

## MTRN4230 Lab 09

### 1. Aim

This lab provides students with time to work on Project-2 as well as providing an attempt at determining the torque applied at each joint of a two link robotic arm.

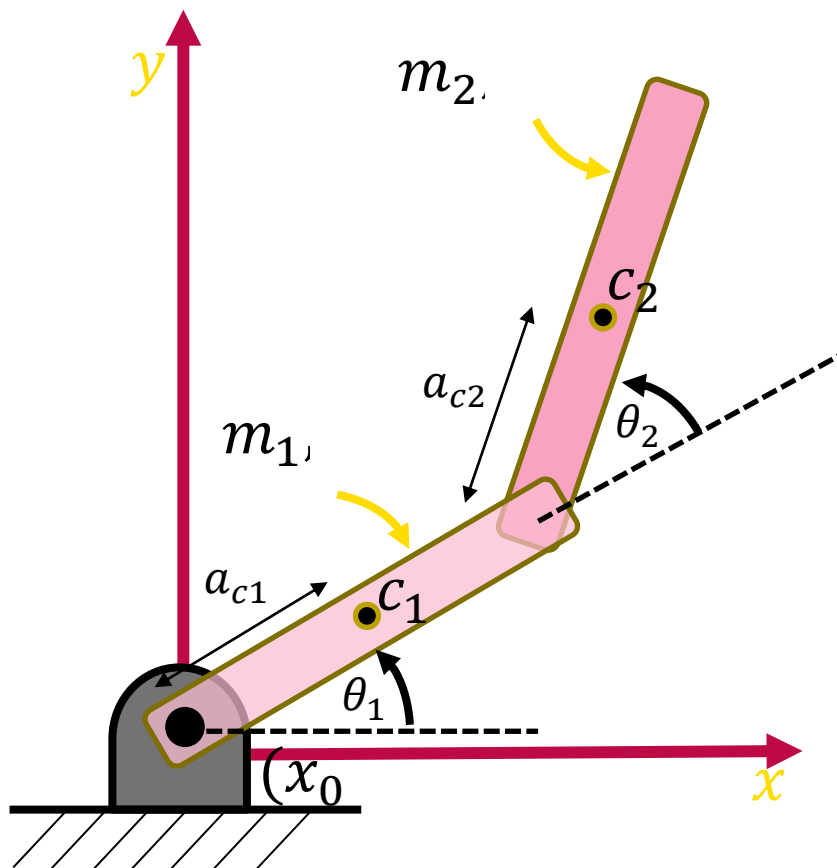
### 2. Pre-lab

Watch the lecture, to understand the equations required to solve Part A.

### 3. Lab Activities

#### Part A:

Consider a two-link robot as shown in this figure. The centre of mass of each link locates at their geometrical centre ( $a_{c1} = L1/2$ ;  $a_{c2}=L2/2$ ).



Assume that  $m_1 = 4\text{kg}$ ;  $m_2 = 2\text{kg}$ ;  $L_1 = 80\text{cm}$ ;  $L_2 = 60\text{cm}$ ;  $\text{width}_1 = \text{width}_2 = 10\text{cm}$ ; height of both links =  $10\text{cm}$ . Assume that the links are rectangular prisms.

I. Use the RVC toolbox to calculate the Jacobian of the centre of mass of each link:

- (i) Translational velocity Jacobian [ $J_{v01}$   $J_{v02}$ ]
- (ii) Rotational velocity Jacobian [ $J_{\omega_1}$   $J_{\omega_2}$ ]

II. Use the RVC toolbox to generate a trapezoidal trajectory for this robot to move from  $(q_1, q_2)_{\text{start}} = (0, 0)$  to  $(q_1, q_2)_{\text{final}} = (60, 90)$  in 6s (the blend time is  $t_b = 2\text{s}$ ). What is the angular velocity and acceleration at  $t = 3\text{s}$ ;

III. Calculate the torques applied to each joint at  $t = 3\text{s}$ , using the equation in the lecture. Note: The RVC toolbox uses a different approach (Recursive Newton-Euler algorithm) to calculate the dynamics equation; therefore, we do not apply this method in this lab.

Hints:

- Try to calculate the formula for (I) using matlab syms. Once the formula that you have matches with that presented on the lecture slides, sub in the parameters.
- For the calculation of the OT1 and OT2 matrix. Consider creating a one link and a two link robotic arm and calculating the fkine solution for both at the desired joint configurations.
- For (II) generating the trapezoidal trajectory, have a look at the 'trajectories.m' example code provided on moodle. For the mstraj function use '@lspb' instead of '@tpoly'

**Part B:**

Spend the remainder of the lab working on Project-2. The demonstrator will go through a summary of the task.

## 4. Post-lab:

- Further Resources:
  - [https://www.youtube.com/watch?v=1U6y\\_68CjeY&ab\\_channel=NorthwesternRobotics](https://www.youtube.com/watch?v=1U6y_68CjeY&ab_channel=NorthwesternRobotics)
  - [https://www.youtube.com/watch?v=BjD-pL819LA&ab\\_channel=NorthwesternRobotics](https://www.youtube.com/watch?v=BjD-pL819LA&ab_channel=NorthwesternRobotics)
  - <https://robotacademy.net.au/masterclass/robot-joint-control/?lesson=364>
  - <https://robotacademy.net.au/masterclass/rigid-body-dynamics/?lesson=391>

## List of equations:

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i=1}^n \sum_{j=1}^n c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k$$

Inertia Matrix:

$$d_{11} = m_1 a_{c1}^2 + m_2 (a_1^2 + a_{c2}^2 + 2a_1 a_{c2} \cos(q_2)) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2 (a_{c2}^2 + a_1 a_{c2} \cos(q_2)) + I_2$$

$$d_{22} = m_2 a_{c2}^2 + I_2$$

Coriolis and centripetal coupling matrix



$$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = -m_2 a_1 a_{c2} \sin(q_2) = h$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = h$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = -h$$

$$c_{122} = c_{212} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_2} = 0$$

### Gravity Loading

$$g_1 = \frac{\partial P}{\partial q_1} = (m_1 a_{c1} + m_2 a_1) g \cos(q_1) + m_2 a_{c2} g \cos(q_1 + q_2)$$

$$g_2 = \frac{\partial P}{\partial q_2} = m_2 a_{c2} g \cos(q_1 + q_2)$$