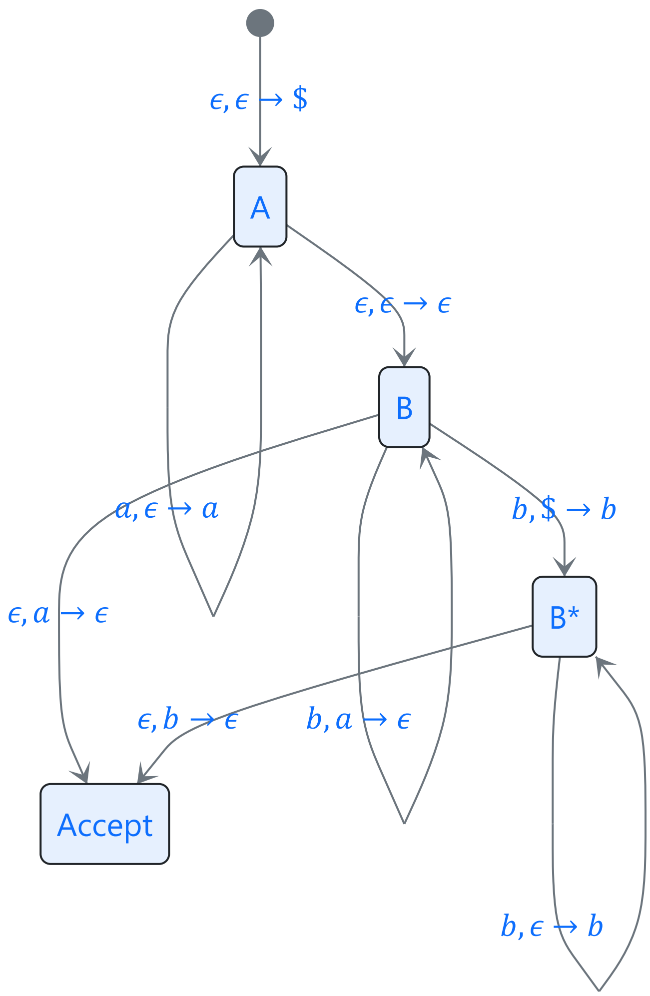
HW6CSDS343

1

Prove the following language is context free $E = \{a^i b^j | i \neq j\}$

A

Give a pushdown automaton that decides E



B

Give a context free grammar for E

> $S \to aSb$
>
> $S \to a$

> $S \to b$

2

Prove the concatenation $(xy|x \in L_1 \land y \in L_2)$ of two context free languages L_1 and L_2 is context free

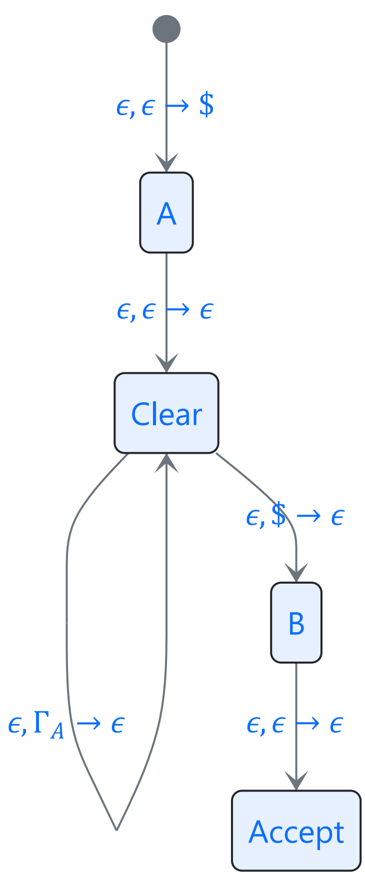
A

Assume you have the context free grammar for each language, give the context free grammar for the concatenation (create the CFG directly)

> $S \to XY$ where X is the context free grammar for L_1 and Y is the context free grammar for L_2

B

Assume you have the pushdown automata for each language, give the pushdown automata for the concatenation (create the PDA directly)



> A is the machine for L_1, B is the machine for L_2
>
> We map the accept state of the first automata to a clear state, which clears the stack, then move to the initial state of the second automata.

> That is, we only move to the other machine if the first machine accepts. We only transition to the accept state if both machines accept. This machine only accepts if we end at the accept state and there is nothing left on the string.

3

Prove that each of the following languages are not context free

A

$L_1 = \{w\bar{w} | w \in \{0,1\}\}$ where $\bar{w}$ is the bit compliment of w

> We can prove this language is not context free using proof by contradiction with the pumping lemma
>
> Suppose L_1 is context free, then $\exists$ a p s.t. all strings s in L_1 with $|s| > p$ can be divded into $s = uvxyz$ such that $|vxy| \leq p$ and $|vy| > 0$
>
> Given the string $0^p1^p0^p1^p0^p1^p$ and that $|vxy| \leq p$, there are 5 cases of vxy
>
> 1. $vxy = 0^*$ first
> 2. $vxy = 0^*1^*$ first
> 3. $vxy = 1^*$ first
> 4. $vxy = 1^*0^*$ first
> 5. $vxy = 0^*$ second
> 6. $vxy = 0^*1^*$ second
> 7. $vxy = 1^*$ second
> 8. $vxy = 1^*0^*$ second
> 9. $vxy = 0^*$ third
> 10. $vxy = 0^*1^*$ third
> 11. $vxy = 1^*$ third
>
> For all cases suppose the string uv^0xy^0z. This string should be in L_1 according to the pumping lemma but we can prove this is not true for all cases.
>
> In case 1, our string is now of the form $0^{p-|vy|}1^p0^p1^p0^p1^p$. This shifts the midpoint to the right by $|vy|/2$. Thus if we split the string in half we get $w = 0^{p-|vy|}1^p0^p1^{|vy|/2}$ and the second half as $1^{p-|vy|/2}0^p1^p$. Since the second half is not the compliment of w this string is not in L_1.
>
> Case 5 and 9 follow similar logic, with case 9 shifting the midpoint left.
>
> In case 3, our string is now of the form $0^p1^{p-|vy|}0^p1^p0^p1^p$. This shifts the midpoint to the right by $|vy|/2$. Thus if we split the string in half we get $w = 0^p1^{p-|vy|}0^p1^{|vy|/2}$ and the second half as $1^{p-|vy|/2}0^p1^p$. Since the second half is not the compliment of w this string is not in L_1.
>
> Case 7 and 11 follow similar logic, with case 11 shifting the midpoint left.

> Cases 2 and 4 follow similar logic, that is decreasing the number of elements on the left by $|vy|$ and then shifting the midpoint to the right and creating strings 0101 and 101 with some number of 1's and 0's (less than or equal to p). Since the second half is not the compliment of w this string is not in L_1.
>
> Similar logic for cases 8 and 10 except these cases decrease the number of elements on the right, shifting the midpoint left and creating strings 010 and 0101 (with some number of 1's and 0's less than or equal to p) which are not compliments of eachother.
>
> Case 6 is at the midpoint, thus if it takes away more 0's than 1's the midpoint will shift to the right and follow similar logic as cases above. If it takes away more 1's than 0's the midpoint will shift to the left and follow similar logic as cases above. However if it takes away the same number of 0's and 1's the midpoint will not shift and we will split the string into halves $w = 0^p 1^p 0^{p-|vy|/2}$ and $1^{p-|vy|/2} 0^p 1^p$. Since the second half is not the compliment of w this string is not in L_1.
>
> We have shown for all cases that uv^0xy^0z is not in L_1, thus L_1 is not context free as it contradicts the pumping lemma

B

$L_2 = \{a^m b^n c^{m\times n} | m, n \in \mathbf{Z}_{\geq 0}\}$

> We can prove this language is not context free using proof by contradiction with the pumping lemma
>
> Suppose L_2 is context free, then $\exists$ a p s.t. all strings s in L_2 with $|s| > p$ can be divded into $s = uvxyz$ such that $|vxy| \leq p$ and $|vy| > 0$
>
> Given the string $a^p b^p c^{p\times p}$ and that $|vxy| \leq p$, there are 5 cases of vxy
>
> 1. $vxy = a^*$
> 2. $vxy = a^* b^*$
> 3. $vxy = b^*$
> 4. $vxy = b^* c^*$
> 5. $vxy = c^*$
>
> In case 1 / 2 / 3, suppose the string uv^0xy^0z, then we have less a's / b's without changing the number of c's, thus our new string is not in L_2.
>
> In case 5, suppose the string uv^0xy^0z, then we have less c's without changing the number of a's or b's, thus our new string is not in L_2.
>
> In case 4, suppose the string uv^0xy^0z, then we linearly decrease the number of b's and c's, but is we decrease the number of b's by one then we need to decrease the number of c's by p. However we cannot decrease c by a factor of p because $|vxy| < p$, thus our new string is not in L_2.
>
> Therefore since all of our cases reject the pumping lemma, L_2 is not context free