

lecture 5

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H/W Review - One layer

```
#####
# TODO:                                     #
# Stack linear layers for best accuracy.      #
#####
class LinearModel(nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear1_1 = nn.Linear(in_features=784, out_features=10, bias=True)
        # self.linear1_2 = nn.Linear(in_features=, out_features=, bias=True)
        # self.linear1_3 = nn.Linear(in_features=, out_features=, bias=True)

        self.sigmoid = nn.LogSigmoid()
        # self.softmax = nn.Softmax(dim=1)

    def forward(self, x):
        x = self.linear1_1(x)
        x = self.sigmoid(x)
        # x = self.linear1_2(x)
        # x = self.sigmoid(x)
        # x = self.linear1_3(x)
        # x = self.sigmoid(x)
        # x = self.softmax(x)

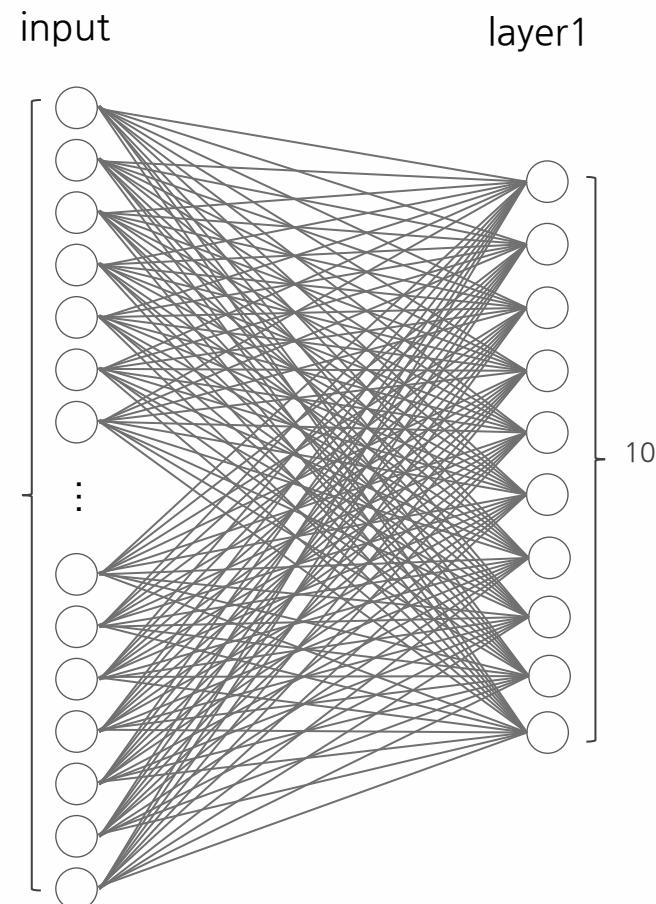
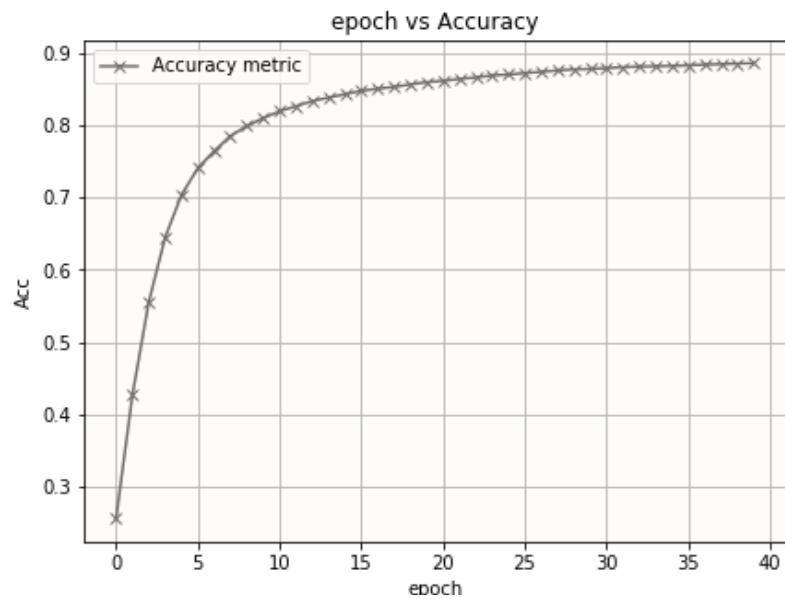
        return x
```

H/W Review - One layer

Layer 1:

input: 784
output: 10

best Acc: 88%



Params: 7850

H/W Review - Two layers

```
class LinearModel(nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear1_1 = nn.Linear(in_features=784, out_features=128, bias=True)
        self.linear1_2 = nn.Linear(in_features=128, out_features=10, bias=True)

        self.sigmoid = nn.LogSigmoid()

    def forward(self, x):
        x = self.linear1_1(x)
        x = self.sigmoid(x)
        x = self.linear1_2(x)
        x = self.sigmoid(x)

    return x
```

H/W Review - Two layers

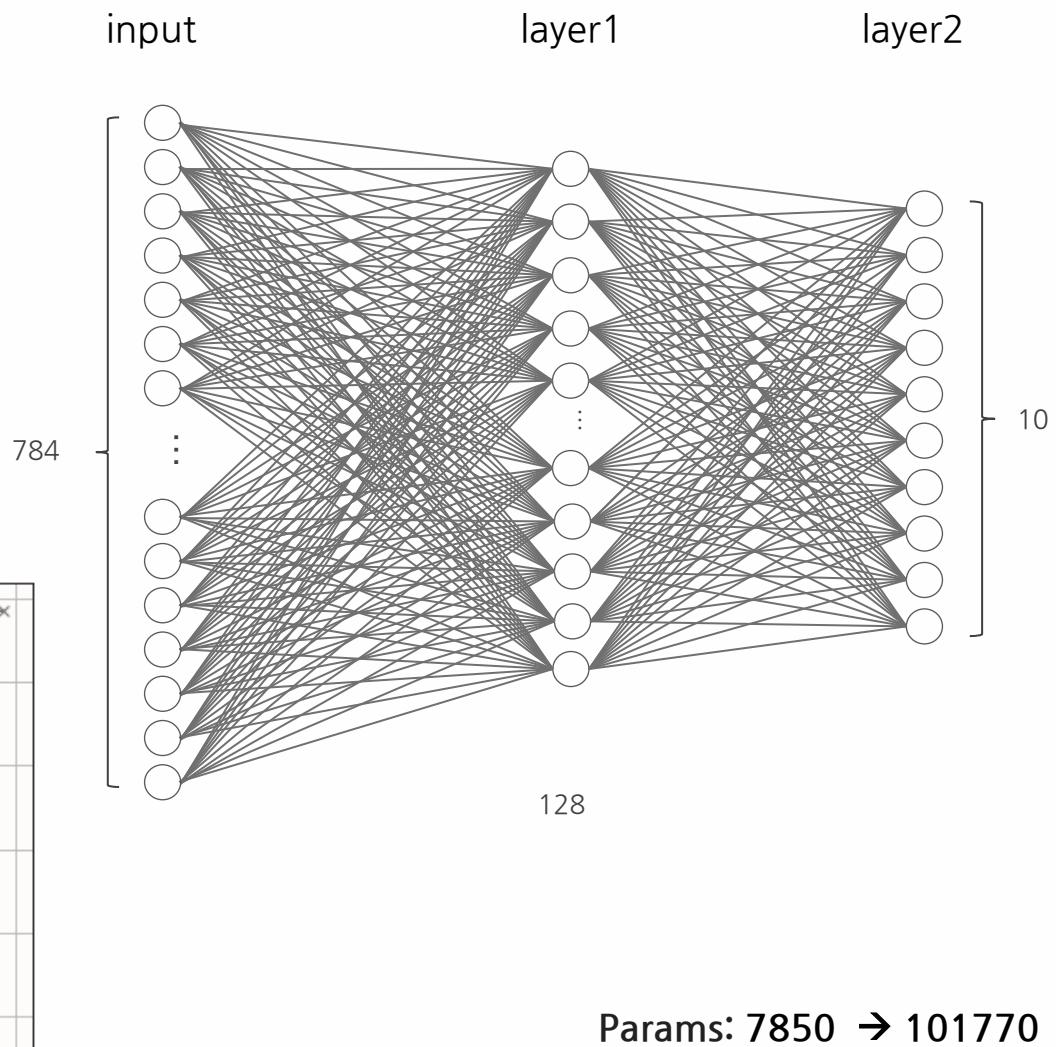
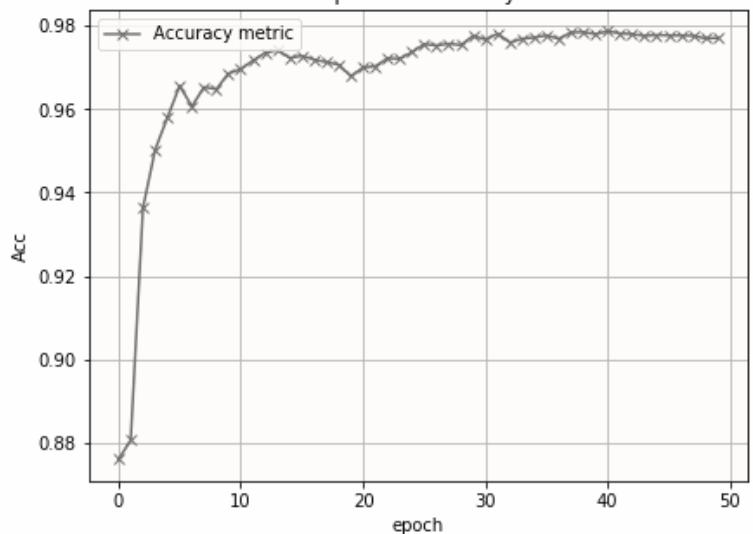
Layer 1:

input: 784
output: 128

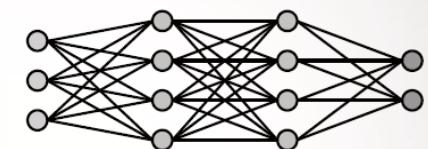
Layer 2:

input: 128
output: 10

best Acc: 97.9%
epoch vs Accuracy



H/W Review - Three layers

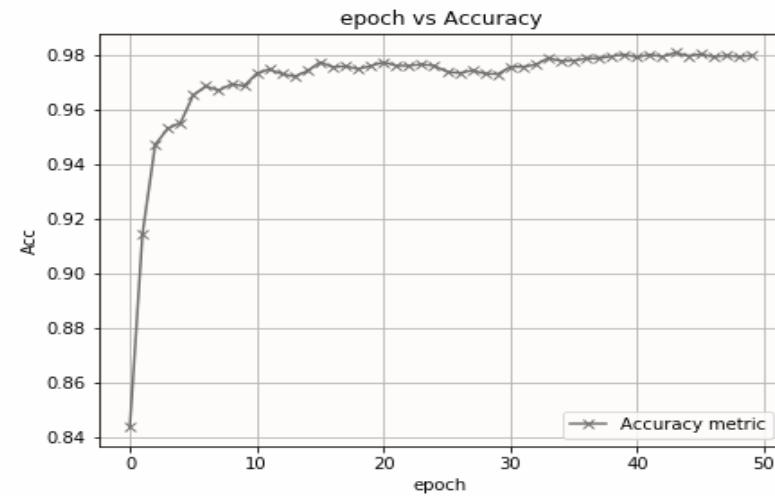
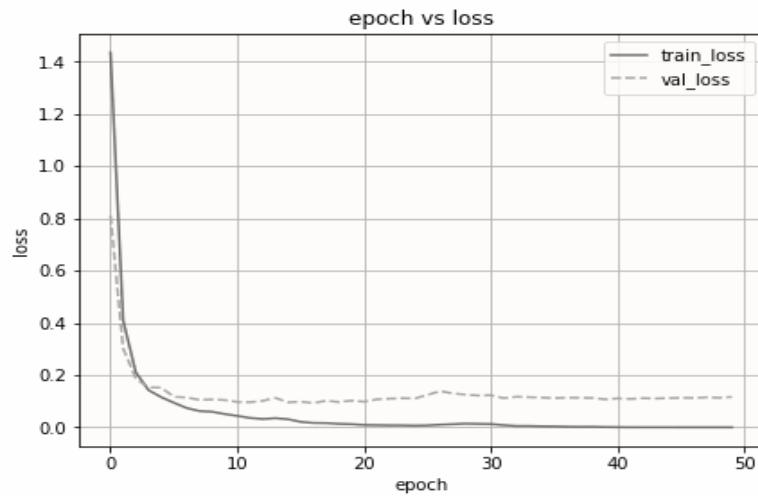


```
class LinearModel(nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear1_1 = nn.Linear(in_features=784, out_features=512, bias=True)
        self.linear1_2 = nn.Linear(in_features=512, out_features=256, bias=True)
        self.linear1_3 = nn.Linear(in_features=256, out_features=10, bias=True)

        self.sigmoid = nn.LogSigmoid()

    def forward(self, x):
        x = self.linear1_1(x)
        x = self.sigmoid(x)
        x = self.linear1_2(x)
        x = self.sigmoid(x)
        x = self.linear1_3(x)
        x = self.sigmoid(x)

        return x
```



Params: 7850 → 101770 → 535818

H/W Review - Three layers, More params

```
class LinearModel(nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear1_1 = nn.Linear(in_features=784, out_features=1024, bias=True)
        self.linear1_2 = nn.Linear(in_features=1024, out_features=2048, bias=True)
        self.linear1_3 = nn.Linear(in_features=2048, out_features=10, bias=True)

        self.sigmoid = nn.LogSigmoid()

    def forward(self, x):
        x = self.linear1_1(x)
        x = self.sigmoid(x)
        x = self.linear1_2(x)
        x = self.sigmoid(x)
        x = self.linear1_3(x)
        x = self.sigmoid(x)

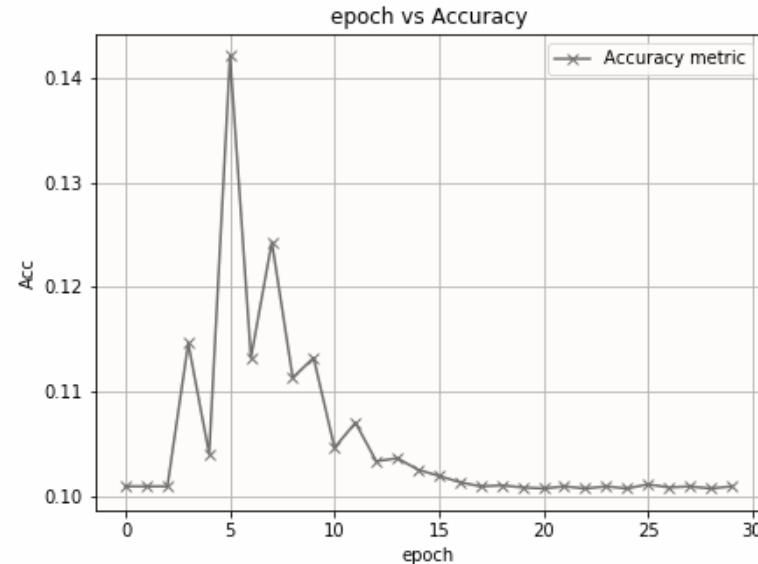
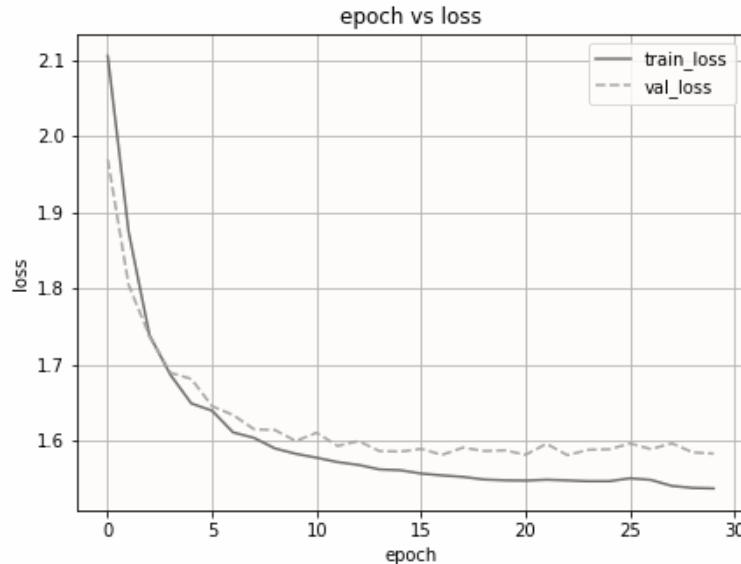
        return x
```

Layer 1:
input: 784
output: 1024

Layer 2:
input: 1024
output: 2048

Layer 3:
input: 2048
output: 10

Best Acc: 14.2%



Params: 7850 → 101770 → 535818 → 2923530

H/W Review - Three layers, More, more params

```
class LinearModel(nn.Module):
    def __init__(self):
        super(LinearModel, self).__init__()
        self.linear1_1 = nn.Linear(in_features=784, out_features=2048, bias=True)
        self.linear1_2 = nn.Linear(in_features=2048, out_features=4096, bias=True)
        self.linear1_3 = nn.Linear(in_features=4096, out_features=10, bias=True)

        self.sigmoid = nn.LogSigmoid()

    def forward(self, x):
        x = self.linear1_1(x)
        x = self.sigmoid(x)
        x = self.linear1_2(x)
        x = self.sigmoid(x)
        x = self.linear1_3(x)
        x = self.sigmoid(x)

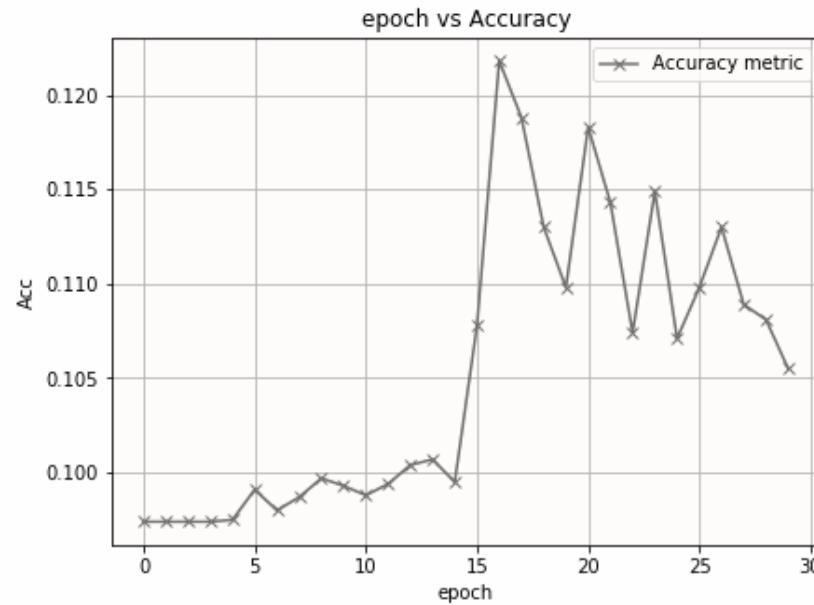
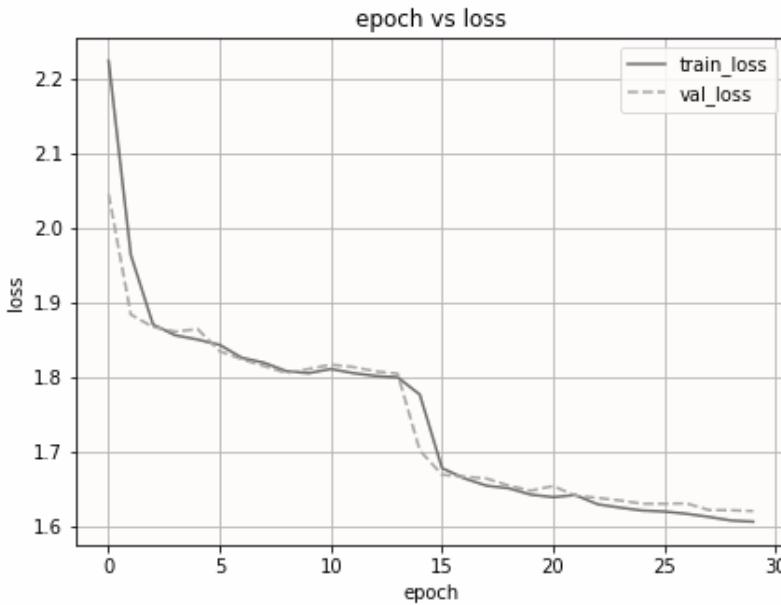
    return x
```

Layer 1:
input: 784
output: 2048

Layer 2:
input: 2048
output: 4096

Layer 3:
input: 4096
output: 10

best Acc: 12.5%

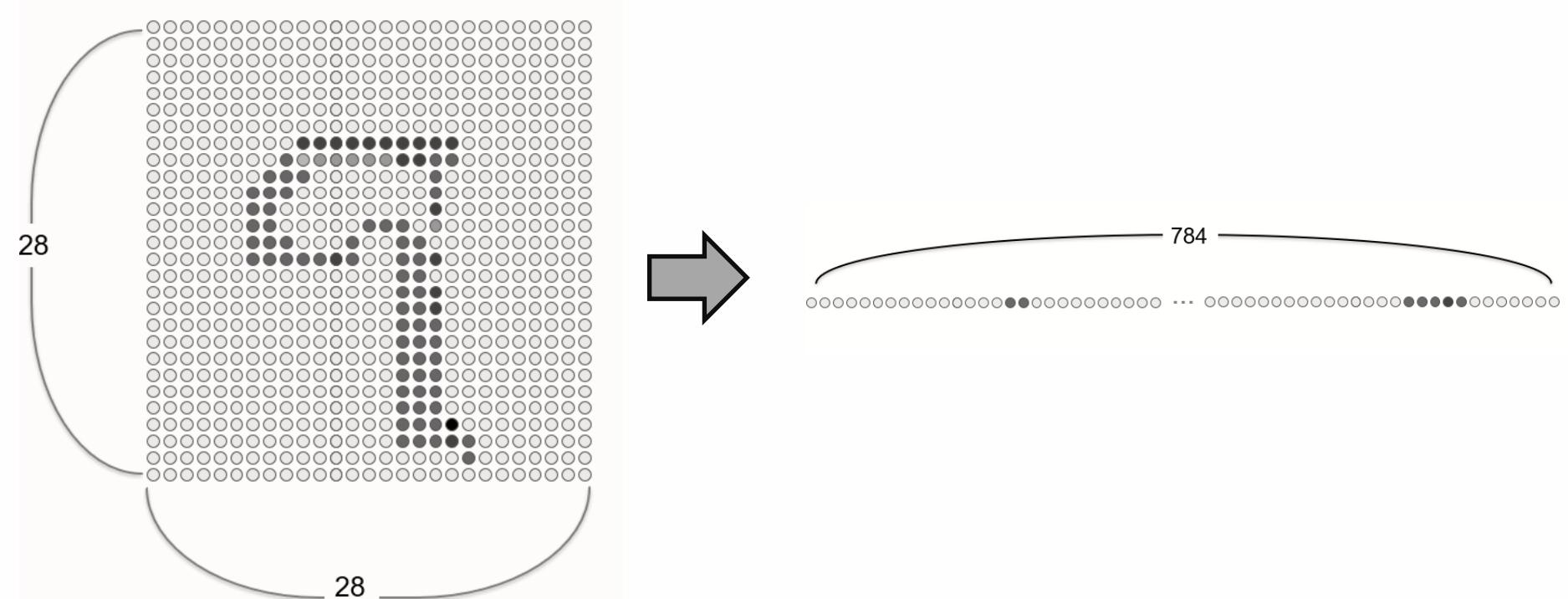


Params: 7850 → 101770 → 535818 → 2923530 → 10041354

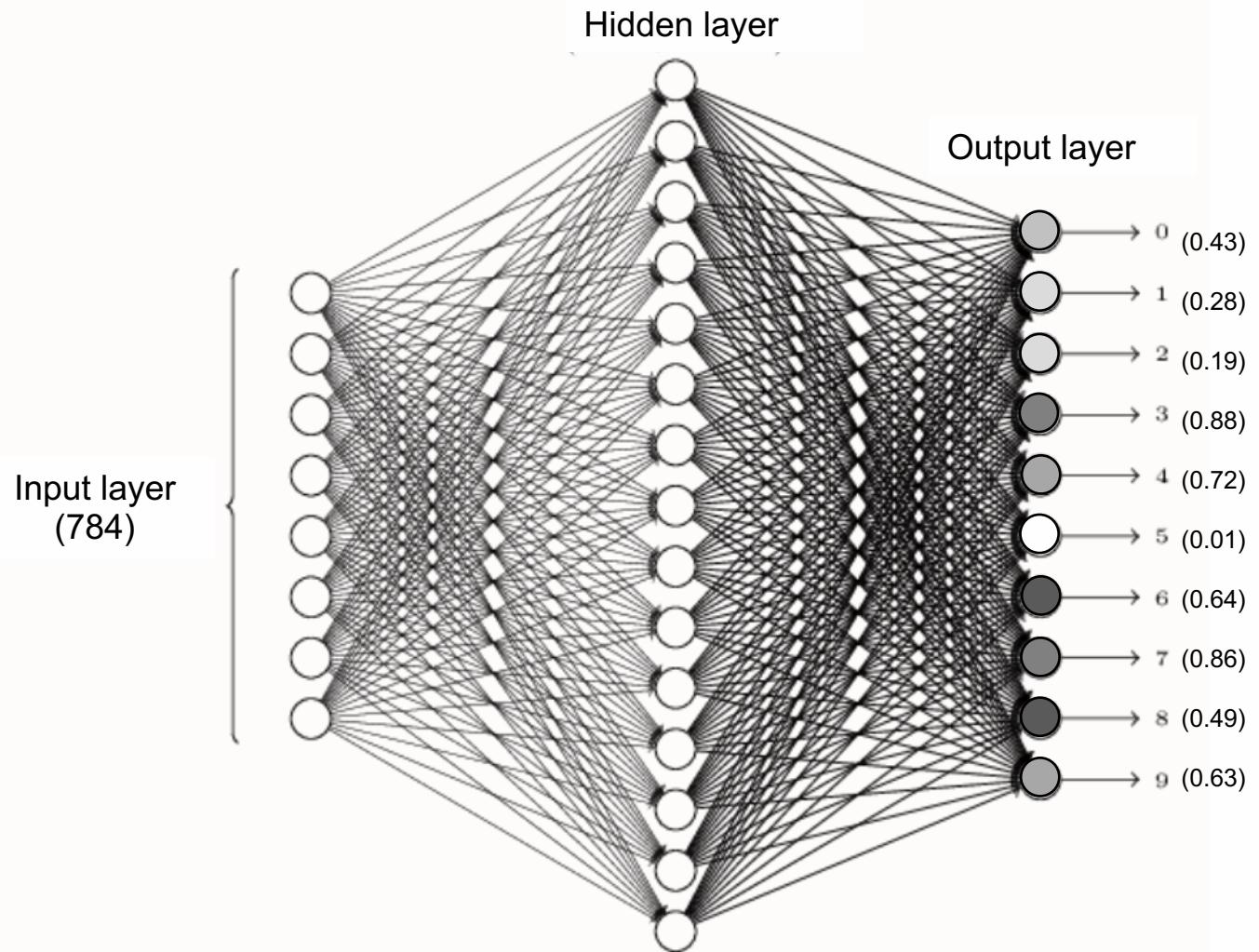
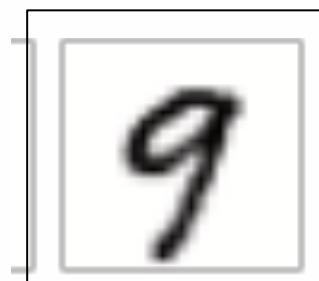
H/W Summary

1. Stacking multiple layers deepens the network for higher accuracy.
2. Adjust the dimensions properly to get good accuracy.
3. If the number of dimensions in the network is too large, the curse of the dimensions will make learning difficult.

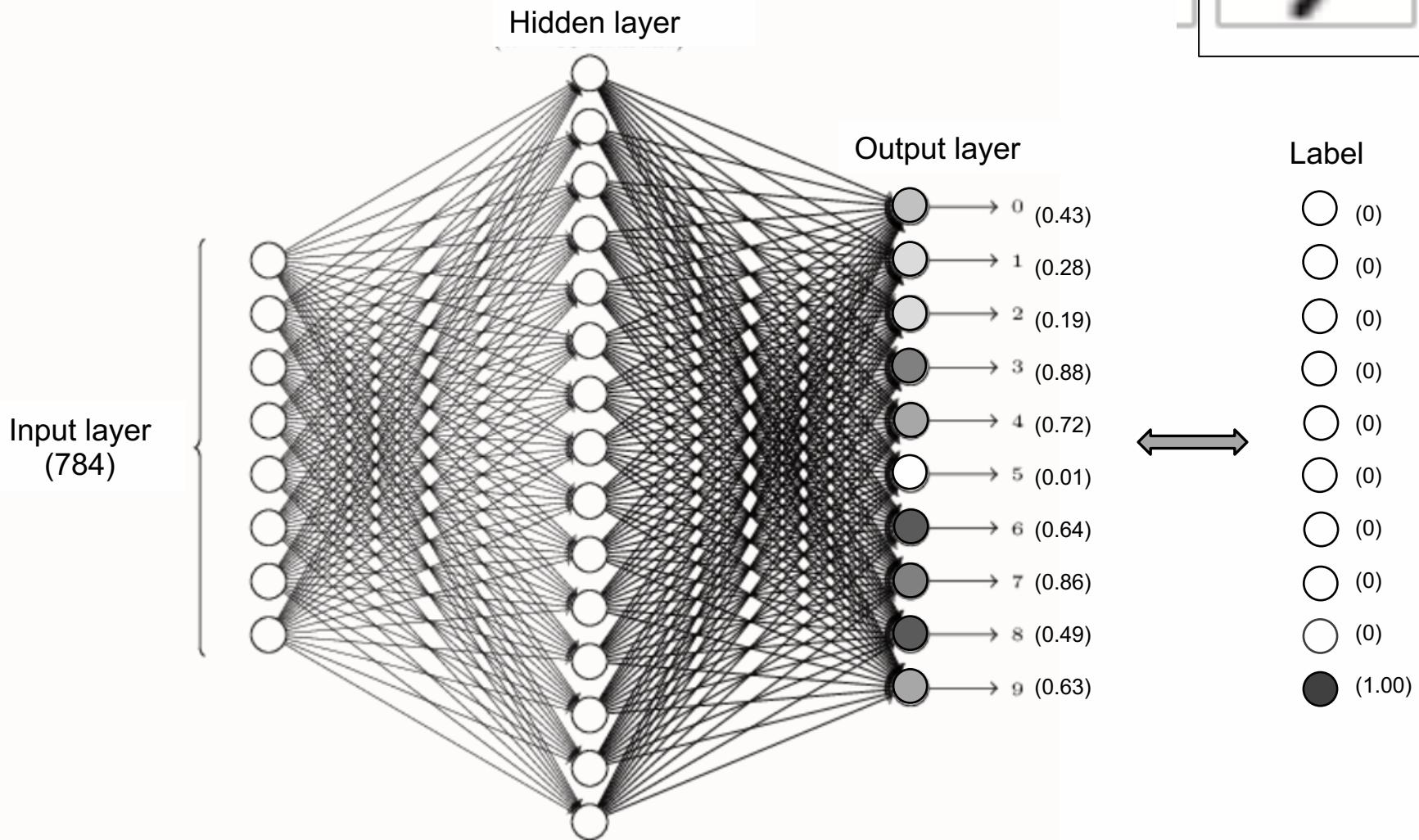
Remind – Forward propagation



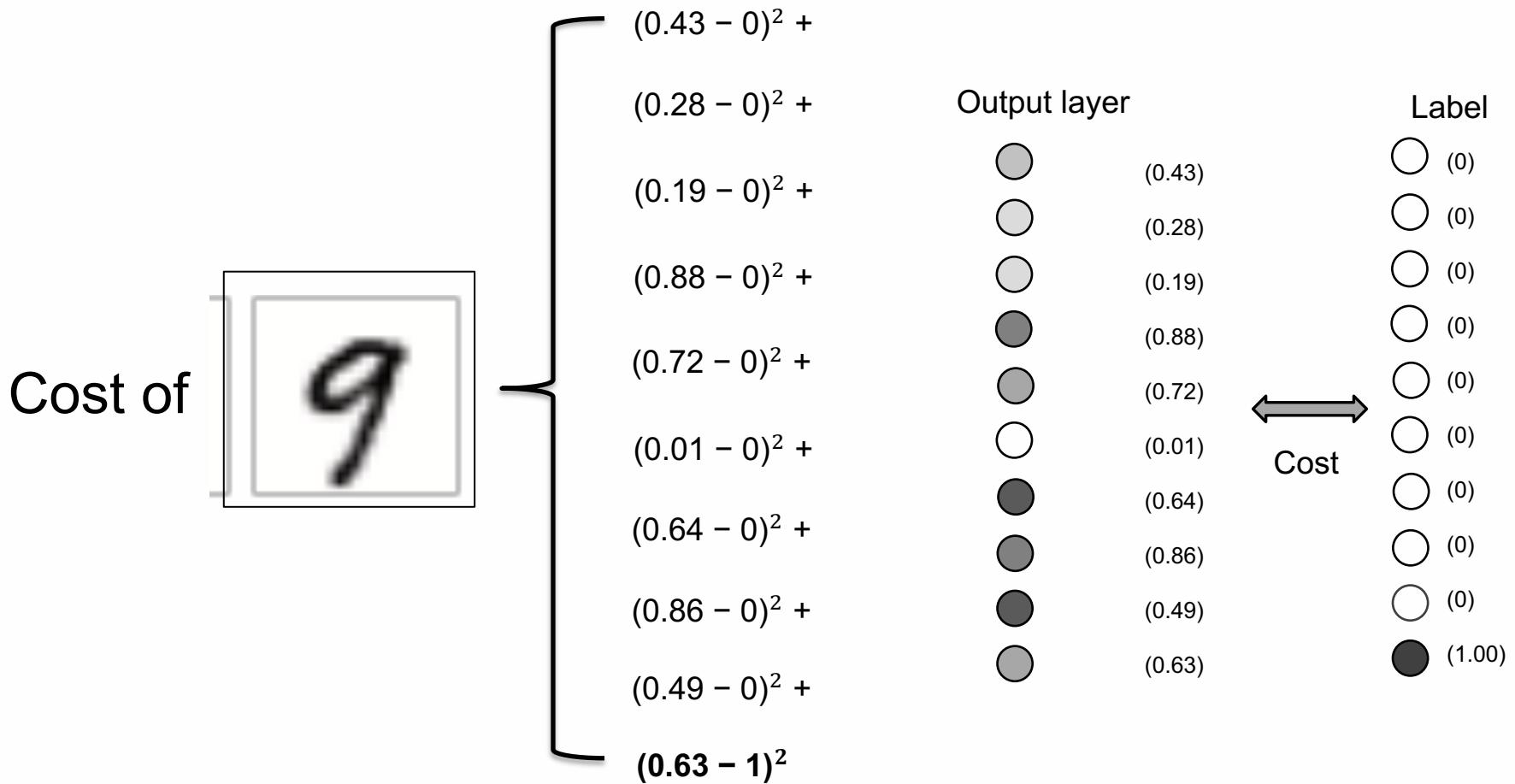
Remind – Forward propagation



Remind – Forward propagation

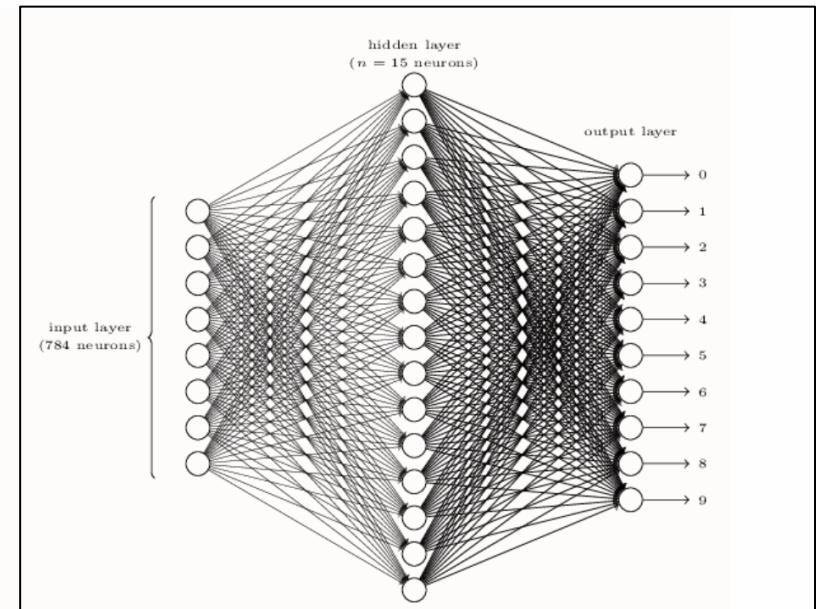


Remind – Forward propagation



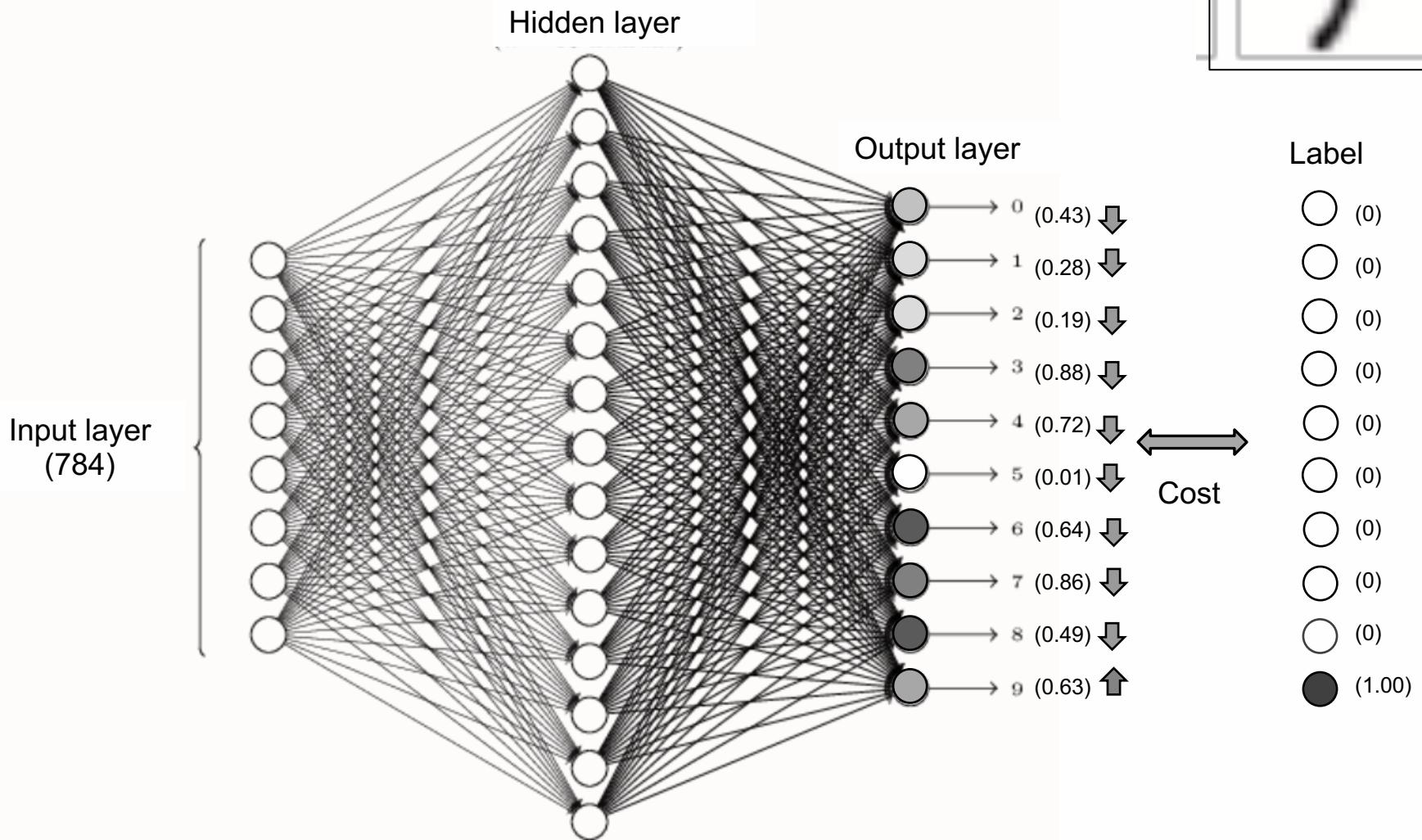
Remind – Forward propagation

$$-\nabla C(\dots) = \underbrace{\quad}_{\text{All weights and biases}} = \begin{matrix} (0.43 - 0)^2 + \\ (0.28 - 0)^2 + \\ (0.19 - 0)^2 + \\ (0.88 - 0)^2 + \\ \vdots \\ (0.72 - 0)^2 + \\ \vdots \\ (0.01 - 0)^2 + \\ (0.64 - 0)^2 + \\ (0.86 - 0)^2 + \\ \vdots \\ (0.49 - 0)^2 + \\ (0.63 - 1)^2 \end{matrix} = \begin{bmatrix} & \\ & \\ & 0.19 \\ & 0.86 \\ & -0.85 \\ & \vdots \\ & . \\ & -0.05 \\ & 0.24 \\ & 0.13 \end{bmatrix}$$

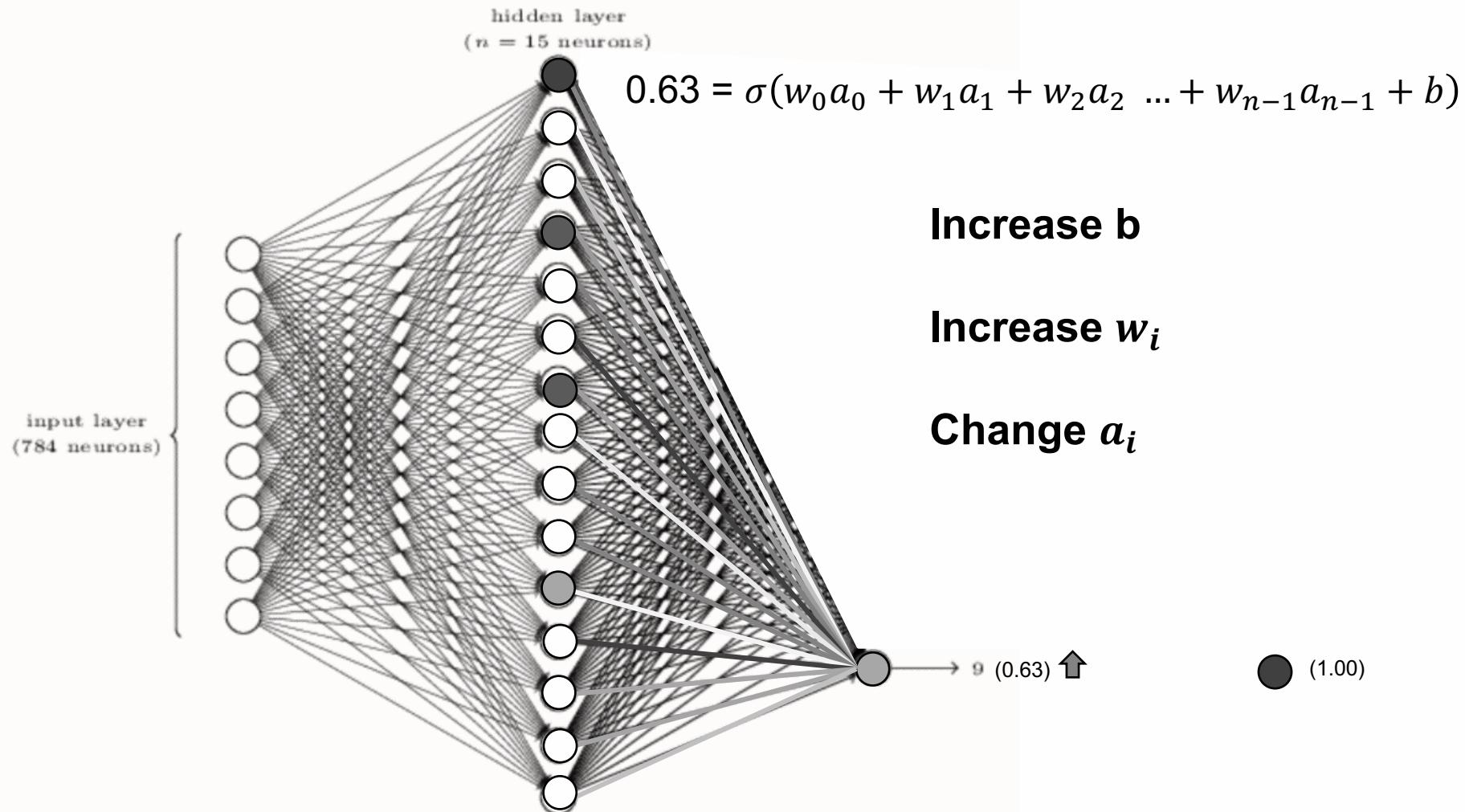


$$C(W_0, W_1, W_2, W_3, W_4, \dots, W_{11910}) = 3.27$$

Remind – Forward propagation



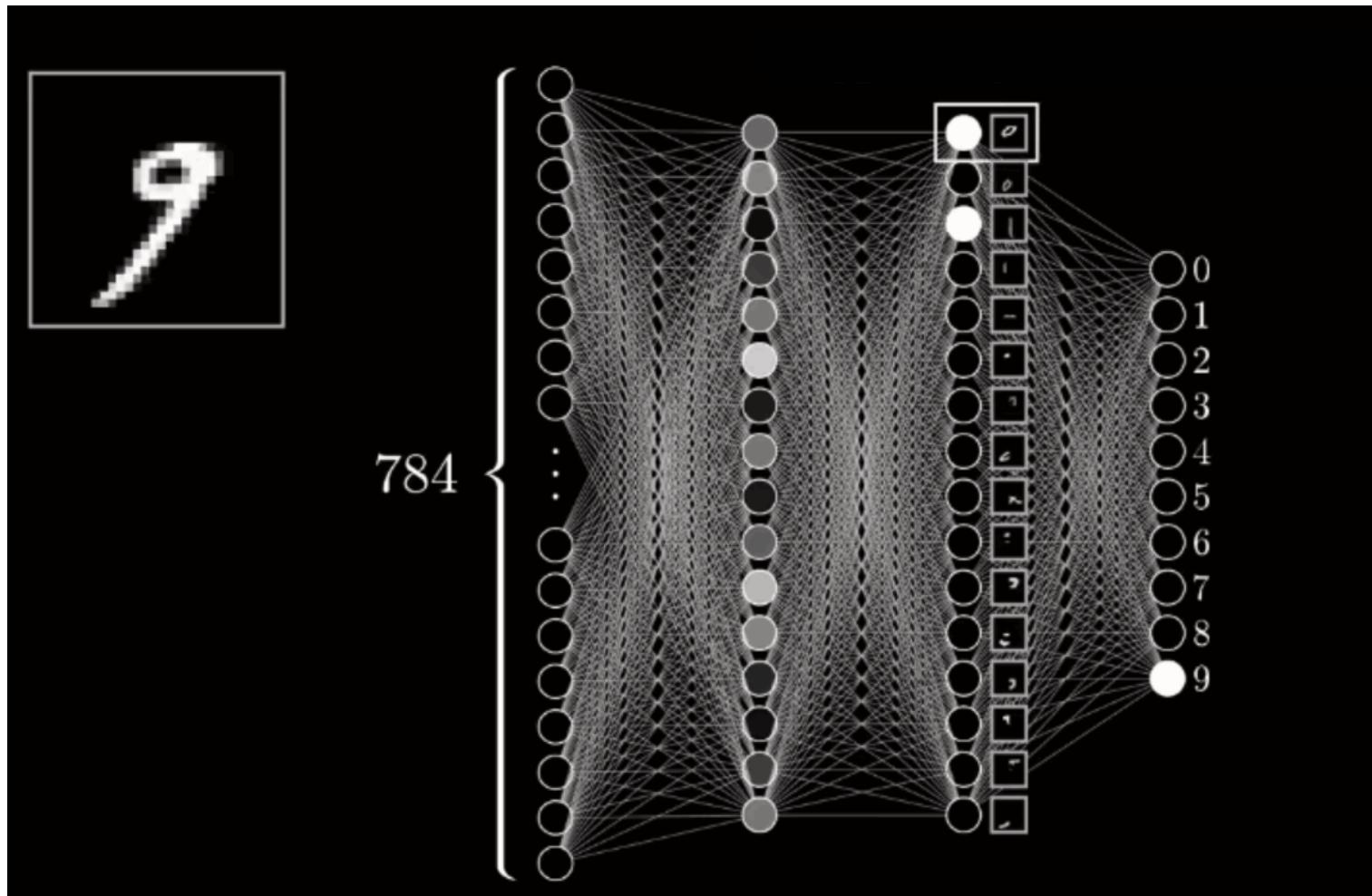
Remind – Forward propagation



Backpropagation



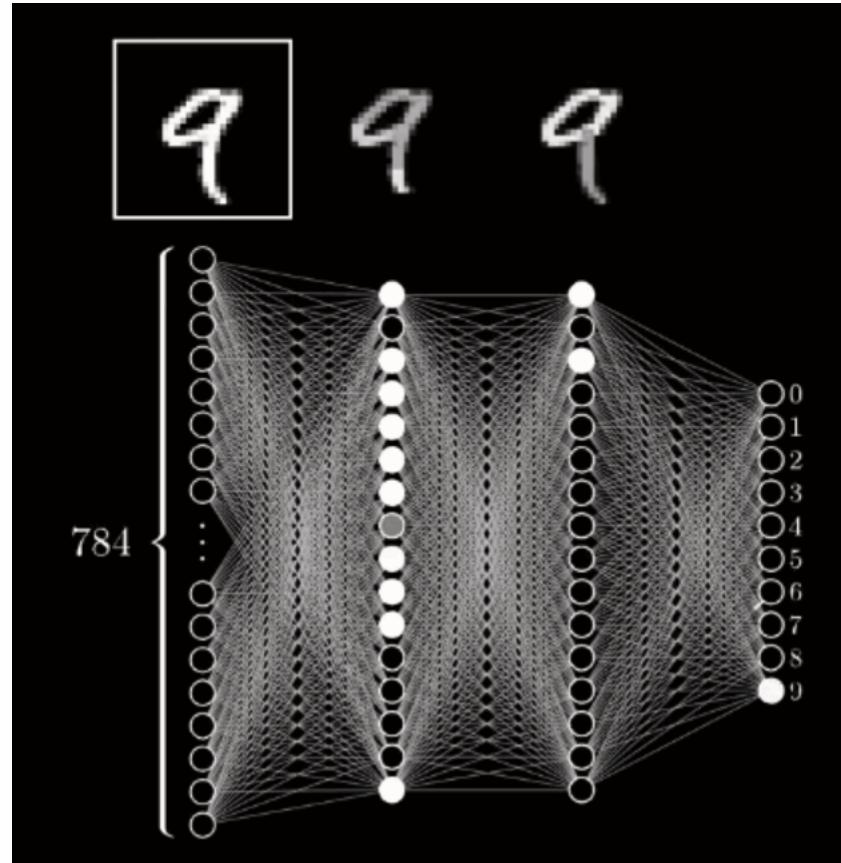
Backpropagation



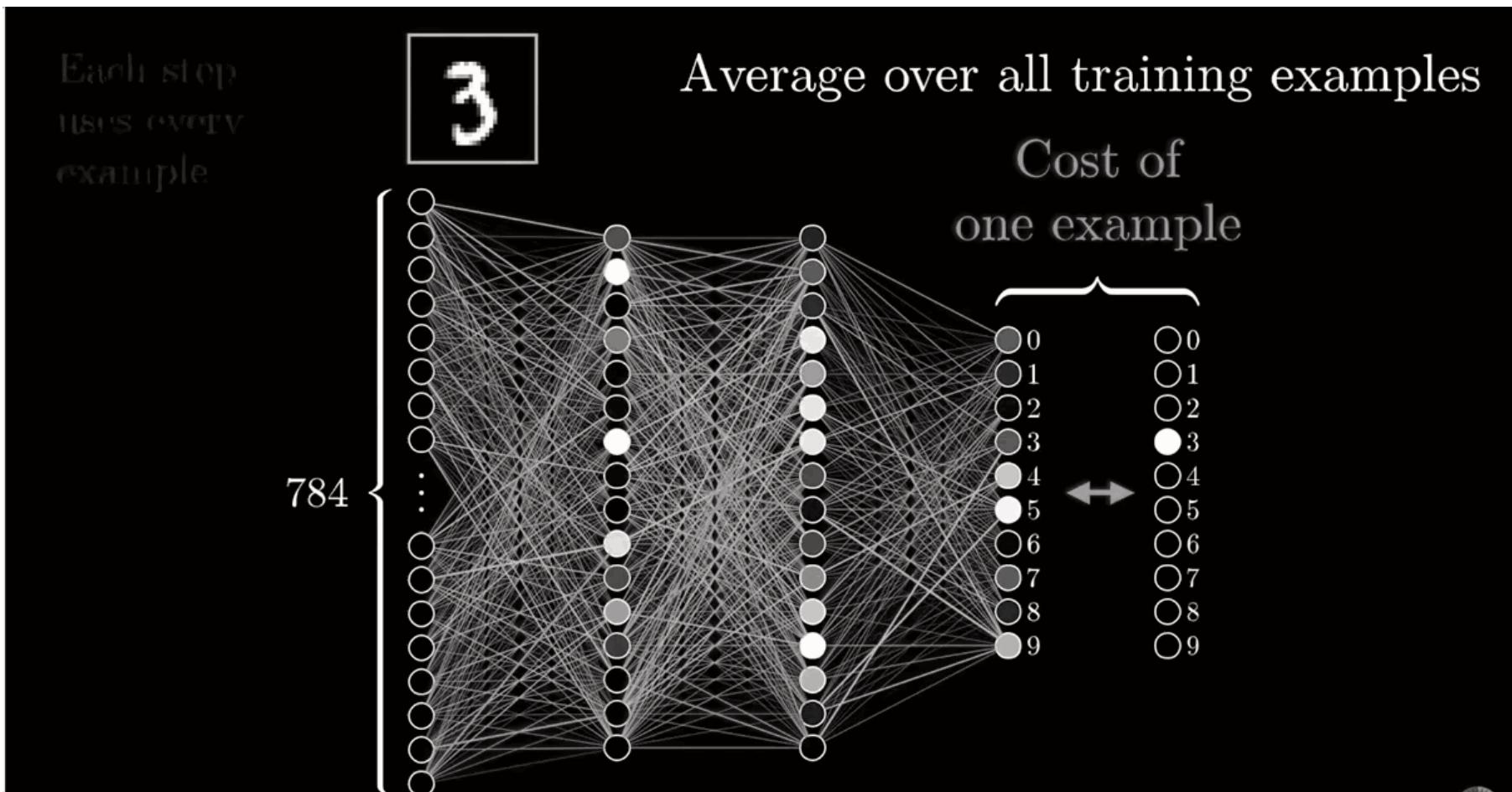
Backpropagation

$$\begin{aligned} \textcircled{1} &= \boxed{\text{stroke}} + \boxed{\text{tail}} + \boxed{\text{body}} + \boxed{\text{head}} + \boxed{\text{tail}} \\ \textcircled{2} &= \boxed{\text{dot}} + \boxed{\text{tail}} + \boxed{\text{body}} \end{aligned}$$

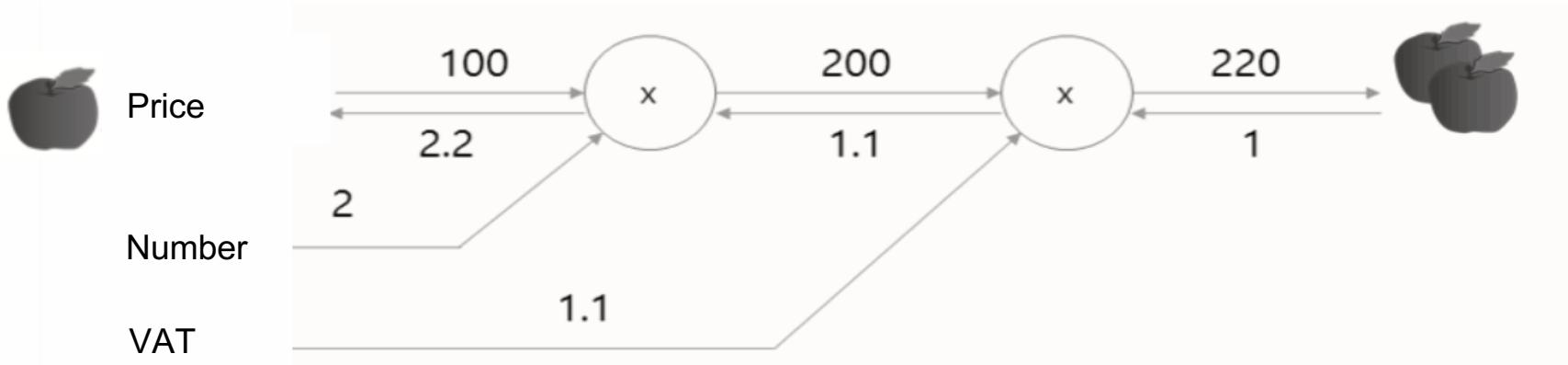
Backpropagation



Backpropagation



Backpropagation

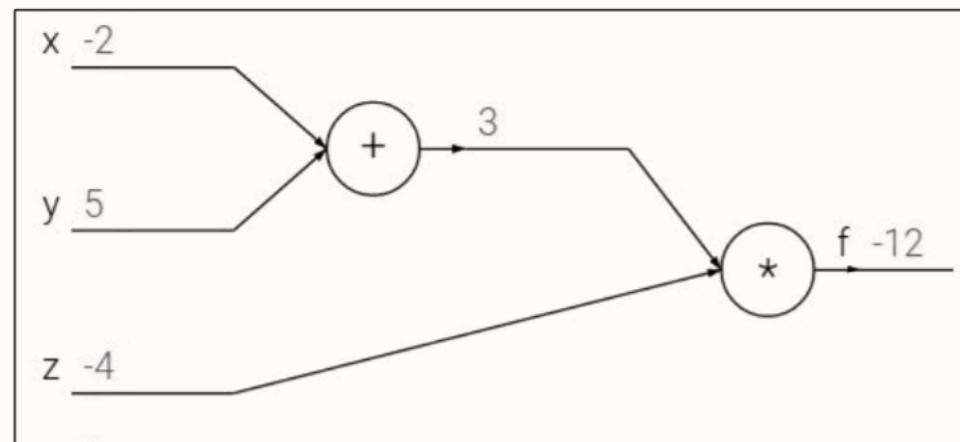


Backpropagation

Backpropagation: a simple example

$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$



Backpropagation

Backpropagation: a simple example

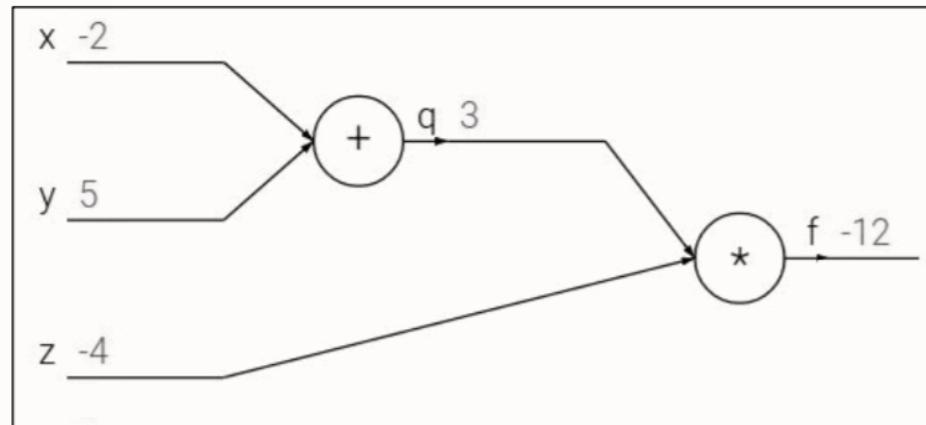
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

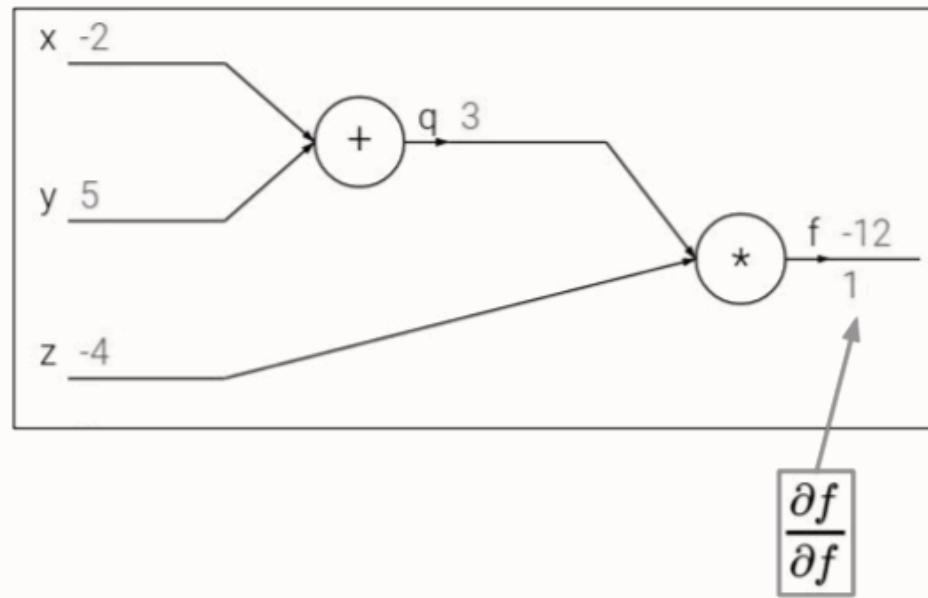
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$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

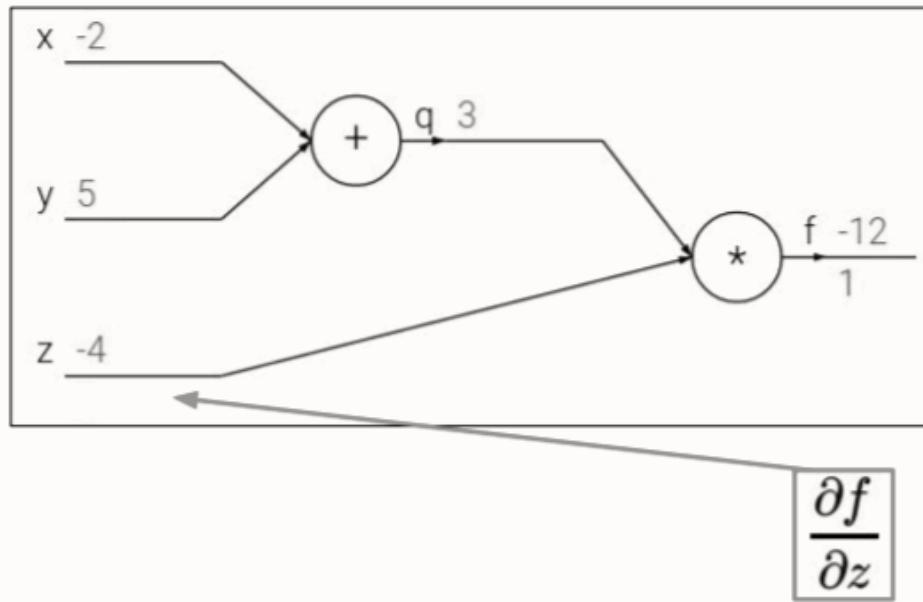
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

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$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

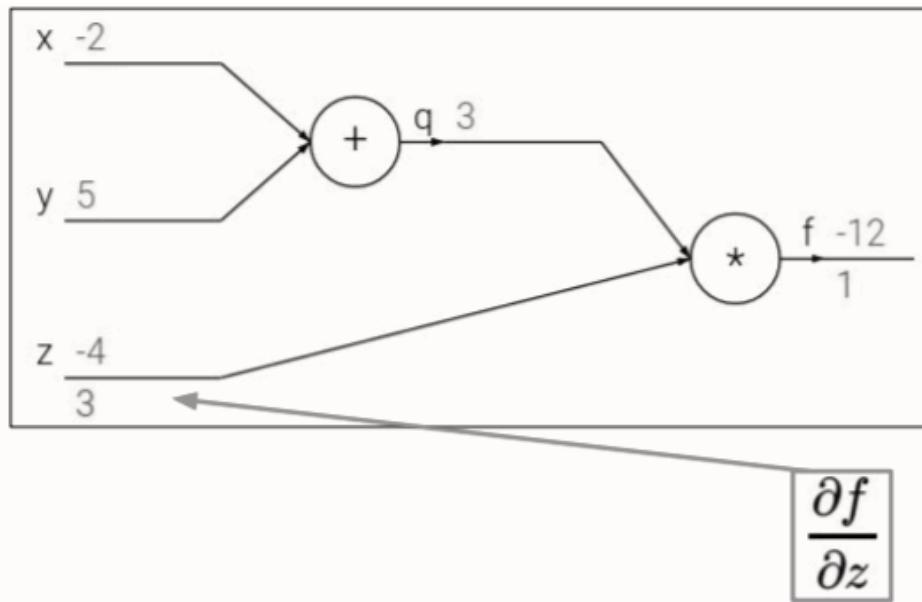
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

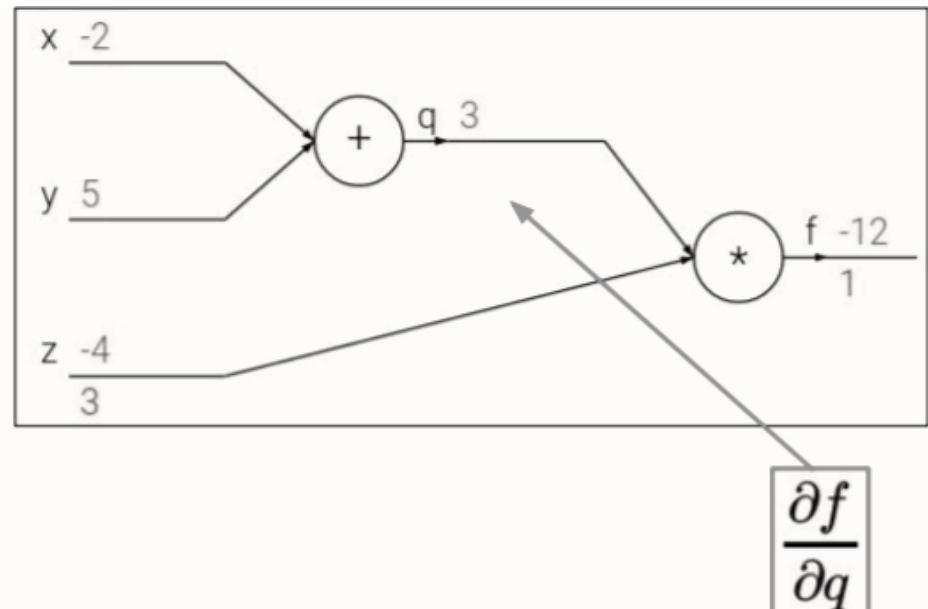
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

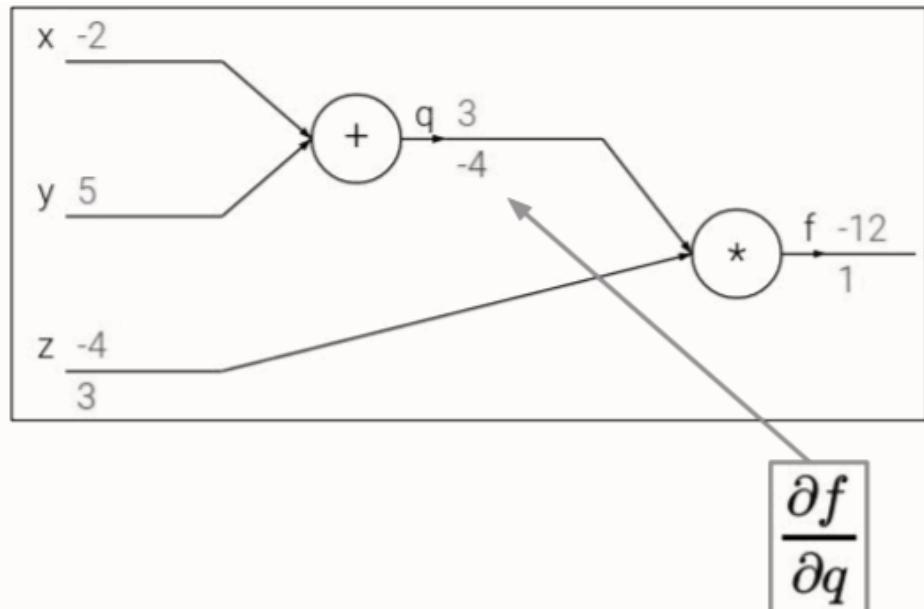
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

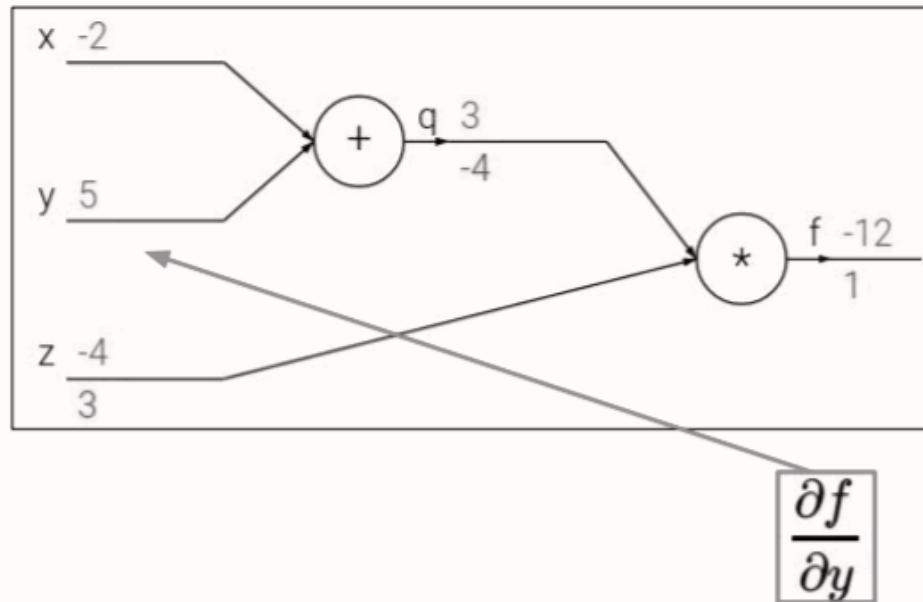
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

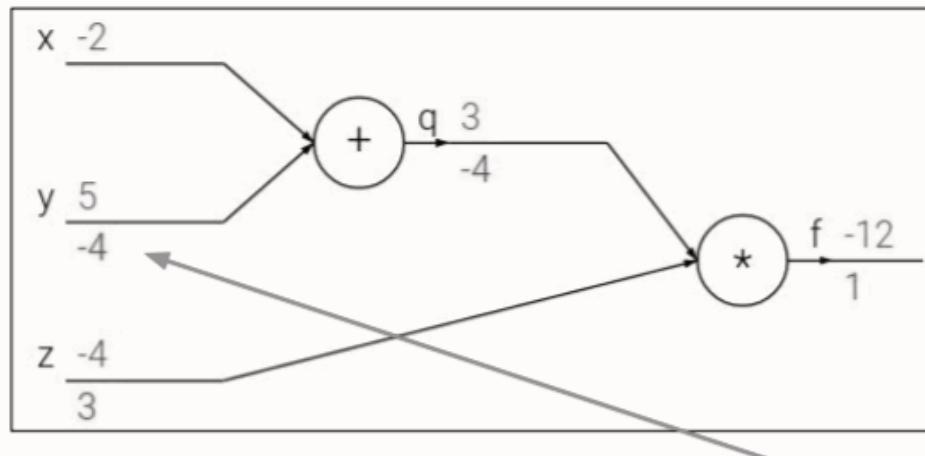
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

Backpropagation

Backpropagation: a simple example

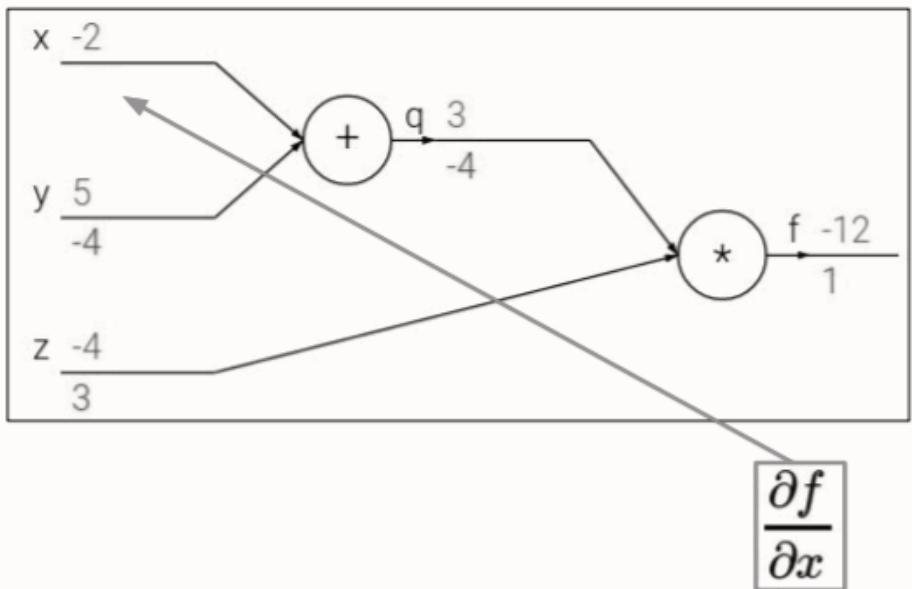
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation

Backpropagation: a simple example

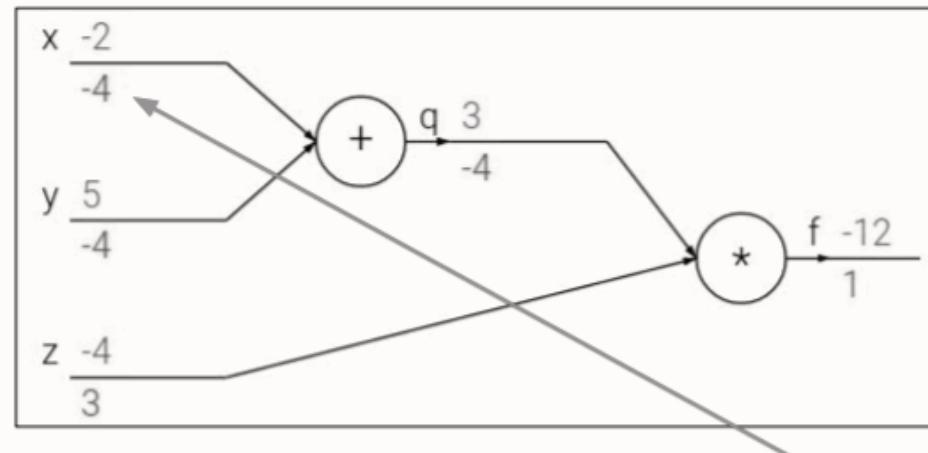
$$f(x, y, z) = (x + y)z$$

e.g. $x = -2$, $y = 5$, $z = -4$

$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

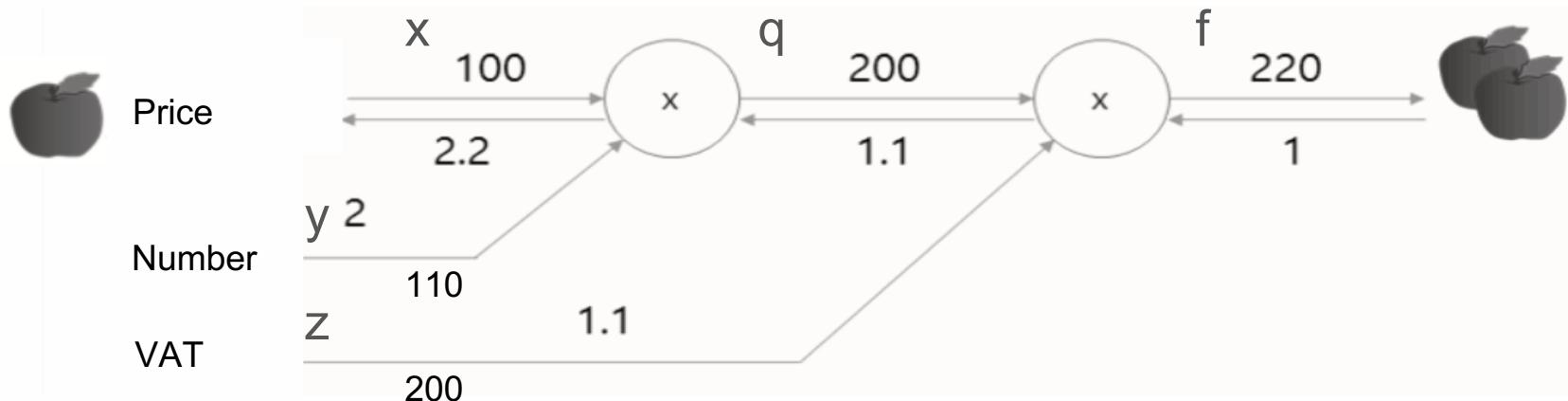
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

Backpropagation



$$q = x \times y$$

$$f = q \times z$$

$$\frac{\partial q}{\partial x} = y, \quad \frac{\partial q}{\partial y} = x$$

$$\frac{\partial f}{\partial q} = z, \quad \boxed{\frac{\partial f}{\partial z}} = q$$

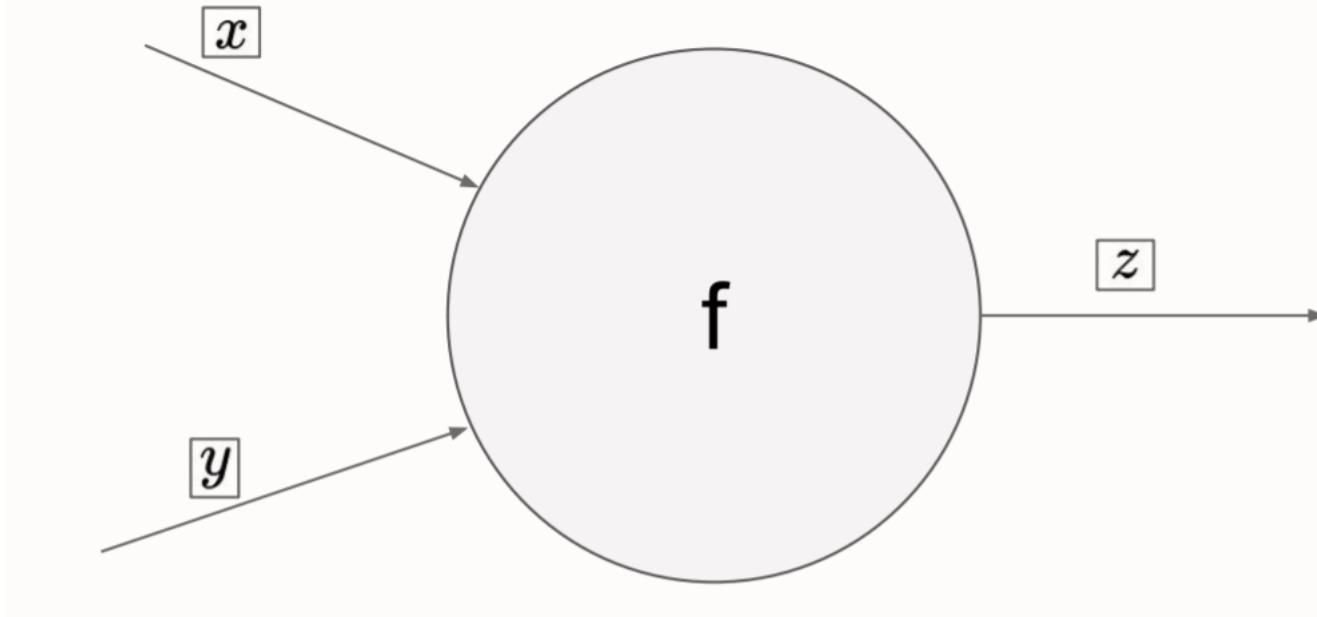
$$\boxed{\frac{\partial f}{\partial x}} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x} = z \times y, \quad \boxed{\frac{\partial f}{\partial y}} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y} = z \times x$$

if $x = 100 + 1 = 101$, then $f = 220 + 2.2 = 222.2$
 prof. $101 \times 2 \times 1.1 = 222.2$

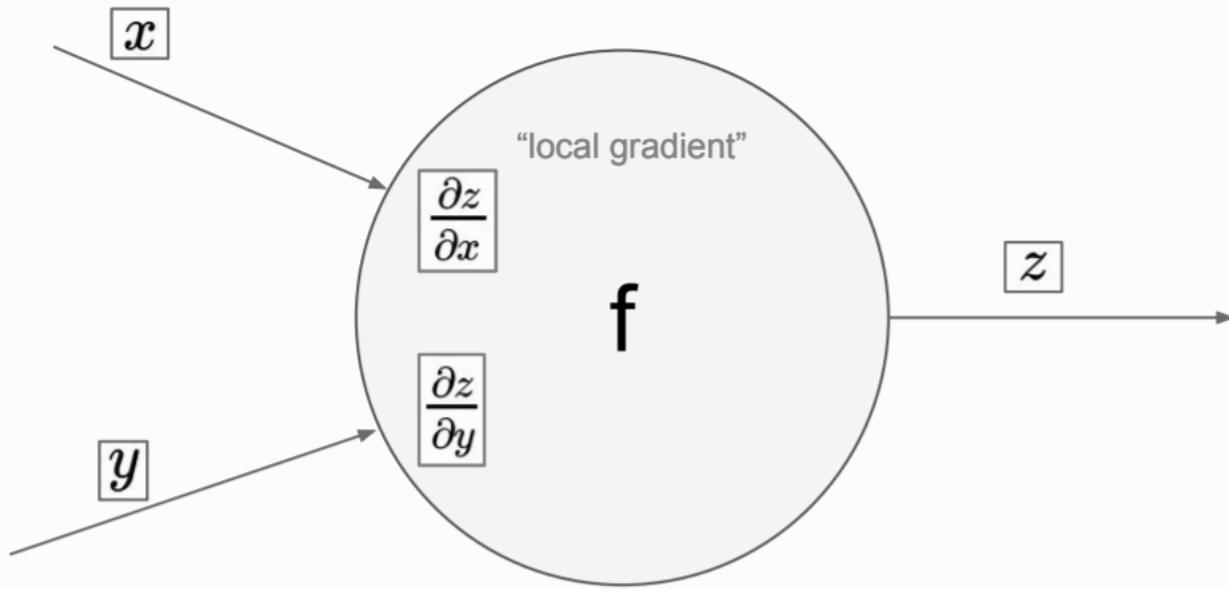
if $y = 2 + 1 = 3$, then $f = 220 + 110 = 330$
 prof. $100 \times 3 \times 1.1 = 330$

if $z = 1.1 + 1 = 2.1$, then $f = 220 + 200 = 420$
 prof. $100 \times 2 \times 2.1 = 420$

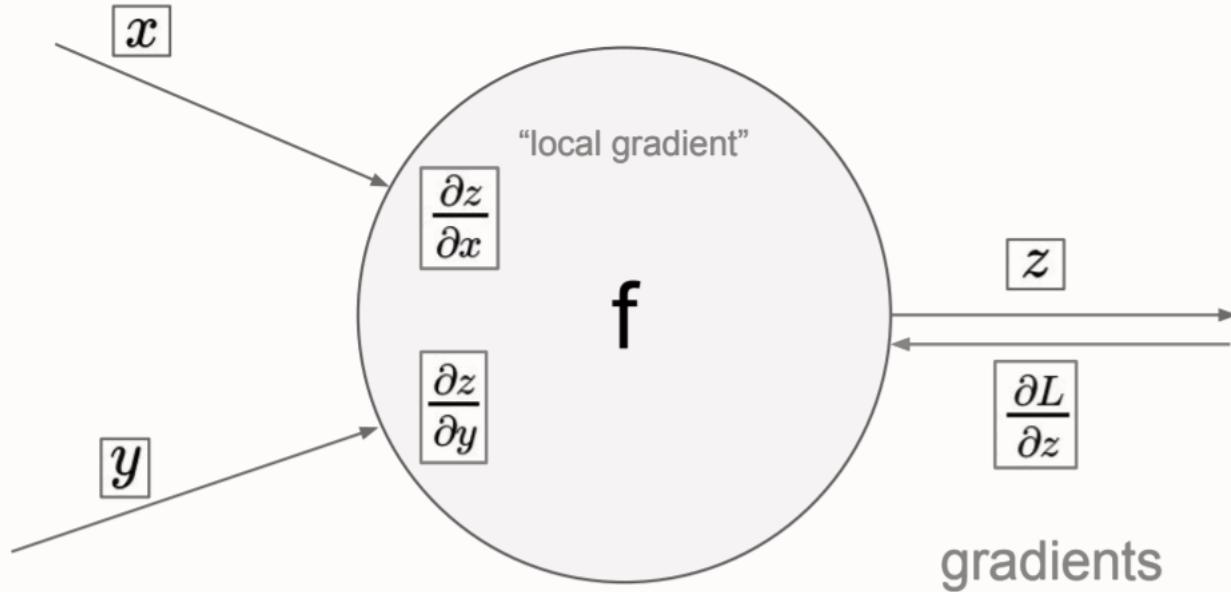
Backpropagation



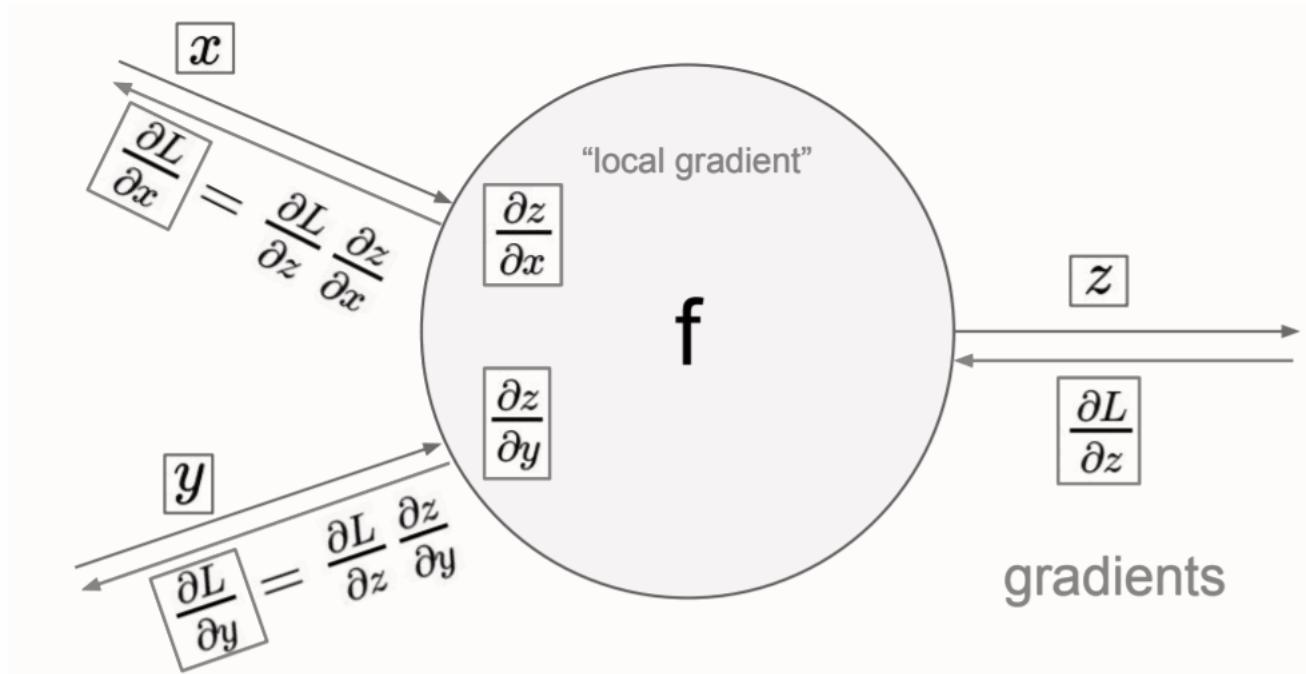
Backpropagation



Backpropagation

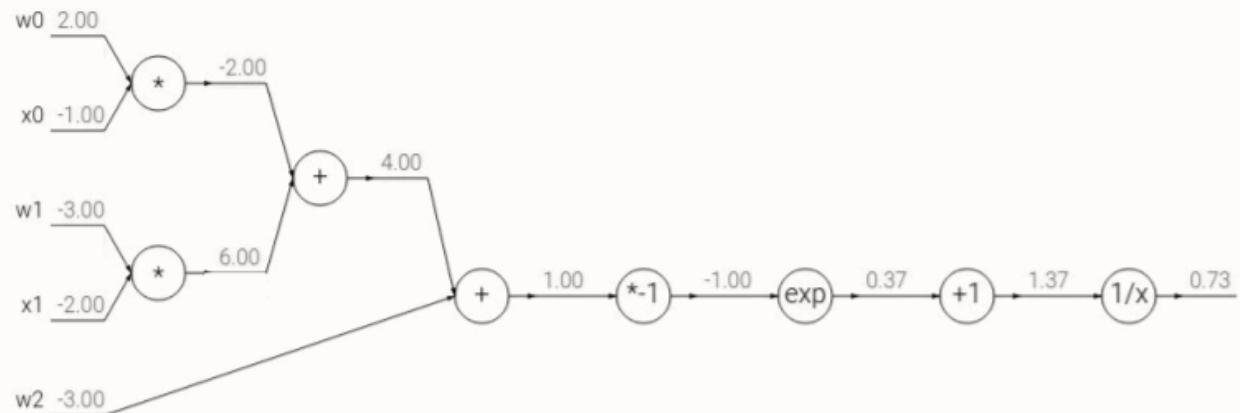


Backpropagation



Backpropagation

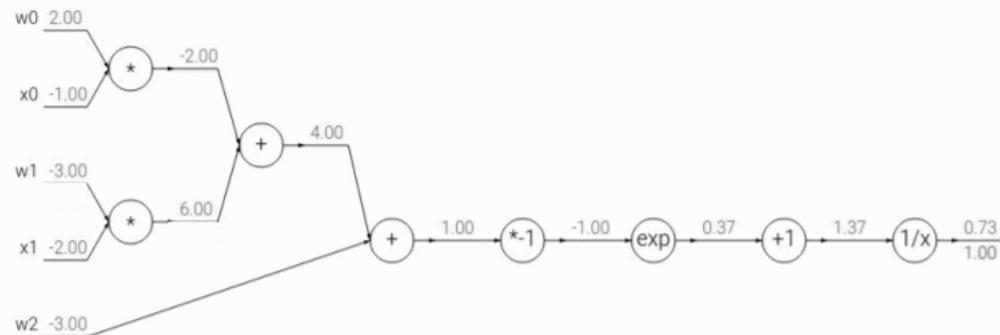
Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



Backpropagation

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f_c(x) = c + x$$

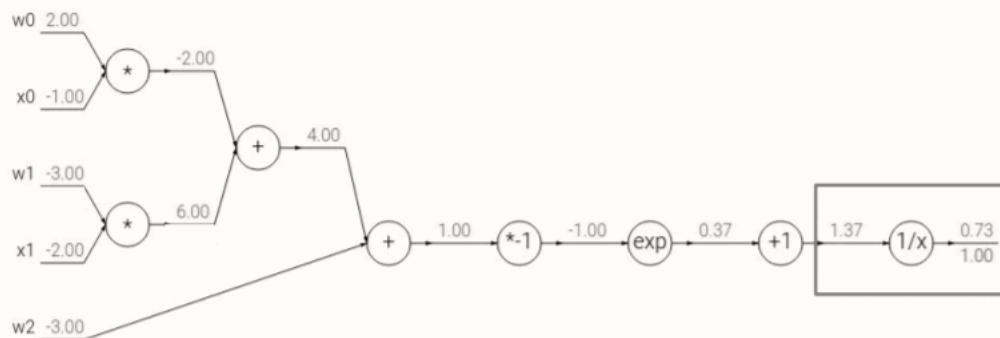
→

$$\frac{df}{dx} = 1$$

Backpropagation

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

→

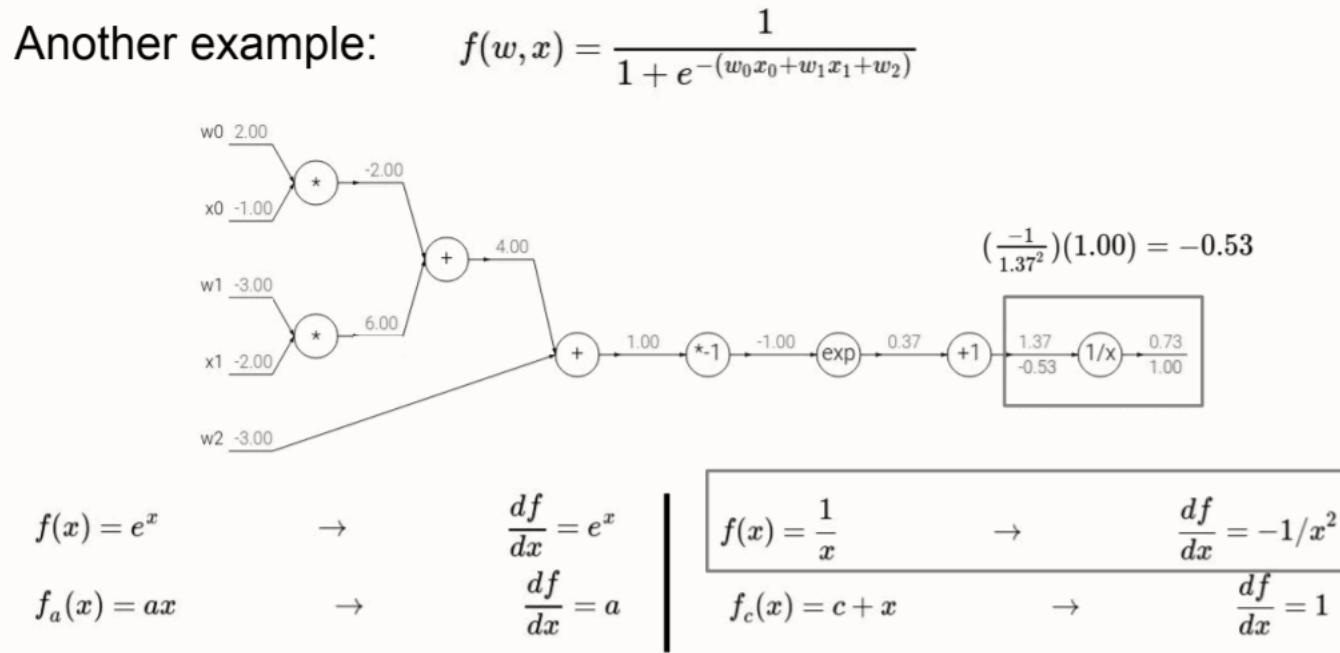
$$\frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

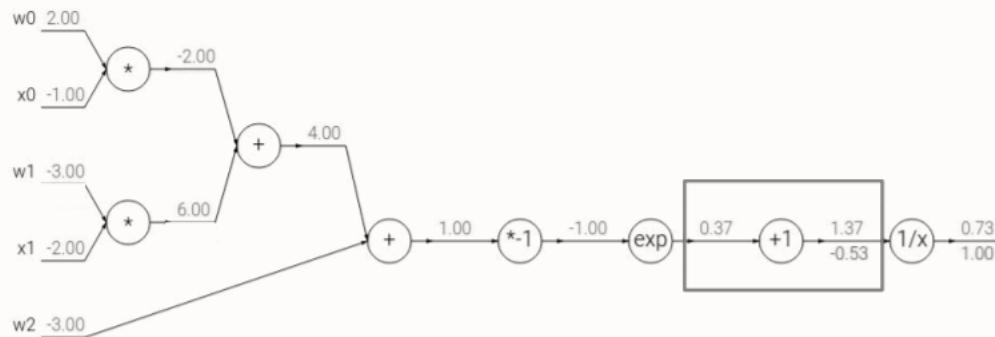
Backpropagation



Backpropagation

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x$$

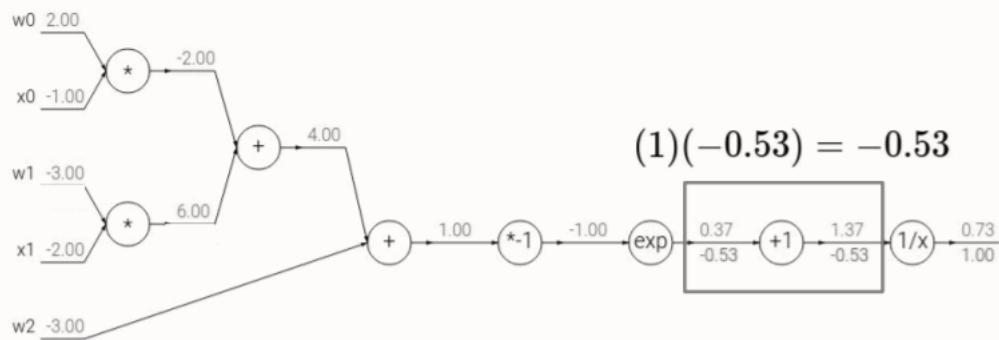
→

$$\frac{df}{dx} = 1$$

Backpropagation

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

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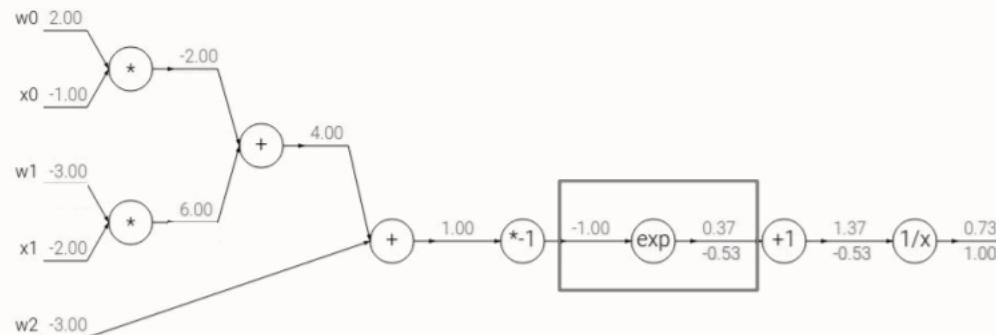
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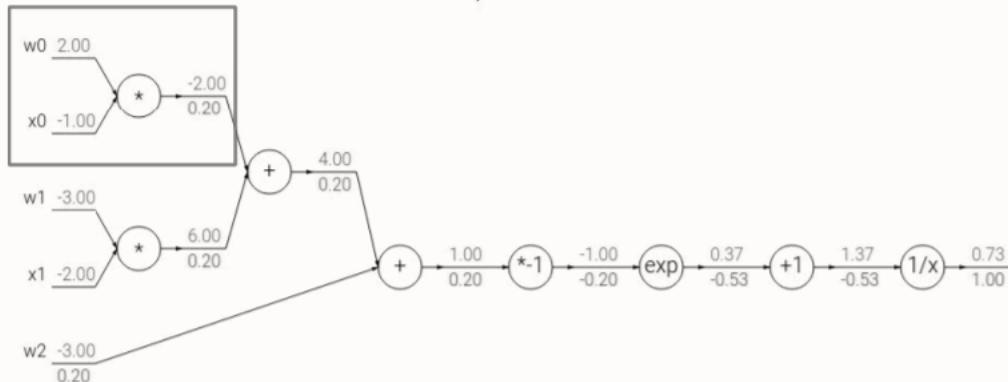
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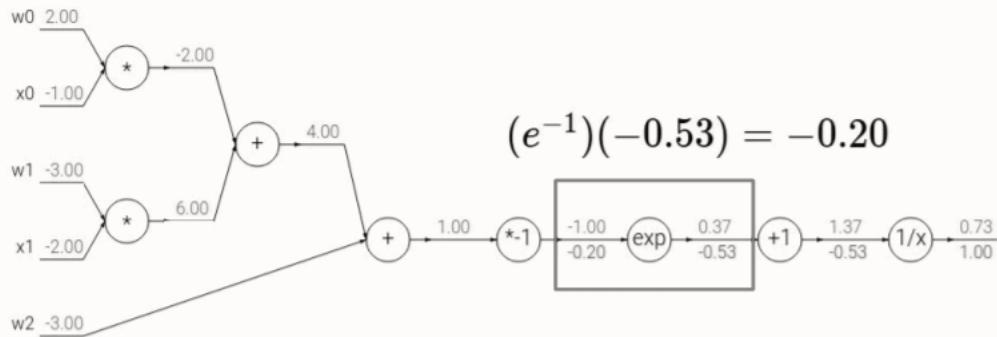
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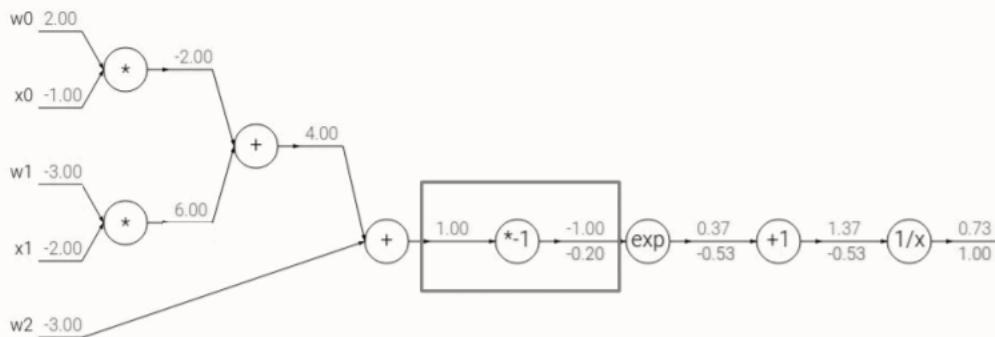
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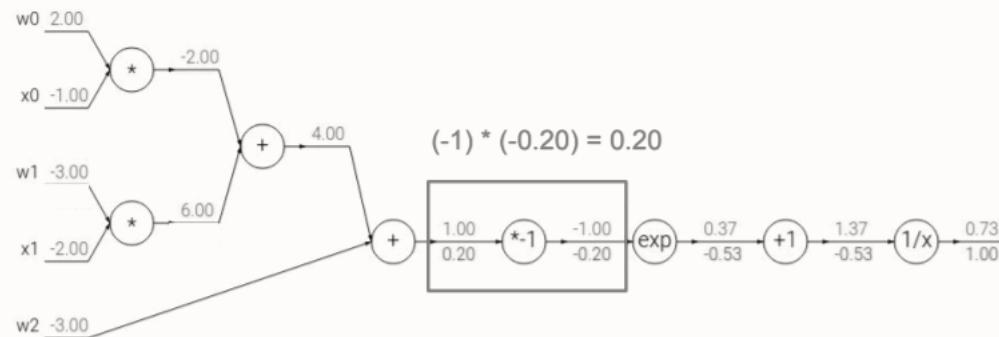
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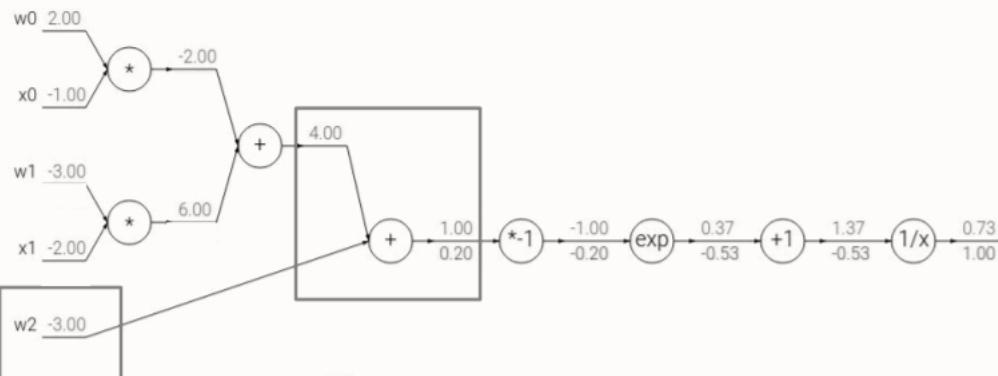
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Backpropagation

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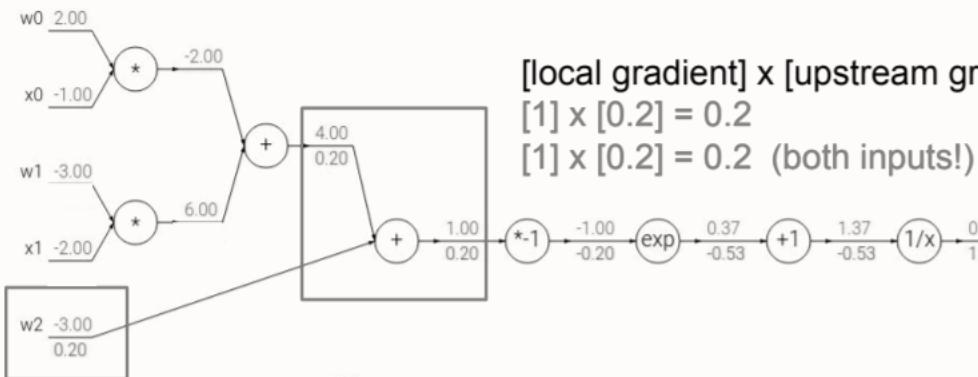
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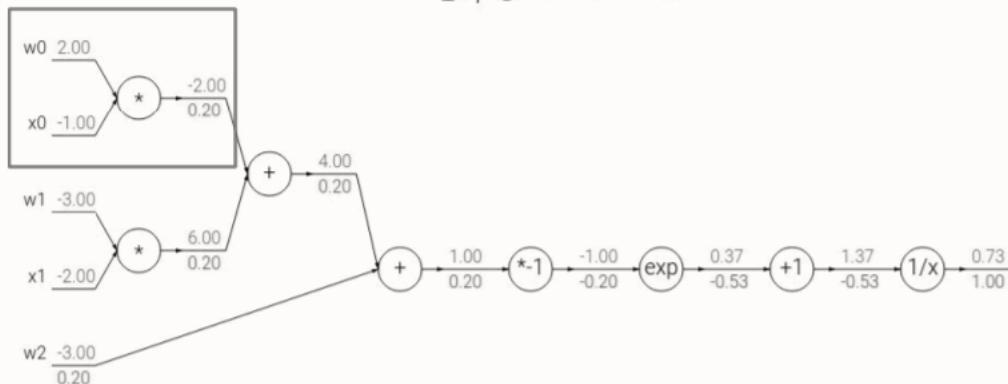
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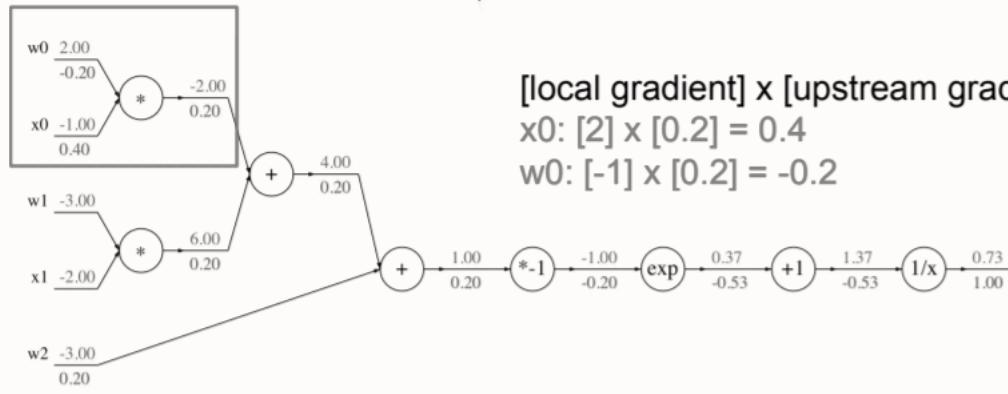
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Backpropagation

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$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



[local gradient] x [upstream gradient]

$$x_0: [2] \times [0.2] = 0.4$$

$$w_0: [-1] \times [0.2] = -0.2$$

$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

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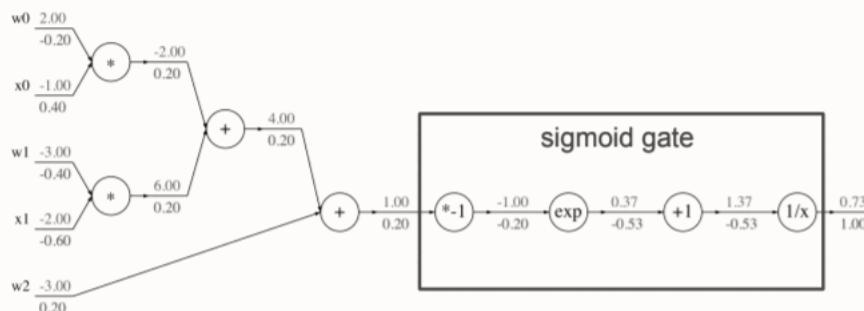
$$\frac{df}{dx} = 1$$

Backpropagation

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad \text{sigmoid function}$$

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left(\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left(\frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$

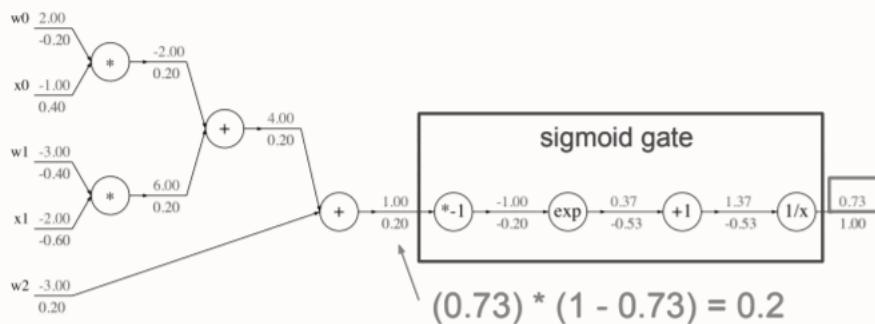


Backpropagation

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad \text{sigmoid function}$$

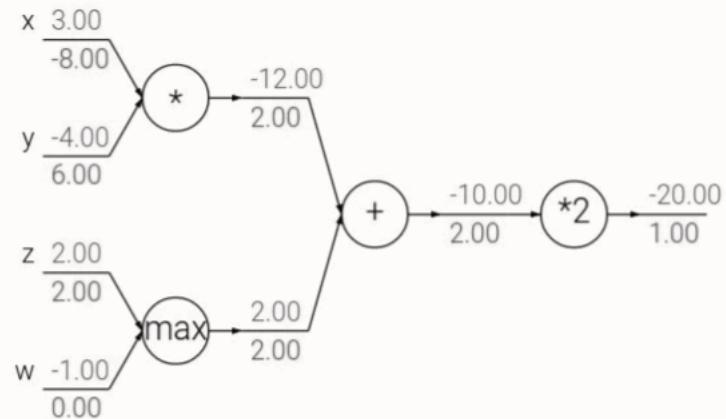
$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left(\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left(\frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$



Backpropagation

Patterns in backward flow

add gate: gradient distributor

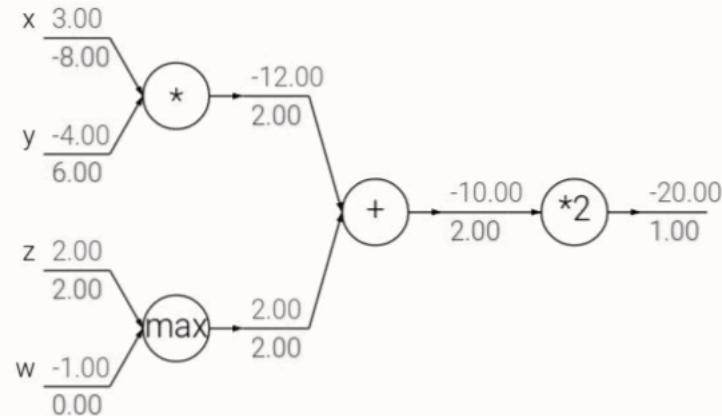


Backpropagation

Patterns in backward flow

add gate: gradient distributor

Q: What is a **max** gate?

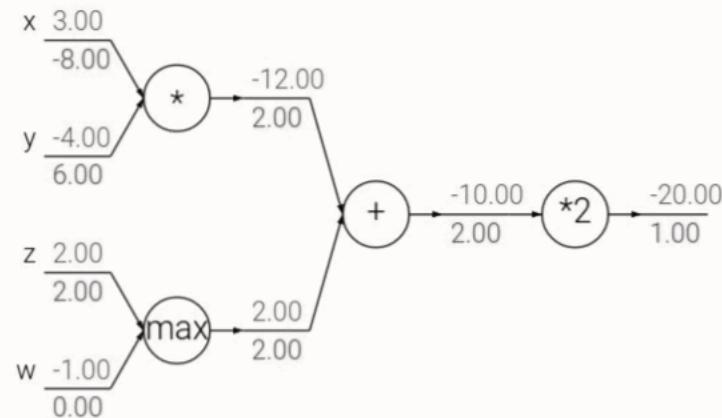


Backpropagation

Patterns in backward flow

add gate: gradient distributor

max gate: gradient router



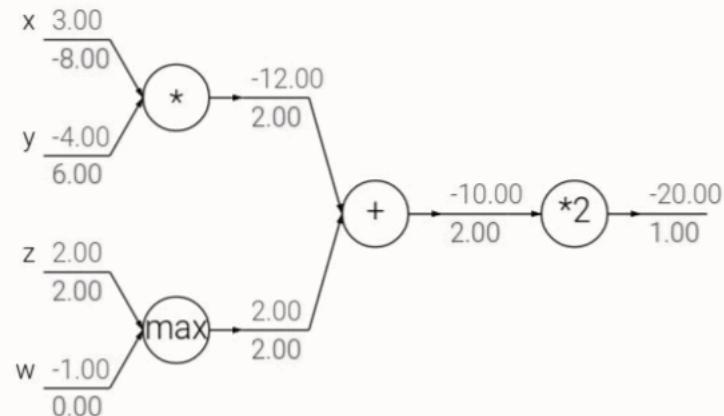
Backpropagation

Patterns in backward flow

add gate: gradient distributor

max gate: gradient router

Q: What is a **mul** gate?



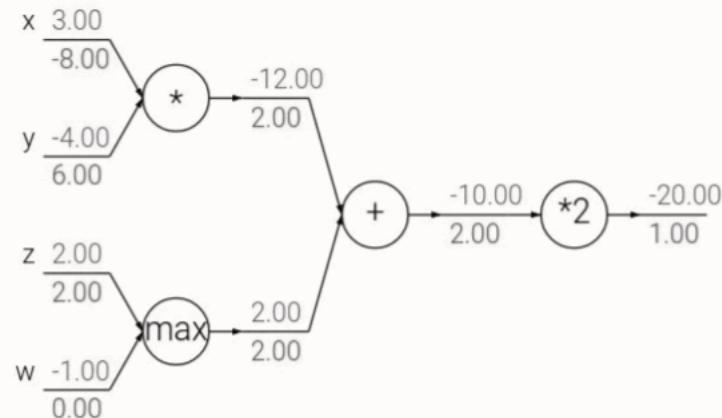
Backpropagation

Patterns in backward flow

add gate: gradient distributor

max gate: gradient router

mul gate: gradient switcher

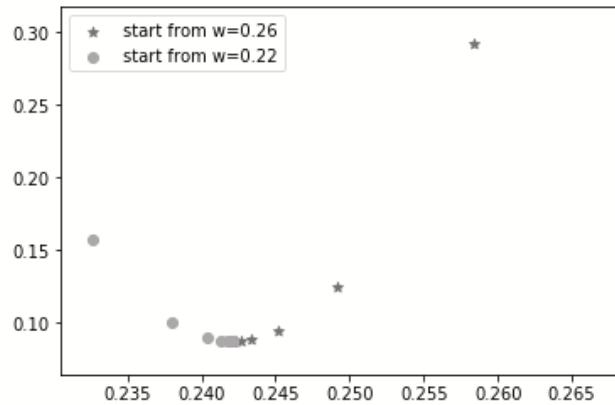
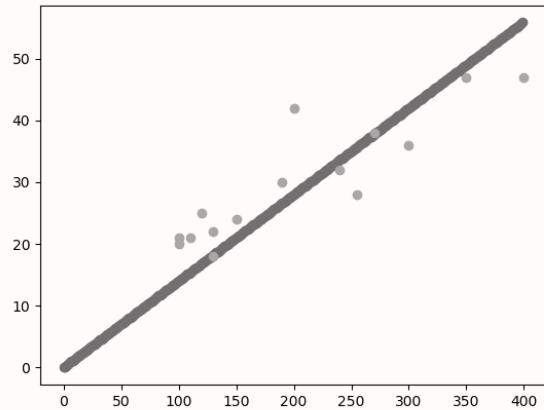
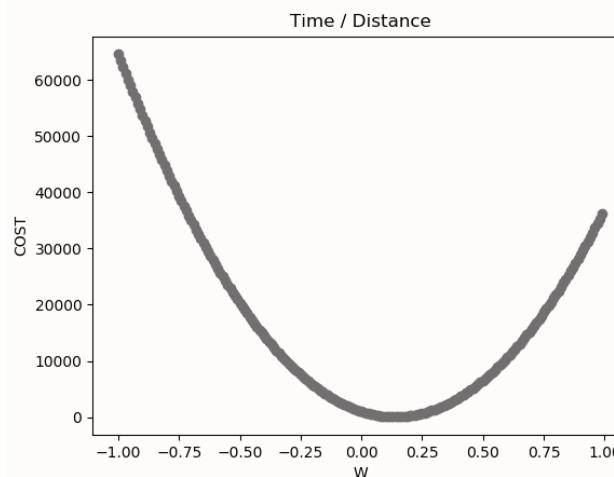
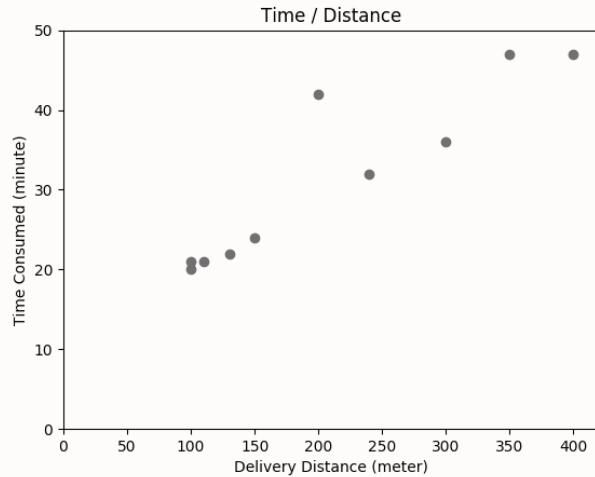


SUMMARY

1. Linear regression
2. Multi input Linear regression
3. Logistic Classification
4. Back Propagation

SUMMARY

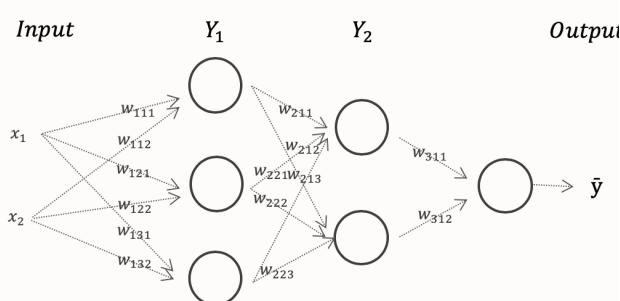
1. Linear regression



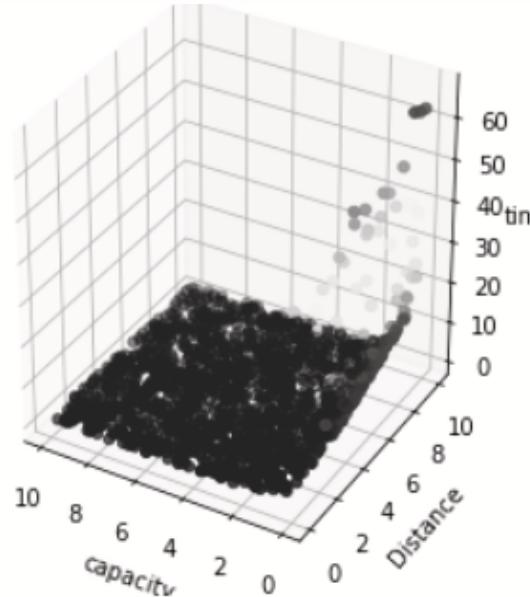
SUMMARY

2. Multi input Linear regression

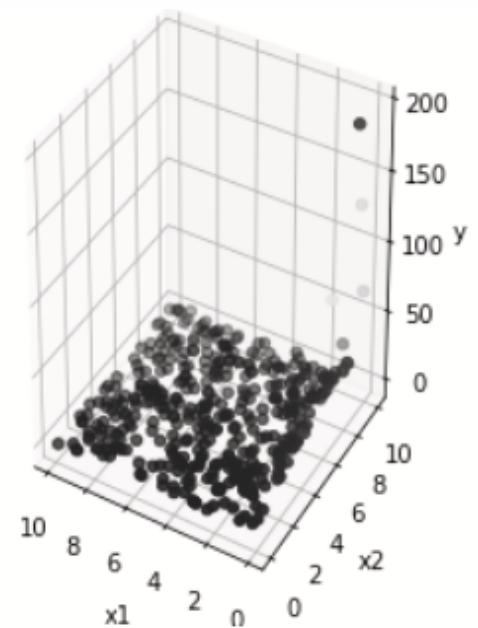
model



model predicted

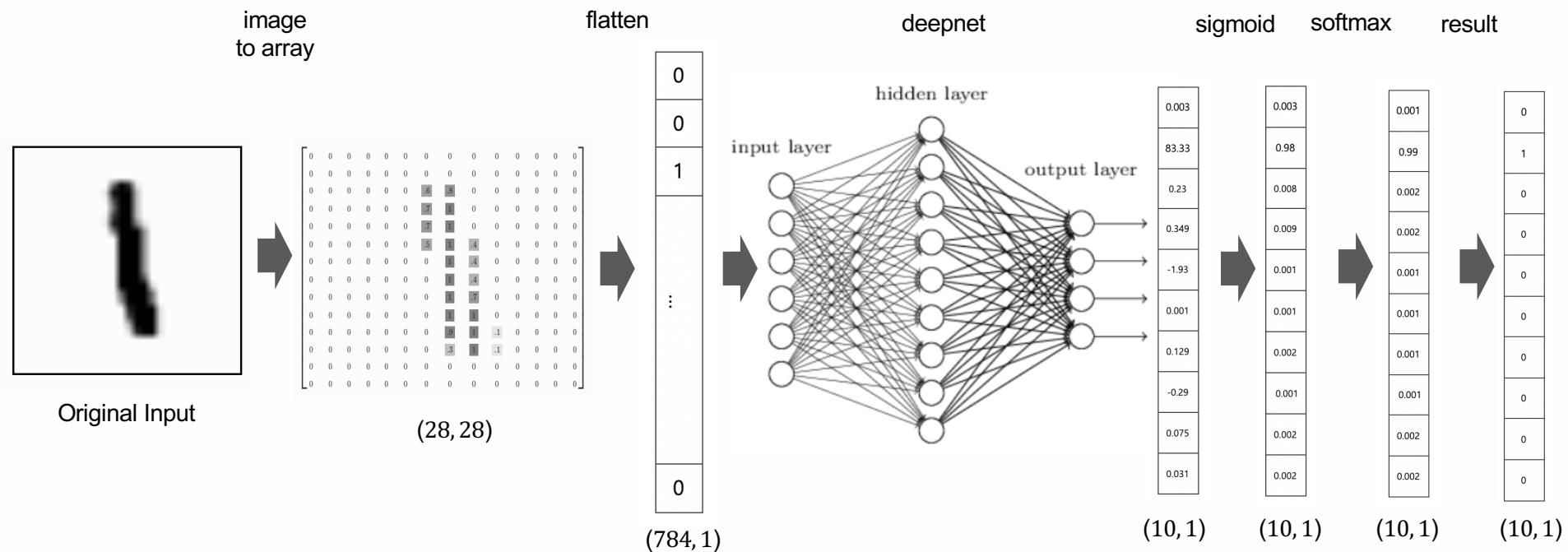


real data



SUMMARY

3. Logistic Classification



SUMMARY

4. Back Propagation

