Path Planning Challenge

Magicc Lab, Summer 2023

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Abstract

Convex optimization challenge for summer 2023.

1 Introduction

The convex optimization challenge will use the VTOL example from [1], which is describe below for convenience.

In this design study we will explore the control design for a simplified planar version of a quadrotor following a ground target. In particular, we will constrain the dynamics to be in two a dimensional plane. In Cartesian space, the position of the VTOL can be defined by a variable describing the vertical position, one describing the horizontal position, and one angular variable describing the orientation, as shown in 1. The planar vertical take-off and landing (VTOL) system is comprised of a center pod of mass m_c and inertia J_c, a right motor/rotor that is modeled as a point mass m_r that exerts a force f_r at a distance d from the center of mass, and a left motor/rotor that is modeled as a point mass m_ℓ that exerts a force f_ℓ at a distance $-d$ from the center of mass. The position of the center of mass of the planar VTOL system is given by horizontal position z_v and altitude h. The airflow through the rotor creates a change in the direction of flow of air and causes what is called "momentum drag." Momentum drag can be modeled as a viscous drag force that is proportional to the horizontal velocity $\dot{z}_v$. In other words, the drag force is $F_{\text{drag}} = -\mu \dot{z}_v$. The target on the ground will be modeled as an object with position z_t and altitude $h = 0$. We will not explicitly model the dynamics of the target.

Use the following physical parameters: $m_c = 1$ kg, $J_c = 0.0042$ kg m^2, $m_r = 0.25$ kg, $m_\ell = 0.25$ kg, $d = 0.3$ m, $\mu = 0.1$ kg/s, $g = 9.81$ m/s^2.

2 Dynamics

The dynamics are given by

$$\begin{pmatrix} m_c + 2m_r & 0 & 0 \\ 0 & m_c + 2m_r & 0 \\ 0 & 0 & J_c + 2m_r d^2 \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} -(f_r + f_\ell)\sin\theta - \mu\dot{z} \\ -(m_c + 2m_r)g + (f_r + f_\ell)\cos\theta \\ d\,(f_r - f_\ell) \end{pmatrix}. \tag{1}$$

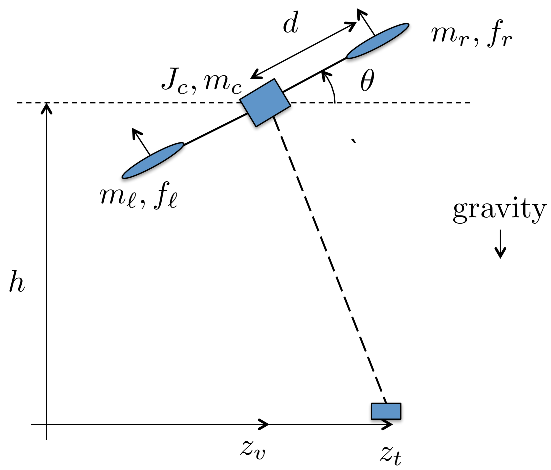

Figure 1: Planar vertical take-off and landing (VTOL)

We will assume that the right and left rotor forces are constrained as follows

$$0 \leq f_r, f_\ell \leq 0.75(m_c + 2m_r)g. \tag{2}$$

If the thrust was limited to $0.5(m_c + 2m_r)g$, then at full throttle, the VTOL could just hover. Therefore, the limit allows some additional control authority, but is somewhat constraining.

3 Trajectory Following

We will assume that a state-space controller with integrator is designed for the system as described in Chapter 12 of [1], and that a disturbance observer is used to estimate the states as described in Chapter 14 of [1]. The input to the system is the desired forward position $z^r(t)$ and the desired altitude $h^r(t)$.

References

[1] R. W. Beard, T. W. McLain, C. Peterson, and M. Killpack, *Introduction to Feedback Control Using Design Studies*. Amazon, 2016.