Phys122 HW10

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a

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$
$$u = \frac{1}{2}\epsilon_0\varepsilon^2\cos^2(kx-\omega t) + \frac{1}{2}\frac{1}{\mu}_0(\frac{\varepsilon}{c})^2\cos^2(kx-\omega t)$$
$$u = \frac{1}{2}\epsilon_0\varepsilon^2\cos^2(kx-\omega t) + \frac{1}{2}\epsilon_0\varepsilon^2\cos^2(kx-\omega t)$$
$$\therefore u = \epsilon_0\varepsilon^2\cos^2(kx-\omega t)$$
$$\langle u\rangle = \frac{1}{T}\int_0^T udt$$
$$\langle u\rangle = \frac{1}{T}\int_0^T \epsilon_0\varepsilon^2\cos^2(kx-\omega t)dt$$
$$\therefore \langle u\rangle = \frac{1}{2}\epsilon_0\varepsilon^2$$

b

$$\mathbf{S} = \frac{1}{\mu}_0(\mathbf{E}\times\mathbf{B})$$
$$\mathbf{S} = \frac{1}{\mu}_0(\varepsilon\cos(kx-\omega t)\hat{j}\times\frac{\varepsilon}{c}\cos(kx-\omega t)\hat{k})$$
$$\mathbf{S} = \frac{1}{\mu_0 c}\varepsilon^2\cos^2(kx-\omega t)\hat{i}$$
$$c^2 = \frac{1}{\mu_0\epsilon_0} \rightarrow c\epsilon_0 = \frac{1}{\mu_0 c}$$
$$\therefore \mathbf{S} = \epsilon_0 c\varepsilon^2\cos^2(kx-\omega t)\hat{i}$$
$$\langle\mathbf{S}\rangle = \frac{1}{T}\int_0^T \mathbf{S}dt$$
$$\therefore \langle\mathbf{S}\rangle = \frac{c\epsilon_0\varepsilon^2}{2}\hat{i}$$

c

$$\Pi = \frac{S}{c^2} \rightarrow \therefore \Pi = \frac{\epsilon_0\varepsilon^2}{c}\cos^2(kx-\omega t)\hat{i}$$
$$\langle\Pi\rangle = \frac{1}{T}\int_0^T \Pi dt$$
$$\therefore \langle\Pi\rangle = \frac{\epsilon_0\varepsilon^2}{2c}\hat{i}$$

d

i

$$P_{rad} = \frac{\langle S\rangle}{c} \rightarrow F = PA = \frac{\langle S\rangle A}{c}$$
$$\therefore F = \frac{c\epsilon_0\varepsilon^2}{2}\hat{i}\frac{A}{c} = \frac{A\epsilon_0\varepsilon^2}{2}\hat{i}$$

ii

$$P_{rad} = \frac{2\langle S\rangle}{c} \rightarrow F = PA = \frac{2\langle S\rangle A}{c}$$
$$\therefore F = \frac{c\epsilon_0\varepsilon^2}{2}\hat{i}\frac{2A}{c} = A\epsilon_0\varepsilon^2\hat{i}$$

e

i

$\langle u\rangle c = \frac{J}{m^3}\frac{m}{s} = \frac{J}{m^2 s}$
$\langle u\rangle /c = \frac{\frac{J}{m^3}}{m/s} = \frac{Js}{m^4}$
$I = \frac{W}{m^2} = \frac{J}{m^2 s}$ from question f
$\therefore I = \langle u\rangle c$

ii

$\langle u\rangle /c = \frac{\frac{J}{m^3}}{m/s} = \frac{Js}{m^4}$
$\langle u\rangle c = \frac{J}{m^3}\frac{m}{s} = \frac{J}{m^2 s}$
$J = \frac{kgm^2}{s^2}$
$\langle u\rangle c = \frac{kg}{s^3}, \langle u\rangle /c = \frac{kg}{m^2 s}$
$\langle \Pi\rangle = \frac{kg}{m^2 s}$ from question f iii
$\therefore \langle \Pi\rangle = \langle u\rangle /c$

f

$I = 1360\frac{W}{m^2}$

i

$1360 = \frac{c\epsilon_0\varepsilon^2}{2}\hat{i}$
$\therefore \varepsilon = \sqrt{\frac{2*1360}{(3.0\times10^8)(8.85\times10^{-12})}} = 1012.17\frac{V}{m}$
$\therefore \frac{\varepsilon}{c} = \frac{1012.17}{3.0\times10^8} = 3.37\times10^{-6}T$

ii

$1360 = \langle u\rangle c \rightarrow \langle u\rangle = \frac{1360}{c}$
$\therefore \langle u\rangle = 4.53\times10^{-6}\frac{J}{m^3}$

iii

$\langle \Pi\rangle = \langle u\rangle /c = \frac{4.53\times10^{-6}}{3.0\times10^8}$
$\therefore \langle \Pi\rangle = 1.51\times10^{-14}\frac{kg}{m^2 s}$

iv

$1mi^2 = 2.59\times10^6 m^2$
$F = A\epsilon_0\varepsilon^2$ from d
$\therefore F = (2.59\times10^6)(8.85\times10^{-12})(1012.17)^2 = 23.48N$

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a

$I = \frac{W}{m^2} \rightarrow I = \frac{L}{A}$
$\therefore I = \frac{L}{4\pi R^2}$

b

$F = \frac{IA}{c}$
$A =$ Cross Section Area
$A = \pi r^2$
$F = \frac{L\pi r^2}{4\pi R^2 c}$
$\therefore F = \frac{Lr^2}{4R^2 c}$

c

$F = G\frac{m_1 m_2}{R^2} - \frac{IA}{c} = 0$
$G\frac{M\rho\frac{4}{3}\pi r^3}{R^2} = \frac{Lr^2}{4R^2 c}$
$\therefore r = \frac{3L}{16cMG\rho\pi}$ meters

d

$r = \frac{3L}{16cMG\rho\pi} \rightarrow r = \frac{(3)(4\times10^{26}W)}{(16\pi)(3\times10^8\frac{m}{s})(2\times10^{30}kg)(6.67\times10^{-11}\frac{Nm^2}{kg^2})(2.5\times10^3\frac{kg}{m^3})}$
$\therefore r = 2.386 \times 10^-7\frac{Ws}{N} = 2.386 \times 10^-7\frac{Nm}{s}\frac{s}{N} = 2.386 \times 10^-7m$

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a

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$
$\Phi_E = \oint \vec{E} \cdot d\vec{a}$
$\Phi_E = \vec{E} \oint d\vec{a}$
$\Phi_E = E_0 \sin(\omega t)\hat{i}(2\pi R^2)$
$\frac{d\Phi_E}{dt} = E_0 \cos(\omega t)(2\pi R^2)\hat{i}$
$\vec{B} \oint d\vec{s} = \mu_0(I_{enc} + \varepsilon_0 E_0 \cos(\omega t)(2\pi R^2)\hat{i})$
$\vec{B} \oint d\vec{s} = \mu_0 \varepsilon_0 E_0 \cos(\omega t)(2\pi R^2)\hat{i}$
$\vec{B} 2\pi R = \mu_0 \varepsilon_0 E_0 \cos(\omega t)(2\pi R^2)\hat{i}$
$\vec{B} = \mu_0 \varepsilon_0 E_0 R \cos(\omega t)$ Counter Clockwise

b

$\vec{B} 2\pi(2R) = \mu_0 \varepsilon_0 E_0 \cos(\omega t)(2\pi(2R)^2)$
$\vec{B} = 2\mu_0 \varepsilon_0 E_0 R \cos(\omega t)$ Counter Clockwise