HW6CSDS343

1

Prove the following language is context free $E = \{a^i b^j | i \neq j\}$

A

Give a pushdown automaton that decides E

B

Give a context free grammar for E

2

Prove the concatenation $(xy | x \in L_1 \wedge y \in L_2)$ of two context free languages L_1 and L_2 is context free

A

Assume you have the context free grammar for each language, give the context free grammar for the concatenation (create the CFG directly)

B

Assume you have the pushdown automata for each language, give the pushdown automata for the concatenation (create the PDA directly)

3

Prove that each of the following languages are not context free

A

$L_1 = \{w\bar{w} | w \in \{0,1\}\}$ where $\bar{w}$ is the bit compliment of w

We can prove this language is not context free using proof by contradiction with the pumping lemma

Suppose L_1 is context free, then $\exists$ a p s.t. all strings s in L_1 with $|s| > p$ can be divded into $s = uvxyz$ such that $|vxy| \leq p$ and $|vy| > 0$

Given the string $0^p1^p0^p1^p0^p1^p$ and that $|vxy| \leq p$, there are 5 cases of vxy

1. $vxy = 0^*$ first
2. $vxy = 0^*1^*$ first
3. $vxy = 1^*$ first
4. $vxy = 1^*0^*$ first
5. $vxy = 0^*$ second
6. $vxy = 0^*1^*$ second
7. $vxy = 1^*$ second
8. $vxy = 1^*0^*$ second
9. $vxy = 0^*$ third
10. $vxy = 0^*1^*$ third
11. $vxy = 1^*$ third

For all cases suppose the string uv^0xy^0z. This string should be in L_1 according to the pumping lemma but we can prove this is not true for all cases.

In case 1, our string is now of the form $0^{p-|vy|}1^p0^p1^p0^p1^p$. This shifts the midpoint to the right by $|vy|/2$. Thus if we split the string in half we get $w = 0^{p-|vy|}1^p0^p1^{|vy|/2}$ and the second half as $1^{p-|vy|/2}0^p1^p$. Since the second half is not the compliment of w this string is not in L_1.

Case 5 and 9 follow similar logic, with case 9 shifting the midpoint left.

In case 3, our string is now of the form $0^p1^{p-|vy|}0^p1^p0^p1^p$. This shifts the midpoint to the right by $|vy|/2$. Thus if we split the string in half we get $w = 0^p1^{p-|vy|}0^p1^{|vy|/2}$ and the second half as $1^{p-|vy|/2}0^p1^p$. Since the second half is not the compliment of w this string is not in L_1.

Case 7 and 11 follow similar logic, with case 11 shifting the midpoint left.

B

$L_2 = \{a^m b^n c^{m \times n} | m, n \in \mathbf{Z}_{\geq 0}\}$

We can prove this language is not context free using proof by contradiction with the pumping lemma

Suppose L_2 is context free, then $\exists$ a p s.t. all strings s in L_2 with $|s| > p$ can be divded into $s = uvxyz$ such that $|vxy| \leq p$ and $|vy| > 0$

Given the string $a^p b^p c^{p \times p}$ and that $|vxy| \leq p$, there are 5 cases of vxy

1. $vxy = a^*$
2. $vxy = a^* b^*$
3. $vxy = b^*$
4. $vxy = b^* c^*$
5. $vxy = c^*$

In case 1 / 2 / 3, suppose the string uv^0xy^0z, then we have less a's / b's without changing the number of c's, thus our new string is not in L_2.

In case 5, suppose the string uv^0xy^0z, then we have less c's without changing the number of a's or b's, thus our new string is not in L_2.

In case 4, suppose the string uv^0xy^0z, then we linearly decrease the number of b's and c's, but is we decrease the number of b's by one then we need to decrease the number of c's by p. However we cannot decrease c by a factor of p because $|vxy| < p$, thus our new string is not in L_2.

Therefore since all of our cases reject the pumping lemma, L_2 is not context free