

Sampling Technique (3 types)

②

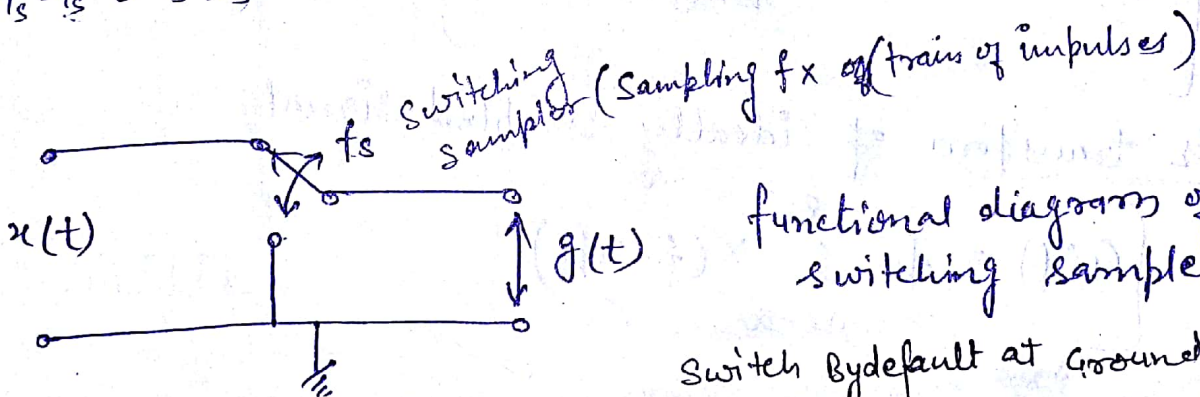
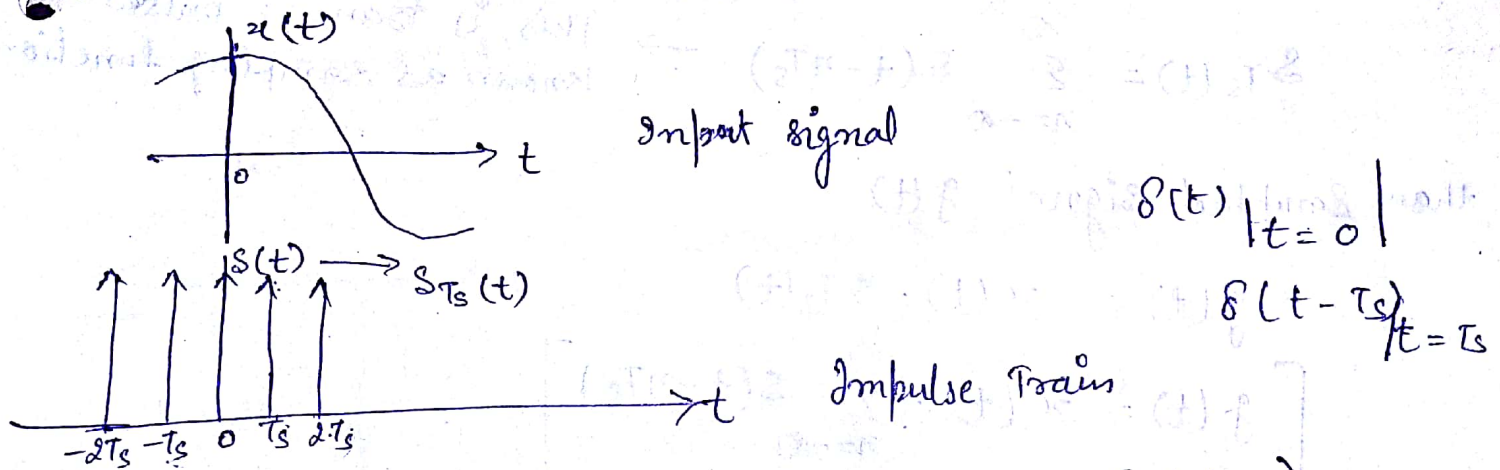
(1) Instantaneous sampling (ideal case)

② Natural sampling.
③ Flat top sampling.] Practical case.

① Instantaneous Sampling.

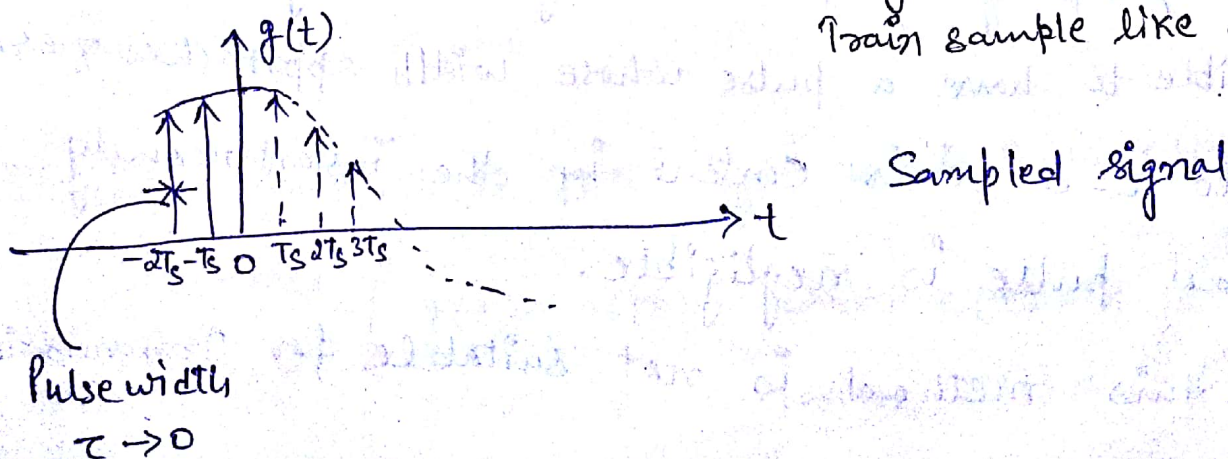
Also called Ideal or Impulse Sampling.

→ In this type of sampling the sampling f_x is a train of impulses.



functional diagram of a switching sampler.

Switch by default at ground level and change position corresponding to impulse train sample like $0, T_s, 2T_s, \dots$



working - ① Ckt consists of a switch. Now if we assume that the closing time ' τ ' of the switch approaches zero, then the o/p $g(t)$ of this ckt. will contain only instantaneous value of the input signal $x(t)$.

② Since the width of the pulse approaches zero, the instantaneous sampling gives a train of impulses of height equal to the instantaneous value of the input signal $x(t)$ at the sampling instant.

$$S_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \rightarrow \text{This is train of pulses and known as sampling function.}$$

then sampled signal $g(t)$

$$g(t) = x(t) \cdot S_{Ts}(t)$$

$$\left[g(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right]$$

Fourier transform of ideally sampled signal

$$\left[G(f) = f_s \sum_{n=-\infty}^{\infty} x(f - n f_s) \right]$$

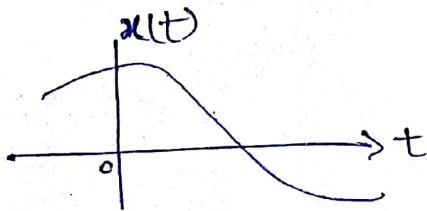
Note - ① Ideal sampling is possible only in theory since it is impossible to have a pulse whose width approaches zero.

② Due to $\tau=0$ power content in the instantaneously sampled pulse is negligible.

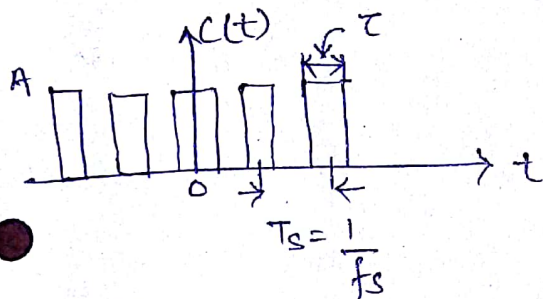
③ Thus this method is not suitable for Transmission.

Natural Sampling - ✓ In Natural sampling pulse has a finite width equal to τ .

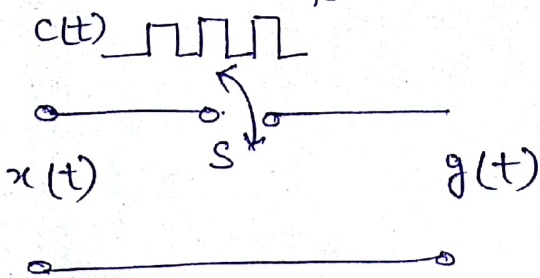
- ① Consider analog continuous time signal $x(t)$.
- ② f_s is higher than Nyquist rate (sampling theorem satisfied)
- ③ sampling fx $c(t)$ which is a train of periodic pulses of width τ and freq. equal to $f_s \text{ Hz}$.
- ④ with the help of natural sampler a sampled signal $g(t)$ is obtained by multiplication of sampling function $c(t)$ and input signal $x(t)$.



Continuous time signal.



Periodic pulse train

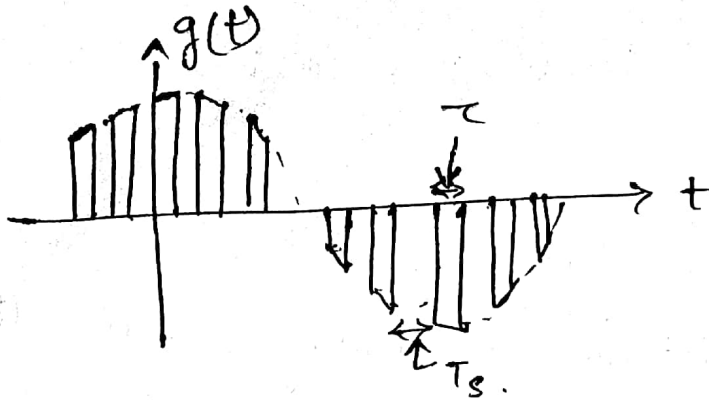


Natural sampler.
using chopping ~~etc~~ Principle.

working: When $c(t)$ goes high switch s is closed.

Therefore $\left[\begin{array}{ll} g(t) = x(t) & \text{when } c(t) = A \\ g(t) = 0 & \text{when } c(t) = 0 \end{array} \right]$ } Amplitude of c

$$g(t) = c(t) \cdot x(t)$$



Naturally sampled
signal waveform $g(t)$

* Noise interference is minimum.

Q. ① An Analog signal is expressed by the eq.

$$x(t) = 3 \cos(50\pi t) + 10 \sin(300\pi t) - \cos(100\pi t)$$

calculate Nyquist rate for this signal.

Solu:

$$x(t) = 3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$$

New eqn for signal

$$x(t) = 3 \cos \omega_1 t + 10 \sin \omega_2 t - \cos \omega_3 t$$

$$\therefore \boxed{\omega = 2\pi f}$$

$$\omega = 2\pi f$$

for ω_1 ~~$2\pi f = 50\pi$~~
 $2\pi f = 50\pi$
 $f = \frac{50\pi}{2\pi}$
 $\boxed{f = 25 \text{ Hz}}$

for ω_2 $2\pi f = 300\pi$
 $f = \frac{300\pi}{2\pi} = \boxed{150 \text{ Hz}}$ ✓

for ω_3 $2\pi f = 100\pi$
 $f = \frac{100\pi}{2\pi}$
 $\boxed{f = 50 \text{ Hz}}$

so that $f_m = 150 \text{ Hz}$

Nyquist rate $f_s = 2 \times 150 \text{ Hz}$

Ans:

$$\boxed{f_s = 300 \text{ Hz}}$$
 ✓

Nyquist Rate and Nyquist Interval

→ when sampling rate exactly equal to $2f_m$ sample per sec. then it is called Nyquist rate.

→ Nyquist rate is also called the minimum sampling rate.

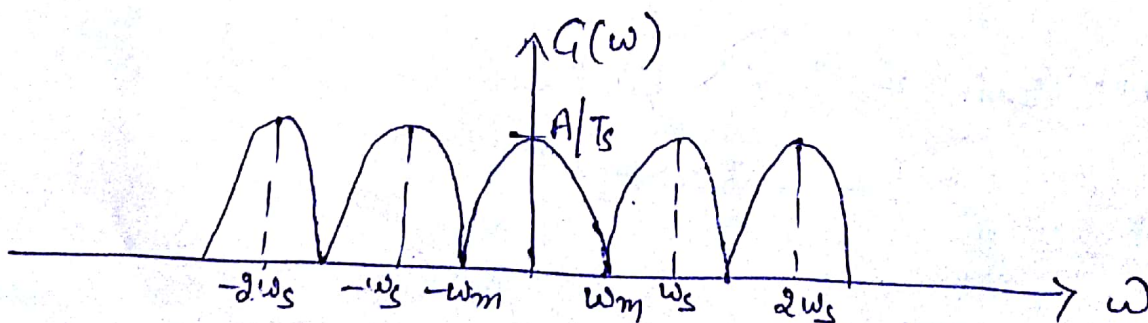
$$f_s = 2f_m$$

→ Similarly maximum sampling interval is called Nyquist interval.

$$T_s = \frac{1}{2f_m}$$

→ when $f_s = 2f_m$, sampled spectrum $G(\omega)$ contain non-overlapping $G(\omega)$ repeating periodically. But successive cycle of $G(\omega)$ touch each other.

→ Therefore original spectrum $x(\omega)$ can be recovered from the sampled spectrum by using a low pass filter with a cut off frequency ω_m .



Q. 2) Find the Nyquist rate and Nyquist interval for the signal.

$$x(t) = \frac{2 \times 1}{2\pi} \underbrace{\cos(4000\pi t)}_A \underbrace{\cos(1000\pi t)}_B$$

Soln : $x(t) = \frac{1}{4\pi} [2 \cos(4000\pi t) \cos(1000\pi t)]$

Since $[2 \cos A \cos B] = \cos(A+B) + \cos(A-B)$

$$x(t) = \frac{1}{4\pi} [\cos(4000\pi t + 1000\pi t) + \cos(4000\pi t - 1000\pi t)]$$

$$x(t) = \frac{1}{4\pi} [\underbrace{\cos 5000\pi t}_{\omega_1} + \underbrace{\cos 3000\pi t}_{\omega_2}]$$

$$x(t) = \frac{1}{4\pi} [\cos \omega_1 t + \cos \omega_2 t]$$

$$\omega_1(t) \rightarrow 2\pi f_1 = 5000\pi$$

$$\boxed{f_1 = 2500 \text{ Hz}}$$

$$\omega_2(t) = 3000\pi$$

$$2\pi f_2 = 3000\pi$$

$$\boxed{f_2 = 1500 \text{ Hz}}$$

So that max. freq. present in the signal is

$$\boxed{f_1 = 2500 \text{ Hz}}$$

Therefore Nyquist rate $f_s = \underline{2f_m} = 2 \times 2500 = 5000 \text{ Hz}$

$$\text{or } \boxed{5 \text{ kHz}} \checkmark$$

Nyquist interval

$$T_s = \frac{1}{2f_m} = \frac{1}{2 \times 2500} = \frac{1}{5000}$$

$$T_s = 0.2 \times 10^{-3} \text{ sec.} = \boxed{0.2 \text{ m sec.}}$$

Q.3 Determine the Nyquist rate for a continuous time signal

$$x(t) = 6 \cos 50\pi t + 20 \sin 300\pi t - 10 \cos 100\pi t$$

Soln: General form of any continuous time signal

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t + A_3 \cos \omega_3 t$$

Compare given signal with general form of signal.

$$\omega_1 \Rightarrow 2\pi f_1 = 50\pi$$

$$f_1 = 25 \text{ Hz}$$

$$\omega_2 \Rightarrow 2\pi f_2 = 300\pi$$

$$f_2 = 150 \text{ Hz}$$

$$\omega_3 \Rightarrow 2\pi f_3 = 100\pi$$

$$f_3 = 50 \text{ Hz}$$

highest freq. component of given msg signal will be

$$f_{\max} = 150 \text{ Hz}$$

Therefore

$$\text{Nyquist rate} = 2f_{\max}$$

$$= 2 \times 150 \text{ Hz}$$

$$\text{Nyquist rate} = 300 \text{ Hz} \quad \text{Ans}$$

Nyquist Interval -

$$T_s = \frac{1}{2f_m}$$

$$= \frac{1}{300}$$

$$T_s = .003 \text{ sec.}$$