

# CSDS 343 HW5

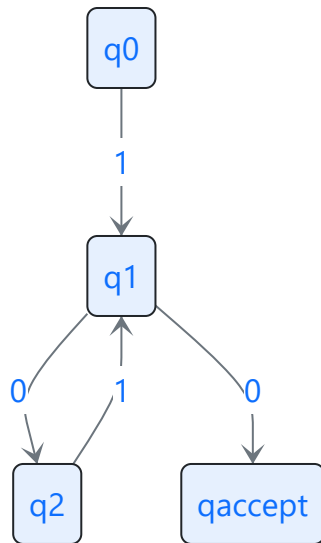
1

a

Construct a FSA equivalent to the REG EX

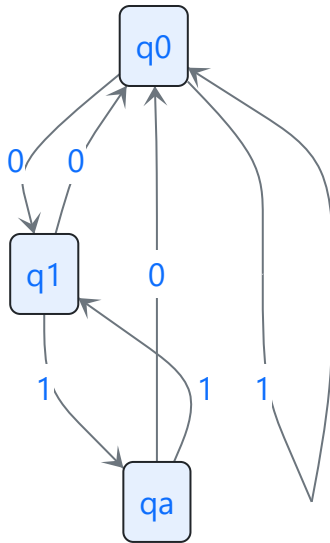
$1[((01)^* + 1 + 01)^* + 1]^*0$

The above REG EX is equivalent to this regex:  $1(01 + 1)^*0$  Below is the FSA equivalent to the REG EX



b

Construct a REG EX equivalent to the FSA



Regex Equivalent:  $[1 + 0(0 + 10)]^*01(11)^*$

## 2

Prove the reverse of a regular language is also a regular language,  $(A, A^R)$

There are 2 ways to prove this true (please grade using the 2nd method):

If  $A$  is a regular language then there exists a FSA  $F$  that can decide  $A$ .  $A^R$  would then be decided by the transpose of  $F$ , that is  $F^T$  can decide  $A^R$ . This is done by having the  $q_0$  of  $F$  be the accept state of  $F^T$  and the various accept states of  $F$  are the initial states of  $F^T$ . However we cannot have multiple initial states of  $F^T$ , so to circumvent this we create a  $q_0$  that  $\epsilon$  transitions to each of the accept states of  $F$ .

Alternatively, if  $A$  is a regular language then there exists a regular expression for  $A$ .  $A^R$  would then be decided by the reverse of the regular expression. We can prove this by showing the reverses of each regular expression rule:

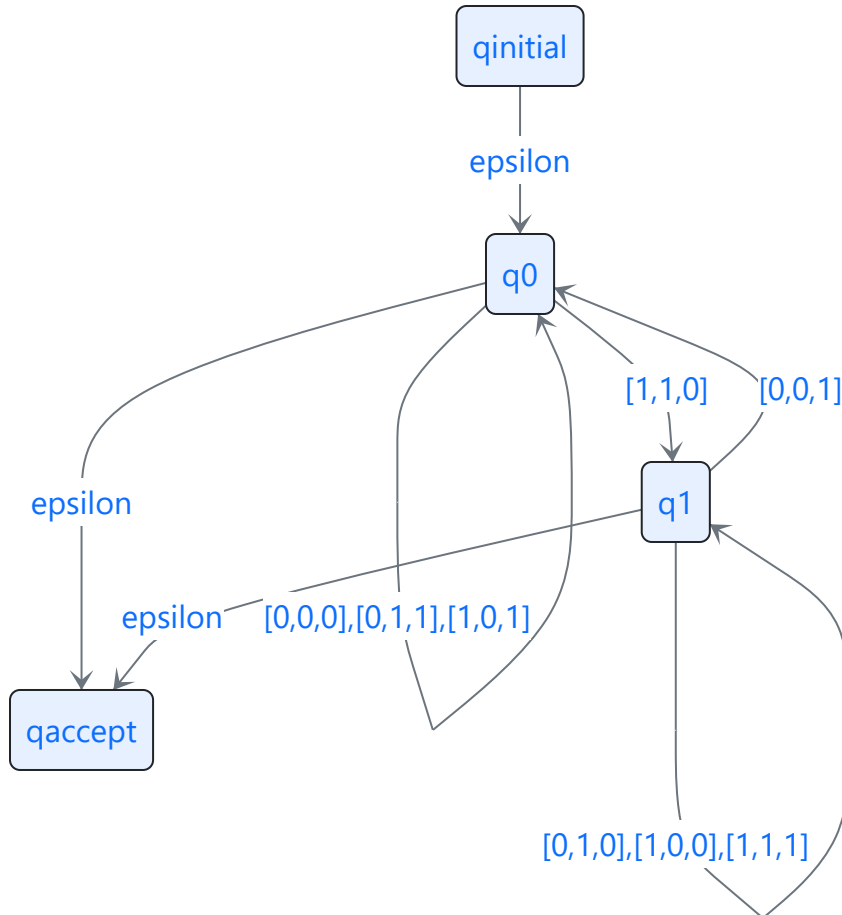
1.  $\epsilon \rightarrow^R \epsilon$  holds true
2.  $a \rightarrow^R a$ , where  $a \in \Sigma$ , holds true
3.  $(x \cup y) \rightarrow^R (x^R \cup y^R)$ , where  $x$  and  $y$  are regular expressions, holds true
4.  $(xy) \rightarrow^R (y^R x^R)$ , where  $x$  and  $y$  are regular expressions, holds true
5.  $x^* \rightarrow^R x^*$ , where  $x$  is a regular expression, holds true

Thus since we have created a reverse function  $\rightarrow^R$  we can show that  $A^R$  is a regular language by reversing the regular expression for  $A$  and using the reverse regex for strings in  $A^R$ .

## 3

Prove  $B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top 2 rows}\}$  is a regular language. Hint prove  $B^R$  is regular.

We can show this language is regular by proving its reverse is regular. A language can be proven regular if it can be decided by a FSA.



How this FSA works is we alternate between q0 (no carry) and q1 (carry over) if there is a 1 to carry over from the previous string. Then we only accept if there are no more columns to read. Note that any columns not mentioned at a particular state lead to a reject. This FSA will recognize any string in  $B^R$ .

Thus since  $B^R$  is a regular language we know that  $B$  is a regular language by question 2.

## 4

Prove the following languages are not regular languages:

a

Palindromes over the alphabet  $\Sigma = \{a, b, c\}$

We can show this by proof by contradiction of the pumping lemma.

Suppose palindromes are regular then  $\exists p$  s.t. all strings  $s$  in  $L$  with  $|s| > p$  can be divided into  $s = xyz$  such that  $|xy| \leq p$  and  $|y| > 0$

Given the string  $a^p b a^p$  is a valid palindrome, we can divide it into string  $xy = a^p$  because  $|xy| = |a^p| < p$  and  $|y| = 0$ .

The pumping lemma suggests  $xy^k z$  is a palindrome for all  $k \in \mathbb{N}$

But what about  $xy^2 z$ ? In order for  $xy^2 z = a^{p+|y|} b a^p$  to be a palindrome  $p + |y| = p$  so  $|y| = 0$  which is a contradiction because  $|y| > 0$ .

Therefore palindromes are not regular.

## b

The language  $L = \{abbaaabb...a^{k-2}b^{k-1}a^k | k \geq 3\}$

We can show this by proof by contradiction of the pumping lemma.

Suppose this language is regular. The the reverse of this language ( $L^R$ ) is also regular. If  $L^R$  is regular then  $\exists p$  s.t. all strings  $s$  in  $L$  with  $|s| > p$  can be divided into  $s = xyz$  such that  $|xy| \leq p$  and  $|y| > 0$

Given the string  $a^p b^{p-1} a^{p-2} \dots \in L^R$ , we can divide it into string  $xy = a^p$  because  $|xy| = |a^p| < p$  and  $|y| = 0$ .

The pumping lemma suggests  $xy^k z \in L^R$  for all  $k \in \mathbb{N}$

But what about  $xy^2 z$ ? In order for  $xy^2 z = a^{p+|y|} b^{p-1} a^{p-2}$  to be in  $L$   $p - |y| = p$  so  $|y| = 0$  which is a contradiction because  $|y| > 0$ .

Therefore  $L$  is not regular because its reverse is not regular