HW5 STAT 312

1

The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce. ($N(12.4, 0.1^2)$)

A

What is the probability that a fill volume is less than 12 fluid ounces?

$z = \frac{x-\mu}{\sigma} = \frac{12-12.4}{0.1} = -4$

$P(Z \leq -4) = 0.000033$

B

If all cans less than 12.1 or more than 12.6 ounces are scrapped, what proportion of cans is scrapped?

$z = \frac{x-\mu}{\sigma} = \frac{12.1-12.4}{0.1} = -3$

$P(Z \leq -3) = 0.00135$

$z = \frac{x-\mu}{\sigma} = \frac{12.6-12.4}{0.1} = 2$

$P(Z \geq 2) = 0.0.02275$

The percent of cans that are scrapped is $0.00135 + 0.02275 = 0.0241 = 2.41$

C

Determine specifications that are symmetric about the mean that include 99% of all cans.

$0.99 = P(-z \leq Z \leq z) = P(Z \leq z) - P(Z \leq -z)$

Alternatively, $0.99 = \int_{-z}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

This means to find z such that the first statement is true we need to find z such that $P(Z \leq z) = 0.995$ and $P(Z \leq -z) = 0.005$.

Using the z table we find that $z = \pm 2.58$.

We can then use this to find the range by solving $z = \frac{x-\mu}{\sigma}$ for x.

$\pm 2.58 = \frac{x-12.4}{0.1} \rightarrow x = 12.4 \pm 2.58 \times 0.1 = 12.658$ or 12.142

The specifications are $12.142 \leq x \leq 12.658$

D

Use R to find solutions for parts (a) and (c). Attach the R codes and output.

```
print(paste("1A: ", pnorm(12,12.4,0.1)))
```

```
[1] "1A:  3.16712418331194e-05"
```

```
print(paste("1C: ", "(",qnorm(0.005,12.4,0.1),",",qnorm(0.995,12.4,0.1), ")"))
```

```
[1] "1C:  ( 12.1424170696451 , 12.6575829303549 )"
```

2

The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.002 inch and a standard deviation of 0.0004 inch. ### (a) What is the probability that the diameter of a dot exceeds 0.0026?

(b)

What is the probability that a diameter is between 0.0014 and 0.0026?

(c)

What standard deviation of diameters is needed so that the probability in part (b) is 0.995?

3

Suppose that X has a lognormal distribution with parameters = 5 and 2 = 9.

(a)

Use Table III to find P(X < 13,300).

(b)

Use Table III to find the value of x such that P(X   x) = 0.95.

(c)

Find the mean and variance of X.

(d)

Use R to find solutions for parts (a) and (b). Attach the R codes and output. Please note that your answers in this part may not be exactly the same as your answers in parts a and b when using table III.

4

Suppose that X has a lognormal distribution and that the mean and variance of X are 100 and 85,000, respectively. ### (a) Determine the parameters   and 2 of the lognormal distribution. You can use the method that we discussed in class or the hint below to solve the problem.

Hint: define x= exp(   ) and y= exp(2) and write two equations in terms of x and y.

(b)

Use R to find P(X   5). Attach the R code and output.