## **Chapter 1**

## Example problem: Steady flow in a curved tube

The problem of steady flow in a curved tube is considered with a prescribed Poiseuille flow at the inlet and a traction-free outlet condition. It is not clear that the latter is appropriate, but the main aim of this example is to check that the TubeMesh works correctly.

A detailed comparison between the flow field and the Dean solution should be performed for validation purposes, but the qualitative features seem reasonable.

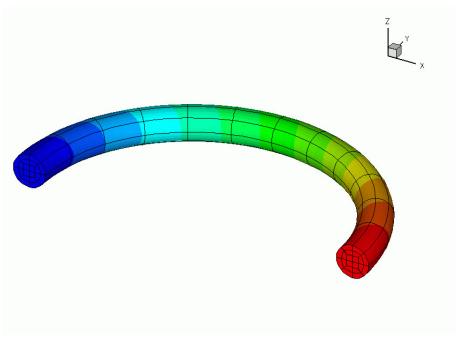


Figure 1.1 Sketch of the problem with pressure contours.

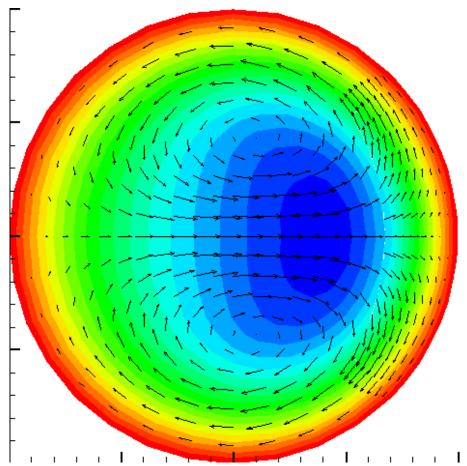


Figure 1.2 Contours of axial velocity and secondary streamlines.

## Detailed documentation to be written. Here's the driver code...

```
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
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//LIC// 02110-1301 USA.
//LIC//
//LIC// The authors may be contacted at oomph-lib@maths.man.ac.uk.
//LIC//
///Driver for a 3D navier stokes steady entry flow problem in a
///uniformly curved tube
//Generic routines
#include "generic.h"
#include "navier_stokes.h"
// The mesh
#include "meshes/tube_mesh.h"
using namespace std;
using namespace oomph;
//=start_of_MyCurvedCylinder=======
//A geometric object that represents the geometry of the domain
```

```
class MyCurvedCylinder : public GeomObject
public:
 /// Constructor that takes the radius and curvature of the tube
 /// as its arguments
MyCurvedCylinder(const double &radius, const double &delta) :
 GeomObject(3,3), Radius(radius), Delta(delta) { }
/// Destructor
virtual~MyCurvedCylinder(){}
///Lagrangian coordinate xi
void position (const Vector<double>& xi, Vector<double>& r) const
r[0] = (1.0/Delta)*cos(xi[0]) + xi[2]*Radius*cos(xi[0])*cos(xi[1]);
r[1] = (1.0/Delta)*sin(xi[0]) + xi[2]*Radius*sin(xi[0])*cos(xi[1]);
r[2] = -xi[2]*Radius*sin(xi[1]);
/// Return the position of the tube as a function of time
/// (doesn't move as a function of time)
void position(const unsigned& t,
             const Vector<double>& xi, Vector<double>& r) const
  position(xi,r);
private:
 ///Storage for the radius of the tube
double Radius;
 //Storage for the curvature of the tube
double Delta;
/// Namespace for physical parameters
namespace Global_Physical_Variables
 /// Reynolds number
double Re=50;
 /// The desired curvature of the pipe
double Delta=0.1;
} // end_of_namespace
/// Entry flow problem in tapered tube domain
               ______
template<class ELEMENT>
class SteadyCurvedTubeProblem : public Problem
public:
 /// Constructor: Pass DocInfo object and target errors
SteadyCurvedTubeProblem(DocInfo& doc_info, const double& min_error_target,
                const double& max_error_target);
 /// Destructor (empty)
 ~SteadyCurvedTubeProblem() {}
 /// \short Update the problem specs before solve
 void actions_before_newton_solve();
 /// After adaptation: Pin redudant pressure dofs.
 void actions_after_adapt()
   // Pin redudant pressure dofs
  RefineableNavierStokesEquations<3>::
   pin_redundant_nodal_pressures(mesh_pt()->element_pt());
 /// Doc the solution
void doc_solution();
 /// \short Overload generic access function by one that returns
 /// a pointer to the specific mesh
 RefineableTubeMesh<ELEMENT>* mesh_pt()
  return dynamic_cast<RefineableTubeMesh<ELEMENT>*>(Problem::mesh_pt());
private:
 /// Doc info object
DocInfo Doc_info;
 ///Pointer to GeomObject that specifies the domain volume
```

```
GeomObject *Volume_pt;
}; // end_of_problem_class
//-start_of_constructor------
/// Constructor: Pass DocInfo object and error targets
template<class ELEMENT>
SteadyCurvedTubeProblem<ELEMENT>::SteadyCurvedTubeProblem(DocInfo& doc_info,
                                                const double& min_error_target,
                                                const double& max_error_target)
 : Doc_info(doc_info)
{
 // Setup mesh:
 // Create GeomObject that specifies the domain geometry
 //The radius of the tube is one and the curvature is specified by
 //the global variable Delta.
Volume_pt=new MyCurvedCylinder(1.0,Global_Physical_Variables::Delta);
const double pi = MathematicalConstants::Pi;
 //Set the centerline coordinates spanning the mesh
Vector<double> centreline_limits(2);
centreline_limits[0] = 0.0;
centreline_limits[1] = pi;
 //Set the positions of the angles that divide the outer ring
 //These must be in the range -pi,pi, ordered from smallest to
 //largest
 Vector<double> theta_positions(4);
theta_positions[0] = -0.75*pi;
theta_positions[1] = -0.25*pi;
 theta_positions[2] = 0.25*pi;
 theta_positions[3] = 0.75*pi;
 //Define the radial fraction of the central box (always halfway
 //along the radius)
Vector<double> radial_frac(4,0.5);
 // Number of layers in the initial mesh
unsigned nlayer=6;
 // Build and assign mesh
Problem::mesh_pt() = new RefineableTubeMesh<ELEMENT>(Volume_pt,
                                                         centreline limits,
                                                         theta_positions,
                                                         radial_frac,
                                                         nlayer);
 // Set error estimator
Z2ErrorEstimator* error_estimator_pt=new Z2ErrorEstimator;
{\tt mesh\_pt()->spatial\_error\_estimator\_pt()=error\_estimator\_pt;}
 // Error targets for adaptive refinement
mesh_pt()->max_permitted_error()=max_error_target;
mesh_pt()->min_permitted_error()=min_error_target;
 ^{\prime\prime} Set the boundary conditions for this problem: All nodal values are
 // free by default -- just pin the ones that have Dirichlet conditions
 //Choose the conventional form by setting gamma to zero
 //The boundary conditions will be pseudo-traction free (d/dn = 0)
 ELEMENT::Gamma[0] = 0.0;
ELEMENT::Gamma[1] = 0.0;
ELEMENT::Gamma[2] = 0.0;
 //Loop over the boundaries
 unsigned num_bound = mesh_pt()->nboundary();
 for(unsigned ibound=0;ibound<num_bound;ibound++)</pre>
  unsigned num_nod= mesh_pt()->nboundary_node(ibound);
for (unsigned inod=0;inod<num_nod;inod++)</pre>
    {
     // Boundary 0 is the inlet symmetry boundary:
     // Boundary 1 is the tube wall
     // Pin all values
     if((ibound==0) || (ibound==1))
       mesh pt()->boundary node pt(ibound,inod)->pin(0);
       mesh_pt()->boundary_node_pt(ibound,inod)->pin(1);
       mesh_pt()->boundary_node_pt(ibound,inod)->pin(2);
  } // end loop over boundaries
 // Loop over the elements to set up element-specific
// things that cannot be handled by constructor
 unsigned n_element = mesh_pt()->nelement();
 for (unsigned i=0;i<n_element;i++)</pre>
   // Upcast from GeneralisedElement to the present element
ELEMENT* el_pt = dynamic_cast<ELEMENT*>(mesh_pt()->element_pt(i));
```

```
//Set the Reynolds number, etc
  el_pt->re_pt() = &Global_Physical_Variables::Re;
 // Pin redudant pressure dofs
RefineableNavierStokesEquations<3>::
 pin_redundant_nodal_pressures(mesh_pt()->element_pt());
 //Attach the boundary conditions to the mesh
 cout «"Number of equations: " « assign_eqn_numbers() « std::endl;
} // end_of_constructor
//=start_of_actions_before_newton_solve=====
/// Set the inflow boundary conditions
//============
template<class ELEMENT>
void SteadyCurvedTubeProblem<ELEMENT>::actions_before_newton_solve()
 // (Re-)assign velocity profile at inflow values
unsigned ibound=0;
unsigned num_nod= mesh_pt()->nboundary_node(ibound);
 for (unsigned inod=0;inod<num_nod;inod++)</pre>
  // Recover coordinates of tube relative to centre position
  double x=mesh_pt()->boundary_node_pt(ibound,inod)->x(0) -
   1.0/Global Physical Variables::Delta;
  double z=mesh_pt()->boundary_node_pt(ibound, inod)->x(2);
  //Calculate the radius
  double r=sqrt(x*x+z*z);
  // Poiseuille-type profile for axial velocity (component 1 at the inlet)
  mesh_pt()->boundary_node_pt(ibound,inod)->
   set_value(1,(1.0-pow(r,2.0)));
} // end_of_actions_before_newton_solve
//=start_of_doc_solution=======
/// Doc the solution
//=====
template<class ELEMENT>
void SteadyCurvedTubeProblem<ELEMENT>::doc solution()
 //Output file stream
ofstream some_file;
char filename[100];
 // Number of plot points
unsigned npts;
npts=5;
 //Need high precision for large radii of curvature
 //some_file.precision(10);
// Output solution labelled by the Reynolds number
sprintf(filename,"%s/soln_Re%g.dat",Doc_info.directory().c_str(),
        Global Physical Variables::Re);
some_file.open(filename);
mesh_pt()->output(some_file,npts);
some_file.close();
} // end_of_doc_solution
/// Driver for 3D entry flow into a curved tube. If there are
/// any command line arguments, we regard this as a validation run
/// and perform only a single adaptation
int main(int argc, char* argv[])
 // Store command line arguments
CommandLineArgs::setup(argc,argv);
 // Allow (up to) two rounds of fully automatic adapation in response to
 // error estimate
unsigned max_adapt;
double max_error_target,min_error_target;
// Set max number of adaptations in black-box Newton solver and // error targets for adaptation
 if (CommandLineArgs::Argc==1)
  // Up to two adaptations
  max adapt=2;
  // Error targets for adaptive refinement
  max_error_target=0.001;
  min_error_target=0.00001;
 // Validation run: Only one adaptation. Relax error targets
 // for faster solution
else
```

```
// Validation run: Just one round of adaptation
  max_adapt=1;
  // Error targets for adaptive refinement \max\_error\_target=0.02;
  min_error_target=0.002;
// end max_adapt setup
// Set up doc info
DocInfo doc_info;
// Do Taylor-Hood elements
//
{
    // Set output directory
    doc_info.set_directory("RESLT_TH");
 // Step number
 doc_info.number()=0;
 // Build problem
 SteadyCurvedTubeProblem<RefineableQTaylorHoodElement<3>>
  problem(doc_info,min_error_target,max_error_target);
 cout « " Doing Taylor-Hood elements " « std::endl;
 // Solve the problem
 problem.newton_solve(max_adapt);
// Doc solution after solving
 problem.doc_solution();
// Do Crouzeix-Raviart elements
 // Set output directory
 doc_info.set_directory("RESLT_CR");
 // Step number
 doc_info.number()=0;
 // Build problem
 SteadyCurvedTubeProblem<RefineableQCrouzeixRaviartElement<3> >
  problem(doc_info,min_error_target,max_error_target);
 cout « " Doing Crouzeix-Raviart elements " « std::endl;
 \ensuremath{//} Solve the problem
 problem.newton_solve(max_adapt);
// Doc solution after solving
 problem.doc_solution();
  // end_of_main
```

## 1.1 PDF file

A pdf version of this document is available.