CSCI 3104 Fall 2022
Instructors: Prof. Grochow and Nagesh

Problem Set 3

Due Date .... September 19, 2022 8pm MT
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Collaborators .... **Vikky Wong, Miles Sanders, Anna Rahn, Al Haddad**

Contents

Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

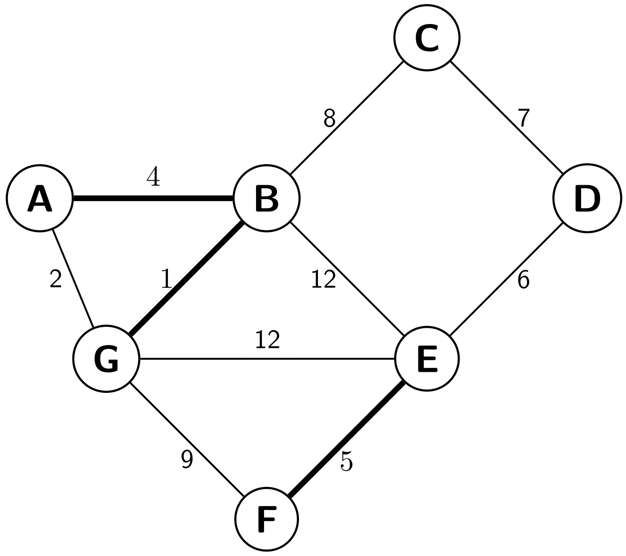
Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (Aidan Reese). I agree to the above □

6 Standard 6 – Safe and Useless Edges

Problem 6. Consider the weighted graph $G(V, E, w)$ below. Let $\mathcal{F} = \{\{A, B\}, \{B, G\}, \{E, F\}\}$ be an intermediate spanning forest (indicated by the thick edges below). Label each edge that is **not** in $\mathcal{F}$ as safe, useless, or undecided. Provide a 1-2 sentence explanation for each such edge.



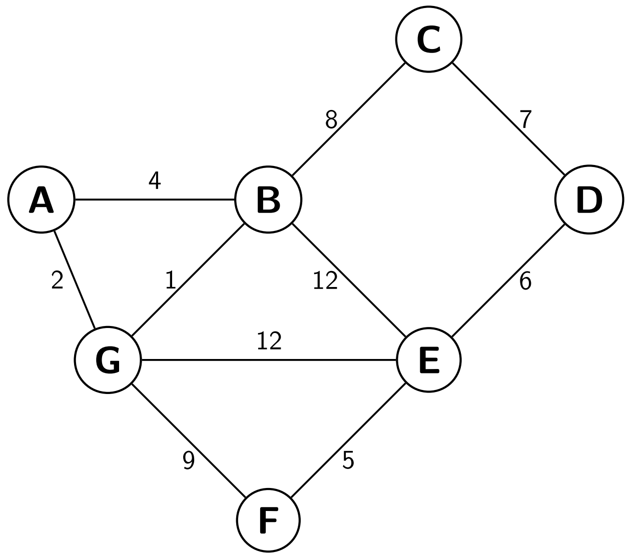
Answer.

- The edge $\{A, G\}$ is... Useless because ... Adding this edge would form a cycle.
- The edge $\{G, F\}$ is... Undecided because ... (G,F) does not create a cycle but is not a minimum edge that connects a vertex to T.
- The edge $\{G, E\}$ is... Undecided because ... (G,E) does not create a cycle but is not a minimum edge that connects a vertex to T.
- The edge $\{B, E\}$ is... Undecided because ... (G,F) does not create a cycle but is not a minimum edge that connects a vertex to T.
- The edge $\{B, C\}$ is...Safe because ... It is a minimum edge that connects a vertex to an endpoint of F
- The edge $\{C, D\}$ is... Safe because ... After the connection at (E,D), this would have an endpoint to the spanning tree at D
- The edge $\{D, E\}$ is... Safe because ... Minimum edge that is connected to an endpoint of F

□

7 Standard 7: Kruskal's MST Algorithm

Problem 7. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Kruskal's algorithm adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Kruskal's algorithm takes as it selects the first **three** edges.



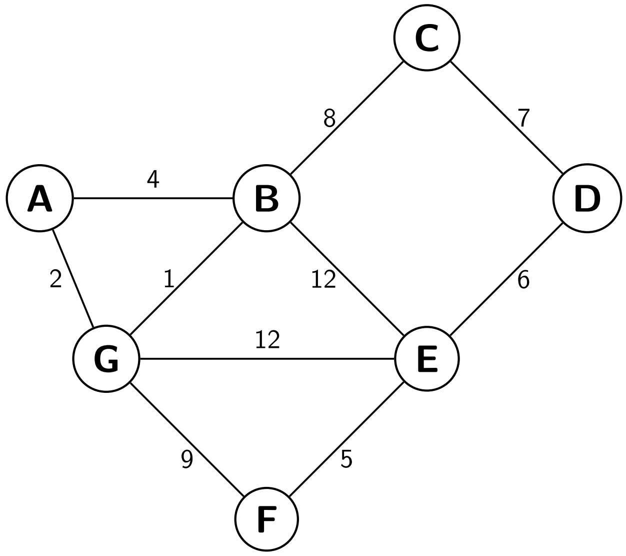
Proof.
(G,B)
(G,A)
(A,B) Does not get added as it makes a cycle
(F,E)
(E,D)
(D,C)
(C,B)
(G,F) Does not get added
(G,E) Does not get added
(B,E) Does not get added

The algorithm looks at all of the minimum edges, checks to see if the minimum edge creates a cycle, if not it gets added to T. Else if it does make a cycle, the edge does not get added to T and the algorithm continues to the next minimum edge. Kruskal's exits once the number of edges is equal to n-1, n being the number of nodes in the graph

□

8 Standard 8: Prim's MST Algorithm

Problem 8. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Prim's algorithm, **using the source vertex** A, adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Prim's algorithm takes as it selects the first **three** edges.



Proof.
(A,G)
(G,B)
(B,A) Does not get added because it makes a cycle
(B,C)
(C,D)
(D,E)
(E,F)
(F,G) Does not get added
(E,G) Does not get added
(E,B) Does not get added

Starting from A, the minimum node is (A,G), from that tree it is then (G,B), from this tree the next minimum edge is (A,B) but this creates a cycle and this edge is not added to the tree. From here the graph wraps around from nodes (B,C,D,E,F) as these are all of the minimum edges available in order. (G,F),(G,E), and (B,E) are not added to T as Prims exits after the number of edges = (n-1), n being the number of nodes. □

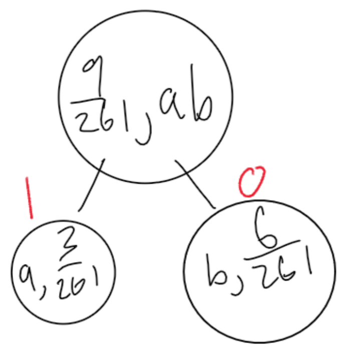
9 Standard 9: Huffman Coding

Problem 9. Consider the following sequence of numbers:

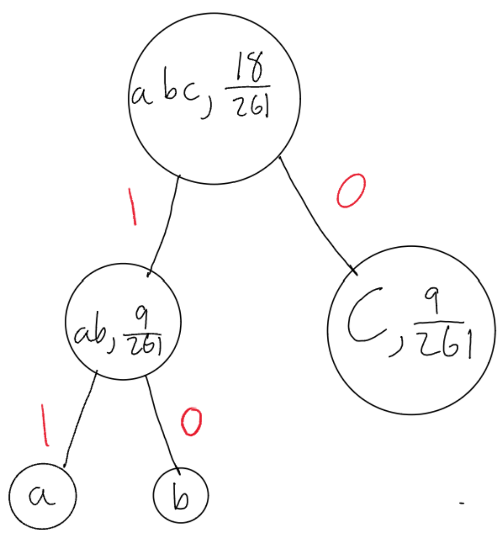
$$S_n = \begin{cases} 3 & n = 0 \\ 6 & n = 1 \\ S_{n-1} + S_{n-2} & n \geq 2. \end{cases}$$

For an alphabet $\Sigma = \{a, b, c, d, e, f, g, h\}$ with frequencies given by the first $|\Sigma|$ many numbers $S_0, S_1, \dots, S_{|\Sigma|-1}$, give an optimal Huffman code and its corresponding encoding tree. Specify the frequencies of each letter, and for each stage of the algorithm, the subtrees merged at that stage, and the resulting total frequency for the new merged subtree.

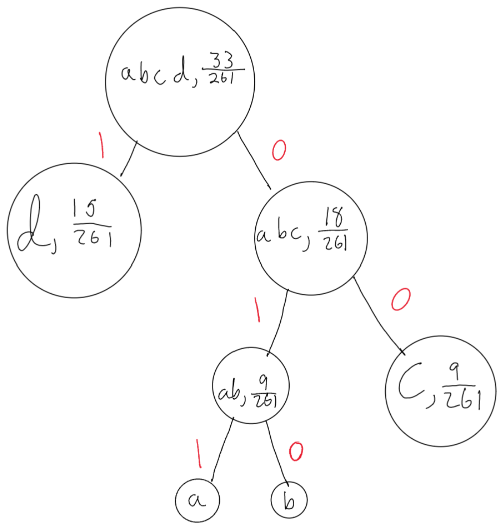
Proof.
((A,$\frac{3}{261}$), (B,$\frac{6}{261}$), (C,$\frac{9}{261}$), (D,$\frac{15}{261}$), (E,$\frac{24}{261}$), (F,$\frac{39}{261}$), (G,$\frac{63}{261}$), (H,$\frac{102}{261}$))



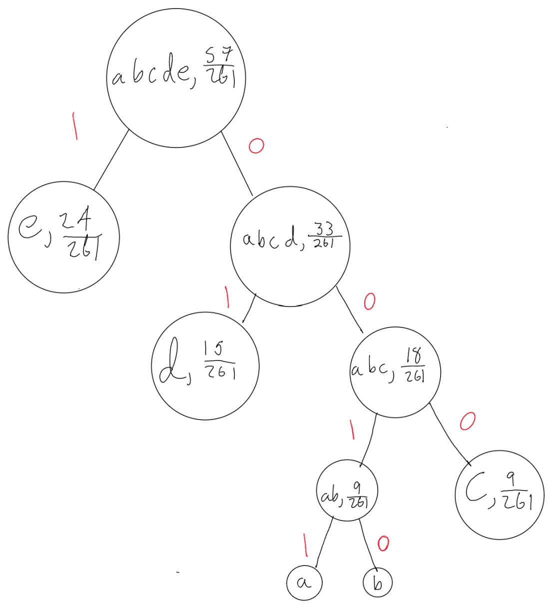
((AB,$\frac{9}{261}$), (C,$\frac{9}{261}$), (D,$\frac{15}{261}$), (E,$\frac{24}{261}$), (F,$\frac{39}{261}$), (G,$\frac{63}{261}$), (H,$\frac{102}{261}$))



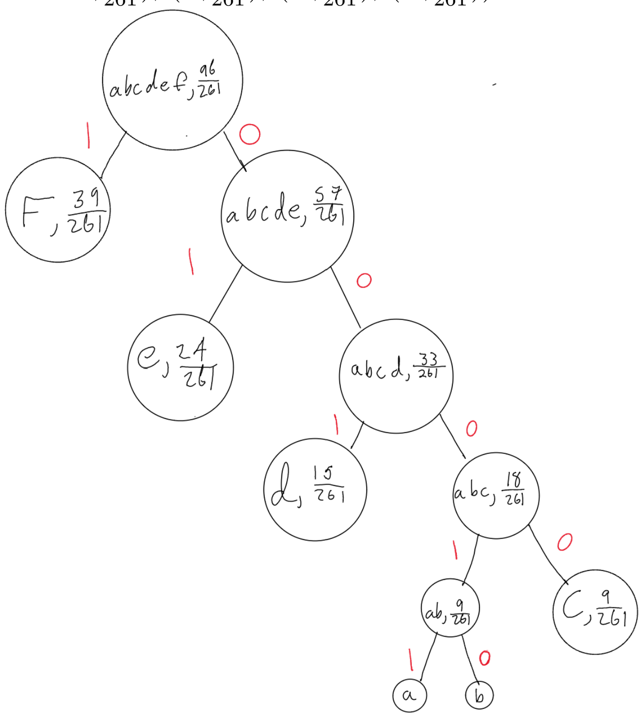
$((\text{ABC},\frac{18}{261}), (\text{D},\frac{15}{261}), (\text{E},\frac{24}{261}), (\text{F},\frac{39}{261}), (\text{G},\frac{63}{261}), (\text{H},\frac{102}{261}))$



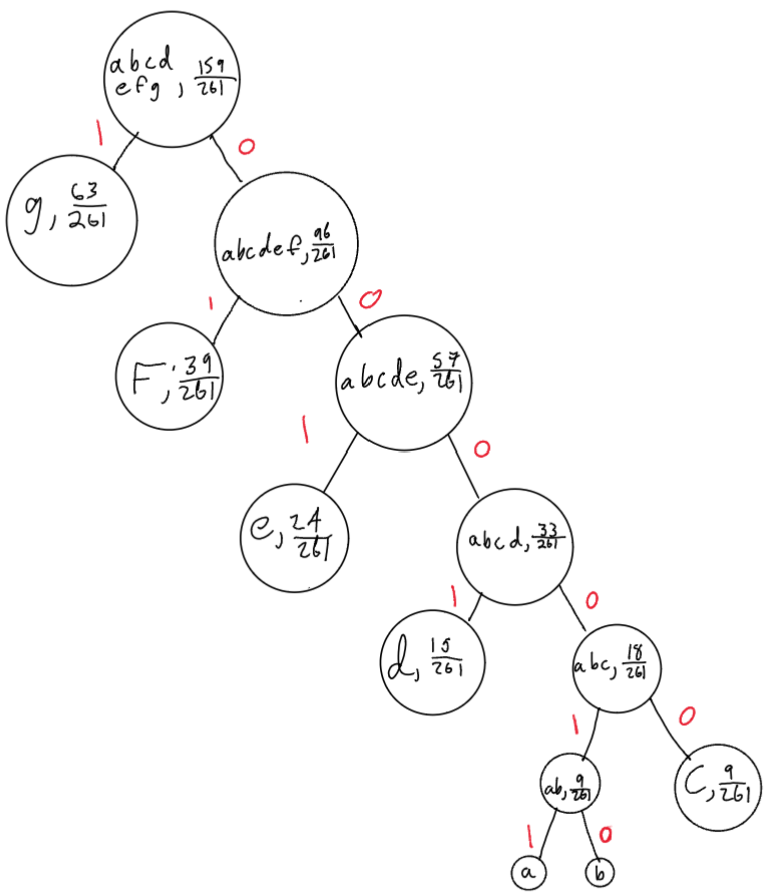
$((\text{ABCD},\frac{33}{261}), (\text{E},\frac{24}{261}), (\text{F},\frac{39}{261}), (\text{G},\frac{63}{261}), (\text{H},\frac{102}{261}))$



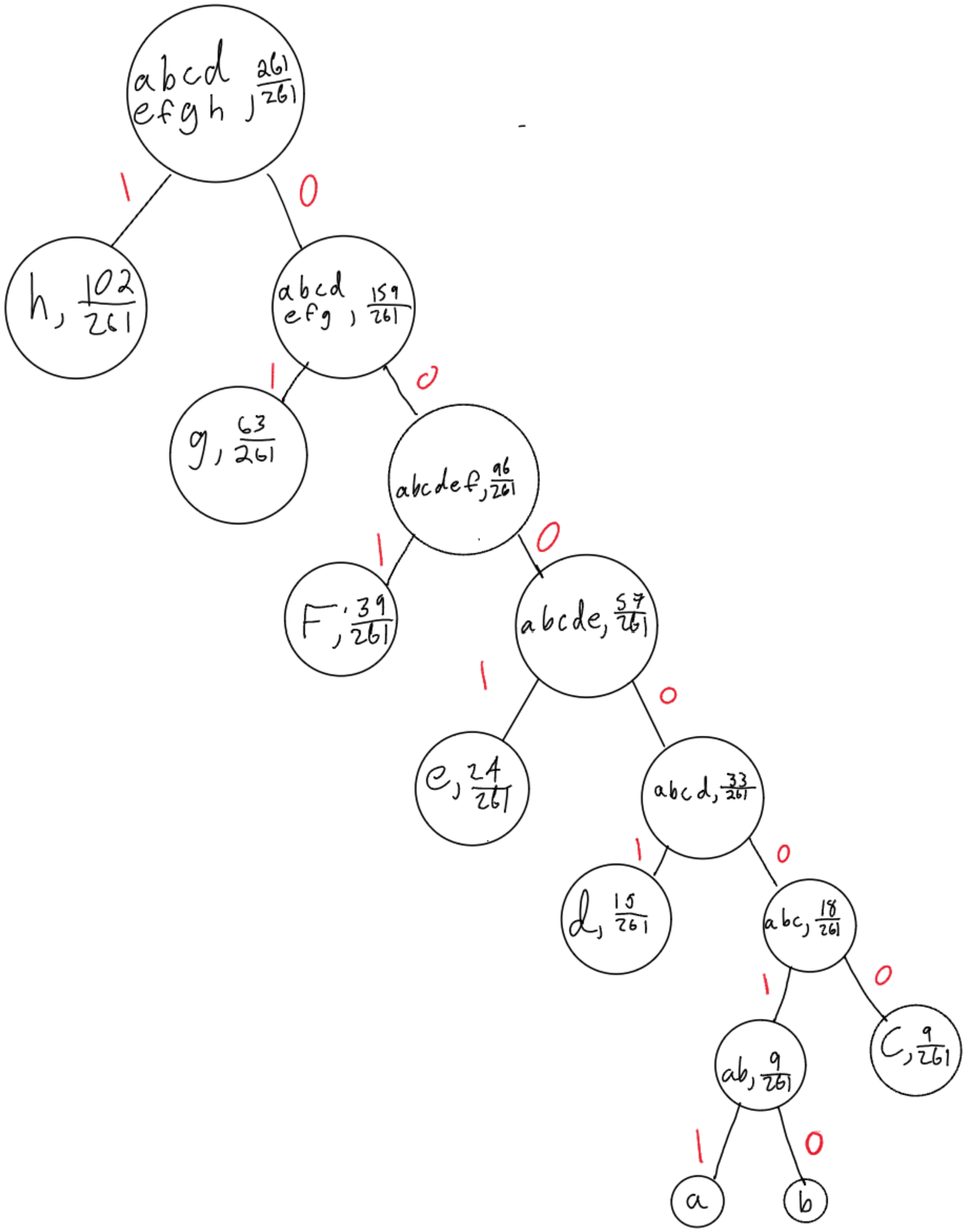
$((\text{ABCDE}, \frac{57}{261}), (\text{F}, \frac{39}{261}), (\text{G}, \frac{63}{261}), (\text{H}, \frac{102}{261}))$



$((\text{ABCDEF}, \frac{96}{261}), (\text{G}, \frac{63}{261}), (\text{H}, \frac{102}{261}))$



((ABCDEFG,$\frac{159}{261}$), (H,$\frac{102}{261}$))



$\square$

A	0000011
B	0000010
C	000000
D	00001
E	0001
F	001
G	01
H	1