HW4

1

In order to prove this, we can prove there exists a function f such that $x \in A_{TM} \leftrightarrow f(x) \in L_{343}$

Remember $x =< M, w >$ and $f(x) =< M' >$

M' on input y:

- Run M on w
- If M accepts:
  - Check if y is equal to "CSDS 343 is fun"
  - If yes -> Accept
  - Else
    - Check if y is equal to "MATH 343 is fun"
    - If yes -> Accept
    - Else -> Reject
- If M rejects:
  - M' rejects

If $x \in A_{TM}$ show $f(x) \in L_{343}$

- If $f(x) \in L_{343}$ then M' must accept y if it is "CSDS 343 is fun" or "MATH 343 is fun", otherwise reject.
- In the case y is "CSDS 343 is fun" or $MATH 343 is fun"
  - M' runs M on w which accepts because $x \in A_{TM}$. Next M' checks if y is "CSDS 343 is fun" if it is M' accepts. Then checks if y is "MATH 343 is fun" if it is M' accepts. Thus M' accepts when it should when $x \in A_{TM}$
- In the case y is not $CSDS 343 is fun" or "MATH 343 is fun"
  - M' runs M on w which accepts because $x \in A_{TM}$. Next M' checks if y is "CSDS 343 is fun", then checks if y is "MATH 343 is fun". Since y is neither "CSDS 343 is fun" or "MATH 343 is fun" M' will reject. Thus M' rejects when it should when $x \in A_{TM}$

If $x \notin A_{TM}$ Show $f(x) \notin L_{343}$

- Since $x \notin A_{TM}$, M' will run M on w which rejects so M' will reject. Thus, even if $y =$ "CSDS 343 is fun" M' will reject so $< M' > \notin L_{343}$

Thus because $\exists f$ s.t. $x \in A_{TM} \leftrightarrow f(x) \in L_{343}$ then $A_{TM} \leq_m L_3 43$. Thus since we know A_{TM} is reducible to L_{343} and A_{TM} is unddecidable, then L_{343} is undecidable.

2

Proof by contradiction. So assume $DIFF\_BY\_1$ is decidable. Then $\exists M_{DIFF}$ which decides $DIFF\_BY\_1$

Create a TM M which takes a TM $< m >$ as input

M on input $< m >$

- Run M_{DIFF} on $< M, m >$
- If M_{DIFF} accepts, M rejects
- If M_{DIFF} rejects, M accepts

Create a TM M' which takes y as input

M' on input y

- If $y = M'$ reject
- Else run M on y
  - If M accepts, M' accepts
  - If M rejects, M' rejects

Now suppose we run M on M', which will then run M_{DIFF} on $< M, M' >$

If M_{DIFF} accepts $< M, M' >$, then M will reject on M'. However this creates a contradiction because M_{DIFF} says $< M, M' >$ are different by 1, however M and M' both accept / reject on the same input (they are not different by 1) because M' runs M and rejects on input M'. Thus they are not different by 1 and M_{DIFF} should reject.

If M_{DIFF} rejects $< M, M' >$, then M will accept on M'. However this creates a contradiction because M_{DIFF} says $< M, M' >$ are not different by 1, however M and M' both accept / reject on the same input except for input M' where M' rejects and M accepts. Thus they are different by one and M_{DIFF} should accept.

3

A Prove $A_{TM} \leq_M L_{add}$

In order to prove this, we must prove there exists a function f such that $x \in A_{TM} \leftrightarrow f(x) \in L_{add}$

Remember $x =< M, w >$ and $f(x) =< M' >$

M' on input y:

- Run M on w

- If M accepts:
  - Run a M in Ladd
- If M rejects:
  - M' rejects

B Prove $\bar{A_{TM}} \leq_M L_{add}$

In order to prove this, we must prove there exists a function f such that $x \in \bar{A_{TM}} \leftrightarrow f(x) \in L_{add}$

Remember $x =< M, w >$ and $f(x) =< M' >$

M' on input y:

- Run M on w
- If M rejects:
  - Run M_{add} on y // $M_{add} \in L_{add}$
- If M accepts:
  - M' rejects