Which variables could predict the Equity Risk Premium?

Abstract

This paper conducts variable selection, parameters estimation and models comparison in order to detect which of given 13 explanatory variables could better explain the equity risk premium. We take SP 500 index returns and Treasury-bill rates. We use multiple linear regression model as our benchmark model. The variable selection is done by Forward and Backward variable selection methods, the parameters estimation is done by Ordinary Least Squares and to compare models we use R^2, Adjusted R^2, AIC and BIC. We find that the model with *Net Equity Expansion* and *Dividend Yield* variables is best in terms of AIC and BIC. However, the model with all explanatory variables is better in terms of R^2 and Adjusted R^2.

1 Introduction

The Equity Risk Premium (ERP) is the extra return that's available to equity investors above the return they could get by investing in a risk-free investment. The higher ERP, the more odds that investors would fill their portfolio with equities rather than with bonds. Hence, the accurate prediction of ERP is crucial in order to make better investment decisions. So the aim of our research is to find which variables could predict ERP more accurately and which adjustments we could make to improve the accuracy of the models.

Our research use SP 500 index and Treasury Bill as a risky and risk-free investment, respectively. Our data have 13 explanatory variables and the variable which we want to predict *Next Log Equity Risk Premium*. To make predictions we use multiple linear regression and OLS method to obtain estimations for unknown parameters.

The results of this paper found that model with all 13 explanatory variables have the best results for R^2, Adjusted R^2, while the model with *Net Equity Expansion* and *Dividend Yield* variables have the best results for AIC and BIC. Also we found that models' prediction accuracy could be improved by adding powers of fitted values and by splitting data into two dataset, the breakpoint is 1980.

In section 3 we provide more information about the variables, in section 4 we describe the models, methods and tests which we use in our research. Finally, in section 5 we have results of our research.

2 Theory

In order to find the variables which predict the ERP accurately we perform variable selection methods as Backward and Forward variable selection. We use the lowest p-value as the selection criterion

To compare models we use the following prediction accuracy measurements R^2, Adjusted R^2, AIC and BIC. Asymptotically, the BIC is consistent, it will select the true model if, among other assumptions, the true model is among the candidate models considered. The AIC is not consistent under these circumstances. When the true model is not in the candidate model set the AIC is efficient, so it will asymptotically choose whichever model minimizes the error of prediction. The BIC is not efficient under these circumstances (Vrieze, 2012).

Next, we also need to investigate if we have a model misspecification. Misspecified models create: biases to parameter estimates, inconsistent standard errors and an invalid asymptotic distribution of the χ^2 test statistic (White, 1982).

Finally, the potential set of explanatory variables was taken from Welch and Goyal (2008).

3 Data

To perform our research we choose SP 500 index and Treasury Bill as a risky and risk-free investment, respectively. We use SP 500 index returns from Center for Research in Security Press(CRSP) and the Treasury-bill rates, from 1926 to 2005, which gives 86 observations. The data contains 13 explanatory variables and dependent variable *Next Log Equity Risk Premium.*

The first group of explanatory variables give information that relates to stock:

- *Dividend Price Ratio* is the difference between the log of dividend and the log of prices
- *Dividend Yield* is the difference between the log of dividends and the log of lagged prices
- *Earnings Price Ratio* is the difference between the log of earnings and the log of prices
- *Stock Variance* is calculated as sum of squared daily returns on the S&P 500
- *Book-to-Market Ratio* is the ratio of book value to market value
- *Net Equity Expansion* is the ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total market capitalization of NYSE stocks
- *Percent Equity Issuing* is the ratio of equity issuing activity as a fraction of total issuing

The second group of explanatory variables provide information about interest-rate :

- *Treasury-bill Rates*
- *The Default Return Spread* is the difference between long-term corporate bond and long-term government bond returns
- *Default Yield Spread*) is the difference between BAA and AAA-rated corporate bond yields
- *Long Term Rate of Returns*
- *The Term Spread* is the difference between the long term yield on government bonds and the Treasury-bill
- *Inflation*

Summary descriptive statistics of variables is shown in Table 1

	Mean	Median	Max	Min	Stdev	Skewness	Kurtosis
BOOKMARKETRATIO	0.577	0.557	1.442	0.131	0.262	0.564	3.317
DEFAULTRETSPREAD	0.003	0.011	0.179	-0.171	0.045	-0.190	7.068
DEFAULTYIELDSPR	0.012	0.010	0.051	0.003	0.008	2.326	9.873
DIVPRICERATIO	-3.347	-3.333	-2.293	-4.478	0.463	-0.373	2.804
DIVYIELD	-3.293	-3.237	-2.434	-4.503	0.445	-0.681	2.984
EARNPRICERATIO	-2.723	-2.767	-1.893	-4.106	0.413	-0.338	3.796
INFLATION	0.031	0.028	0.179	-0.104	0.041	0.218	6.259
LONGTERMRATERETURNS	0.061	0.037	0.404	-0.149	0.098	1.015	4.388
NETEQUITYEXPANSION	0.019	0.019	0.166	-0.042	0.027	2.402	14.525
NEXTLOGEQPREM	0.057	0.077	0.422	-0.564	0.192	-0.763	3.714
PERCEQISSUING	0.193	0.165	0.721	0.037	0.108	1.757	8.488
STOCKVARIANCE	0.036	0.019	0.309	0.003	0.049	3.357	16.116
TERMSPREAD	0.016	0.016	0.045	-0.035	0.014	-0.459	3.906
TREASBILLRATE	0.036	0.031	0.155	0.000	0.031	1.046	4.425

Table 1: Summary statistics of data

Figure 1 presents the plot of *nextlogeqpremium* and *bookmarketratio*. We can observe that the very often the direction of lines for both variables is similar. The variables follow opposite directions in the beginning of 1930 and 1970.

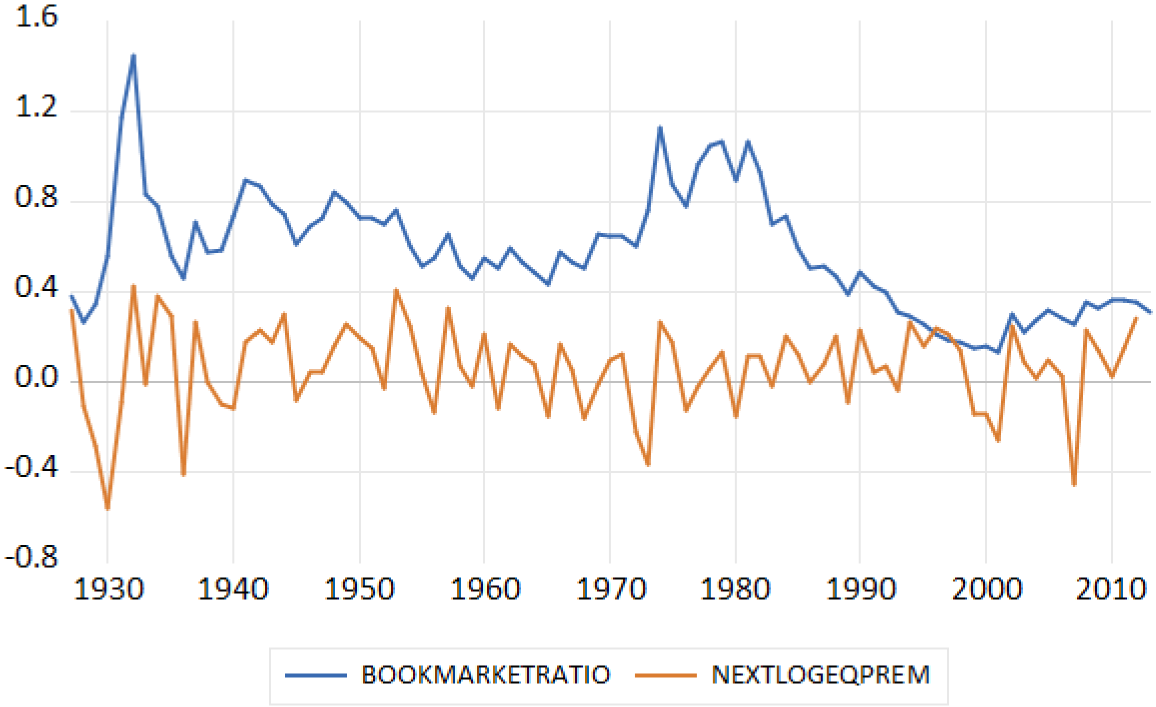

Figure 1: Plot of nextlogeqprem with bookmarketratio

4 Methodology

Our benchmark model is the multiple linear regression model(MLR) including the full base set of predictors, which we estimate using ordinary least square (OLS) estimation. The OLS method, under certain assumptions, has some convenient statistical properties which made OLS one of the most popular methods for regression analysis. The estimates obtained by OLS are unbiased, consistent, normally distributed and have minimum variance among unbiased linear estimators. We can reasonably assume that the variables satisfy the following necessary assumptions:

1. Fixed regressors: x_1, ... , x_n are fixed variables, satisfying $\sum_{i=1}^{n}(x_i - \bar{x})^2 > 0$
2. Random disturbances,zero mean: $\epsilon_1, ..., \epsilon_n$ are random variables satisfying $E(\epsilon_i) = 0$
3. Homoskedasticity: the variances of $\epsilon_1, ..., \epsilon_n$ exist and are all equal, $E(\epsilon_i^2) = \sigma^2$
4. No correlation: all pairs (ϵ_i, ϵ_j) are uncorrelated, $E(\epsilon_i \epsilon_j) = 0, i \neq j$
5. Constant parameters: β and σ are fixed unknown numbers with $\sigma > 0$
6. The linear regression model is "linear in parameters."

In contrast to the first method, the second method we use results in a MLR model with only a subset of our base set of variables as predictors. We perform Stepwise regression (SR), using backward and forward selection. Forwards selection starts with no additional regressors in the regression, then it adds the variable with the lowest p-value. The variable with the next lowest p-value given that the first variable has already chosen, is then added. The procedure ends when the lowest p-value of the variables not yet included is greater than the specified forwards stopping criterion. Backwards selection procedure starts with all possible variables included in the model. The variable with the highest p-value is first removed. The variable with the next highest p-value, given the removal of the first variable, is also removed. The routine ends when the largest p-value of the variables inside the model is less than the specified backwards stopping criterion.

We perform Regression Specification Error Test(RESET) which was proposed by Ramsey (Ramsey, 1969). RESET is a general test for the following types of specification errors: omitted variables, incorrect functional form, i.e, some or all of the variables in model should be transformed to logs, powers or in some other way and correlation between explanatory variables and residuals. Output from the test reports the test regression and the F-statistic and log likelihood ratio for testing the hypothesis that the coefficients on the powers of fitted values are all zero. In our test we used as omitted variables powers of fitted values from two to four. We set the significance level at five percent.

Also we perform Chow Forecast Test and Chow's Breakpoint Test which were proposed by Chow (Chow, 1960). The Chow Forecast Test tells us if the regression coefficients are different for split data sets. So, it tests whether one regression line or two separate regression lines best fit a split set of data. The Chow Breakpoint Test compares the sum of squared residuals obtained by fitting a model to the entire sample with the sum of squared residuals obtained when separate models are fit to each subsample of the data. The significance level is five percent.

5 Results

In this section we provide the results obtained in our research.

Table 2 shows the estimated parameters for the model with all 13 explanatory variables, for the model with selected variables obtained using general-to-specific and specific-to-general methods. In the Table 2 M1 is a model with all 13 explanatory variables, M2 is a model which use a Backward Stepwise variable selection method and M3 is a model which apply a Forward Stepwise. After performing Backward selection only two explanatory variables are significant, namely NETEQUITYEXPANSION and DIVYIELD. In case of Forward selection PERCEQISSUING and DIVPRICERATIO are significant.

Variable	M1	M2	M3
C	0.481669	0.424261	0.539543
NETEQUITYEXPANSION	-1.239569	-2.150287	-
LONGTERMRATERETURNS	0.384551	-	-
INFLATION	0.209157	-	-
PERCEQISSUING	-0.375861	-	-0.600545
STOCKVARIANCE	0.875183	-	-
TERMSPREAD	0.906696	-	-
TREASBILLRATE	-0.530363	-	-
EARNPRICERATIO	0.037276	-	-
DIVYIELD	0.021542	0.099274	-
DIVPRICERATIO	0.055556	-	0.109530
DEFAULTYIELDSPR	-8.058484	-	-
DEFAULTRETSPREAD	0.305552	-	-
BOOKMARKETRATIO	0.120908	-	-

Table 2: Estimated parameters

Table 5 presents the statistics that measure the success of the regression in predicting the values of the dependent variable within the given sample. In the table 3, Model 1 is the model with all 13 explanatory variables included; Model 2 is the model with only a constant and *bookmarketratio* included; Model 3 is the model with a constant, *netequityexpansion* and *divyield* included. We can see that R^2 and Adjusted R^2 prefer model with all explanatory variables to models with selected variables. For model comparison, the model with the lowest Akaike information criterion(AIC) and Bayesian information criterion(BIC) score is preferred. The absolute values of the AIC and BIC scores do not matter. Hence, AIC and BIC both prefer Model 3.

	Model 1	Model 2	Model 3
R^2	0.226136	0.046744	0.121176
Adjusted R^2	0.226136	0.035396	0.100000
AIC	-0.401559	-0.472141	-0.530184
BIC	-0.002014	-0.415064	-0.444568

Table 3: Models Result

Table 4 provides the test statistics and p-values of conducted RESET tests for Model 1, Model 2 and Model 3. At five percent level of significance we do not reject the null hypothesis in all three models, i.e the coefficients on the powers of fitted values are not equal to zero. In our test we used as omitted variables powers of fitted values from two to four.

	Model 1	Model 2	Model 3
F-statistic	0.561395	0.374598	2.347239
p-value	0.6423	0.7715	0.0789

Table 4: RESET test results

We perfrom Chow Forecast Test and Chow's Breakpoint Test for Model 2. At five percent level of significance we do not reject the null hypothesis, i.e there is a structural change in equity risk premium before 1980 and after 1980, both tests give the same results.

	Chow Forecast Test	Chow's Breakpoint Test
F-statistic	0.784466	0.7684
p-value	2.318593	0.1048

Table 5: Chow test results

6 Conclusion

In conclusion, there are two models which can be used to predict equity risk premium, the full model and the model obtained by variable selection with *Net Equity Expansion* and *Dividend Yield* as explanatory variables(Model 2). However, the conducted tests tell that both models have omitted variables as powers of fitted values from two to four. Thus, we can improve our models' prediction accuracy by adding those omitted variables.

Moreover, the Chow's Tests detected that we have a structural change in our data before 1980 and after 1980. So, two separate regression lines best fit a split set of data.

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