

## 高等数学模拟试卷（二） 参考答案

$$\begin{aligned} 1. \text{ 解: } \because \lim_{x \rightarrow 0} \frac{e^x + \ln(1-x) - 1}{x^k} &= \lim_{x \rightarrow 0} \frac{e^x - \frac{1}{1-x}}{kx^{k-1}} = \lim_{x \rightarrow 0} \frac{(x-1)e^x + 1}{kx^{k-1}(x-1)} \\ &= -\lim_{x \rightarrow 0} \frac{(x-1)e^x + 1}{kx^{k-1}} = -\lim_{x \rightarrow 0} \frac{e^x + (x-1)e^x}{k(k-1)e^{k-2}} \\ &= -\lim_{x \rightarrow 0} \frac{e^x}{k(k-1)(k-2)x^{k-3}} = c \neq 0 \end{aligned}$$

$\because \lim_{x \rightarrow 0} e^x = 1 \neq 0$ , 所以必有  $\lim_{x \rightarrow 0} k(k-1)(k-2)x^{k-3}$  存在且不为 0

$$\therefore k-3=0, \quad k=3$$

2. 解: 间断点  $x=0, x=1, x=-1$

将  $x=0$  代入分子中, 分子等于 0, 但式中含有  $|x-0|$

所以  $x=0$  为第一类跳跃间断点

将  $x=1$  代入分子中, 分子等于 0, 所以  $x=1$  为第一类可去间断点

将  $x=-1$  代入分子中, 分子不等于 0, 所以  $x=-1$  为第二类无穷型间断点

所以第一类间断点共有 2 个

3. 解:  $f'(x) = ae^{ax} - 2x - 2$

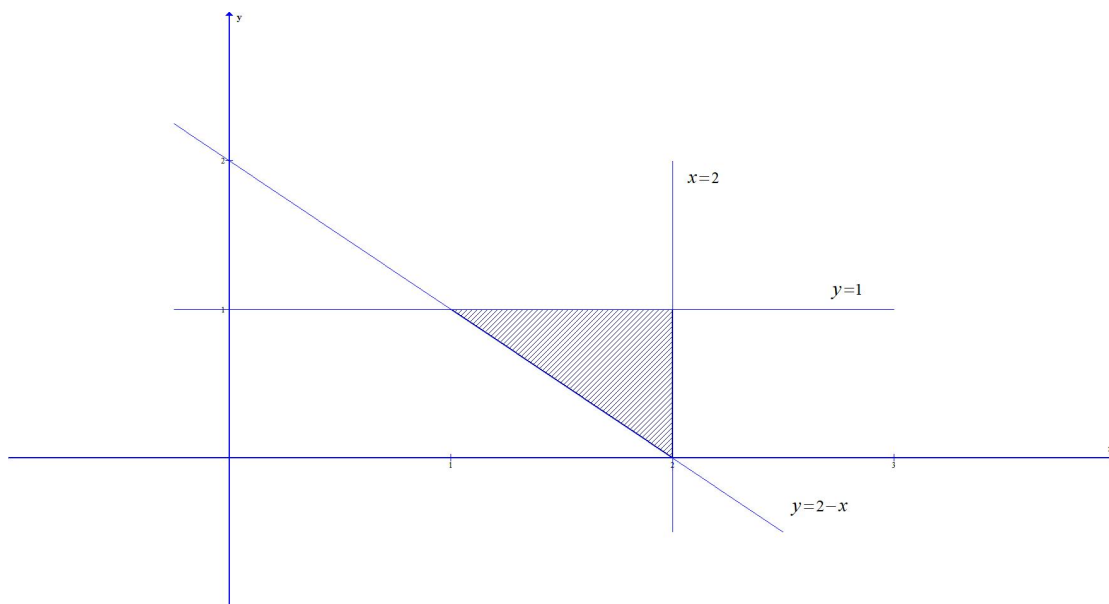
$\because f(x)$  在  $x=0$  处取得极值,  $\therefore f'(0) = a - 2 = 0$ , 即  $a=2$

4. 解:  $f(x) = (xe^x)' = (x+1)e^x$

$$\begin{aligned} \int f'(2x+1)dx &= \frac{1}{2} \int f'(2x+1)d(2x+1) = \frac{1}{2} f(2x+1) + c \\ &= \frac{1}{2} [(2x+1)+1] e^{2x+1} + c = (x+1)e^{2x+1} + c \end{aligned}$$

5. 解: 积分区域  $\begin{cases} 2-x \leq y \leq 1 \\ 1 \leq x \leq 2 \end{cases}$

$$\int_1^2 dx \int_{2-x}^1 f(x, y) dy = \int_0^1 dy \int_{2-y}^2 f(x, y) dx$$



6. 解: 对于 A:  $\lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{3n+2}} = \sqrt{\frac{1}{3}} \neq 0$  发散

对于 B: 不易看出, 先跳过

对于 C:  $\sum_{n=1}^{\infty} \frac{1+(-1)^n}{\sqrt{n+1}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}} + \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$  发散

对于 D: 取  $V_n = \frac{n\sqrt{n}}{n^2} = \frac{1}{\sqrt{n}}$ , 发散

A, C, D 都发散, 所以选 B

7. 解:  $|-3A^{-1}| = (-3)^3 |A^{-1}| = \frac{-27}{|A|} = -9$

8. 解:  $f(x) = \begin{vmatrix} x & -1 & 0 \\ 2 & 2 & 3 \\ -2 & 5 & 4 \end{vmatrix}_{c_1+xc_2} = \begin{vmatrix} 0 & -1 & 0 \\ 2+2x & 2 & 3 \\ -2+5x & 5 & 4 \end{vmatrix} \xrightarrow{\text{按第1行展开}} \begin{vmatrix} 2+2x & 3 \\ -2+5x & 4 \end{vmatrix} = -7x+14$

9. 解:  $\because y=2$  是其水平渐近线

$$\therefore \lim_{x \rightarrow \infty} \left( \frac{a-x}{2a-x} \right)^x = \lim_{x \rightarrow \infty} e^{\left( \frac{a-x}{2a-x} - 1 \right)x} = \lim_{x \rightarrow \infty} e^{\frac{-ax}{2a-x}} = e^a = 2$$

$$a = \ln 2$$

10. 解:  $\varphi'(x) = -x^2 \ln(1+x^2) \cdot 2x = -2x^3 \ln(1+x^2)$

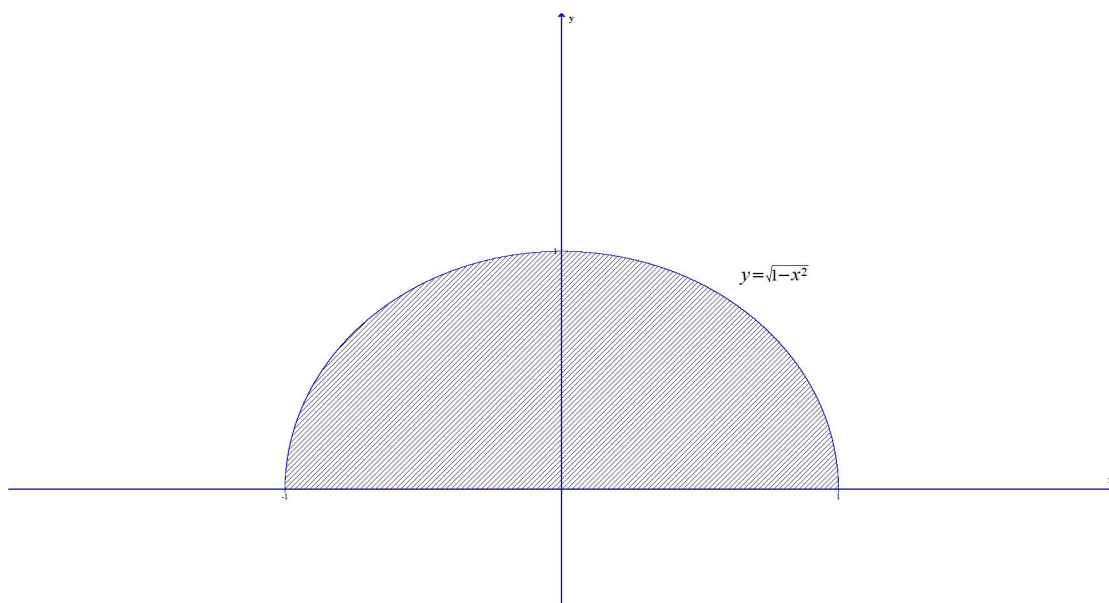
11. 解: 方程两边对  $x$  求偏导数

$$2z \cdot \frac{\partial z}{\partial x} - 2yz - 2xy \cdot \frac{\partial z}{\partial x} - 2x = 0$$

$$\therefore \frac{\partial z}{\partial x} = \frac{x + yz}{z - xy}$$

12. 解:  $D$  对称于  $y$  轴,  $xy^2$  关于  $x$  为基函数, 所以  $\iint_D xy^2 dx dy = 0$

$$\iint_D (xy^2 + 2) dx dy = \iint_D 2 dx dy = 2 \iint_D dx dy = \pi$$



13. 解: 令  $\lim_{n \rightarrow \infty} \left| \frac{(x-1)^{n+1}}{n+2} \cdot \frac{n+1}{(x-1)^n} \right| = |x-1| < 1$ , 得  $0 < x < 2$

当  $x = 0$  时, 级数为  $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{n+1} = \sum_{n=1}^{\infty} \frac{1}{n+1}$  发散

当  $x = 2$  时, 级数为  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$  收敛

收敛域为  $(0, 2]$

$$14. \text{ 解: } C = A^T B = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 1 \\ 4 & -2 & 2 \\ -2 & 1 & -1 \end{pmatrix}$$

$$15. \text{ 解: } \lim_{x \rightarrow 0} \frac{\arctan x - \sin x}{x^2 \ln(1-x)} = \lim_{x \rightarrow 0} \frac{\arctan x - \sin x}{-x^3} = \lim_{x \rightarrow 0} \frac{\frac{1}{1+x^2} - \cos x}{-3x^2}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{1 - (1 + x^2) \cos x}{-3x^2} \\
&= \lim_{x \rightarrow 0} \frac{-2x \cos x + (1 + x^2) \sin x}{-6x} \\
&= \lim_{x \rightarrow 0} \left[ \frac{-2x \cos x}{-6x} + \frac{(1 + x^2) \sin x}{-6x} \right] = \frac{1}{6}
\end{aligned}$$

16. 解:  $\frac{dx}{dt} = (t+1)e^t$

$$e^t + e^y = 2e \text{ 两边对 } t \text{ 求导得 } e^t + e^y \frac{dy}{dt} = 0, \quad \frac{dy}{dt} = -\frac{e^t}{e^y}$$

$$\left. \frac{dy}{dx} \right|_{t=1} = \left. \frac{dy}{dt} \middle/ \frac{dx}{dt} \right|_{t=1} = \left. \frac{-e^t}{e^y(t+1)e^t} \right|_{t=1} = -\frac{1}{2e^y}$$

由原方程可知, 当  $t=1$  时  $y=1$

$$\therefore \left. \frac{dy}{dx} \right|_{t=1} = -\frac{1}{2e^y} \bigg|_{y=1} = -\frac{1}{2e}$$

17. 解: 令  $\sqrt{x} = t$ , 则  $x = t^2$

$$\begin{aligned}
\int \sqrt{x} \sin 2\sqrt{x} dx &= \int t \sin 2t dt^2 = 2 \int t^2 \sin 2t dt = \int t^2 \sin 2t d2t \\
&= -\int t^2 d \cos 2t = -t^2 \cos 2t + \int \cos 2t dt^2 \\
&= -t^2 \cos 2t + \int 2t \cos 2t dt \\
&= -t^2 \cos 2t + \int t d \sin 2t \\
&= -t^2 \cos 2t + t \sin 2t - \int \sin 2t dt \\
&= -t^2 \cos 2t + t \sin 2t + \frac{1}{2} \cos 2t + c \\
&= -x \cos 2\sqrt{x} + \sqrt{x} \sin 2\sqrt{x} + \frac{1}{2} \cos 2\sqrt{x} + c
\end{aligned}$$

18. 解: 令  $x = \sin t$ , 当  $x=0$  时  $t=0$ , 当  $x=1$  时  $t = \frac{\pi}{2}$

$$\int_0^1 \frac{dx}{1 + \sqrt{1-x^2}} = \int_0^{\frac{\pi}{2}} \frac{d \sin t}{1 + \cos t} = \int_0^{\frac{\pi}{2}} \frac{\cos t}{1 + \cos t} dt = \int_0^{\frac{\pi}{2}} dt - \int_0^{\frac{\pi}{2}} \frac{1}{1 + \cos t} dt$$

$$= t \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{dt}{2 \cos^2 \frac{t}{2}} = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \frac{1}{\cos^2 \frac{t}{2}} d \frac{t}{2}$$

$$= \frac{\pi}{2} - \tan \frac{t}{2} \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$$

19. 解:  $\frac{\partial z}{\partial x} = f_1' \cdot 2x + f_2' \cdot y = 2xf_1' + yf_2'$

$$\frac{\partial^2 z}{\partial x \partial y} = 2x[f_{11}''(-2y) + f_{12}'' \cdot x] + f_2' + y[f_{21}''(-2y) + f_{22}'' \cdot x]$$

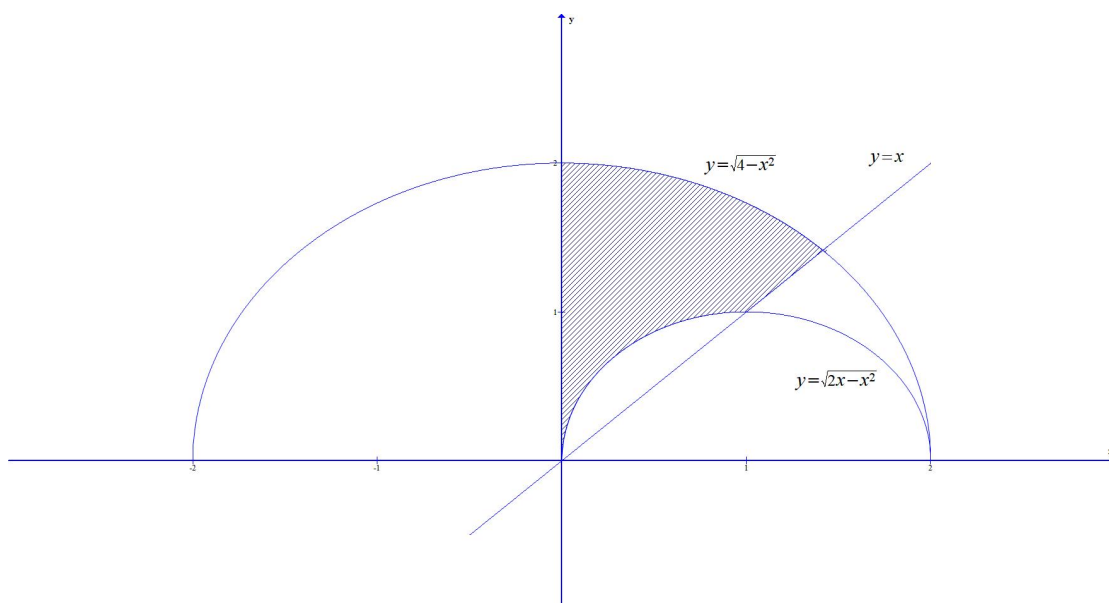
$$= f_2' - 4xyf_{11}'' + 2(x^2 - y^2)f_{12}'' + xyf_{22}''$$

20. 解:  $\iint_D y dx dy = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_{2\cos\theta}^2 r \sin\theta r dr = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \cdot \sin\theta \cdot \frac{1}{3} r^3 \Big|_{2\cos\theta}^2$

$$= \frac{8}{3} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta (1 - \cos^3\theta) d\theta$$

$$= \frac{8}{3} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta d\theta - \frac{8}{3} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta \cos^3\theta d\theta$$

$$= -\frac{8}{3} \cos\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{2}{3} \cos^4\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{4\sqrt{2}}{3} - \frac{1}{6}$$



21. 解: 对应齐次方程的特征方程  $r^2 - 3r + 2 = 0$ , 特征根  $r_1 = 1, r_2 = 2$

$$\text{通解 } \bar{y} = c_1 e^x + c_2 e^{2x}$$

$$\text{设原方程的一个特解 } y^* = x(ax+b)e^x = (ax^2 + bx)e^x$$

$$y^{*'} = (2ax+b)e^x + (ax^2 + bx)e^x$$

$$y^{*''} = 2ae^x + (2ax+b)e^x + (2ax+b)e^x + (ax^2 + bx)e^x$$

$$\text{将 } y^* \text{ 代入原方程得解得 } a = -1, b = -2$$

$$\therefore y^* = (-x^2 - 2x)e^x$$

$$\text{原方程通解 } y = \bar{y} + y^* = c_1 e^x + c_2 e^{2x} - (x^2 + 2x)e^x$$

$$22. \text{ 解: } AX = B \text{ 两边左乘 } A^{-1} \text{ 得 } X = A^{-1}B$$

$$(AE) = \begin{pmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{r_1+2r_2} \begin{pmatrix} 1 & 0 & -3 & 1 & 0 & -3 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\substack{r_1+3r_3 \\ r_2+2r_3}} \begin{pmatrix} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 0 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ -2 & 3 \\ -1 & 1 \end{pmatrix}$$

$$23. \text{ 证明: } f(0) = 2$$

$$f(0-0) = \lim_{x \rightarrow 0^-} \frac{e^{2x} - \cos x}{x} = \lim_{x \rightarrow 0^-} (2e^{2x} - \sin x) = 2$$

$$f(0+0) = \lim_{x \rightarrow 0^+} (2x + 2) = 2$$

$$\therefore f(0-0) = f(0+0) = f(0), \therefore f(x) \text{ 在 } x=0 \text{ 处连续}$$

$$f'_-(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^-} \frac{\frac{e^{2x} - \cos x}{x} - 2}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{e^{2x} - \cos x - 2x}{x^2} = \lim_{x \rightarrow 0^-} \frac{2e^{2x} + \sin x - 2}{2x}$$

$$= \lim_{x \rightarrow 0^-} \frac{4e^{2x} + \cos x}{2} = \frac{5}{2}$$

$$f_+'(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^+} \frac{2x + 2 - 2}{x} = 2$$

$\therefore f_-'(0) \neq f_+'(0)$ ,  $\therefore f(x)$  在  $x=0$  处不可导

24. 解: (1)  $y' = 3ax^2 + 2bx + c$ ,  $y'' = 6ax + b$

$\therefore$  函数在  $x=2$  处取得极值, 且  $(1,6)$  是其图形的拐点

$$\therefore \begin{cases} y'|_{x=2} = 0 \\ y''|_{x=1} = 0 \\ y|_{x=1} = 6 \end{cases} \Rightarrow \begin{cases} 2a + 4b + c = 0 \\ 6a + b = 0 \\ a + b + c + 4 = 6 \end{cases}$$

解得  $a = -1, b = 3, c = 0$

$$(2) y' = -3x^2 + 6x = -3x(x-2)$$

令  $y' = 0$ , 得  $x = 0, x = 2$

$x$	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, +\infty)$
$y'$	-	0	+	0	-
$y$	$\searrow$	极小值	$\nearrow$	极大值	$\searrow$

由表可知, 函数的单调递增区间为  $(0, 2)$

单调递减区间为  $(-\infty, 0)$ ,  $(2, +\infty)$

极小值  $y(0) = 4$ , 极大值  $y(2) = 8$

$$(3) y'' = -6x + 6 = -6(x-1), \text{ 令 } y'' = 0 \text{ 得 } x = 1$$

$x$	$(-\infty, 1)$	1	$(1, +\infty)$
$y''$	+	0	-

$y$	$\cup$	拐点(1,6)	$\cap$
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由表可知, 曲线得凹区间为 $(-\infty, 1)$ , 凸区间为 $(1, +\infty)$ , 拐点(1,6)

$$\begin{aligned}
 25. \text{ 解: } \bar{A} &= \begin{pmatrix} 1 & 1 & 0 & -1 & 4 \\ -1 & 0 & 2 & 0 & -1 \\ 2 & 1 & -3 & 0 & 5 \\ 1 & 2 & 1 & -1 & 7 \end{pmatrix} \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} \begin{pmatrix} 1 & 1 & 0 & -1 & 4 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & -1 & -3 & 2 & -3 \\ 0 & 1 & 1 & 0 & 3 \end{pmatrix} \\
 &\xrightarrow{\substack{r_1-r_2 \\ r_3+r_2 \\ r_4+r_2}} \begin{pmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix} \xrightarrow{\substack{r_1-2r_3 \\ r_2+2r_3 \\ r_4-r_3}} \begin{pmatrix} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
 &\xrightarrow{-r_3} \begin{pmatrix} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
 \end{aligned}$$

$x_1, x_2, x_3$  为约束变量,  $x_4$  为自由变量

$$\text{一般解} \begin{cases} x_1 = 1 + 2x_4 \\ x_2 = 3 - x_4 \\ x_3 = x_4 \end{cases}, \text{ 令 } x_4 = 0 \text{ 得原方程组的一个特解 } \eta^* = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{导出组的一般解} \begin{cases} x_1 = 2x_4 \\ x_2 = -x_4 \\ x_3 = x_4 \end{cases}, \text{ 令 } x_4 = 1 \text{ 得基础解系 } \xi = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{原方程组的通解} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix} \quad (k \text{ 为任意常数})$$