# Efficient Magnetic Localization and Orientation Technique for Capsule Endoscopy *

Chao Hu[1, 2, 3], Max Qinghu Meng[2], Mrinal Mandal[1]

[1] Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada
[2] Department of Electronic Engineering, The Chinese University of Hong Kong, Shatin, N.T. Hong Kong
[3] Department of Electrical Engineering & Automation, Ningbo University, Ningbo, Zhejiang, 315211, PR China
hcsfds@ece.ualberta.ca, max@ee.cuhk.edu.hk, mandal@ece.ualberta.ca

***Abstract* -To build a new wireless robotic capsule endoscope with external guidance for controllable and interactive GI tract examination, a sensing system is needed for tracking 3D location and 2D orientation of the capsule movement. An appropriate sensing approach is to enclose a small permanent magnet in the capsule. The magnet establishes a magnetic field around the patient's body. With the sensing data of magnetic sensor array outside the patient's body, the 3D location and 2D orientation of the capsule can be calculated. Higher localization and orientation accuracy can be obtained if more sensors and proper optimization algorithm are applied. In this paper, different nonlinear optimization algorithms are evaluated, and we have found that Levenberg-Marquardt method provides higher accuracy and faster speed. Simulations were done for investigating the de-noise ability of this algorithm based on different sensor arrays. Furthermore, the real experiment shows that the results are satisfactory with high accuracy.**

***Index Terms* - Wireless Capsule Endoscopy, Localization and Orientation, Magnet, Optimization.**

## I. INTRODUACTION

One recent great breakthrough in endoscopy is the development of wireless capsule endoscope. This new capsule endoscope makes it possible for doctor to inspect the entire gastrointestinal (GI) tract without causing much pain to the patient, while the traditional endoscope cannot reach entire small intestine and causes much discomfort. A clinical product, known as M2A [1, 2], based on this technique, has been developed by Given Image Ltd, and approved by U.S. Food and Drug Administration [4]. Fig. 1 shows the composition of the capsule endoscope, which is small (11×28 mm) enough to be swallowed. During the endoscopic test, the capsule moves through the GI tract by natural peristalsis, while transmitting out captured images using wireless technique.

However, M2A takes long time (between 20 to 36 hours) to go through the entire human GI tract. In addition, it is impossible for the doctor to control the capsule's location and orientation, and hence it is possible to miss capturing images of some important spots. To address these limitations, we try to make a new type of wireless robotic capsule endoscopic system, where the capsule movement can be externally controlled [3,4]. Therefore, the capsule is guided to stop at any important spots and is moved through unimportant areas in a much faster pace. To realize such a system, an actuation and guidance system is required. In our design, a small permanent magnet is included in the capsule, which can be driven by an externally applied magnetic field [5]. In order to realize such external guidance and locomotion, it is a prerequisite that the doctor know the location or orientation of the capsule. Hence, a localization and orientation system of the capsule is needed.

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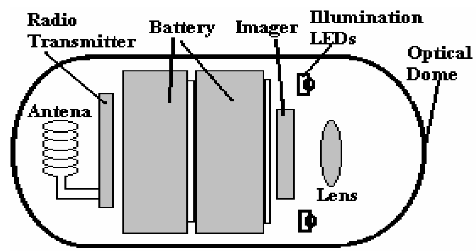



Fig. 1 Wireless Capsule Endoscope

The M2A endoscopic system includes a localization module [4, 6, 7]. The module is based on the RF (radio frequency) signals received by eight antennas on the exterior of the human abdomen corresponding to the emitting RF signals sent by the capsule. The localization algorithm is based on the assumption that the closest sensor receives the strongest signal. Because the RF system is also used to transmit the captured images, this module has the advantage that it does not need any additional equipment. However, the measurement accuracy is too low (average error is 37.7 mm [6, 7]), and we need to find more efficient localization method.

Our implementation is to do magnetic localization using the magnet enclosed in the capsule. Because human body has magnetic permeability very similar to that of air, water, and other non-ferromagnetic materials, it does not influence the static magnetic field. Therefore, it is possible to achieve higher localization accuracy with static magnetic method. A few magnetic methods have been used for medical applications. For example, it has been applied to monitor the movement of solid oral dosage through GI tract by Weitschies et al [8]. Here the magnetic measurement was realized by a commercial seven-channel DC-SQUID device. Yoshiaki et al [9] used a magnetic system consisting of a small magnet and four magnetic sensors, to measure both tooth rotation and rectilinear movements. Akutagawa et al [10] used a magnet and a neural network sensing system to measure the mandibular movement.

In this paper, we propose a localization and orientation technique using a permanent magnet inside the capsule. The magnetic field produced by the magnet is determined by its location (3D) and orientation (2D). Therefore, we measure the magnetic intensity and direction in some spatial points using a few properly arranged sensors outside of the patient's abdomen, and determine the magnet's (or capsule's) location and orientation by these sensor data. This idea was influenced by the work reported by Prakash et al [11] and Schlageter et al [12]. The system in [11] was used to localize a magnetic marker in GI motility studies, based on the measurement data of eight fluxgate magneto-meters. The system in [12] was built with a 2D-array of 16 cylindrical Hall sensors and is used for tracking the location and orientation of a general magnetic marker. However, we cannot find detailed implementation schemes and algorithms in these two papers, and it is our idea to apply magnetic technique to track the capsule.

The magnetic intensity in a spatial point produced by the magnet is a high-order nonlinear function of 5D magnet's location and orientation parameters. To solve such a problem, an appropriate optimization method must be found, which should have large tolerance of initial guess of the parameters, fast speed, and strong de-noising ability. In addition, a reliable magnetic sensing system should be built.

The organization of this paper is as follows. In section II, we present the mathematical model of the magnetic field produced by the permanent magnet, and evaluate several nonlinear algorithms to determine the best algorithm in term of calculation accuracy and speed. In section III, we present the performance of the different sensor arrays, which is followed by the conclusions in section IV.

## II Mathmatic Model and Minimization Algorithms

### *A. Mathematic Model*

Fig. 2 shows a cylindrical magnet with length $L$, diameter $b$, and a uniform magnetization $M = M_0$ (Amp/meter) on its upper and bottom surfaces. Let **P** be the vector that represents a spatial point $p=(x_l, y_l, z_l)^T$ with respect to the magnet position $(a, b, c)^T$. The magnetic flux density at $p$ can be calculated using the following equation [13]

$$\mathbf{B} = \frac{\mu_r \mu_0 M_T}{4\pi}\left(\frac{3(\mathbf{H}_0 \cdot \mathbf{P})\mathbf{P}}{R_l^5} - \frac{\mathbf{H}_0}{R_l^3}\right) = N_T\left(\frac{3(\mathbf{H}_0 \cdot \mathbf{P})\mathbf{P}}{R_l^5} - \frac{\mathbf{H}_0}{R_l^3}\right) \quad (1)$$

where $\mu_r$ is the relative permeability of the medium, $\mu_0$ is the air magnetic permeability (T·m/A), $M_T = \pi b^2 L M_0$, $N_T = \frac{\mu_r \mu_0 M_T}{4\pi}$, and $\mathbf{H}_0 ==(m, n, p)^T$ is the normalized vector of the magnetic intensity of the magnet.

Assume that there are N sensors, with $l$-th sensor located at $(x_l, y_l, z_l)^T$, $1 \le l \le N$. The magnetic flux density at the $l$-th sensor

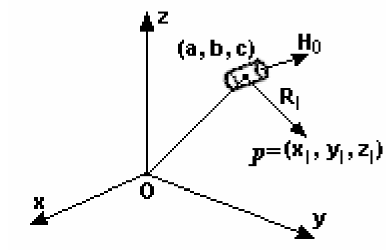



Fig. 2 Coordinate System for Magnet's Localization

location can be represented by

$$\mathbf{B}_l = B_{lx}\mathbf{i} + B_{ly}\mathbf{j} + B_{lz}\mathbf{k} \qquad (l = 1,2,\ldots, N) \quad (2)$$

with three orthogonal components:

$$B_{lx} = N_T\left\{\frac{3[m(x_l-a)+n(y_l-b)+p(z_l-c)]\cdot(x_l-a)}{R_l^5} - \frac{m}{R_l^3}\right\} \quad (3)$$

$$B_{ly} = N_T\left\{\frac{3[m(x_l-a)+n(y_l-b)+p(z_l-c)]\cdot(y_l-b)}{R_l^5} - \frac{n}{R_l^3}\right\} \quad (4)$$

$$B_{lz} = N_T\left\{\frac{3[m(x_l-a)+n(y_l-b)+p(z_l-c)]\cdot(z_l-c)}{R_l^5} - \frac{p}{R_l^3}\right\} \quad (5)$$

where $R_l = \sqrt{(x_l-a)^2 + (y_l-b)^2 + (z_l-c)^2}$,

In the proposed endoscopic system, the flux intensities $B_{lx}$, $B_{ly}$, and $B_{lz}$ are measured using the $l$-th magnetic sensor located around position $(x_l, y_l, z_l)^T$, which is known in advance. Note that there are six unknown parameters $(a, b, c, m, n, p)$. The flux intensity is invariant to the rotation of the magnet along its major axis. Hence the magnet's orientation $\mathbf{H}_0$ is in two dimensions. Therefore we add following constraint for $(m, n, p)^T$:

$$m^2 + n^2 + p^2 = 1 \quad (6)$$

Equations (3)~(5) show the relationship between the flux intensities at the $l$-th sensor and the magnet location and orientation. For each sensor, we measure three magnetic intensities $B_{lx}$, $B_{ly}$, and $B_{lz}$, and for N sensors, we obtain 3×N measured flux intensity values. Therefore, with two 3-axis magnetic sensors or five 1-axis sensors, the six parameters of the magnet location and orientation could be calculated by solving (3)-(6).

Five sensors are the minimum to solve the 5 unknown localization and orientation parameters. When N>5, the solution is not unique (in the presence of noise), and we try to obtain a solution of parameters $(a, b, c, m, n, p)$ that minimize an objective function. In this paper, we define an error objective function as follows:

$$E_x = \sum_{l=1}^{N}[B'_{lx} - B_{lx}]^2 \quad (7)$$

$$E_y = \sum_{l=1}^{N} [B_{ly}^{'} - B_{ly}]^2 \quad (8)$$

$$E_z = \sum_{l=1}^{N} [B_{lz}^{'} - B_{lz}]^2 \quad (9)$$

where $B_{lx}, B_{ly}$, and $B_{lz}$ are defined by (3)~(5), and $B_{lx}^{'}$, $B_{ly}^{'}$, and $B_{lz}^{'}$ are the three measured data of the *l-th* 3-axis magnetic sensor. The total error is the summation of above three errors.

$$E = E_x + E_y + E_z \quad (10)$$

We need to find optimal parameters (*a, b, c, m, n, p*) to minimize this error *E*. This is the least square error problem, which can be solved by some minimization algorithms. To evaluate the performance of an algorithm, we define two parameters, localization error and orientation error as follows:

$$E_p = \sqrt{(a_c - a_t)^2 + (b_c - b_t)^2 + (c_c - c_t)^2} \quad (11)$$

$$E_o = \sqrt{(m_c - m_t)^2 + (n_c - n_t)^2 + (p_c - p_t)^2} \quad (12)$$

where ($a_c$, $b_c$, $c_c$, $m_c$, $n_c$, $p_c$) represent the calculated location and orientation parameters, and ($a_t$, $b_t$, $c_t$, $m_t$, $n_t$, $p_t$) represent the true location and orientation parameters of the magnet.

### B. Localization and Orientation Algorithms

There exist several minimization algorithms to solve the high-order nonlinear equations (7)~(10). To choose an appropriate minimization algorithm, the following factors must be considered:

1) A nonlinear optimization algorithm normally needs an initial guess of the parameters, or their bounds, to begin the search for the minimum. If the initial parameters are chosen with large error, the algorithms may fail to give correct global solution in case that there are many local minima. Therefore, the chosen minimization algorithm should have large tolerance of the initial guess of the parameters.

2) The minimization algorithm should be fast enough to enable real time implementation.

3) The algorithm and system implementation should be robust to the noises in the sensor data.

In this paper, we compare the performance of several algorithms with respect to the localization error, orientation error, and execution time. Our main objective here is to show how these parameters change with the error level of the initial guess of the parameters. We do the simulation with five x-axis scheme as shown in Fig. 3, where $S_1$~$S_5$ [$S_1$=(0 0 0), $S_2$=(0.1m 0 0), $S_3$=(0 0.1m 0), $S_4$=(-0.1m 0 0), $S_5$=(0 -0.1m 0)] are the locations of the five sensors, $\mathbf{P_i}$ is the initial guess of the magnet's location, and $\mathbf{P_t}$ (0.1m, 0.1m, 0.12m) is the true magnet's location. The error in the initial guess for magnet's location is defined by $\boldsymbol{d} = \| \mathbf{P_i} - \mathbf{P_t} \|$.

In the following, we present several optimization algorithms and their performances.

*1. Powell's Algorithm* [14]

Powell's algorithm is a local minimization method with

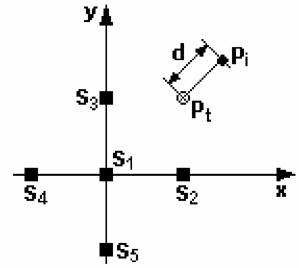



Fig. 3 Localization with 5 Sensors under Initial Guess of Magnet's Location which has error ***d*** with respect to true magnet's location.

fast searching speed. In this algorithm, some searching directions are defined based on quadratically convergent method. With these directions, the multi-dimensional minimization problem is changed to one-dimensional search problem, and the minimized parameters are found when the searches along these directions are completed. However, in our trials, the calculation error of Powell's algorithm is too large to be acceptable. The localization error, orientation error, and execution time are shown in Fig. 4, Fig. 5, and Fig. 6. It is observed that the localization error reaches 8 cm under the error levels (0.1 ~ 20 cm) of the initial location guess. The orientation error reaches 30%. The average execution time is 0.367 second.

*2. Downhill Simplex Algorithm* [14]

The downhill simplex method is also a local minimization method. Starting with an initial guess for the parameters, and applying operation steps, e.g., reflection, expansion, contraction and multi-contraction, the algorithm is then supposed to make its own way downhill until it encounters a minimum. As shown in Fig. 4, the results with downhill simplex method are good with localization error smaller than 5mm when the initial guess error is smaller then 1.7 cm. When the initial guess error is larger than 2.3 cm, the results are unacceptable. Its average execution time is 0.290 second.

*3. DIRECT* [15]

DIRECT is an algorithm for finding the global minimum of a multi-variant function subject to simple bounds. The algorithm is a modification of the standard Lipschitzian approach that eliminates the need to specify a Lipschitz constant. The algorithm is called **DIRECT** because it is a direct search technique and is an acronym for ***di***viding ***rect***angles, a key step in the algorithm. As shown in Fig. 4, the localization error is smaller than 10 mm under the location level 0~20 cm of the search bounds. Also in Fig. 5 and Fig. 6, we can see the orientation error is within 16 %, and its average execution time is 0.503 second.

*4. Multilevel Coordinate Search (MCS)* [16]

MCS is similar to the DIRECT method for global optimization based on multilevel coordinate search. Some shortcomings of DIRECT are remedied in MCS so that it can provide higher accuracy. The calculation errors using this method are pretty small. As shown in Fig. 4 and Fig. 5, the localization error is smaller than 0.2 mm, and orientation error is smaller than 0.5%, under different values (0~20 cm) of the location bounds. However, the average execution time is 0.688 second, as shown in Fig. 6, which is longer than other methods.

*5. Levenberg-Marquardt method* [17]

The Levenberg-Marquardt method is a general nonlinear downhill minimization algorithm. It dynamically mixes Gauss-Newton algorithms and the gradient iterations. Both the MATLAB functions "lsqnonlin", the nonlinear least squares solution, and "fsolve", solving for roots of the nonlinear equations, can provide the calculation of Levenberg-Marquardt method. In our simulations, we have found that "lsqnonlin" provides a marginally better accuracy than "fsolve". Excellent localization and orientation accuracy is obtained with this algorithm. As shown in Fig. 4 and Fig.5, the localization and orientation errors are zero when the error levels of the initial guess of location are within 20 cm. Moreover, this method provides faster speed with average execution time 0.106 second.

Comparing above results, we can conclude that:

1) The local minimization methods, e.g. Powell's and downhill simplex, are not suitable because Powell's method results large error and downhill simplex has too small tolerance for initial guess parameters.

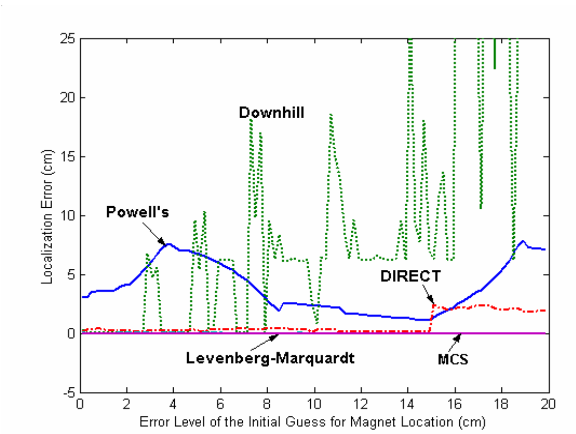



Fig.4 Localization Errors of Minimization Algorithms under Different Error Level of the Initial guess of Magnet Location.

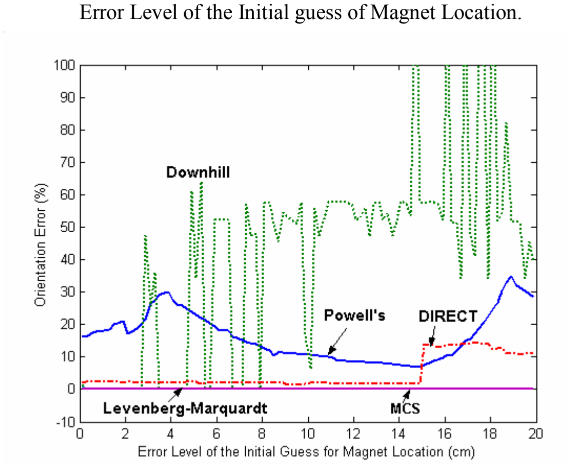



Fig.5 Orientation Errors of Minimization Algorithms under Different Error Level of the Initial guess of Magnet Location.

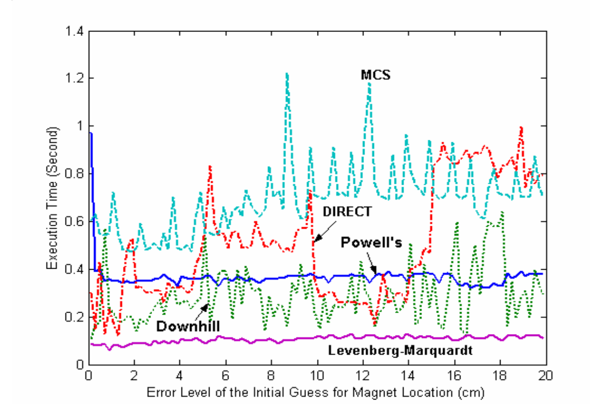



Fig.6 Execution Time of Minimization Algorithms under Different Error Level of the Initial guess of Magnet Location.

2) The global minimization methods, e.g. DIRECT and MCS, can provide high search accuracy, but their execution speed is low.

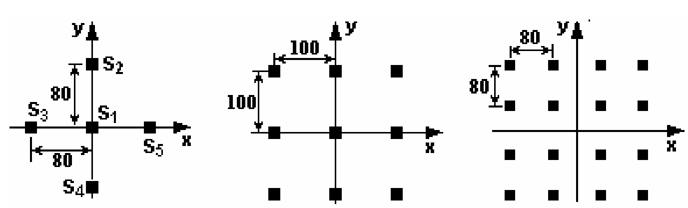
3) The Levenberg-Marquardt method is an appropriate algorithm. It provides satisfactory tolerance for initial guess parameters, and faster speed (<0.11 Second).

## III. PERFORMANCE EVALUATION

The techniques discussed in section 3 are typically valid when there is no noise in the sensors. However, in a practical system, there are always some noises due to the influences in the sensors, the amplification circuit, human body, and the environment. In this section, we present the simulation results for de-noising abilities of different sensor arrays, and the results obtained by a real testing system with sixteen 1-, 2- and 3-axis Hall sensors. We use the MATLAB function "lsqnonlin" to perform the optimization calculations.

### *A. Simulation Results of Sensor Array*

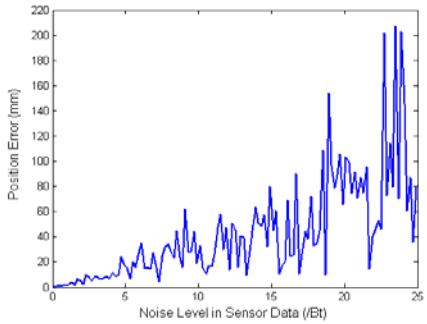
Although five sensors is the minimum to solve the 5 unknown localization/orientation parameters, the noise response of 5-sensor implementation (Fig. 7a) might often be unacceptable. Fig. 8(a) shows the localization error using five 1-axis (only x-axis) sensors via random noises with level 0.1 to 25 (25 is about 2~3 % full output range of the sensors). The localization error might reach 200 mm. To improve the accuracy, we use a denser sensor arrangement. In our implementation, nine and sixteen 1, 2, and 3-axis sensors are also tried. As shown in Fig.7b, 7c, for nine-sensor case, the sensors are uniformly arranged on the 200×200 mm$^2$ square plane; whereas for 16-sensors' case, the sensors are uniformly arranged on the 240×240 mm$^2$ square plane.



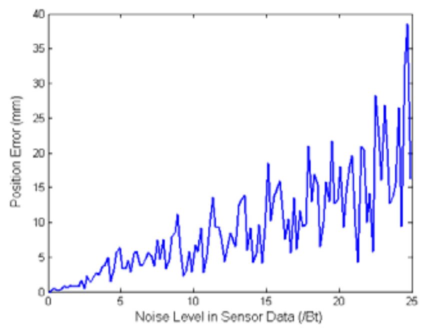
(a) 5-sensors (b) 9-sensors (c) 16-sensors

Fig.7 Disposition of the Sensor Array

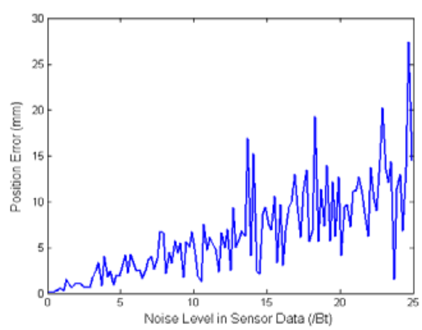
Table 1 shows the resulting average localization error (mm) and orientation error (in percentage, relative to the unit orientation vector) via random noises, where the distance between the magnet to the sensor plane is 12cm. Fig. 8(b), (c), and (d) show the localization errors with the cases of five, nine, and sixteen 3-axis sensors.



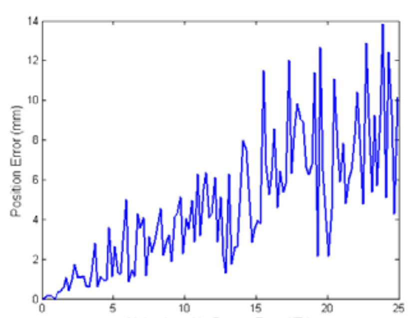

(a) Five 1-axis (x-axis) sensors




(b) Five 3-axis sensors




(c) Nine 3-axis sensors




(d) Sixteen 3-axis sensors

Fig. 8 Localization Error sensors via noise level

Table 1 Average Localization/Orientation Errors via Noise Level 0.1:0.2:25

| | 1-axis (x-axis) | 2-axis (x & y axes) | 3-axis |
|---|---|---|---|
| 5-sensors | 42.7mm; 17.9% | 16.8mm; 16.6% | 9.1mm; 8.2% |
| 9-sensors | 26.4mm; 27.1% | 10.8mm; 9.3% | 6.4mm; 6.0% |
| 16-sensors | 14.2mm; 15.8% | 6.6mm; 5.8% | 4.6mm; 4.5% |

It is observed that higher localization accuracy is obtained with larger number of sensors. For the case of sixteen 3-axis sensors, the average localization error is 4.6mm for and orientation error is 4.5%.

### *B. Real Experiment and Testing Results*

We built a real experiment system, which includes magnetic sensors, precision amplification circuit, stabilized power source, AD Converters, and PC computer. The magnetic sensors are made of 3-axis Hall sensors. We tried the sixteen-sensor scheme such that there are sixteen groups of 1, 2, or 3-axis magnetic intensity data on the sixteen sensor positions. Using the Levenberg-Marquardt algorithm (in MATLAB function "lsqnonlin"), the localization and orientation parameters are calculated.

Fig. 9 and Fig.10 show the localization and orientation errors. We see that, for sixteen 1, 2, and 3-axis sensors respectively, the localization/ orientation errors are smaller than (21mm /33%), (16mm/25%), and (12.6mm/12%); and the average localization/orientation errors are (7.8mm/6.14%), (7.3mm/5.35%), and (5.6mm /4.26%). Fig. 11 and Fig.12 show the 3D and 2D (x and y plane) localization results with sixteen 3-axis sensors, and its average execution time of each sample is 0.137 second. These results are pretty satisfactory, comparing with the RF method using in M2A. The latter has 87.0% samples with error smaller than 60 mm; and the average error was 37.7 mm.

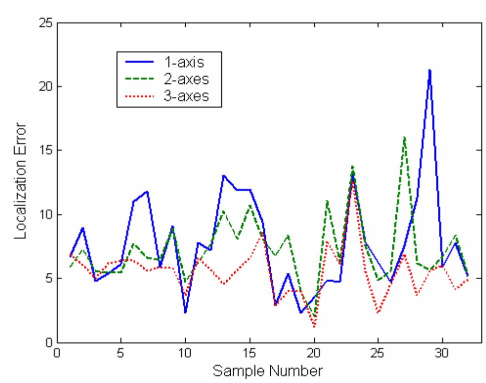



Fig. 9 Localization error (mm) via Different Samples for 1, 2, & 3-axis Sensors

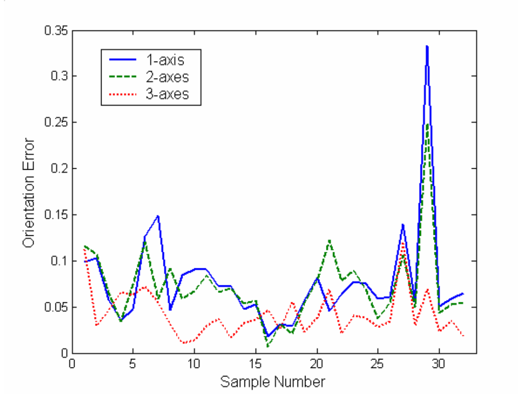


Fig. 10 Orientation error (relative to orientation vector) via Different Samples for 1, 2, & 3-axis Sensors

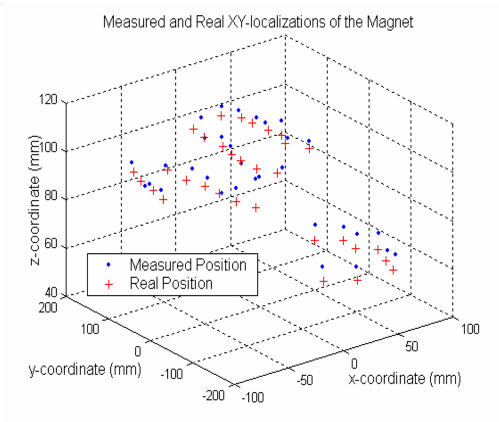


Fig. 11 3D Localization Result

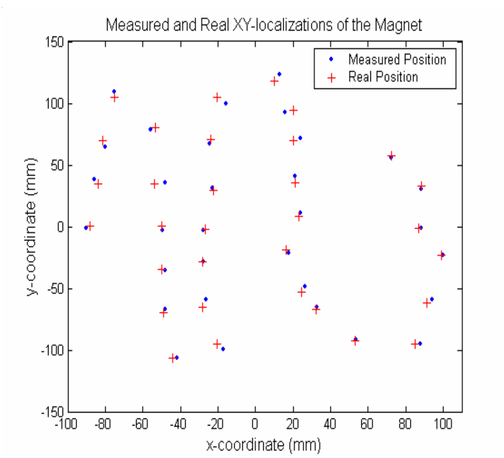


Fig.12 2D Localization Result

## IV. Conclusions

An efficient magnetic technique is proposed for localization and orientation of the capsule endoscope using a permanent magnet. The detection system is composed of a magnetic sensor array of five, or more number of, magnetic sensors. Higher localization and orientation accuracy can be obtained with denser sensor arrangement and the appropriate optimization algorithm. Our experimental results show that the Levenberg-Marquardt optimization method provides the best performance in terms of calculation accuracy and execution speed. In addition, a real test system has been built by an array of sixteen 3-axis Hall sensors and results are satisfactory with the localization error smaller than 10mm, which is acceptable for tracking the capsule. In the future, we will apply this system to the real localization and orientation of the capsule in animal and human body. We are promising for this localization and orientation system to be merged a new system of the capsule endoscopy.

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