

```
%% Self-Balancing Reaction Wheel Robot: True Physical Model
```

```
clear all; close all; clc;
```

```
Mm = 0.231; % [kg] motor mass
```

```
Mb = 0.8; % structure mass
```

```
Mr = 0.100; % rotor mass
```

```
lm = 0.09; % [m]
```

```
lb = 0.07; % COM
```

```
lr = lm; % rotor at same point at mass
```

```
g = 9.81; % [m/s2]
```

```
k_mgl = g * (Mm*lm + Mb*lb + Mr*lr)
```

```
k_mgl = 0.8416
```

```
Iro = Mr*lr^2 % rotor inertia
```

```
Iro = 8.1000e-04
```

```
Imo = Mm*lm^2 % motor "
```

```
Imo = 0.0019
```

```
Ibo = Mb*lb^2 % struct "
```

```
Ibo = 0.0039
```

```
Iso = Iro + Imo + Ibo % total intertia about o
```

```
Iso = 0.0066
```

```
tau_m = 0.05; % [s] control loop response time to motor speed input on odrive
```

```
dt = 0.01; % Sampling time [s]
```

```
%% State-Space Model
```

```
% States: x = [theta, theta_d, omega_m]
```

```
% Input: u = commanded motor speed, w_mc
```

```
A = [0          1          0;  
      k_mgl/Iso  0          -Iro/(Iso*tau_m);  
      0          0          -1/tau_m];
```

```
B = [0;  
      Iro/(Iso*tau_m);  
      1/tau_m];
```

```
C = eye(3); % We can measure all states
D = zeros(3, 1);
```

```
% Discretize
```

```
sys_c = ss(A, B, C, D);
sys_d = c2d(sys_c, dt, 'zoh');
Ad = sys_d.A
```

```
Ad = 3×3
    1.0064    0.0100   -0.0001
    1.2776    1.0064   -0.0223
         0         0    0.8187
```

```
Bd = sys_d.B
```

```
Bd = 3×1
    0.0001
    0.0223
    0.1813
```

```
%% LQR Design
```

```
Q = diag([100, 10, 1]); % High cost on angle and angular velocity
R = 10; % Cost on motor speed (control effort)
```

```
[K, P, e] = dlqr(Ad, Bd, Q, R);
```

```
disp('LQR Gain:');
```

```
LQR Gain:
```

```
disp(K);
```

```
279.8065    24.7953   -1.9319
```

```
%% Simulate
```

```
t = 0:dt:5;
x0 = [5*pi/180; 0; 10];
```

```
% Closed-loop system
```

```
sys_cl = ss(Ad - Bd*K, Bd, C, D, dt);
```

```
% Simulate
```

```
[y, t] = initial(sys_cl, x0, t);
```

```
% Plot
```

```
figure;
subplot(3,1,1);
plot(t, y(:,1)*180/pi); ylabel('\theta [deg]'); title('Body Angle');
xlim([0 1])
subplot(3,1,2);
plot(t, y(:,2)*180/pi); ylabel('\omega_{body} [deg/s]'); title('Body Angular Velocity');
```

```

xlim([0 1])
subplot(3,1,3);
plot(t, y(:,3)); ylabel('\omega_{wheel} [rad/s]'); title('Wheel Speed');
xlim([0 1])
xlabel('Time [s]');
sgtitle('Response to 10-degree Initial Angle');

```

