-0.5}-\frac{\frac{7}{8}}{z+0.3}\\&=\frac{\frac{15}{8}z^{-1}}{1-0.5z^{-1}}-\frac{\frac{7}{8}z^{-1}}{1+0.3z^{-1}}\end{aligned}$$

..................2

Finally, the inverse z-transform of the above right-sided transfer function can be obtained as

$$h[n]=\frac{15}{8}(0.5)^{n-1}u[n-1]-\frac{7}{8}(-0.3)^{n-1}u[n-1]$$

......................2

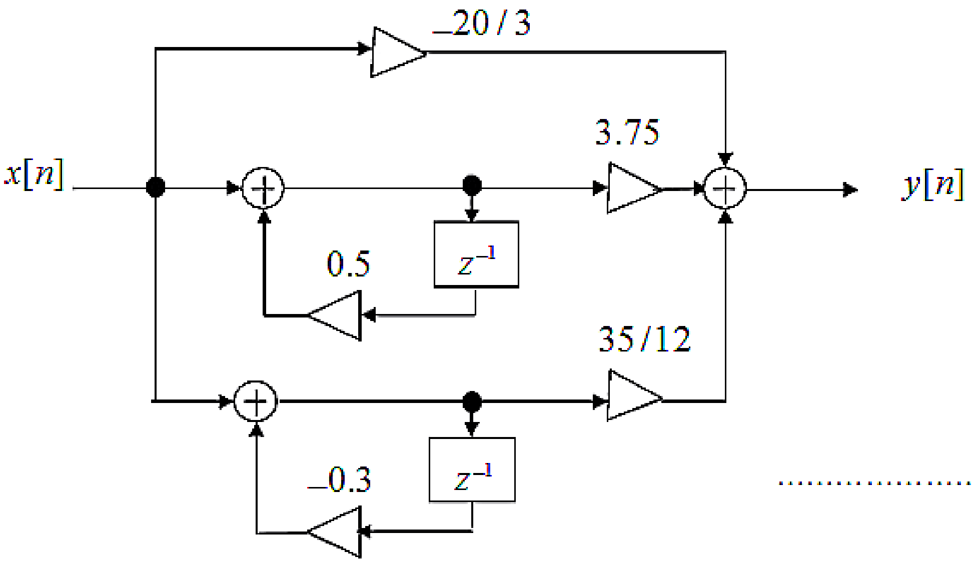
(3) (5 分)

Method1: From the result obtained by **Method 1** in (2), we can decompose the transfer function H(z) into the following partial-fraction expression

$$H(z)=\frac{3.75}{1-0.5z^{-1}}+\frac{\frac{35}{12}}{1+0.3z^{-1}}-\frac{20}{3}$$

.......................1

Thus, one of the parallel forms of the above transfer function can be realized as follows



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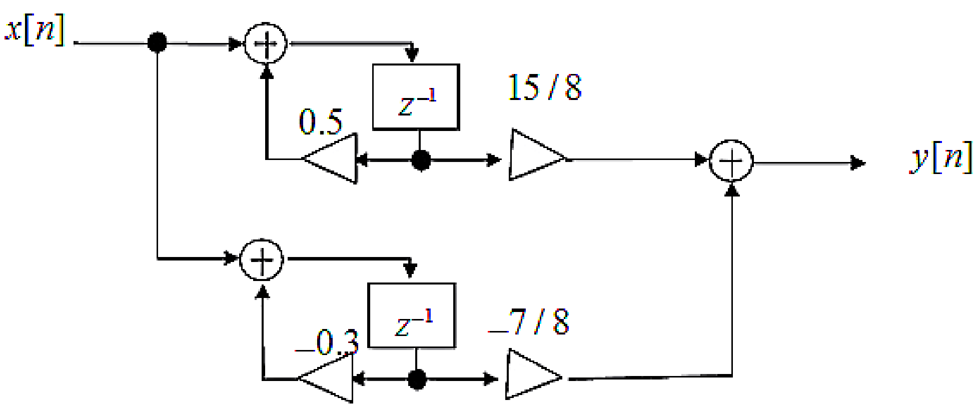
Method2: From the result obtained by **Method 2** in (2), we can decompose the transfer function H(z) into the following partial-fraction expression

$$H(z) = \frac{\frac{15}{8} z^{-1}}{1 - 0.5z^{-1}} - \frac{\frac{7}{8} z^{-1}}{1 + 0.3z^{-1}}$$ ……………………1

Thus, one of the parallel forms of the above transfer function can be realized as follows



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三、计算题（共 15 分，共 1 小题）

(1) （15 分）Utilize the bilinear transformation to design a Butterworth lowpass **digital filter** with the given specifications: passband edge 2.5 kHz, stopband edge frequency 5.5 kHz, passband ripple 0.5 dB, minimum stopband attenuation 15 dB, and sampling frequency 20kHz.

The transfer function of the Butterworth lowpass **analog filter** of the first three orders with 3-dB cutoff frequency **Ωc=1** is given as below:

The 1^{st} order: $$H(s)=\frac{1}{s+1}$$

The 2^{nd} order: $$H(s)=\frac{1}{s^2+\sqrt{2}s+1}$$

The 3^{rd} order: $$H(s)=\frac{1}{(s+1)(s^2+s+1)}$$

and their corresponding frequency responses are given by

$$|H_a(j\Omega)|^2=\frac{1}{1+(\Omega/\Omega_c)^{2N}},\quad N=1,2,3$$

Solution:

(1) First, we determine the normalized edge frequencies of the lowpass digital filter as

$$\omega_p=\frac{2\pi f_p}{F_s}=\frac{2\pi\times 2.5}{20}=0.25\pi$$
$$\omega_s=\frac{2\pi f_s}{F_s}=\frac{2\pi\times 5.5}{20}=0.55\pi$$
........................1

By prewarping the frequency, we have the two edge frequencies of the analog lowpass filter

$$\Omega_p=\tan(\frac{\omega_p}{2})=\tan(\frac{0.25\pi}{2})=0.4142136$$
$$\Omega_s=\tan(\frac{\omega_s}{2})=\tan(\frac{0.55\pi}{2})=1.1708496$$
........................1

The inverse transition ratio is

$$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = \frac{1.1708496}{0.4142136} = 2.82668 \qquad \text{........................2}$$

From the specified passband ripple and the minimum stopband attenuation and the formula

$$10\log_{10}\left(\frac{1}{1+\varepsilon^2}\right) = -\alpha_p = -0.5,$$
$$10\log_{10}\left(\frac{1}{A^2}\right) = -\alpha_s = -15$$

we obtain

$$\varepsilon^2 = 0.1220185$$
$$A^2 = 31.622777$$

Thus, the inverse discrimination ratio is

$$\frac{1}{k_1} = \frac{\sqrt{A^2 - 1}}{\varepsilon} = 15.841979 \qquad \text{............2}$$

Then, the filter order N can be estimated as

$$N = ceil\left(\frac{\log_{10}(1/k_1)}{\log_{10}(1/k)}\right) = ceil\left(\frac{\log_{10}(15.841979)}{\log_{10}(2.8266814)}\right) = ceil(2.6586997) = 3 \qquad \text{...............2}$$

where ceil(x) is the function that takes the nearest higher interger of x. Therefore, we take N=3 as the filter order.

Next, we calculate the 3-dB cutoff frequency Ωc of the analog Butterworth filter with the given formula

$$|H_a(j\Omega_p)|^2 = \frac{1}{1+(\Omega_p/\Omega_o)^{2N}} = \frac{1}{1+\varepsilon^2}$$

or

$$|H_a(j\Omega_s)|^2 = \frac{1}{1+(\Omega_s/\Omega_o)^{2N}} = \frac{1}{A^2}$$

Substituting the filter order N =3 and $\varepsilon^2 = 0.1220185$ into the above equation, we get

$$\frac{1}{1+(0.4142136/\Omega_o)^6} = \frac{1}{1+(0.1220185)^2},$$

or

$$\frac{1}{1+(1.1708496/\Omega_o)^6} = \frac{1}{(31.622777)^2},$$

whose solution yields

$$\Omega_o = 0.588148 \qquad \text{............1}$$

or

$$\Omega_o = 0.661953$$

By using the spectral transformation from LP filter to LP filter:

$$\hat{s} = \frac{s}{\Omega_o} \qquad \text{.........................2}$$

and substituting it into the 3rd order transfer function of the Butterworth analog filter

$$H(s) = \frac{1}{(s+1)(s^2+s+1)},$$

We can obtain the desired transfer function of the 3-order Butterworth analog filter with the 3-dB cutoff frequency $\Omega_c = 0.588148$ or $\Omega_c = 0.661953$ as follows

$$H_a(\bar{s}) = \frac{0.290056}{(\bar{s}+0.661953)(\bar{s}^2+0.661953\bar{s}+0.438182)}, \quad for\ \Omega_c = 0.588148 \qquad \ldots\ldots\ldots\ldots\ldots 2$$

or

$$H_a(\bar{s}) = \frac{0.203451}{(\bar{s}+0.588148)(\bar{s}^2+0.588148\bar{s}+0.345918)}, \quad for\ \Omega_c = 0.661953$$

Applying the bilinear transformation,

$$s = \frac{1-z^{-1}}{1+z^{-1}}$$

we finally arrive at the desired expression for the digital lowpass transfer function

$$H_{LP}(z) = H_a(\bar{s})\big|_{\bar{s}=\frac{1-z^{-1}}{1+z^{-1}}}$$

$$= \frac{0.203451}{\left(\frac{1-z^{-1}}{1+z^{-1}}+0.588148\right)\left(\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+0.588148\frac{1-z^{-1}}{1+z^{-1}}+0.345918\right)} \qquad \ldots\ldots 2$$

$$= \frac{0.0662272(1+z^{-1})^3}{(1-0.2593284z^{-1})(1-0.6762858z^{-1}+0.345918z^{-2})}, \quad for\ \Omega_c = 0.588148$$

$$H_{LP}(z) = H_c(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$
$$= \frac{0.290056}{\left(\frac{1-z^{-1}}{1+z^{-1}} + 0.661953\right)\left(\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.661953\frac{1-z^{-1}}{1+z^{-1}} + 0.438182\right)},$$

$$for\ \Omega_c = 0.661953$$

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四、 计算题（共 20 分，共 2 小题）

Develop a set of time-domain equations describing the digital filter structure of Figure 1 in terms of the input x[n], output y[n], and the intermediate variables $w_k[n]$ in a sequential order.

(1) （10 分）Justify whether this set of equations describe a valid computational algorithm by examining **the matrix F** of the matrix representation of the digital filter structure.

(2) （10 分）Develop the **signal flow-graph representation** of this digital filter structure, and determine **its procedure graph**.

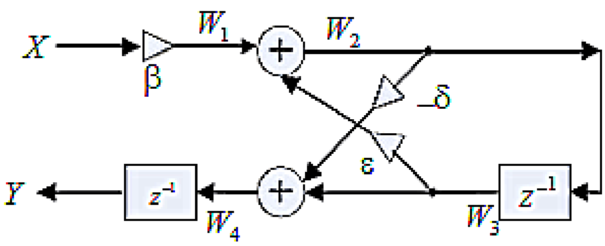

Figure 1. The structure of a digital filter

Solution:

(1) From the figure, we obtain the difference equations as follows

$$
\begin{aligned}
w_1[n] &= \beta x[n] \\
w_2[n] &= w_1[n] + \varepsilon w_3[n] \\
w_3[n] &= w_2[n-1] \\
w_4[n] &= -\delta w_2[n] + w_3[n] \\
y[n] &= w_4[n-1]
\end{aligned}
$$

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From the above equations, the matrix **F** can be written as

$$
F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\delta & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

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Since there are some entries of the upper diagonal F with nonzero values, it implies that the equations are **noncomputable**.

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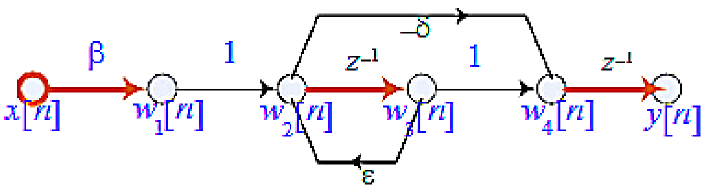
(2) The following figure shows the signal flow-graph of the filter structure



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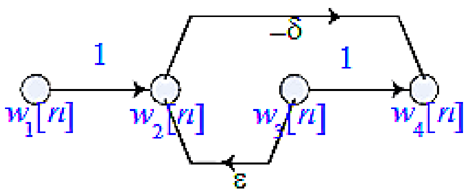
The reduced signal-flow graph obtained by removing the branches going out of the input node and the delay branches from the above signal flow-graph is shown



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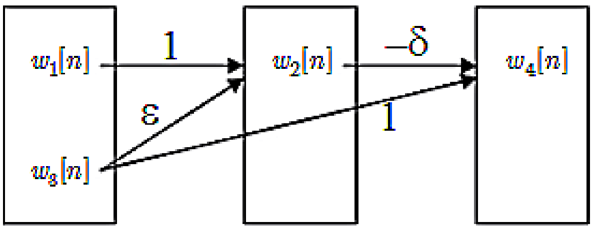
From the above reduced signal flow-graph, we can group the nodes as follows:

$$\{N_1\} = \{w_1[n], w_3[n]\}$$
$$\{N_2\} = \{w_2[n]\}$$
$$\{N_3\} = \{w_4[n]\}$$

……………………2

The following diagram shows the procedure graph.



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五、 计算题（共 20 分，共 2 小题）

(1) （10 分）Determine the DFT of a sequence $x[n] = \{1, 2, 2, 1\}$ of length $N = 4$ using **2-point FFT modules**.

(a) Develop two length-2 subsequences formed from the even-indexed samples $x_0[n] = x[2n]$ and the odd-indexed samples $x_1[n] = x[2n+1]$.

(b) Compute the DFTs $X_0[k]$ *and* $X_1[k]$ of the new length-2 subsequences $x_0[n]$ and $x_1[n]$ with **the definition of the DFT**, respectively.

(c) Compute the DFT $X[k]$ of the original length-4 sequence $x[n]$ by forming a weighted sum of the two 2-point DFTs $X_0[k]$ *and* $X_1[k]$ with the following formula

$$X[k] = X_0[< k >_{N/2}] + e^{-j\frac{2\pi}{N}k} X_1[< k >_{N/2}],\ \ 0 \le k \le N-1$$

(2) （10 分）Compare the computational efficiency of the 2-point FFT method with the approach of the **DFT definition** by examining the total computations of the complex multiplication and the complex addition.

Solution:

(1) （10 分）

(a) The new length-2 subsequences $x_0[n]$ and $x_1[n]$ are

Solution:

(1) （10 分）

(a) The new length-2 subsequences $x_0[n]$ and $x_1[n]$ are

$$x_0[n] = \{1,2\}$$
$$x_1[n] = \{2,1\}$$
……………………..2

(b) The DFT of the new length-2 subsequences $x_0[n]$ are calculated as

$$X_0[0] = x_0[0] + x_0[1]e^{-j\frac{2\pi\times1\times0}{2}} = 1+2 = 3$$
$$X_0[1] = x_0[0] + x_0[1]e^{-j\frac{2\pi\times1\times1}{2}} = 1-2 = -1$$
……………………..2

the DFT of the new length-2 subsequences $x_1[n]$ are calculated as

$$X_1[0] = x_1[0] + x_1[1]e^{-j\frac{2\pi\times 1\times 0}{2}} = 2+1=3$$
$$X_1[1] = x_1[0] + x_1[1]e^{-j\frac{2\pi\times 1\times 1}{2}} = 2-1=1$$
........................2

(c) the DFT of the original length-4 subsequences $x[n]$ are calculated as

$$X[0] = X_0[<0>_2] + e^{-j\frac{2\pi}{4}\times 0}X_1[<0>_2] = 3+3=6$$
$$X[1] = X_0[<1>_2] + e^{-j\frac{2\pi}{4}\times 1}X_1[<1>_2] = -1-j$$
$$X[2] = X_0[<2>_2] + e^{-j\frac{2\pi}{4}\times 2}X_1[<2>_2] = 3-3=0$$
$$X[3] = X_0[<3>_2] + e^{-j\frac{2\pi}{4}\times 3}X_1[<3>_2] = -1+j$$
..................4

(2) （10 分） The total complex multiplication of the above 2-point FFT is

$$N\times\log_2 N = 4\times\log_2 4 = 4\times 2 = 8$$
.....................2

The total complex addition of the above 2-point FFT is

$$N\times\log_2 N = 4\times\log_2 4 = 4\times 2 = 8$$
........................2

Thus, the total computation of the 2-point FFT is 8+8=16.

The total complex multiplication of the direct DFT is

$$N\times N = 4\times 4 = 16$$
........................2

The total complex addition of the direct DFT is

$$N\times(N-1) = 4\times(4-1) = 12$$
........................2

Thus, the total computation of the direct DFT is 16+12=28.

Since 16<28, we find that the 2-point FFT has better computation efficiency than the direct DFT.

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【It should be noted that any operation related to $e^{-j\frac{2\pi}{N}nk}$ is regarded as complex calculation.】

The answer sheet is formatted by Deshuang Zhao on Jan. 03, 2013 at UESTC. Any questions can be sent to me via dszhao@uestc.edu.cn.