The `tikz-3dplot-circleofsphere` Package: Drawing circles of a sphere with `tikz-3dplot`

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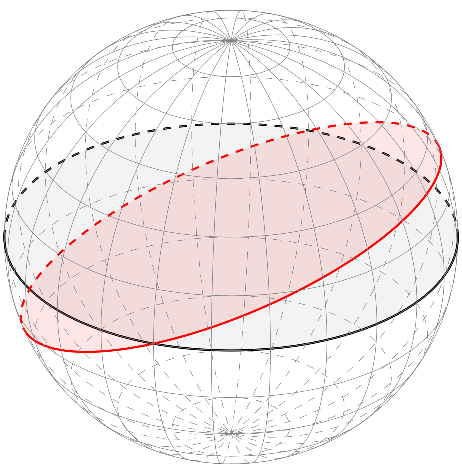
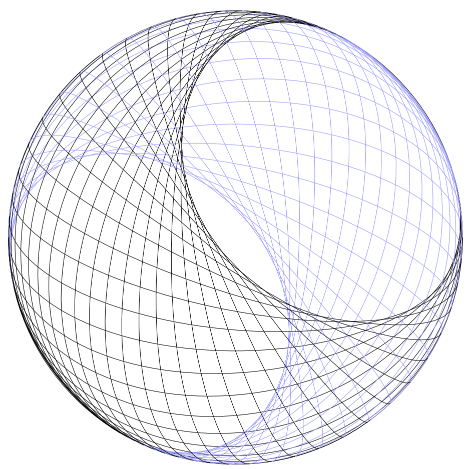
Abstract

A *circle of a sphere* is a circle drawn on a spherical surface like, for instance, circles of latitude or longitude. Circles in arbitrary 3D positions can be drawn with TikZ [2] very easily using a transformed coordinate system provided by the `tikz-3dplot` package [1] (that is because TikZ can only draw circles on the xy-plane). However, automatically distinguishing the parts of the circle lying on the front and back sides of the sphere, e.g. by drawing a solid arc on the front side and a dashed one on the back side, is a somewhat tricky feat. The `tikz-3dplot-circleofsphere` package will perform that feat for you.

Note: Package and documentation are under construction!





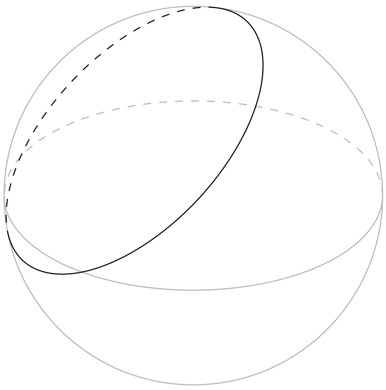
```latex
\documentclass{standalone}
\usepackage{tikz-3dplot-circleofsphere}
\begin{document}
  \centering
  \def\R{3}
  \tdplotsetmaincoords{60}{125}
  \begin{tikzpicture}[tdplot_main_coords]
    \draw[tdplot_screen_coords,very thin,gray] (0,0,0) circle (\R);
    \tdplotCsDrawLatCircle%
      [thick,tdplotCsFill/.style={opacity=0.05}]{\R}{0}
    \tdplotCsDrawGreatCircle%
      [red,thick,tdplotCsFill/.style={opacity=0.1}]{\R}{105}{-23.5}
    \foreach \a in {-75,-60,...,75}
      {\tdplotCsDrawLatCircle[very thin,gray]{\R}{\a}}
    \foreach \a in {0,15,...,165}
      {\tdplotCsDrawLonCircle[very thin,gray]{\R}{\a}}
  \end{tikzpicture}
\end{document}
```

```latex
\documentclass{standalone}
\usepackage{tikz-3dplot-circleofsphere}
\begin{document}
  \centering
  \def\R{3}
  \tdplotsetmaincoords{60}{125}
  \begin{tikzpicture}[tdplot_main_coords]
    \def\e{80};
    \draw[tdplot_screen_coords,very thin] (0,0,0) circle (\R);
    \foreach \a in {0,5,...,175} {
      \tdplotCsDrawGreatCircle%
        [very thin, tdplotCsBack/.style={very thin,blue!40}]%
        {\R}{\a}{90*sin(\a)*sin(\e)}
    }
  \end{tikzpicture}
\end{document}
```

Contents

1 Just Looking for the Minimalist Code?

There you go!



```
\documentclass{standalone}
\usepackage{tikz,tikz-3dplot}
%% >> MINIMALIST CIRCLE OF SHPERE DRAWING CODE _______________________________________________________________
\newcommand\scircle[4]{%
  \tdplotsetrotatedcoords{#2}{#3}{0}                                     % Rotate coordinate system
  \let\a\tdplotalpha                                                     % alpha (rotated coord. system)
  \let\b\tdplotbeta                                                      % beta (rotated coord. system)
  \let\p\tdplotmainphi                                                   % phi (main coord. system)
  \let\t\tdplotmaintheta                                                 % theta (main coord. system)
  \pgfmathsetmacro\azx{cos(\a)*cos(\b)*sin(\p)*sin(\t) - sin(\b)*cos(\t) - cos(\b)*cos(\p)*sin(\a)*sin(\t)}
  \pgfmathsetmacro\azy{-cos(\a)*cos(\p)*sin(\t) - sin(\a)*sin(\p)*sin(\t)}
  \pgfmathsetmacro\azz{cos(\b)*cos(\t) + cos(\a)*sin(\b)*sin(\p)*sin(\t) - cos(\p)*sin(\a)*sin(\b)*sin(\t)}
  \pgfmathsetmacro\Re {#1*cos(#4)}                                       % Radius of circle
  \pgfmathsetmacro\ze {#1*sin(#4)}                                       % z-coordinate of drawing plane
  \pgfmathsetmacro\coX{\ze*cos(#2)*sin(#3)}                              % x-coordinate offset for ze
  \pgfmathsetmacro\coY{\ze*sin(#2)*sin(#3)}                              % y-coordinate offset for ze
  \pgfmathsetmacro\coZ{\ze*cos(#3)}                                      % z-coordinate offset for ze
  \coordinate (coffs) at (\coX,\coY,\coZ);                               % Offset as coordinate value
  \tdplotsetrotatedcoordsorigin{(coffs)}                                 % Offset coordinate system
  \begin{scope}[tdplot_rotated_coords]                                   % Drawing scope >>
    \pgfmathsetmacro\tanEps{tan(#4)}                                     %   Tangent of elevation angle
    \pgfmathsetmacro\bOneside{((\tanEps)^2)>=(((\azx)^2+(\azy)^2)/(\azz)^2)} %   Circle entirely on one side?
    \ifthenelse{\bOneside=1}{%                                           %   Circle on one side of sphere >>
      \pgfmathsetmacro\bFrontside{(\azx*\Re+\azz*\ze)>=0}                %     Circle entirely on front side?
       \ifthenelse{\bFrontside=1}                                        %     |
         {\draw (0,0) circle (\Re);}                                     %     Draw on front side
         {\draw[dashed] (0,0) circle (\Re);}                             %     Draw on back side
    }{%                                                                  %   << Circle on both sides >>
      \pgfmathsetmacro\u{\azy}                                           %     Substitution u=...
      \pgfmathsetmacro\v{sqrt( (\azx)^2 + (\azy)^2 - (\azz)^2*(\tanEps)^2 )} %     Substitution v=...
      \pgfmathsetmacro\w{\azx - \azz*\tanEps}                            %     Substitution w=...
      \pgfmathsetmacro\aPhiBf{2*atan2(\u-\v,\w)}                         %     Back->front crossing angle
      \pgfmathsetmacro\aPhiFb{2*atan2(\u+\v,\w)}                         %     Front->back crossing angle
      \pgfmathsetmacro\bUnwrapA{(\aPhiFb-\aPhiBf)>360}                   %     Unwrap front->back angle #1?
      \pgfmathsetmacro\bUnwrapB{\aPhiBf>\aPhiFb}                         %     Unwrap front->back angle #2?
      \ifthenelse{\bUnwrapA=1}{\pgfmathsetmacro\aPhiBf{\aPhiBf+360}}{}   %     Unwrap front->back angle #1
      \ifthenelse{\bUnwrapB=1}{\pgfmathsetmacro\aPhiBf{\aPhiBf-360}}{}   %     Unwrap front->back angle #2
      \draw[dashed] (\aPhiFb:\Re) arc (\aPhiFb:{\aPhiBf+360}:\Re);       %     Draw back side arc
      \draw (\aPhiBf:\Re) arc (\aPhiBf:\aPhiFb:\Re);                     %     Draw back side arc
    }                                                                    %   <<
  \end{scope}                                                            % << (Drawing scope)
}
%% << ________________________________________________________________________________________________________
\begin{document}
  \tdplotsetmaincoords{60}{125}                                          % Set main coordintate system
  \begin{tikzpicture}[tdplot_main_coords]                                % TikZ picture >>
    \begin{scope}[black!30]                                              %   Draw in gray >>
      \draw[tdplot_screen_coords] (0,0,0) circle (2.5);                    %     Sphere outline
      \scircle{2.5}{0}{0}{0}                                               %     Equator
    \end{scope}                                                          %   <<
    \scircle{2.5}{-40}{40}{30}                                             %   Draw another sphere circle
  \end{tikzpicture}                                                      % <<
\end{document}
```

Want some more convenience or interested in what we did? Read on...

2 The tikz-3dplot-circleofsphere Package

2.1 Installation

Download `tikz-3dplot-circleofsphere.sty` from [3] file into your project folder and include the package with `\usepackage{tikz-3dplot-circleofsphere}`. That's all.

2.2 Drawing Commands

```
\tdplotCsDrawCircle[style]{r}{alpha}{beta}{epsilon}
```

[TODO: . . .]

```
\tdplotCsDrawPoint[style]{r}{alpha}{beta}{epsilon}
```

[TODO: . . .]

2.3 Geographic Drawing Commands

```
\tdplotCsDrawCircleLL[style]r}{lat}{lon}{elev}
```

[TODO: . . .]

```
\tdplotCsDrawLatitudeCircleLL[style]r}{lat}
```

[TODO: . . .]

```
\tdplotCsDrawLongitudeCircleLL[style]r}{lon}
```

[TODO: . . .]

```
\tdplotCsDrawPointLL[style]{r}{lat}{lon}
```

[TODO: . . .]

2.4 Auxiliary Commands

```
\tdtdplotCsFrontsidePoint
```

Invoked by `\tdplotCsDrawPoint` to draw a point on the front side of a sphere. Redefine to customize.

```
\tdtdplotCsBacksidePoint
```

Invoked by `\tdplotCsDrawPoint` to draw a point on the back side of a sphere. Redefine to customize.

```
\tdplotCsComputeTransformRotScreen
```

Computes the elements of the full rotation matrix

$$A = \begin{pmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{pmatrix}.$$

See Section 3.1 for details.

Parameters

none

Output

`\axx`	Element a_{xx} of full rotation matrix
`\axy`	Element a_{xy} of full rotation matrix
...	
`\azz`	Element a_{zz} of full rotation matrix

Remarks

The command uses some internal variables of `tikz-3dplot`, namely `\tdplotalpha`, `\tdplotbeta`, `\tdplotmainphi`, and `\tdplotmaintheta`.

2.5 Examples

Examples **??** and **??** (see below) demonstrate the usage of the `tikz-3dplot-circleofsphere` package.

[**TODO:** Fix examples!]

3 Implementation Details

3.1 The Maths

Circles on a Sphere

[**TODO:** Briefly explain!]

[**TODO:** Make a picture!]

Coordinate Transforms with `tikz-3dplot`

For drawing circles on a sphere, we use the `circle` and `arc` path construction operations of TikZ. As TikZ will only draw circles and arcs on the xy-plane, we need to rotate and possibly offset the coordinate system for drawing circles of spheres. This functionality is provided by the `tikz-3dplot` [1] package.

First, `tikz-3dplot` provides a *main coordinate system* which is basicly defining the view point a 3D coordinate system. Denote by $P = (x\,y\,z)^\top$ a point in the 3D coordinate system. `tikz-3dplot` transforms that point in to screen coordinates $P' = (x'\,y'\,z')^\top$ by

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R^d(\phi, \theta) \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

with the rotation matrix[1]

$$
\begin{aligned}
R^d(\phi,\theta) &= \left(R^{z'}(\phi)\, R^x(\theta)\right)^\top && (2)\\
&= \left(\begin{pmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{pmatrix}\right)^\top\\
&= \begin{pmatrix}\cos\phi & \sin\phi & 0\\ -\cos\theta\sin\phi & \cos\theta\cos\phi & +\sin\theta\\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta\end{pmatrix}.
\end{aligned}
$$

Second, for drawing circles and arcs outside the xy-plane, we need to rotate the coordinate system further. To this end, we use `tikz-3dplot`'s *rotated coordinate system*[2]

$$
\begin{pmatrix}x'\\ y'\\ z'\end{pmatrix} = R^d(\phi,\theta)\, D(\alpha,\beta,\gamma) \begin{pmatrix}x\\ y\\ z\end{pmatrix} \tag{3}
$$

with the rotation matrix (cf. [1, p. 7])

$$
\begin{aligned}
D(\alpha,\beta,0) &= R^z(\alpha)R^y(\beta) && (4)\\
&= \begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{pmatrix}\\
&= \begin{pmatrix}\cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta\\ \sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta\\ -\sin\beta & 0 & \cos\beta\end{pmatrix}
\end{aligned}
$$

where we deliberately omitted the last rotation $R^z(\gamma)$ by choosing $\gamma = 0$. Thus, the full rotation matrix for drawing a great circle is

$$
\begin{aligned}
A = \begin{pmatrix}a_{xx} & a_{xy} & a_{xz}\\ a_{yx} & a_{yy} & a_{yz}\\ a_{zx} & a_{zy} & a_{zz}\end{pmatrix} &= R^d(\phi,\theta)\, D(\alpha,\beta,0) && (5)\\
= \begin{pmatrix}\cos\phi & \sin\phi & 0\\ -\cos\theta\sin\phi & \cos\theta\cos\phi & \sin\theta\\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta\end{pmatrix} &\begin{pmatrix}\cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta\\ \sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta\\ -\sin\beta & 0 & \cos\beta\end{pmatrix}\\
= \left(\begin{matrix}\cos\alpha\cos\beta\cos\phi + \cos\beta\sin\alpha\sin\phi\\ \cos\beta\cos\phi\sin\alpha\cos\theta - \cos\alpha\cos\beta\cos\theta\sin\phi - \sin\beta\sin\theta\\ \cos\alpha\cos\beta\sin\phi\sin\theta - \sin\beta\cos\theta - \cos\beta\cos\phi\sin\alpha\sin\theta\end{matrix}\right.\\
\begin{matrix}\cos\alpha\sin\phi - \cos\phi\sin\alpha\\ \cos\alpha\cos\phi\cos\theta + \sin\alpha\cos\theta\sin\phi\\ -\cos\alpha\cos\phi\sin\theta - \sin\alpha\sin\phi\sin\theta\end{matrix}\\
\left.\begin{matrix}\cos\alpha\cos\phi\sin\beta + \sin\alpha\sin\beta\sin\phi\\ \cos\beta\sin\theta - \cos\alpha\sin\beta\cos\theta\sin\phi + \cos\phi\sin\alpha\sin\beta\cos\theta\\ \cos\beta\cos\theta + \cos\alpha\sin\beta\sin\phi\sin\theta - \cos\phi\sin\alpha\sin\beta\sin\theta\end{matrix}\right)
\end{aligned}
$$

With the coordinate transforms described so far, we can only draw circles and arcs whose center is the origin of the main coordinate systems. For drawing other circles on a sphere, we additionally need to offset the origin of the rotated coordinate system. This is provided by the `\tdplotsetrotatedcoordsorigin` command of `tikz-3dplot`.

[**TODO:** Describe how!]

[1] Equation (2.1) in [1] seems to be incorrect. I used a version with changes marked in red: Since $\left(R^{z'}(\phi)\, R^x(\theta)\right)^\top = R^x(\theta)^\top\, R^{z'}(\phi)^\top$, rotations are performed on opposite order and direction.

[2] Equation (2.4) in [1] seems to be incorrect. I used a version with changes marked in red: Rotations are performed in opposite order.

Drawing Circles of a Sphere

The parametric representation of a circle at a plane parallel to the xy-plane is

$$\begin{pmatrix} x(\varphi) \\ y(\varphi) \\ z(\varphi) \end{pmatrix} = \begin{pmatrix} r_e \cos\varphi \\ r_e \sin\varphi \\ z_e \end{pmatrix}, \tag{6}$$

where $-180° < \varphi \leq 180°$ the angle parameter,

$$r_e = \cos\epsilon \tag{7}$$

the radius,

$$z_e = \sin\epsilon \tag{8}$$

the height above the xy-plane, and ϵ the elevation angle. Fig. 1 shows an illustration.

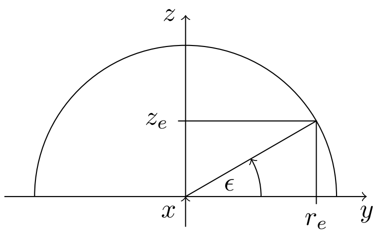


Figure 1: Illustration of z-coordinate and radius of an elevated circle on a sphere

Note that we actually *draw* this circle in the rotated *and offset* coordinate system where it takes the form

$$\begin{pmatrix} x(\varphi) \\ y(\varphi) \\ z(\varphi) \end{pmatrix} = \begin{pmatrix} r_e \cos\varphi \\ r_e \sin\varphi \\ 0 \end{pmatrix}. \tag{9}$$

However, we will stick to Eqn. (6) for simplicity. The screen coordinates for Eqn. (6) are

$$\begin{aligned} \begin{pmatrix} x'(\varphi) \\ y'(\varphi) \\ z'(\varphi) \end{pmatrix} &= A \begin{pmatrix} x(\varphi) \\ y(\varphi) \\ z(\varphi) \end{pmatrix} = \begin{pmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{pmatrix} \begin{pmatrix} r\cos\epsilon\cos\varphi \\ r\cos\epsilon\sin\varphi \\ r\sin\epsilon \end{pmatrix} \\ &= \begin{pmatrix} a_{xx}\cdot r\cos\epsilon\cos\varphi + a_{xy}\cdot r\cos\epsilon\sin\varphi + a_{xz}\cdot r\sin\epsilon \\ a_{yx}\cdot r\cos\epsilon\cos\varphi + a_{yy}\cdot r\cos\epsilon\sin\varphi + a_{yz}\cdot r\sin\epsilon \\ a_{zx}\cdot r\cos\epsilon\cos\varphi + a_{zy}\cdot r\cos\epsilon\sin\varphi + a_{zz}\cdot r\sin\epsilon \end{pmatrix}. \end{aligned} \tag{10}$$

The $z'(\varphi)$ coordinates are not plotted. However, they are useful for determining which parts of the circle are

$$\begin{aligned} \text{on the front side} \quad & z'(\varphi) \geq 0 \quad \text{and} \\ \text{on the back side} \quad & z'(\varphi) < 0 \end{aligned} \tag{11}$$

of the sphere. We denote by φ_0 the crossing angles between the front and back sides. In order to determine them we solve

$$0 \stackrel{!}{=} z'(\varphi_0) = a_{zx}\cdot r\cos\epsilon\cos\varphi_0 + a_{zy}\cdot r\cos\epsilon\sin\varphi_0 + a_{zz}\cdot r\sin\epsilon. \tag{12}$$

I must admit that I was too lazy to puzzle this out myself. . . ;-) Matlab says:

$$\tan\left(\frac{\varphi_0}{2}\right) = \frac{a_{zy}\cos\epsilon \pm \sqrt{a_{zx}^2\cos^2\epsilon + a_{zy}^2\cos^2\epsilon - a_{zz}^2\sin^2\epsilon}}{a_{zx}\cos\epsilon - a_{zz}\sin\epsilon} \tag{13}$$

$$= \frac{a_{zy} \pm \sqrt{a_{zx}^2 + a_{zy}^2 - a_{zz}^2\tan^2\epsilon}}{a_{zx} - a_{zz}\tan\epsilon}, \tag{14}$$

where

$$a_{zz}^2 \sin^2 \epsilon \geq (a_{zx}^2 + a_{zy}^2) \cos^2 \epsilon \quad \rightsquigarrow \quad \tan^2 \epsilon \geq \frac{a_{zx}^2 + a_{zy}^2}{a_{zz}^2} \tag{15}$$

must hold. With the substitutions

$$u = a_{zy}, \tag{16}$$

$$v = \sqrt{a_{zx}^2 + a_{zy}^2 - a_{zz}^2 \tan^2 \epsilon} \quad \text{and} \tag{17}$$

$$w = a_{zx} - a_{zz} \tan \epsilon \tag{18}$$

we get

$$\tan\left(\frac{\varphi_0}{2}\right) = \frac{u \pm v}{w} \quad \rightsquigarrow \quad \varphi_0 = \begin{cases} 2 \operatorname{arctan2}(u + v, w) \\ 2 \operatorname{arctan2}(u - v, w) \end{cases} \tag{19}$$

Here we used the $\operatorname{arctan2}(x, y)$ function which is defined as

$$\operatorname{arctan2}(x, y) = \begin{cases} \arctan\left(\frac{x}{y}\right) & y > 0 \\ \arctan\left(\frac{x}{y}\right) + \pi & y < 0, x \geq 0 \\ \arctan\left(\frac{x}{y}\right) - \pi & y < 0, x < 0 \\ \frac{\pi}{2} & y = 0, x > 0 \\ -\frac{\pi}{2} & y = 0, x < 0 \\ 0 & y = 0, x = 0 \end{cases} \tag{20}$$

Iff condition (15) holds, Eqn. (12) has exactly two solutions,[3]

$$\varphi_{0,\mathrm{bf}} : \text{angle of back to front side crossing and}$$
$$\varphi_{0,\mathrm{fb}} : \text{angle of front to back side crossing,}$$

Otherwise it has no solutions, which means that the circle lies entirely either on the front side or on the back side of the sphere.

3.2 The Package Source Code

```
%% == LaTeX PACKAGE tikz-3dplot-circleofsphere ===============================
%%    Drawing circles of a sphere with tikz-3dplot
%%
%% Matthias Wolff, BTU Cottbus-Sentenberg
%% July 27, 2018
%%
%% References:
%% [1] J. Hein. The tikz-3dplot package. 2012. Online, retrieved July 20, 2018.
%%     http://mirror.ctan.org/graphics/pgf/contrib/tikz-3dplot/tikz-3dplot_documentation.pdf
%% [2] T. Tantau. TikZ & PGF - Manual for Version 3.0.1a. 2015. Online, retrieved July 22, 2018.
%%     http://mirror.ctan.org/graphics/pgf/base/doc/pgfmanual.pdf
%% [3] Drawing Great Circles
%%     https://tex.stackexchange.com/questions/168521/spherical-triangles-and-great-circles

%% == REQUIRED PACKAGES ======================================================

\RequirePackage{xifthen}
\RequirePackage{tikz}
\RequirePackage{tikz-3dplot}

%% == TikZ STYLES ============================================================

\tikzset{
  tdplotCsFront/.style={solid},
```

[3] which coincide iff the left and right sides of condition (15) are equal

```
  tdplotCsBack/.style={dashed},
  tdplotCsFill/.style={opacity=0},
  tdplotPtFront/.style={},
  tdplotPtBack/.style={},
  tdplotCsDrawAux/.style={}
}

%% == COMMANDS ===============================================================

\newcommand{\tdplotCsComputeTransformRotScreen}{%
  % Computes the elements of the full rotation matrix
  %
  %   A = [\axx \axy \axz]
  %       [\ayx \ayy \ayz]
  %       [\azx \azy \azz].
  %
  % Ouput:
  %   \axx - Element A(1,1) of rotation matrix
  %   \axy - Element A(1,2) of rotation matrix
  %   ...
  %   \azz - Element A(3,3) of rotation matrix
  %
  \let\a\tdplotalpha
  \let\b\tdplotbeta
  \let\p\tdplotmainphi
  \let\t\tdplotmaintheta
  % Row 1: [\axx \axy \axz]
  \pgfmathsetmacro\axx{cos(\a)*cos(\b)*cos(\p) + cos(\b)*sin(\a)*sin(\p)}
  \pgfmathsetmacro\axy{cos(\a)*sin(\p) - cos(\p)*sin(\a)}
  \pgfmathsetmacro\axz{cos(\a)*cos(\p)*sin(\b) + sin(\a)*sin(\b)*sin(\p)}
  % Row 2: [\ayx \ayy \ayz]
  \pgfmathsetmacro\ayx{cos(\b)*cos(\p)*sin(\a)*cos(\t) - cos(\a)*cos(\b)*cos(\t)*sin(\p) - sin(\b)*sin(\t)}
  \pgfmathsetmacro\ayy{cos(\a)*cos(\p)*cos(\t) + sin(\a)*cos(\t)*sin(\p)}
  \pgfmathsetmacro\ayz{cos(\b)*sin(\t) - cos(\a)*sin(\b)*cos(\t)*sin(\p) + cos(\p)*sin(\a)*sin(\b)*cos(\t)}
  % Row 3: [\azx \azy \azz]
  \pgfmathsetmacro\azx{cos(\a)*cos(\b)*sin(\p)*sin(\t) - sin(\b)*cos(\t) - cos(\b)*cos(\p)*sin(\a)*sin(\t)}
  \pgfmathsetmacro\azy{-cos(\a)*cos(\p)*sin(\t) - sin(\a)*sin(\p)*sin(\t)}
  \pgfmathsetmacro\azz{cos(\b)*cos(\t) + cos(\a)*sin(\b)*sin(\p)*sin(\t) - cos(\p)*sin(\a)*sin(\b)*sin(\t)}
}

\newcommand{\tdplotCsDrawCircleOfSphere}[5][]{%
  % Draws a circle of a sphere.
  %
  % Input:
  %   #1 - TikZ style
  %        - use tdplotCsFront/.style={blub} to style the visible semicircle
  %        - use tdplotCsBack/.style={blah} to style the invisible semicircle
  %        - use tdplotCsFill/.style={foo} to style the fill of the circle
  %        - use tdplotCsDrawAux to draw some auxiliary information
  %   #2 - Radius of sphere
  %   #3 - Azimutal angle of drawing plane 1)
  %   #4 - Polar angle of drawing plane 2)
  %   #5 - Elevation angle of circle above the drawing plane. Permissible
  %        values are -90 < #5 < 90. Use 0 for drawing a great circle.
  %
  % Ouput:
  %   none
  %
  % Footnotes:
  %   1) passed as alpha to \tdplotsetrotatedcoords{alpha}{beta}{gamma}
  %   2) passed as beta to \tdplotsetrotatedcoords{alpha}{beta}{gamma}
  \begin{scope}[#1]                                                  % Macro scope >>
    % Do some computation                                            #   ----------------------------------
    \pgfmathsetmacro\R   {#2}                                        %   Parse radius
    \pgfmathsetmacro\aAlp{#3}                                        %   Parse azimuthal angle (alpha)
    \pgfmathsetmacro\aBet{#4}                                        %   Parse polar angle (beta)
    \pgfmathsetmacro\aEps{#5}                                        %   Parse elevation angle (epsilon)
    \pgfmathsetmacro\Re  {\R*cos(\aEps)}                             %   Radius of circle
    \pgfmathsetmacro\ze  {\R*sin(\aEps)}                             %   z-coordinate of drawing plane
    \pgfmathsetmacro\coX {\ze*cos(\aAlp)*sin(\aBet)}                 %   x-coordinate offset for ze
    \pgfmathsetmacro\coY {\ze*sin(\aAlp)*sin(\aBet)}                 %   y-coordinate offset for ze
    \pgfmathsetmacro\coZ {\ze*cos(\aBet)}                            %   z-coordinate offset for ze
```

```
    \coordinate (coffs) at (\coX,\coY,\coZ);                                 %  Offset as coordinate value
    % Rotate and offset coordinate system                                    %  ------------------------------------
    \tdplotsetrotatedcoords{\aAlp}{\aBet}{0}                                 %  Rotate coordinate system
    \tdplotsetrotatedcoordsorigin{(coffs)}                                   %  Offset coordinate system
    % Draw                                                                   %  ------------------------------------
    \begin{scope}[tdplot_rotated_coords]                                     %  Drawing scope >>
      \tdplotCsComputeTransformRotScreen                                     %    Compute full rotation matrix
      \pgfmathsetmacro\tanEps{tan(\aEps)}                                    %    Tangent of elevation angle
      \pgfmathsetmacro\bOneside{((\tanEps)^2)>=(((\azx)^2+(\azy)^2)/(\azz)^2)} %    Circle entirely on one side?
      \fill[tdplotCsFill] (0,0) circle (\Re);                                %    Draw fill of circle
      \ifthenelse{\bOneside=1}{                                              %    Circle on one side of sphere >>
        \pgfmathsetmacro\bFrontside{(\azx*\Re+\azz*\ze)>=0}                   %      Circle entirely on front side?
         \ifthenelse{\bFrontside=1}                                          %      |
          {\draw[tdplotCsFront] (0,0) circle (\Re);}                         %      Draw on front side
          {\draw[tdplotCsBack]  (0,0) circle (\Re);}                         %      Draw on back side
      }{                                                                     %    << Circle on both sides >>
        \pgfmathsetmacro\u{\azy}                                             %      Substitution u=...
        \pgfmathsetmacro\v{sqrt( (\azx)^2 + (\azy)^2 - (\azz)^2*(\tanEps)^2 )} %      Substitution v=...
        \pgfmathsetmacro\w{\azx - \azz*\tanEps}                              %      Substitution w=...
        \pgfmathsetmacro\aPhiBf{2*atan2(\u-\v,\w)}                           %      Back->front crossing angle
        \pgfmathsetmacro\aPhiFb{2*atan2(\u+\v,\w)}                           %      Front->back crossing angle
        \pgfmathsetmacro\bUnwrapA{(\aPhiFb-\aPhiBf)>360}                     %      Unwrap front->back angle #1?
        \pgfmathsetmacro\bUnwrapB{\aPhiBf>\aPhiFb}                           %      Unwrap front->back angle #2?
        \ifthenelse{\bUnwrapA=1}{\pgfmathsetmacro\aPhiBf{\aPhiBf+360}}{}     %      Unwrap front->back angle #1
        \ifthenelse{\bUnwrapB=1}{\pgfmathsetmacro\aPhiBf{\aPhiBf-360}}{}     %      Unwrap front->back angle #2
        \draw[tdplotCsBack]  (\aPhiFb:\Re) arc (\aPhiFb:{\aPhiBf+360}:\Re);  %      Draw back side arc
        \draw[tdplotCsFront] (\aPhiBf:\Re) arc (\aPhiBf:\aPhiFb:\Re);        %      Draw back side arc
      }                                                                      %    <<
      % Auxliliary drawing (for debugging and illustration)                  %    - - - - - - - - - - - - - - - - -
      \ifthenelse{\isin{tdplotCsDrawAux}{#1}}{                               %    Auxiliary drawing activated >>
        \draw[red!40,->] (-\Re,0,0) -- (\Re,0,0) node[anchor=north] {$x_d$}; %      x-axis of drawing corrd. system
        \draw[red!40,->] (0,-\Re,0) -- (0,\Re,0) node[anchor=north] {$y_d$}; %      y-axis of drawing corrd. system
        \draw[red!40,->] (0,0,0)    -- (0,0,\Re) node[anchor=north] {$z_d$}; %      z-axis of drawing corrd. system
        \ifthenelse{\bOneside=0}{                                            %      Circ.on both sides of sphere >>
          \node[red] at (\aPhiBf:\Re) {$\circ$};                             %        Indicate back-front crossing
          \node[red] at (\aPhiFb:\Re) {$\times$};                            %        Indicate front-back crossing
        }{}                                                                  %      <<
        \coordinate (coffs) at (-\coX,-\coY,-\coZ);                          %      HACK: Forcibly reset ...
        \tdplotsetrotatedcoordsorigin{(coffs)}                               %      ... coordinate system
        \begin{scope}[tdplot_rotated_coords]                                 %      Aux. display scope >>
          \node[tdplot_screen_coords,red,anchor=north west] at (0.7*\R,-0.9*\R) %        Make a litte display ...
            {\parbox{200pt}{\footnotesize                                    %        ... >>
              $\theta=\tdplotmaintheta^\circ, \phi=\tdplotmainphi^\circ$\\   %          Main coord. sys. parameters
              $\alpha=\aAlp^\circ, \beta=\aBet^\circ,                         %          Rot. coord. sys. parameters
               \epsilon\!=\!\aEps^\circ\!$\\                                  %          Drawing plane elev. angle
              $a_{zx}=\azx, a_{zy}=\azy, a_{zz}=\azz$\\                       %          Elems. of full rot. matrix
              $r_e\!=\!\Re, z_e\!=\!\ze$\\                                    %          Radius and z-elevation
              $\texttt{\textbackslash bOneside}\!=\!\bOneside$,              %          One-side circle flag
              \ifthenelse{\bOneside=1}{                                      %          One-side circle >>
                $\texttt{\textbackslash bFrontside}\!=\!\bFrontside$\\       %            Front-side flag
              }{                                                             %          << Two-side circle >>
                $\texttt{\textbackslash bUnwrapA}\!=\!\bUnwrapA$,            %            Angle unwrap flag #1
                $\texttt{\textbackslash bUnwrapB}\!=\!\bUnwrapB$\\           %            Angle unwrap flag #2
                $\circ\!: \!\texttt{\textbackslash aPhiBf}\!=\!\aPhiBf^\circ\!, %            Back-front crossing angle
                 \times\!:\!\texttt{\textbackslash aPhiFb}\!=\!\aPhiFb^\circ$\\ %            Front-back crossing angle
              }                                                              %          <<
            }};                                                              %        <<
        \end{scope}                                                          %      << (Aux. display scope)
      }{}                                                                    %    << (Auxiliary drawing activated)
    \end{scope}                                                              %  << (Drawing scope)
  \end{scope}                                                                % << (Macro scope)
}

\newcommand{\tdplotCsDrawGreatCircle}[4][]{%
  % TODO: ...
  \tdplotCsDrawCircleOfSphere[#1]{#2}{#3}{#4}{0}
}

\newcommand{\tdplotCsDrawLatCircle}[3][]{%
  % TODO: ...
  \tdplotCsDrawCircleOfSphere[#1]{#2}{0}{0}{#3}
}
```

```

\newcommand{\tdplotCsDrawLonCircle}[3][]{%
  % TODO: ...
  \tdplotCsDrawCircleOfSphere[#1]{#2}{{#3+90}}{90}{0}
}

%% == EOF ===================================================================
```

3.3 An Auxiliary Matlab Script

```
%% == LaTeX PACKAGE tikz-3dplot-circleofsphere ================================
%     Drawing circles of a sphere with tikz-3dplot
%
%  Matthias Wolff, BTU Cottbus-Sentenberg
%  July 26, 2018
%
%  References:
%  [1] J. Hein. The tikz-3dplot package. 2012. Online, retrieved July 20, 2018.
%      https://mirror.hmc.edu/ctan/graphics/pgf/contrib/tikz-3dplot/tikz-3dplot_documentation.pdf
%

%% Rotation matrices ==========================================================
syms a b p t

% R rotation matrix -----------------------------------------------------------
Rz = [  cos(p) -sin(p)   0
        sin(p)  cos(p)   0
        0       0        1      ];

Rx = [  1       0        0
        0       cos(t)  -sin(t)
        0       sin(t)   cos(t) ];

% - [1] eq. (2.1) line 2
% R = Rz*Rx; disp(R);

% - [1] eq. (2.1) line 3
% R = [  cos(p)         sin(p)         0
%       -cos(t)*sin(p)  cos(t)*cos(p) -sin(t)
%        sin(t)*sin(p) -sin(t)*cos(p)  cos(t) ];

% - [1] eq. (2.1) line 3, corrected
R = (Rz*Rx).';

% -- D rotation matrix ---------------------------------------------------------
Dz = [  cos(a) -sin(a)  0
        sin(a)  cos(a)  0
        0       0       1      ];

Dy = [  cos(b)  0       sin(b)
        0       1       0
       -sin(b)  0       cos(b) ];

Dx = [  1       0       0
        0       cos(b) -sin(b)
        0       sin(b)  cos(b) ];

D = Dz*Dy; disp(D);

% -- Full rotation matrix ------------------------------------------------------
A = R*D; disp(A);
axx = A(1,1); axy = A(1,2); axz = A(1,3);
ayx = A(2,1); ayy = A(2,2); ayz = A(2,3);
azx = A(3,1); azy = A(3,2); azz = A(3,3);

%% == Transform a vector (world -> screen) ====================================
syms x y z
p = [ x
      y
      z ];
q=A*p;
disp(q);
```

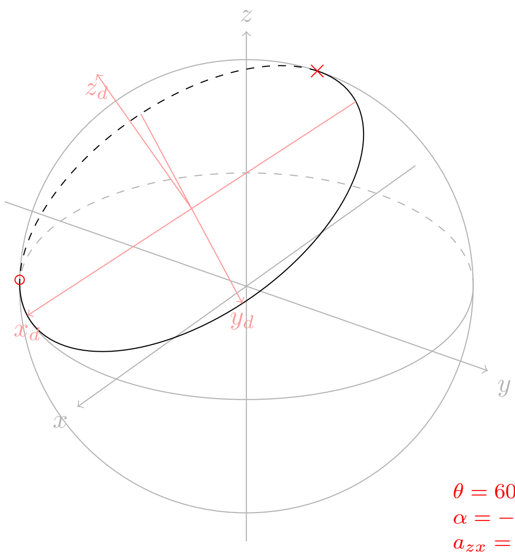
```
%% == View angle ===============================================================
syms p0 r eps azx azy azz
assume(p0,'real');
assume(r,'real');
assume(eps,'real');
assume(azx,'real');
assume(azy,'real');
assume(azz,'real');
eqn = azx*r*cos(eps)*cos(p0) + azy*r*cos(eps)*sin(p0) + azz*r*sin(eps) == 0
solve(eqn,p0,'Real',true)

% syms p0 u v w
% assume(p0,'real');
% assume(u,'real');
% assume(v,'real');
% assume(w,'real');
% eqn = u*cos(p0) + v*sin(p0) + w == 0;
% solve(eqn,p0,'Real',true)

%% == EOF ======================================================================
```

References

[1] Jeff Hein. The tikz-3dplot package. http://mirror.ctan.org/graphics/pgf/contrib/tikz-3dplot/tikz-3dplot_documentation.pdf, 2012. Retrieved: July 27, 2018.

[2] Till Tantau. Tikz & pgf - manual for version 3.0.1a. http://mirror.ctan.org/graphics/pgf/base/doc/pgfmanual.pdf, 2015. Retrieved: July 27, 2018.

[3] Matthias Wolff. The tikz-3dplot-circleofsphere package: Drawing circles of a sphere with tikz-3dplot. https://github.com/matthias-wolff/tikz-3dplot-circleofsphere, 2018. Retrieved: July 27, 2018.



$\theta = 60.0°, \phi = 125.0°$
$\alpha = -40.0°, \beta = 30°, \epsilon = 30°$
$a_{zx} = -0.05588, a_{zy} = 0.8365, a_{zz} = 0.54507$
$r_e = 2.59808, z_e = 1.5$
`\bOneside` $= 0$, `\bUnwrapA` $= 0$, `\bUnwrapB` $= 1$
$\circ$: `\aPhiBf` $= -18.22858°$, $\times$: `\aPhiFb` $= 205.86197°$

```latex
\documentclass{standalone}
\usepackage[dvipsnames]{xcolor}
\usepackage{tikz-3dplot-circleofsphere}

\begin{document}

  \def\elev{  30} \pgfmathsetmacro{\tdpTheta}{90-\elev}
  \def\azim{  35} \pgfmathsetmacro{\tdpPhi}{90+\azim}
  \def\R{3}
  \tdplotsetmaincoords{\tdpTheta}{\tdpPhi}
  \begin{tikzpicture}[scale=1,tdplot_main_coords]
    \begin{scope}[black!30,name=auxiliary]
      \draw[tdplot_screen_coords] (0,0,0) circle (\R);
      \draw[->] (-1.3*\R,0,0) -- (1.3*\R,0,0) node[anchor=north east]{$x$};
      \draw[->] (0,-1.3*\R,0) -- (0,1.3*\R,0) node[anchor=north west]{$y$};
      \draw[->] (0,0,-1.3*\R) -- (0,0,1.3*\R) node[anchor=south]{$z$};
      \tdplotCsDrawCircleOfSphere{\R}{0}{0}{0};
    \end{scope}
    \begin{scope}
%       \tdplotCsDrawLatCircle[tdplotCsDrawAux]{\R}{-30}
      % --
      \tdplotCsDrawCircleOfSphere[tdplotCsDrawAux]{\R}{-40}{30}{30}
      % --
%       \foreach \a in {0,15,...,345}
%         { \tdplotCsDrawCircleOfSphere[very thin,gray]{\R}{\a}{90}{0} }
%       \foreach \a in {-75,-60,...,75}
%         { \tdplotCsDrawCircleOfSphere[very thin,gray]{\R}{0}{0}{\a} }
      % -- Pathologic cases -->
%       \tdplotCsDrawCircleOfSphere{\R}{35}{60}{0}
      % <--
    \end{scope}
  \end{tikzpicture}

\end{document}
```