$\begin{array}{c} {\bf Linear~Algebra~Optimization~in~C~using}\\ {\bf OpenMPI} \end{array}$

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1 Introduction

In today's data driven world, there is a constant demand for faster and most efficient computing methods. Traditional sequential computing, where tasks are executed one after another by a singular processor, often falls short when tackling complex and large scale problems. While this model remains useful for certain tasks and small scale applications, it fails to meet the performance needs of large scale computations in modern research, analytics, and simulation based fields.

Parallel computing addresses this limitation by distributing workload across multiple processors or cores, enabling simultaneous execution of sub tasks. This approach significantly reduces execution time and increases efficiency, especially for complex mathematical operations and data heavy computations. Parallelism has become integral in many different cases—from national defense and physics simulations to artificial intelligence and high frequency trading—where performance constraints dictate the outcomes of solutions (Barney & Frederick). The Open Message Passing Interface (OpenMPI) implementation has become a core element of solving this problem by facilitating communication and coordination among distributed processes. OpenMPI is widely used in both academic and industry settings due to its scalability, ease of use, and support for high performance clusters.

This research project investigates the use of OpenMPI to optimize linear algebra algorithms written in C, which is a very low level language. Linear algebra remains a central pillar of computer science, with applications ranging from computer graphics and machine learning to economic modeling and engineering. One historic example includes Professor Leontief's use of linear equations in 1949 to model the structure of the United States economy, which demonstrated the importance of these systems (Lay et al., 2020). In this study, emphasis is placed on the efficient execution of operations such as matrix multiplication, row echelon form (REF), reduced row echelon form (RREF), and LU decomposition, which serve as critical tools in solving systems of linear equations. These algorithms are implemented in both sequential and parallel forms to allow for direct performance comparisons. Timing benchmarks collected during testing reveal the extent to which parallelism accelerates execution and under what conditions those gains are most significant. These performance results will be discussed in detail in later sections.

In addition to algorithm optimization, the project involved constructing a custom parallel computing environment. The process of configuring and deploying a multi node OpenMPI cluster provided valuable learning experiences into the logistics and technical requirement of distributed computing. This included configuration of operating systems, secure shell (SSH) communication setup between nodes, implementation of shared storage, and fine tuning of the scripts. The resulting cluster not only enabled controlled benchmarking of parallel code but also served as a replicable model for lightweight high performance computing environments.

2 Terminology and MPI Overview

To aid in understanding the parallel implementations discussed in this study, this section defines key terms and MPI (message passing interface) functions that appear throughout the

algorithms discussed. MPI, message passing interface, is a standardized and portable message passing system that allows processes to run on distributed memory systems. The CPUs are physically separated from each other, but still communicate and transmit data between themselves. According to Spickler (2025), the following diagram displays how distributed memory systems look:

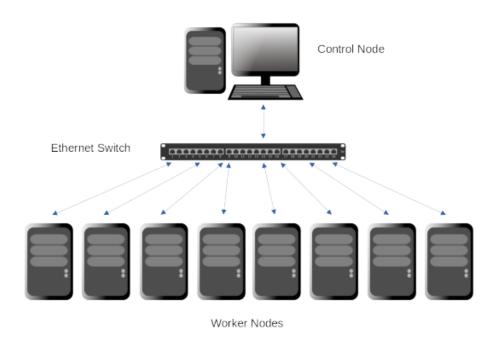


Figure 1: Data Flow in OpenMPI Cluster

An MPI process is a single instance of a running program in an MPI application. Each process has a unique rank, which is an integer associated with that process, separating it from the other processes. Rank 0 is typically the root process, the one that initializes the program. MPI_Bcast is a broadcast function that is used to send data from one process, usually the root, to all other processes in a communicator. This ensures that all processes have the same updated data before continuing. Without the usage of MPI_Bcast, the data would now be shared across all nodes. The following diagram displays how MPI_Bcast distributes memory:

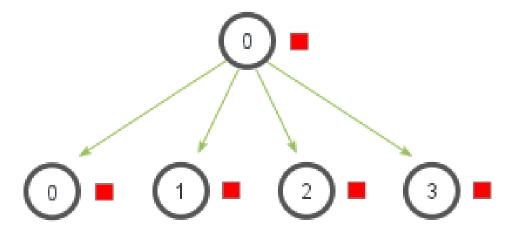


Figure 2: Data Flow using MPI_Bcast

Additionally, MPI_Barrier is a synchronization function that blocks all processes until every process in the communicator has reached the barrier. This makes sure that all previous operations are complete across all processes before proceeding. While this tool is necessary, large overhead can occur because it pauses all processes until every one is complete. In large matrices, this build up can accumulate over time. The following diagram demonstrates how MPI_Barrier waits for all processes to complete:

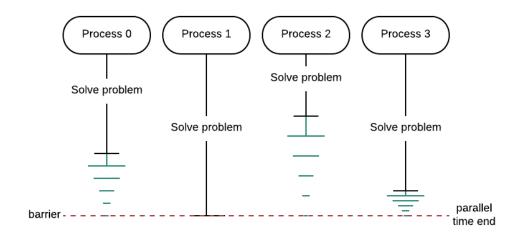


Figure 3: Data Flow using MPI_Barrier

MPI_Allreduce is a collective communication operation that sums values from all running processed using a specified operation, and distributes the result back to all processes. This command is used to pull all results together and re-send data back out to all processes. Here is a diagram displaying how MPI_Allreduce works:

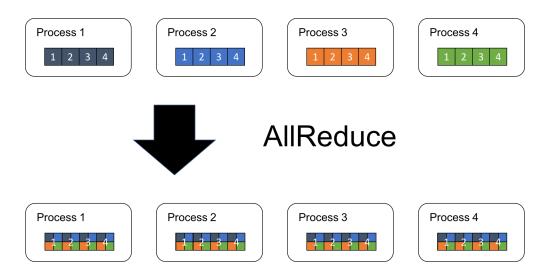


Figure 4: Data Flow using MPI_Allreduce

3 Mathematics and Algorithms

The implementation of parallelized linear algebra routines requires not only a theoretical understanding of the algorithms but also practical insight into how these operations work under distributed computation. This section outlines the core mathematical algorithms used in the project: matrix multiplication, REF, RREF, and LU decomposition, as well as comparing the differences between their sequential and OpenMPI forms. Matrix multiplication is a foundational operation in linear algebra with applications ranging from linear transformations to machine learning. Given two matrices, A of size m x n and B of size n x p, the produce C = A*B is defined as:

$$C_{i,j} = \sum_{k=1}^{n} A_{i,k} \cdot B_{k,j}$$

In the sequential implementation, three nested loops are used to iterate through each row i of A, each column j of B, and each inner dimension k, summing the partial products in Ci,j. The parallel implementation distributes the row range of matrix A across all MPI processes. Each process computes a subset of the output matrix C, then gets called by MPI_Allreduce to consolidate the partial results across all nodes. Load balancing is managed by evenly dividing rows, and the implementations accounts for division with remainders by assigning an extra row to the first few ranks. This design allows each core to perform independent multiplication on the assigned rows, significantly reducing computation time while leveraging all available nodes. The correctness of results is guaranteed by the summation of MPI_Allreduce.

Additionally, the REF form of a matrix is another fundamental transformation in linear algebra, and is commonly used for solving systems of linear equations and preparing a matrix for further operations such as back substitution, RREF conversion, and LU decomposition. A matrix is said to be in row echelon form when all nonzero rows are above any rows of all zeroes; the leading coefficient (pivot) of a nonzero row is strictly to the right of the pivot in the row above it; and all entries below a pivot are zero. The sequential algorithm follows the

Gaussian elimination process. The function iteratively scans for nonzero pivots, performs row swaps if needed, normalizes the pivot row so the leading entry becomes 1, and eliminates all entries below the pivot by subtracting a multiple of the pivot row from each target row. In the parallel implementation, the primary logic is preserved, but the elimination step is distributed across multiple MPI processes. Each process is responsible for updating a subset of rows. More specifically, rows are assigned based on their index modulus the total number of MPI processes:

If $i \mod p = \text{rank}$, then process rank handles row i.

Each process waits until the pivot row is broadcast from rank 0, then applies the elimination step locally to its assigned rows. After completing a round of elimination, processes synchronize using MPLBarrier and then broadcast the updated matrix using MPLBcast to ensure all nodes have a consistent view before moving to the next pivot row. This approach preserves correctness while achieving partial parallelism. Although some overhead exists due to the global communication at each iteration, the performance gains from dividing the row work outweigh the costs, especially on large matrices. Building upon the REF, RREF represents a stricter standard that fully solves a system of linear equations in a matrix form. In addition to the requirements for REF, RREF imposes two additional conditions: Each leading '1' is the only nonzero entry in its column; and each leading '1' is the first nonzero entry in its row. The RREF algorithm is typically used as a second phase after REF. Once the matrix is in upper triangular form, the algorithm proceeds in reverse, starting from the bottom row and moving upwards, eliminating entries above each pivot. In sequential form, the implementation iterations over the rows in reverse order. For each row, it locates the pivot, then eliminates all entries above that pivot by subtracting multiples of the pivot row from each row above. This back substitution stage ensures that each pivot is isolated. In the parallel implementation using OpenMPI, the reverse pass is distributed similarly to REF. After converting the matrix to REF form, the backward elimination steps assigns work to MPI processes based on row indices. Each process eliminates entries above pivots in rows it is responsible for, using the modulus rank pattern seen before. Once a set of updates is applied, all processes synchronize with MPI_Barrier, and the updated matrix is broadcasted again using MPI_Bcast to maintain consistency. While this introduces communication overhead, particularly in the upper triangular back propagation phase, the division of labor across processors still yields speedup in large matrices.

Furthermore, LU decomposition is another rudimentary matrix operation used to repeatedly solve systems of linear equations, especially when the coefficient matrix remains constant. The process factors a square matrix, A, into two triangular matrices—a lower triangular matrix L and an upper triangular matrix U such that A = L * U. In the sequential algorithm, Gaussian elimination is used to progressively zero out entries below the pivots, simultaneously creating the L matrix with the multipliers used during the elimination process and assigning the resulting upper triangle to U. This involves searching through each pivot position, calculating scalars for the rows below, and updating both matrices in place. In the parallel OpenMPI version, the decomposition is distributed across multiple processes by partitioning the matrix rows and assigning each subset to a different rank. During each iteration, the pivot row is broadcast from the root process to all other using MPI_Bcast,

and each process computes updates for its assigned rows independently. As with REF and RREF, synchronization is required after each iteration using MPI_Barrier to ensure consistent progress. The challenge in the parallel LU decomposition lies in how the managing data dependencies between the L and U matrices are handled.

4 Results

To evaluate the performance of sequential versus parallel implementations, a series of timing benchmarks were conducted on square matrices of varying sizes. Each algorithm- matrix multiplication, REF, RREF, and LU decomposition—was tested using both a single process mode and a multi processes OpenMPI mode on the constructed cluster. Matrix sized ranged from 10 x 10 to 2,000 x 2,000 and execution times were recorded for each operation. The results revealed that parallel matrix multiplication, REF, and RREF achieved significant speedups compared to their sequential counterparts, particularly as matrix size increased. For example, on a REF test of a 2,000 x 2,000 matrix, the sequential algorithm took 9.422 seconds, whereas the parallel process with 8 processes took 1.971 seconds to compute, which is roughly a 4.75x speedup. Similarly, the parallel RREF and matrix multiplication implementations showed substantial time reductions on larger matrices, although some communication overhead was observed during pivot synchronization. However, the parallel LU decomposition algorithm was almost consistently slower than the sequential version across all tested matrix sizes. For example, on a 2,000 x 2,000 matrix, the sequential time for LU Decomposition took 8.153 seconds, whereas the parallel time with 8 processes took 19.306 seconds. As discussed earlier, the method in which the algorithm has to be built and the frequent use of MPI_Bcast and MPI_Barrier caused the communication and synchronization overhead to dominate execution time, negating the benefits of multiprocessing.

Below are the timing results of all four operations being tested on the cluster in the HPCL. Variables that could affect the speeds were minimized— for example, there were no background processes running during any of the runtime tests. As seen, the sequential algorithms run slower in matrix multiplication, REF, and RREF compared to the parallel versions, whereas the sequential algorithm for LU Decomposition runs faster in comparison to the parallel version.

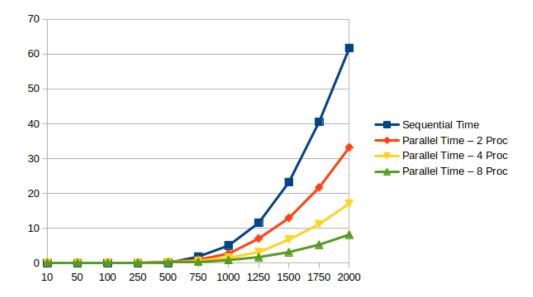


Figure 5: Matrix Multiplication Timings

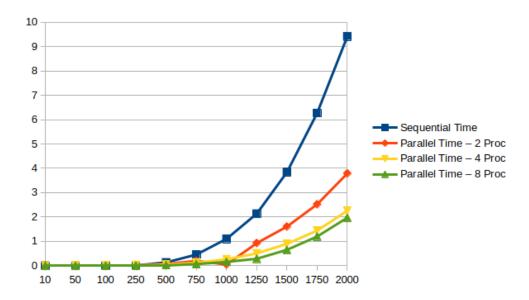


Figure 6: REF Timings

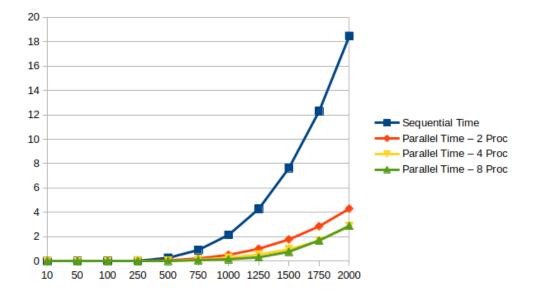


Figure 7: RREF Timings

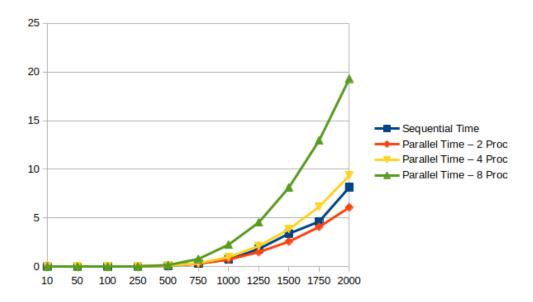


Figure 8: LU Decomposition Timings

5 Conclusion

The study demonstrated that parallel computing using OpenMPI can significantly accelerate linear algebra operations that naturally allow independent work distribution, such as matrix multiplication, REF, and RREF. Timing benchmarks confirmed that parallel implementations of these algorithms achieved substantial speedup compared to sequential versions, particularly as matrix sizes increased. However, not all algorithms benefited equally from parallelization. LU decomposition, which has strong data dependencies between elimination

steps, experienced performance degradation in the parallel implementation due to communication and synchronization overhead. This highlighted a key limitation in naive parallelization strategies— operations that frequently require inter process communication may not scale well without more sophisticated techniques or algorithms. Overall, this project provided valuable insights into both the potential and challenges of parallel computing. Future work could explore more advanced decomposition methods, such as block LU decomposition, or hybrid models that combine MPI with multithreading to better balance computation and communication.

References

- Frederick, (n.d.). Introduction[1] Barney, В., & D. to parallel computing tutorial.Lawrence Livermore National Laboratory. Retrieved April https://hpc.llnl.gov/documentation/tutorials/ 2025, from introduction-parallel-computing-tutorial
- [2] Lay, D. C., Lay, S. R., & McDonald, J. J. (2020). Linear algebra and its applications (6th ed.). Pearson.
- [3] Spickler, D. (2025). COSC 420: High-Performance Computing. Salisbury University.

MatrixC.c

```
=1
    1
#include "MatrixC.h"
3 / *
4 Name: error()
5 Parameters: const char*
6 Return: void
7 Description: Prints error message and exits program
9 void error(const char *message)
    fprintf(stderr, "Error: %s\n", message);
    exit(EXIT_FAILURE);
13 }
15 / *
16 Name: isValid()
17 Parameters: const Matrix*
18 Return: bool
19 Description: Returns true if cols and rows are greater than 0
21 bool isValid(const Matrix *m)
    return (m->rows > 0 && m->cols > 0);
24 }
25
26 / *
27 Name: getRows()
28 Parameters: const Matrix*
29 Return: int
30 Description: Returns the number of rows in a matrix
32 int getRows(const Matrix *m) { return m->rows; }
34 / *
35 Name: getCols()
36 Parameters: const Matrix*
37 Return: int
38 Description: Returns the number of cols in a matrix
40 int getCols(const Matrix *m) { return m->cols; }
41
```

```
42 / *
43 Name: init()
44 Parameters: Matrix*
45 Return: void
46 Description: Default constructor
47 */
48 void init (Matrix *m)
49 {
  m->rows = 0;
  m->cols = 0;
51
52 }
53
54 / *
55 Name: initSize()
56 Parameters: Matrix*, const int r, const int c
57 Return: void
58 Description: Constructor with set size parameters
60 void initSize(Matrix *m, const int r, const int c)
    m->rows = r;
62
    m->cols = c;
63
64
    for (int i = 0; i < r; i++)</pre>
66
      for (int j = 0; j < c; j++)
67
68
        m->matrix[i][j] = 0;
70
    }
71
 }
72
73
74 / *
75 Name: initValue();
76 Parameters: Matrix*, const int r, const int c, const double
     defval
77 Return: void
78 Description: Constructor with set size parameters and default
     value
79 */
so void initValue (Matrix *m, const int r, const int c, const double
     defval)
81 {
  m->rows = r;
  m->cols = c;
```

```
84
     for (int i = 0; i < r; i++)</pre>
85
86
       for (int j = 0; j < c; j++)
87
         m->matrix[i][j] = defval;
89
       }
91
92
93
94 / *
95 Name: copyMatrix()
96 Parameters: Matrix *dest, const Matrix *src
  Return: void
  Description: Copies src matrix into dest matrix
  void copyMatrix(Matrix *dest, const Matrix *src)
101
     if (dest == src)
102
       return;
103
104
     if (!isValid(dest) || !isValid(src))
105
106
       error("Cannot copy invalid matrix");
107
108
109
     dest->rows = src->rows;
110
     dest->cols = src->cols;
111
112
     for (int i = 0; i < src->rows; i++)
113
114
       for (int j = 0; j < src->cols; j++)
115
116
         dest->matrix[i][j] = src->matrix[i][j];
117
118
119
120
121
122 / *
123 Name: display()
124 Parameters: Matrix
125 Return: void
126 Description: Displays the matrix
127 */
128 void display(const Matrix *m)
```

```
129
     int maxRows = 10, maxCols = 10;
130
     int showAllRows = m->rows <= maxRows;</pre>
131
     int showAllCols = m->cols <= maxCols;</pre>
132
133
     for (int i = 0; i < (showAllRows ? m->rows : maxRows); i++)
134
135
       printf("[");
136
       for (int j = 0; j < (showAllCols ? m->cols : maxCols); j++)
137
138
         printf(" %7.2f", m->matrix[i][j]);
139
140
       if (!showAllCols)
141
         printf(" ...");
142
       printf(" ]\n");
143
144
     if (!showAllRows)
145
146
       int width = (showAllCols ? m->cols : maxCols) * 8 + (
147
          showAllCols ? 2 : 6);
       for (int i = 0; i < width; i++)</pre>
148
         printf(" ");
149
       printf("...\n");
150
152
153
154 / *
155 Name: rowScale()
156 Parameters: Matrix*, int row, double scalar
  Return: void
  Description: Multiplies a row by a non-zero scalar
159
  void rowScale(Matrix *m, const int row, const double scalar)
160
161
     if (!isValid(m) || row >= m->rows || row < 0)
162
163
       error("Invalid matrix size for row scaling");
164
     }
165
166
     for (int i = 0; i < m->cols; i++)
167
168
       m->matrix[row][i] *= scalar;
169
170
171
172
```

```
173 / *
174 Name: rowSwap()
175 Parameters: Matrix*, int row1, int row2
  Return: void
  Description: Swaps two valid rows of the matrix
178
  void rowSwap(Matrix *m, const int row1, const int row2)
179
180
     if (!isValid(m) || row1 >= m->rows || row2 >= m->rows || row1 <</pre>
181
         0 \mid | row2 < 0)
182
       error("Invalid row access for row swap");
183
     }
184
185
     for (int i = 0; i < m->cols; i++)
186
187
       double temp = m->matrix[row1][i];
188
       m->matrix[row1][i] = m->matrix[row2][i];
189
       m->matrix[row2][i] = temp;
190
     }
192
193
194
195 Name: rowReplace()
196 Parameters: Matrix*, int targetRow, int sourceRow, double scalar
197 Return: void
  Description: Replaces target row with itself plus scalar
     multiplied by source row
  */
199
200 void rowReplace (Matrix *m, const int targetRow, const int
     sourceRow, const double scalar)
201
     if (!isValid(m) || targetRow >= m->rows || sourceRow >= m->rows
202
         || targetRow < 0 || sourceRow < 0)
     {
203
       error("Invalid row access for row replace");
204
205
206
     for (int i = 0; i < m->cols; i++)
207
208
       m->matrix[targetRow][i] += scalar * m->matrix[sourceRow][i];
209
210
  }
211
212
213 / *
```

```
214 Name: setValue()
215 Parameters: Matrix*, int r, int c, double value
216 Return: void
  Description: Sets an element of the row to a value
219 void setValue (Matrix *m, const int r, const int c, const double
     value)
220
     if (!isValid(m) || r >= m->rows || c >= m->cols || r < 0 || c <</pre>
221
         0)
     {
222
       error("Invalid row or column access for set value");
223
     }
224
225
    m->matrix[r][c] = value;
226
227
228
  /*
229
230 Name: setRandom()
231 Parameters: Matrix*, const int maxRand
  Return: void
  Description: Sets matrix to random values
  void setRandom(Matrix *m, const int maxRand)
236
     if (!isValid(m))
237
238
       error("Invalid matrix in set random");
239
     }
240
241
     for (int i = 0; i < m->rows; i++)
242
243
       for (int j = 0; j < m->cols; j++)
244
245
         m->matrix[i][j] = rand() % maxRand;
246
247
248
  }
249
250
251
252 Name: getValue()
253 Parameters: Matrix*, const int r, const int c
254 Return: double
255 Description: Returns matrix element at a slot
256 */
```

```
double getValue(Matrix *m, const int r, const int c)
258
     if (!isValid(m) || r >= m->rows || c >= m->cols || r < 0 || c <</pre>
259
         0)
     {
260
       error("Invalid row or column access in get value");
261
262
263
     return m->matrix[r][c];
264
265
266
  /*
267
268 Name: displayDimensions()
  Parameters: Matrix*
  Return: void
  Description: Displays dimensions of the matrix
  void displayDimensions(const Matrix *m)
274
     if (!isValid(m))
     {
276
       error("Invalid matrix");
277
     }
278
279
     printf("[%d rows, %d cols]\n", m->rows, m->cols);
280
  }
281
282
  /*
283
284 Name: isSquare()
285 Parameters: const Matrix* A
  Return: bool
287 Description: Returns true if the matrix is square
  */
  bool isSquare(const Matrix *A)
289
290
     return A->rows == A->cols;
291
292
293
  /*
295 Name: transpose()
296 Parameters: const Matrix *input, Matrix *output
  Return: void
  Description: Transposes the input matrix, stores result in output
      matrix
299 */
```

```
void transpose(const Matrix *input, Matrix *output)
301
302
     if (!isValid(input))
303
304
       error("Invalid matrix in transpose");
305
     }
306
307
     int rank, size;
308
     MPI_Comm_rank(MPI_COMM_WORLD, &rank);
309
     MPI_Comm_size(MPI_COMM_WORLD, &size);
310
     MPI_Bcast((void *)input, sizeof(Matrix), MPI_BYTE, 0,
311
        MPI COMM WORLD);
312
     output->rows = input->cols;
313
     output->cols = input->rows;
314
315
     int rows_per_proc = input->rows / size;
316
     int remaining_rows = input->rows % size;
317
     int start = rank * rows_per_proc + (rank < remaining_rows ?</pre>
318
        rank : remaining rows);
     int end = start + rows_per_proc + (rank < remaining_rows ? 1 :</pre>
319
        0);
320
     for (int i = start; i < end; i++)
321
322
       for (int j = 0; j < input->cols; j++)
323
       {
324
         output->matrix[j][i] = input->matrix[i][j];
325
       }
326
     }
327
328
     // back to all processes
329
     for (int proc = 0; proc < size; proc++)</pre>
330
331
     {
       int proc_start = proc * rows_per_proc + (proc <</pre>
332
          remaining_rows ? proc : remaining_rows);
       int proc_end = proc_start + rows_per_proc + (proc <</pre>
333
          remaining_rows ? 1 : 0);
334
       for (int row = 0; row < input->cols; row++)
335
336
         for (int col = proc_start; col < proc_end; col++)</pre>
337
338
           MPI_Bcast(&output->matrix[row][col], 1, MPI_DOUBLE, proc,
339
```

```
MPI_COMM_WORLD);
340
341
342
343
344
  /*
345
346 Name: writeToFile()
347 Parameters: const Matrix*, const char* filename
  Return: void
  Description: Writes a matrix to a file
350
  void writeToFile(const Matrix *m, const char *filename)
352
     if (!isValid(m))
353
354
       error("Invalid matrix");
355
356
357
     FILE *file = fopen(filename, "w");
358
     if (file == NULL)
359
360
       error("Unable to open file for writing");
361
362
363
     fprintf(file, "Rows: %d, Cols: %d\n", m->rows, m->cols);
364
     for (int i = 0; i < 25; i++)</pre>
365
       fprintf(file, "-");
366
     fprintf(file, "\n");
367
368
     for (int i = 0; i < m->rows; i++)
369
370
       for (int j = 0; j < m->cols; j++)
371
       {
372
         fprintf(file, "%lf ", m->matrix[i][j]);
373
374
       fprintf(file, "\n");
375
     }
376
     fclose(file);
378
379
   }
380
  /*
381
382 Name: solve()
383 Parameters: const Matrix*, const double[], const int size
```

```
384 Return: bool
385 Description: Returns true if the solution set works with the
     matrix
386
387 bool solve (const Matrix *m, const double solutions[], const int
      size)
388
     if (!isValid(m) || size < 1)</pre>
389
     {
390
       error ("Cannot check solution set on empty matrix or empty
391
          solution set");
     }
392
393
     const double EPSILON = 1e-5;
394
     double value = 0;
395
396
     for (int i = 0; i < m->rows; i++)
397
398
       value = 0;
399
       for (int j = 0; j < m->cols - 1; j++)
400
401
         value += m->matrix[i][j] * solutions[j];
402
403
       if (fabs(value - m->matrix[i][m->cols - 1] > EPSILON))
404
         return false;
405
406
     return true;
407
408
409
410 / *
411 Name: multiplyMatrix()
412 Parameters: const Matrix *A, const Matrix *B, Matrix *C
413 Return: void
414 Description: Multiplies matrix A and B using OpenMPI and stores
      result in matrix C.
416 void multiplyMatrix(const Matrix *A, const Matrix *B, Matrix *C)
417 {
     int rank, size;
     MPI_Comm_rank(MPI_COMM_WORLD, &rank);
419
     MPI_Comm_size(MPI_COMM_WORLD, &size);
420
421
     int a_rows = A->rows;
422
     int a_cols = A->cols;
423
     int b_cols = B->cols;
424
```

```
if (rank == 0)
426
427
       if (!isValid(A) || !isValid(B))
428
         error("Invalid input matrices");
429
       if (a_cols != B->rows)
430
         error("Incompatible dimensions for multiplication");
431
432
433
     // Broadcast matrix dimensions
434
    MPI_Bcast(&a_rows, 1, MPI_INT, 0, MPI_COMM_WORLD);
435
     MPI_Bcast(&a_cols, 1, MPI_INT, 0, MPI_COMM_WORLD);
436
    MPI_Bcast(&b_cols, 1, MPI_INT, 0, MPI_COMM_WORLD);
437
438
     // Broadcast B (whole matrix)
439
    MPI_Bcast((void *)B->matrix, MAX_ROWS * MAX_COLS, MPI_DOUBLE,
440
        0, MPI_COMM_WORLD);
441
     // Calculate local rows
442
     int rows_per_proc = a_rows / size;
443
     int remainder = a_rows % size;
444
     int local_rows = (rank < remainder) ? rows_per_proc + 1 :</pre>
445
        rows_per_proc;
     printf("Rank %d received %d rows, computing...\n", rank,
446
        local_rows);
447
     // Allocate only as much as needed
448
     double local_A[local_rows][a_cols];
449
     double local_C[local_rows][b_cols];
450
     memset(local_C, 0, sizeof(local_C));
451
452
     // Prepare scatter metadata
453
     int sendcounts[size];
454
     int displs[size];
455
     int offset = 0;
456
     for (int i = 0; i < size; i++)</pre>
457
458
       int count = ((i < remainder) ? rows_per_proc + 1 :</pre>
459
          rows_per_proc) * a_cols;
       sendcounts[i] = count;
460
       displs[i] = offset;
461
       offset += count;
462
     }
463
464
     // Scatter A rows to processes
465
```

```
MPI_Scatterv(&(A->matrix[0][0]), sendcounts, displs, MPI_DOUBLE
466
                    &(local_A[0][0]), local_rows * a_cols, MPI_DOUBLE,
467
                    0, MPI_COMM_WORLD);
468
469
     // Compute local result
470
     for (int i = 0; i < local_rows; i++)</pre>
471
472
       for (int j = 0; j < b_cols; j++)</pre>
473
474
          for (int k = 0; k < a_cols; k++)</pre>
475
          {
476
            local_C[i][j] += local_A[i][k] * B->matrix[k][j];
477
478
       }
479
     }
480
481
     // Prepare gather metadata
482
     int recvcounts[size];
483
     int recvdispls[size];
484
     offset = 0;
485
     for (int i = 0; i < size; i++)</pre>
486
487
       int count = ((i < remainder) ? rows_per_proc + 1 :</pre>
488
          rows_per_proc) * b_cols;
       recvcounts[i] = count;
489
       recvdispls[i] = offset;
490
       offset += count;
491
     }
492
493
     // Gather results into final matrix C
494
     MPI_Gatherv(&(local_C[0][0]), local_rows * b_cols, MPI_DOUBLE,
495
                   &(C->matrix[0][0]), recvcounts, recvdispls,
496
                      MPI_DOUBLE,
                   0, MPI_COMM_WORLD);
497
498
     // Set dimensions on root
499
     if (rank == 0)
500
501
       C->rows = a_rows;
502
       C->cols = b_cols;
503
504
  }
505
506
507 / *
```

```
508 Name: ref()
509 Parameters: Matrix *m
510 Return: void
511 Description: Computes the Row Echelon Form (REF) of matrix m
      using OpenMPI.
512 */
513 void ref(Matrix *m)
514
     if (!isValid(m))
515
       error("Invalid matrix");
516
517
     int rank, size;
518
     MPI_Comm_rank (MPI_COMM_WORLD, &rank);
519
     MPI_Comm_size(MPI_COMM_WORLD, &size);
520
521
     int rows = m->rows;
522
     int cols = m->cols;
523
524
     int rows_per_proc = rows / size;
525
     int remainder = rows % size;
526
     int local_rows = (rank < remainder) ? rows_per_proc + 1 :</pre>
527
        rows_per_proc;
     printf("Rank %d preparing to allocate %d rows
                                                           d = 0.2f
528
         MB\n", rank, local_rows, cols, (local_rows * cols * sizeof(
        double)) / (1024.0 * 1024.0));
529
     int start_row = (rank < remainder)</pre>
530
                           ? rank * (rows_per_proc + 1)
531
                           : rank * rows_per_proc + remainder;
532
533
     double local matrix[local rows][cols];
534
     printf("Rank %d received %d rows, computing...\n", rank,
535
        local_rows);
536
     // Scatter rows of m into local_matrix
537
     int sendcounts[size], displs[size];
538
     int offset = 0;
539
     for (int i = 0; i < size; i++)</pre>
540
     {
541
       int count = ((i < remainder) ? rows_per_proc + 1 :</pre>
542
          rows_per_proc) * cols;
       sendcounts[i] = count;
543
       displs[i] = offset;
544
       offset += count;
545
     }
546
```

```
547
     MPI_Scatterv(&(m->matrix[0][0]), sendcounts, displs, MPI_DOUBLE
548
                    &(local_matrix[0][0]), local_rows * cols,
549
                       MPI DOUBLE,
                    0, MPI_COMM_WORLD);
550
551
     // Temporary pivot row buffer
552
     double pivot_row[cols];
553
554
     for (int r = 0; r < rows; r++)
555
     {
556
       int owner = -1, local_r = -1;
557
       for (int i = 0; i < size; i++)</pre>
558
       {
559
         int start = (i < remainder) ? i * (rows_per_proc + 1) : i *</pre>
560
              rows_per_proc + remainder;
         int end = start + ((i < remainder) ? rows_per_proc + 1 :</pre>
561
             rows_per_proc);
         if (r >= start && r < end)
562
563
            owner = i;
564
            local_r = r - start;
565
           break;
566
567
       }
568
569
       // Broadcast pivot row
570
       if (rank == owner)
571
         memcpy(pivot_row, local_matrix[local_r], sizeof(double) *
572
             cols);
573
       MPI_Bcast(pivot_row, cols, MPI_DOUBLE, owner, MPI_COMM_WORLD)
574
          ;
575
       // Eliminate below pivot
576
       for (int i = 0; i < local_rows; i++)</pre>
577
       {
578
         int global_row = start_row + i;
579
         if (global_row <= r)</pre>
580
            continue;
581
582
         double factor = local_matrix[i][r] / pivot_row[r];
583
         for (int j = r; j < cols; j++)
584
          {
585
```

```
local_matrix[i][j] -= factor * pivot_row[j];
586
587
       }
588
     }
589
590
     // Gather results back to m
591
    MPI_Gatherv(&(local_matrix[0][0]), local_rows * cols,
592
        MPI DOUBLE,
                  &(m->matrix[0][0]), sendcounts, displs, MPI_DOUBLE,
593
                  0, MPI_COMM_WORLD);
594
595
596
  /*
597
598 Name: rref()
599 Parameters: Matrix *m
600 Return: void
601 Description: Computes Reduced Row Echelon Form (RREF) using
      OpenMPI.
  */
602
  void rref(Matrix *m)
604
     if (!isValid(m))
605
       error("Invalid matrix");
606
607
     int rank, size;
608
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
609
     MPI_Comm_size(MPI_COMM_WORLD, &size);
610
611
     int rows = m->rows;
612
     int cols = m->cols;
613
614
     int rows_per_proc = rows / size;
615
     int remainder = rows % size;
616
     int local_rows = (rank < remainder) ? rows_per_proc + 1 :</pre>
617
        rows_per_proc;
618
     int start_row = (rank < remainder)</pre>
619
                            ? rank * (rows_per_proc + 1)
620
                            : rank * rows_per_proc + remainder;
621
622
     double local_matrix[local_rows][cols];
623
624
     printf("Rank %d received %d rows, computing...\n", rank,
625
        local_rows);
626
```

```
// Setup for scatter/gather
627
     int sendcounts[size], displs[size];
628
     int offset = 0;
629
     for (int i = 0; i < size; i++)
630
631
       int count = ((i < remainder) ? rows_per_proc + 1 :</pre>
632
          rows_per_proc) * cols;
       sendcounts[i] = count;
633
       displs[i] = offset;
634
       offset += count;
635
     }
636
637
     // Scatter matrix
638
     MPI_Scatterv(&(m->matrix[0][0]), sendcounts, displs, MPI_DOUBLE
639
                    &(local_matrix[0][0]), local_rows * cols,
640
                       MPI_DOUBLE,
                    0, MPI_COMM_WORLD);
641
642
     double pivot_row[cols];
643
644
     for (int r = 0; r < rows; r++)
645
646
       int owner = -1, local_r = -1;
647
       for (int i = 0; i < size; i++)</pre>
648
649
          int s = (i < remainder) ? i * (rows_per_proc + 1) : i *</pre>
650
             rows_per_proc + remainder;
          int e = s + ((i < remainder) ? rows_per_proc + 1 :</pre>
651
             rows_per_proc);
         if (r >= s \&\& r < e)
652
653
            owner = i;
654
            local_r = r - s;
655
            break;
656
657
       }
658
659
       // Normalize pivot row on owner
660
       if (rank == owner)
661
       {
662
         double pivot = local_matrix[local_r][r];
663
         if (pivot != 0)
664
          {
665
            for (int j = r; j < cols; j++)</pre>
666
```

```
local_matrix[local_r][j] /= pivot;
667
668
         memcpy(pivot_row, local_matrix[local_r], sizeof(double) *
669
            cols);
       }
670
671
       // Broadcast normalized pivot row
672
       MPI_Bcast(pivot_row, cols, MPI_DOUBLE, owner, MPI_COMM_WORLD)
673
          ;
674
       // Eliminate all other rows (above and below)
675
       for (int i = 0; i < local rows; i++)
676
677
         int global_i = start_row + i;
678
         if (global_i == r)
679
           continue;
680
681
         double factor = local_matrix[i][r];
682
         for (int j = r; j < cols; j++)
683
           local_matrix[i][j] -= factor * pivot_row[j];
684
       }
685
     }
686
687
     // Gather result to root
688
     MPI_Gatherv(&(local_matrix[0][0]), local_rows * cols,
689
        MPI_DOUBLE,
                  &(m->matrix[0][0]), sendcounts, displs, MPI_DOUBLE,
690
                  0, MPI_COMM_WORLD);
691
692
693
  / *
694
695 Name: addMatrix()
  Parameters: const Matrix *A, const Matrix *B, Matrix *result
  Return: Void
697
  Description: Stores result of addition of A and B
699
  void addMatrix(const Matrix *A, const Matrix *B, Matrix *result)
700
  {
701
     if (!isValid(A) || !isValid(B) || A->cols != B->cols || A->rows
702
         ! = B -> rows)
703
       error("Invalid matrix size");
704
705
706
     int rank, size;
707
```

```
MPI_Comm_rank(MPI_COMM_WORLD, &rank);
708
     MPI_Comm_size(MPI_COMM_WORLD, &size);
709
710
     int rows_per_proc = A->rows / size;
711
     int remaining rows = A->rows % size;
712
     int start = rank * rows_per_proc + (rank < remaining_rows ?</pre>
713
        rank : remaining_rows);
     int end = start + rows_per_proc + (rank < remaining_rows ? 1 :</pre>
714
        0);
715
     for (int i = start; i < end; i++)</pre>
716
717
       for (int j = 0; j < A->cols; j++)
718
719
         result->matrix[i][j] = A->matrix[i][j] + B->matrix[i][j];
720
       }
721
     }
722
723
    MPI_Barrier(MPI_COMM_WORLD);
724
    MPI_Allreduce(MPI_IN_PLACE, result->matrix, MAX_ROWS * MAX_COLS
        , MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
  }
726
727
728 / *
729 Name: subtractMatrix()
730 Parameters: const Matrix *A, const Matrix *B, Matrix *result
731 Return: Void
732 Description: Stores result of subtraction of A and B
733
734 void subtractMatrix(const Matrix *A, const Matrix *B, Matrix *
      result)
735
     if (!isValid(A) || !isValid(B) || A->cols != B->cols || A->rows
736
         ! = B -> rows)
     {
737
       error("Invalid matrix size");
738
739
740
     if (!isValid(A) || !isValid(B) || A->cols != B->cols || A->rows
741
         ! = B -> rows)
742
       error("Invalid matrix size");
743
744
745
     int rank, size;
746
```

```
MPI_Comm_rank(MPI_COMM_WORLD, &rank);
     MPI_Comm_size(MPI_COMM_WORLD, &size);
748
749
     int rows_per_proc = A->rows / size;
750
     int remaining rows = A->rows % size;
751
     int start = rank * rows_per_proc + (rank < remaining_rows ?</pre>
752
        rank : remaining_rows);
     int end = start + rows_per_proc + (rank < remaining_rows ? 1 :</pre>
753
        0);
754
     for (int i = start; i < end; i++)</pre>
755
756
       for (int j = 0; j < A->cols; j++)
757
758
         result->matrix[i][j] = A->matrix[i][j] - B->matrix[i][j];
759
       }
760
761
762
     MPI_Barrier(MPI_COMM_WORLD);
763
     MPI_Allreduce(MPI_IN_PLACE, result->matrix, MAX_ROWS * MAX_COLS
764
        , MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
  }
765
766
  /*
767
768 Name: LU
769 Parameters: Matrix* A, Matrix* L, Matrix* U
  Return: void
  Description: Performs LU decomposition
  void LU(Matrix *A, Matrix *L, Matrix *U)
774
     int n = A->rows;
775
776
     // Initialize L and U matrices
777
     for (int i = 0; i < n; ++i)</pre>
778
779
       for (int j = 0; j < n; ++j)
780
781
         L->matrix[i][j] = 0.0;
782
         U->matrix[i][j] = A->matrix[i][j];
783
       }
784
     }
785
786
     for (int k = 0; k < n; ++k)
787
     {
788
```

```
for (int i = k + 1; i < n; ++i)
789
790
         double multiplier = U->matrix[i][k] / U->matrix[k][k];
791
         L->matrix[i][k] = multiplier;
792
793
          for (int j = k; j < n; ++j)
794
795
            U->matrix[i][j] -= multiplier * U->matrix[k][j];
796
797
       }
798
     }
799
800
     // Set the diagonal of L to 1
801
     for (int i = 0; i < n; ++i)</pre>
802
     {
803
       L->matrix[i][i] = 1.0;
804
805
806
807
808
809
810
811
812
813
  Sequentual functions below
814
815
816
817
818
819
820
821
  /*
822
823 Name: Seq_multiplyMatrix()
824 Parameters: const Matrix *A, const Matrix *B, Matrix *C
825 Return: void
826 Description: Multiplies matrix A and matrix B sequentially and
      stores result in matrix C.
828 void Seq_multiplyMatrix(const Matrix *A, const Matrix *B, Matrix
      *C)
  {
829
     if (!isValid(A) || !isValid(B))
830
       error("Invalid input matrices");
831
```

```
if (A->cols != B->rows)
832
       error("Incompatible dimensions for multiplication");
833
834
     for (int i = 0; i < A->rows; i++)
835
836
       for (int j = 0; j < B->cols; j++)
837
838
         C->matrix[i][j] = 0;
839
          for (int k = 0; k < A \rightarrow cols; k++)
840
841
            C->matrix[i][j] += A->matrix[i][k] * B->matrix[k][j];
842
843
844
845
   }
846
847
  /*
848
  Name: Seq_ref()
850 Parameters: Matrix *m
  Return: void
  Description: Puts the matrix into row echelon form sequentially.
  */
853
  void Seq_ref(Matrix *m)
854
     if (!isValid(m))
856
       error("Invalid matrix");
857
858
     int lead = 0;
859
     int rowCount = m->rows;
860
     int colCount = m->cols;
861
862
     for (int r = 0; r < rowCount; r++)
863
864
       if (lead >= colCount)
865
         return;
866
867
       int i = r;
868
       while (m->matrix[i][lead] == 0)
869
       {
870
         i++;
871
          if (i == rowCount)
872
          {
873
            i = r;
874
            lead++;
875
            if (lead == colCount)
876
```

```
return;
877
878
        }
879
880
        if (i != r)
881
882
          for (int j = 0; j < colCount; j++)</pre>
883
884
            double temp = m->matrix[r][j];
885
            m\rightarrow matrix[r][j] = m\rightarrow matrix[i][j];
886
            m->matrix[i][j] = temp;
887
888
        }
889
890
        double lv = m->matrix[r][lead];
891
        if (lv != 0)
892
893
          for (int j = 0; j < colCount; j++)</pre>
894
            m->matrix[r][j] /= lv;
895
        }
896
897
        for (int i = r + 1; i < rowCount; i++)</pre>
898
899
          double lv2 = m->matrix[i][lead];
900
          for (int j = 0; j < colCount; j++)</pre>
901
            m->matrix[i][j] -= lv2 * m->matrix[r][j];
902
        }
903
904
        lead++;
905
906
907
908
910 Name: Seq_rref()
911 Parameters: Matrix *m
912 Return: void
913 Description: Converts matrix to reduced row echelon form without
      MPI.
914 */
915 void Seq_rref(Matrix *m)
916
  {
     if (!isValid(m))
917
        error("Invalid matrix");
918
919
     Seq_ref(m);
920
```

```
921
     int rowCount = m->rows;
922
     int colCount = m->cols;
923
924
     for (int r = rowCount - 1; r >= 0; r--)
925
     {
926
       int leadCol = -1;
927
       for (int j = 0; j < colCount; j++)
928
929
         if (fabs(m->matrix[r][j] - 1.0) < 1e-6)
930
931
            leadCol = j;
932
            break;
933
934
       }
935
936
       if (leadCol == -1)
937
         continue;
938
939
       for (int i = 0; i < r; i++)</pre>
940
941
         double factor = m->matrix[i][leadCol];
942
         for (int j = 0; j < colCount; j++)
943
            m->matrix[i][j] -= factor * m->matrix[r][j];
944
945
946
947
948
949
950 Name: Seq_LU()
  Parameters: const Matrix *A, Matrix *L, Matrix *U
  Return: void
  Description: Performs LU decomposition sequentially.
954
  void Seq_LU(const Matrix *A, Matrix *L, Matrix *U)
956
     if (!isValid(A))
957
       error("Invalid matrix");
958
959
     int n = A->rows;
960
961
     for (int i = 0; i < n; ++i)
962
963
       for (int j = 0; j < n; ++j)
964
       {
965
```

```
L->matrix[i][j] = 0;
966
         U->matrix[i][j] = 0;
967
       }
968
     }
969
970
     for (int k = 0; k < n; ++k)
971
972
       for (int j = k; j < n; ++j)
973
       {
974
         double sum = 0;
975
         for (int s = 0; s < k; ++s)
976
            sum += L->matrix[k][s] * U->matrix[s][j];
977
         U->matrix[k][j] = A->matrix[k][j] - sum;
978
979
980
       L->matrix[k][k] = 1.0;
981
982
       for (int i = k + 1; i < n; ++i)</pre>
983
       {
984
         double sum = 0;
985
         for (int s = 0; s < k; ++s)
986
            sum += L->matrix[i][s] * U->matrix[s][k];
987
         L->matrix[i][k] = (A->matrix[i][k] - sum) / U->matrix[k][k]
988
             ];
989
990
991 }
```

multiplyTest.c

```
=1
     1
#include <stdio.h>
2 #include <stdlib.h>
3 #include <mpi.h>
4 #include <time.h>
5 #include "MatrixC.h"
7 void divisor(const char *msg)
    printf("n----%s ----nn", msq);
10
  }
11
12 Matrix A, B, C_parallel, C_seq;
13
int main(int argc, char **argv)
  {
15
    MPI_Init(&argc, &argv);
16
17
    int rank, size;
18
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
19
    MPI_Comm_size(MPI_COMM_WORLD, &size);
20
21
    srand(time(NULL) + rank);
22
23
    int N = 100, use_parallel = 0;
24
25
    if (rank == 0)
26
27
      printf("Enter matrix size: ");
28
      scanf("%d", &N);
29
30
      printf("Run sequential too? (1 = yes, 0 = no): ");
31
      scanf("%d", &use_parallel);
32
33
34
    // Broadcast settings
35
    MPI_Bcast(&N, 1, MPI_INT, 0, MPI_COMM_WORLD);
36
    MPI_Bcast(&use_parallel, 1, MPI_INT, 0, MPI_COMM_WORLD);
37
38
    initSize(&A, N, N);
39
    initSize(&B, N, N);
40
    initSize(&C_parallel, N, N);
```

```
initSize(&C_seq, N, N);
43
    // Rank O generates the random data
    if (rank == 0)
45
    {
46
      setRandom(&A, 100);
47
      setRandom(&B, 100);
49
50
    // Broadcast A and B
51
    MPI_Bcast(&(A.matrix[0][0]), N * N, MPI_DOUBLE, 0,
52
      MPI_COMM_WORLD);
    MPI_Bcast(&(B.matrix[0][0]), N * N, MPI_DOUBLE, 0,
53
       MPI_COMM_WORLD);
54
    // ----- Parallel -----
55
    if (rank == 0)
56
      printf("Running parallel multiplyMatrix()...\n");
57
58
    double start = MPI_Wtime();
59
    multiplyMatrix(&A, &B, &C_parallel);
60
    double end = MPI_Wtime();
61
62
    if (rank == 0)
63
64
      printf("multiplyMatrix() done in %.3f seconds\n", end - start
65
      divisor("multiplyMatrix");
66
    }
67
68
    // ----- Sequential -----
69
    if (rank == 0 && use_parallel == 1)
70
71
      printf("Running sequential Seq_multiplyMatrix()...\n");
72
73
      start = MPI_Wtime();
74
      Seq_multiplyMatrix(&A, &B, &C_seq);
75
      end = MPI_Wtime();
76
77
      printf("Seq_multiplyMatrix() done in %.3f seconds\n", end -
78
         start);
      divisor("Seq_multiplyMatrix");
79
    }
80
81
    MPI_Finalize();
```

```
return 0;
s4 }
```

refTest.c

```
=1
    1
#include <stdio.h>
2 #include <stdlib.h>
3 #include <mpi.h>
4 #include <time.h>
5 #include "MatrixC.h"
7 void divisor(const char *msg)
    printf("n----%s ----nn", msq);
10 }
11
12 // Global matrices
13 Matrix A_parallel, A_seq;
int main(int argc, char **argv)
16
    MPI_Init(&argc, &argv);
17
18
    int rank, size;
19
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
20
    MPI_Comm_size(MPI_COMM_WORLD, &size);
21
22
    srand(time(NULL) + rank);
23
24
    int N = 10, use_parallel = 1;
25
26
    if (rank == 0)
27
28
      printf("Enter matrix size: ");
29
      scanf("%d", &N);
30
31
      printf("Run sequential too? (1 = yes, 0 = no): ");
32
      scanf("%d", &use_parallel);
33
    }
34
35
    // Broadcast settings
36
    MPI_Bcast(&N, 1, MPI_INT, 0, MPI_COMM_WORLD);
37
    MPI_Bcast(&use_parallel, 1, MPI_INT, 0, MPI_COMM_WORLD);
38
39
    initSize(&A_parallel, N, N);
40
    initSize(&A_seq, N, N);
```

```
// Rank 0 generates the random matrix
43
    if (rank == 0)
44
45
      setRandom(&A_parallel, 100);
46
      copyMatrix(&A_seq, &A_parallel);
47
    }
48
49
    // Broadcast A_parallel to all ranks
50
    MPI_Bcast(&(A_parallel.matrix[0][0]), N * N, MPI_DOUBLE, 0,
51
       MPI_COMM_WORLD);
52
    // ----- Parallel ref() -----
53
    if (rank == 0)
54
      printf("Running parallel ref()...\n");
55
56
    double start = MPI_Wtime();
57
    ref(&A_parallel);
58
    double end = MPI_Wtime();
59
60
    if (rank == 0)
61
62
      printf("ref() done in %.3f seconds\n", end - start);
63
      divisor("ref");
    }
65
66
    // ----- Sequential Seq_ref() -----
67
    if (rank == 0 && use_parallel == 1)
68
69
      printf("Running sequential Seq_ref()...\n");
70
71
      start = MPI_Wtime();
72
      Seq_ref(&A_seq);
73
      end = MPI_Wtime();
74
75
      printf("Seq_ref() done in %.3f seconds\n", end - start);
76
      divisor("Seq_ref");
77
    }
78
    MPI_Finalize();
80
    return 0;
82 }
```

rrefTest.c

```
=1
    1
#include <stdio.h>
2 #include <stdlib.h>
3 #include <mpi.h>
4 #include <time.h>
5 #include "MatrixC.h"
7 void divisor(const char *msg)
    printf("n----%s ----nn", msq);
10 }
11
12 // Global matrices
13 Matrix A_parallel, A_seq;
int main(int argc, char **argv)
16
    MPI_Init(&argc, &argv);
17
18
    int rank, size;
19
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
20
    MPI_Comm_size(MPI_COMM_WORLD, &size);
21
22
    srand(time(NULL) + rank);
23
24
    int N = 10, use parallel = 1;
25
26
    if (rank == 0)
27
28
      printf("Enter matrix size: ");
29
      scanf("%d", &N);
30
31
      printf("Run sequential too? (1 = yes, 0 = no): ");
32
      scanf("%d", &use_parallel);
33
    }
34
35
    // Broadcast settings
36
    MPI_Bcast(&N, 1, MPI_INT, 0, MPI_COMM_WORLD);
37
    MPI_Bcast(&use_parallel, 1, MPI_INT, 0, MPI_COMM_WORLD);
38
39
    initSize(&A_parallel, N, N);
40
    initSize(&A_seq, N, N);
```

```
// Rank 0 sets data
43
    if (rank == 0)
44
45
      setRandom(&A_parallel, 100);
46
      copyMatrix(&A_seq, &A_parallel);
47
    }
48
49
    // Broadcast to all processes
50
    MPI_Bcast(&(A_parallel.matrix[0][0]), N * N, MPI_DOUBLE, 0,
51
       MPI_COMM_WORLD);
52
    // ----- Parallel rref() -----
53
    if (rank == 0)
54
      printf("Running parallel rref()...\n");
55
56
    double start = MPI_Wtime();
57
    rref(&A_parallel);
58
    double end = MPI_Wtime();
59
60
    if (rank == 0)
61
62
      printf("rref() done in %.3f seconds\n", end - start);
63
      divisor("rref");
    }
65
66
    // ----- Sequential Seq_rref() -----
67
    if (rank == 0 && use_parallel == 1)
68
69
      printf("Running sequential Seq_rref()...\n");
70
71
      start = MPI_Wtime();
72
      Seq_rref(&A_seq);
73
      end = MPI_Wtime();
74
75
      printf("Seq_rref() done in %.3f seconds\n", end - start);
76
      divisor("Seq_rref");
77
    }
78
79
    MPI_Finalize();
80
    return 0;
82 }
```

luTest.c

```
=1
     1
#include <stdio.h>
2 #include <stdlib.h>
3 #include <mpi.h>
4 #include <time.h>
5 #include "MatrixC.h"
7 void divisor(const char *msg);
9 Matrix A_original, L_parallel, U_parallel;
10 Matrix L_seq, U_seq;
int main(int argc, char **argv)
13
    MPI_Init(&argc, &argv);
14
15
    int rank, size;
16
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
17
    MPI_Comm_size(MPI_COMM_WORLD, &size);
18
19
    srand(time(NULL) + rank);
20
21
    int N = 10, use_sequential = 0;
22
23
    if (rank == 0)
24
25
      printf("Enter matrix size (N for NxN): ");
26
      scanf("%d", &N);
27
28
      printf("Run sequential also? (1 = yes, 0 = no): ");
29
      scanf("%d", &use_sequential);
30
    }
31
32
    MPI_Bcast(&N, 1, MPI_INT, 0, MPI_COMM_WORLD);
33
    MPI_Bcast(&use_sequential, 1, MPI_INT, 0, MPI_COMM_WORLD);
34
35
    initSize(&A_original, N, N);
36
    initSize(&L_parallel, N, N);
37
    initSize(&U_parallel, N, N);
38
39
    if (rank == 0)
40
    {
41
```

```
initSize(&L_seq, N, N);
      initSize(&U_seq, N, N);
43
      setRandom(&A_original, 100);
44
    }
45
46
    MPI_Bcast(&(A_original.matrix[0][0]), N * N, MPI_DOUBLE, 0,
47
       MPI_COMM_WORLD);
48
    // ----- Parallel LU ------
49
    if (rank == 0)
50
      printf("Running parallel LU()...\n");
51
52
    double start = MPI_Wtime();
53
    LU(&A_original, &L_parallel, &U_parallel);
54
    double end = MPI_Wtime();
55
56
    if (rank == 0)
57
58
      printf("Parallel LU() done in %.3f seconds\n", end - start);
59
      divisor("Parallel LU");
60
61
      if (use_sequential == 1)
62
      {
63
        Matrix A_copy;
        initSize(&A_copy, N, N);
65
        copyMatrix(&A_copy, &A_original);
67
        printf("Running sequential Seq_LU()...\n");
68
69
        start = MPI_Wtime();
70
        Seq_LU(&A_copy, &L_seq, &U_seq);
71
        end = MPI_Wtime();
72
73
        printf("Sequential Seq_LU() done in %.3f seconds\n", end -
74
           start);
        divisor("Sequential LU");
75
      }
76
    }
77
78
    MPI_Finalize();
79
    return 0;
80
81 }
83 void divisor(const char *msg)
84 {
```

```
85  printf("\n---- %s ----\n\n", msg);
86 }
```

runTest.sh

```
=1
    1
1 #!/bin/bash
3 echo "Enter matrix size (N for NxN): "
4 read N
6 echo "Run sequential also? (1 = yes, 0 = no): "
7 read USE_SEQ
9 echo "Enter number of MPI processes to use: "
10 read PROCS
12 echo "Which test to run?"
13 echo "1. multiplyTest"
14 echo "2. refTest"
15 echo "3. rrefTest"
16 echo "4. luTest"
17 read CHOICE
19 # Pick executable
20 case $CHOICE in
  1) EXEC="./multiplyTest" ;;
   2) EXEC="./refTest" ;;
22
  3) EXEC="./rrefTest";;
  4) EXEC="./luTest";;
    *) echo "Invalid choice"; exit 1 ;;
26 esac
27
28 echo
29 echo "Running $EXEC with:"
30 echo "- Matrix size: $N"
31 echo "- Sequential: $USE_SEQ"
32 echo "- Processes: $PROCS"
33 echo
35 mpirun --oversubscribe -np $PROCS $EXEC <<< "$N
36 $USE_SEQ"
```