4.1

a. Node A is the root node
b. Nodes G, H, I, L, M, and K are the leaves.

4.2

Node A
a. No parent node
b. Node B & C
c. No siblings
d. Depth of 0
e. Height of 4

Node B
a. Node A
b. Node D & E
c. Node C
d. Depth of 1
e. Height of 3

Node C
a. Node A
b. Node F
c. Node B
d. Depth of 1
e. Height of 2

Node D
a. Node B
b. Node G & H
c. Node E
d. Depth of 2
e. Height of 1

Node E
a. Node B
b. Node I & J
c. Node D
d. Depth of 2
e. Height of 2

Node F
a. Node C
b. Node K
c. No siblings
d. Depth of 2
e. Height of 1

Node G
a. Node D
b. No children
c. Node H
d. Depth of 3
e. Height of 0

Node H
a. Node D
b. No children
c. Node G
d. Depth of 3
e. Height of 0

Node I
a. Node E
b. No children
c. Node J
d. Depth of 3
e. Height of 0

Node J
a. Node E
b. Node L & M
c. Node I
d. Depth of 3
e. Height of 1

Node K
a. Node F
b. No children
c. No siblings
d. Depth of 3
e. Height of 0

Node L
a. Node J
b. No children
c. Node M
d. Depth of 4
e. Height of 0

Node M
a. Node J
b. No children
c. Node M
d. Depth of 4
e. Height of 0

4.3

The tree has a depth of 4.

4.4

Base Case: <u>n = 1</u>

1 + 1 = 2, One node will have two nullptr link representing children; True

Induction Case: <u>(n+1)</u>

<u>Show n+1 is true</u>

(n+1)+1 = (n+2)

(n+2)-(n+1) = 1

The difference between the amount of nodes and nullptrs will always equal one.

Therefore, a binary tree of N nodes have N+1 nullptr links representing children

4.5

Base Case: <u>h = 1</u>

2^(1+1)-1 = 4-1 = 3,The maximum number of nodes in a binary tree of height 1 is 3; True

Induction Case: <u>h = k+1</u>

Induction hypothesis states that the maximum number of nodes of each subtree is:

2^(k+1)-1

The maximum number of node can be with height h=k+1:

1 + 2(2^(k+1)-1) = 2^(k+2)-2+1

= 2^((k+1)+1)-1 = 2^(h +1) -1

So that would mean that the maximum number of nodes in a binary tree of height h is

2^(h +1) -1

4.6

n = number of full nodes

Base Case: <u>n = 1</u>

1 + 1 = 2, 1 full node will result in 2 leaves; True

Induction Case: <u>(n+1)</u>

<u>Show n+1 is true</u>

(n+1)+1 = (n+2)

(n+2)-(n+1) = 1

The difference between the full nodes and the number of leaves will always equal one.

Therefore, the number of full nodes plus one is equal to the number of leaves in a nonempty binary tree.