4+4 ~~pts~~ 8 points

decision region $R_1 = \{x : p(x|\mu_1, \sigma_1) \geq p(x|\mu_2, \sigma_2)\}$

4.21 (a)

i.e x has to satisfy

$$\frac{1}{\cancel{(2\pi)^{1/2}}\sigma_1} \exp\left(\frac{-1}{2\sigma_1^2}(x-\mu_1)^2\right) \geq \frac{1}{\cancel{(2\pi)^{1/2}}\sigma_2} \exp\left(\frac{-1}{2\sigma_2^2}(x-\mu)^2\right)$$

taking log

$$-\log\sigma_1 - \frac{1}{2\sigma_1^2}(x-\mu_1)^2 \geq -\log\sigma_2 - \frac{1}{2\sigma_2^2}(x-\mu_2)^2$$

given $\mu_1 = 0$, $\mu_2 = 1$, $\sigma_1^2 = 1$, $\sigma_2^2 = 10^6$

$$\frac{-x^2}{2} \geq -\log\sigma_2 - \frac{x^2}{2\sigma_2^2} - \frac{\mu_2^2}{2\sigma_2^2} + \frac{\mu_2 x}{\sigma_2^2}$$

$$-x^2 \geq -2\log\sigma_2 - \frac{x^2}{\sigma_2^2} + \frac{2x}{\sigma_2^2} - \frac{1}{\sigma_2^2} \quad -(1)$$

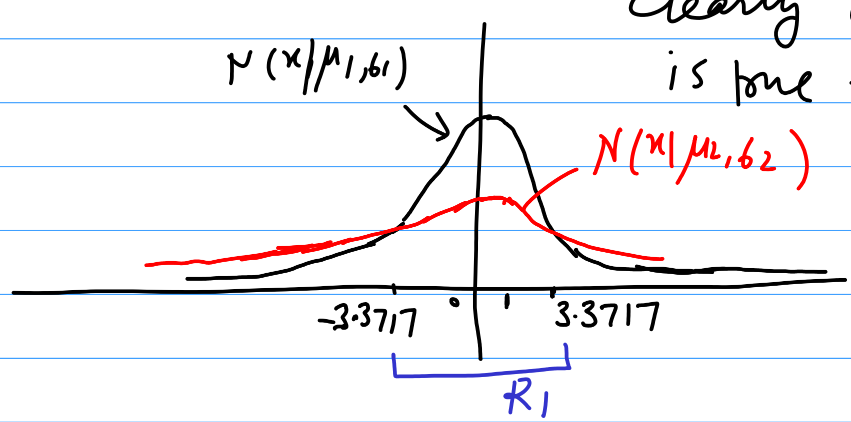
$$\left(\frac{1}{\sigma_2^2} - 1\right)x^2 - \frac{2}{\sigma_2^2}x + \frac{1}{\sigma^2} + 2\log\sigma_2 \geq 0$$

using any quadratic solver for equality part we get

$x = \pm 3.3717$

$(x+3.3717)(-x+3.3717) \geq 0$

clearly above inequality is true for $x \in [-3.3717, +3.3717]$



(b) If $\sigma_2 = 1$ then equation (1) becomes

$$-x^2 \geq 0 - x^2 + 2x - 1$$

$$-2x + 1 \geq 0$$

$$2x - 1 \leq 0$$

$$x \leq 1/2$$

Hence $x \in R_1$ if $x \leq .5$

(4+4 = 8 points)

4.22 $$P(y=c|X) = \frac{P(X|y=c)\, P(y=c)}{P(X)}$$

(a) using given values, we should get

$$P(y=1/X_1) = 0.46$$

$$P(y=2|X_1) = 0.145)$$

$$P(y=3|X_1) = 0.39$$

also

$$R(\alpha_1|X_1) = 0.145 + 0.39 = 0.53$$

$$R(\alpha_2|X_2) = 0.85 \text{ and } R(\alpha_3|X_3) = 0.60$$

Clearly class 1 has minimal risk/maximum posterior probability.

(b) Similarly

$$P(y=1|x_2) = 0.45, \quad P(y=2|x_2) = 0.46$$
$$P(y=3|x_2) = 0.09$$

Also $R(\alpha_1|x_2) = 0.55$, $R(\alpha_2|x_2) = .54$, $R(\alpha_3|x_2) = 0.91$

clearly $\underline{\underline{\text{class } 2}}$

(4+4 = 8 points)

5.2 Clearly posterior expected loss is ~~Risk~~

(a)

$$R(\hat{y}=0|x) = \lambda_{01} P(y=1|x) = \lambda_{01} P_1$$

and $R(\hat{y}=0|x) = \lambda_{10} P(y=0|x) = \lambda_{10} P_0$
$= \lambda_{10}(1-P_1)$

So we will predict $\hat{y}=0$

if $R(\hat{y}=0|x) < R(\hat{y}=1|x)$

$$\lambda_{01} P_1 < \lambda_{10}(1-P_1)$$

$$\underline{\underline{P_1 < \frac{\lambda_{10}}{\lambda_{01}+\lambda_{10}} = \theta}}$$

(b) if $\dfrac{\lambda_{10}}{\lambda_{01}+\lambda_{10}} = 0.1 = \dfrac{1}{10} = \dfrac{1}{1+9}$

then $\lambda_{10} = 1$ and $\lambda_{01} = 9$ (Note: Not unique) clearly loss matrix will be

predicted $\hat{y}$	True y 0	1
0	0	9
1	1	0

(Note any multiple of 1 and 9 will also give same threshold 0.1)

5.3 (4+4=8) posterior expected loss/Risk
(4 points)

(a) cost of rejecting is λ_r

cost of picking most probable class is

$$\hat{j} = \arg\max_{l} P(y=l|x) \text{ is}$$

$$\sum_{i \neq j} \lambda_s P(y=i|x)$$ [cost of picking right class is 0]

so pick $\hat{j}$ if

$$\lambda_r \geq \sum_{i \neq j} \lambda_s P(y=i|x)$$

$$\frac{\lambda_r}{\lambda_s} \geq 1 - P(y=j|x)$$ [probability sum to one]

or $P(y=j|x) \geq 1 - \frac{\lambda_r}{\lambda_s}$

otherwise choose reject.

Note if we decide to choose a class we has to choose

$$j = \arg\max_i P(y=i|x)$$

if we choose other class $K \neq j$ we will incur more cost.

ie cost of choosing K will be

$$\sum_{i \neq K} \lambda_s P(y=i|x) = \lambda_s \left(1 - P(y=K|x)\right)$$

$$\geq \lambda_s \left(1 - P(y=j|x)\right)$$

because $j = \arg\max_i P(y=i|x)$

(b) (4 points) if $\frac{\lambda_r}{\lambda_s} = 0$ there is no cost of rejecting.

as $\frac{\lambda_r}{\lambda_s} \to 1$

$$P(y=j|x) \geq 1 - 1 \geq 0$$

cost of rejecting increases. Above inequality for most probable class is satisfied more and more. we always accept the most probable class.

Total 32 points