CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Nagesh

Problem Set 4

Due Date .... September 26, 2022 8pm MT
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Collaborators .... **DJ Richardson, Vikki Wong, Zane Perry**

Contents

Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

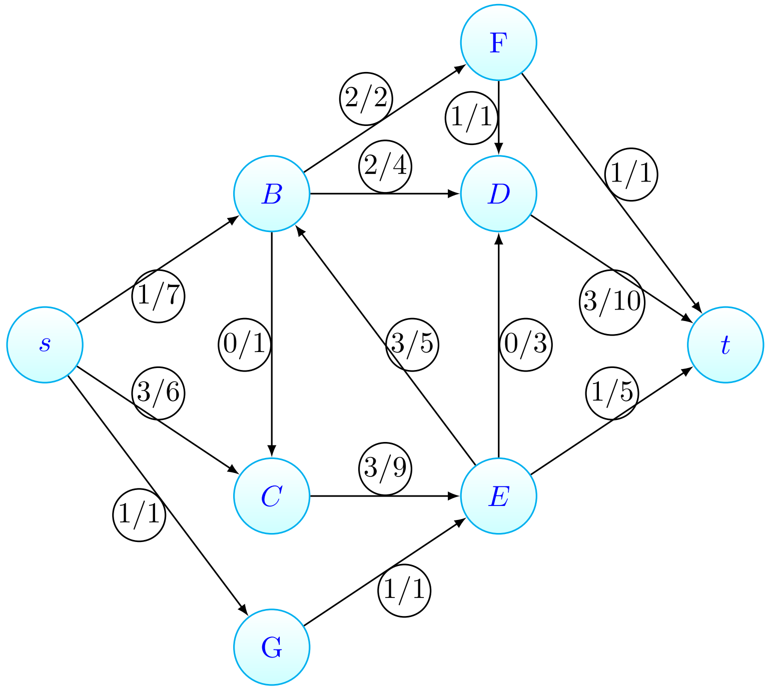
Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (Aidan Reese). I agree to the above $\square$

10 Standard 10 - Flow Networks: Terminology

Problem 10. Consider the following flow network, with the following flow configuration f as indicated below.



Do the following (there are five parts, (a)–(e), continuing on to the next page).

(a) Given the current flow configuration f, what is the maximum *additional* amount of flow that we can push across the edge (B, D) from $B \to D$? Justify using 1-2 sentences.

Answer. Because the capasity of (E,D) edge is 4, and it currently has 2 in it, then there can be a flow of 2 added to this edge to reach capacity. □

(b) Given the current flow configuration f, what is the maximum amount of flow that B can push backwards to E? Do **not** consider whether E can reroute that flow elsewhere; just whether B can push flow backwards. Justify using 1-2 sentences.

Answer. There is only 3 available flow to redirect backwards elsewhere. □

(c) Given the current flow configuration f, what is the maximum amount of flow that D can push backwards to E? Do **not** consider whether D can reroute that flow elsewhere; just whether D can push flow backwards. Justify using 1-2 sentences.

Answer. There is now flow existing in E,D so there is no flow to push backwards and the edge only faces the other direction. □

(d) How much *additional* flow can be pushed along the flow-augmenting path $s \to B \to E \to t$? Do not include the current flow along these edges. Justify using 1-2 sentences.

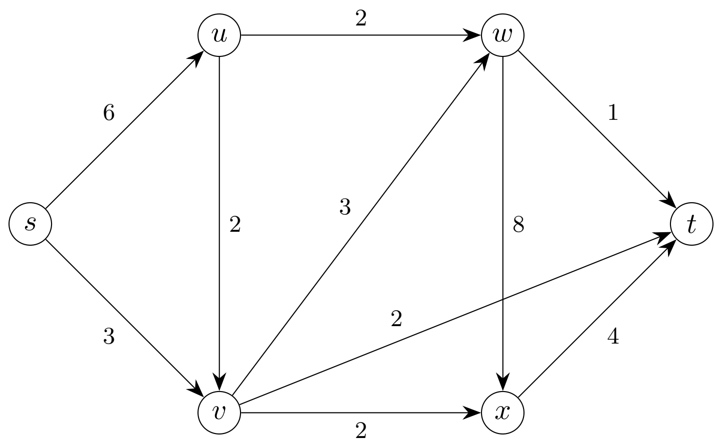
Answer. Instead of having the flow going from $E \to B$, we push this flow backwards and have it directed towards t. $\square$

(e) Find a second flow-augmenting path and indicate the maximum amount of additional flow that can be pushed along the path. Assume that the flow-augmenting path from part (d) has **not** been applied. Justify using 1-2 sentences.

Answer. $S \to C \to E \to t$ can all take an additionall 3 units of flow to augment the path to increase overall flow by 3. $\square$

11 Standard 11 - Flow Networks: Ford–Fulkerson

Problem 11. For Problem 11, consider the following flow network.

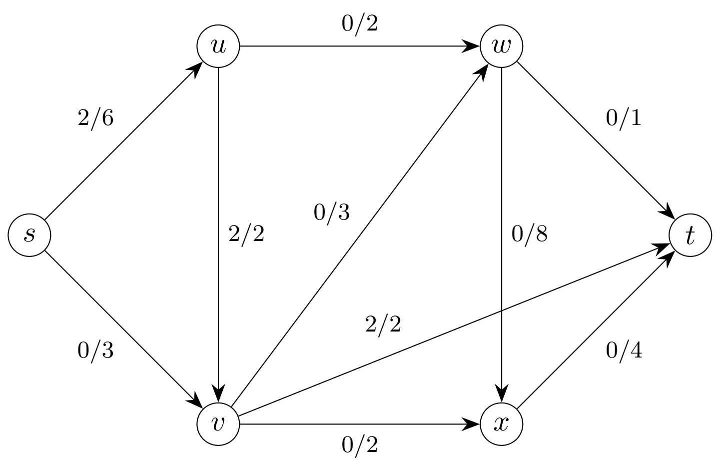


Do the following.

11.1 Problem 11(a)

(a) Consider the flow-augmenting path $s \to u \to v \to t$. Push as much flow through the flow-augmenting path and draw the updated flow network below (we have provided a tikzpicture in the LaTeX comments that you can use and just modify the labels on the edges, or you may hand-draw and embed an image).

Answer.

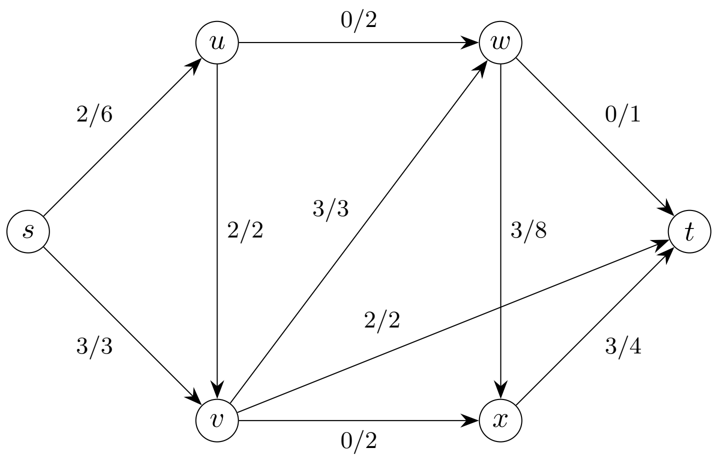


□

11.2 Problem 11(b)

(b) Find a flow-augmenting path starting from the updated flow configuration from your answer to part (a). Then do the following: (i) clearly identify both the new flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer. We could add the augmented path from $s \to v \to w \to x \to t$ this being the most amount of flow possibly added at 3.

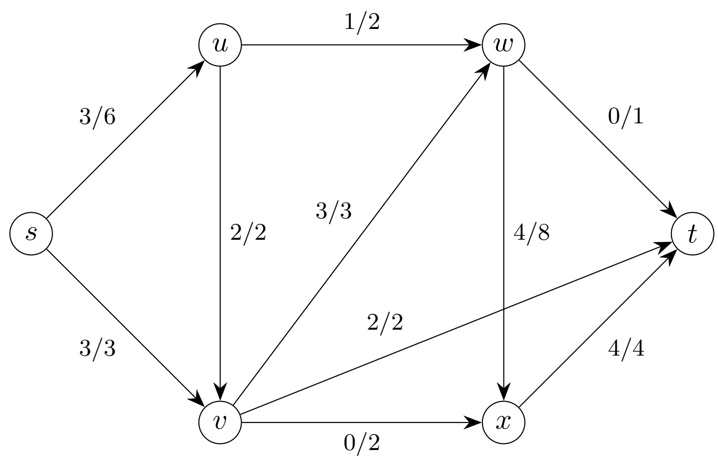


$\square$

11.3 Problem 11(c)

(c) Find a flow-augmenting path starting from the updated flow configuration from your answer to part (b). Then do the following: (i) clearly identify both the new flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer. The next augmented flow that we can add will be $s \to u \to w \to x \to t$, only carrying 1 flow.

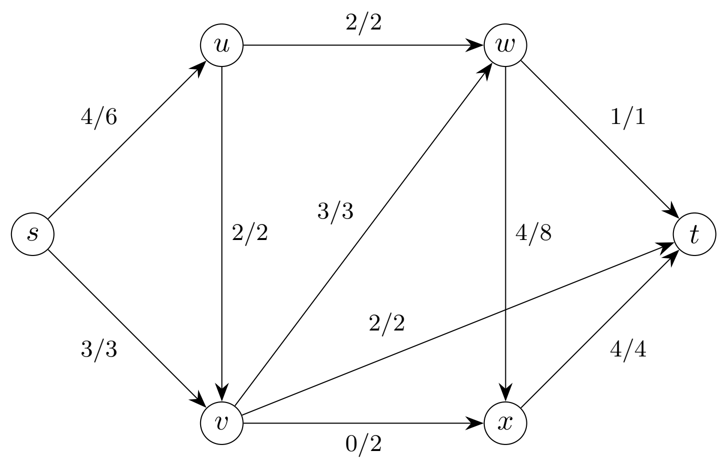


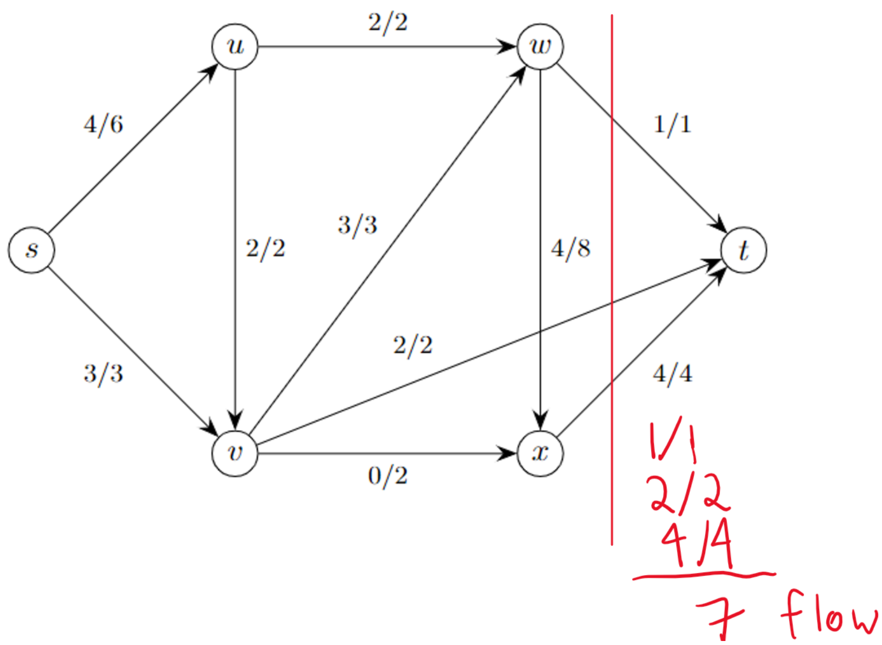
□

11.4 Problem 11(d)

(d) Using the flow configuration from part (c), finish executing the Ford–Fulkerson algorithm. Include the following here: (i) your flow network, reflecting the maximum-valued flow configuration you found, and (ii) the corresponding minimum capacity cut. There may be multiple minimum capacity cuts, but you should identify the one corresponding to your maximum-valued flow configuration. Then (iii) finally, compare the value of your flow to the capacity of the cut.

Answer. The very last augmented path to be added is $s \to u \to w \to t$, adding only 1 flow to the graph.





Shown here, at the cut there is 7 flow, and exiting s there is also 7. At the cut all edges are at capasity for the amount of flow they can handle. This is why the maximum flow of the graph and the flow on the cut are the same at 7.

□