CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Nagesh

Problem Set 6

Due Date .... October 17, 2022
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Contents

Instructions

- The solutions **must be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Useful links and references on LaTeXcan be found here on Canvas.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
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16 Standard 16 - Analyzing Code: Writing Down Recurrences

Problem 16. For each algorithm, write down the recurrence relation for the number of times they print "Welcome". (In this case, this is big-Θ of the runtime - do you see why?) **Don't forget to include the base cases.**

16(a). Problem 16(a)

Algorithm 1 Writing Recurrences 1

(a)
```
procedure Foo(Integer n)
    if n ≤ 5 then return n

    Foo(n/10)
    Foo(n/5)
    Foo(n/5)

    for i ← 1; i ≤ 3 * n; i ← i + 1 do
        print "Welcome"
```

Answer.

$$T(n) = \begin{cases} \Theta(1) & : n \leq 5, \\ T(n/10) + T(n/5) + T(n/5) + (3n) & : n > 5 \end{cases}$$

$$T(n) = \begin{cases} \Theta(1) & : n \leq 5, \\ T(n/10) + 2T(n/5) + \Theta(n) & : n > 5 \end{cases}$$

$$T(n) = \Theta(n)$$

$$T(n) = 3n$$

□

16(b). Problem 16(b)

Algorithm 2 Writing Recurrences 2

(b)
```
procedure Foo2(Integer n)
    if n ≤ 7 then return

    Foo2(n/14)
    Foo2(n/14)

    for i ← 1; i ≤ n; i ← i * 2 do
        for j ← 1; j ≤ n; j ← j * 3 do
            print "Welcome"
```

Answer.

$$T(n) = \begin{cases} \Theta(1) & : n \leq 7, \\ 2T(n/14) + (...) & : n > 7 \end{cases}$$

(...) represents the nested j loop inside of the i loop

j loop:
1 operation outside of the loop (j =1)
1 operation inside the loop (j $\leq$ n)
2 operations inside the loop (j=j*3)
1 operation inside loop (print)

$(1 + \sum_{j=1}^{\log_3(n)+1} 4) = 1 + 4(\lceil \log_3(n) + 1 \rceil)$

i loop :
1 operation outside of the loop (i =1)
1 operation outside of the loop (i =1)
1 operation inside the loop (i $\leq$ n)
2 operations inside the loop (i=i*2)
$1 + 4(\lceil \log_3(n) + 1 \rceil)$ operations inside the loop for the j loop
$1 + \sum_{i=1}^{\lceil \log_2(n)+1 \rceil} (1 + 2 + (1 + 4(\lceil \log_3(n) \rceil + 1)))$
$= 9 + \lceil 8 \log_2(n) \rceil + 4((\lceil log_2(n) * \log_3(n) \rceil))$

For values less than or equal to 7 the function will print 0 times,
For values greater than 7 the function will print $O(\log^2(n))$ times

□

17 Standard 17 - Solving Recurrences I: Unrolling

Problem 17. For each of the following recurrences, solve them using the unrolling method (i.e. find a suitable function $f(n)$ such that $T(n) \in \Theta(f(n))$). **Note:** Show all the work.

17(a). Problem 17(a)

a.

$$T(n) = \begin{cases} 3n & : n < 3, \\ 2T(n/2) + 4n & : n \geq 3. \end{cases}$$

Answer. (i):

$$T(n) = \begin{cases} 3n & : n < 3, \\ 2T(n/2) + 4n & : n \geq 3. \end{cases}$$

$$T(n) = \begin{cases} O(n) & : n < 3, \\ 2T(n/2) + O(n) & : n \geq 3. \end{cases}$$

$$T(n) = \begin{cases} nc & : n < 3, \\ 2T(n/2) + nc & : n \geq 3. \end{cases}$$

(ii):
T(n/2)+nC
T(n/4)+nC+nC
T(n/8)+nC+nC+nC
T(n) = T(n/2^k)+knC

Solve for k : n/2^k=1
k = $\log_2(n)$

(iii):
T(n) = T(n/2^k)+ (ck)n
T(n) = T(1) + $\log_2(n) * n$
T(n) = $O(n \log(n))$

□

17(b). Problem 17(b)

b.

$$T(n) = \begin{cases} 5 & : n < 2, \\ 7T(n-2) + 9 & : n \geq 2. \end{cases}$$

Answer. (i):

$$T(n) = \begin{cases} O(1) & : n < 2, \\ T(n-2) + O(1) & : n \geq 2. \end{cases}$$

$$T(n) = \begin{cases} c & : n < 2, \\ T(n-2) + c & : n \geq 2. \end{cases}$$

(ii):

T(n-2)+c
T(n-4)+c+c
T(n-6)+c+c+c
T(n-2k)+ck

(iii):
T(1) : k=n
T(1)+kc
T(n) = $O(n)$

□

18 Standard 18 - Divide and Conquer: Counterexamples

Problem 18. Consider the following problem:

> MAX PAIR SUM
> *Input:* A list L of integers
> *Output:* An index $i \in \{1, \ldots, len(L) - 1\}$ such that $L[i] + L[i+1]$ is maximized (that is, such that $L[i] + L[i+1] \geq L[j] + L[j+1]$ for all j), and the value of $L[i] + L[i+1]$

(Note the list here is 1-indexed, so the problem simply does not consider the last element, as it has nothing to pair it with.)

Consider the algorithm below that attempts to solve this problem. **Give an instance** of input (preferably a list of length at most 6) for which it fails to output the correct value for the above problem, and **explain why it fails**.

Algorithm 3 Proposed divide-and-conquer algorithm for the Max Pair Sum problem

1: **procedure** MAXPAIRSUM(List L) $n \leftarrow len(L)$
2:     **if** $n \leq 1$ **then return** ;
3:     **if** $n = 2$ **then return** $(1, L[1] + L[2])$;
4:     $(i1, sum1) \leftarrow \text{MaxPairSum}(L[1..\lfloor n/2 \rfloor])$;
5:     $(i2, sum2) \leftarrow \text{MaxPairSum}(L[\lfloor n/2 \rfloor + 1..n])$;
6:     **if** $i1 \geq i2$ **then**
7:         **return** $(i1, sum1)$;
8:     **else**
9:         **return** $(i2, sum2)$;

Proof. Take a list of len(L) = 4
Let $L = [1, 3, 2, 1]$
The algorithm is meant to find the largest sum of the list of adjacent elements and in this case it should be $3 + 2 = 5$.
The algorithm would split these into L1 = $[1, 3]$ and L2 = $[2, 1]$.
On lines 7 and 9 the algorithm would return 1 and the sum of the larger list (4)
This is where the algorithm fails as it is not returning the largest sum, aswell as it is returning (1) and the sub list, which is not the greatist sum.

□