Chapter 1

Demo problem: The (axisymmetric) static free surface bounding a layer of viscous fluid.

1.1 Overview of the problem

We consider an open circular container of unit radius containing a still viscous fluid of prescribed volume $\mathcal{V}$ that meets the wall of the container at a contact angle θ_c. The problem is extremely similar to that described in the `two-dimensional static cap tutorial.` The exact solution corresponds to a free surface of constant curvature that is the arc of a circle rotated about the axis to give a section of a sphere. The mean curvature of the interface in this problem is $\kappa = 2/r = 4\cos\theta_c$, which differs from the two-dimensional problem in which it was $2\cos\theta_c$.

The only differences between the axisymmetric and the `two-dimensional` driver codes are that:

1. two-dimensional elements are replaced by the equivalent axisymmetric elements;
2. specified volume is different;
3. swirl (theta) component of velocity is pinned;
4. analytic pressure drop is changed.

In other words we make the following changes:

	Two-dimensional problem	Axisymmetric problem
Bulk Fluid Element	QCrouzeixRaviartElement<2>	AxisymmetricQCrouzeixRaviart↩ Element
Pseudo-Solid Free Surface Face Element	ElasticLineFluidInterfaceElement	ElasticAxisymmetricFluid↩ InterfaceElement
Spine Free Surface Face Element	SpineLineFluidInterfaceElement	SpineAxisymmetricFluid↩ InterfaceElement
Pseudo-Solid Volume Constraint Face Element	ElasticLineVolumeConstraint↩ BoundingElement	ElasticAxisymmetricVolume↩ ConstraintBoundingElement
Spine Volume Constraint Face Element	SpineLineVolumeConstraint↩ BoundingElement	SpineAxisymmetricVolume↩ ConstraintBoundingElement
Specified Volume	$xh = 0.5$	$r^2h/2 = 0.125$
Analytic pressure drop	$2\cos\theta_c$	$4\cos\theta_c$

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1.2 Comments and Exercises

1.2.1 Comments

- The formula for the specified volume in the axisymmetric case is the true volume $\mathcal{V}$ divided by 2π because all the axisymmetric equations are divided by the common factor of 2π. The `AxisymmetricVolumeConstraintBoundingElement` class must be used so that the volume is correctly calculated.
- The swirl velocity is pinned on the boundaries by including the additional code
```
Bulk_mesh_pt->boundary_node_pt(b,n)->pin(2);
```

1.2.2 Exercises

1. Confirm that the computed pressure differences agree with the analytic expression. Verify that the interface shape is unaffected by the capillary number, but that the pressure difference across the interface changes in inverse proportion to it. Check that the pressure difference is unaffected by the choice of reference pressure.

2. Investigate what happens when the two-dimensional volume constraint elements are used. Explain your result.

1.3 Source files for this tutorial

- Source files for this tutorial are located in the directory:

  demo_drivers/axisym_navier_stokes/axi_static_cap/

- The driver code is:

  demo_drivers/axisym_navier_stokes/axi_static_cap/axi_static_cap.cc

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1.4 PDF file

A pdf version of this document is available.