Problem 1 (7 pt)

For the hypothesis test $H_0 : \mu = 10$ against $H_1 : \mu < 10$ with variance unknown and $n = 20$, let the value of the test statistic be $t_0 = 1.25$.

a. Use table V to approximate the P-value.

> $t_0 = 1.25$ and we have $20 - 1 = 19$ degrees of freedom.
>
> A t value of 1.25 is between 0.688 and 1.328, which results in $0.25 < \alpha < 0.10$ or $0.75 < P < 0.9$. Since 1.25 is a lot closer to 1.328 than 0.688 then we can approximate $P \approx 0.88$

b. Use R to compute the P-value. Attach the code and output.

```
pt(1.25, df=19, lower.tail=TRUE)
```

```
[1] 0.8867613
```

c. Does your answer in part b agree with your answer in part a? Why or why not?

> Yes they do because R got an answer of 0.8867613 which is in between the range specified above and my approximation of 0.88 is close to the R output.

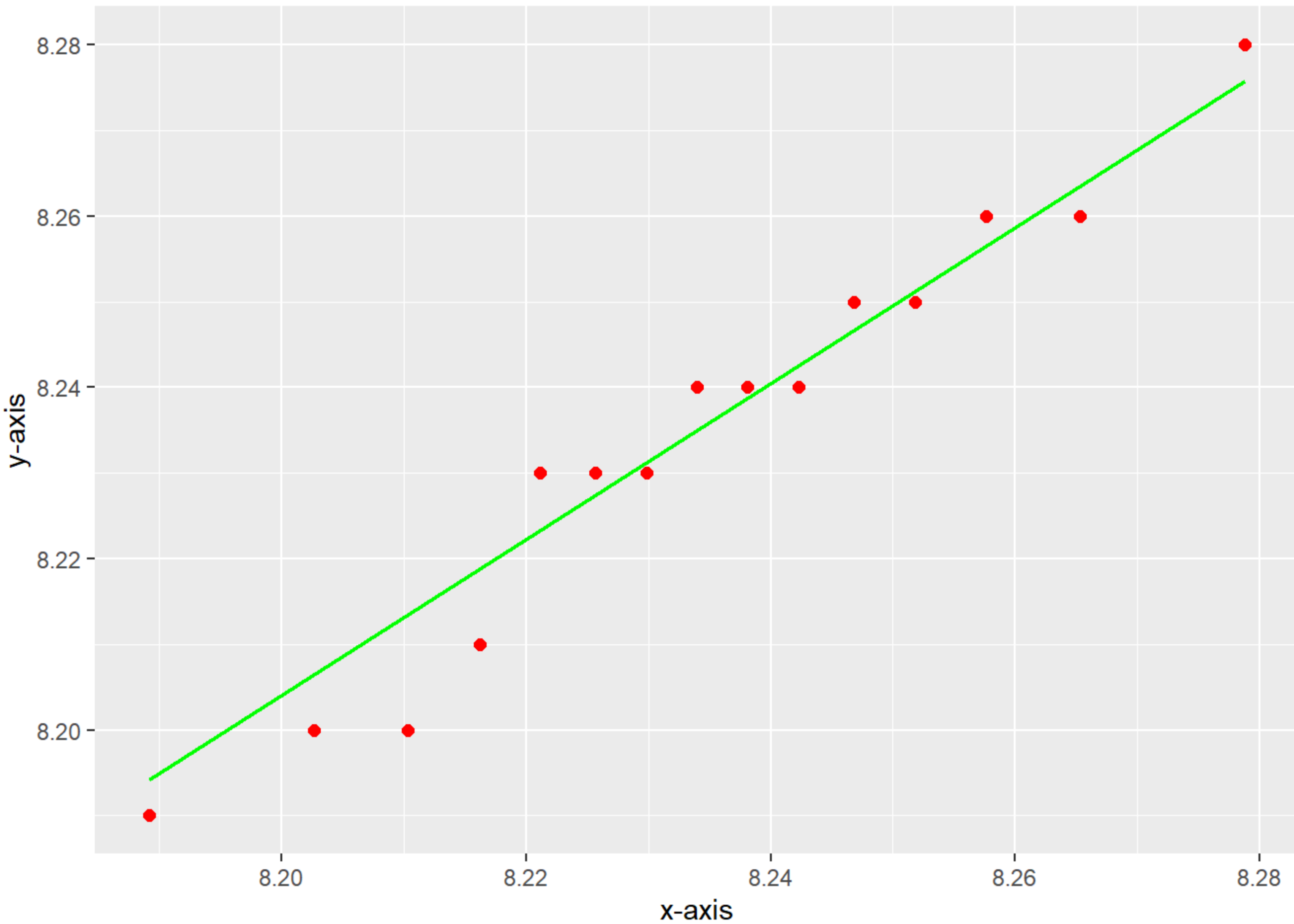
P287 8-40 (Modified) (37 pt)

A machine produces metal rods used in an automobile suspension system. A random sample of 15 rods is selected, and the diameter is measured. The resulting data (in millimeters) are as follows: 8.24 8.21 8.23 8.25 8.26 8.23 8.20 8.26 8.19 8.23 8.20 8.28 8.24 8.25 8.24

a. Check the assumption of normality for rod diameter in the population using a normal probability plot. What is your conclusion and why? Attach the plot.

```
data = c(8.24, 8.21, 8.23, 8.25, 8.26, 8.23, 8.20, 8.26, 8.19, 8.23, 8.20, 8.28, 8.24, 8.25, 8.24
```

```
ggplot(mapping = aes(sample = data)) + stat_qq_point(size = 2,color = "red") + stat_qq_line(color=
```



> All points are pretty close to the line and there is no clear curve to indicate a skewed distribution. Therefore the population is normal.

b. Calculate a 95% two-sided confidence interval for the mean rod diameter of all the metal rods produced by this machine.

```
print(mean(data))
```

```
[1] 8.234
```

```
print(sd(data))
```

```
[1] 0.02529822
```

> $df = 15 - 1 = 14$, 2 tailed 95% t for $df = 14$ is $t = 2.145$

> $\mu = \bar{x} \pm t \times \frac{s}{\sqrt{n}}$
>
> $8.234 \pm 2.145 \times \frac{0.02529822}{\sqrt{15}}$
>
> $\mu = [8.22, 8.248]$

c. Based on the data given, can you conclude that the mean rod diameter of all the metal rods produced by this machine is different from 8.25 mm at a significant level of 0.01? Use the 8-step P-value approach.

1. Identify the parameter of interest

> μ is mean diameter of rods

2. State the hypotheses to be tested.

> $H_0 : \mu = 8.25$
>
> $H_1 : \mu \neq 8.25$

3. Specify the level of significance α.

> 2 tailed so $\alpha = 0.005$

4. Determine the test statistic and its distribution.

> t test:
>
> $t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{\bar{x}-8.25}{s/\sqrt{15}}$
>
> $t \approx t_{14}$ when H_0 is true

5. Calculate the observed value for the test statistic.

> $t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{\bar{x}-8.25}{s/\sqrt{15}}$
>
> $= \frac{8.234-8.25}{0.02529822\sqrt{15}} = -2.4495$

6. Compute the P-value

> $P(t \leq -2.4495) = P(t \geq 2.4495)$

> $2.145 < 2.4495 < 2.624$
>
> So $0.025 < p < 0.01$

7. Make a statistical decision.

> Fail to reject H_0 because $p > 0.005$

8. State the conclusion.

> At the 0.01 significance level there is not enough evidence that the mean diameter of all rods produced by this machine is different than 8.25

d. Use R to conduct the same test given in part c. The resulting output should include all important information that can be used for 3 testing approaches, the rejection region approach, the p-value approach and the confidence interval approach. Attach the code and output.

```
t.test(data, mu=8.25, conf.level =0.99)
```

```

	One Sample t-test

data:  data
t = -2.4495, df = 14, p-value = 0.02807
alternative hypothesis: true mean is not equal to 8.25
99 percent confidence interval:
 8.214555 8.253445
sample estimates:
mean of x 
    8.234 
```

e. Can you use the confidence interval in part b to conduct the test in part c? Why or why not?

> No because they have different confidence intervals

f. Based on the data given, can you conclude that the variance of rod diameter of all the metal rods produced by this machine is

greater than 0.0002 mm at a significant level of 0.01? Use the 8-step rejection region approach.

1. Identify the parameter of interest

> σ^2 is standard deviation of the diameter of all rods in the population

2. State the hypotheses to be tested.

> $H_0 : \sigma^2 = 0.0002$
>
> $H_1 : \sigma^2 \geq 0.0002$

3. Specify the level of significance α.

> 1 tailed so $\alpha = 0.01$

4. Determine the test statistic and its distribution.

> χ^2 test:
>
> $\chi^2 = (n-1)\frac{s^2}{\sigma^2} = 14\frac{s^2}{0.0002} = 70,000s^2$
>
> $\chi^2 \approx \chi^2_{14}$ when H_0 is true and when the population distribution is normal

5. Determine the rejection region

> $\chi^2_{0.01,14} = 29.14$
>
> If $\chi^2 > 29.14$ then we reject

6. Compute the χ^2-value

> $\chi^2 = (n-1)\frac{s^2}{\sigma^2} = 14\frac{s^2}{0.0002^2} = 70,000s^2$
>
> $= 70,000(0.02529822^2) = 44.80$

7. Make a statistical decision.

> Reject H_0 because $\chi^2 > 29.14$

8.

> At the 0.01 significance level there is enough evidence that the population variance of the diameter of all rods produced by this machine is greater than 0.0002

g. Do you need any additional assumptions to justify your test in part f? Why or why not?

> No additional assumptions because normailty was verified in part a

h. Use an appropriate table to approximate the P- value for the test in part f.

> $44.80 > 31.32$ so $p < 0.005$

i. Use R to calculate the P-value for the test in part f. Attach the code and output.

```
pchisq(44.8, 14, lower.tail=FALSE)
```

```
[1] 4.395692e-05
```