Homework #3 | **Coffee Automaton**
CS 217 @ SJTU | ACM Class, Zhiyuan College, SJTU
Prof. **Dominik Scheder** | Due Date: September 30, 2019
TA. **Tang Shuyang** | Submit Date: September 30, 2019

Exercise 1

It's trivial that T is a subgraph of G, and T_c is a subgraph of G_c.

So for two vertices u, v, if u, v are connected in T_c, they are must connected in G_c, because T_c is a subgraph of G_c.

If u, v are not connected in T_c, they are not connected in G_c. A simple proof: because T is a minimum spanning tree of G, so all vertices are connected in T. If u, v are not connected in T_c, there is at least an edge that connect u and v which weights bigger than c. Assume u, v are connected in G_c, then there is an path to connect u and v that every edge's weight is less or equal to c. So we can find a tree that is a spanning tree of G, which has smaller weight than T. But T is the MST, so it is a contradiction.

So T_c and G_c have exactly the same connected components.

Exercise 2

Considering the Kruskal's algorithm, all MSTs have $n-1$ edges, and have same amount of edges with weight 1, weight 2, ... , weight c. So all T_c have same amount of edges, that $m_c(T) = m_c(T')$.

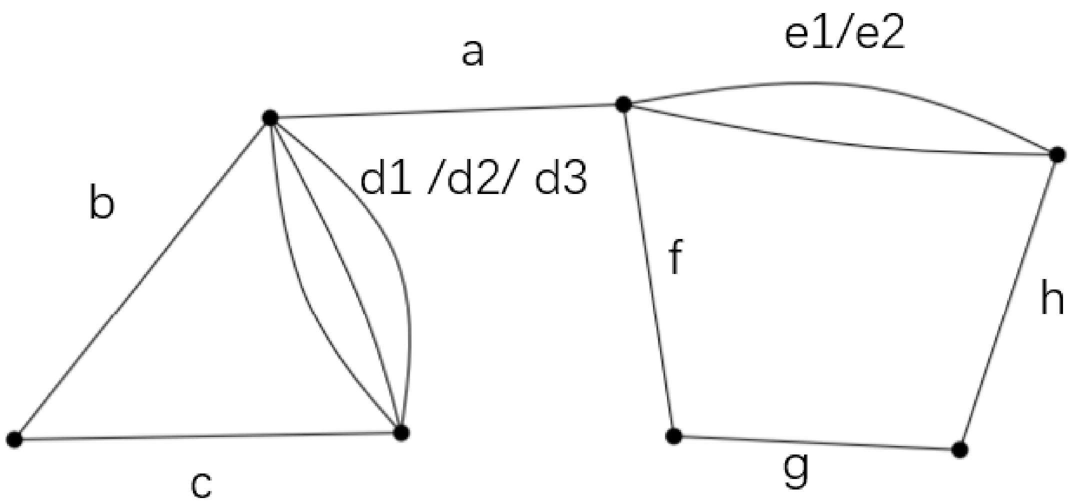
Exercise 3

Considering the Kruskal's algorithm, each time we chooose edge of fixed weight and cycle $n-1$ times. Because no two of edges have same weight, what we choose is determinate and unique. So there is exactly one MST.

Exercise 4

The multigraph is showed in figure with labeled.

Because we want a spanning tree, so edge a is a must, and choose two edges of b, c, d, three edges of e, f, g, h. And we have three choices in d, two choices in e, so we have $(2*3+1)*(3*2+1) = 49$ spanning trees.



Exercise 5

Let T be a minimum spanning tree of G.

By exercise 2, we know that the connectivity of T_c is uniquely determined.

The overall algorithm can be obtained by making a little modification to the Kruskal's Algorithm.

1. Sort all edges in the graph.
2. Traverse the sorted edges and group them together for the same edge weight w.
3. Calculate the number of spanning trees with the weight of w, and multiply the answer by this number. Then go to step 2.