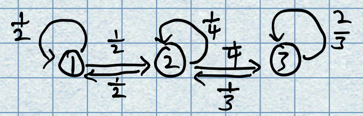
1.解: $N(t)\sim P(5t)$ $\quad P\{N(t)=k\}=\frac{(5t)^k}{k!}e^{-5t}$

2.解: (1)



(2) $P=\begin{pmatrix}\frac12 & \frac12 & 0\\ \frac12 & \frac14 & \frac14\\ 0 & \frac13 & \frac23\end{pmatrix}$ $\quad P^{(2)}=P^2=\begin{pmatrix}\frac12 & \frac38 & \frac18\\ \frac38 & \frac{19}{48} & \frac{11}{48}\\ \frac16 & \frac{11}{36} & \frac{19}{36}\end{pmatrix}$ 满足遍历性.

(3) 遍历态:从马氏链的极限分布即为平稳分布. 令 $\Pi=(\pi_1,\pi_2,\pi_3)$

则有

$$(\pi_1,\pi_2,\pi_3)=(\pi_1,\pi_2,\pi_3)\begin{pmatrix}\frac12 & \frac12 & 0\\ \frac12 & \frac14 & \frac14\\ 0 & \frac13 & \frac23\end{pmatrix} \Rightarrow \begin{cases}\pi_1=\frac12\pi_1+\frac12\pi_2\\ \pi_2=\frac12\pi_1+\frac14\pi_2+\frac13\pi_3\\ \pi_3=\frac14\pi_2+\frac23\pi_3\\ \pi_1+\pi_2+\pi_3=1\end{cases} \Rightarrow \begin{cases}\pi_1=\frac{4}{11}\\ \pi_2=\frac{4}{11}\\ \pi_3=\frac{3}{11}\end{cases}$$

$$\Rightarrow V=\Pi=\left(\frac{4}{11},\frac{4}{11},\frac{3}{11}\right)$$

(4) $P\{X(4)=3\mid X(1)=1,X(2)=1\}=P\{X(4)=3\mid X(2)=1\}=P_{13}^{(2)}=\frac18$

$P\{X(2)=1,X(3)=2\mid X(1)=1\}=P\{X(3)=2\mid X(2)=1\}P\{X(2)=1\mid X(1)=2\}=P_{12}^{(1)}P_{21}^{(1)}=\frac12\times\frac12=\frac14$

3.解: (1) $P\{N(20)=2\mid N(60)=2\}=\frac{P\{N(20)=2,N(60)=2\}}{P\{N(60)=2\}}=\frac{P\{N(20)=2\}P\{N(60)-N(20)=0\}}{P\{N(60)=2\}}=\frac19$

(2) $P\{N(20)=1\mid N(60)=2\}+P\{N(20)=2\mid N(60)=2\}=\frac49+\frac19=\frac59$

4.解: $R_X(s,t)=E[X(s)X(t)]=E[(As^2+Bs+C)(At^2+Bt+C)]=s^2t^2E(A^2)+stE(B^2)+E(C^2)=s^2t^2+st+1$

则 $R'_s(s,t)=2st^2+t$ $\quad R''_{st}(s,t)=4st+1$

$R'_t(s,t)=2ts^2+s$ $\quad R''_{ts}(s,t)=4st+1$ $\quad$ 则 $R_X(s,t)$ 广义二阶可微.

$\Rightarrow$ $X(t)$ 均方可导, 均方连续, 均方可积.

5.解: (1) $M_X(t)=E(A\cos t)=\frac12\cos A$

$R_X(s,t)=E[X(s)X(t)]=E(A^2\cos s\cos t)=\frac13\cos s\cos t$ $\quad$ 不是平稳过程.

(2) $Y(t)=\frac1T\int_{t-T}^{t}A\cos u\,du$

$=\frac{A}{T}[\sin(t)-\sin(t-T)]=\frac{A}{T}2\sin\frac{T}{2}\cos\left(t-\frac{T}{2}\right)=A\frac{\sin\frac{T}{2}}{\frac{T}{2}}\cos\left(t-\frac{T}{2}\right)$

(3). $m_Y(t)=E[Y(t)]=\frac{\sin\frac{T}{2}}{\frac{T}{2}}\cos\left(t-\frac{T}{2}\right)E(A)=\frac12\frac{\sin\frac{T}{2}}{T/2}\cos\left(t-\frac{T}{2}\right)$

$R_Y(s,t)=E[Y(s)Y(t)]=\sim.$ $\quad$ 不是平稳过程

6.解: (1) $E=\{-2,-1,0,1,2\}$

(2) $P=\begin{pmatrix}1 & 0 & 0 & 0 & 0\\ q & r & p & 0 & 0\\ 0 & q & r & p & 0\\ 0 & 0 & q & r & p\\ 0 & 0 & 0 & 0 & 1\end{pmatrix}$ $\quad$ (3). $P^{(2)}=P^2=\begin{pmatrix} & \sim & \end{pmatrix}$

7.解: 由泊松过程的分解可知. $N_1(t)\sim P(p\lambda t)$. $\quad N_2(t)\sim P(q\lambda t)$.

8.解:　考虑一个周期T内,

9.解:

① $m_X(t)=E[X(t)]=E[A\cos(\omega t+\theta)]=E(A)\cdot E[\cos(\omega t+\theta)]$

$=2\cdot\int_{-5}^{5}\frac{1}{10}d\omega\int_{-\pi}^{\pi}\frac{1}{2\pi}\cos(\omega t+\theta)d\theta \quad =\frac{1}{10\pi}\cdot\int_{-5}^{5}2\cos\omega\pi\, d\omega$

$=0$

②. $R_X(s,t)=E[X(s)X(t)]=E(A^2)E[\cos(\omega s+\theta)\cos(\omega t+\theta)] \quad =4\cos[\omega(s+t)+2\theta]+4\cos[\omega(s-t)]$

$=\begin{cases}4 & t-s=0\\ \frac{4\sin5(t-s)}{5(t-s)} & t-s\neq0\end{cases}$

只与 $t-s$ 有关.

③. $E[X^2(t)]=R_X(t,t)=4<+\infty$

则 $X(t)$ 为平稳过程.　　均值具有遍历性.

13.解:

(1).

$$\begin{tikzcd} 1 \arrow[loop above, "1"] & 2 \arrow[l, "q"'] \arrow[r, shift left, "p"] & 3 \arrow[l, shift left, "q"] \arrow[r, "p"] & 4 \arrow[loop above, "1"] \end{tikzcd}$$

(2). ①: $f_{11}^{(1)}=1$　$f_{11}^{(n)}=0$, $n\geq2$时,　则 $f_{11}=\sum_{n=1}^{\infty}f_{11}^{(n)}=1$

$\mu_1=\sum_{n=1}^{\infty}nf^{(n)}=1<+\infty$　$f_{11}^{(1)}=1>0$.　则①为非周期正常返遍历态

②: $f_{22}^{(1)}=0$.　$f_{22}^{(2)}=pq$　$f_{22}^{(n)}=0$, $n\geq3$.　则 $f_{22}=pq<1$　则②为非常返态.

③: ②与③互通，则③为非常返态.

④: $f_{44}^{(1)}=1$.　$f_{44}^{(n)}=0$. $n\geq2$.　则 $f_{44}=1$

$\mu_4=1<+\infty$.　$f_{44}^{(1)}=1>0$,　则④为非周期正常返遍历态.

则 $E=\{②③\}+\{①\}+\{④\}$