

Digital Logic & Turing Machine

- ① Arith : $+$ \equiv $(-, \times, \div)$ (反变如) (此外三机表字)
- ② $+$ \equiv AND . OR . NOT
- ③ Logical AND/OR/NOT can do everything

Rmk of ②:

$$\begin{array}{r}
 a_3 \quad a_2 \quad a_1 \quad a_0 \\
 0 \quad 0 \quad 1 \quad 0 \\
 + \quad b_3 \quad b_2 \quad b_1 \quad b_0 \\
 \quad 0 \quad 1 \quad 1 \quad 1 \\
 \hline
 1 \quad 0 \quad 0 \quad 1 \quad c_0 \\
 c_1 \quad c_2 \quad c_3 \quad c_4
 \end{array}$$

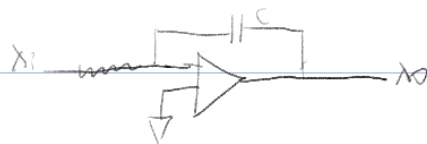
exclusive OR

$$\begin{aligned}
 c_0 &= a_0 + b_0 \equiv a_0 \text{ XOR } b_0 \\
 c_1 &= a_1 + b_1 \equiv a_1 \text{ XOR } b_1 \\
 \text{Carry}_1 &= 1 \equiv a_1 \text{ AND } b_1
 \end{aligned}$$

数字逻辑 V.S 模拟机算 in 1940s ?

- 模拟机算 可行性在1940s 已被验证 (英国雷达)

(物理世界天生是非线性的)

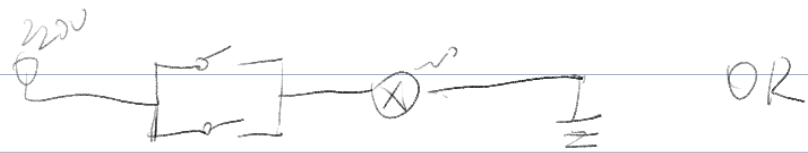


- 冯·诺依曼 结构 纯数字化

- 工程师上场:

Logical AND, OR, NOT

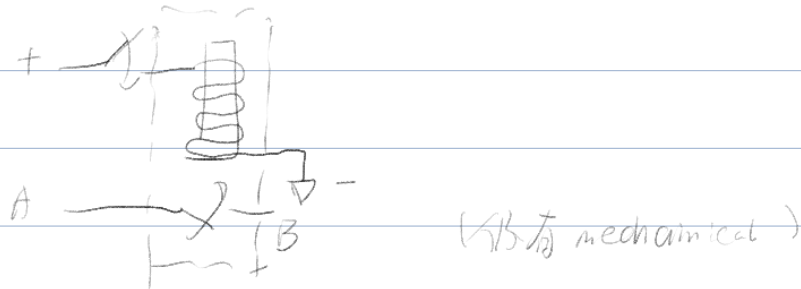




! 亦可做状态逻辑!

然而电子系统 不能依赖机械力

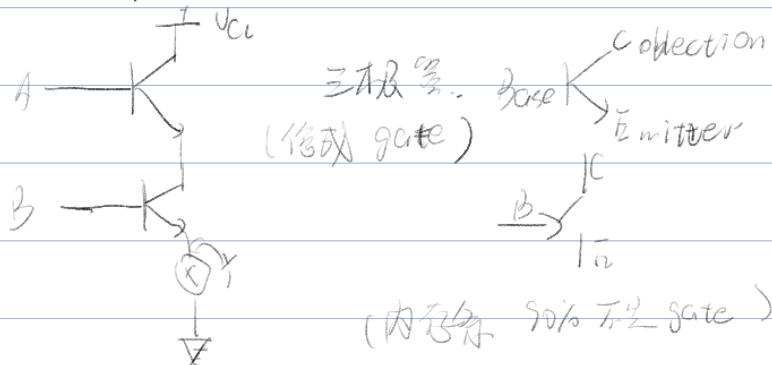
Relay (继电器)



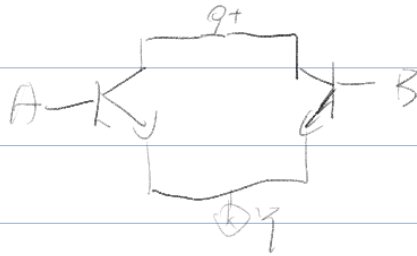
纯电子: 电阻、电容、电感、二极管

怎么用其搭建 AND、OR、NOT?

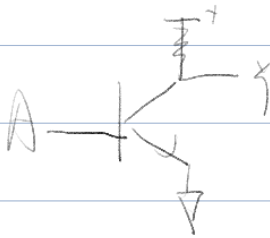
① AND: $Y = A \cdot B$



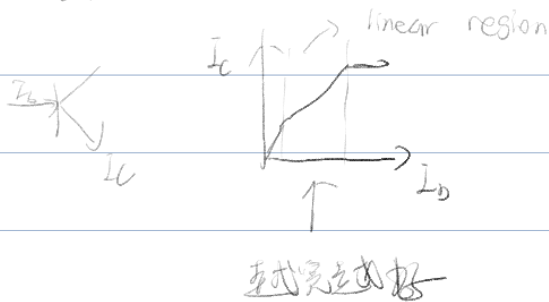
② OR: $Y = A + B$



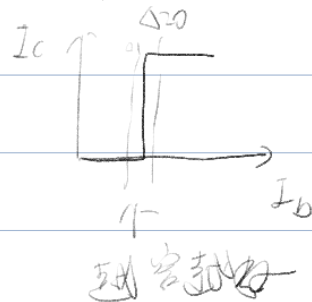
③ NOT: $Y = \bar{A}$



Dist. In $\bar{b}\bar{z}$.



In CS:



Mem Tech

(黑板为理 Mem Tech)

stateful logic

Volatile (C)

Volatile int x; ↗ unique address

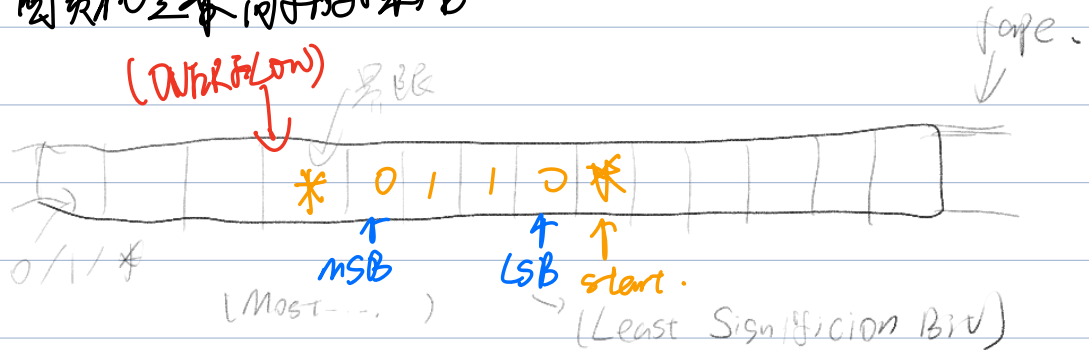
size
 被篡改 (本来先在黑板上写, 风一吹就没了)

Turing Machine. $[T_m = \{ \sum_{i=1}^K, \frac{K}{state}, s, b, b \}]$

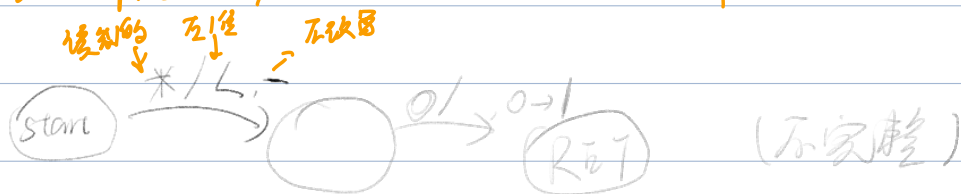
其一开始发现是解决数学问题

只是后来被广泛应用于计算机

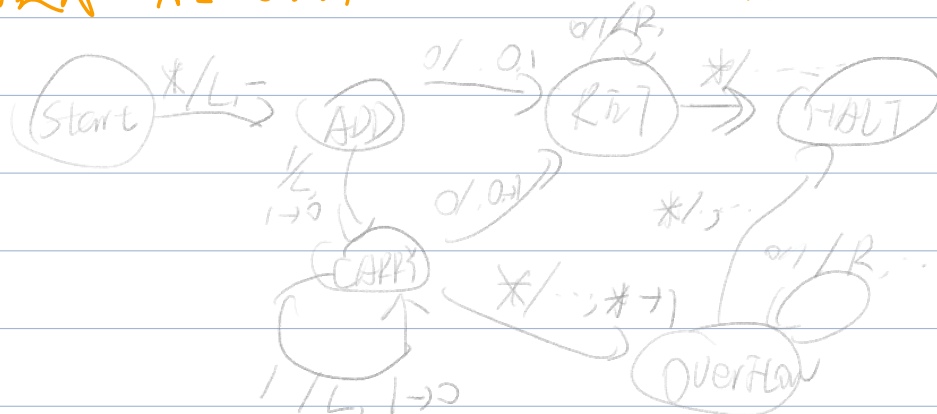
图灵机是最简单的模型



Example. $X = 0110$ $Y = X + 1$



模拟 $X = 011$ * 0 1 1 *



图灵机图 (ARI)

| Card state | ^{0, 1, *} input | ^(0 → 1, 1 → 0, 0.1 → *) Write | ^(L, R, or -) shift. |
|------------|--------------------------|--|--------------------------------|
| ADD | 0 | | |
| | 1 | | |
| | * | | |
| RST | | | |
| CARRY | | | |
| HALT | | | |
| OVERFLOW | | | |

Conclusion

- Discrete Logic: 加法可以实现任何运算
 - + 加法可被 AND, OR, NOT 实现
 - ⇒ 逻辑门可实现任何运算
 - 最终选用 三极管 (开关效应)

- Turing Machine: 输入 → 输出

$$T_m = \{ \underbrace{\Sigma_{0,1,*}}_{\text{符号}}, \underbrace{K}_{\text{state}}, \underbrace{s, h, b}_{\text{移动}} \}$$

Why Digital Logic?

冯·诺依曼: 相信技术成熟后成本会下来

(芯片 1k 35% → 23%)