高等数学模拟试卷（五）参考答案

1. 解：将$x=2$代入分子中，分子等于 0，且式中含有$|x-2|$

所以$x=2$是$f(x)$的跳跃间断点

2. 解：$f_-'(0)=\lim\limits_{x\to 0^-}\dfrac{f(x)-f(0)}{x}=\lim\limits_{x\to 0^-}\dfrac{x^2+ax}{x}=a$

$f_+'(0)=\lim\limits_{x\to 0^+}\dfrac{f(x)-f(0)}{x}=\lim\limits_{x\to 0^+}\dfrac{e^x-1}{x}=1$

$\because f(x)$在$x=0$处可导，$\therefore$应有$f_-'(0)=f_+'(0)$，即$a=1$

3. 解：$\varphi'(x)=-e^{\frac{1}{x}}\sin\dfrac{1}{x}\cdot\dfrac{-1}{x^2}=\dfrac{1}{x^2}e^{\frac{1}{x}}\sin\dfrac{1}{x}$

4. 解：$f'(x)=e^{-x}-xe^{-x}=(1-x)e^{-x}$

$f''(x)=-e^{-x}-(1-x)e^{-x}=(x-2)e^{-x}$

令$\begin{cases}f'(x)<0\\f''(x)<0\end{cases}$，即$\begin{cases}(1-x)e^{-x}<0\\(x-2)e^{-x}<0\end{cases}\Rightarrow\begin{cases}1-x<0\\x-2<0\end{cases}\Leftarrow\begin{cases}x>1\\x<2\end{cases}$

5. 解：$\int_1^{\sqrt{2}}dx\int_0^{\sqrt{2-x^2}}f(x,y)dy=\int_0^{\frac{\pi}{4}}d\theta\int_{\frac{1}{\cos\theta}}^{\sqrt{2}}f(r\cos\theta,r\sin\theta)rdr$

6. 解：对于 B：取$V_n=\dfrac{n}{n^2\sqrt{n}}=\dfrac{1}{n\sqrt{n}}$，$\because\sum\limits_{n=1}^{\infty}\dfrac{1}{n\sqrt{n}}$收敛，所以$\sum\limits_{n=1}^{\infty}\dfrac{n+1}{n^2\sqrt{n+1}}$收敛

7. 解：$|A|\neq 0\Rightarrow A$可逆，$AB=AC$两边左乘A^{-1}

$A^{-1}AB=A^{-1}AC\Rightarrow B=C$

8. 解：个数大于维数必相关，选 C

9. 解：$\lim\limits_{x\to+\infty}\left(\dfrac{1+x}{2+x}\right)^{2x}=\lim\limits_{x\to+\infty}e^{\left(\frac{1+x}{2+x}-1\right)\cdot 2x}=e^{-2}$

$\therefore$水平渐近线方程为$y=e^{-2}$

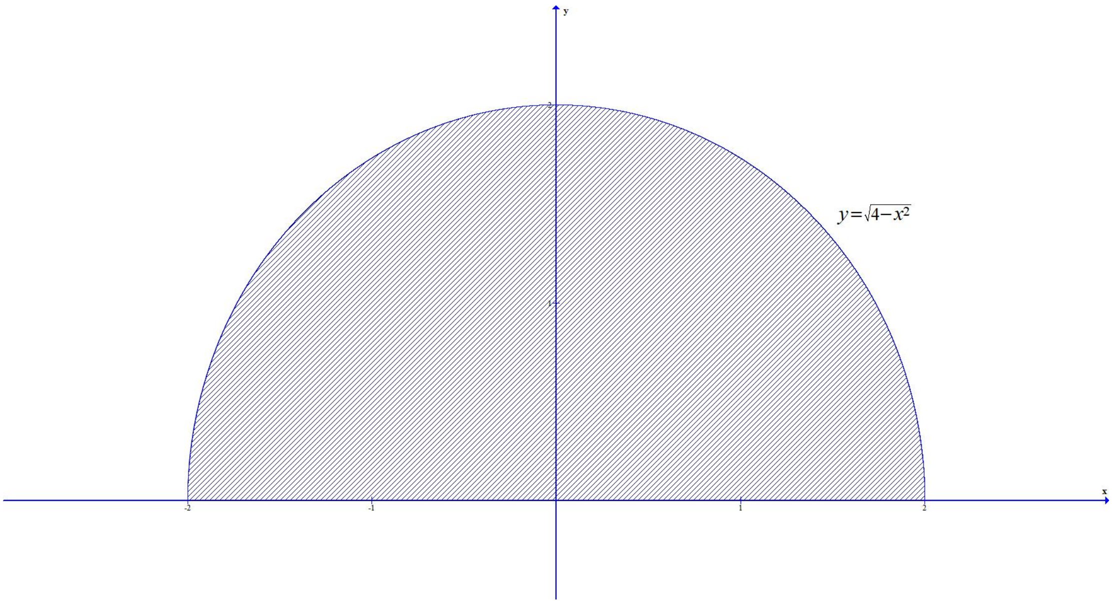
10. 解：$f(0)=a$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\sin x}{\sqrt{1+x}-1}=\lim_{x\to 0}\frac{x}{\frac{1}{2}x}=2$$

$\because f(x)$在$x=0$处连续，$\therefore$应有$\lim\limits_{x\to 0} f(x)=f(0)$，故$a=2$

11. 解：$\int_{-2}^{2}\left(x^2\sin x+1\right)\sqrt{4-x^2}dx=\int_{-2}^{2}x^2\sin x\sqrt{4-x^2}dx+\int_{-2}^{2}\sqrt{4-x^2}dx$

$$=0+\int_{-2}^{2}\sqrt{4-x^2}dx=2\pi$$



12. 解：方程两边对x求偏导数，得：$2z\frac{\partial z}{\partial x}-2yz-2xy\frac{\partial z}{\partial x}+2x=0$

$\therefore \frac{\partial z}{\partial x}=\frac{yz-x}{z-xy}$

13. 解：令$\lim\limits_{n\to\infty}\left|\frac{(x+1)^{n+1}}{(n+1)\cdot 2^{n+1}}\cdot\frac{n\cdot 2^n}{(x+1)^n}\right|=\frac{1}{2}|x+1|<1$

$-2<x+1<2$，$-3<x<1$

当$x=-3$时，级数为$\sum\limits_{n=1}^{\infty}\frac{(-2)^n}{n\cdot 2^n}=\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n}$收敛

当$x=1$时，级数为$\sum\limits_{n=1}^{\infty}\frac{2^n}{n\cdot 2^n}=\sum\limits_{n=1}^{\infty}\frac{1}{n}$发散

收敛域$[-3,1)$

14. 解：$A^2=\begin{pmatrix}2&-2\\0&2\end{pmatrix}\begin{pmatrix}2&-2\\0&2\end{pmatrix}=\begin{pmatrix}4&-8\\0&4\end{pmatrix}$

$$\therefore B=A^2-2A+E_2=\begin{pmatrix}4&-8\\0&4\end{pmatrix}-2\begin{pmatrix}2&-2\\0&2\end{pmatrix}+E_2$$

$$=\begin{pmatrix}4&-8\\0&4\end{pmatrix}-\begin{pmatrix}4&-4\\0&4\end{pmatrix}+E_2=\begin{pmatrix}1&-4\\0&1\end{pmatrix}$$

15. 解：$\lim\limits_{x\to0}\dfrac{x-x^3-\sin x}{x^2\arctan x}=\lim\limits_{x\to0}\dfrac{x-x^3-\sin x}{x^3}=\lim\limits_{x\to0}\dfrac{1-3x^2-\cos x}{3x^2}$

$$=\lim_{x\to0}\frac{-6x+\sin x}{6x}=\lim_{x\to0}\frac{-6+\cos x}{6}=-\frac{5}{6}$$

16. 解：方程两边对 x 求导，得：$e^y\cdot y'+6y+6xy'+2x=0$ ①

①式两边再对 x 求导，得：

$e^y\cdot(y')^2+e^y\cdot y''+6y'+6y'+6xy''+2=0$ ②

当 $x=0$ 时，由原方程可知 $y=0$

将 $x=0$， $y=0$ 代入①式，得：$y'(0)=0$

将 $x=0$， $y=0$， $y'(0)=0$ 代入②式，得：$y''(0)+2=0$， $y''(0)=-2$

17. 解：$\displaystyle\int\frac{x\sin x}{\cos^3 x}dx=\frac{1}{2}\int xd\frac{1}{\cos^2 x}dx=\frac{x}{2\cos^2 x}-\frac{1}{2}\int\frac{1}{\cos^2 x}dx$

$$=\frac{x}{2\cos^2 x}-\frac{1}{2}\tan x+c$$

18. 解：令 $\sqrt{2x+1}=t$，则 $x=\dfrac{t^2-1}{2}$，当 $x=-\dfrac{1}{2}$ 时 $t=0$，当 $x=\dfrac{3}{2}$ 时 $t=2$

$$\int_{-\frac{1}{2}}^{\frac{3}{2}}\frac{\sqrt{2x+1}}{2x+5}dx=\int_0^2\frac{t}{t^2+4}d\frac{t^2-1}{2}=\int_0^2\frac{t^2}{t^2+4}dt$$

$$=\int_0^2 dt-4\int_0^2\frac{1}{t^2+2^2}dt=t\Big|_0^2-2\arctan\frac{t}{2}\Big|_0^2$$

$$=2-\frac{\pi}{2}$$

19. 解：$\dfrac{\partial z}{\partial y}=f_1'\cdot1+f_2'\cdot e^{x-y}\cdot(-1)=f_1'-e^{x-y}f_2'$

$$\frac{\partial^2 z}{\partial y\partial x}=f_{11}''\cdot1+f_{12}''\cdot e^{x-y}-e^{x-y}f_2'-e^{x-y}\left(f_{21}\cdot1+f_{22}\cdot e^{x-y}\right)$$

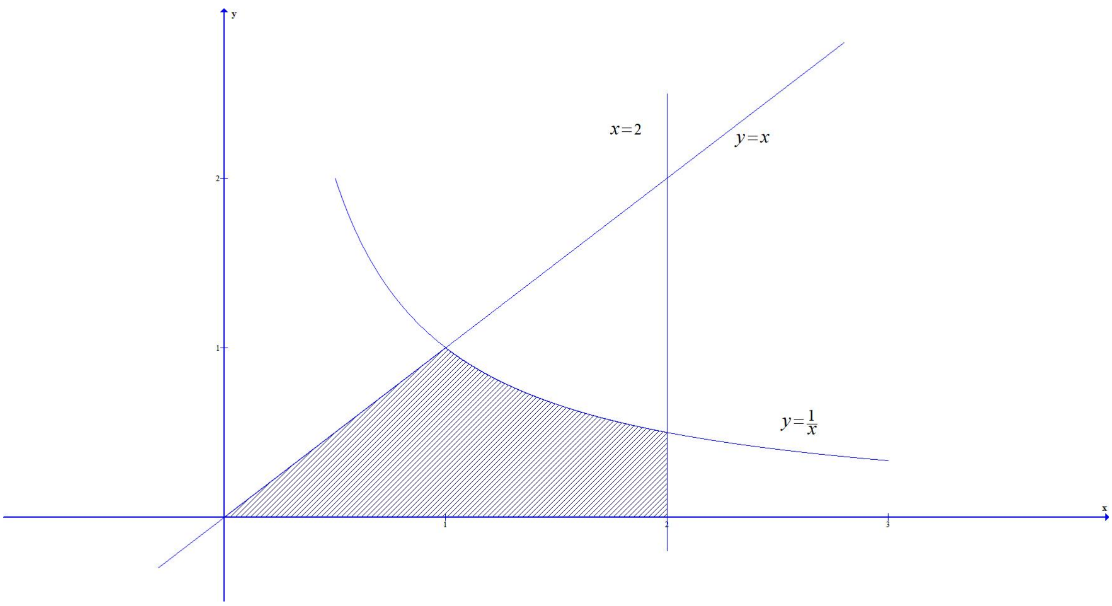
$$=f_{11}''-e^{x-y}f_2'+e^{2(x-y)}f_{22}''$$

20. 解：$\iint\limits_D (x+y)dxdy=\int_0^1 dx\int_0^x (x+y)dy+\int_1^2 dx\int_0^{\frac{1}{x}}(x+y)dy$

$$=\int_0^1 dx\left(xy+\frac{1}{2}y^2\right)\Bigg|_0^x+\int_1^2 dx\left(xy+\frac{1}{2}y^2\right)\Bigg|_0^{\frac{1}{x}}$$

$$=\int_0^1 \frac{3}{2}x^2dx+\int_1^2\left(1+\frac{1}{2x^2}\right)dx$$

$$=\frac{1}{2}x^3\Bigg|_0^1+\left(x-\frac{1}{2x}\right)\Bigg|_1^2=\frac{7}{4}$$



21. 解：$\int\frac{f(x)}{x}dx=e^{2x}+c$ 两边对 x 求导，得 $\frac{f(x)}{x}=2e^{2x}$，$\therefore f(x)=2xe^{2x}$

所求微分方程为 $y''+y'-2y=2xe^{2x}$

对应齐次方程的特征方程 $r^2+r-2=0$，特征根 $r_1=1,r_2=-2$

其通解 $\overline{y}=c_1e^x+c_2e^{-2x}$

设原方程一个特解 $y^*=(ax+b)e^{2x}$

$y^*{}'=ae^{2x}+2(ax+b)e^{2x}$，$y^*{}''=4ae^{2x}+4(ax+b)e^{2x}$

将 y^* 代入原方程，得：

$$4ae^{2x}+4(ax+b)e^{2x}+ae^{2x}+2(ax+b)e^{2x}-2(ax+b)e^{2x}=2xe^{2x}$$

$$\begin{cases}4a=2\\5a+4b=0\end{cases}\Rightarrow\begin{cases}a=\dfrac{1}{2}\\b=-\dfrac{5}{8}\end{cases}，\ y^*=\left(\frac{1}{2}x-\frac{5}{8}\right)e^{2x}$$

原方程通解 $y=\bar{y}+y^*=c_1e^x+c_2e^{-2x}+\left(\frac{1}{2}x-\frac{5}{8}\right)e^{2x}$

22. 解：$|A|=\begin{vmatrix}3&-2&1&4\\-7&5&-3&-6\\2&1&-1&3\\4&-3&2&8\end{vmatrix}\overset{\substack{r_2-3r_1\\r_3+r_2\\r_4-2r_1}}{=}\begin{vmatrix}3&-2&1&4\\2&-1&0&6\\5&-1&0&7\\-2&1&0&0\end{vmatrix}\overset{按c_3展开}{=}\begin{vmatrix}2&-1&6\\5&-1&7\\-2&1&0\end{vmatrix}$

$\overset{c_1+2c_2}{=}\begin{vmatrix}0&-1&6\\3&-1&7\\0&1&0\end{vmatrix}\overset{按r_3展开}{=}-\begin{vmatrix}0&6\\3&7\end{vmatrix}=18$

$\therefore |A^{-1}|=\dfrac{1}{|A|}=\dfrac{1}{18}$

23. 证明：令 $F(x)=x^3+(1-x)^3-\dfrac{1}{4}$

$F'(x)=3x^2-3(1-x)^2$，令 $F'(x)=0$，得 $x=\dfrac{1}{2}$

当 $0<x<\dfrac{1}{2}$ 时，$F'(x)<0$，当 $\dfrac{1}{2}<x<1$ 时，$F'(x)>0$

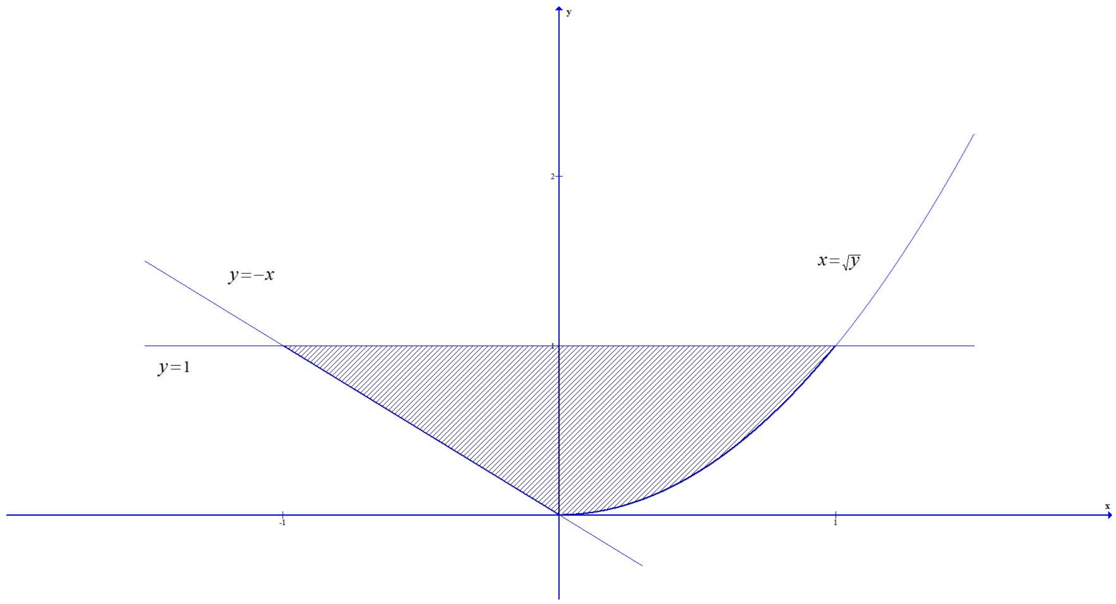
所以 $F(x)$ 在 $x=\dfrac{1}{2}$ 处取得最小值，最小值 $F\left(\dfrac{1}{2}\right)=0$

故当 $0\le x\le 1$ 时，$F(x)\ge 0$，即 $x^3+(1-x)^3\ge\dfrac{1}{4}$

24. 解：(1) $S=\dfrac{1}{2}\times1\times1+\int_0^1(1-x^2)dx=\dfrac{1}{2}-\left(x-\dfrac{1}{3}x^3\right)\Big|_0^1=\dfrac{7}{6}$

(2) $V_x=\pi\int_{-1}^{1}1^2dx-\pi\int_{-1}^{0}(-x)^2dx-\pi\int_0^1\left(x^2\right)^2dx$

$=\pi x\Big|_{-1}^{1}-\dfrac{\pi}{3}x^3\Big|_{-1}^{0}-\dfrac{\pi}{5}x^5\Big|_0^1=\dfrac{22}{15}\pi$



25. 解：$\overline{A}=\begin{pmatrix}1&1&-1&-1&1&0\\2&2&1&0&1&1\\3&3&0&-1&2&1\\1&1&2&1&0&1\end{pmatrix}\xrightarrow[r_4-r_1]{\substack{r_2-2r_1\\r_3-3r_1}}\begin{pmatrix}1&1&-1&-1&1&0\\0&0&3&2&-1&1\\0&0&3&2&-1&1\\0&0&3&2&-1&1\end{pmatrix}$

$\xrightarrow{\substack{r_3-r_2\\r_4-r_2}}\begin{pmatrix}1&1&-1&-1&1&0\\0&0&3&2&-1&1\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}\xrightarrow{\frac{1}{3}r_2}\begin{pmatrix}1&1&-1&-1&1&0\\0&0&1&\frac{2}{3}&-\frac{1}{3}&\frac{1}{3}\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}$

$\xrightarrow{r_1+r_2}\begin{pmatrix}1&1&0&-\frac{1}{3}&\frac{2}{3}&\frac{1}{3}\\0&0&1&\frac{2}{3}&-\frac{1}{3}&\frac{1}{3}\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}$

x_1,x_3为约束变量，x_2,x_4,x_5为自由变量

$\begin{cases}x_1=\frac{1}{3}-x_2+\frac{1}{3}x_4-\frac{2}{3}x_5\\x_2=\frac{1}{3}-\frac{2}{3}x_4+\frac{1}{3}x_5\end{cases}$ 令$\begin{cases}x_2=0\\x_4=0\\x_5=0\end{cases}$，得原方程组的一个特解$\eta^*=\begin{pmatrix}\frac{1}{3}\\0\\\frac{1}{3}\\0\\0\end{pmatrix}$

导出组一般解$\begin{cases} x_1 = -x_2 + \frac{1}{3}x_4 - \frac{2}{3}x_5 \\ x_2 = -\frac{2}{3}x_4 + \frac{1}{3}x_5 \end{cases}$

令$\begin{cases} x_2 = 1 \\ x_4 = 0 \\ x_5 = 0 \end{cases}$，得$\xi_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$，令$\begin{cases} x_2 = 0 \\ x_4 = 1 \\ x_5 = 0 \end{cases}$，得$\xi_2 = \begin{pmatrix} \frac{1}{3} \\ 0 \\ -\frac{2}{3} \\ 1 \\ 0 \end{pmatrix}$

令$\begin{cases} x_2 = 0 \\ x_4 = 0 \\ x_5 = 1 \end{cases}$，得$\xi_3 = \begin{pmatrix} -\frac{2}{3} \\ 0 \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix}$

原方程组通解$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ s_5 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ \frac{1}{3} \\ 0 \\ 0 \end{pmatrix} + k_1 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + k_2 \begin{pmatrix} \frac{1}{3} \\ 0 \\ -\frac{2}{3} \\ 1 \\ 0 \end{pmatrix} + k_3 \begin{pmatrix} -\frac{2}{3} \\ 0 \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix}$（其中$k_1, k_2, k_3$为任意常数）