Homework #6 **Coffee Automaton**
CS 217 @ SJTU ACM Class, Zhiyuan College, SJTU
Prof. **Dominik Scheder** Due Date: November 7, 2019
TA. **Tang Shuyang** Submit Date: November 7, 2019

Exercise 1

Let $\nu(G)$ denote the size of a maximum matching of G.

For every cycle $x_1, \cdots, x_k$ in the bipartite graph (every cycle has an even length), we can modify the cycle by shaking the weights to $x_1 + \epsilon, x_2 - \epsilon, x_3 + \epsilon, \cdots, x_k - \epsilon$ until some weight becomes 0 or 1.

Repeat the process above until we get a integral graph.

Therefore, $\nu(G) = \text{val}(MLP(G))$ for all bipartite graphs G.

Exercise 2

For every edge $e = (u, v)$ in the match we choose both u and v, then we can obtain a vertex cover with size $2\nu(G)$.

Therefore, $\tau(G) \leq 2\nu(G)$.

Exercise 3

Let $C = \{v \in V | x_v \geq \frac{1}{2}\}$, then C is a vertex cover, and $tau(G) \leq |C| \leq 2\text{opt}(\text{VCLP}(G))$

Exercise 5

Suppose C is a vertex cover, and F be the set of tight edges.

We have

- $i \in I(x)$ iff e_i is a tight edge for $1 \leq i \leq m$
- $i \in I(x)$ iff $v_{i-m} \notin C$ for $1 \leq i \leq n$

1. If $y_u \in \{0, 1\}$ for all $u \in V$ and C is a minimal vertex cover, then $I(x)$ contains vertices that are not covered and all tight edges. We have $\text{rank}(A_{I(x)}) = n$ so y is a basic solution of VCLP(G).

2. If y is a basic solution of VCLP(G), y is the solution of $A_{I(y)}y = b_{I(y)}$. Then $I(y)$ contains the vertices that are not covered and all tight edges. To satisfy all the constraints, y must be an integral vector, that is, $y_u \in \{0, 1\}$ for all $u \in V$, and C is a vertex cover of G. To show C is a minimal vertex cover, consider removing some vertex u from C, the constraints can't be satisfied, then $C \backslash \{u\}$ can't be a vertex cover. Therefore, C is a minimal vertex cover.

In conclusion, y is a basic solution of VCLP(G) iff $y_u \in \{0, 1\}$ for all $u \in V$ and C is a minimal vertex cover.