P320 9-10 (Modified) (9 pt)

The heat evolved in calories per gram of a cement mixture is approximately normally distributed with a standard deviation of 2. We wish to test $H_0 : \quad = 100$ versus $H_1 : \quad\quad 100$ with a sample of size 5. If the rejection region is defined as $\bar{x} < 98.5$ and $\bar{x} > 101.5$, then find

a. α , the type I error probability;

$\alpha = P(\text{type 1 error}) = P(\text{reject } H_0 \text{ when } H_0 \text{ is true})$

$= P(\bar{x} < 98.5 \vee \bar{x} > 101.5)$

z $= \pm\frac{1.5}{2/\sqrt{5}} = \pm 1.677$

$= P(Z < -1.677) + P(Z > 1.677)$

$= 0.04648 + 0.04648 = 0.09296$

b. β, the type II error probability, for the case when the true mean heat evolved is 103; and

$\beta(103) = P(\text{type 2 error}) = P(\text{fail to reject } H_0 \text{ when } \mu = 103)$

$= P(98.5 < \bar{x} < 101.5)$

z_{low} $= \frac{-4.5}{2/\sqrt{5}} = -5.031$

z_{high} $= \frac{-1.5}{2/\sqrt{5}} = -1.677$

$= P(Z < z_{high}) - P(Z < z_{low}) \rightarrow P(Z < -1.677) - P(Z < -5.031)$

$= 0.04648 - 0 = 0.04648$

c. the power of the test when the true mean heat evolved is 103.

$1 - \beta(103) = 0.95352$

P330 9-42 (Modified) (17 pt)

Humans are known to have a mean gestation period of 280 days (from last menstruation) with a standard deviation of about 9 days. A hospital wondered whether there was any evidence that their patients were at risk for giving birth prematurely. In a random sample of 70 women from this hospital, the average gestation time was 274.3 days.

a. Can you conclude that the sample data provides enough evidence that the average gestation time of all the patients at this hospital was less than 280 days at a significant level of 0.05? Use the 8-step critical region approach.

1. Identify the parameter of interest

   μ

2. State the hypotheses to be tested.

   $H_0 = \mu = 280$

   $H_1 = \mu \leq 280$

3. Specify the level of significance .

   $\alpha = 0.05$

4. Determine the test statistic and its distribution.

   z-test

   $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{x}-280}{9/\sqrt{70}}$

   Assuming H_0 is true

5. Determine the rejection region.

   $z < -1.64$

6. Calculate the observed value for the test statistic.

   $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{274.3-280}{9/\sqrt{70}}$

   $z = \frac{-5.7}{9/\sqrt{70}} = -5.299$

7. Make a statistical decision.

   Reject H_0 because z is in the reject region ($z < 1.64$)

8. State the conclusion.

   At the 0.05 significance level there is enough evidence to conclude that the population mean is less than 280

b. Can you use the P-value to conclude that the sample data provides enough evidence that the average gestation time of all the patients at this hospital was less than 280 days at a significant level of 0.05? Why or why not?

1. Identify the parameter of interest

   μ

2. State the hypotheses to be tested.

   $H_0 = \mu = 280$

   $H_1 = \mu \leq 280$

3. Specify the level of significance .

   $\alpha = 0.05$

4. Determine the test statistic and its distribution.

   z-test

   $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{x}-280}{9/\sqrt{70}}$

   Assuming H_0 is true

5. Calculate the observed value for the test statistic.

   $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{274.3-280}{9/\sqrt{70}}$

   $z = \frac{-5.7}{9/\sqrt{70}} = -5.299$

6. Compute the p-value

   $P(z < -5.299) = 0$

7. Make a statistical decision.

   Reject H_0 because P-value is less than 0.05

8. State the conclusion.

   At the 0.05 significance level there is enough evidence to conclude that the population mean is less than 280

c. Can you conclude that the sample data provides enough evidence that the average gestation time of all the patients at this hospital was less than 280 days at a significant level of 0.05 using an appropriate confidence interval? State you reason.

$z_\alpha = 1.64$

The range for μ is $(-\infty, 274.3 + 1,64 * 9/\sqrt{70}] = (-\infty, 276.06]$

$\mu = 280$ is outside of this range

At the 0.05 significance level there is enough evidence to conclude that the population mean is less than 280 because 280 is outside of the range of μ

d. What additional assumptions are necessary for the method you used in part a-c to be valid? Explain.

Assumption of normality and random sampling

Problem 3 (8 pt)

For the hypothesis test H0: $= 10$ against H1: > 10 with variance known and n = 15,

a. Use table III to find the P-value for each of the following values of test statistic.

(1) z_0 **= - 2.05**

$P(Z \leq -2.05) = 0.020182$

(2) z_0 **= 1.84**

$P(Z \leq 1.84) = 0.967116$

b. Use R to compute the P-values in part a. Attach the codes and output.

```
pnorm(-2.05)
```

```
[1] 0.02018222
```