高等数学模拟试卷（十）参考答案

1. 选 A

解：$\lim\limits_{x\to 0}\dfrac{f(x)}{\varphi(x)}=\lim\limits_{x\to 0}\dfrac{x-\sin ax}{x^2\ln(1-bx)}=\lim\limits_{x\to 0}\dfrac{x-\sin ax}{-bx^3}=\lim\limits_{x\to 0}\dfrac{1-a\cos ax}{-3bx^2}=1$

因$\lim\limits_{x\to 0}\left(-3bx^2\right)=0$，$\therefore$必有$\lim\limits_{x\to 0}(1-a\cos ax)=0$，$a=1$

于是有$\lim\limits_{x\to 0}\dfrac{1-\cos x}{-3bx^2}=\lim\limits_{x\to 0}\dfrac{\frac{1}{2}x^2}{-3bx^2}=-\dfrac{1}{6b}=1$，$b=-\dfrac{1}{6}$

2. 选 B

解：$f(0-0)=\lim\limits_{x\to 0^-}\dfrac{(x-1)\left(1-e^{-x}\right)}{|x|\left(x^2-1\right)}=\lim\limits_{x\to 0^-}\dfrac{(x-1)x}{-x\left(x^2-1\right)}=-1$

$f(0+0)=\lim\limits_{x\to 0^+}\dfrac{(x-1)\left(1-e^{-x}\right)}{|x|\left(x^2-1\right)}=\lim\limits_{x\to 0^+}\dfrac{(x-1)x}{x\left(x^2-1\right)}=1$

因$f(0-0),f(0+0)$都存在，但不相等

$\therefore x=0$是$f(x)$的跳跃间断点

3. 选 A

解：$\lim\limits_{x\to 0}\dfrac{f(0)-f(x)}{x}=-\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x}=-f'(0)$

4. 选 C

解：$\because f'(x)=e^{-x}$，$\therefore\int f'(x)dx=\int e^{-x}dx=-e^{-x}+c$

即$f(x)=-e^{-x}+c$

由$f(0)=0$，可知$c=1$，$\therefore f(x)=-e^{-x}+1$，$f(-x)=-e^{x}+1$

于是$\int f(-x)dx=\int\left(-e^{x}+1\right)dx=-e^{x}+x+c$

5. 选 B

解：$f_x'(x,y)=3ax^2+cy$，$f_y'(x,y)=3by^2+cx$

$\because f(1,1)=-1$ 是 $f(x,y)$ 的极值

$$\therefore \begin{cases} f_x{}'(1,1)=3a+c=0 \\ f_y{}'(1,1)=3b+c=0 \\ a+b+c=-1 \end{cases}，解得 \begin{cases} a=1 \\ b=1 \\ c=-3 \end{cases}$$

6. 选 D

解：由条件收敛与绝对收敛的概念即知 D 正确

7. 选 D

解：$D_1=8\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \end{vmatrix} \overset{r_2\leftrightarrow r_3}{=} -8\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=-8m$

8. 选 C

解：$|-A|=(-1)^4|A|=|A|$

9. 应填 2

解：$\because \lim\limits_{x\to 0}\left(\dfrac{2+x}{2-x}-1\right)\cdot\dfrac{k}{x}=\lim\limits_{x\to 0}\dfrac{2x}{2-x}\cdot\dfrac{k}{x}=k$

$$\therefore \lim_{x\to 0}\left(\frac{2+x}{2-x}\right)^{\frac{k}{x}}=e^k$$

由 $e^k=e^2$，得 $k=2$

10. 应填 $\dfrac{1}{2}$

解：方程两边对 y 求偏导数，得：

$$3z^2\cdot\frac{\partial z}{\partial y}-3z-3y\cdot\frac{\partial z}{\partial y}=0，\ \therefore \frac{\partial z}{\partial y}=\frac{z}{z^2-y}$$

当 $x=0, y=0$ 时，由原方程可得 $z=2$

$$\therefore \left.\frac{\partial z}{\partial y}\right|_{\substack{x=0\\y=0}}=\left.\frac{z}{z^2-y}\right|_{\substack{x=0\\y=0}}=\left.\frac{1}{z}\right|_{z=2}=\frac{1}{2}$$

11. 应填 $y=(1+x^2)(\arctan x+c)$

解：方程可化成$\dfrac{dy}{dx}-\dfrac{2x}{1+x^2}y=1$

通解 $y=e^{\int\frac{2x}{1+x^2}dx}\left(\int e^{-\int\frac{2x}{1+x^2}dx}dx+c\right)$

$=e^{\ln(1+x^2)}\left(\int e^{-\ln(1+x^2)}dx+c\right)$

$=(1+x^2)\left(\int\dfrac{1}{1+x^2}dx+c\right)$

$=(1+x^2)(\arctan x+c)$

12. 应填$\left[\dfrac{3}{2},\dfrac{5}{2}\right]$

解：令$\lim\limits_{n\to\infty}\left|\dfrac{u_{n+1}(x)}{u_n(x)}\right|=\lim\limits_{n\to\infty}\left|\dfrac{4^{n+1}\cdot(x-2)^{2n+1}}{2n+1}\cdot\dfrac{2n-1}{4^n(x-2)^{2n-1}}\right|=4(x-2)^2<1$

得$\dfrac{3}{2}<x<\dfrac{5}{2}$，收敛半径$R=\dfrac{1}{2}$

当$x=\dfrac{3}{2}$时，级数为$\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n\cdot4^n}{2n-1}\cdot\left(-\dfrac{1}{2}\right)^{2n-1}=\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n+1}\cdot2}{2n-1}$收敛

当$x=\dfrac{5}{2}$时，级数为$\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n\cdot4^n}{2n-1}\cdot\left(\dfrac{1}{2}\right)^{2n-1}=\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n\cdot2}{2n-1}$收敛

所以收敛域为$\left[\dfrac{3}{2},\dfrac{5}{2}\right]$

13. 应填9

解：$A=\begin{pmatrix}2&3&4\\6&t&2\\4&6&3\end{pmatrix}\xrightarrow[r_3-2r_1]{r_2-3r_1}\begin{pmatrix}2&3&4\\0&t-9&-10\\0&0&-5\end{pmatrix}\xrightarrow{r_2\leftrightarrow r_3}\begin{pmatrix}2&3&4\\0&0&-5\\0&t-9&-10\end{pmatrix}$

$\xrightarrow{r_3-2r_2}\begin{pmatrix}2&3&4\\0&0&-5\\0&t-9&0\end{pmatrix}$

$\because R(A)=2$，$\therefore$应有$t-9=0$，即$t=9$

14. 应填$\dfrac{3}{2}$

解：$AB=\begin{pmatrix}1&3\\x&1\end{pmatrix}\begin{pmatrix}2&0\\0&1\end{pmatrix}=\begin{pmatrix}2&3\\2x&1\end{pmatrix}$

$\because AB$ 为对称矩阵，$\therefore$ 应有 $2x=3$，$x=\dfrac{3}{2}$

15. 解：$\lim\limits_{x\to0}\left(\dfrac{1}{x\arcsin x}-\dfrac{1}{x^2}\right)=\lim\limits_{x\to0}\dfrac{x-\arcsin x}{x^2\arcsin x}=\lim\limits_{x\to0}\dfrac{1-\dfrac{1}{\sqrt{1-x^2}}}{3x^2}$

$$=\lim_{x\to0}\frac{\sqrt{1-x^2}-1}{3x^2\sqrt{1-x^2}}=\lim_{x\to0}\frac{-\frac{1}{2}x^2}{3x^2}=-\frac{1}{6}$$

16. 解：$\dfrac{dx}{dt}=2t+2$

$t^2-y+\sin y=1$，两边对 t 求导，得：

$2t-\dfrac{dy}{dt}+\cos y\cdot\dfrac{dy}{dt}=0$

$\therefore\dfrac{dy}{dt}=\dfrac{2t}{1-\cos y}$

$\dfrac{dy}{dx}=\dfrac{dy}{dt}\Big/\dfrac{dx}{dt}=\dfrac{t}{(t+1)(1-\cos y)}$

17. 解：$\int(x\ln x)^2dx=\int x^2\ln^2 xdx=\dfrac{1}{3}\int\ln^2 xdx^3$

$$=\frac{1}{3}x^3\ln^2 x-\frac{1}{3}\int x^3d\ln^2 x$$
$$=\frac{1}{3}x^3\ln^2 x-\frac{2}{3}\int x^2\ln xdx$$
$$=\frac{1}{3}x^3\ln^2 x-\frac{2}{9}\int\ln xdx^3$$
$$=\frac{1}{3}x^3\ln^2 x-\frac{2}{9}x^3\ln x+\frac{2}{9}\int x^3d\ln x$$

18. 解：令 $\sqrt{2-x}=t$，则 $x=2-t^2$

当 $x=-1$ 时 $t=\sqrt{3}$，当 $x=1$ 时 $t=1$

$\therefore\int_{-1}^{1}\dfrac{dx}{(3-x)\sqrt{2-x}}=\int_{\sqrt{3}}^{1}\dfrac{d(2-t^2)}{(1+t^2)\,t}=-2\int_{\sqrt{3}}^{1}\dfrac{1}{1+t^2}dt$

$$=-2\arctan t\Big|_{\sqrt{3}}^{1}=-2\left(\frac{\pi}{4}-\frac{\pi}{3}\right)=\frac{\pi}{6}$$

19. 解：$\dfrac{\partial z}{\partial x}=y\left(2f_1'+2xyf_2'\right)=2yf_1'+2xy^2f_2'$

$$\frac{\partial^2 z}{\partial x\partial y}=2f_1'+2y\left(f_{11}''\cdot 3+f_{12}''\cdot x^2\right)+4xyf_2'+2xy^2\left(f_{21}''\cdot 3+f_{22}''\cdot x^2\right)$$

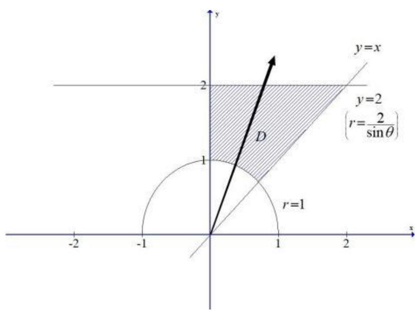
$$=2f_1'+4xyf_2'+6yf_{11}''+\left(2x^2y+6xy^2\right)f_{12}''+2x^3y^2f_{22}''$$

20. 解：$\iint\limits_D xydxdy=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}d\theta\int_1^{\frac{2}{\sin\theta}}r^3\cos\theta\sin\theta dr=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}d\theta\cos\theta\sin\theta\cdot\frac{1}{4}r^4\Big|_1^{\frac{2}{\sin\theta}}$

$$=4\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos\theta}{\sin^3\theta}d\theta-\frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos\theta\sin\theta d\theta$$

$$=4\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1}{\sin^3\theta}d\sin\theta-\frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin\theta d\sin\theta$$

$$=\frac{-2}{\sin^2\theta}\Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}}-\frac{1}{8}\sin^2\theta\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}=\frac{31}{16}$$



21. 解：对应齐次方程的特征方程 $r^2+4r+4=0$，特征根 $r_1=-2$， $r_2=-2$

其通解 $\overline{y}=\left(c_1+c_2x\right)e^{-2x}$

设原方程的一个特解 $y^*=Ax^2e^{-2x}$

将 y^* 代入原方程，解得 $A=\dfrac{1}{2}$， $\therefore y^*=\dfrac{1}{2}x^2e^{-2x}$

所以原方程通解为 $y=\overline{y}+y^*=\left(c_1+c_2x\right)e^{-2x}+\dfrac{1}{2}x^2e^{-2x}$

22. 解：$A=\begin{pmatrix}1&1&3&4\\2&1&4&5\\1&1&3&6\\0&2&4&4\end{pmatrix}\xrightarrow[r_3-r_1]{r_2-2r_1}\begin{pmatrix}1&1&3&4\\0&-1&-2&-3\\0&0&0&2\\0&2&4&4\end{pmatrix}$

$$\xrightarrow{r_4+2r_2}\begin{pmatrix}1&1&3&4\\0&-1&-2&-3\\0&0&0&2\\0&0&0&-2\end{pmatrix}\rightarrow\begin{pmatrix}1&1&3&4\\0&-1&-2&-3\\0&0&0&1\\0&0&0&0\end{pmatrix}$$

向量组的一个极大无关组为$\alpha_1,\alpha_2,\alpha_4$，秩$r=3$

进一步化成简化阶梯形

$$A\xrightarrow[-r_2]{r_1-r_2}\begin{pmatrix}1&0&1&1\\0&1&2&3\\0&0&0&1\end{pmatrix}\xrightarrow[r_2-3r_3]{r_1-r_3}\begin{pmatrix}1&0&1&0\\0&1&2&0\\0&0&0&1\end{pmatrix}$$

$\therefore\alpha_3=\alpha_1+2\alpha_2$

23. 证明：令$F(x)=\ln(1+x)-xe^{-x}$

$F'(x)=\dfrac{1}{1+x}-e^{-x}+xe^{-x}=\dfrac{1}{1+x}+(x-1)e^{-x}$

令$F'(x)=0$，得$x=0$，$F''(x)=\dfrac{-1}{(1+x)^2}+e^{-x}-(x-1)e^{-x}$

$F''(0)=1>0$，$\therefore F(x)$在$x=0$处取得最小值，最小值为$F(0)=0$

故当$x>-1$时，$F(x)\geq 0$，即$\ln(1+x)-xe^{-x}\geq 0$

所以$xe^{-x}\leq\ln(1+x)$

24. 解：设切点为$\left(a,\sqrt{-a}\right)$

$y'=\dfrac{-1}{2\sqrt{-x}}$，$k_{切}=y'\big|_{x=a}=\dfrac{-1}{2\sqrt{-a}}$

切线方程为$y-\sqrt{-a}=\dfrac{-1}{2\sqrt{-a}}(x-a)$

$\because$切线过点$(1,0)$，$\therefore$有$-\sqrt{-a}=\dfrac{-1}{2\sqrt{-a}}(1-a)$，解得$a=-1$

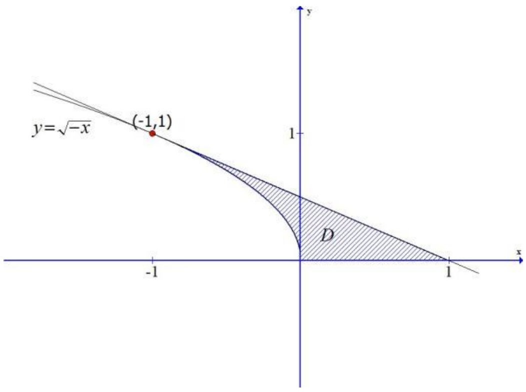
切线方程为 $y-1=\frac{-1}{2}(x+1)$，即 $y=-\frac{1}{2}x+\frac{1}{2}$

(1) $S=\frac{1}{2}\times 2\times 1-\int_{-1}^{0}\sqrt{-x}dx=1+\frac{2}{3}(-x)^{\frac{3}{2}}\Big|_{-1}^{0}=\frac{1}{3}$

(2) $V_x=\int_{-1}^{1}\pi\left(-\frac{x-1}{2}\right)^2dx-\int_{-1}^{0}\pi\left(\sqrt{-x}\right)^2dx$

$$=\frac{\pi}{4}\int_{-1}^{1}(x-1)^2dx+\int_{-1}^{0}\pi x dx$$

$$=\frac{\pi}{12}(x-1)^3\Big|_{-1}^{1}+\frac{\pi}{2}x^2\Big|_{-1}^{0}=\frac{\pi}{6}$$



25. 解：$\overline{A}=\begin{pmatrix}1&1&k&4\\-1&k&1&k^2\\1&-1&2&-4\end{pmatrix}\xrightarrow[r_3-r_1]{r_2+r_1}\begin{pmatrix}1&1&k&4\\0&k+1&k+1&k^2+4\\0&-2&2-k&-8\end{pmatrix}$

$$\xrightarrow{r_2\leftrightarrow r_3}\begin{pmatrix}1&1&k&4\\0&-2&2-k&-8\\0&k+1&k+1&k^2+4\end{pmatrix}\xrightarrow{-\frac{1}{2}r_2}\begin{pmatrix}1&1&k&4\\0&1&\frac{k-2}{2}&4\\0&k+1&k+1&k^2+4\end{pmatrix}$$

$$\xrightarrow{r_3-(k+1)r_2}\begin{pmatrix}1&1&k&4\\0&1&\frac{k-2}{2}&4\\0&0&\frac{(k+1)(4-k)}{2}&k(k-4)\end{pmatrix}$$

(1)当 $k\neq -1$ 和 $k\neq 4$ 时，$R(A)=R(\overline{A})=3$，方程组有唯一解

(2)当 $k=-1$ 时，$R(A)=2$，$R(\overline{A})=3$，$R(A)\neq R(\overline{A})$，方程组无解

(3)当 $k=4$ 时，$R(A)=R(\overline{A})=2<3$，方程组有无穷多组解

当$k=4$时，$\overline{A}=\begin{pmatrix}1&1&4&4\\0&1&1&4\\0&0&0&0\end{pmatrix}\xrightarrow{r_1-r_2}\begin{pmatrix}1&0&3&0\\0&1&1&4\\0&0&0&0\end{pmatrix}$

x_1,x_2为约束变量，x_3为自由变量

一般解$\begin{cases}x_1=-3x_3\\x_2=4-x_3\end{cases}$，令$x_3=0$得方程组的一个特解$\eta^*=\begin{pmatrix}0\\4\\0\end{pmatrix}$

导出组一般解$\begin{cases}x_1=-3x_3\\x_2=-x_3\end{cases}$，令$x_3=1$得基础解系$\xi=\begin{pmatrix}-3\\-1\\1\end{pmatrix}$

方程组通解为$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\4\\0\end{pmatrix}+k\begin{pmatrix}-3\\-1\\1\end{pmatrix}$ (其中k为任意常数)