高等数学模拟试卷（九）参考答案

1. 选 A

解：$\because f(x)$在$x=1$处连续，$\therefore$应有$\lim\limits_{x\to 1} f(x)=f(1)=1$

$\because \lim\limits_{x\to 1}\dfrac{2x^2+ax+b}{x-1}=1$且$\lim\limits_{x\to 1}(x-1)=0$

$\therefore$必有$\lim\limits_{x\to 1}\left(2x^2+ax+b\right)=2+a+b=0$　　①

于是有$\lim\limits_{x\to 1}\dfrac{2x^2+ax+b}{x-1}=\lim\limits_{x\to 1}(4x+a)=4+a=1$

$\therefore a=-3$，代入①式得$b=1$

2. 选 B

解：$f(2-0)=\lim\limits_{x\to 2^-}\dfrac{|x-2|(x-1)}{x^3-3x+2}=\lim\limits_{x\to 2^-}\dfrac{-(x-2)(x-1)}{(x-2)(x-1)}=-1$

$f(2+0)=\lim\limits_{x\to 2^+}\dfrac{|x-2|(x-1)}{x^3-3x+2}=\lim\limits_{x\to 2^+}\dfrac{(x-2)(x-1)}{(x-2)(x-1)}=1$

因$f(2-0),f(2+0)$都存在，但不相等

$\therefore x=2$是$f(x)$的跳跃间断点

3. 选 B

解：$\lim\limits_{x\to 0}\dfrac{f(a+x)-f(a-x)}{x}=[1-(-1)]f'(a)=2f'(a)$

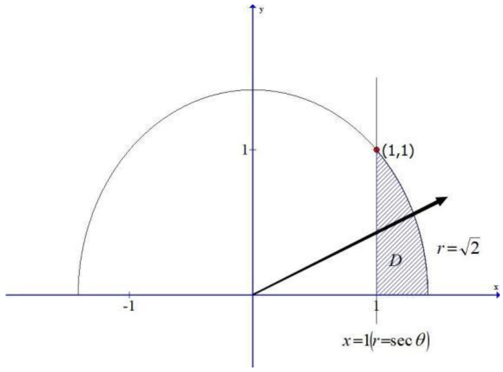
4. 选 C

解：依据不定积分与求导的关系，易知 C 是正确的，应选 C

5. 选 A

解：积分区域$D:\begin{cases}0\le y\le\sqrt{2-x^2}\\ 1\le x\le\sqrt{2}\end{cases}$

$$\therefore \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x,y)dy = \int_0^{\frac{\pi}{4}} d\theta \int_{\sec\theta}^{\sqrt{2}} f(r\cos\theta, r\sin\theta) rdr$$



6. 选 C

解：$\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} (-1)^n \sin\frac{1}{\sqrt{n}}$，$\sum_{n=1}^{\infty} u_n^2 = \sum_{n=1}^{\infty} \sin^2\frac{1}{\sqrt{n}}$

当 $n \to \infty$ 时 $\sin\frac{1}{\sqrt{n}} \sim \frac{1}{\sqrt{n}}$，$\sin^2\frac{1}{\sqrt{n}} \sim \frac{1}{n}$

因 $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ 收敛，所以 $\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} (-1)^n \sin\frac{1}{\sqrt{n}}$ 收敛

因 $\sum_{n=1}^{\infty} \frac{1}{n}$ 发散，所以 $\sum_{n=1}^{\infty} u_n^2 = \sum_{n=1}^{\infty} \sin^2\frac{1}{\sqrt{n}}$ 发散

7. 选 D

解：$ABC = BAC = BCA$

8. 选 D

解：$$\begin{vmatrix} a_{11} & 2a_{12}-3a_{11} & a_{13} \\ a_{21} & 2a_{22}-3a_{21} & a_{23} \\ a_{31} & 2a_{23}-3a_{31} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & 2a_{12} & a_{13} \\ a_{21} & 2a_{22} & a_{23} \\ a_{31} & 2a_{23} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & -3a_{11} & a_{13} \\ a_{21} & -3a_{21} & a_{23} \\ a_{31} & -3a_{31} & a_{33} \end{vmatrix}$$

$$= 2\begin{vmatrix} a_{11} & 2a_{12} & a_{13} \\ a_{21} & 2a_{22} & a_{23} \\ a_{31} & 2a_{23} & a_{33} \end{vmatrix} + 0 = -2$$

$$\therefore \begin{vmatrix} a_{11} & 2a_{12} & a_{13} \\ a_{21} & 2a_{22} & a_{23} \\ a_{31} & 2a_{23} & a_{33} \end{vmatrix} = -1$$

9. 应填$\dfrac{2}{e}$

解：$\because \lim\limits_{x\to\infty}\left(1-\dfrac{1}{x}-1\right)\cdot x=-1$

$\therefore \lim\limits_{x\to\infty}\left(1-\dfrac{1}{x}\right)^{x}=e^{-1}$

又$\lim\limits_{x\to 0}\dfrac{\sqrt{1+kx}-1}{x}=\lim\limits_{x\to 0}\dfrac{\frac{1}{2}kx}{x}=\dfrac{1}{2}k$

由$e^{-1}=\dfrac{1}{2}k$，得$k=\dfrac{2}{e}$

10. 应填$\dfrac{-1}{\sin x}+c$

解：$\int f(x)\sin x dx=\ln\sin x+c$，两边对$x$求导，得：

$f(x)\sin x=\dfrac{\cos x}{\sin x}$，$\therefore f(x)=\dfrac{\cos x}{\sin^2 x}$

于是$\int f(x)dx=\int\dfrac{\cos x}{\sin^2 x}dx=\int\dfrac{1}{\sin^2 x}d\sin x=\dfrac{-1}{\sin x}+c$

11. 应填 5

解：$y=x\left[e^{x}+(2x+1)^{3}\right]=xe^{x}+x(2x+1)^{3}$

$=xe^{x}+p_4(x)$ （$p_4(x)$是四次多项式）

$\therefore y^{(n)}=(x+n)e^{x}$(当$n\geq 5$)，$y^{(5)}(0)=5$

12. 应填$2R$

解：$\because \sum\limits_{n=1}^{\infty}a_n x^n$ 收敛半径为R，$\therefore \lim\limits_{n\to\infty}\left|\dfrac{a_n}{a_{n+1}}\right|=R$

对于$\sum\limits_{n=1}^{\infty}a_n\left(\dfrac{x-2}{2}\right)^{n}$

令$\lim\limits_{n\to\infty}\left|\dfrac{u_{n+1}(x)}{u_n(x)}\right|=\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}\left(\dfrac{x-2}{2}\right)^{n+1}}{a_n\left(\dfrac{x-2}{2}\right)^{n}}\right|=\dfrac{1}{R}\left|\dfrac{x-2}{2}\right|<1$，$|x-2|<2R$

$\therefore \sum\limits_{n=1}^{\infty}a_n\left(\dfrac{x-2}{2}\right)^{n}$ 收敛半径为$2R$

13. 应填20

解：$$\begin{vmatrix}2a_1 & 2b_1 & c_1+d_1\\2a_2 & 2b_2 & c_2+d_2\\2a_3 & 2b_3 & c_3+d_3\end{vmatrix}=\begin{vmatrix}2a_1 & 2b_1 & c_1\\2a_2 & 2b_2 & c_2\\2a_3 & 2b_3 & c_3\end{vmatrix}+\begin{vmatrix}2a_1 & 2b_1 & d_1\\2a_2 & 2b_2 & d_2\\2a_3 & 2b_3 & d_3\end{vmatrix}$$

$$=4\begin{vmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{vmatrix}+4\begin{vmatrix}a_1 & b_1 & d_1\\a_2 & b_2 & d_2\\a_3 & b_3 & d_3\end{vmatrix}=4\times4+4\times1=20$$

14. 应填$\dfrac{1}{4}$

解：$$\begin{pmatrix}k & 0 & 1\\1 & 2 & 2\\1 & 3 & 1\end{pmatrix}\xrightarrow{r_1\leftrightarrow r_2}\begin{pmatrix}1 & 2 & 2\\k & 0 & 1\\1 & 3 & 1\end{pmatrix}\xrightarrow[r_2-r_1]{r_2-kr_1}\begin{pmatrix}1 & 2 & 2\\0 & -2k & 1-2k\\0 & 1 & -1\end{pmatrix}$$

$$\xrightarrow{r_2\leftrightarrow r_3}\begin{pmatrix}1 & 2 & 2\\0 & 1 & -1\\0 & -2k & 1-2k\end{pmatrix}\xrightarrow{r_3-2kr_2}\begin{pmatrix}1 & 2 & 2\\0 & 1 & -1\\0 & 0 & 1-4k\end{pmatrix}$$

要使$\alpha_1,\alpha_2,\alpha_3$线性相关，则应有$1-4k=0$，即$k=\dfrac{1}{4}$

15. 解：$$\lim_{x\to0}\frac{\tan x-\sin x}{x^2\ln(1+x)}=\lim_{x\to0}\frac{\tan x-\sin x}{x^3}$$

$$=\lim_{x\to0}\frac{\dfrac{1}{\cos^2 x}-\cos x}{3x^2}$$

$$=\lim_{x\to0}\frac{1-\cos^3 x}{3x^2\cdot\cos^2 x}$$

$$=\lim_{x\to0}\frac{3\cos^2 x\sin x}{6x}=\frac{1}{2}$$

16. 解：令$\sqrt{x}=t$，则$x=t^2$

$$\int\sqrt{x}\sin 2\sqrt{x}dx=\int t\sin 2tdt^2=2\int t^2\sin 2tdt=-\int t^2d\cos 2t$$

$$=-t^2\cos 2t+\int\cos 2tdt^2=-t^2\cos 2t+2\int t\cos 2tdt$$

$$=-t^2\cos 2t+\int td\sin 2t=-t^2\cos 2t+t\sin 2t-\int\sin 2tdt$$

$$=-t^2\cos 2t+t\sin 2t+\frac{1}{2}\cos 2t+c$$

$$=-x\cos 2\sqrt{x}+\sqrt{x}\sin 2\sqrt{x}+\frac{1}{2}\cos 2\sqrt{x}+c$$

17. 解：令 $x=\sin t$，当 $x=0$ 时 $t=0$，当 $x=\frac{\sqrt{3}}{2}$ 时 $t=\frac{\pi}{3}$

$$\therefore \int_0^{\frac{\sqrt{3}}{2}}\frac{x(1-x)}{\sqrt{1-x^2}}dx=\int_0^{\frac{\pi}{3}}\frac{\sin t(1-\sin t)}{\cos t}d\sin t$$

$$=\int_0^{\frac{\pi}{3}}\left(\sin t-\sin^2 t\right)dt=\int_0^{\frac{\pi}{3}}\sin tdt-\int_0^{\frac{\pi}{3}}\sin^2 tdt$$

$$=-\cos t\Big|_0^{\frac{\pi}{3}}-\int_0^{\frac{\pi}{3}}\frac{1-\cos 2t}{2}dt$$

$$=\frac{1}{2}-\frac{1}{2}t\Big|_0^{\frac{\pi}{3}}+\frac{1}{4}\sin 2t\Big|_0^{\frac{\pi}{3}}$$

$$=\frac{1}{2}-\frac{1}{6}\pi+\frac{\sqrt{3}}{8}$$

18. 解：方程两边对 x 求偏导数，得：

$$\frac{z-x\frac{\partial z}{\partial x}}{z^2}=e^{y+z}\cdot\frac{\partial z}{\partial x}，\ \therefore \frac{\partial z}{\partial x}=\frac{z}{x(z+1)}$$

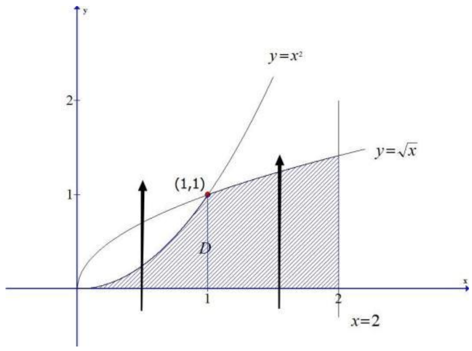
方程两边对 y 求偏导数，得：

$$\frac{-x}{z^2}\cdot\frac{\partial z}{\partial y}=e^{y+z}\cdot\left(1+\frac{\partial z}{\partial y}\right)，\ \therefore \frac{\partial z}{\partial y}=-\frac{z}{z+1}$$

19. 解：$\iint\limits_D xydxdy=\int_0^1 dx\int_0^{x^2}xydy+\int_1^2 dx\int_0^{\sqrt{x}}xydy$

$$=\int_0^1 dx\cdot\frac{1}{2}xy^2\Big|_0^{x^2}+\int_1^2 dx\cdot\frac{1}{2}xy^2\Big|_0^{\sqrt{x}}$$

$$=\frac{1}{2}\int_0^1 x^5dx+\frac{1}{2}\int_1^2 x^2dx=\frac{5}{4}$$



20. 解：对应齐次方程的特征方程 $r^2-5r+6=0$，特征根 $r_1=2$， $r_2=3$

其通解 $\overline{y}=c_1e^{2x}+c_2e^{3x}$

设原方程的一个特解 $y^*=x(ax+b)e^{2x}=(ax^2+bx)e^{2x}$

将 y^* 代入原方程，解得 $a=-1$， $b=-2$， $\therefore y^*=-(x^2+2x)e^{2x}$

所以原方程通解为 $y=\overline{y}+y^*=c_1e^{2x}+c_2e^{3x}-(x^2+2x)e^{2x}$

21. 解：由 $X=AX+B$，得 $X-AX=B$， $\therefore(E-A)X=B$

两边左乘 $(E-A)^{-1}$，得 $X=(E-A)^{-1}B$

而求得 $(E-A)^{-1}=\begin{pmatrix}0 & \frac{2}{3} & \frac{1}{3}\\ -1 & \frac{2}{3} & \frac{1}{3}\\ 0 & -\frac{1}{3} & \frac{1}{3}\end{pmatrix}$

所以 $X=\begin{pmatrix}0 & \frac{2}{3} & \frac{1}{3}\\ -1 & \frac{2}{3} & \frac{1}{3}\\ 0 & -\frac{1}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}1 & -1\\ 2 & 0\\ 5 & -3\end{pmatrix}=\begin{pmatrix}3 & -1\\ 2 & 0\\ 1 & -1\end{pmatrix}$

22. 解：$\overline{A}=\begin{pmatrix}2 & 1 & -1 & 1 & 1\\ 1 & 2 & 1 & -1 & 2\\ 1 & 1 & 2 & 1 & 3\end{pmatrix}\xrightarrow{r_1\leftrightarrow r_2}\begin{pmatrix}1 & 2 & 1 & -1 & 2\\ 2 & 1 & -1 & 1 & 1\\ 1 & 1 & 2 & 1 & 3\end{pmatrix}$

$\xrightarrow[r_3-r_1]{r_2-2r_1}\begin{pmatrix}1 & 2 & 1 & -1 & 2\\ 0 & -3 & -3 & 3 & -3\\ 0 & -1 & 1 & 2 & 1\end{pmatrix}\xrightarrow{-\frac{1}{3}r_2}\begin{pmatrix}1 & 2 & 1 & -1 & 2\\ 0 & 1 & 1 & -1 & 1\\ 0 & -1 & 1 & 2 & 1\end{pmatrix}$

$\xrightarrow[r_3-r_2]{r_1-2r_2}\begin{pmatrix}1 & 0 & -1 & 1 & 0\\ 0 & 1 & 1 & -1 & 1\\ 0 & 0 & 2 & 1 & 2\end{pmatrix}\xrightarrow{\frac{1}{2}r_3}\begin{pmatrix}1 & 0 & -1 & 1 & 0\\ 0 & 1 & 1 & -1 & 1\\ 0 & 0 & 1 & \frac{1}{2} & 1\end{pmatrix}$

$$\xrightarrow[r_2-r_3]{r_1+r_3}\begin{pmatrix} 1 & 0 & 0 & \frac{3}{2} & 1 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 1 \end{pmatrix}$$

x_1,x_2,x_3为约束变量，x_4为自由变量

一般解$\begin{cases} x_1=1-\frac{3}{2}x_4 \\ x_2=\frac{3}{2}x_4 \\ x_3=1-\frac{1}{2}x_4 \end{cases}$，令$x_4=0$得原方程组的一个特解$\eta^*=\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

导出组一般解$\begin{cases} x_1=-\frac{3}{2}x_4 \\ x_2=\frac{3}{2}x_4 \\ x_3=-\frac{1}{2}x_4 \end{cases}$，令$x_4=1$得基础解系$\xi=\begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$

原方程组通解为$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}+k\begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$（其中$k$为任意常数）

23. 证明：令$F(x)=4x\ln x-x^2-2x+3$，则$F(1)=0$

$F'(x)=4\ln x+4-2x-2=4\ln x-2x+2$

$F'(1)=0$

$F''(x)=\frac{4}{x}-2=\frac{4-2x}{x}$

当$1<x<2$时，$F''(x)>0$，$\therefore F'(x)$单调递增

$F'(x)>F'(1)=0$

由$F'(x)>0$又可知，$F(x)$单调递增

$\therefore F(x)>F(1)=0$

即 $4x\ln x - x^2 - 2x + 3 > 0$

$\therefore 4x\ln x > x^2 + 2x - 3$

24. 解：由 $\begin{cases} x=\sqrt{y} \\ x-y+2=0 \end{cases}$，解得交点坐标 $(2,4)$

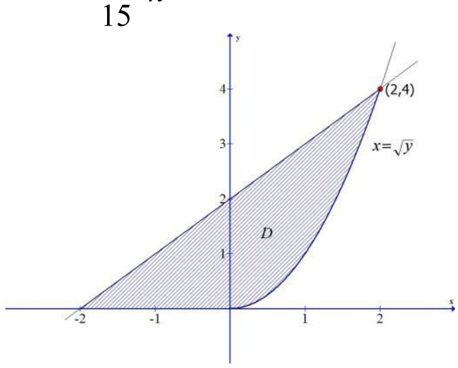
(1) $S=\dfrac{1}{2}\times 4\times 4-\int_0^2 x^2dx$

$=8-\dfrac{1}{3}x^3\Big|_0^2=\dfrac{16}{3}$

(2) $V_x=\pi\int_{-2}^{2}(x+2)^2dx-\pi\int_0^2(x^2)^2dx$

$=\dfrac{\pi}{3}(x+2)^3\Big|_{-2}^{2}-\dfrac{\pi}{5}x^5\Big|_0^2$

$=\dfrac{224}{15}\pi$



25. 解：(1) $f'(x)=3ax^2+2bx+c$，$f''(x)=6ax+2b$

$\because f(x)$ 在 $x=-1$ 处取得极值，且 $(1,-1)$ 是其图形的拐点

$\therefore$ 有 $\begin{cases} f'(-1)=0 \\ f''(1)=0 \\ f(1)=-1 \end{cases}$，即 $\begin{cases} 3a-2b+c=0 \\ 6a+2b=0 \\ a+b+c-12=-1 \end{cases}$

解得 $a=-1, b=3, c=9$

(2)令 $f'(x)=-3x^2+6x+9=0$，得 $x=-1, x=3$

x	$(-\infty,-1)$	-1	$(-1,3)$	3	$(3,+\infty)$
$f'(x)$	+	0	−	0	+
$f(x)$	↘	极小值	↗	极大值	↘

由表可知，$f(x)$的单调递减区间为$(-\infty,-1)$，$(3,+\infty)$

单调递增区间为$(-1,3)$

极小值$f(-1)=-17$，极大值$f(3)=15$

(3) $f''(x)=-6x+6$，令$f''(x)=0$得$x=1$

x	$(-\infty,1)$	1	$(1,+\infty)$
$f''(x)$	$+$	0	$-$
$y=f(x)$	$\cup$	拐点	$\cap$

由表可知，曲线$y=f(x)$得凹区间为$(-\infty,1)$，凸区间为$(1,+\infty)$