

## Pushdown Automata

A machine where the memory is a stack of infinite capacity.

On each step the machine reads on input (or not) reads top of the stack and pops / pushes or keeps stack.

$\text{PDA} = (Q, \Sigma, \Gamma, \delta)$   $\delta : Q \times \{\Sigma \cup \{\epsilon\}\} \times \{\Gamma \cup \{\epsilon\}\} \rightarrow P(Q \times \{\Gamma \cup \{\epsilon\}\})$

Watch class video on 02/24 for an example of a drawn out PDA (I am too lazy to draw it :sob:)

## Pumping Lemma for context free languages

Think of giant tree, some states must repeat so we are left with the following pumping lemma:

If  $L$  is context free,  $\exists p$  such that for all  $s \in L, |s| > p$ ,

- $s = uvxyz$
- $|vxy| \leq p$
- $|vy| > 0$

$uv^kxy^kz \in L \forall k = 0, 1, 2..$

### EX

$L = \{a^n b^n c^n | n \geq 0\}$

Prove  $L$  is not context free

Assume  $L$  is context free, thus there exists a  $p$  such that for all  $s, |s| > p : s = uvxyz$

Let  $s = a^p b^p c^p$

Since  $|vxy| \leq p$ , there are 5 cases.

$vy = a^*, vy = a^*b^*, vy = b^*, vy = b^*c^*, vy = c^*$