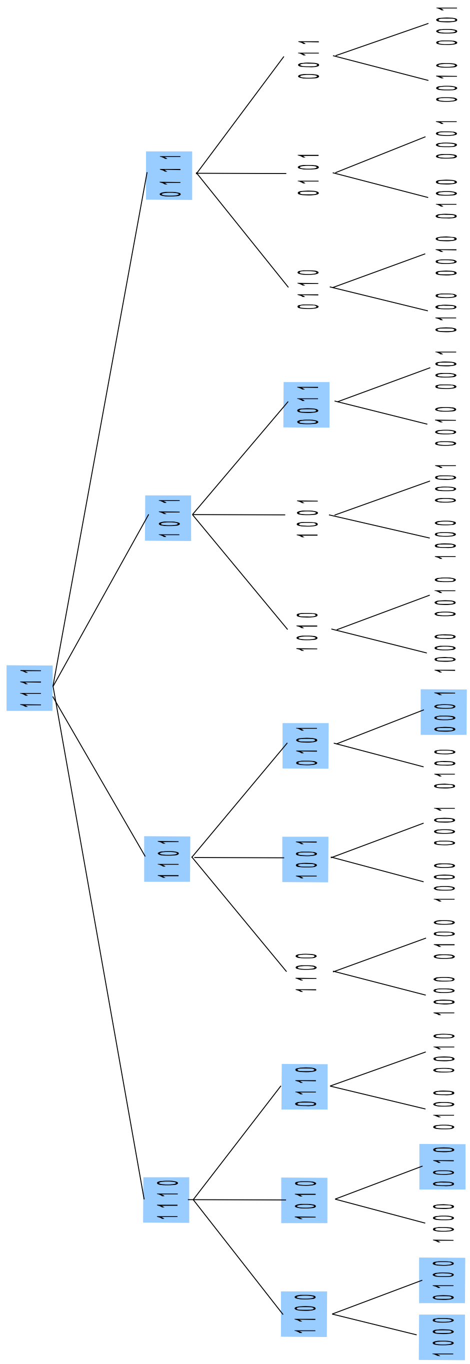
Add two integers x and y. Albeit have zeros for coefficients of an arbitrary n, and obvious g,h, quadratics, or general polynomials, of x and y are equivalently added. Suppose appropriate non-zero integer coefficients,a,b,c,d,e,f, for an arbitrary n. A non-trivial arithmetic equivalence is defined..

$$0\mathrm{n}^2+0\mathrm{n}+g=x \qquad an^2+bn+c=x$$
$$0\mathrm{n}^2+0\mathrm{n}+h=y \qquad dn^2+en+f=y$$
$$(g+h)=x+y \qquad (a+d)n^2+(b+e)n+(c+f)=x+y$$

fig 1

Generate a count of terms (combinations) from fig 2. as follows:



The graph at left represents all subsets of an arbitrary four element set. Lower members are contained in upper members. Each level of the graph represents all the subsets with a certain size. To identify a unique image of the power set, a counting algorithm suffices.

That is, from the root element, there are four unique, and only four, subsets with three elements. Of two element subsets, three can be identified from one three element subset, two more can be identified from another three element subset, the final unique two element subset can be identified from a third three element subset and the final three element subset offers no unique two element subsets. A similar counting rule can extract all subsets with one element from the two element subsets.

This counting takes the arithmetic form:

1
4
3+2+1
2+1 +1 = 15 + 1 = 16.

Likewise, all the subsets of a five element set can be counted as:

1
5
4+3+2+1
3+2+1+2+1+1
2+1+1+1 = 31 + 1 = 32.

Importantly, this counting can be generalized. The size and membership of a power set can be written as a linear combination of series of consecutive integers. It remains to define scalars to quantify multiplicity of a particular series and to prove the total sum is the size of a power set of N elements.

Fig 2

The counting rule can be written out as $N + \sum_{m=1}^{N} f(m) \sum_{n=1}^{m} n$

with $f(m) \mid f(1)=1, f(2)=1, f(3)=2, f(n)=2f(n-1)$.

fig 4

Each inner-series evaluate according to the well known formula (n)(n+1)/2. Tables below exhibit terms of function for N = 4,5,6.

										Result				
f(m)	**N=4**				**n^2**	**N**	**a**	**b**	**c**	**a**	**b**	**c**	**Total**	**T/2**
1	4		4	n										4
1	6	3+2+1		n(n-1)/2	16	4	1	1	0	16	4	0	12	6
1	3	2+1		(n-1)(n-2)/2	16	4	1	3	2	16	12	2	6	3
3	1	1		(n-2)(n-3)/2	16	4	1	5	6	48	60	18	6	3
										Total				16

										Result				
f(m)		**N=5**			**n^2**	**N**	**a**	**b**	**c**	**a**	**b**	**c**	**Total**	**T/2**
1	5 =	5		n										5
1	10 =	4+3+2+1		n(n-1)/2	25	5	1	1	0	25	5	0	20	10
1	6 =	3+2+1		(n-1)(n-2)/2	25	5	1	3	2	25	15	2	12	6
2	3 =	2+1		(n-2)(n-3)/2	25	5	1	5	6	50	50	12	12	6
5	1 =	1		(n-3)(n-4)/2	25	5	1	7	12	125	175	60	10	5
										Total				32

										Result				
f(m)		**N=6**			**n^2**	**N**	**a**	**b**	**c**	**a**	**b**	**c**	**Total**	**T/2**
1	6 =	6	6	n										6
1	15 =	5+4+3+2+1	15	n(n-1)/2	36	6	1	1	0	36	6	0	30	15
1	10 =	4+3+2+1	10	(n-1)(n-2)/2	36	6	1	3	2	36	18	2	20	10
2	6 =	3+2+1	12	(n-2)(n-3)/2	36	6	1	5	6	72	60	12	24	12
4	3 =	2+1	12	(n-3)(n-4)/2	36	6	1	7	12	144	168	48	24	12
9	1 =	1	9	(n-4)(n-5)/2	36	6	1	9	20	324	486	180	18	9
										Total				64

fig 5 (note the integers 1,2 are defined as the constants 1 and 2 because they are not applicable numbers to the counting formula)

The quadratics described above evaluate to 2^n for N, the size of its native power set. Each multi-linear system can be rewritten for evaluation at an arbitrary $m = n + x$ This is possible using a corollary of the sum of first n integers formula:

$$\sum_{1}^{m-x}(m-x) = \frac{(m-x)(m-x+1)}{2}$$

Inductively speaking, for the base condition if x = 0 then m=n and

$$\sum_{1}^{n} n = \frac{n(n+1)}{2}$$

which is the familiar sum of first n positive integers.

For the inductive step, if the arbitrary m implies the arbitrary m+1, and am looking to prove equivalence, then with , $(m+1) = n + (x+1)$

$$\frac{(m-x)(m-x+1)}{2} = \frac{((m+1)-(x+1))((m+1)-(x+1)+1)}{2}$$
$$(m-x)(m-x+1) = (m+1-x-1)(m+1-x-1+1)$$
$$(m-x)(m-x+1) = (m-x)(m-x+1)$$

which are equivalent.

To show a doubling of the counting formula from N to N+1 it its sufficient to inspect linear terms between two consecutive iterations. There are four considerations. A set of terms double between two iterations and therefor need no further analysis. There is an new term in N+1. It is (n+1)(n)/2. This along with the smallest inner-series term, which does not double but is deficit by one must reconcile with two other pairs of linear terms. The first is an invariant term in N and N+1. It is n(n-1)/2 and has not doubled. The other are two distinct linear terms in N and N+1. N needs to double but only appears as N+1. There is a deficit of 2N-(N+1). Fig. 6 proves the non-explicitly doubling terms are equivalent to a multiplication by 2.

$$\frac{(n+1)n}{2} - 1 = \frac{n(n-1)}{2} + 2n - (n+1)$$
$$(n+1)n - 2 = n(n-1) + 4n - 2n + 2$$
$$n^2 + n = n^2 - n + 2n$$
$$n^2 + n = n^2 + n$$

Fig. 6