HW5 STAT 312

1

The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce. ($N(12.4, 0.1^2)$)

A

What is the probability that a fill volume is less than 12 fluid ounces?

$z = \frac{x-\mu}{\sigma} = \frac{12-12.4}{0.1} = -4$

$P(Z \leq -4) = 0.000033$

B

If all cans less than 12.1 or more than 12.6 ounces are scrapped, what proportion of cans is scrapped?

$z = \frac{x-\mu}{\sigma} = \frac{12.1-12.4}{0.1} = -3$

$P(Z \leq -3) = 0.00135$

$z = \frac{x-\mu}{\sigma} = \frac{12.6-12.4}{0.1} = 2$

$P(Z \geq 2) = 0.0.02275$

The percent of cans that are scrapped is $0.00135 + 0.02275 = 0.0241 = 2.41\%$

C

Determine specifications that are symmetric about the mean that include 99% of all cans.

$0.99 = P(-z \leq Z \leq z) = P(Z \leq z) - P(Z \leq -z)$

Alternatively, $0.99 = \int_{-z}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

This means to find z such that the first statement is true we need to find z such that $P(Z \leq z) = 0.995$ and $P(Z \leq -z) = 0.005$.

Using the z table we find that $z = \pm 2.58$.

We can then use this to find the range by solving $z = \frac{x-\mu}{\sigma}$ for x.

$\pm 2.58 = \frac{x-12.4}{0.1} \to x = 12.4 \pm 2.58 \times 0.1 = 12.658$ or 12.142

The specifications are $12.142 \leq x \leq 12.658$

D

Use R to find solutions for parts (a) and (c). Attach the R codes and output.

```
print(paste("1A: ", pnorm(12,12.4,0.1)))
```

```
[1] "1A:  3.16712418331194e-05"
```

```
print(paste("1C: ", "(",qnorm(0.005,12.4,0.1),",",qnorm(0.995,12.4,0.1), ")"))
```

```
[1] "1C:  ( 12.1424170696451 , 12.6575829303549 )"
```

2

The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.002 inch and a standard deviation of 0.0004 inch. ### (a) What is the probability that the diameter of a dot exceeds 0.0026?

$z = \frac{x-\mu}{\sigma} = \frac{0.0026-0.002}{0.0004} = 1.5$

$P(Z > 1.5) = 1 - P(Z \leq 1.5) = 1 - 0.93319 = 0.06681$

(b)

What is the probability that a diameter is between 0.0014 and 0.0026?

$z = \frac{x-\mu}{\sigma} = \frac{0.0026-0.002}{0.0004} = 1.5$

$z = \frac{x-\mu}{\sigma} = \frac{0.0014-0.002}{0.0004} = -1.5$

$P(-1.5 \leq Z \leq 1.5) = P(Z \leq 1.5) - P(Z \leq -1.5) = 0.93319 - 0.06681 = 0.86638$

(c)

What standard deviation of diameters is needed so that the probability in part (b) is 0.995?

> $0.995 = P(-z \leq Z \leq z) = P(Z \leq z) - P(Z \leq -z)$ so $0.9975 = P(Z \leq z)$ and $0.0025 = P(Z \leq -z)$
>
> Using the Ztable, we find that $z = \pm 2.81$
>
> We can then use this to find the range by solving $z = \frac{x-\mu}{\sigma}$ for σ.
>
> $\pm 2.81 = \frac{\pm 0.0006}{\sigma} \rightarrow \sigma = \frac{0.0006}{2.81} = 0.0002135231$

3

Suppose that X has a lognormal distribution with parameters $\theta = 5$ and $\omega^2 = 9$.

(a)

Use Table III to find P(X < 13,300).

> $P(X < 13,300) = P(\ln(X) < \ln(13,300)) = P(\ln(X) < 9.4955)$
>
> $z = \frac{\ln(13{,}300)-5}{3} = 1.4985$
>
> Using the z-table with $z = 1.4985$ we find that $P(\ln(X) < 1.4985) = 0.933193$.

(b)

Use Table III to find the value of x such that P(X x) = 0.95.

> The corresponding z-score for a probability of 0.95 is $z = 1.65$.
>
> Thus we need to solve $z = \frac{\ln(x)-5}{3}$ for x.
>
> $z = \frac{\ln(x)-5}{3} = 1.65 \rightarrow \ln(x) = 5 + 3 \times 1.65 = 20952.22$

(c)

Find the mean and variance of X.

> $\mu = E(X) = e^{\theta+\omega^2/2} = 13359.7268$
>
> $\sigma^2 = V(X) = e^{2\theta+\omega^2}(e^{\omega^2} - 1) = 1.446 \times 10^{12}$

(d)

Use R to find solutions for parts (a) and (b). Attach the R codes and output. Please note that your answers in this part may not be exactly the same as your answers in parts a and b when using table III.

```
print(paste("3A: ", plnorm(13300,5,3)))
```

```
[1] "3A:  0.932999139402293"
```

```
print(paste("3B: ", qlnorm(0.95,5,3)))
```

```
[1] "3B:  20631.222875142"
```

4

Suppose that X has a lognormal distribution and that the mean and variance of X are 100 and 85,000, respectively.

(a)

Determine the parameters θ and ω^2 of the lognormal distribution. You can use the method that we discussed in class or the hint below to solve the problem.

Hint: define $x = exp(\theta)$ and $y = exp(\omega^2)$ and write two equations in terms of x and y.

> $\theta = x\sqrt{y} = 100$, $\omega^2 = x^2y(y-1) = 85000$
>
> $x\sqrt{y} = 100 \rightarrow x^2y = 10000 \rightarrow x^2y(y-1) = 10000(y-1) = 85000 \rightarrow 10000y = 95000 \rightarrow y = 9.5$
>
> $x\sqrt{9.5} = 100 \rightarrow x = 32.444$
>
> Thus $\theta = 3.47952$ and $\omega^2 = 2.25129$.

(b)

Use R to find P($X \leq 5$). Attach the R code and output.

```
print(paste("4B: ", plnorm(5,3.47952,2.25129)))
```

```
[1] "4B:  0.203079659773497"
```