Curtailment of Economic Activity and Labor Inequalities *

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Abstract

We develop a macroeconomic model with heterogeneous labor to study how the business closure orders enforced by several states during the pandemic affected the fragile segments of the labor force in the US. We estimate the model using data from the Current Population Survey and the Bureau of Economic Analysis. The model generates differential effects of the business closure orders and productivity on workers depending on age, skills, and origin. Middle-aged natives with low skills are the most affected, while less-educated young foreign workers experience the largest fall in total hours worked. In terms of wages, middle-aged workers experience a larger increase than young workers.

Keywords: productivity shock, labor inequalities, heterogeneous labor, business closure orders.
JEL Codes: E20, E24, J01.

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1 Introduction

The Covid-19 outbreak had a significant impact on the U.S. labor market. Total hours worked in the nonfarm business sector dropped sharply in 2020 after several states ordered citizens to stay home and imposed a shutdown of non-essential businesses. There has been a well-documented decline in total productivity during the Covid-19 lockdown. This decline in hours and productivity is perhaps the greatest in terms of its nature.[1] Many economists were claiming that the business closure orders may exacerbate the pre-existing labor inequalities. Labor market inequalities in the U.S. appear as one of the most worrying trends, particularly among the most vulnerable workers (Yasenov, 2020). A similar conclusion is reached by Adams-Prassl et al. (2020), who show that the pandemic hit harder the fragile segments of the labor force. Given the scale of the crisis, it seems that certain groups of workers such as young, low-skilled, and non-native workers would be lagging far behind other segments in how they fully recover from the crisis. Quantifying the labor market implications of business closure orders requires a framework that captures well the distributional effect of productivity shock on hours worked and wages as well as its repercussions on different types of workers.

In this paper, we study the effects of the business closure orders and productivity shock on the US labor markets, in particular, the responses of labor supply and hourly earnings during the pandemic. We do so by developing a model with heterogeneous labor, nested CES production function, and three generations. These key elements allow the model to generate a differential effect of the productivity shock across workers that differs by age, skill, and origin. The model is estimated using the Current Population Survey (CPS) and the Bureau of Economic Analysis (BEA) data. We focus on the years 2019-2021 (pre- and post-Covid-19 crisis) to estimate the elasticity of substitution between type of labor.

Our model generates realistic responses consistent with the observed facts. We find that the fall in productivity during the pandemic crisis led to heterogeneous impact on labor supply and hourly earnings. Higher responsiveness to a negative productivity

[1] The recent data from the U.S. Bureau of Labor Statistics confirm this thought. The U.S. second and third quarters' total hours (Y-o-Y) fell by 13.7 and 7%, respectively, relative to the second and third quarters of 2019, a year not affected by the Covid-19 crisis. The US economy experienced a huge decrease in the gross value added of nonfarm businesses (in the second quarter of 2020, it was 11.86% lower than it was in 2019)

shock is observed among less-educated young foreign workers who experienced the largest decline in hours worked, as well as among young foreign workers with high skills. We find that among native workers, middle-aged workers with low skills are the most disadvantaged as they experience the largest fall in total hours worked. Furthermore, we show that the response of hours worked by foreign workers to a fall in productivity is larger than the responses of hours worked by native workers. This disturbance most obviously affects hours worked and adds to the existing disparities between workers.

Considering the wage responses to the productivity shock, we show that all type of workers enjoyed a rise in wage rates in response to the negative productivity shock. Age makes a difference in terms of wage growth, with middle-aged workers appearing to be the most advantageous as they experience a wage increase above the level of young workers. According to our model simulation, the business closure shocks contribute to higher volatility in wage rates.

This paper relates, first, to the literature on the effects of Covid-19 on the labor markets, which shows that the disparity between workers in the reduced hours of work and employment is mostly due to the lower flexibility of jobs (Borjas and Cassidy, 2020; Yasenov, 2020) and to the higher spread of the virus among less "remotable" jobs (Basso et al., 2020).[2] Recent important contributions to this literature also include Cajner et al. (2020), Matias and Forsythe (2020), Cortes and Forsythe (2020), Adams-Prassl et al. (2020), and Leyva and Urrutia (2022). While the literature on the distributional effects of Covid-19 on labor markets mainly focuses on some segments of the labor force and could not address the macroeconomic implications of this shock, we instead propose an Overlapping Generations (OLG) model with heterogeneous labor to estimate and understand the impact of the productivity shock on labor markets.

Our study follows the literature adopting a structural approach to model heterogeneous labor. Our main reference is a recent paper by Busch et al. (2020), who employs an overlapping generations model to estimate the impact of refugees flow to Germany between 2015 and 2016 on different categories of workers. In particular, the authors assess the gross wage responses to the immigration inflow.[3] We instead develop a model with

[2] Fasani and Mazza (2020) use European data and find that extra-EU migrants are more likely to be exposed to a high risk of losing their job than natives, but only within industry and occupation, as many foreign workers are employed in what they identified as *essential occupations*.

[3] See also Imrohoḡlu et al. (2017) who focuses on how immigration policy based on attracting highly skilled

heterogeneous workers to assess the effects of the business closure orders on different types of workers, as we assume heterogeneity by age, origin, and skill. We also augment the model with a business closure shock to understand how the adoption of these orders in the US during the crisis contributed to the curtailment of productivity and affected substantially the dynamics in wages and hours worked. Other contemporaneous papers that explain the post-pandemic dynamics in the labor market are Faccini et al. (2022) who attribute the dynamics in the labor market to the high propensity of workers searching for a new job; Peri and Zaiour (2022) who emphasize the fall in the number of foreign workers as the main driver of US labor market dynamics; Domash and Summers (2022) who attempt to explain the labor market tightness to the increase in several vacant jobs in the US; and Tüzemen (2022) who documents a decline in the participation rate due to a large number of new retirees.[4] Our paper differs from the aforementioned studies, as our model can generate differential effects of business closure orders on workers and suggest that the labor market is substantially tight.

Finally, we contribute to the literature that analyzes the labor inequality,[5] and estimate the elasticity of substitution between the type of labor. This growing literature includes Berger et al. (2022) who focus on the differential effects of minimum wage. Other papers in this vein include Engbom and Moser (2021) and Hurst et al. (2022). To name a few of the active literature on the estimation of elasticity of substitution, see for instance, León-Ledesma et al. (2011) who employ a normalized multi-level production function; Borjas (2003) who use a simple supply-demand framework; Alvarez-Cuadrado et al. (2018) who account for sectoral differences; and Gechert et al. (2022) who revisit the existing approach in the estimation of the elasticity of substitution. We add to this literature by providing estimates of labor-labor elasticity which is crucial to understanding the distributional effects of business closure orders on the labor market. Furthermore, the productivity shock mainly hit non-essential and less remotable industries, those industries are more

can mitigate Japan's fiscal imbalances problem. Storesletten (2000) employs an overlapping generations model to examine whether a selective immigration policy based on skills and age can have strong quantitative implications for U.S. public finance. He shows that an increased inflow of middle-aged and high and medium-skilled immigrants can sustain U.S. fiscal policy. Other papers adopting a structural approach to model immigration are Chassamboulli and Palivos (2014), Moreno-Galbis and Tritah (2016), and Ortega (2000), who focus on the effects of immigration on destination countries' labor markets.

[4] See also Muehlemann and Leiser (2018) who show that labor market tightness is positively associated with hiring costs using Swiss data.

[5] See for example Card et al. (2018), who interrogate whether differences in wages are driven by firm-specific productivity differentials.

likely to have a high share of low-skilled, young, and non-native workers. The model we present in this paper takes a stance by including these key features. We thus view our work as partly related to past papers which analyze the concentration of immigrants in specific sectors. Among these Foged and Peri (2015), who show that migrants are concentrated in some low-skill sectors with high manual and low communication content, and their entry into the labor market leads low-skilled natives to shift towards non-manual (communication intensive) jobs that allow for native upgrading and an increase in natives' wages. In addition, Burstein et al. (2020) explore the labor impacts of immigration with worker heterogeneity in occupational productivity. They find that the effect varies across occupations, because of the concentration of high-skilled immigrant workers in computer-related jobs, and low-skilled immigrants are concentrated in agriculture and manufacturing sectors.

Layout. The rest of the paper is as follows. Section 2 presents the trends in productivity and labor inequalities in the US. Section 3 laid out the theoretical model. Section 4 describes the data and the estimation of model parameters. Section 5 discusses the effects of productivity shocks on hours worked and wages. Section 6 concludes.

2 Labor Inequality and Business Closure: What Does the Data Tell Us?

We start our analysis by documenting the recent trends in labor inequality. Next, we examine how the business closure orders make labor inequality worse, which we will later explain its implication on production and the labor market.

Productivity and Hours Worked. As the U.S. economy emerged from the recent pandemic, it is worth examining the phases of this unprecedented crisis. Figure 1 plots the gross domestic product and total hours worked in the US over the period Q1-1964 to Q4-2021. Though the 2020 recession is the shortest, it has driven down the total hours. Indeed total hours worked fell dramatically in the second quarter of 2020 after most states across the US shut down the non-essential business to curb the spread of the virus. During this period, the gross domestic product reached the bottom in the second quarter.

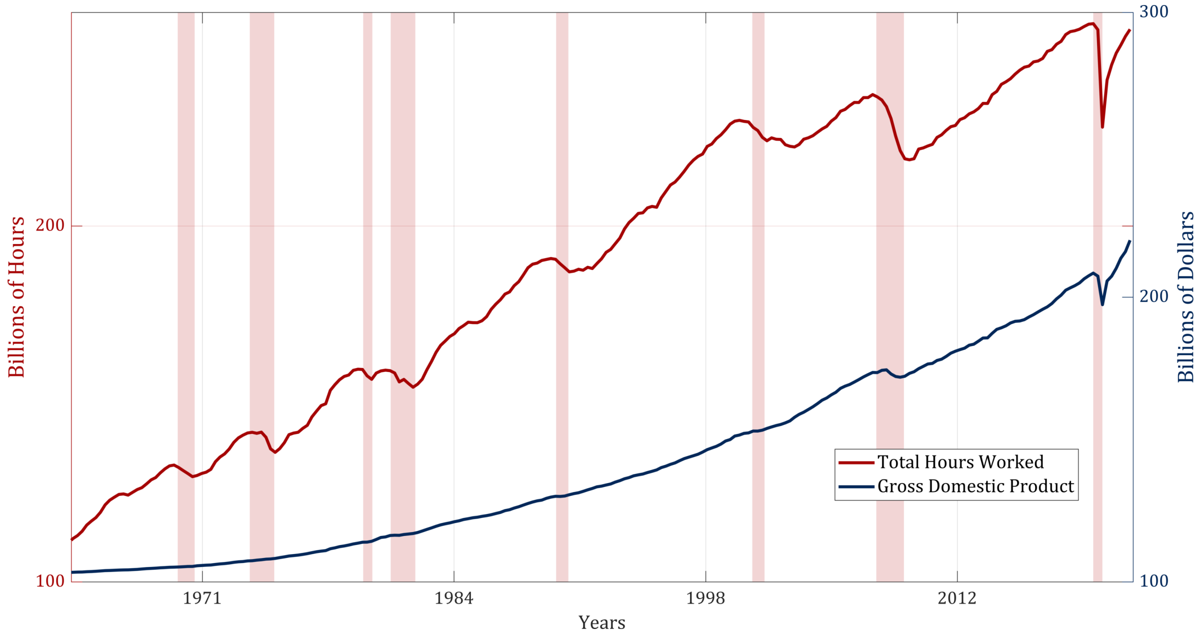


Figure 1: Hours Worked and Output Dynamics

Notes: The figure depicts the dynamics in gross domestic product and hours worked between 1964 and 2021. The shaded red areas are NBER defined recessions. Source: Federal Reserve Economic Data.

Many states have relaxed their restrictions on economic activities starting from the third quarter of the same year. Then, the total hours worked went up, and following this trend, the gross domestic product rose rapidly giving a sign of economic recovery. This crisis brings attention to the impact of the reduction in productivity during the pandemic on the labor market. As shown in Figure 2, the impact of the crisis varied by sector, and this effect is likely due to the possibility of teleworking by sector and its *emergency* nature. For example, sectors such as Construction, Merchandise Stores, and Manufacturing, which require the physical presence of a high share of workers, were hardly hit by the crisis, whereas it had a small (or even null) impact on the production of the Data Processing sector, which has a high share of workers who can work remotely.

Distribution of the Average Hourly Wage Before turning to the distribution of wages across types of workers, we should first note that this analysis is not intended to reflect the impact of the pandemic on wage inequality. Rather, it is meant to underline the degree to which hourly earnings are unequally distributed. Essentially, we present the unequal distribution of hourly earnings over the whole period of 2018-2021, and the conclusions drawn from Figures 3 may not necessarily reflect the effects of the pandemic.

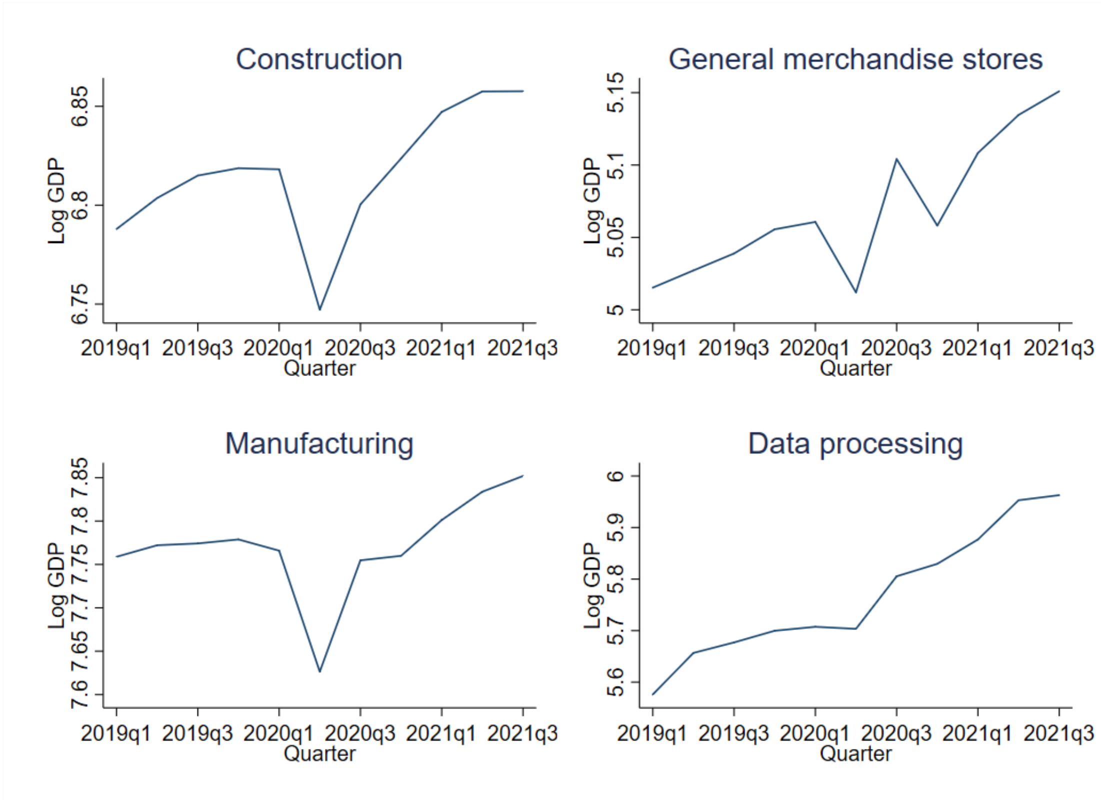
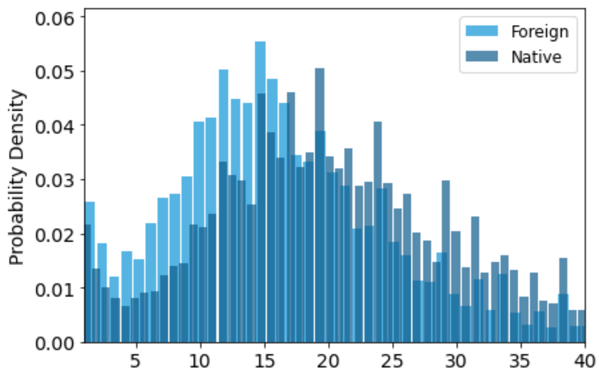


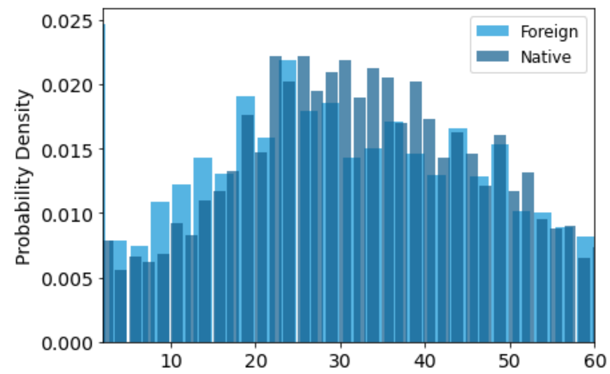
Figure 2: Log-GDP by Sector: Some Examples

Notes: The left-hand side panel describes the dynamics of log value added in the construction sector. The right-hand side panel describes the dynamics of log value added in the general merchandise stores sector. The left-hand side panel describes the dynamics of log value added in the manufacturing sector. The right-hand side panel describes the dynamics of log value added in the data processing sector. Time: 2019-2021. Source: Bureau of Economic Analysis.

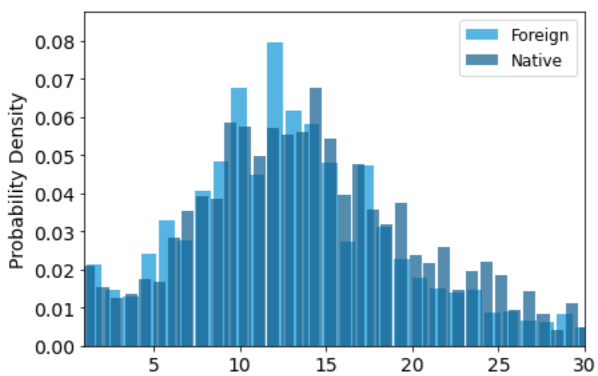
Exploring the most recent data from the CPS during the period 2018-2021 reveals the potential difference in the hourly earning distribution of different types of workers. Figures 3 show the average wage distribution of natives and foreign workers in four broad classes. Two important facts appear from these figures. Between 2018 and 2021, there is a much more unequal distribution of wages between native and foreign workers when they are less skilled and middle-aged (Figure 3, panel A). Though this divide between these two types of workers (native vs foreigner) is less evident in the other classes. We doubt that this difference, observed in Figure 3 panel A, is because foreign workers newly arriving in the labor force in the US may face difficulties integrating the labor market. They may see their job experience not recognized or evaluated differently, and then continue to earn less than their counterparts.



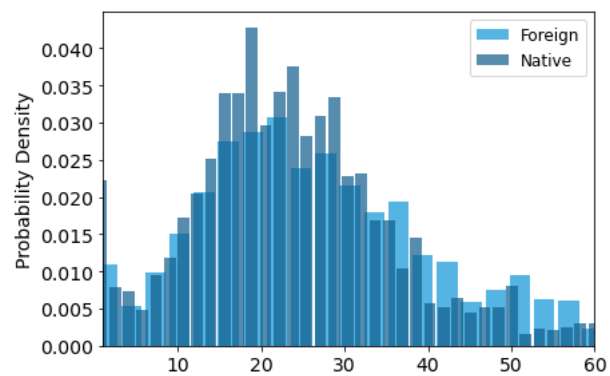
A. Middle-aged and Low Skill



B. Middle-aged and High Skill



C. Young and Low Skill



D. Young and High Skill

Figure 3: Distribution of Hourly Wage by Middle-aged and Young Native and Foreign Workers of Different Skill

Notes: The left-hand panel describes the distribution of hourly wages across native and foreign workers when they are middle-aged and low-skilled. The right-hand-side panel describes the distribution of hourly wages across native and foreign workers when they are middle-aged and high-skilled. The left-hand panel describes the distribution of hourly wages across native and foreign workers when they are young and low-skilled. The right-hand-side panel describes the distribution of hourly wages across native and foreign workers when they are young and high-skilled. Time: 2018-2021. Source: Current Population Survey

Figure 3 Panel B reports a small difference in the distribution of average wage rate between natives and foreigners when middle-aged and with high educational levels. It appears that there is a small shift of the native worker into the upper echelons of the hourly earnings distribution. Figure 3 Panel C illustrates that the average wage rate of workers with lower educational levels lags further behind workers with high educational levels. However, among young workers with lower educational levels, there is no clear difference in the hourly wage rate between native and foreign workers (Figure 3, Panel C).

Throughout the same period, Figure 3 Panel D indicates that the wage distribution of young workers with high skills is slightly skewed to the right. Reflecting the case that young workers are, on average, paid less than middle-aged workers with similar skills. Contrary to our expectations, we observe that the difference in earnings distribution between natives and foreigners when young and highly skilled is very small and the hourly earnings of foreign workers only become dominant at the top of the distribution (Figure 3, Panel D).[6]

We rely on Covid-19 related questions in the CPS to understand how the pandemic outbreak affected workers' ability to work. We focus on the responses to the question: *Were you unable to work due to Covid-19?* with the responses recorded as *yes* or *no*, over the period May 2020 to December 2021. Figure 4 reports differences in the answers to the question of interest by age, skills, and origin of the respondent. Interestingly, we find that foreign workers were much more affected by the pandemic outbreak than natives. We also show that middle-aged workers are more exposed to the pandemic compared with young workers, while the ability to work for low-skilled workers is significantly affected compared to high-skilled workers. These observations suggest that the pandemic has significantly impacted the most fragile segments of the US labor market, particularly, low-skilled, middle-aged, and foreign workers compared to other worker categories. This evidence is in line with our thesis that Covid-19 hit workers differently depending on age, skills, and origin.

Labor Inequalities: A Suggestive Evidence. If we think of labor inequalities, we might consider separately the average wage rate and hours worked by type of worker. Table 1 provides an example that illustrates substantial differences between wages and hours worked between comparable workers.

Productivity shocks may contribute to labor inequalities between workers of different origins, skills, and ages. For instance, low-skilled workers performing non-remote-able jobs can be more vulnerable to government-mandated business closure. Temporary closing of non-essential businesses can lead to worsening the pre-existing disparities between workers with similar characteristics. Yet, in Table 1, we find that the average

[6] Interested readers can consult Appendix G, which contains the distribution of hourly earnings before and after the pandemic.

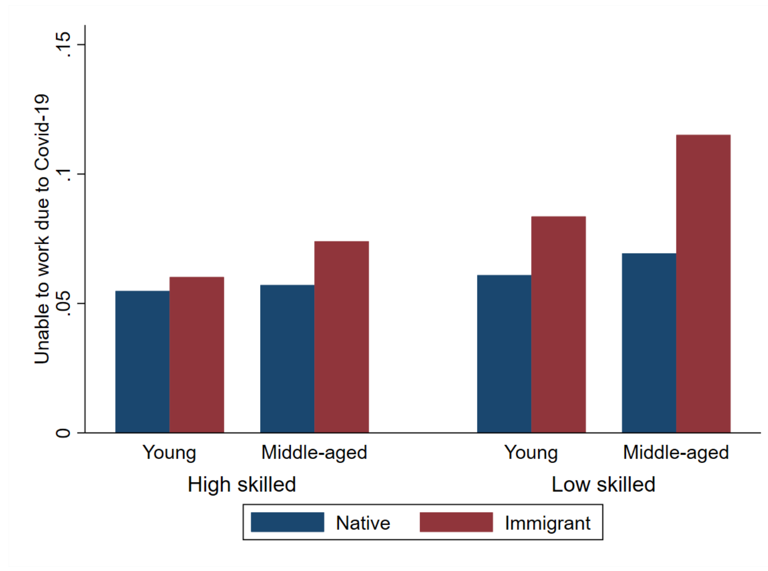


Figure 4: The effect of Covid-19 on the ability to work

Notes: The Figure shows the answers to the question "Were you unable to work due to Covid-19?"from the CPS from May 2020 to December 2021.

Table 1: Weekly Hourly Wage and Hours Worked 2018-2021

	Hourly Wage				*Hours*			
	Low skill		High skill		Low skill		High skill	
	Immigrant	Native	Immigrant	Native	Immigrant	Native	Immigrant	Native
Young workers								
2018	14.30	15.77	36.30	25.58	35.55	34.57	39.15	39.44
2019	14.27	15.89	36.32	25.65	35.55	34.54	39.15	39.41
2020	14.34	16.06	36.37	25.83	35.47	34.48	39.10	39.40
2021	14.35	16.10	37.18	25.88	35.47	34.56	39.16	39.41
Middle aged workers								
2018	17.65	21.97	37.84	38.37	38.82	40.03	40.07	41.07
2019	17.89	22.39	37.65	38.98	38.83	40.07	40.15	41.13
2020	17.82	23.70	38.04	39.13	38.61	39.45	39.98	40.70
2021	18.07	24.07	38.26	39.95	38.79	39.76	40.13	40.83

Notes: The Table describes the weekly hourly wages and hours worked across workers categories between 2018 and 2021. Source: Current Population Survey.

hours per week worked fell for all age and skill groups in 2020. A conceivable explanation for this variation may be the restrictions on economic activities during the pandemic outbreak. We also see a large difference between average hours worked per week by type of worker, these differences among workers are also intriguing in how they exacerbate labor disparities. As evidenced in the table, the decline in average hours worked could be attributed to the reduced working hours of workers who remained employed, as well as those who have lost their jobs during the pandemic.[7]

In Table 1, we report the average weekly wage rate from the CPS from 2018 to 2021. This table shows reliable evidence of an increase in hourly earnings inequality between native and immigrant workers within age and skill categories. Throughout the period, middle-aged foreign workers earn lower wages than native workers with similar skills and age. It is also true that a young foreign worker with low skills makes less than a comparable native worker. Perhaps surprisingly, the young immigrant with high skills earned more than what the native earned.

We provide an intuition of the patterns of wages and hours worked during the years 2018-2021 in Table 1. There are many plausible factors explaining the discrepancy between native and foreign workers. One explanation is the concentration of immigrants in low-paying occupations or sectors, it is also possible that there are some barriers to getting into some occupations that require some professional licensing. Not only the concentration of immigrants in specific sectors may be important in explaining low wages among these workers, but also those foreign workers may be less informed about the market wages and thus can accept lower wages.

We are not claiming that all variations in hours worked stem from business closure orders only. Numerous factors, including, the collapse in international trade, the decline in participation rate due to workers leaving the labor force, the increase in the retired share of the US population, the decrease in immigration inflow, and the expansion of the unemployment insurance claims, among others, may potentially explain labor market dynamics. Furthermore, a focus on the effects of business closure orders is crucial in providing insight into post-pandemic dynamics in the US labor market. In terms of the quantitative relevance of this channel, several studies investigated business closures during the pandemic. For instance, Bartik et al. (2020) decomposed the percent change in hours using high-frequency data, and show that the decline in hours is mainly due to businesses shutdowns and layoffs, which appear to have the same quantitative effect on total hours, whereas cuts in hours at firm level explained a minor share of the variation. In a different study Chetty et al. (2020) find that re-openings have little impact on

[7] It is worth to mention that the CPS has surveyed both employed and unemployed individuals.

employment. While (Kong and Prinz, 2020) indicates that non-essential business closures explain only 6.4 percent of unemployment insurance claims, using high-frequency Google search data.

Based on the empirical facts on hourly earnings and hours worked, we propose a model that ideally accounts not only for labor heterogeneity but also considers the feedback from business closure to labor markets through the productivity channel. Our macroeconomic model allows us to generate the differential impact of productivity shocks and business closures across workers with different education levels, ages, and origins.

3 An OLG Model with Heterogeneous Labor

We start this section by presenting an overlapping generations model. In this model, we adopt a nested constant elasticity of substitutions (CES) production function, which allows us to consider the differential effect of a shock on productivity across workers. We allow for imperfect substitutability (i.e., the elasticity of substitution different from one) among workers of a given age (young vs. middle-aged) to understand how changes in productivity increase inequalities in the labor market, and later between workers of a given age (young vs. middle-aged), origin (native vs. foreign), and skills (high-skill vs. low-skill).

One can question why we need a general equilibrium model and why not simply try to causally identify the effect of business closures on natives vs. non-natives using standard reduced-form methods. One reason for conducting this study is to capture the distributional general equilibrium effects of productivity shocks and business closure orders on US labor markets. The impact of the pandemic on workers was unequally distributed, and many vulnerable segments of the labor force were more exposed to the business closure order than others. We attempt to propose a model that can generate distributional effects consistent with this reality. The use of the three-period overlapping generations model with heterogeneous labor in which household decisions are age-dependent is convenient for our analysis since the age category of the worker is an important feature of heterogeneity in the model.

Households We start with a simple overlapping generations model and we consider three generations, each is alive at any point in time and maximizes the life time utility

$$U(c^j_{s,k,t}, l^j_{s,k,t})$$

Let the instantaneous utility function $U(c)$ be a logarithmic function and $U(l)$ be a constant relative risk aversion $(l^j_t)^{1+\eta}/1+\eta$, where η is the curvature on the disutility of labor. We assume that the parameter η is the same for all types of workers. We have three generations at each time t where $j \in \{y, m, o\}$ identifies age groups: young, middle, and old age. We let s denote the skill of the agent with $s \in \{h, l\}$, and k represent the origin of the agent with $k \in \{f, n\}$. Note that each agent faces a specific budget constraint but the marginal utilities are the same across the three age groups. The budget constraint of each agent is therefore given by:

$$\begin{aligned} c^y_{s,k,t-1} + a^y_{s,k,t-1} &= \omega^y_{s,k,t-1} l^y_{s,k,t-1} \\ c^m_{s,k,t} + a^m_{s,k,t} &= \omega^m_{s,k,t} l^m_{s,k,t} + a^y_{s,k,t-1}(1 + r_{t-1}) \\ c^o_{s,k,t+1} &= a^m_{s,k,t}(1 + r_t) \end{aligned}$$

A household has units of time to spend on labor, let $l^y_{s,k}$ and $l^m_{s,k}$ represent the labor supply by young and middle-aged individuals, respectively. Moreover, old individuals (retirees) do not participate in the labor market. Let $\omega^j_{s,k}$ denote the average hourly earnings of a worker. $c^y_{s,k}$, $c^m_{s,k}$, $c^o_{s,k}$ denote the consumption of an agent belonging to the young, middle-aged and old generations. We further assume that both young and middle-aged agents save ($a^y_{s,k}$, $a^m_{s,k}$), but only the middle-aged and old-age agents receive the return at risk-free interest rate r.[8] The young and middle-aged agents consume and work in each period, in the first and second period of life, while old agents earn no labor income in the third period of life but receive retirement income $a^m_{s,k,t}(1 + r)$ and consume. Finally, in this economy, we assume that old agents are non-altruistic and consume all they have.

Labor-Labor Substitution. Now, we shall present the labor force equation by assuming that the elasticity of substitution among workers for given age, origin, and skill are

[8] Note that the real interest rate is given by the demand and supply of assets a. If assets are perfectly elastically supplied then a no-arbitrage condition should determine interest rate r.

different and show that nesting the labor subgroups does change the results we have. In this economy, we distinguish labor by age (young and middle-aged), national origin (native and foreign), and sectors (high and low-skilled sectors). Let l_t^y denote the total hours worked by young individuals and consider the separability of the labor factor. The hours worked by younger individuals are given by:

$$l_t^y = [(\phi_{h,f}(l_{h,f,t}^y)^{\frac{1}{\varrho}} + \phi_{h,n}(l_{h,n,t}^y)^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,f}(l_{l,f,t}^y)^{\frac{1}{v}} + \phi_{l,n}(l_{l,n,t}^y)^{\frac{1}{v}})^{\frac{v}{\sigma}}]^{\sigma},$$

this equation emphasizes that hours worked by younger individuals young is broken down into various category: origin (native or foreign) and skill (high or low) groups. We assume that labor inputs l_t^y is a vector of four types of labor: hours worked by foreign workers in high-skilled sector $l_{h,f,t}^y$; hours worked by native worker in high skilled sector $l_{h,n,t}^y$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^y$; and hours worked by native worker in low skilled sector $l_{l,n,t}^y$. Let ϱ (v) denote the parameter of substitution between native and foreign workers of young age and working in the high (low) skill sector. The parameter σ denotes the parameter of substitution between high and low-skilled workers when young.
Similarly, the hours worked by middle-aged individuals are given by:

$$l_t^m = [(\theta_{h,f}(l_{h,f,t}^m)^{\frac{1}{\varepsilon}} + \theta_{h,n}(l_{h,n,t}^m)^{\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\xi}} + (\theta_{l,f}(l_{l,f,t}^m)^{\frac{1}{\zeta}} + \theta_{l,n}(l_{l,n,t}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}}]^{\xi},$$

the term l_t^m represents the total hours worked by young (middle-aged) individuals and encompasses four labor groups. The hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled. The labor inputs l_t^m is then a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^m$; hours worked by native worker in high skilled sector $l_{h,n,t}^m$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^m$; and hours worked by native worker in low skilled sector $l_{l,n,t}^m$. We also distinguish between the elasticity of substitution between middle-aged natives and foreign workers, with ε (ζ) that represents the parameter of substitution between middle-aged workers with high (low) skill. We also assume that the parameter of substitution between middle-aged high and low-skill workers is given by ξ.

Labor is disaggregated according to several criteria. That is, the relative supply elasticities between native and foreign workers are not identical given the age and the sector. A notable study on the substitution between workers is Ottaviano and Peri (2008), which adopted a nested constant elasticity of substitution production function. The key assumption in their nested CES framework is that for labor inputs, workers of different education levels are split into specific education subgroups, and those groups are then nested into groups with different experience levels. Within the same education and experience group, they identify US-born and foreign-born workers. Our framework instead considers age categories (young vs middle-aged workers) and assumes a split between high-skilled native workers, high-skilled foreign workers, low-skilled native workers, and low-skilled foreign workers. Moreover, we do not restrict the elasticity of substitution to be the same between high and low-skilled workers of different ages (young and middle-aged). We also do not restrict the elasticity of substitution to be the same between foreign and native workers with different ages (young and middle-aged) and skills (high skill and low skill). Nonetheless, the framework provides new estimates of labor-labor elasticities, adding features such as age, skill, and origin of workers.

Technology Firms choose labor to maximize profits subject to the production function. The production function that relates outputs to inputs takes the constant elasticity of substitution (CES) form:

$$y_t = z_t(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \tag{3.1}$$

where α is a distribution parameter that determines how important the two labor factors are in aggregate production, z_t is the productivity shock, and s denotes the elasticity of substitution between hours worked by young (l_t^y) and middle-aged workers (l_t^m). For each age group, we distinguish between natives and immigrants, and high and low skilled workers. In this case, we consider sectors that employ high numbers of high-skilled workers and sectors with a high share of low-skilled workers. When $s > 1$, the two types of workers are gross substitutes, and when $s < 1$, the two types of workers are complements. Following Gourio (2013), we assume

$$\ln(z_t) = \rho^z \ln(z_{t-1}) + \delta d_t + \epsilon_t^z; \tag{3.2}$$

where ρ^z is the productivity smoothing parameter, δ is the productivity response to the business closure orders. d_t is our measure of business closure order variable, and ϵ_t^z is the productivity shock. We assume that d_t and ϵ_t^z are i.i.d. N(0,1). Following (Koren and Pető, 2020) we construct a measure that reflects the share of workers affected by the business closure restrictions. We can interpret this measure as the occurrence of the business closure orders and how it affects economic activity. For example, prior and after to the pandemic crisis, this share will be equal to zero, but when the Covid shock hits, the share will have a value between 0 and 1 and will vary across industries, given that some industries may have high or low affected shares.

Agents do not anticipate a negative productivity shock ϵ_t or a possibility of business closure order occurring d_t. Agents do observe changes in productivity z_t which is driven by these two innovations. Note that the business closure order shock is not persistent, but agents can anticipate a decline or an increase of z_t in the next period.

Asset Market The total amount of asset in the economy evolves as follow $a_t = a_{t-1} + \tau_t$, where τ_t is the new allocated saving in this economy. The saving flow equation is given by $a_{t-1}(1 + r_{t-1}) = a_t$, $a_t \geq 0$, that simplifies to $r_{t-1}a_{t-1} = \tau_t$. This condition states that the return on saving equals the newly allocated saving. The market clearing condition is given by

$$a_t = y_t - c^y_{s,k,t} - c^m_{s,k,t} - c^o_{s,k,t}$$

where $a_t = a^y_{s,k,t} + a^m_{s,k,t}$, this equation reveals that total savings is this economy is equal to the difference between total output and the consumption of the three generations at time t.

The Definition of Equilibrium. We define an equilibrium as a collection of quantities, and prices such that, (i) Households choose $\{c^y_t, c^m_t, c^o_t, l^y_t, l^m_t, a_t\}$ in order to maximize their utility subject to the budget constraints; (ii) Producers choose how much of each input to employ $\{l^y_{h,f,t}, l^m_{h,f,t}, l^y_{l,f,t}, l^m_{l,f,t}, l^y_{h,n,t}, l^m_{h,n,t}, l^y_{l,n,t}, l^m_{l,n,t}\}$ to minimize their production cost. The first order conditions yields the market prices at the equilibrium $\{\omega^y_{h,f,t}, \omega^m_{h,f,t}, \omega^y_{l,f,t}, \omega^m_{l,f,t}, \omega^y_{h,n,t}, \omega^m_{h,n,t}, \omega^y_{l,n,t}, \omega^m_{l,n,t}\}$. (iii) Equilibrium requires that $a_t = y_t - c_t$, holds and by definition we have saving accumulation condition $a_t = a_{t-1} + \tau_t$, and the market for assets clears $a_t = a_{t-1}(1 + r_{t-1})$.[9]

4 Data and Estimation

In this section, we set the value of the main parameters of the model such that it matches the key facts of the U.S. labor market. We measure the remaining parameters directly from U.S. data.

4.1 Data, Summary Statistics and Construction of Variables.

Data. Our main data source of workers' data is the Current Population Survey conducted by the Census Bureau (Flood et al., 2021). This data seeks to collect information on each individual of households over 15 years of age. This dataset has several advantages and collects information about the U.S. labor force characteristics every month, such as the average hours worked per week, education level, age (ages 14-24, ages 25-64), origin, industry, and county.

Additionally, we use quarterly data on Gross Domestic Product (Value Added) by industry at the county level as released by the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce, and focus on the period 2019-2021. The total factor productivity is calculated using the value-added and labor input at the industry level.

In the US, several states have enforced business closure orders starting from the second quarter of 2020 to curb the spread of the virus, these restrictions have been largely lifted in the third quarter of 2020. This is why we use a sample that covers 2019-2021, as it covers the period before and after the public health restrictions on businesses.

Summary Statistics. The summary statistics in Table 5 shows the weekly average hours worked which is equal to 37 with a standard deviation of 11 hours. There is a substantial variation in hours worked and wages among individuals surveyed. Our initial dataset contains 4,274,781 individuals. We drop observations with missing or zero usual hours worked and missing wages and, then, winsorize wages at 1 and 99 percentiles. This leaves us with 1,910,679 (monthly) observations. 16 percent of individuals are immigrants,

[9] Detailed computations are contained in the appendix A and B.

Table 2: Summary statistics

Variable	Mean	Std. Dev.	N
Immigrant	0.158	0.365	1,910,679
Young (15-30)	0.232	0.422	1,910,679
Middle-aged (30-64)	0.692	0.462	1,910,679
Old (> 64)	0.076	0.265	1,910,679
High skilled (Degree or higher)	0.395	0.489	1,910,679
Low skilled (Less than degree)	0.605	0.489	1,910,679
Hours worked last week	38.653	13.034	1,910,679
Wage and salary income (yearly data)	53322.105	52007.357	222,240

23% are young, and slightly less than 40% are high-skilled (i.e., at least with a bachelor's degree). The yearly wage exhibits substantial volatility, with the mean equal to \$53,322 and the standard deviation being \$52,007.

Data for the estimation of the elasticities Using the CPS data, we computed weighted averages by skill level (we consider those who earned a degree as high-skilled and the others as low-skilled), age category (young: 15-30, middle-aged: 30-64, old: >64), industry, origin (foreigner or native), and county quarterly between 2019 and 2021. This collapse leaves us with a sample of 449,556 observations. To proceed with the estimation of elasticities, we reshape the data from long to wide and use the year-month-industry code-state-county as the cross-section identifiers. We will end up with data that is linked using these identifiers. There are also empty cells, for instance, it may happen that a specific industry does not exist in a specific county. To overcome this issue, we replace missing values with the mean of each column. Our sample thus reshaped now contains 98,709 observations.

Data for estimation of the productivity shock. To estimate the productivity shock, we start by computing the total hours worked. We need to collapse the CPS data by quarter and industry. Starting from the same initial dataset of 1,910,679 of monthly observations at the individual level, we collapse the CPS dataset by quarter (from Q1 2019 to Q3 2021) and industry. For what concerns the Covid-19 shock, we follow Koren and Pető (2020), which used the Occupational Information Network dataset. This dataset includes the share of workers affected by Covid-19 restrictions in each industry at 3-digit NAICS. Some industries may have been more severely affected than others affected. One can see that the affected share is a measure of the percentage of workers hit by the restrictions, which in this case is an appropriate proxy of business closure orders.

Using the Bureau of Economic Analysis (BEA) data, we can observe the total output at the quarter and sector level (with 671 observations). A sector is identified by the 3-digit NAICS code[10].For the collapsed CPS data at the quarter level from Q1 2019 to Q3 2021 (with 3052 observations), we can observe the hours worked in each quarter and within each sector or sub-sector. We assign the specific industry (3-digit NAICS) code for each sub-sector to standardize it to the format of the BEA dataset. We drop data that contain 0 in the quarter column, this leaves us with 2782 observations. We then, aggregate the data at the sector level (3-digit NAICS code), to make the CPS data comparable to the BEA data, such that observation is an industry-quarter. After this, we end up with 693 observations. Additionally, we assign the affected share in each quarter and within each sector to the appropriate quarter-industry level.

We finally merge data from the BEA with the CPS data to obtain quarterly information on output, hours, and affected share at the industry level (3-digit NAICS code) between Q1-2019 to Q3-2021. As a result, our sample contains 671 observations that we will use for the estimation of the productivity shock parameters.

Construction of Variables. As the CPS includes questions on wages only in the ASEC module (the March module) and not monthly, we construct a measure of the hourly wage rate (measured in \$) as follows:

$$\text{Hourly Wage Rate} = \frac{\text{Wage Income Annual}}{\text{Weeks} \times \text{Hours Worked per Week}}$$

where *Hours Worked per Week* is computed from the monthly CPS.
To model the productivity shock due to the Covid-19 pandemic, we followed (Koren and Pető, 2020) which used the Occupational Information Network (ONET) dataset to measure the share of workers affected by the restrictions by industry. To compute this share, the authors build three indices (Teamwork, Customer, and Presence) that account for face-to-face interactions along these dimensions in more than 1,000 occupations included in the ONET dataset. Then, they calculate the share of affected workers based

[10]For a complete description of the industry code see Appendix C.

on the share of workers in each occupation by industry (NAICS).

Note that for our exercise the aggregate productivity shock z_t will depend on our measure of business closure order variable d_t, which is defined as follows:

$$d_{i,t} = \text{Affected Share}_{i,t}$$

the variable $d_{i,t}$ is proxied by the affected share in industry i at time t from the ONET dataset. Additionally, we approximate the value of total factor productivity z_t using the production function equation as follows:

$$\ln z_{i,t} = \ln y_{i,t} - \ln l_{i,t}$$
$$\ln \text{Total Factor productivity}_{i,t} = \ln \text{Value Added}_{i,t} - \ln \text{Hours Worked}_{i,t}$$

here again, we use the value added at the industry level from the BEA data and the total hours worked from the CPS data which is aggregated at the industry level.

4.2 Estimation

Preset parameters. We first set those parameters that are commonly used in the literature. We set the value of the discount factor $\beta = 0.97$ to target an interest rate of 3 percent. We set the value of the curvature on the disutility of labor $\eta = 1.5$ based on a standard value in the literature, see for example (Chetty et al., 2011).

Estimated parameters for labor-labor Substitution. Our sample contains weighted averages of hours worked and wage rate by skill level, age category, industry, origin, county, and quarter-year. We reshaped the data in a way that we can estimate the elasticity of substitution between different types of labor. We first calculate the sum of hours supplied by young and middle-aged workers

$$l^y = l^y_{h,n} + l^y_{l,n} + l^y_{h,f} + l^y_{l,f} \qquad l^m = l^m_{h,n} + l^m_{l,n} + l^m_{h,f} + l^m_{l,f}$$

The youth labor share parameter is given by $\alpha = l^y/(l^y + l^m)$, we use this equation to pin down α, implying a value of 0.48 (see Panel B in Table 4).

In the second step, we estimate the distribution parameters measuring the specific labor intensity of given labor to total labor supplied by younger workers with

$$\theta_{h,n} = l^y_{h,n,t}/l^y_t = 0.27, \qquad \theta_{h,f} = l^y_{h,f,t}/l^y_t = 0.26,$$
$$\theta_{l,n} = l^y_{l,n,t}/l^y_t = 0.23, \qquad \theta_{l,f} = l^y_{l,f,t}/l^y_t = 0.24.$$

The intensity of a type of labor can be simply measured by taking the ratio of a given type of labor to total labor supplied by middle-aged workers

$$\phi_{h,n} = l^m_{h,n,t}/l^m_t = 0.26, \qquad \phi_{h,f} = l^m_{h,f,t}/l^m_t = 0.25,$$
$$\phi_{l,n} = l^m_{l,n,t}/l^m_t = 0.25, \qquad \phi_{l,f} = l^m_{l,f,t}/l^m_t = 0.24.$$

Note that Panel C in Table 4 presents the values assigned to each parameter.
If we assume constant labor intensity parameters $\{\theta, \phi\}$ over time, one can easily estimate the elasticity parameters. The estimation of the elasticity of substitution $\{\varrho, v, \varepsilon, \zeta\}$ between the type of labor can be identified using the Euler equations from the producer optimization. Our estimates of elasticity use the CPS data between 2018 and 2021. The optimality conditions that we will use to estimate the elasticity of substitution parameters are shown in Appendix A. More formally, we take the logarithm of the Euler equations: $\ln(\omega^y_{h,f,t}) - \ln(\omega^y_{h,n,t}) = (1/\varrho - 1)\left(\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})\right)$. We introduce county α_i and year α_t fixed effects to absorb time trends and county characteristics that may affect the wage differentials. We can find the elasticity of substitution using the following specification

$$\ln \omega^y_{i,h,f,t} - \ln \omega^y_{i,h,n,t} = \alpha + 0.14092(\ln l^y_{i,h,f,t} - \ln l^y_{i,h,n,t}) + \alpha_t + \alpha_i + \epsilon_{i,t}.$$

The estimation of the elasticity of substitution between foreign and native workers with equal skill and age shows that young-high skilled foreign and native workers are substitutes (when the elasticity is positive, $1/\varrho - 1 = 0.14092$), this is different from the estimates of Ottaviano and Peri (2008) (see Table 3). The estimate suggests some degree of substitutability between foreign and native workers. An increase in the supply of labor by foreign workers relative to natives, specifically when they are both young and highly skilled, leads to an increase in wages of foreign workers relative to natives with the same characteristics.

In a simple form, we can also estimate the elasticity of substitution between young-low skilled foreign and native workers using $\ln(\omega^{y}_{l,f,t}) - \ln(\omega^{y}_{l,n,t}) = (1/v - 1)\,(\ln(l^{y}_{l,f,t}) - \ln(l^{y}_{l,n,t}))$, We augment the equation with a constant α and error term $\epsilon_{i,t}$. Our empirical specification includes the time fixed effects as well as the county fixed effects.

$$\ln \omega^{y}_{i,l,f,t} - \ln \omega^{y}_{i,l,n,t} = \alpha + 0.11723(\ln l^{y}_{i,l,f,t} - \ln l^{y}_{i,l,n,t}) + \alpha_t + \alpha_i + \epsilon_{i,t}.,$$

We also find a positive elasticity ($1/v - 1 = 0.11723$) between young-low skilled foreign and native workers, this reflects some degree of substitution between foreign and native workers. An increase by 1 percent of labor supply by foreign workers relative to natives leads to an increase in wages of foreign workers relative to native workers by 0.117 percent.

In addition, the elasticity between middle-aged foreign and native workers with high skills is captured by $\ln(\omega^{m}_{h,f,t}) - \ln(\omega^{m}_{h,n,t}) = (1/\epsilon - 1)\,(\ln(l^{m}_{h,f,t}) - \ln(l^{m}_{h,n,t}))$. We augment this specification with a constant term, time fixed effects, county fixed effects, and an error term, we rewrite as follows:

$$\ln(\omega^{m}_{i,h,f,t}) - \ln(\omega^{m}_{i,h,n,t} = \alpha + 0.05898(\ln l^{m}_{i,h,f,t} - \ln l^{m}_{i,h,n,t}) + \alpha_t + \alpha_i + \epsilon_{i,t}.$$

Our estimate favors a some degree of substitution between between the two categories. A positive elasticity $1/\varepsilon - 1 = 0.05898$ implies that an increase by 1 percent of labor supply by foreign workers relatives to native workers leads to a increase by 0.058 percent.

We also use the Euler equation to estimate the elasticity of substitution between middle-aged foreign and native workers with low skill $\ln(\omega^{m}_{l,f,t}) - \ln(\omega^{m}_{l,n,t}) = (1/\zeta - 1)\,(\ln(l^{m}_{l,f,t}) - \ln(l^{m}_{l,n,t}))$. We augment the specification with a constant term, time fixed effects, county fixed effects, and an error term. Hence we rewrite the specification as follows:

$$\ln(\omega^{m}_{i,l,f,t}) - \ln(\omega^{m}_{i,l,n,t}) = \alpha + 0.11216(\ln l^{m}_{i,l,f,t} - \ln l^{m}_{i,l,n,t}) + \alpha_t + \alpha_i + \epsilon_{i,t}.$$

We also find evidence of substitutability between middle-aged foreign and native workers with low skills, with positive elasticity of substitution $1/\zeta - 1 = 0.11216$. An increase by 1 percent of labor supply by foreign workers relative to native workers, especially when they are both low-skilled and middle-aged workers, leads to an increase in wages of foreign workers relative to native workers with the same characteristics by 0.112 percent. We report all the estimates of the elasticity of substitutions in Table 3, while detailed estimation results under various specification are provided in Appendix D.

Before moving to the estimation of the elasticity between young and middle-aged workers, it is important to explain how our estimates relate to the emerging literature trying to estimate the labor-supply elasticity. In particular, the question of imperfect substitution between native and immigrant workers has been raised early on in Ottaviano and Peri (2008) and D'Amuri et al. (2009), which find imperfect substitutability between natives and immigrants. Though our estimates differ substantially, because we have a different specification of the estimated Euler equations.

An additional parameter that turns out to be important for our empirical analysis is the elasticity of substitution between middle-aged and young workers s. Note that the producer optimization implies an Euler equation which is given by $\omega_t^m/\omega_t^y = (1 - \alpha/\alpha)(l_t^m/l_t^y)^{-1/s}$. From the first order condition for producer's cost minimization, one can write it in logarithmic form and augment the specification with a constant term, county fixed effects, time fixed effects, and an error term as follows:

$$\ln(\omega_{i,t}^m) - \ln(\omega_{i,t}^y) = \alpha + 0.07658(\ln l_{i,t}^m - \ln l_{i,t}^y) + \alpha_t + \alpha_i + \epsilon_{i,t}.$$

Table 3 presents the estimates of the elasticity of substitution $1/s = 0.07658$, we include both county and year fixed effects in the regression. Here, we note that the estimated $\frac{1}{s}$ is positive and statistically significant. An increase by 1 percent of labor supply by middle-aged workers relative to young workers leads to an increase in wages of middle-aged workers relative to young workers by 0.076 percent.

When we turn to the elasticity of substitution between high and low-skilled workers when middle-aged, our model yields a key Euler equation to estimate $(1/\xi - 1)$ using the expression: $\ln(\omega_{h,t}^m) - \ln(\omega_{l,t}^m) = (1/\xi - 1)\,(\ln(l_{h,t}^m) - \ln(l_{l,t}^m))$. We also augment the specification with a constant term, a country fixed effects, a time fixed effects, and an error term. Hence, the model to be estimated rewrites as follows:

$$\ln(\omega_{i,h,t}^m) - \ln(\omega_{i,l,t}^m) = \alpha + 0.07146(\ln l_{i,h,t}^m - \ln l_{i,l,t}^m) + \alpha_t + \alpha_i + \epsilon_{i,t}.$$

We find that the elasticity of substitution between high and low-skill workers is $1/\xi - 1 =$ 0.0.07146 among middle-aged workers (see Table 3), this gsuggests that high and low-skill middle-aged workers are substitutes. An increase by 1 percent of labor supply by high-skilled workers relative to low-skilled workers when they are both middle-aged workers leads to an increase in wages of high-skilled workers relative to low-skilled workers by 0.071 percent.

Finally, we run the regression for the Euler equation that captures the elasticity of substitution between high-skilled and low-skilled young workers $\ln(\omega^y_{h,t}) - \ln(\omega^y_{l,t}) = (1/\sigma - 1)\,(\ln(l^y_{h,t}) - \ln(l^y_{l,t}))$. The specification account for a constant term, a time fixed effects, and county fixed effects, and an error term, which rewrites as follows:

$$\ln(\omega^y_{i,h,t}) - \ln(\omega^y_{i,l,t}) = \alpha + 0.03924(\ln l^y_{i,h,t} - \ln l^y_{i,l,t}) + \alpha_t + \alpha_i + \epsilon_{i,t}.$$

Table 3 also presents the estimates of $1/\sigma - 1$. This specification includes county and year fixed effects to remove time trends and specific county factors that can explain wage differentials. Ignoring the origin of the worker, once again the elasticity of substitution between high and low-skilled workers of young workers $1/\sigma - 1$ is positive and equals 0.03924. An increase by 1 percent of labor supply by high skilled workers relative to low-skilled workers when they are both young leads to an increase in wages of high skilled workers relative to low-skilled workers by 0.039 percent. These results imply that the two groups are perfect substitutes. These estimates are substantially different from the estimates of McAdam et al. (2011), who find high elasticity of substitution between skilled and unskilled labor.

One could reasonably question the model identification for the estimation of the labor-labor elasticities. Our model assumes an upward-sloping labor supply curve and downward-sloping labor demand curve. The optimality conditions for firms in our model determine the optimum demand for labor and thus determine wages. These conditions are both necessary and convenient for our study since the main focus is to account for key ingredients that identify the equilibrium relationships between wages and hours worked, and to inform us of the degree of substitutability between workers. Nevertheless, we should mention that estimating the elasticities of substitution is difficult, and it is challenging to come with a clean identification for these elasticities. Our specification

Table 3: Elasticity of substitution parameters

Elasticity of substitution						
$1/\varrho-1$	$1/\upsilon-1$	$1/\varepsilon-1$	$1/\zeta-1$	$1/s$	$1/\sigma-1$	$1/\xi-1$
0.14092*** (0.00558)	0.11723*** (0.00604)	0.05898*** (0.00593)	0.11216*** (0.00680)	0.07658*** (0.00623)	0.03924*** (0.00512)	0.07146*** (0.00638)
Elasticity of substitution parameters						
ϱ	υ	ε	ζ	s	σ	ξ
0.88	0.89	0.94	0.89	13.06	0.96	0.93

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, the number of observation is equal to 98709. Homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

could potentially suffer from major omitted variable biases, that we cannot completely rule out. Therefore, we include various checks of our estimates with time and country fixed effects. We also run additional regressions to check the robustness of our estimates, and find that the estimates are not sensitive to the exclusion of pandemic sample period and are largely robust to specifications that allows for county fixed effects and year fixed effects (see Appendix D and E for more details).

Empirical specification of the productivity shock. Turning to the productivity shock, we now estimate the total factor productivity using the production function

$$y_{i,t} = F(z_{i,t}, l_{i,t})$$

where $y_{i,t}$ the amount of gross domestic product in industry i at time t depends on productivity shock $z_{i,t}$ in industry i at time t and labor supply $l_{i,t}$ in industry i at time t.[11] Taking the logs we can write $\ln(z_{i,t}) = \ln(y_{i,t}) - \ln(l_{i,t})$ which can be estimated using data on gross domestic product and labor supply at industry level.

Due to the limitation in the data, we abstract from dis-aggregating labor by type for the estimation of the total factor productivity z. We use quarterly data on gross domestic product and labor supply (hours worked) at the industry level to compute z. Then, we

[11] One could also consider industries with different production functions and assume that each industry has a specific return to scale parameter. This is an interesting approach to model industry heterogeneity that we can explore, but we believe it is beyond the scope of this paper.

Table 4: Summary of Parameters

Parameter	Description	Value	Source
Panel A: Household			
β	Discount factor	0.95	Standard
η	The curvature on the disutility of labor	1.5	Chetty et al. (2011)
Panel B: Producer			
α	Youth labor share parameter	0.48	CPS
s	Elasticity of substitution between labor factor l^y and l^m	13.06	CPS
ρ^z	Autocorrelation parameter of technology shock	0.9832	BEA
σ^z	Standard deviation of technology shock	0.0835	BEA
δ	The responses of technology shock to the activity specific affected share	-0.0017	BEA/ ONET
Panel C: Labor - Distribution parameters			
$\phi_{h,f}$	high-skilled middle-aged foreign labor share parameter	0.25	CPS
$\theta_{h,f}$	high-skilled young foreign labor share parameter	0.26	CPS
$\phi_{h,n}$	high-skilled middle-aged native labor share parameter	0.26	CPS
$\theta_{h,n}$	high-skilled young native labor share parameter	0.27	CPS
$\phi_{l,f}$	low-skilled middle-aged foreign labor share parameter	0.24	CPS
$\theta_{l,f}$	low-skilled young foreign labor share parameter	0.24	CPS
$\phi_{l,n}$	low-skilled middle-aged native labor share parameter	0.25	CPS
$\theta_{l,n}$	low-skilled young native labor share parameter	0.23	CPS
Panel D: Labor - Elasticity of substitution parameters (high vs low-skilled)			
σ	Elasticity of substitution between labor factor l_h^y and l_l^y	0.96	CPS
ξ	Elasticity of substitution between labor factor l_h^m and l_l^m	0.93	CPS
Panel E: Labor - Elasticity of substitution parameters (natives vs foreigners)			
ϱ	Elasticity of substitution between labor factor $l_{h,f}^y$ and $l_{h,n}^y$	0.88	CPS
υ	Elasticity of substitution between labor factor $l_{l,f}^y$ and $l_{l,n}^y$	0.89	CPS
ε	Elasticity of substitution between labor factor $l_{h,f}^m$ and $l_{h,n}^m$	0.94	CPS
ζ	Elasticity of substitution between labor factor $l_{l,f}^m$ and $l_{l,n}^m$	0.89	CPS

estimate a regression of the following form

$$\ln(z_{i,t}) = \rho^z \ln(z_{i,t-1}) + \delta d_{i,t} + \epsilon_{i,t}^z;$$

where ρ^z captures the persistence of productivity shocks and $\epsilon_{i,t}^z$ is an i.i.d. normal random variable with mean zero and variance σ^z. We aim to capture the idea that business closures can affect the total factor productivity and thus affect aggregate supply. To do so, we add a variable $d_{i,t}$ to this specification, which reflect the affected share in industry i at time t. The term d measures the share of economic activities that have been disrupted by COVID-19. Lastly, we feed the estimates for $\rho^z = 0.98$, $\delta = -0.00173$ and $\sigma^z = 0.08$ estimated from the BEA and CPS data into our model. For more details, refer to Appendix F.[12]

5 Results

We divide our analysis of the impact of the productivity shock on the labor markets into three parts. First, we study the responses of total hours worked to a decline in productivity and business closure orders. Second, we investigate the implications of a negative productivity shock on the wage rate. Finally, we analyze how a business closure order affects the volatility of wages.

5.1 Labor Response to Productivity and Business Closure Shocks

In Figure 5, we document that aggregate hours worked declined considerably during the pandemic for all types of workers. The primary economic explanation of the responses of hours to a drop in productivity shock is that strict restrictions on businesses have suspended or entirely ceased business activity and which causes a significant decrease in labor market activity. As panel A of Figure 5 illustrates, a business closure order that affects sectors disproportionately generates a fall in total hours worked of -1.4 percent for young foreign workers with high skills. On the other hand, middle-aged workers experienced a decline in the number of hours worked by -1.2 percent. The same economic forces led to a significant decline in hours worked by less-educated foreign workers (plotted in figure bottom-left), a decrease by -0.9 percent when middle-aged and -1.2 percent when young. These disparities between this labor group show that young workers suffered more heavily from the business closure than middle-aged workers and high-skill workers experienced the largest reduction in hours worked. Certainly, the conclusion from this figure is that age and education, to a certain extent, made a difference between foreign workers' responses to a negative productivity shock.

We now turn to the impact of productivity shock on native workers. Figure 5 panel B depicts the responses of hours worked. One consequence of productivity disturbance has

[12] One could also use total factor productivity (TFP) from the KLEMS data to estimate the parameters of the productivity shock. This dataset provides TFP at the industry level with only two-digit NAICS codes. However, for our analysis, we use the Bureau of Economic Analysis data, which is available at three-digit NAICS codes, and this is convenient since the business closure orders data from Occupational Information Network (ONET) are also available at three-digit NAICS.

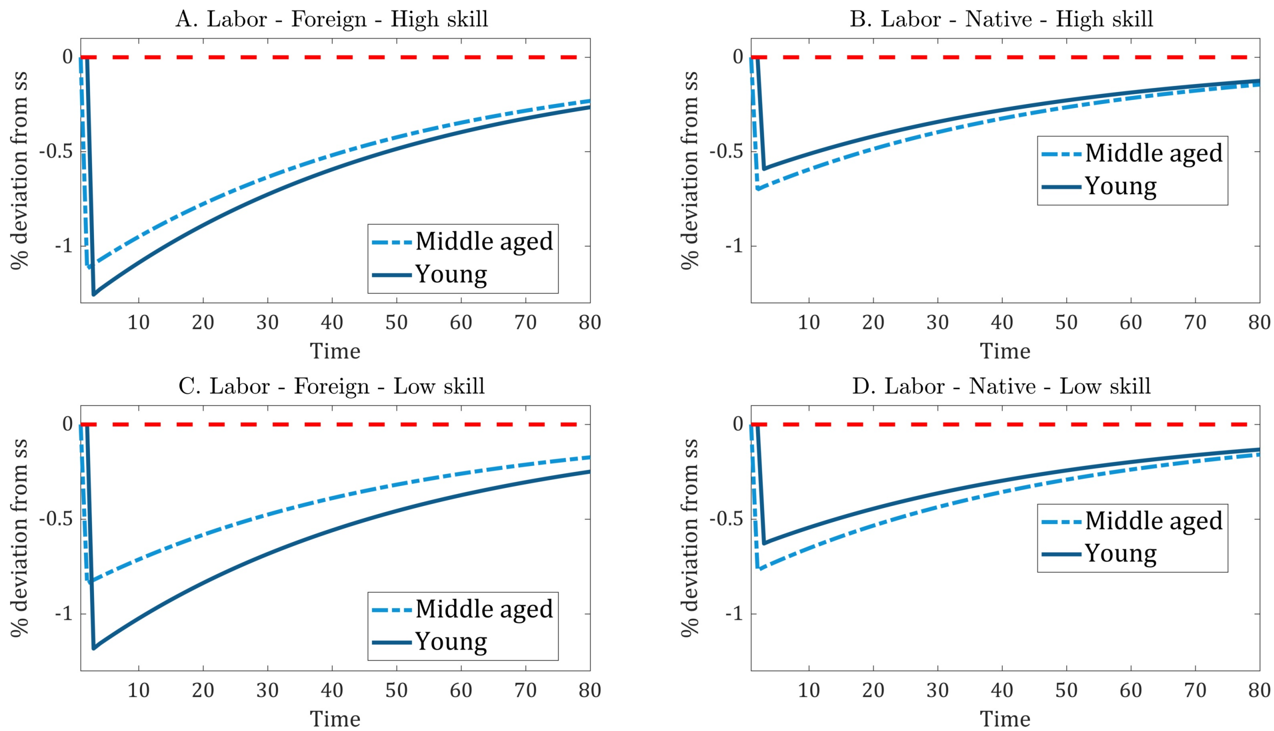


Figure 5: Hours responses to a negative productivity shock

Notes: The left panel A and C show impulse responses of hours worked by foreign workers of different skill and age to a negative productivity shock. The left panel B and D show impulse responses of hours worked by native workers of different skill and age to a negative productivity shock.

been a considerable decline in hours worked by highly-educated middle-aged workers by -0.7 percent. At the same time, young workers experienced a fall in hours of -0.6 percent. Furthermore, the decline in productivity affects low-skilled workers substantially (see Figure 5 panel C). A negative productivity shock depresses the total hours worked by middle-aged workers by -0.8 percent and -0.6 percent for young workers (see Figure 5 panel D). The response of hours worked by middle-aged workers to a fall in productivity is larger than the responses of hours worked by young workers.

This disturbance most obviously affects hours worked and merely adds to the existing disparities between workers. An interesting result is that the fall in productivity during the pandemic crisis lead to heterogeneous responses of hours worked. Higher exposure to productivity shock is observed among less-educated foreign young workers who experienced the largest decline in hours worked. We find that among native workers, middle-aged workers with low skills are the most disadvantaged by having the largest fall in total hours worked.

The key result that stands out from this analysis is that a negative shock to productivity driven by business closure orders leads to a contraction in total hours worked. This is consistent with (Cajner et al., 2020) who find that business closure explains 18 percent of the decline in aggregate employment. Baek et al. (2021) who use state-level data and conclude that one-week stay-at-home orders raise unemployment claims by 1.9 percent of state-level employment. However, there are many reasons to believe that workers throughout the pandemic period became less willing to work due to several reasons not related to business restrictions, for instance, unemployment benefits extensions, fear of the virus, and lack of childcare, among other factors. In a landmark study, Aum et al. (2021) explore an interesting case as it pertains to the South Korean context where measures such as lockdowns or business closures were not implemented. As most governmental policies were centered on social distancing and school closures. They show that the primary drivers of employment decline were the contraction in economic activity and international factors, particularly the economic downturn in China. Forsythe et al. (2020) find that vacancy postings declined by 44 percent and a 13 percent collapse in employment between February and April 2020. They argue that the decline in employment is not solely caused by stay-at-home orders.

5.2 Hourly Earnings and Productivity Shocks

Consider the response of highly skilled native workers to the productivity shock, in panel B of Figure 6. A one percent decrease in productivity increases the wage rate of young workers by 2.9 percent and raises the hourly wage rate of the middle-aged by more than 4 percent. A similar effect is observed for the low-skilled native workers (See panel D of Figure 6). A one percent increase in productivity inflate the wage rate of young workers by 3 percent and middle-aged workers by 4.1 percent. Our model predicts that the effect on young workers is small compared to middle-aged workers. At the margin, one explanation for this result is that middle-aged workers are much more exposed to productivity shock, suggesting that they experience a shrink in their hourly wage. Young and middle-aged workers should be distinguished, and the fact that the hourly wage rate of middle-aged workers tends to be higher means that firms may have an incentive to reduce their labor costs in response to the chronic shock to productivity. This issue is

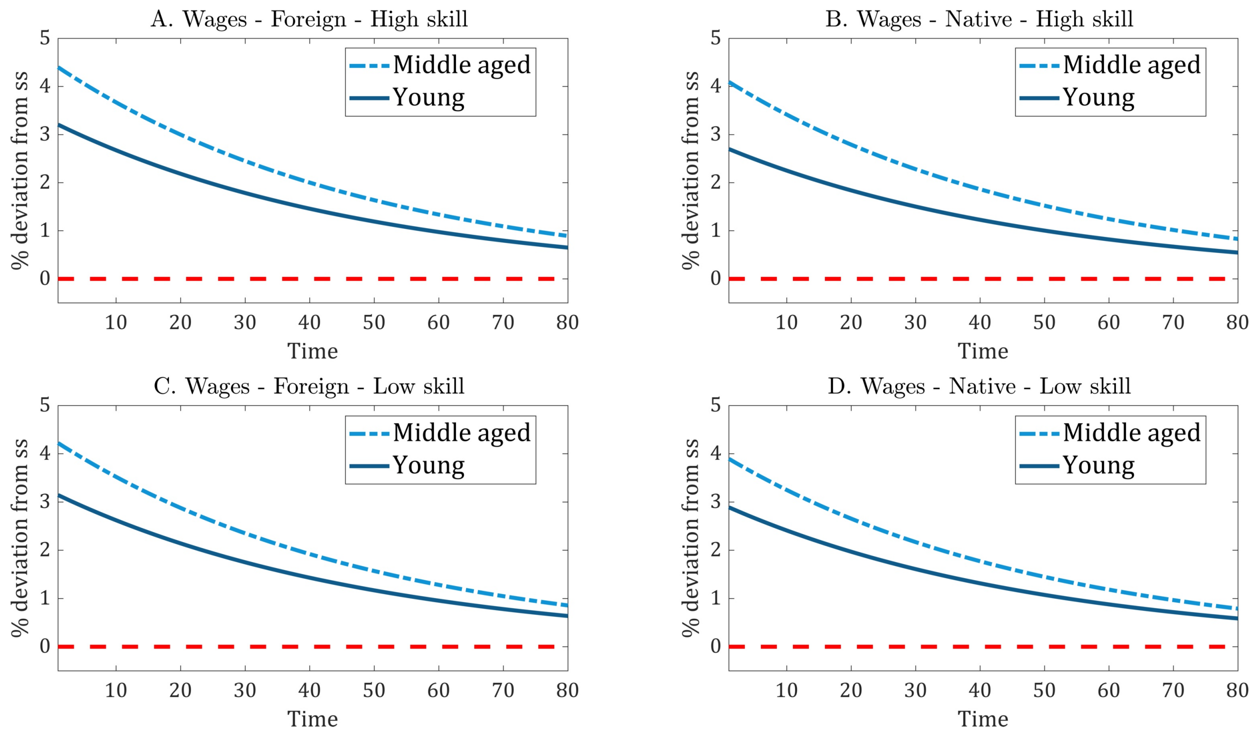


Figure 6: Wages responses to a negative productivity shock

Notes: The left panel A and C show impulse responses of wages by foreign workers of different skill and age to a negative productivity shock. The left panel B and D show impulse responses of wages by native workers of different skill and age to a negative productivity shock.

especially apparent among workers with higher hourly earnings.

Returning to the case of foreign workers, we show that middle-aged workers experience rapid wage growth in response to a fall in productivity a result not very different from the case of native workers. The increase in the hourly wage rate is higher for middle-aged workers with an increase of 4.7 percent, and 3.4 percent for young workers(figure 6 panel A). Similarly, low-skilled workers enjoyed a rise in wage rate in response to the negative productivity shock, with an increase of 4.5 percent for middle-aged workers and an increase of 3.2 percent for young workers (figure 6 panel C).

Again age is making the difference in terms of wage growth. Middle-aged workers appear to be the most affected by the decline in productivity, whereas the wage growth of young workers generated by the pandemic is below the level of middle-aged workers. There are other possible explanations for these results. Intuitively, young workers may have suffered low and stagnant wages in the past which makes the rise in wages for this category less pronounced. This conclusion is also supported by Cortes and Forsythe

Table 5: Least Squares Regression: Log Wages and Worker characteristics

VARIABLES	(1) Log Wage
Post Covid	0.043***
	(0.012)
Middle-aged & Post Covid	0.016
	(0.013)
Low skilled & Post Covid	0.020*
	(0.011)
Middle-aged & Low skilled & Post Covid	-0.016
	(0.013)
Immigrant & Post Covid	0.034
	(0.039)
Middle-aged & Immigrant & Post Covid	-0.029
	(0.040)
Middle-aged & Immigrant & Low skilled & Post Covid	0.051
	(0.057)
Observations	289,157
Adjusted R-squared	0.310
Industry FE	Yes
County FE	Yes
Number of counties	281
Mean Dep. Var.	10.545

Notes Standard error clustered at county level. Standard error in parentheses *p<.05; **p<.01; ***p<.001

(2021) who document heterogeneous employment losses across types of jobs and groups of workers, and find that the pandemic disproportionately affected low-wage jobs.

The US labor market has suffered significant tightness, manifested by both an increase in labor demand and a shortfall in labor supply (Lee et al., 2023). Table 5 provides empirical evidence concerning nominal wage growth after the pandemic crisis. Our estimate reveals that, on average, workers enjoyed a 4 percent nominal wage growth during the post-COVID period of 2021 in comparison to the pre-COVID period. One plausible explanation for the observed wage growth could be attributed to the decline in labor supply, as evidenced by a reduction in hours worked and labor shortage Faberman et al. (2022). As echoed by Hobijn and Şahin (2022), the labor supply is further restrained due to a decline in the population growth rate since the pandemic. Hobijn and Şahin (2022) clarify that the fall in the participation rate was evident even before the pandemic, and argue that this decline in labor supply, coupled with a scarcity of workers dampen job creation. The acute labor shortage may induce firms to be eager to hire and increase wages to attract prospective workers.

Table 6: Wage Volatility

	Benchmark	High δ
Standard deviation of average wages		
High-skilled young foreign worker ω^y_{hf}	0.157	0.424
High-skilled young native worker ω^y_{hn}	0.151	0.406
Low-skilled young foreign worker ω^y_{lf}	0.154	0.415
Low-skilled young native worker ω^y_{ln}	0.154	0.416
High-skilled middle-aged foreign worker ω^m_{hf}	0.221	0.595
High-skilled middle-aged native worker ω^m_{hn}	0.217	0.584
Low-skilled middle-aged foreign worker ω^m_{lf}	0.207	0.557
Low-skilled middle-aged native worker ω^m_{ln}	0.207	0.558

Notes: We report the standard deviation of the average hourly wage rate from the model simulation. High δ reflect the case of high responsiveness of productivity to covid shock.

5.3 Wages Volatility in Unstable Times

Our model assumes that aggregate productivity shock conveys the economic impact of the pandemic, with important implications for workers' hourly earnings. The business closure shocks can lead to higher volatility in labor markets.

Table 6 reports the standard deviation of the average wage rate across labor categories generated by the model under the assumption that productivity shock is highly responsive to the business order closure shock by setting $\delta = 0.2$. The unprecedented supply shock across US industries has curtailed economic activity and caused a reduction in total hours worked. It is evident from the table that the pandemic increases the volatility of wage rates, firms face a combination of two economic phenomena, falling hours worked and rising wage rates. Somewhat surprising is that business closure orders contributed significantly to the excess wage inflation in the US.

Yet the problem of labor market tightness and skill shortage in the US is becoming increasingly worrying. As Domash and Summers (2022) suggested, there is evidence that the US labor market is extremely tight and the inflationary pressure from the labor market will persist. Mounting inflationary pressure on firms will push these firms to adjust their price and margin and eventually will pass through the wage cost to price inflation. This model shows the mechanical effects of the pandemic as we assume that the responsiveness of productivity to this shock is high, and the underlying hourly earnings of all types of workers tend to become more volatile. This is consistent with a recent empirical study that documents large volatility of wages during heightened times as this improves workers' bargaining power (Den Haan et al., 2021).

6 Conclusion

In this paper, we study the effects of business closures during the pandemic crisis on the US labor market. The inclusion of restrictions on US business to the specification of the aggregate productivity shock generates a reduction in labor market activity and translates into a severe episode of economic downturn. The intuition for this result is that the business closure orders deliver a fall in total hours worked associated with a skill shortage. Recent work by Peri and Zaiour (2022) points out a presumably fall in the number of foreign workers in 2021. A similar conclusion is emphasized in Tüzemen (2022) who documents a decline in the labor force and the participation rate in the US at the onset of the pandemic, and Coibion et al. (2020) who study the implication of the pandemic on the labor force participation rate and unemployment using large scale household survey data. This combination of labor shortage and restrictions on businesses tends to push wages up.

Our paper emphasizes the estimation of labor-labor elasticity which is deemed crucial to map the wage differentials and differences in hours worked across the type of workers. Our model makes it clear that a negative shock to aggregate productivity leads to a contraction in total hours worked and sustained wage inflation. Though the magnitude of the shock responses differs by age, skill, and origin of the worker. This suggests that the productivity shock is unequally distributed over the type of workers during the pandemic.

One of the key contributions of this paper is to develop a model that generates labor market fluctuations consistent with what we observe in the real world. An interesting extension of our framework would be the inclusion of capital into the nested CES production function, we believe including this feature in our model is important. One reason is that US firms in general have experienced a substantial change in the allocation of capital when several states imposed a shutdown of non-essential businesses. We believe our paper provides many directions for future investigation.

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Technical Appendix

Curtailment of Economic Activity and Labor Inequalities

Erminia Florio and Aicha Kharazi

Appendix A Model Computations

A.1 Nested production function

Technology We introduce a standard CES production function

$$y_t = z_t(\alpha(l_t^y)^{\frac{s-1}{s}} + (1-\alpha)(l_t^m)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \tag{A.1}$$

where z_t is the productivity shock, α is the share parameter of labor input. For simplicity, the producer use labor factor only as in input. l_t^y and l_t^m denote the hours worked by young and middle-aged workers, respectively. The parameter s represent the elasticity of substitution.
The hours worked by young is given by

$$l_t^y = [((\phi_{h,f}l_{h,f,t}^y)^{\frac{1}{\varrho}} + (\phi_{h,n}l_{h,n,t}^y)^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + ((\phi_{l,f}l_{l,f,t}^y)^{\frac{1}{\upsilon}} + (\phi_{l,n}l_{l,n,t}^y)^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}}]^{\sigma}$$

where l_t^y denotes the total hours worked by young individuals. This equation emphasizes that hours worked by younger individuals young is broken down into various origin (native or foreign) and skill (high or low) groups. We assume that labor inputs l_t^y is a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^y$; hours worked by native worker in high skilled sector $l_{h,n,t}^y$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^y$; and hours worked by native worker in low skilled sector $l_{l,n,t}^y$.
Similarly, the hours worked by middle-aged individuals is given by:

$$l_t^m = [((\theta_{h,f}l_{h,f,t}^m)^{\frac{1}{\varepsilon}} + (\theta_{h,n}l_{h,n,t}^m)^{\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\xi}} + ((\theta_{l,f}l_{l,f,t}^m)^{\frac{1}{\zeta}} + (\theta_{l,n}l_{l,n,t}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}}]^{\xi}$$

the term l_t^m represents the total hours worked by young (middle-aged) individuals. The hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled. The labor inputs l_t^m is then a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^m$; hours worked by native worker in high skilled sector $l_{h,n,t}^m$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^m$; and hours worked by native worker in low skilled sector $l_{l,n,t}^m$.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega^y_{h,f,t} l^y_{h,f,t} + \omega^y_{h,n,t} l^y_{h,n,t} + \omega^y_{l,f,t} l^y_{l,f,t} + \omega^y_{l,n,t} l^y_{l,n,t} + \omega^m_{h,f,t} l^m_{h,f,t} + \omega^m_{h,n,t} l^m_{h,n,t} + \omega^m_{l,f,t} l^m_{l,f,t} + \omega^m_{l,n,t} l^m_{l,n,t}$$

subject to the production function that relates outputs to inputs. The first order condition with respect to $l^y_{h,f,t}$ and $l^y_{h,n,t}$ are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l^y_{h,f,t}} : \quad & \omega^y_{h,f,t} - \mu_t \alpha z_t (\phi_{h,f} l^y_{h,f,t})^{\frac{1}{\varrho}-1} ((\phi_{h,f} l^y_{h,f,t})^{\frac{1}{\varrho}} + (\phi_{h,n} l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1} \\ & (((\phi_{hf} l^y_{h,f,t})^{\frac{1}{\varrho}} + (\phi_{hn} l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + ((\phi_{l,n} l^y_{l,n,t})^{\frac{1}{\upsilon}} + (\phi_{l,f} l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1} \\ & (l^y_t)^{\frac{s-1}{s}-1} (\alpha (l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l^y_{h,n,t}} : \quad & \omega^y_{h,n,t} - \mu_t \alpha z_t (\phi_{h,n} l^y_{h,n,t})^{\frac{1}{\varrho}-1} ((\phi_{hn} l^y_{h,n,t})^{\frac{1}{\varrho}} + (\phi_{hf} l^y_{hft})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1} \\ & (((\phi_{h,n} l^y_{h,n,t})^{\frac{1}{\varrho}} + (\phi_{h,f} l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + ((\phi_{l,n} l^y_{l,n,t})^{\frac{1}{\upsilon}} + (\phi_{l,f} l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1} \\ & (l^y_t)^{\frac{s-1}{s}-1} (\alpha (l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

The Euler equation is

$$\frac{\omega^y_{h,f,t}}{\omega^y_{h,n,t}} = \left(\frac{\phi_{h,f} l^y_{h,f,t}}{\phi_{h,n} l^y_{h,n,t}} \right)^{\frac{1}{\varrho}-1}$$

The first order condition with respect to $l^y_{l,f,t}$ and $l^m_{l,n,t}$ are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l^y_{l,f,t}} : \quad & \omega^y_{l,f,t} - \mu_t \alpha z_t (\phi_{lf} l^y_{l,f,t})^{\frac{1}{\upsilon}-1} ((\phi_{lf} l^y_{l,f,t})^{\frac{1}{\upsilon}} + (\phi_{ln} l^y_{lnt})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1} \\ & (((\phi_{lf} l^y_{l,f,t})^{\frac{1}{\upsilon}} + (\phi_{ln} l^y_{lnt})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + ((\phi_{hn} l^y_{hnt})^{\frac{1}{\varrho}} + (\phi_{hf} l^y_{hft})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1} \\ & (l^y_t)^{\frac{s-1}{s}-1} (\alpha (l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial l^y_{l,n,t}} : \quad & \omega^y_{l,n,t} - \mu_t \alpha z_t (\phi_{ln} l^y_{l,n,t})^{\frac{1}{\upsilon}-1} ((\phi_{ln} l^y_{l,n,t})^{\frac{1}{\upsilon}} + (\phi_{lf} l^y_{lft})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1} \\ & (((\phi_{ln} l^y_{l,n,t})^{\frac{1}{\upsilon}} + (\phi_{lf} l^y_{lft})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + ((\phi_{hn} l^y_{hnt})^{\frac{1}{\varrho}} + (\phi_{hf} l^y_{hft})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1} \\ & (l^y_t)^{\sigma})^{\frac{s-1}{s}-1} (\alpha (l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0 \end{aligned}$$

The Euler equation is given by

$$\frac{\omega^y_{l,f,t}}{\omega^y_{l,n,t}} = \left(\frac{\phi_{l,f} l^y_{l,f,t}}{\phi_{l,n} l^y_{l,n,t}}\right)^{\frac{1}{v}-1}$$

The first order condition with respect to $l^m_{h,f,t}$ and $l^m_{h,n,t}$ are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{h,f,t}} : \qquad & \omega^m_{h,f,t} - \mu_t(1-\alpha)z_t(\theta_{hf} l^m_{h,f,t})^{\frac{1}{\epsilon}-1}((\theta_{hf} l^m_{h,f,t})^{\frac{1}{\epsilon}} + (\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1} \\ & (((\theta_{hf} l^m_{h,f,t})^{\frac{1}{\epsilon}} + (\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + ((\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{s-1}{s}-1}(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{h,n,t}} : \qquad & \omega^m_{h,n,t} - \mu_t(1-\alpha)z_t(\theta_{hn} l^m_{h,n,t})^{\frac{1}{\epsilon}-1}((\theta_{hn} l^m_{h,n,t})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1} \\ & (((\theta_{hn} l^m_{h,n,t})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + ((\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{s-1}{s}-1}(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0\end{aligned}$$

The Euler equation is given by

$$\frac{\omega^m_{h,f,t}}{\omega^m_{h,n,t}} = \left(\frac{\theta_{h,f} l^m_{h,f,t}}{\theta_{h,n} l^m_{h,n,t}}\right)^{\frac{1}{\epsilon}-1}$$

The first order condition with respect to $l^m_{l,f,t}$ and $l^m_{l,n,t}$ are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{l,f,t}} : \qquad & \omega^m_{l,f,t} - \mu_t(1-\alpha)z_t(\theta_{lf} l^m_{l,f,t})^{\frac{1}{\zeta}-1}((\theta_{lf} l^m_{l,f,t})^{\frac{1}{\zeta}} + (\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1} \\ & ((\theta_{lf}(l^m_{l,f,t})^{\frac{1}{\zeta}} + (\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + ((\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{s-1}{s}-1}(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{l,n,t}} : \qquad & \omega^m_{l,n,t} - \mu_t(1-\alpha)z_t(\theta_{ln} l^m_{l,n,t})^{\frac{1}{\zeta}-1}((\theta_{ln} l^m_{l,n,t})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1} \\ & (((\theta_{ln} l^m_{l,n,t})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + ((\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{s-1}{s}-1}(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0\end{aligned}$$

The Euler equation

$$\frac{\omega^m_{l,f,t}}{\omega^m_{l,n,t}} = \left(\frac{\theta_{l,f} l^m_{l,f,t}}{\theta_{l,n} l^m_{l,n,t}}\right)^{\frac{1}{\zeta}-1}$$

A.2 Households

There are three generations, each is alive at any point in time. The lifetime utility is given by

$$\begin{aligned} maximize \quad U(c^y_{s,k,t-1}, c^m_{s,k,t}, c^o_{s,k,t+1}, l^y_{s,k,t-1}, l^m_{s,k,t}) &= \left\{ \ln(c^y_{s,k,t-1}) - \frac{(l^y_{s,k,t-1})^{1+\eta}}{1+\eta} \right\} \\ + \beta \left\{ \ln(c^m_{s,k,t}) - \frac{(l^m_{s,k,t})^{1+\eta}}{1+\eta} \right\} &+ \beta^2 \left\{ \ln(c^o_{s,k,t+1}) \right\} \end{aligned}$$

where $0 \leq \beta \leq 1$ is the discount factor and η is the curvature on the disutility of labor. All agents maximize the utility subject to the budget constraint and capital accumulation.

$$\begin{aligned} &c^y_{s,k,t-1} + a^y_{s,k,t-1} = \omega^y_{s,k,t-1} l^y_{s,k,t-1} \\ &c^m_{s,k,t} + a^m_{s,k,t} = \omega^m_{s,k,t} l^m_{s,k,t} + a^y_{s,k,t-1}(1 + r_{t-1}) \\ &c^o_{s,k,t+1} = a^m_{s,k,t}(1 + r_t) \end{aligned}$$

We let $c^j_{s,k}$ denote the per capita consumption across age groups j at time t where $j \in \{y, m, o\}$ identifies age groups: young, middle-aged, old age. l^y and $l^m_{s,k}$ denote labor supply by young and middle-aged individuals at wage rate $\omega^j_{s,k}$. The young and middle-aged agents consume and work in each period, in the first and second period of life, while old agent earn no income in the third period of life but receive retirement income and consume. Both young and middle-aged agent save, but only the middle-aged and old age agents receive asset earnings at risk free interest rate r.

Using the substitution method we can then write the Lagrangian as follows

$$\begin{aligned} \mathcal{L}_t = \Bigg[&\left\{ \ln(c^y_{s,k,t-1}) - \frac{(l^y_{s,k,t-1})^{1+\eta}}{1+\eta} \right\} + \beta \left\{ \ln(c^m_{s,k,t}) - \frac{(l^m_{s,k,t})^{1+\eta}}{1+\eta} \right\} + \beta^2 \left\{ \ln(c^o_{s,k,t+1}) \right\} \\ &+ \lambda_t \left(\omega^m_{s,k,t} l^m_{s,k,t} + (\omega_{s,k,t-1} l^y_{s,k,t-1} - c^y_{s,k,t-1})(1 + r_{t-1}) - c^m_{s,k,t} - \frac{c^o_{s,k,t+1}}{1 + r_t} \right) \Bigg] \end{aligned}$$

The household optimality conditions with respect to consumption: $c^m_{s,k,t}$, $c^y_{s,k,t-1}$, $c^o_{s,k,t+1}$, and labor: $l^m_{s,k,t}$, and $l^y_{s,k,t-1}$ are derived as follow

$$\frac{\partial \mathcal{L}_t}{\partial c^y_{s,k,t-1}} : \qquad c^y_{s,k,t-1} = \frac{1}{\lambda_t (1 + r_{t-1})} \tag{A.2}$$

$$\frac{\partial \mathcal{L}_t}{\partial c^m_{s,k,t}} : \qquad \frac{\beta}{c^m_{s,k,t}} = \lambda_t \tag{A.3}$$

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{s,k,t-1}} : \qquad (l^y_{s,k,t-1})^\eta = \lambda_t \omega^y_{s,k,t-1}(1+r_{t-1}) \tag{A.4}$$

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{s,k,t}} : \qquad \beta(l^m_{s,k,t})^\eta = \lambda_t \omega^m_{s,k,t} \tag{A.5}$$

$$\frac{\partial \mathcal{L}_t}{\partial c^o_{s,k,t+1}} : \qquad \frac{\beta^2}{c^o_{s,k,t+1}} = \frac{\lambda_t}{(1+r_t)} \tag{A.6}$$

from A.3 and A.5 we obtain

$$\frac{1}{c^m_{s,k,t}} = \frac{(l^m_{s,k,t})^\eta}{\omega^m_{s,k,t}}$$

using A.2 and A.4 we obtain

$$\frac{1}{c^y_{s,k,t-1}} = \frac{(l^y_{s,k,t-1})^\eta}{\omega^y_{s,k,t-1}}$$

A.3 Alternative Specification of the Production Function

Suppose now that producers have access to the following production function

$$y_t = z_t(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \tag{A.7}$$

We assume that firm combine heterogeneous labor to produce good y. The hours worked by younger individuals is given by:

$$l^y_t = \phi_{h,f} l^y_{h,f,t} + \phi_{h,n} l^y_{h,n,t} + \phi_{l,f} l^y_{l,f,t} + \phi_{l,n} l^y_{l,n,t}$$

This equation emphasizes that hours worked by younger individuals young is broken down into various origin (native or foreign) and skill (high or low) groups. Similarly, the hours worked by middle-aged individuals is given by:

$$l^m_t = \theta_{h,f} l^m_{h,f,t} + \theta_{h,n} l^m_{h,n,t} + \theta_{l,f} l^m_{l,f,t} + \theta_{l,n} l^m_{l,n,t}$$

the hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega^y_{h,f,t} l^y_{h,f,t} + \omega^y_{h,n,t} l^y_{h,n,t} + \omega^y_{l,f,t} l^y_{l,f,t} + \omega^y_{l,n,t} l^y_{l,n,t} + \omega^m_{h,f,t} l^m_{h,f,t} + \omega^m_{h,n,t} l^m_{h,n,t} + \omega^m_{l,f,t} l^m_{l,f,t} + \omega^m_{l,n,t} l^m_{l,n,t}$$

subject to the production function that relates outputs to inputs. The first order condition with respect to $l^y_{h,f,t}$ and $l^m_{h,f,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{h,f,t}} : \qquad \omega^y_{h,f,t} - \mu_t \alpha z_t \phi_{h,f} (l^y_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial l^m_{h,f,t}} : \qquad \omega^m_{h,f,t} - \mu_t (1-\alpha) z_t \theta_{h,f} (l^m_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0.$$

We derive the Euler equation

$$\frac{\omega^y_{h,f,t}}{\omega^m_{h,f,t}} = \frac{\alpha}{(1-\alpha)} \frac{\phi_{h,f}}{\theta_{h,f}} \left(\frac{l^y_t}{l^m_t} \right)^{-\frac{1}{s}},$$

The first order condition wrt $l^y_{h,n,t}$ and $l^m_{h,n,t}$ are:

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{h,n,t}} : \qquad \omega^y_{h,n,t} - \mu_t \alpha z_t \phi_{h,n} (l^y_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial l^m_{h,n,t}} : \qquad \omega^m_{h,n,t} - \mu_t (1-\alpha) z_t \theta_{h,n} (l^m_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0.$$

Combining these two equations gives the following Euler equation

$$\frac{\omega^y_{h,n,t}}{\omega^m_{h,n,t}} = \frac{\alpha}{1-\alpha} \frac{\phi_{h,n}}{\theta_{h,n}} \left(\frac{l^y_t}{l^m_t} \right)^{-\frac{1}{s}},$$

first order condition with respect to $l^y_{l,f,t}$ and $l^m_{l,f,t}$

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,f,t}} : \qquad \omega^y_{l,f,t} - \mu_t \alpha z_t \phi_{l,f} (l^y_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial l^m_{l,f,t}} : \qquad \omega^m_{l,f,t} - \mu_t (1-\alpha) z_t \theta_{l,f} (l^m_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$

The Euler equation

$$\frac{\omega^y_{l,f,t}}{\omega^m_{l,f,t}} = \frac{\alpha}{1-\alpha} \frac{\phi_{l,f}}{\theta_{l,f}} \left(\frac{l^y_t}{l^m_t} \right)^{-\frac{1}{s}},$$

first order condition with respect to $l^y_{l,n,t}$ and $l^m_{l,n,t}$

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,n,t}} : \qquad \omega^y_{l,n,t} - \mu_t \alpha z_t \phi_{l,n} (l^y_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{l,n,t}}: \qquad \omega^m_{l,n,t} - \mu_t(1-\alpha)z_t\theta_{l,n}(l^m_t)^{\frac{s-1}{s}-1}((l^y_t)^{\frac{s-1}{s}} + (l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,$$

The Euler equation

$$\frac{\omega^y_{l,n,t}}{\omega^m_{l,n,t}} = \frac{\alpha}{1-\alpha}\frac{\phi_{l,n}}{\theta_{l,n}}\left(\frac{l^y_t}{l^m_t}\right)^{-\frac{1}{s}},$$

The producer marginal cost is

$$mc = \mu_t = \frac{\omega_t(l^y_t + l^m_t)}{z_t(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}}},$$

Appendix B Steady State

As a first step we assume that $a^m_t = a^y_t$, given that the middle-aged budget constraint is $c^m_t + a^m_t = \omega_t l^m_t + a^y_{t-1}(1 + r_{t-1})$, and the total saving in this economy equals to $a_t = a^m_t + a^y_t$. The first order condition with respect to a_y is

$$\lambda_t = \lambda_{t+1}\beta(1 + r_t)$$

We can easily compute the steady state value of interest rate:

$$\lambda_{ss} = \lambda_{ss}\beta(1 + r_{ss})$$
$$r_{ss} = \frac{1}{\beta} - 1$$

We set an initial value for the household's marginal utility λ_{ss} and we calculate the consumption value of middle-aged agent

$$\lambda_t = \frac{\beta}{c^m_t}$$
$$c^m_{ss} = \frac{\beta}{\lambda_{ss}}$$

the consumption of young agents

$$c^y_{t-1} = \frac{1}{\lambda_t(1 + r_{t-1})}$$
$$c^y_{ss} = \frac{1}{\lambda_{ss}(1 + r_{ss})}$$

the consumption of the older

$$\frac{\beta^2}{c^o_{t+1}} = \frac{\lambda_t}{(1+r_t)}$$
$$c^o_{ss} = \frac{\beta^2(1+r_{ss})}{\lambda_{ss}}$$

We set an initial value for ω and we compute the value of labor l^m and l^y using the optimality conditions

$$\beta\,(l^m_t)^\eta = \lambda_t\omega_t \qquad l^{y\eta}_t = \lambda_t\omega_{t-1}(1+r_{t-1})$$
$$l^m_{ss} = \left(\frac{(\lambda_{ss}\omega_{ss})}{\beta}\right)^{\frac{1}{\eta}} \qquad l^y_{ss} = (\lambda_{ss}\omega_{ss}(1+r_{ss}))^{\frac{1}{\eta}}$$

using the labor intensity parameters we can derive the value of labor by types of workers

$$\begin{aligned}
l^y_{t,h,f} &= \phi_{h,f}l^y_t & l^y_{ss,h,f} &= \phi_{h,f}l^y_{ss}\\
l^y_{t,h,n} &= \phi_{h,n}l^y_t & l^y_{ss,h,n} &= \phi_{h,n}l^y_{ss}\\
l^y_{t,l,f} &= \phi_{l,f}l^y_t & l^y_{ss,l,f} &= \phi_{l,f}l^y_{ss}\\
l^y_{t,l,n} &= \phi_{l,n}l^y_t & l^y_{ss,l,n} &= \phi_{l,n}l^y_{ss}\\
l^m_{t,h,f} &= \theta_{h,f}l^m_t & l^m_{ss,h,f} &= \theta_{h,f}l^m_{ss}\\
l^m_{t,h,n} &= \theta_{h,n}l^m_t & l^m_{ss,h,n} &= \theta_{h,n}l^m_{ss}\\
l^m_{t,l,f} &= \theta_{l,f}l^m_t & l^m_{ss,l,f} &= \theta_{l,f}l^m_{ss}\\
l^m_{t,l,n} &= \theta_{l,n}l^m_t & l^m_{ss,l,n} &= \theta_{l,n}l^m_{ss}
\end{aligned}$$

Then, we set a initial value for the producer marginal cost μ we use first order conditions with respect to to labor to compute the hourly wage rate

$$\begin{aligned}
&\omega^y_{h,f,ss} - \mu_{ss}\alpha z_{ss}(\phi_{h,f}l^y_{h,f,ss})^{\frac{1}{\varrho}-1}((\phi_{h,f}l^y_{h,f,ss})^{\frac{1}{\varrho}} + (\phi_{h,n}l^y_{h,n,ss})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}\\
&\quad(((\phi_{hf}l^y_{h,f,ss})^{\frac{1}{\varrho}} + (\phi_{hn}l^y_{h,n,ss})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + ((\phi_{l,n}l^y_{l,n,ss})^{\frac{1}{\upsilon}} + (\phi_{l,f}l^y_{l,f,ss})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}\\
&\quad(l^y_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,\\
&\omega^y_{h,n,ss} - \mu_{ss}\alpha z_{ss}(\phi_{h,n}l^y_{h,n,ss})^{\frac{1}{\varrho}-1}((\phi_{hn}l^y_{h,n,ss})^{\frac{1}{\varrho}} + (\phi_{hf}l^y_{hft})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}\\
&\quad(((\phi_{h,n}l^y_{h,n,ss})^{\frac{1}{\varrho}} + (\phi_{h,f}l^y_{h,f,ss})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + ((\phi_{l,n}l^y_{l,n,ss})^{\frac{1}{\upsilon}} + (\phi_{l,f}l^y_{l,f,ss})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}\\
&\quad(l^y_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0,\\
&\omega^y_{l,f,ss} - \mu_{ss}\alpha z_{ss}(\phi_{lf}l^y_{l,f,ss})^{\frac{1}{\upsilon}-1}((\phi_{lf}l^y_{l,f,ss})^{\frac{1}{\upsilon}} + (\phi_{ln}l^y_{lnt})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}
\end{aligned}$$

$$
\begin{aligned}
&(((\phi_{lf} l^y_{l,f,ss})^{\frac{1}{v}} + (\phi_{ln} l^y_{lnt})^{\frac{1}{v}})^{\frac{v}{\sigma}} + ((\phi_{hn} l^y_{hnt})^{\frac{1}{\varrho}} + (\phi_{hf} l^y_{hft})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1} \\
&(l^y_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega^y_{l,n,ss} &- \mu_{ss}\alpha z_{ss}(\phi_{ln} l^y_{l,n,ss})^{\frac{1}{v}-1}((\phi_{ln} l^y_{l,n,ss})^{\frac{1}{v}} + (\phi_{lf} l^y_{lft})^{\frac{1}{v}})^{\frac{v}{\sigma}-1} \\
&(((\phi_{ln} l^y_{l,n,ss})^{\frac{1}{v}} + (\phi_{lf} l^y_{lft})^{\frac{1}{v}})^{\frac{v}{\sigma}} + ((\phi_{hn} l^y_{hnt})^{\frac{1}{\varrho}} + (\phi_{hf} l^y_{hft})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1} \\
&(l^y_{ss})^{\sigma})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega^m_{h,f,ss} &- \mu_{ss}(1-\alpha)z_{ss}(\theta_{hf} l^m_{h,f,ss})^{\frac{1}{\epsilon}-1}((\theta_{hf} l^m_{h,f,ss})^{\frac{1}{\epsilon}} + (\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1} \\
&(((\theta_{hf} l^m_{h,f,ss})^{\frac{1}{\epsilon}} + (\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + ((\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1} \\
&(l^m_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega^m_{h,n,ss} &- \mu_{ss}(1-\alpha)z_{ss}(\theta_{hn} l^m_{h,n,ss})^{\frac{1}{\epsilon}-1}((\theta_{hn} l^m_{h,n,ss})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1} \\
&(((\theta_{hn} l^m_{h,n,ss})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + ((\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1} \\
&(l^m_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega^m_{l,f,ss} &- \mu_{ss}(1-\alpha)z_{ss}(\theta_{lf} l^m_{l,f,ss})^{\frac{1}{\zeta}-1}((\theta_{lf} l^m_{l,f,ss})^{\frac{1}{\zeta}} + (\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1} \\
&((\theta_{lf}(l^m_{l,f,ss})^{\frac{1}{\zeta}} + (\theta_{ln} l^m_{lnt})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + ((\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1} \\
&(l^m_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0, \\
\omega^m_{l,n,ss} &- \mu_{ss}(1-\alpha)z_{ss}(\theta_{ln} l^m_{l,n,ss})^{\frac{1}{\zeta}-1}((\theta_{ln} l^m_{l,n,ss})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1} \\
&(((\theta_{ln} l^m_{l,n,ss})^{\frac{1}{\zeta}} + (\theta_{lf} l^m_{lft})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + ((\theta_{hn} l^m_{hnt})^{\frac{1}{\epsilon}} + (\theta_{hf} l^m_{hft})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1} \\
&(l^m_{ss})^{\frac{s-1}{s}-1}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}-1} = 0
\end{aligned}
$$

Then we compute the total cost

$$
\begin{aligned}
\mathrm{TC}_t &= l^m_{t,h,f}\omega^m_{t,h,f} + l^m_{t,h,n}\omega^m_{t,h,n} + l^m_{t,l,f}\omega^m_{t,l,f} + l^m_{t,l,n}\omega^m_{t,l,n} \\
&+ l^y_{t,h,f}\omega^y_{t,h,f} + l^y_{t,h,n}\omega^y_{t,h,n} + l^y_{t,l,f}\omega^y_{t,l,f} + l^y_{t,l,n}\omega^y_{t,l,n} \\
\mathrm{TC}_{ss} &= l^m_{ss,h,f}\omega^m_{ss,h,f} + l^m_{ss,h,n}\omega^m_{ss,h,n} + l^m_{ss,l,f}\omega^m_{ss,l,f} + l^m_{ss,l,n}\omega^m_{ss,l,n} \\
&+ l^y_{ss,h,f}\omega^y_{ss,h,f} + l^y_{ss,h,n}\omega^y_{ss,h,n} + l^y_{ss,l,f}\omega^y_{ss,l,f} + l^y_{ss,l,n}\omega^y_{ss,l,n}
\end{aligned}
$$

we compute the value of y

$$
\begin{aligned}
y_t &= z_t(\alpha(l^y_t)^{\frac{s-1}{s}} + (1-\alpha)(l^m_t)^{\frac{s-1}{s}})^{\frac{s}{s-1}} \\
y_{ss} &= z_{ss}(\alpha(l^y_{ss})^{\frac{s-1}{s}} + (1-\alpha)(l^m_{ss})^{\frac{s-1}{s}})^{\frac{s}{s-1}}
\end{aligned}
$$

and marginal cost

$$\text{marginal cost}_t = \frac{\text{TC}_t}{y_t}$$
$$\text{marginal cost}_{ss} = \frac{\text{TC}_{ss}}{y_{ss}}$$

and using market clearing condition we can compute the total saving

$$a_t = y_t - c_t^m - c_t^y - c_t^o$$
$$a_{ss} = y_{ss} - c_{ss}^m - c_{ss}^y - c_{ss}^o$$

Appendix C Sample

We describe the 3 digits NIACS code for each sector of our sample.

Table 7: Industry code

Industry description	Code	Industry description	Code
Accommodation	721	Wood products	321
Administrative and support services	561	Air transportation	481
Ambulatory health care services	621	Broadcasting and telecommunications	515-517
Amusements, gambling, and recreation industries	713	Data processing, internet publishing, and other information services	518-519
Apparel and leather and allied products	315-316	Educational services	611
Chemical products	325	Farms	111-112
Computer and electronic products	334	Food and beverage stores	445
Electrical equipment, appliances, and components	335	Forestry, fishing, and related activities	113-114-115
Fabricated metal products	332	General merchandise stores	451-452-453-454
Federal Reserve banks, credit intermediation, and related activities	521-522	Management of companies and enterprises	551
Food and beverage and tobacco products	311-312	Mining, except oil and gas	212
Food services and drinking places	722	Motion picture and sound recording industries	512
Furniture and related products	337	Motor vehicle and parts dealers	441
Hospitals	622	Oil and gas extraction	211
Insurance carriers and related activities	524	Other retail	442-443-444-446-447-448
Machinery	333	Other transportation and support activities	487-488-491-492
Miscellaneous manufacturing	339	Pipeline transportation	486
Nonmetallic mineral products	327	Professional, scientific, and technical services	541
Nursing and residential care facilities	623	Publishing industries, except internet (includes software)	511
Paper products	322	Rail transportation	482
Performing arts, spectator sports, museums, and related activities	711-712	Support activities for mining	213
Petroleum and coal products	324	Transit and ground passenger transportation	485
Plastics and rubber products	326	Truck transportation	484
Primary metals	331	Warehousing and storage	493
Printing and related support activities	323	Water transportation	483
Real estate	531	Construction	23
Rental and leasing services and lessors of intangible assets	532-533	Utilities	221
Social assistance	624	Wholesale trade	423-424-425
Textile mills and textile product mills	313-314	Motor vehicles, bodies and trailers, and parts and Other transportation equipment	336
Waste management and remediation services	562	Funds, trusts, and other financial vehicles and Securities, commodity contracts, and investments	523-525
		Other services, except government	811-812-813

[a] Note: We assign ”Not specified utilities” to the 221 NAICS code, ”Not specified food industries” to 312 NAICS code. We assign ”Knitting mills, and apparel knitting mills” to 315 NAICS code, and 315M to 315. We assign ”Not specified metal industries” to 332 NAICS code, and 333MS to 333. We assign ”Not specified wholesale” trade to 424. We assign 44511 to 445, 45121 to 452, 454110 to 454. We assign ”Not specified retail trade” to 454. We assign ”Banking and related activities” to 522, ”Securities, commodities, funds, trusts, and other financial investments” to 523, and 532M2 to 532. We assign ”Commercial, industrial, and other intangible assets rental and leasing” to 533, we also assign 55 to 551.

[b] In the Bureau of Economic Analysis data, the ”Construction” industry is aggregated by sector, whereas we have affected share observations by three sub sectors of the ”Construction” industry. To overcome this limitation, we assign the value of affected share of the sub-sector ”Construction of buildings” to the overall sector ”construction”. Data sources: The Bureau of Economic Analysis.

Appendix D Regression results: the elasticity of substitution

Table 8: Estimation of the elasticity of substitution $1/\zeta - 1$

	Dependent variable: $\ln(\omega^m_{l,f,t}) - \ln(\omega^m_{l,n,t})$			
	(1)	(2)	(3)	(4)
Constant	0.00465**	0.00469**	0.00464**	0.00470***
	(0.00181)	(0.00182)	(0.00181)	(0.00182)
$\ln(l^m_{l,f,t}) - \ln(l^m_{l,n,t})$	0.11263***	0.11628***	0.11216***	0.11672***
	(0.00680)	(0.00683)	(0.00680)	(0.00683)
Fixed effects	County, Year		County	Year
F-statistic	274.1	289.5	271.7	291.8
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 9: Estimation of the elasticity of substitution $1/\xi - 1$

	Dependent variable: $\ln(\omega^m_{h,t}) - \ln(\omega^m_{l,t})$			
	(1)	(2)	(3)	(4)
Constant	0.64794***	0.64793***	0.64797***	0.64790***
	(0.00117)	(0.00118)	(0.00117)	(0.00117)
$\ln(l^m_{h,t}) - \ln(l^m_{l,t})$	0.07146***	0.07175***	0.07064***	0.07258***
	(0.00638)	(0.00639)	(0.00638)	(0.00639)
Fixed effects	County, Year		County	Year
F-statistic	125.5	126.1	122.5	129.2
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 10: Estimation of the elasticity of substitution $1/\varrho - 1$

	Dependent variable: $\ln(\omega^y_{h,f,t}) - \ln(\omega^y_{h,n,t})$			
	(1)	(2)	(3)	(4)
Constant	-0.09227***	-0.09226***	-0.09227***	-0.09226***
	(0.00126)	(0.00126)	(0.00126)	(0.00126)
$\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})$	0.14092***	0.14080***	0.14093***	0.14078***
	(0.00558)	(0.00558)	(0.00558)	(0.00558)
Fixed effects	County, Year		County	Year
F-statistic	637.8	635.9	638.0	635.8
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 11: Estimation of the elasticity of substitution $1/\varepsilon - 1$

	Dependent variable: $\ln(\omega^m_{h,f,t}) - \ln(\omega^m_{h,n,t})$			
	(1)	(2)	(3)	(4)
Constant	-0.16092***	-0.16091***	-0.16091***	-0.16092***
	(0.00241)	(0.00242)	(0.00241)	(0.00242)
$\ln(l^m_{h,f,t}) - \ln(l^m_{h,n,t})$	0.05898***	0.05850***	0.05774***	0.05972***
	(0.00593)	(0.00596)	(0.00594)	(0.00595)
Fixed effects	County, Year		County	Year
F-statistic	98.84	96.49	94.62	100.7
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 12: Estimation of the elasticity of substitution $1/\upsilon - 1$

		Dependent variable: $\ln(\omega^y_{l,f,t}) - \ln(\omega^y_{l,n,t})$		
	(1)	(2)	(3)	(4)
Constant	0.35256***	0.35256***	0.35256***	0.35256***
	(0.00075)	(0.00075)	(0.00075)	(0.00075)
$\ln(l^y_{l,f,t}) - \ln(l^y_{l,n,t})$	0.11723***	0.11781***	0.11732***	0.11771***
	(0.00604)	(0.00604)	(0.00604)	(0.00604)
Fixed effects	County, Year		County	Year
F-statistic	377.0	380.6	377.6	380.0
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 13: Estimation of the elasticity of substitution $1/s$

	Dependent variable: $\ln(\omega_t^m) - \ln(\omega_t^y)$			
	(1)	(2)	(3)	(4)
Constant	-0.22813***	-0.22825***	-0.22825***	-0.22813***
	(0.00076)	(0.00077)	(0.00076)	(0.00077)
$\ln(l_t^m) - \ln(l_t^y)$	0.07658***	0.07489***	0.07487***	0.07658***
	(0.00623)	(0.00627)	(0.00624)	(0.00626)
Fixed effects	County, Year		County	Year
F-statistic	151.0	142.9	144.1	149.6
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Table 14: Estimation of the elasticity of substitution $1/\sigma - 1$

	Dependent variable: $\ln(\omega_{h,t}^y) - \ln(\omega_{l,t}^y)$			
	(1)	(2)	(3)	(4)
Constant	0.72139***	0.72155***	0.72142***	0.72153***
	(0.00078)	(0.00079)	(0.00078)	(0.00079)
$\ln(l_{h,t}^y) - \ln(l_{l,t}^y)$	0.03924***	0.03787***	0.03904***	0.03805***
	(0.00512)	(0.00512)	(0.00512)	(0.00512)
Fixed effects	County, Year		County	Year
F-statistic	58.73	54.67	58.13	55.21
No. Observations	98709	98709	98709	98709

Note: The estimates in this table is obtained using a simple OLS with county and year fixed effects, homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

Appendix E Robustness checks: the elasticity of substitution

Table 15: Estimation of the elasticity of substitution

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI	No. Obs.	R^2 (Within)
Estimation of the elasticity of substitution $1/\varrho - 1$; Dep. Variable: $\ln(\omega^y_{h,f,t}) - \ln(\omega^y_{h,n,t})$								
$\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})$ with County, Time FE	0.1394	0.0076	18.264	0.0000	0.1245	0.1544	54895	0.0060
$\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})$	0.1397	0.0076	18.297	0.0000	0.1248	0.1547	54895	0.0060
$\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})$ with County FE	0.1394	0.0076	18.265	0.0000	0.1245	0.1544	54895	0.0060
$\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})$ with Time FE	0.1397	0.0076	18.296	0.0000	0.1248	0.1547	54895	0.0060
Estimation of the elasticity of substitution $1/\upsilon - 1$; Dep. Variable: $\ln(\omega^y_{l,f,t}) - \ln(\omega^y_{l,n,t})$								
$\ln(l^y_{l,f,t}) - \ln(l^y_{l,n,t})$ with Time FE	0.0786	0.0076	10.396	0.0000	0.0638	0.0935	54895	0.0020
$\ln(l^y_{l,f,t}) - \ln(l^y_{l,n,t})$	0.0789	0.0076	10.434	0.0000	0.0641	0.0938	54895	0.0020
$\ln(l^y_{l,f,t}) - \ln(l^y_{l,n,t})$ with County FE	0.0787	0.0076	10.399	0.0000	0.0638	0.0935	54895	0.0020
$\ln(l^y_{l,f,t}) - \ln(l^y_{l,n,t})$ with Time FE	0.0789	0.0076	10.431	0.0000	0.0641	0.0937	54895	0.0020
Estimation of the elasticity of substitution $1/\varepsilon - 1$; Dep. Variable: $\ln(\omega^m_{h,f,t}) - \ln(\omega^m_{h,n,t})$								
$\ln(l^m_{h,f,t}) - \ln(l^m_{h,n,t})$ with County, Time FE	0.0614	0.0084	7.3228	0.0000	0.0449	0.0778	54895	0.0010
$\ln(l^m_{h,f,t}) - \ln(l^m_{h,n,t})$	0.0629	0.0084	7.4795	0.0000	0.0464	0.0793	54895	0.0010
$\ln(l^m_{h,f,t}) - \ln(l^m_{h,n,t})$ with County FE	0.0613	0.0084	7.3143	0.0000	0.0449	0.0777	54895	0.0010
$\ln(l^m_{h,f,t}) - \ln(l^m_{h,n,t})$ with Time FE	0.0629	0.0084	7.4877	0.0000	0.0465	0.0794	54895	0.0010
Estimation of the elasticity of substitution $1/\zeta - 1$; Dep. Variable: $\ln(\omega^m_{l,f,t}) - \ln(\omega^m_{l,n,t})$								
$\ln(l^m_{l,f,t}) - \ln(l^m_{l,n,t})$ with County, Time FE	0.1344	0.0093	14.528	0.0000	0.1163	0.1526	54895	0.0038
$\mathrm{Iln}(l^m_{l,f,t}) - \ln(l^m_{l,n,t})$	0.1408	0.0093	15.155	0.0000	0.1226	0.1591	54895	0.0038
$\ln(l^m_{l,f,t}) - \ln(l^m_{l,n,t})$ with County FE	0.1342	0.0093	14.501	0.0000	0.1161	0.1523	54895	0.0038
$\ln(l^m_{l,f,t}) - \ln(l^m_{l,n,t})$ with Time FE	0.1410	0.0093	15.180	0.0000	0.1228	0.1592	54895	0.0038
Estimation of the elasticity of substitution $1/s$; Dep. Variable: $\ln(\omega^m_t) - \ln(\omega^y_t)$								
$\ln(l^m_t) - \ln(l^y_t)$ with County, Time FE	0.0766	0.0062	12.288	0.0000	0.0644	0.0888	98709	0.0015
$\ln(l^m_t) - \ln(l^y_t)$	0.0749	0.0063	11.952	0.0000	0.0626	0.0872	98709	0.0015
$\ln(l^m_t) - \ln(l^y_t)$ with County FE	0.0749	0.0062	12.004	0.0000	0.0626	0.0871	98709	0.0015
$\ln(l^m_t) - \ln(l^y_t)$ with Time FE	0.0766	0.0063	12.230	0.0000	0.0643	0.0888	98709	0.0015
Estimation of the elasticity of substitution σ; Dep. Variable: $\ln(\omega^y_{h,t}) - \ln(\omega^y_{l,t})$								
$\ln(l^y_{h,t}) - \ln(l^y_{l,t})$ with County, Time FE	0.0392	0.0051	7.6637	0.0000	0.0292	0.0493	98709	0.0006
$\ln(l^y_{h,t}) - \ln(l^y_{l,t})$	0.0379	0.0051	7.3938	0.0000	0.0278	0.0479	98709	0.0006
$\ln(l^y_{h,t}) - \ln(l^y_{l,t})$ with County FE	0.0390	0.0051	7.6241	0.0000	0.0290	0.0491	98709	0.0006
$\ln(l^y_{h,t}) - \ln(l^y_{l,t})$ with Time FE	0.0381	0.0051	7.4307	0.0000	0.0280	0.0481	98709	0.0006
Estimation of the elasticity of substitution $1/\xi - 1$; Dep. Variable: $\ln(\omega^m_{h,t}) - \ln(\omega^m_{l,t})$								
$\ln(l^m_{h,t}) - \ln(l^m_{l,t})$ with County, Time FE	0.0715	0.0064	11.204	0.0000	0.0590	0.0840	98709	0.0012
$\ln(l^m_{h,t}) - \ln(l^m_{l,t})$	0.0717	0.0064	11.230	0.0000	0.0592	0.0843	98709	0.0012
$\ln(l^m_{h,t}) - \ln(l^m_{l,t})$ with County FE	0.0706	0.0064	11.070	0.0000	0.0581	0.0831	98709	0.0012
$\ln(l^m_{h,t}) - \ln(l^m_{l,t})$ with Time FE	0.0726	0.0064	11.365	0.0000	0.0601	0.0851	98709	0.0012

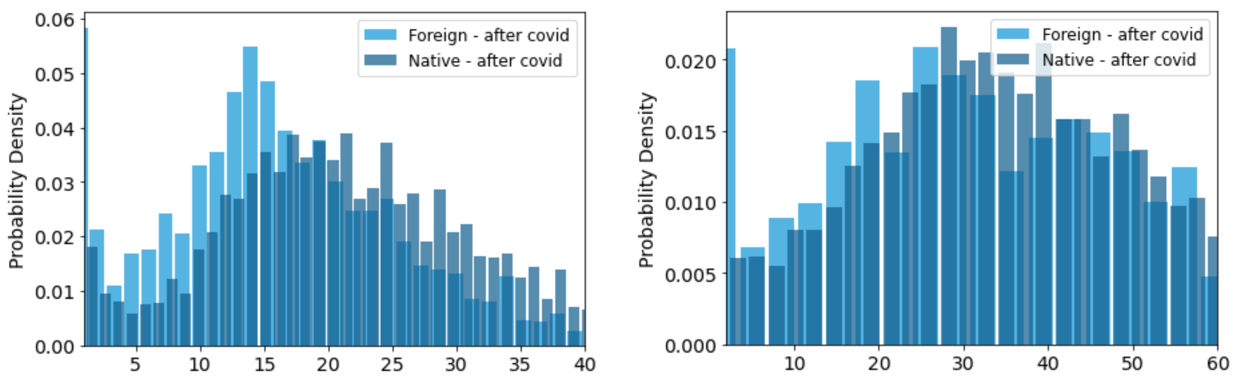
Appendix F Regression results: productivity shock parameters

Table 16: Estimation of the productivity shock parameters

Dependent variable: $\ln(\mathrm{tfp}_{i,t})$	Coefficients
Lagged total factor productivity $\ln(\mathrm{tfp}_{i,t-1})$	0.98319***
	(0.00251)
The affected share $d_{i,t}$	-0.00173***
	(0.00060)
No. Observations:	610

Note: homoscedastic standard error in parentheses $*p < .1, **p < .05, ***p < .01$

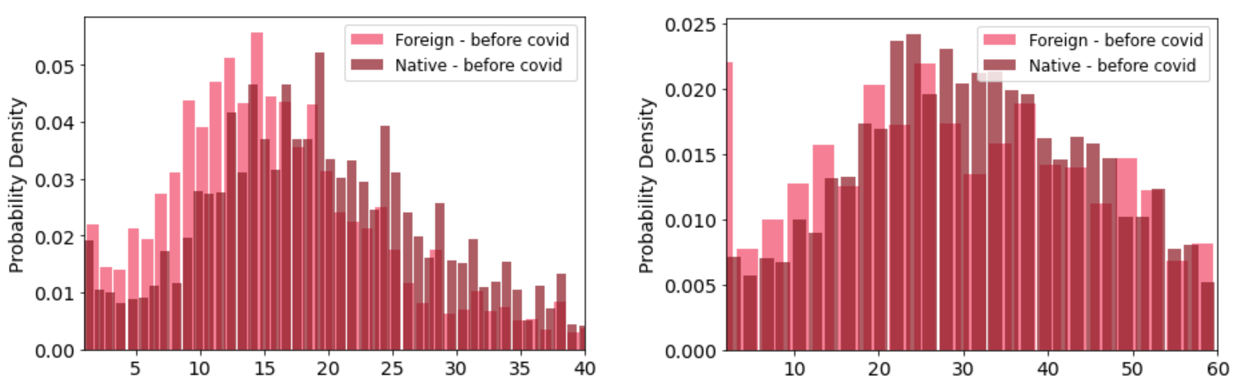
Appendix G Hourly earnings distribution before and after the pandemic



A. Middle-aged and Low Skill B. Middle-aged and High Skill

Figure 7: Distribution of Hourly Wage by Middle-aged Native and Foreign Workers of Different Skill

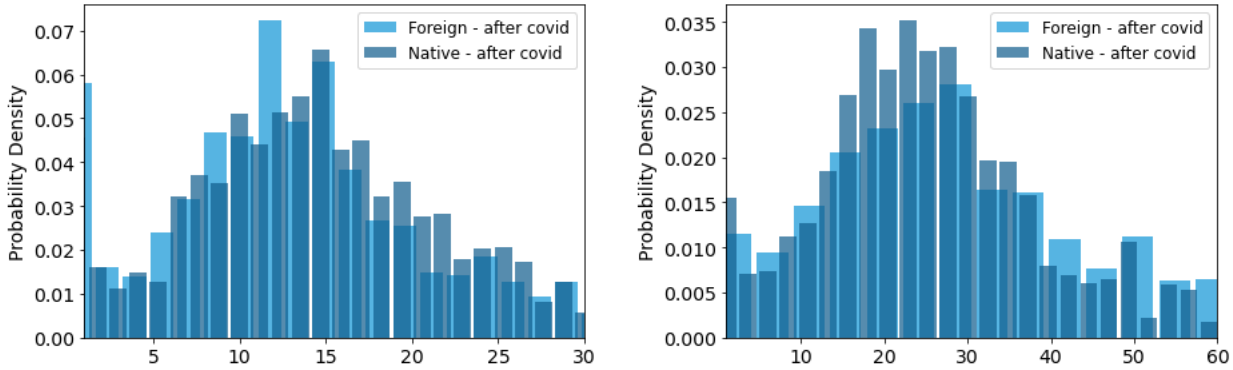
Notes: The left-hand panel describes the distribution of hourly wages of across native and foreign workers when they are middle-aged and low skill. The right hans-side panel describes the distribution of hourly wages of across native and foreign workers when they are middle-aged and high skill. Time: 2018-2021. Source: Current Population Survey.



A. Middle-aged and Low Skill B. Middle-aged and High Skill

Figure 8: Distribution of Hourly Wage by Middle-aged Native and Foreign Workers of Different Skill

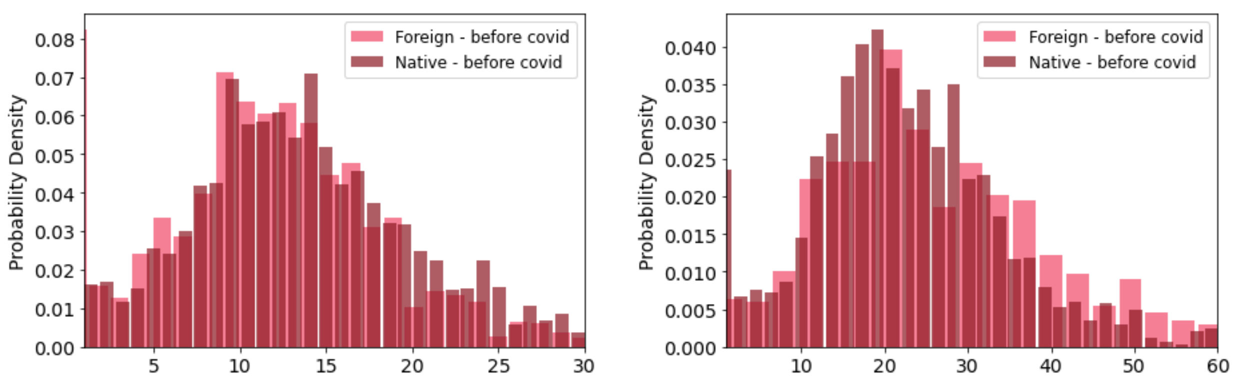
Notes: The left-hand panel describes the distribution of hourly wages of across native and foreign workers when they are middle-aged and low skill. The right hans-side panel describes the distribution of hourly wages of across native and foreign workers when they are middle-aged and high skill. Time: 2018-2021. Source: Current Population Survey.



A. Young and Low Skill B. Young and High Skill

Figure 9: Distribution of Hourly Wage by Young Native and Foreign Workers of Different Skill

Notes: The left-hand panel describes the distribution of hourly wages of across native and foreign workers when they are young and low skill. The right hans-side panel describes the distribution of hourly wages of across native and foreign workers when they are young and high skill. Time: 2018-2021. Source: Current Population Survey



A. Young and Low Skill B. Young and High Skill

Figure 10: Distribution of Hourly Wage by Young Native and Foreign Workers of Different Skill

Notes: The left-hand panel describes the distribution of hourly wages of across native and foreign workers when they are young and low skill. The right hans-side panel describes the distribution of hourly wages of across native and foreign workers when they are young and high skill. Time: 2018-2021. Source: Current Population Survey