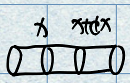
1. 解: 

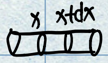
$$SY[u_x(x+dx,t)-u_x(x,t)]=S\rho\,dx\,u_{tt}$$

$$\Rightarrow\quad Yu_{xx}=\rho u_{tt}\qquad\Rightarrow u_{tt}=\frac{Y}{\rho}u_{xx}=a^2u_{xx}$$

2. 解: 令 $u(x,t)=v(x,t)+w(x,t)$ 　设 $w(x,t)=Ax+B$

$$\Rightarrow\begin{cases}h_1(t)=B+kA\\ h_2(t)=B+lA\end{cases}\Rightarrow\begin{cases}A=\dfrac{h_1(t)-h_2(t)}{k-l}\\ B=\dfrac{lh_1(t)-kh_2(t)}{l-k}\end{cases}\quad 则\ w(x,t)=\frac{h_1(t)-h_2(t)}{k-l}x+\frac{lh_1(t)-kh_2(t)}{l-k}$$

$$则\begin{cases}v_{tt}=a^2v_{xx}-\dfrac{h_1''(t)-h_2''(t)}{k-l}x-\dfrac{lh_1''(t)-kh_2''(t)}{l-k}\\ [v+kv_x]|_{x=0}=0\qquad v|_{x=l}=0\\ v|_{t=0}=\varphi(x)-\dfrac{h_1(0)-h_2(0)}{k-l}x-\dfrac{lh_1(0)-kh_2(0)}{l-k}\qquad v_t|_{t=0}=\psi(x)-\dfrac{h_1'(0)-h_2'(0)}{k-l}x-\dfrac{lh_1'(0)-kh_2'(0)}{l-k}\end{cases}$$

3. 解: 

$$dQ=CA\rho\,dx[u(x,t+dt)-u(x,t)]$$

$$=CA\rho\,dx\,u_t\,dt$$

$$dQ_1=-kAu_x(x,t)dt\qquad dQ_2=-kAu_x(x+dx,t)dt$$

$$又\ dQ'=dQ_1-dQ_2\quad\Rightarrow u_t=\frac{k}{c\rho}u_{xx}=a^2u_{xx}$$

$$\Rightarrow\begin{cases}u_t=a^2u_{xx}\qquad 0<x<L,\ t>0\\ u|_{x=0}=0.\quad u|_{x=L}=0\\ u|_{t=0}=0.\end{cases}$$

4. 解: 令 $u(x,t)=v(x,t)+w(x,t)$

$$\begin{cases}v_{tt}=a^2v_{xx}\qquad x\in R,\ t>0\\ v|_{t=0}=\varphi(x)\\ v_t|_{t=0}=\psi(x)\end{cases}\Rightarrow v(x,t)=\frac{1}{2}[\varphi(x+at)+\varphi(x-at)]+\frac{1}{2a}\int_{x-at}^{x+at}\psi(\xi)d\xi$$

$$\begin{cases}w_{tt}=a^2w_{xx}+f(x,t)\qquad x\in R,\ t>0\\ w|_{t=0}=0\\ w_t|_{t=0}=0\end{cases}\qquad w(x,t)=\int_0^t\frac{1}{2a}\int_{x-a(t-\tau)}^{x+a(t-\tau)}f(\xi,\tau)d\xi d\tau$$

则 $u(x,t)=v(x,t)+w(x,t)$

5. 解: $$\hat{f}(\omega)=\int_{-\infty}^{+\infty}f(x)e^{-j\omega x}dx=\int_{-\frac{a}{2}}^{\frac{a}{2}}Ee^{-j\omega x}dx=E\frac{1}{-j\omega}e^{-j\omega x}\Big|_{-a/2}^{a/2}=\frac{2E}{\omega}\sin\frac{a}{2}\omega$$

$$g(s)=\int_0^{+\infty}\sin kx\,e^{-sx}dx=\int_0^{+\infty}\frac{e^{jkx}-e^{-jkx}}{2j}e^{-sx}dx=\frac{k}{s^2+k^2}$$

6. 解: $$u(M_0)=-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\partial G(M,M_0)}{\partial\vec{n}}\varphi(x,y)dxdy$$

又 $G(M,M_0)=\frac{1}{4\pi}\left(\frac{1}{r_{MM_0}}-\frac{1}{r_{MM_1}}\right)$.　$\frac{\partial G(M,M_0)}{\partial n}=-\frac{1}{2\pi}\frac{z_0}{[(x-x_0)^2+(y-y_0)^2+z_0^2]^{3/2}}$

则 $u(M_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{z_0}{[(x-x_0)^2+(y-y_0)^2+z_0^2]^{3/2}}\varphi(x,y)\,dx\,dy$

7. 证:　$J_n(x)=\sum_{m=0}^{\infty}(-1)^m\frac{x^{n+2m}}{2^{n+2m}\,m!\,\Gamma(n+m+1)}$

$\Rightarrow (-1)^n J_n(x)=\sum_{m=0}^{\infty}(-1)^m\frac{(-1)^n x^{n+2m}}{2^{n+2m}\,m!\,\Gamma(n+m+1)}=\sum_{k=0}^{\infty}(-1)^k\frac{x^{-n+2k}}{2^{-n+2k}\,k!\,\Gamma(-n+k+1)}=J_{-n}(x)$

8. 解:　$\int_{-1}^{1}(1-x^2)[P_n'(x)]^2dx$

$=\int_{-1}^{1}(1-x^2)P_n'(x)\,dP_n(x)$

$=(1-x^2)P_n'(x)P_n(x)\Big|_{-1}^{1}-\int_{-1}^{1}P_n(x)\cdot(1-x^2)P_n''(x)-2xP_n'(x)\,dx$

又 $P_n(x)$ 为 $(1-x^2)y''-2xy'+n(n+1)y=0$ 的解

$\Rightarrow (1-x^2)P_n''(x)-2xP_n'(x)=-n(n+1)P_n(x)$

$=\int_{-1}^{1}P_n(x)\,n(n+1)P_n(x)\,dx$

$=n(n+1)\cdot\frac{2}{2n+1}$