CSDS 343 HW5

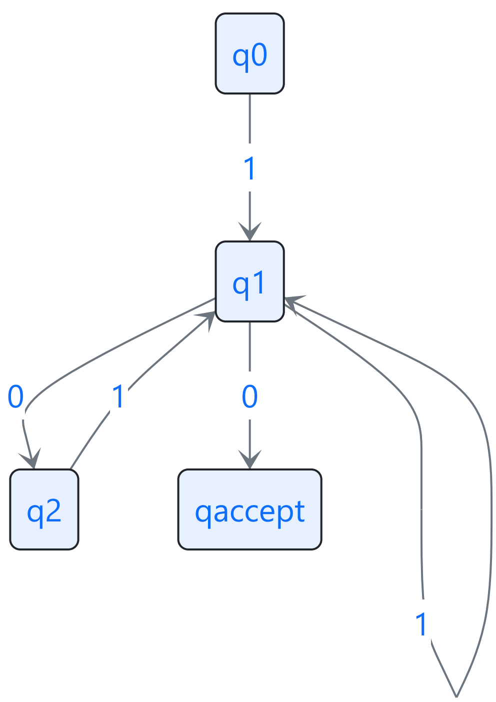
1

a

Construct a FSA equivalent to the REG EX

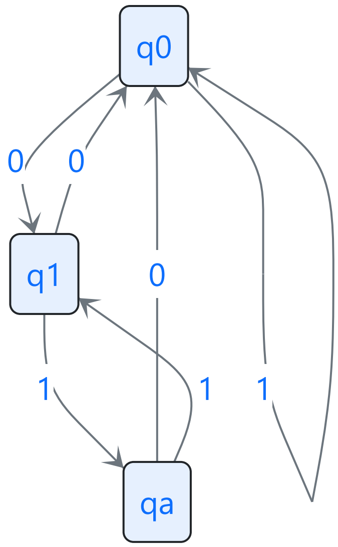
$1[((01)^* + 1 + 01)^* + 1]^*0$

> The above REG EX is equivalent to this regex: $1(01 + 1)^*0$ Below is the FSA equivalent to the REG EX



b

Construct a REG EX equivalent to the FSA



> Regex Equivalent: $[1+0(11)^*(0+10)]^*01(11)^*$

2

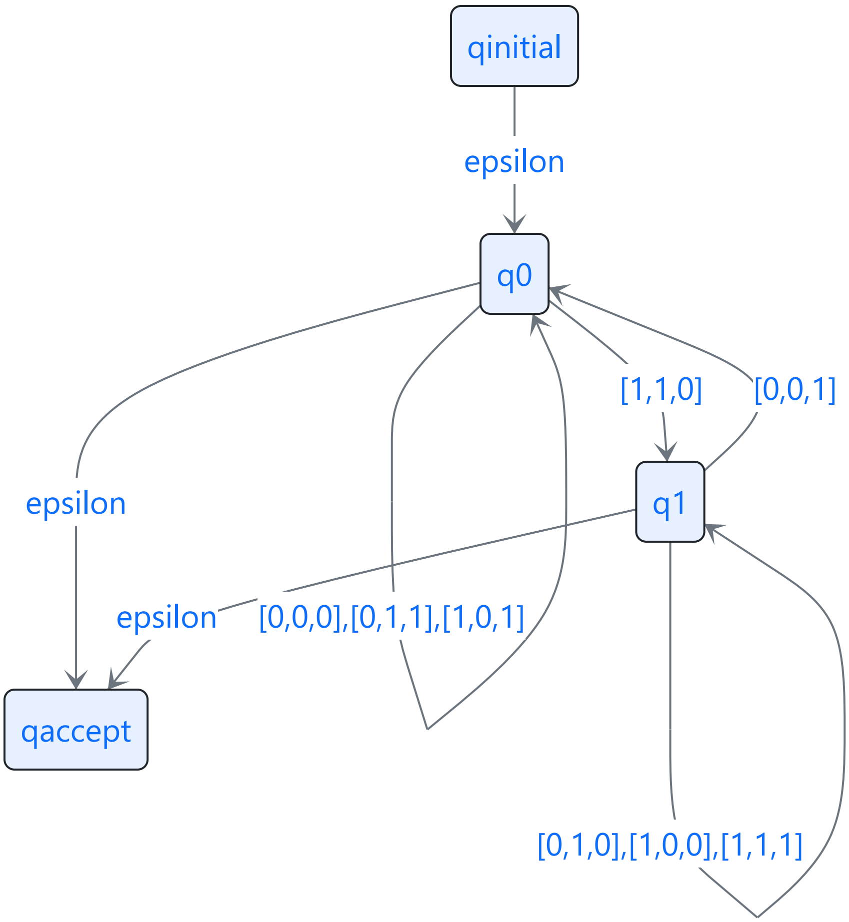
Prove the reverse of a regular langauge is also a regular langauge, (A, A^R)

> There are 2 ways to prove this true (please grade using the 2nd method):
>
> If A is a regular language then there exists a FSA F that can decide A. A^R would then be decided by the transpose of F, that is F^T can decide A^R. This is done by having the q_0 of F be the accept state of F^T and the various accept states of F are the initial states of F^T. However we cannot have multiple initial states of F^T, so to circumvent this we create a q_0 that ϵ transitions to each of the accept states of F.
>
> Alternatively, if A is a regular language then there exists a regular expression for A. A^R would then be decided by the reverse of the regular expression. We can prove this by showing the reverses of each regular expression rule:
>
> 1. $\epsilon \to^R \epsilon$ holds true
> 2. $a \to^R a$, where $a \in \Sigma$, holds true
> 3. $(x \cup y) \to^R (x^R \cup y^R)$, where x and y are regular expressions, holds true
> 4. $(xy) \to^R (y^R x^R)$, where x and y are regular expressions, holds true
> 5. $x^* \to^R x^*$, where x is a regular expression, holds true
>
> Thus since we have created a reverse function $\to^R$ we can show that A^R is a regular language by reversing the regular expression for A and using the reverse regex for strings in A^R.

3

Prove $B = \{w \in \Sigma_3^* | \text{the bottom row of w is the sum of the top 2 rows}\}$ is a regular language. Hint prove B^R is regular.

> We can show this language is regular by proving its reverse is regular. A language can be proven regular if it can be decided by a FSA.



> How this FSA works is we alternate between q0 (no carry) and q1 (carry over) if there is a 1 to carry over from the previous string. Then we only accept if there are no more columns to read. Note that any columns not mentioned at a particular state lead to a reject.This FSA will recognize any string in B^R.
>
> Thus sinse B^R is a regular language we know that B is a regular language by question 2.

4

Prove the following languages are not regular languages:

a

Palindromes over the alphabet $\Sigma = \{a, b, c\}$

> We can show this by proof by contradiction of the pumping lemma.
>
> Suppose palindromes are regular then $\exists p$ s.t. all strings s in L with $|s| > p$ can be divded into $s = xyz$ such that $|xy| \leq p$ and $|y| > 0$
>
> Given the string $a^p b a^p$ is a valid palindrome, we can divide it into string $xy = a^p$ because $|xy| = |a^p| < p$ and $|y| = 0$.
>
> The pumping lemma suggests $xy^k z$ is a palindrome for all $k \in \mathbb{N}$
>
> But what about $xy^2 z$? In order for $xy^2 z = a^{p+|y|} b a^p$ to be a palindrome $p + |y| = p$ so $|y| = 0$ which is a contradiction because $|y| > 0$.
>
> Therefore palindromes are not regular.

b

The language $L = \{abbaaabbbb \dots a^{k-2} b^{k-1} a^k | k \geq 3\}$

> We can show this by proof by contradiction of the pumping lemma.
>
> Suppose this language is regular. The the reverse of this langauge (L^R) is also regular. If L^R is regular then $\exists p$ s.t. all strings s in L with $|s| > p$ can be divded into $s = xyz$ such that $|xy| \leq p$ and $|y| > 0$
>
> Given the string $a^p b^{p-1} a^{p-2} \dots \in L^R$, we can divide it into string $xy = a^p$ because $|xy| = |a^p| < p$ and $|y| = 0$.
>
> The pumping lemma suggests $xy^k z \in L^R$ for all $k \in \mathbb{N}$
>
> But what about $xy^2 z$? In order for $xy^0 z = a^{p-|y|} b^{p-1} a^{p-2}$ to be in L $p - |y| = p$ so $|y| = 0$ which is a contradiction because $|y| > 0$.
>
> Therefore L is not regular because its reverse is not regular