Homework 2

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Question

Given the parameters in the Lorenz Attractor equation:

$$\sigma = 10; \beta = \frac{8}{3}; \rho = 28$$

and the initial values at $t = 0$:

$$x = -7; y = 7; z = 25$$

1. Follow the process told to solve the Lorenz Attractor equation in the time window $[0, 100]$.
2. Plot the result (trajectory) in 3D real space.
3. Plot 2 Lorenz Attractor in the window $[0, 100]$. For the second one, increase the initial value of y by 1.0×10^{-6}, and to observe the separation of their trajectories.

Tip: use `plot3(x, y, z, x2, y2, z2)`

Solution

根据题设条件定义模型参数

```
global sigma beta rho
sigma = 10;
beta = 8/3;
rho = 28;
T = 100;
phi0 = [-7; 7; 25];
```

Lorenz Attractor 方程如下所示：

$$\begin{cases} x' = \sigma(y - x) \\ y' = x(\rho - z) - y \\ z' = xy - \beta z \end{cases}$$

根据 Lorenz Attractor 方程定义函数 `LorenzAttr`

注：MATLAB R2024a 版本引入新特性，可以在脚本和实时脚本中任意位置定义函数。

参见：https://ww2.mathworks.cn/products/new_products/release2024a.html

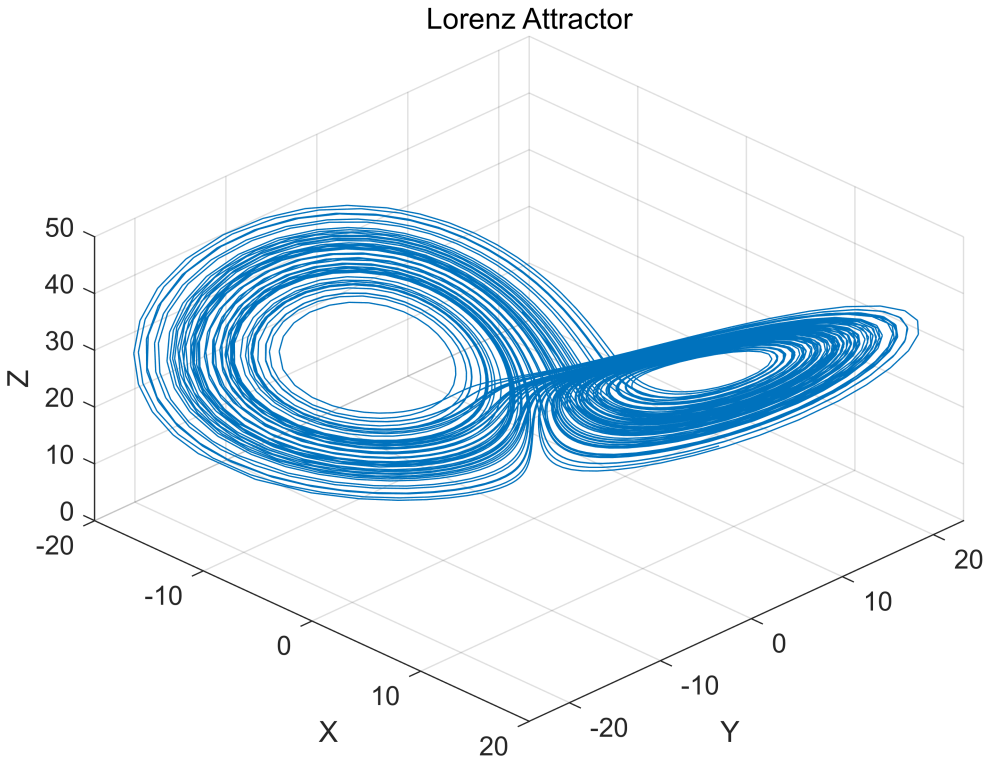
```
function dphidt = LorenzAttr(t, phi)
    global sigma beta rho;
    dphidt = [sigma*(phi(2)-phi(1));        % dx/dt = sigma * (y-x)
              phi(1)*(rho-phi(3))-phi(2);   % dy/dt = x * (rho-z) - y
              phi(1)*phi(2)-beta*phi(3)];   % dz/dt = x * y - beta * z
end
```

调用 ode45 解上述方程

```
[t, phi] = ode45(@LorenzAttr, [0, T], phi0);
```

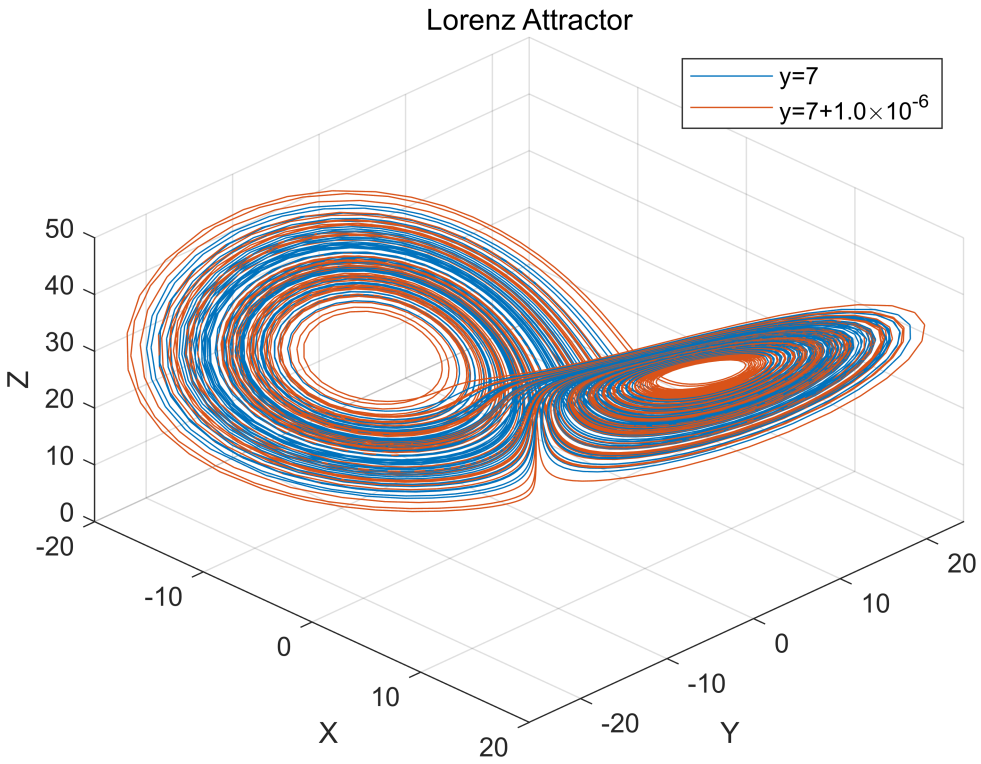
使用 plot3 在三维实空间中绘制 Lorenz Attractor 方程图像

```
figure;
plot3(phi(:,1), phi(:,2), phi(:,3));
title("Lorenz Attractor");
xlabel("X");
ylabel("Y");
zlabel("Z");
grid on;
view([45 45]);
```



更改初始值，在图像上叠加另一个 Lorenz Attractor 轨迹

```
delta = [0; 1e-6; 0]; % 第二条轨迹初始值偏移量
phi0New = phi0 + delta;
[t, phiNew] = ode45(@LorenzAttr, [0, T], phi0New);
figure;
plot3(phi(:,1), phi(:,2), phi(:,3), phiNew(:,1), phiNew(:,2), phiNew(:,3));
legend(["y=7", "y=7+1.0\times10^{-6}"], Interpreter="tex", Location="northeast");
title("Lorenz Attractor");
xlabel("X");
ylabel("Y");
zlabel("Z");
grid on;
view([45 45]);
```


Lorenz Attractor
y=7
y=7+1.0×10^-6
50
40
30
20
10
0
Z
-20
-10
0
10
20
X
20
10
0
-10
-20
Y