Want to prove $\frac{N(n)}{n} \longrightarrow \pi$

$\left| \frac{N(n)}{n} - \pi \right| \longrightarrow 0$

$\Rightarrow |N(n) - n\pi| \longrightarrow 0$

$\Rightarrow |N(n) - \pi r^2| \longrightarrow 0$

(shaded if contained in circle)

Shaded is $N(n)$, some parts outside x^2+y^2, not all of inside is shaded. Smallest circle that is unshaded on the outside is the upper bound to $N(n)$. Diagonal of unit square is $\sqrt{2}$, $\therefore$ shaded $N(n)$ must be contained by $r = \sqrt{n} + \frac{\sqrt{2}}{2}$

$$\Rightarrow N(n) \le \pi\left(\sqrt{n} + \frac{\sqrt{2}}{2}\right)^2 = \pi\left(n + \sqrt{2n} + \frac{1}{2}\right)$$

$$\frac{N(n)}{n} \le \pi\left(1 + \sqrt{\frac{2}{n}} + \frac{1}{2n}\right)$$

$$\frac{N(n)}{n} - \pi \le \pi\left(\sqrt{\frac{2}{n}} + \frac{1}{2n}\right)$$

RHS $\to 0$ as $n \to \infty$ $\Rightarrow \frac{N(n)}{n} - \pi = 0$ as $n \to \infty$

$\Rightarrow \frac{N(n)}{n} \longrightarrow \pi$ as $n \longrightarrow \infty$ ∎