## Quiz 1

PATTERN RECOGNITION, FALL 2018

Name:

UID:

**Problem 1.**(10 =5+3+2 points.)

1a. If $X$ and $Y$ are conditionally independent, then $P(X,Y|Z) = P(X|Z)P(Y|Z)$. —(i)
Show that it is eqivalent to $P(Y|X,Z) = P(Y|Z)$. Hint use product rule of probability conditioned on $Z$.

By product rule of probability

$P(X,Y|Z) = P(X|Z)\,P(Y|X,Z)$ —(ii)

comparing (i) and (ii) gives

$P(Y|X,Z) = P(Y|Z)$

1b What is the support of Beta distribution and Dirichlet distribution in dimension $D$.

Beta distribution has support in $[0\ 1]$

1c If in Gaussian discriminant analysis the covarince matrix $\Sigma_c$ for a class $c$ is digonal then is it equivalent to what classifier?

Naive Bayes

**Problem 2.**(10 points.) Let scalar $x \sim \mathcal{N}(\mu, \sigma^2)$. If we have $N$, I.I.D samples, then compute the MLE estimate of $\mu$.

let $D = \{(x_i)\}_{i=1}^{N}$ be the $N$ I.I.D samples

$$\hat{\mu} = \arg\max_{\mu} \log P(D) \underset{\text{assumption}}{\overset{\text{I.I.D}}{=}} \arg\max_{\mu} \log \prod_{i=1}^{N} P(x_i)$$

$$= \arg\max_{\mu} \sum_{i=1}^{N} \log P(x_i)$$

$$= \arg\max_{\mu} \left[ \sum_{i=1}^{N} \log \frac{1}{(2\pi)^{1/2}\sigma} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right) \right]$$

$$= \arg\max_{\mu} \left[ \sum_{i=1}^{N} \log \frac{1}{(2\pi)^{1/2}\sigma} - \frac{1}{2}\sum_{i=1}^{N}\frac{(x_i-\mu)^2}{\sigma^2} \right]$$

$$= \arg\max_{\mu} -\frac{1}{2}\sum_{i=1}^{N}\frac{(x_i-\mu)^2}{\sigma^2}$$

$$= \arg\min_{\mu} \left[ \frac{1}{2\sigma^2} \sum_{i=1}^{N} (x_i-\mu)^2 \right] \quad (*)$$

minimizer $\mu$ can be computed by taking derivative $(*)$ and equating to zero

$$\frac{1}{2\sigma^2}\, 2\sum_{i=1}^{N}(x_i-\mu)(-1) = 0$$

$$\Rightarrow \sum_{i=1}^{N} x_i - \sum_{i=1}^{N}\mu = 0 \Rightarrow \sum_{i=1}^{N} x_i = N\mu \Rightarrow \mu = \frac{\sum_{i=1}^{N} x_i}{N}$$