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An optimal control model with cost effectiveness analysis of Maize streak virus disease in maize plant

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ABSTRACT

In this paper we formulated and analyzed an optimal deterministic eco-epidemiological model for the dynamics of maize streak virus (MSV) and examine the best strategy to fight maize population from maize streak disease (MSD). The optimal control model is developed with three control interventions, namely prevention (u_1), quarantine (u_2) and chemical control (u_3). To achieve an optimal control strategy, we used the Pontryagin's maximum principle obtain the Hamiltonian, the adjoint variables, the characterization of the controls and the optimality system. Numerical simulations are performed using Forward-backward sweep iterative method. The findings show that each integrated strategy is able to mitigate the disease in the specified time. However due to limited resources, it is important to find a cost-effective strategy. Using Incremental Cost-Effectiveness Ratio(ICER) a cost-effectiveness analysis is investigated and determined that the combination of prevention and quarantine is the best cost-effective strategy from the other integrated strategies. Therefore, policymakers and stakeholders should apply the integrated intervention to stop the spread of MSV in the maize population.

1. Introduction

Maize (*Zea mays* L.), is the most important food source and cash crop for over 100 million peoples in Africa (M'mboyi et al., 2013). It is planted annually over an area of 15.5 million hectares(M'mboyi et al., 2013; Bosque-Pérez, 2000; Magenya et al., 2008). Maize is one of the main cereals grown by small-scale farmers primary for food and income generation (Mesfin et al., 1991). Maize streak disease (MSD) caused by Maize streak virus (genus Mastrevirus, family Geminiviridae) is the most devastating and destructive disease of maize in Sub-Saharan Africa (Mazengia, 2016; Schneider & Anderson, 2010). Maize streak disease (MSD) is a viral disease which has single-component, circular, single-stranded DNA (Bosque-Pérez, 2000; Magenya et al., 2008).

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MSD is a major threat to cereal crops amongst MSV transmitted by species of Cicadulina leafhoppers. Globally, 22 species of Cicadulina leafhoppers have been reported, of which 18 are found in Africa (Bosque-Pérez, 2000). *Cicadulina mbila* is the most predominant vector and the most important in the epidemiology of the virus(Bosque-Pérez, 2000) from the known 8 vector of MSV in the genus (Mesfin et al., 1991).

Mathematical modeling has become an important tool in understanding the dynamics of disease transmission and in decision making processes regarding intervention programs for disease control. To reduce the losses caused by the MSD different alternative tactics are used such as cultural control, biological control and chemical control (M'mboyi et al., 2013; Magenya et al., 2008; Karavina, 2014) and stakeholders are encouraged to combine at least two strategies in dealing with the disease (Jeger et al., 2004; Karavina, 2014).

Currently, vector-borne plant diseases have attracted the interest of many mathematical modeling researchers (Shi et al., 2014). Ordinary differential equations (ODEs) have been used to model plants infected with viruses (Jeger et al., 1998; Jeger et al., 2004; Shi et al., 2014; Alemneh, Makinde, & Theuri, 2019a, 2019b). For instance, the authors in (Shi et al., 2014), formulated and analyzed the dynamics of a vector-borne plant disease model. The study in (Alemneh et al., 2019a, 2019b), developed and analysed a mathematical model for MSV pathogen interaction with pest invasion on maize plant. The study in (Murwayi et al., Owour), formulated and analyzed a dynamical nonlinear plant vector borne dispersion disease model that incorporates insect and plant population at equilibrium and wind as a parameter of climate change. A mathematical model of plant disease with the effect of fungicide and obtained that the optimal control can reduce the number of infected hosts compared to that of without control formulated and analysed by (Anggriani et al., 2018). The study in (Meng & Li, 2010) developed a model to combat plant viruses by continuously removing infected plants and replacing them with healthy plants. A Mathematical model developed by the authors in (Anggrianiet al., 2017), studied the effects of plant disease transmission dynamics with roguing, replanting and additional treatment such as curative. An eco-epidemiological deterministic model for the transmission dynamics of maize MSD in maize plant developed and analyzed by (Alemneh et al., 2019a, 2019b). However, the study did not consider the optimal control theory and cost-effectiveness analysis.

On the other hand, fewer studies have investigated the dynamic optimization, or optimal control of plant diseases. From this, the work in (Kinene et al., 2015), formulated a mathematical model to investigate the transmission dynamics of Cassava brown streak disease and the cost-effectiveness of the control measures. Two time dependent controls(spraying with chemicals and uprooting & burning of infected plants) are used in the model and they found that uprooting and burning of infected plants is cost effective strategy. The Study in (Hugo et al., 2019), formulated a mathematical model for optimal control and cost-effectiveness analysis of tomato yellow leaf curl virus disease. The other work in (Collins & Duffy, 2016), developed a deterministic differential equation model with optimal control and investigated the impacts of foliar diseases on maize plant population. To the best of our knowledge, an eco-epidemiological SIHY maize streak virus dynamical model with application of optimal control and cost-effectiveness technique has not been done. Therefore, in this paper we are interested in filling this gap.

The work starts by developing an optimal control model for the model developed by (Alemneh et al., 2019a, 2019b), to study the optimal and cost-effective integrated strategy from all possible combinations of maize streak virus control strategies that were proposed by (Magenya et al., 2008; Karavina, 2014; Martin & Shepherd, 2009; Alegbejo et al., 2002). In this study we proposed time-dependent control strategies, namely (i) prevention, (ii) quarantine(uprooting and burning) and (iii) chemical control on the model in (Alemneh et al., 2019a, 2019b). In the study Pontryagin's Maximum Principle was used to derive necessary conditions for the optimal control of the disease. Further, we used Incremental cost-effectiveness ratio to investigate the cost-effectiveness of all possible combinations of the proposed strategies to decide the most efficient approach for reducing MSV with minimum costs.

We organized the rest of the work as follows: Section 2, devoted in formulation and description of the MSV model with its time dependent control intervention strategies is presented. In Section 3, we prove the existence of optimal control for the model. On the other hand, in Section 4, the formulation of Hamiltonian and optimality system using Pontryangin's Maximum Principle is done. In Section 5, we illustrate the numerical simulation results on the control strategies. The cost effectiveness analysis intervention strategy in Section 6 using incremental cost-effectiveness ratio (ICER) is performed. Lastly in Section 7 brief conclusion is presented.

2. Model formulation

The disease transmission considers two different populations, the Maize population $N_1(t)$ and the leafhopper vector population$N_2(t)$. Each of the subpopulations has two sub classes: susceptible and infected. At time t, let $S(t)$ denotes the density of the susceptible maize, and $I(t)$ denotes the density of the infected maize, So that

$$N_1(t) = S(t) + I(t) \tag{1}$$

The susceptible and infected leafhopper vector densities are denoted by H(t) and Y(t), respectively. Thus

$$N_2(t) = H(t) + Y(t) \tag{2}$$

Table 1
Description of parameters of the MSV model (3).

Parameter	Description
β_1	Predation and infection rate of infected leafhopper on susceptible maize plant
β_2	Predation and infection rate of susceptible leafhopper on infected maize plant
b	Conversion rate of infected leafhopper
q	Recruitment rate of susceptible leafhopper
K	Carrying capacity
C	Half saturation rate of susceptible leafhopper with infected maize plant
A	Half saturation rate of susceptible maize with infected plant
μ_1	Death rate of infected maize
μ_2	Death rate of susceptible leafhopper
μ_3	Death rate of infected leafhopper
r	Intrinsic growth rate of maize population

In the absence of leafhopper population and with no MSV disease, the maize population grows logistically with intrinsic growth rate r and environmental carrying capacity $K(K > 0)$. When the disease presents in maize population, the infected host population contributes to the susceptible host population growth towards the carrying capacity $K(K > 0)$. The susceptible leafhopper vectors are recruited at rate q and by eating infected maize plant at a rate β_2 moved to infected leafhopper subpopulation. The natural death rate for susceptible and infected leaf hoper is μ_2 and μ_3 respectively.

Susceptible plants are move to the infected subclass following contacts with infected leafhopper at a per capita rate β_1. If the maize plant once became infected, never recovers and gives zero or very low yield of maize. The maize plant population has natural death rate μ_1. Further more, the disease can not transmitted horizontally and vertically in both populations and it is not genetically inherited. The predation functional response of the leafhopper towards susceptible maize assumed Michaelis-Menten kinetics and used a Holling type II functional form with predation and infection coefficient β_1, β_2 and half saturation constant A and C. The description of the parameters are found in Table 1.

$$\begin{cases} \dfrac{dS}{dt} = rS\left(1-\dfrac{S+I}{K}\right) - \dfrac{\beta_1 SY}{A+S} \\[2ex] \dfrac{dI}{dt} = \dfrac{\beta_1 SY}{A+S} - \mu_1 I \\[2ex] \dfrac{dH}{dt} = q - \dfrac{\beta_2 IH}{C+I} - \mu_2 H \\[2ex] \dfrac{dY}{dt} = \dfrac{b\beta_2 IH}{C+I} - \mu_3 Y \end{cases} \tag{3}$$

with nonnegative initial conditions $S(0) = S_0$, $I(0) = I_0$, $H(0) = H_0$, $Y(0) = Y_0$.

Now, we introduced time dependent controls on the model (3), to identify policies that control MSD epidemic. We apply control strategies on the model with the following assumptions. The first strategy is prevention strategy (u_1) that protect susceptible from contacting the disease. Hence, it minimize or eliminate infected leafhopper-susceptible maize contacts by a factor $(1 - u_1(t))$. The control function $u_2(t)$ represents the control effort on the quarantine (i.e uprooting and burning) of infectious maize individuals. When infected maize individuals in the field are uprooted and burned, it increases their removal rate by $u_2(t)$ which again reduces infected maize -susceptible leafhopper contacts by a factor $(1 - u_2(t))$. We have a third control variable $u_3(t)$ for the leafhopper population. The insecticide chemical harms the entire population of leafhopper by raising their mortality rate by $u_3(t)$. On the time interval $[0, t_f]$, the control functions are performed. We used Pontryagin's Maximum Principle to determine the situations under which disease eradication can be attained in a finite moment. After incorporating the assumptions of the controls u_1, u_2 and u_3 in MSV model (3), we obtain the optimal control model:

$$\begin{cases} \dfrac{dS}{dt} = rS\left(1-\dfrac{S+I}{K}\right) - (1-u_1)\dfrac{\beta_1 SY}{A+S} \\[2ex] \dfrac{dI}{dt} = (1-u_1)\dfrac{\beta_1 SY}{A+S} - (u_2+\mu_1) I \\[2ex] \dfrac{dH}{dt} = q - (1-u_2)\dfrac{\beta_2 IH}{C+I} - (u_3+\mu_2) H \\[2ex] \dfrac{dY}{dt} = (1-u_2)\dfrac{b\beta_2 IH}{C+I} - (u_3+\mu_3) Y \end{cases} \tag{4}$$

2.1. Positivity of solution and boundedness

Its given that $N_1(S, I) = S(t) + I(t)$. Differentiating N_1 and simplifying, we get the expression

$$\begin{aligned}\frac{dN_1}{dt} &= rS\left(1-\frac{S+I}{K}\right)-\mu_1 I,\\ &\leq rS-\mu_1 I.\\ &= S(r+1)-(S+\mu_1 I),\\ &\leq \varsigma(r+1)-\varrho N_1.\end{aligned}$$

where $\varsigma = max\{S(0), K\}$ and $\varrho = min\{1, \mu_1\}$. Then

$$\frac{dN_1}{dt}+\varrho N_1 \leq \varsigma(r+1), \tag{5}$$

After solving equation (5) and evaluating it as $t \to \infty$, we got;

$$\Omega_h = \left\{(S,I)\epsilon\mathcal{R}_+^2 : N_1(t) \leq \frac{\varsigma}{\varrho}(r+1)\right\}.$$

Similarly, for leafhopper population $N_2(H, Y) = H(t) + Y(t)$, we get

$$\begin{aligned}\frac{dN_2}{dt} &= q-\mu_2 H-\mu_3 Y,\\ &\leq q-\xi N_2.\end{aligned}$$

Where $\xi = min(\mu_1, \mu_2)$. Then

$$\frac{dN_2}{dt}+\xi N_2 \leq q, \tag{6}$$

After solving equation (6) and evaluating it as $t \to \infty$, we got;

$$\Omega_v = \left\{(H,Y)\epsilon\mathcal{R}_+^2 : N_2(t) \leq \frac{q}{\xi}\right\}.$$

Therefore, the feasible solution set for the MSV model given by

$$\Omega = \Omega_h \times \Omega_v = \left\{(S,I,H,Y) \in \mathcal{R}_+^4 : N_1(t) \leq \frac{\varsigma}{\varrho}(r+1); N_2(t) \leq \frac{q}{\xi}\right\}. \tag{7}$$

Hence, Ω is positively invariant region, inside which the model is considered to be epidemiologically meaningful and mathematically well-posed. Therefore, with in the region the solution of model (3) is bounded.

Theorem 2.1. *Let $\Omega = \{(S,I,H,Y) \in \Re^4 : S(0) > 0, I(0) > 0, H(0) > 0, Y(0) > 0\}$. Then the solution set $(S(t), I(t), H(t), Y(t))$ of system (3) is positive for all $t \geq 0$.*

Proof. From the first equation of the model

$$\begin{aligned}\frac{dS}{dt} &= rS\left(1-\frac{S+I}{K}\right)-\frac{\beta_1 SY}{A+S},\\ &\leq rS\left(1-\frac{S}{K}\right).\end{aligned}$$

Then we have

$$\frac{dS}{S\left(1-\frac{S}{K}\right)} \leq rdt,$$

$\Rightarrow$

$$S(t) \le \frac{KS(0)}{e^{-rt}(K - S(0)) + S(0)}.$$

As $t \to \infty$, we obtain $0 \le S(t) \le K$. By using the same procedure, we obtained

$$I(t) \ge I(0)e^{-\mu_1 t} \ge 0, H(t) \ge H(0)e^{-\mu_2 t} \ge 0, Y(t) \ge Y(0)e^{-\mu_3 t} \ge 0.$$

Thus all solutions of the model are positive for all $t \ge 0$. □

3. Existence of optimal control

The ultimate aim is to find the optimal level of the intervention targeted to minimize infection and cost of the controls. In order to achieve this, the following objective functional is considered:

$$J = \int_{t_0}^{t_f} \left[d_1 I + d_2 Y + \frac{1}{2}\left(w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2 \right) \right] dt \tag{8}$$

where t_f is the final time, d_1 and d_2 are balancing constant coefficients of the infected maize and infected leafhopper respectively while w_1, w_2 and w_3 are weight coefficients for each individual control measure. We choose a nonlinear cost on the controls based on the assumption that the cost takes nonlinear form (Alemneh, 2020; Alemneh et al., 2020; Osman et al., 2020a). The goal is to find the optimal control triple (u_1^*, u_2^*, u_3^*) such that:

$$J(u_1^*, u_2^*, u_3^*) = min\{J(u_1, u_2, u_3) | (u_1, u_2, u_3) \in U\},$$

where the control set $U = \{(u_1, u_2, u_3) \mid u_i(t)$ is measurable on $[0, t_f]$, $0 \le u_i(t) \le 1, i = 1, 2, 3\}$.

The basic setup of the optimal control problem is to check the existence of the optimal controls. Thus, using the result from (Fleming & Rishel, 1976) the existence of an optimal control pair can be proved.

Theorem 3.1. *Given system (4) with $(S_0, I_0, H_0, Y_0) \ge (0, 0, 0, 0)$, and objective functional $J(u)$ of (4), then there exists an optimal control (u_1^*, u_2^*, u_3^*) and state solutions (S^*, I^*, H^*, Y^*) that minimizes $J(u)$ over U i.e*

$$J(u_1^*, u_2^*, u_3^*) = min\{J(u_1, u_2, u_3) | (u_1, u_2, u_3) \in U\}$$

The Proof is based on the following assumptions given in (Fleming & Rishel, 1976):

(i) The set of controls and corresponding state variables is nonempty.
(ii) The measurable control set is convex and closed.
(iii) The right hand side of the state system is bounded by a linear function in the state and control.
(iv) The integrand $g(x, u)$ of the objective functional is convex on U.
(v) There exist constants α_1, $\alpha_2 > 0$, and $\beta^* > 1$ such that the integrand of the objective functional satisfies

$$g \ge \alpha_1 \left(\left((u_1)^2 + (u_2)^2 + (u_3)^2 \right)^{\frac{\beta^*}{2}} - \alpha_2 \right.$$

Proof.

(i) We proved the boundedness of the model in Subsection 2.1. From which it follows that the solutions of the state system are continuous and bounded for each admissible control functions in U. Further, the right hand side functions of the model equation (4) satisfies the Lipschitz condition with respect to state variables. Hence, from Theorem 9.2.1 of Lukes in (Lukes, 1982), the solutions of system (4) exist. Thus, the set U is non empty.
(ii) It suffices to write $U = u_1 \times u_2 \times u_3$. So that $u_1 \times u_2 \times u_3$ is bounded and convex $\forall t \in [0, t_f]$.
(iii) By definition, each right hand side of system (4) is continuous. All variables S, I, H, Y and u are bounded on $[0, t_f]$. To prove the boundedness, we used the method in (Burden et al., 2004). To do so we use the fact that the supersolutions of system in equation (4) given by:

$$\begin{aligned} \frac{d\hat{S}}{dt} &= r\hat{S} \\ \frac{d\hat{I}}{dt} &= \beta_1\hat{S} \\ \frac{d\hat{H}}{dt} &= q+\beta_2\hat{H} \\ \frac{d\hat{Y}}{dt} &= b\beta_2\hat{H} \end{aligned} \tag{9}$$

are bounded on a finite time interval. System (9) can be written as

$$\times = \begin{bmatrix} \frac{d\hat{S}}{dt} \\ \frac{d\hat{I}}{dt} \\ \frac{d\hat{H}}{dt} \\ \frac{d\hat{Y}}{dt} \end{bmatrix} = \begin{bmatrix} r & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 \\ 0 & 0 & b\beta_2 & 0 \end{bmatrix} \begin{bmatrix} \hat{S} \\ \hat{I} \\ \hat{H} \\ \hat{Y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ q \\ 0 \end{bmatrix} \tag{10}$$

The system is linear in finite time with bounded coefficients, and therefore the supersolutions $\hat{S}$,$\hat{I}$,$\hat{H}$, and $\hat{Y}$ are uniformly bounded. Since the solution to each state equation is bounded, we see that,

$$f(t,\ \times,\ u) \le \begin{bmatrix} \frac{dS}{dt} \\ \frac{dI}{dt} \\ \frac{dH}{dt} \\ \frac{dY}{dt} \end{bmatrix} = \begin{bmatrix} r & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 \\ 0 & 0 & b\beta_2 & 0 \end{bmatrix} \begin{bmatrix} \widehat{S} \\ \widehat{I} \\ \widehat{H} \\ \widehat{Y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ q \\ 0 \end{bmatrix} \le K_1 \times + \mathrm{K}_2$$

where K_1 depends on the coefficients of the system. Thus, the assumption holds.

(iv) The integrand of the cost functional

$$J(I,Y,u) = d_1 I + d_2 Y + \frac{1}{2}\left(w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2\right)$$

is the sum of convex function and hence convex with respect to control variables.

(v) There exist constants α_1, $\alpha_2 > 0$ and $\beta* > 1$ such that the integrand L of the objective function

$$L(I,Y,u) = d_1 I + d_2 Y + \frac{1}{2}\left(w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2\right) \ge \frac{\alpha_1}{2}\left(u_1^2 + u_2^2 + u_3^2\right) - \alpha_2$$

where $\alpha_1 = min\{w_1, w_2, w_3\}$; $\beta* = 2$, $\alpha_2 > 0$ and d_1, $d_2 > 0$. Thus, this assumption is justified. Therefore, there exists an optimal control that minimizes the objective function.

4. The Hamiltonian and optimality system

We used Pontryangin's Maximum Principle(Pontryagin, 1987) to drive the necessary conditions that an optimal control must satisfy. This principle converts the system in equation (4) and equation (8) into a problem of minimizing point-wise Hamiltonian ($\mathcal{M}$), with respect to $u_1(t)$, $u_2(t)$ *and* $u_3(t)$ as:

$$\mathcal{M}(S,I,H,Y) = \frac{dJ}{dt} + \lambda_1\frac{dS}{dt} + \lambda_2\frac{dI}{dt} + \lambda_3\frac{dH}{dt} + \lambda_4\frac{dY}{dt}$$

$$\begin{aligned}\mathcal{M}(S,I,H,Y) &= d_1 I + d_2 Y + \frac{1}{2}\left(w_1u_1^2 + w_2u_2^2 + w_3u_3^2\right)\\ &+\lambda_1\left\{rS\left(1-\frac{S+I}{K}\right) - (1-u_1)\frac{\beta_1 SY}{A+S}\right\}\\ &+\lambda_2\left\{(1-u_1)\frac{\beta_1 SY}{A+S} - (u_2+\mu_1)I\right\}\\ &+\lambda_3\left\{q - (1-u_2)\frac{\beta_2 IH}{C+I} - (u_3+\mu_2)H\right\}\\ &+\lambda_4\left\{(1-u_2)\frac{b\beta_2 IH}{C+I} - (u_3+\mu_3)Y\right\}\end{aligned}$$

Where λ_i, $i = 1, 2, 3, 4$ are the adjoint variable associated with S, I, H, *and* Y to be determined suitably applying the principle (Pontryagin, 1987).

Theorem 4.1. *For an optimal control set u_1, u_2, u_3 that minimizes J over U, there are adjoint variables, λ_1, ..., λ_4 such that:*

$$\begin{cases}\frac{d\lambda_1}{dt} = -\left[r\left(1-\frac{2S+I}{K}\right) - (1-u_1)\frac{\beta_1 AY}{(A+S)^2}\right)\Big]\lambda_1 - (1-u_1)\frac{\beta_1 AY}{(A+S)^2}\lambda_2\\ \frac{d\lambda_2}{dt} = -d_1 + \frac{rS}{K}\lambda_1 + (u_2+\mu_1)\lambda_2 + (1-u_2)\frac{\beta_2 CH}{(C+I)^2}\lambda_3 - (1-u_2)\frac{b\beta_2 CH}{(C+I)^2}\lambda_4\\ \frac{d\lambda_3}{dt} = \left[(1-u_2)\frac{\beta_2 I}{C+I} + \mu_2 + u_3\right]\lambda_3 - (1-u_2)\frac{b\beta_2 I}{C+I}\lambda_4\\ \frac{d\lambda_4}{dt} = -d_2 + (1-u_1)\frac{\beta_1 S}{A+S}\lambda_1 - (1-u_1)\frac{\beta_1 S}{A+S}\lambda_2 + (u_3+\mu_3)\lambda_4\end{cases}$$

With transversality conditions, $\lambda_i(t_f) = 0$, $i = 1, ..., 4$. Furthermore, we obtain the control set (u_1^*, u_2^*, u_3^*) characterized by

$$\begin{aligned}u_1^* &= max\{0, min(1,\psi_1)\}\\ u_2^* &= max\{0, min(1,\psi_2)\}\\ u_3^* &= max\{0, min(1,\psi_3)\}\end{aligned}$$

Where

$$\begin{aligned}\psi_1 &= \frac{\beta_1 SY}{w_1(A+S)}(\lambda_2-\lambda_1)\\ \psi_2 &= \frac{\beta_2 IH}{w_2(C+I)}(b\lambda_4-\lambda_3) + \frac{\lambda_2 I}{w_2}\\ \psi_3 &= \frac{\lambda_3 H + \lambda_4 Y}{w_3}\end{aligned}$$

Proof. The adjoint equation and transversality conditions are standard results from Pontryagin's maximum principle (Pontryagin, 1987). We differentiate Hamiltonian with respect to state S, I, H and Y respectively gives then the adjoint system:

$$\begin{cases} \dfrac{d\lambda_1}{dt} = -\dfrac{d\mathcal{M}}{dS} = -\left[r\left(1-\dfrac{2S+I}{K}\right)-(1-u_1)\dfrac{\beta_1 AY}{(A+S)^2}\right]\lambda_1-(1-u_1)\dfrac{\beta_1 AY}{(A+S)^2}\lambda_2 \\ \dfrac{d\lambda_2}{dt} = -\dfrac{d\mathcal{M}}{dI} = -d_1+\dfrac{rS}{K}\lambda_1+(u_2+\mu_1)\lambda_2+(1-u_2)\dfrac{\beta_2 CH}{(C+I)^2}\lambda_3-(1-u_2)\dfrac{b\beta_2 CH}{(C+I)^2}\lambda_4 \\ \dfrac{d\lambda_3}{dt} = -\dfrac{d\mathcal{M}}{dH} = \left[(1-u_2)\dfrac{\beta_2 I}{C+I}+\mu_2+u_3\right]\lambda_3-(1-u_2)\dfrac{b\beta_2 I}{C+I}\lambda_4 \\ \dfrac{d\lambda_4}{dt} = -\dfrac{d\mathcal{M}}{dY} = -d_2+(1-u_1)\dfrac{\beta_1 S}{A+S}\lambda_1-(1-u_1)\dfrac{\beta_1 S}{A+S}\lambda_2+(u_3+\mu_3)\lambda_4 \end{cases}$$

With transversality conditions, $\lambda_i(t_f)=0$, $i=1, \ldots, 4$. Similarly, following the principle (Pontryagin, 1987), the characterization of optimal controls u_1^*, u_2^*, u_3^* (i.e the optimality equations) are obtained based on the conditions:

$$\frac{\partial\mathcal{M}}{\partial u_1}=\frac{\partial\mathcal{M}}{\partial u_2}=\frac{\partial\mathcal{M}}{\partial u_3}=0$$

Thus, we get

$$\begin{aligned} \frac{\partial\mathcal{M}}{\partial u_1} &= w_1u_1+\frac{\beta_1 SY}{A+S}\lambda_1-\frac{\beta_1 SY}{A+S}\lambda_2 \\ \frac{\partial\mathcal{M}}{\partial u_2} &= w_2u_2-I\lambda_2+\frac{\beta_2 IH}{C+I}\lambda_3-\frac{b\beta_2 IH}{C+I}\lambda_4 \\ \frac{\partial\mathcal{M}}{\partial u_3} &= w_3u_3-\lambda_3 H-Y\lambda_4 \end{aligned}$$

Setting $\frac{\partial\mathcal{M}}{\partial u_i}=0$ at $u_i^*, i=1,2,3$, the results are

$$u_1^*=\frac{\beta_1 SY}{w_1(A+S)}(\lambda_2-\lambda_1), u_2^*=\frac{\beta_2 IH}{w_2(C+I)}(b\lambda_4-\lambda_3)+\frac{\lambda_2 I}{w_2}\quad and\quad u_3^*=\frac{\lambda_3 H+\lambda_4 Y}{w_3}.$$

When we write by using standard control arguments involving the bounds on the controls, we conclude

$$u_1^*=\begin{cases}\psi_1, & \text{if } 0<\psi_1<1;\\ 0, & \text{if } \psi_1\le 0;\\ 1, & \text{if } \psi_1\ge 1\end{cases},\ u_2^*=\begin{cases}\psi_2, & \text{if } 0<\psi_2<1;\\ 0, & \text{if } \psi_3\le 0;\\ 1, & \text{if } \psi_3\ge 1\end{cases},\ u_3^*=\begin{cases}\psi_3, & \text{if } 0<\psi_3<1;\\ 0, & \text{if } \psi_3\le 0;\\ 1, & \text{if } \psi_3\ge 1\end{cases}$$

In compact notation

$$u_1^*=max\{0,min(1,\psi_1)\},\quad u_2^*=max\{0,min(1,\psi_2)\},\quad u_3^*=max\{0,min(1,\psi_3)\}$$

The optimality system is formed from the optimal control system (the state system) and the adjoint variable system by incorporating the characterized control set and initial and transversal condition

$$
\begin{cases}
\dfrac{dS}{dt} & = rS\left(1-\dfrac{S+I}{K}\right)-(1-u_1^*)\dfrac{\beta_1 SY}{A+S} \\
\dfrac{dI}{dt} & = (1-u_1^*)\dfrac{\beta_1 SY}{A+S}-(u_2^*+\mu_1)I \\
\dfrac{dH}{dt} & = q-(1-u_2^*)\dfrac{\beta_2 IH}{C+I}-(\mu_2+u_3^*)H \\
\dfrac{dY}{dt} & = (1-u_2^*)\dfrac{b\beta_2 IH}{C+I}-(u_3^*+\mu_3)Y \\
\dfrac{d\lambda_1}{dt} & = -\left[r\left(1-\dfrac{2S+I}{K}\right)-(1-u_1^*)\dfrac{\beta_1 AY}{(A+S)^2}\right)\Big]\lambda_1-(1-u_1^*)\dfrac{\beta_1 AY}{(A+S)^2}\lambda_2 \\
\dfrac{d\lambda_2}{dt} & = -d_1+\dfrac{rS}{K}\lambda_1+(u_2^*+\mu_1)\lambda_2+(1-u_2^*)\dfrac{\beta_2 CH}{(C+I)^2}\lambda_3-(1-u_2^*)\dfrac{b\beta_2 CH}{(C+I)^2}\lambda_4 \\
\dfrac{d\lambda_3}{dt} & = \left[(1-u_2^*)\dfrac{\beta_2 I}{C+I}+\mu_2\right]\lambda_3-(1-u_2^*)\dfrac{b\beta_2 I}{C+I}\lambda_4 \\
\dfrac{d\lambda_4}{dt} & = -d_2+(1-u_1^*)\dfrac{\beta_1 S}{A+S}\lambda_1-(1-u_1^*)\dfrac{\beta_1 S}{A+S}\lambda_2+(u_3^*+\mu_3)\lambda_4 \\
u_1^* & = max\left\{0, min\left(1, \dfrac{\beta_1 SY}{w_1(A+S)}(\lambda_2-\lambda_1)\right)\right\} \\
u_2^* & = max\left\{0, min\left(1, \dfrac{\beta_2 IH}{w_2(C+I)}(b\lambda_4-\lambda_3)+\dfrac{\lambda_2 I}{w_2}\right)\right\} \\
u_3^* & = max\left\{0, min\left(1, \dfrac{\lambda_3 H+\lambda_4 Y}{w_3}\right)\right\}
\end{cases}
$$

$$
\lambda_i\left(t_f\right) = 0, i=1,\ldots,4 \quad S(0)=S_0 \quad, I(0)=I_0 \quad, H(0)=H_0 \quad, Y(0)=Y_0
$$

4.1. Uniqueness of the optimality system

Due to the a priori boundedness of the state, adjoint functions and the resulting Lipschitz structure of the ODEs, we can obtain the uniqueness of solutions of the optimality system for the small time interval. Hence the following theorem.

Theorem 4.2. *For $t \in [0, t_f]$, the bounded solutions to the optimality system are unique. For the Proof of the theorem* (Fister et al., 1998).

5. Numerical simulations

In this section, we studied numerically the effects of optimal control strategies such as prevention strategies, quarantine and chemical control of infected maize in the spread of MSV. The solution of the optimal control problem was obtained by solving the optimality system of state and adjoint systems through forward?backward sweep method. We start by solving the state equations with an initial guess for the controls over the simulated time using forward fourth order Runge-Kutta scheme. Then we proceed solving the adjoint equations by backward fourth order Runge-Kutta scheme using the current iteration solutions of the state equation and the transversality conditions. The controls continues to be updated by combining from the previous result of the controls with the characterization. The solution of the state and adjoint system is repeated by the updated controls. This condition continues repeatedly up to when consecutive iterations are close enough each other (Workman & Lenhart, 2007).

Next, we investigate numerically the effect of the optimal control strategies on the spread of MSV which incorporate more than one intervention are ordered below and compared pairwise:

- **Strategy A:** Using prevention (u_1) and quarantine (u_2) but without chemical control (u_3)
- **Strategy B:**Using prevention (u_1) and chemical control (u_3) but without quarantine (u_2)
- **Strategy C:** Using quarantine (u_2) and chemical control (u_3) but without prevention (u_1)
- **Strategy D:** Using all the three controls prevention (u_1), quarantine (u_2) and chemical control(u_3)

We used balancing constants $d_1 = 40$, $d_2 = 5$, $w_1 = 100$, $w_2 = 40$ and $w_3 = 200$ for simulation of MSV disease model with optimal control and also for cost-effectiveness analysis. In addition to this, we used $S(0) = 1000$, $I(0) = 20$, $H(0) = 100$, $Y(0) = 0$ as initial values in addition to parameter values in Table 2.

5.1. Strategy A: control with prevention and chemical control

Prevention and chemical control are used to optimise the objective functional J while the other control (u_2) is set to zero. Fig. 1 shows that, the number of infected maize goes down in the specified time and the number of infected leafhopper decreased as compared to without control case. Therefore, this strategy is effective in eradicating the disease from the community in a specified period of time.

5.2. Strategy B: control with prevention and quarantine

We used prevention and quarantine controls as intervention strategies to optimise the objective function J while we set the chemical control u_3, to zero. We observed from Fig. 2, that optimal control of the combination of prevention and quarantine helps to bring down the infectious maize in the specified time as well as decrease the number of infectious leafhopper population. Therefore the strategy helps to eradicate the disease in the maize community.

5.3. Strategy C: control with quarantine and chemical control

We used quarantine and chemical as intervention strategy to optimise the objective functional J. Fig. 3 shows that, the number of infective maize decreased in the specified time and the number of infective leafhopper reduced compared to no control which helps in eradicating the disease from the community in a specified period of time.

5.4. Strategy D: control with prevention, quarantine and chemical control

We now implement all the three controls interventions to optimise the objective functional J. Fig. 4 shows that the number of the infectious maize decrease at the specified time and infected leafhopper population reduced as compared with no control due to the intervention strategies. Therefore, applying this strategy helps to eradicate MSD from the maize field in specified period of time.

6. Cost-effective analysis

To rank the implemented strategies in terms of their cost we used cost-effectiveness analysis. To achieve this, we used incremental cost-effectiveness ratio (ICER), which is done dividing the difference of costs between two strategies to the difference of the total number of their infections averted (Alemneh et al., 2020; Osman et al., 2020b). The total number of infections averted for each strategy is estimated by subtracting total infections with control from without control. To get the total cost of each strategy, we used their respective cost function $\left(\left(\frac{1}{2}\right)w_1u_1^2, \left(\frac{1}{2}\right)w_2u_2^2, \text{and } \left(\frac{1}{2}\right)w_3u_3^2 \right)$ to calculate over the time of intervention. We used the parameter values in Table 2 and apply ICER technique, first we ordered the intervention strategies for pairwise comparison as in Table 3 from A to D with increasing order of effectiveness.

First, we compared the cost effectiveness of strategy A and B.

$$ICER(A) = \frac{1074.50067}{633.9179} = 1.695$$

$$ICER(B) = \frac{2088.9449 - 1074.50067}{1872.19088 - 633.9179} = 0.8192$$

Table 2
Parameter values for the MSV model.

Parameter symbol	Value	Source
β_1	0.45	Bosque-Pérez (2000)
β_2	0.04	Bosque-Pérez (2000)
q	0.02	Alemneh et al. (2019a, 2019b)
K	10, 000	Alemneh et al. (2019a, 2019b)
μ_1	0.008	Alemneh et al. (2019a, 2019b)
μ_2	0.0303	Magenya et al. (2008)
μ_3	0.0303	Magenya et al. (2008)
b	0.45	Alemneh et al. (2019a, 2019b)
A	0.4	Alemneh et al. (2019a, 2019b)
C	0.6	Alemneh et al. (2019a, 2019b)
r	0.0005	Alemneh et al. (2019a, 2019b)

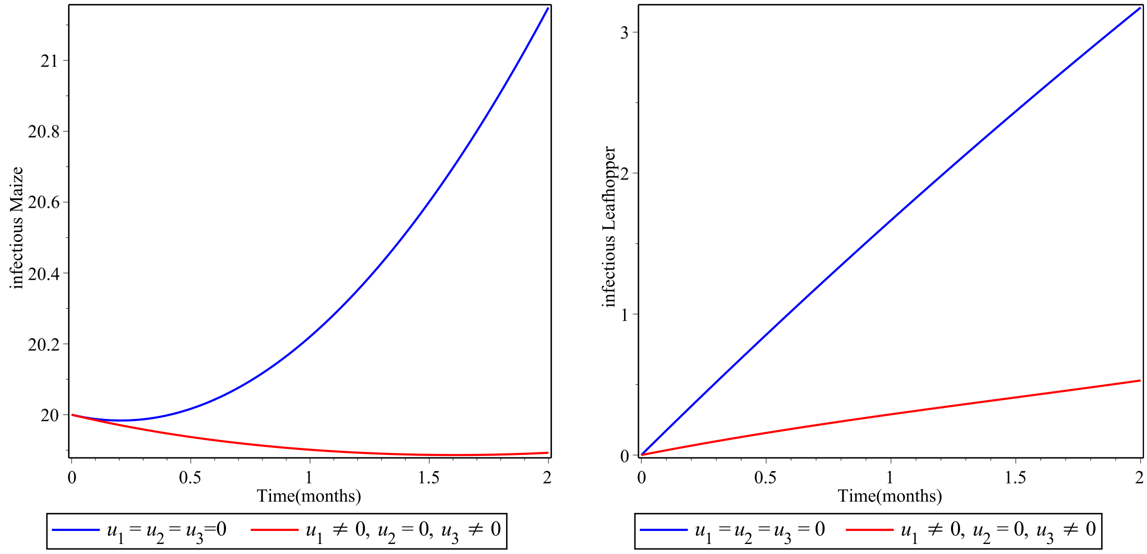

Fig. 1. Simulations of the MSD model with prevention and chemical controls.

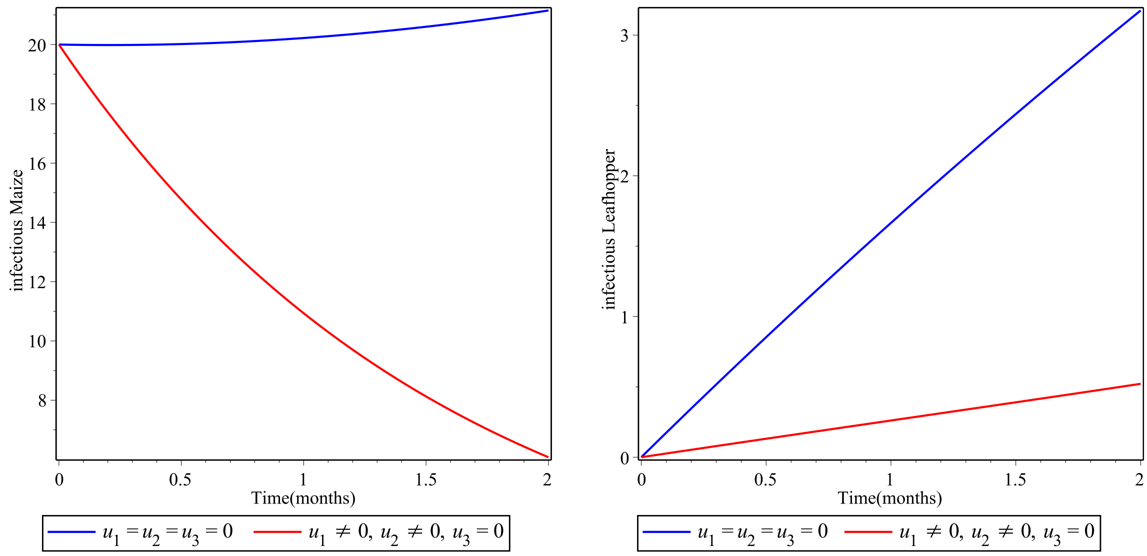

Fig. 2. Simulations of the MSD model with prevention and quarantine controls.

The comparison between strategies A and B indicate that strategy A is strongly dominated and is more costly than strategy B as $ICER(B) < ICER(A)$ then strategy A is excluded in set of alternative hence B and C are compared.

$$ICER(B) = \frac{2088.9449}{1872.19088} = 1.11577$$

$$ICER(C) = \frac{2213.69825 - 2088.9449}{1877.9344 - 1872.19088} = 21.7207$$

Similarly, from ICER (B) and ICER (C) we can see that strategy C is strongly dominated and more costly than B as $ICER(B) < ICER(C)$ then strategy C is excluded in set of alternative hence B and D are compared.

$$ICER(B) = \frac{2088.9449}{1872.19088} = 1.11577$$

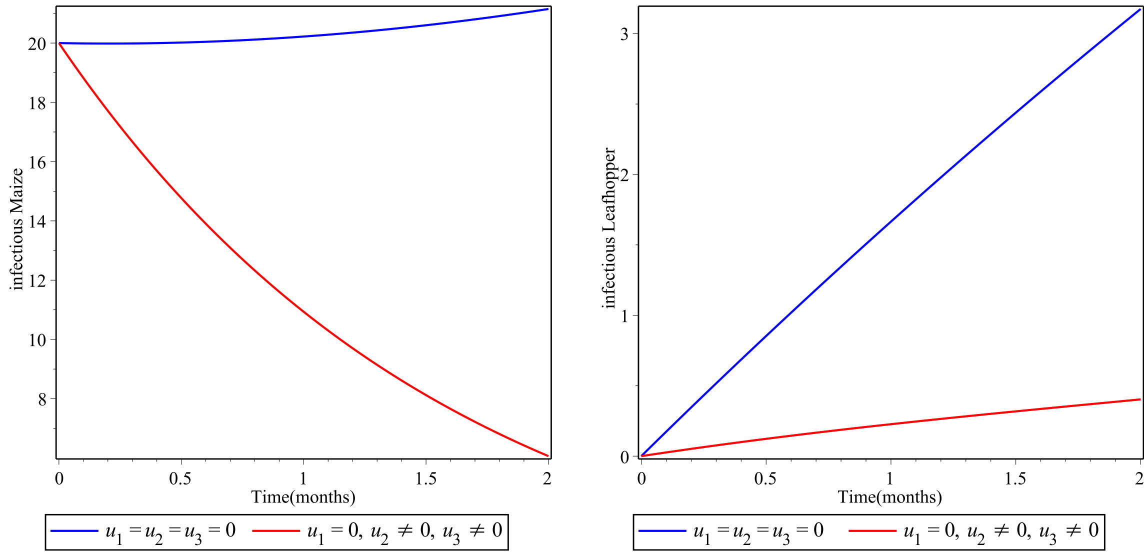

Fig. 3. Simulations of the MSD model with quarantine, and chemical controls.

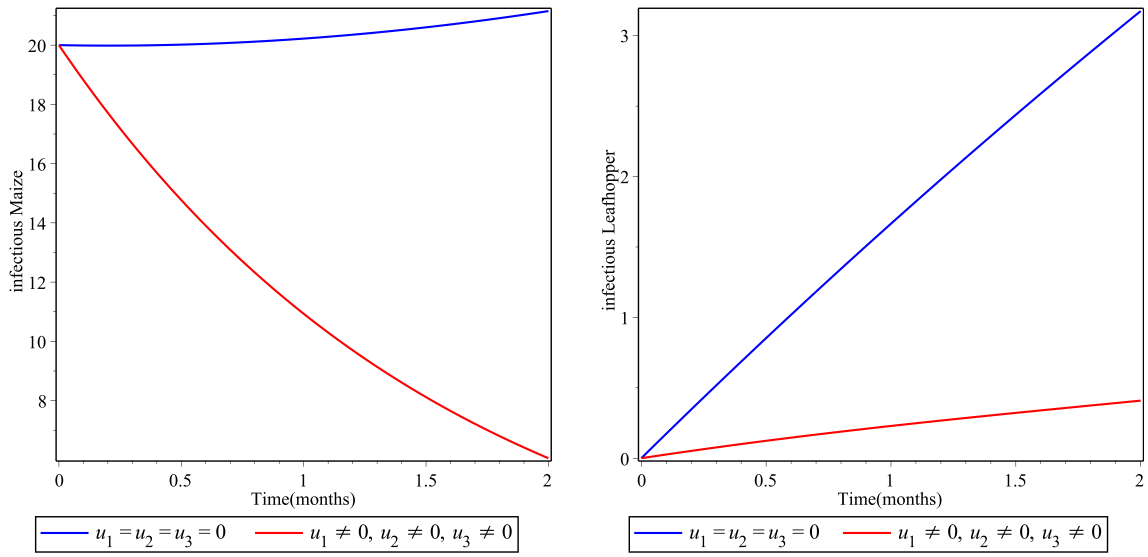

Fig. 4. Simulations of the MSD model with prevention, quarantine, and chemical controls.

Table 3
Number of infections averted and total cost of each strategy.

Strategy	Description	Total infections averted	Total cost (USD)
A	Prevention and chemical	633.9179	$1074.50067
B	Prevention and quarantine	1872.19088	$ 2088.9449
C	Quarantine and chemical	1877.9344	$ 2213.69825
D	Prevention,quarantine and chemical	1880.7669	$ 2248.8348

$$ICER(D) = \frac{2248.8348 - 2088.9449}{1880.7669 - 1872.19088} = 18.6438$$

The comparison shows that $ICER(B) < ICER(D)$, hence strategy D is more costly and excluded in the set of alternatives.
Therefore, we conclude that strategy B(Prevention and quarantine) is the cheapest of all compared strategies, which meant it is the most cost-effective for MSD control interventions strategy's.

7. Discussions and conclusions

In this study, a deterministic mathematical model with optimal control for the dynamics of MSV was formulated and analyzed to find the best strategy for controlling MSD in maize population. We derived and analyzed the necessary conditions for optimal control strategies using Pontryagin's maximum principle (Hugo et al., 2019; Alemneh, 2020). With the principle, we obtained the Hamiltonian, the adjoin variables, the characterization of the controls and the optimality system. For minimizing the spread of MSD, we used intervention strategies such as Prevention (u_1), quarantine (u_2) and chemical control (u_3) as it is suggested by (Magenya et al., 2008; Martin & Shepherd, 2009). In Section 5, we numerically analyzed the optimality system by considering the different integrated strategies. It shows that each approach has the power to manage disease transmission, which is correlated with a similar result for tomatoes and cassava diseases (Kinene et al., 2015; Hugo et al., 2019). In Section 6, we numerically investigated cost-effectiveness to determine, the least and the most expensive strategies by using ICER. From the pairwise result, the integrated disease management strategy called prevention and quarantine is the best cost-effective strategies in terms of cost as well as environmentally welcoming and health benefits(Not poisonous to the soil environment and humans). Present results were agreed with the findings on tomato disease (Hugo et al., 2019). The work done by (Kinene, Luboobi, Nannyonga, & Mwanga, 2015) on cassava concluded that uprooting & burning is a cost-effective strategy that is close to one of our outcomes even though it is a single strategy. The other half strategy prevention, agrees with the findings of study (Anggriani et al., 2018) on plant disease. However, they recommended without applying cost-effectiveness and this made strategy may not be a cost-effective.

To conclude, we established an optimal control SIHY model for Maize streak virus disease with three time dependent control strategies. We obtained the optimality system of the optimal control model. The Numerical results clearly directs that each integrated management strategy have a power to combat the disease. However, from the cost-effectiveness analysis report, prevention and quarantine is the cost-effective integrated strategy from the other options to mitigate MSD in maize plant.

Declaration of competing interest

The authors declare no potential conflict of interest.

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