Project Proposal: Quantum Walk

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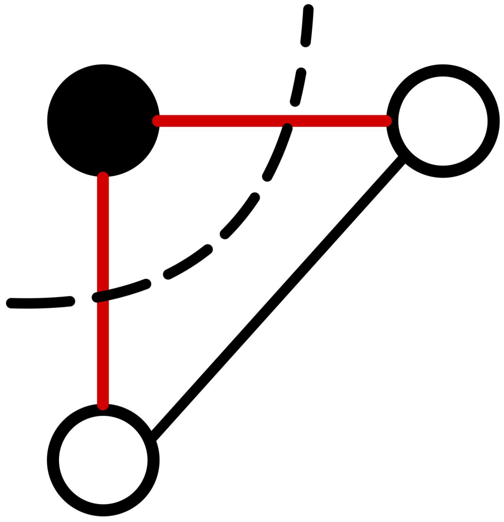
Motivation

- Gaining a deep understanding and giving example usage.
- Demonstrate what was learned in this course.
- Work can be developed into a workshop.
- QW can be used to create new algorithms and simulate complex physical systems.

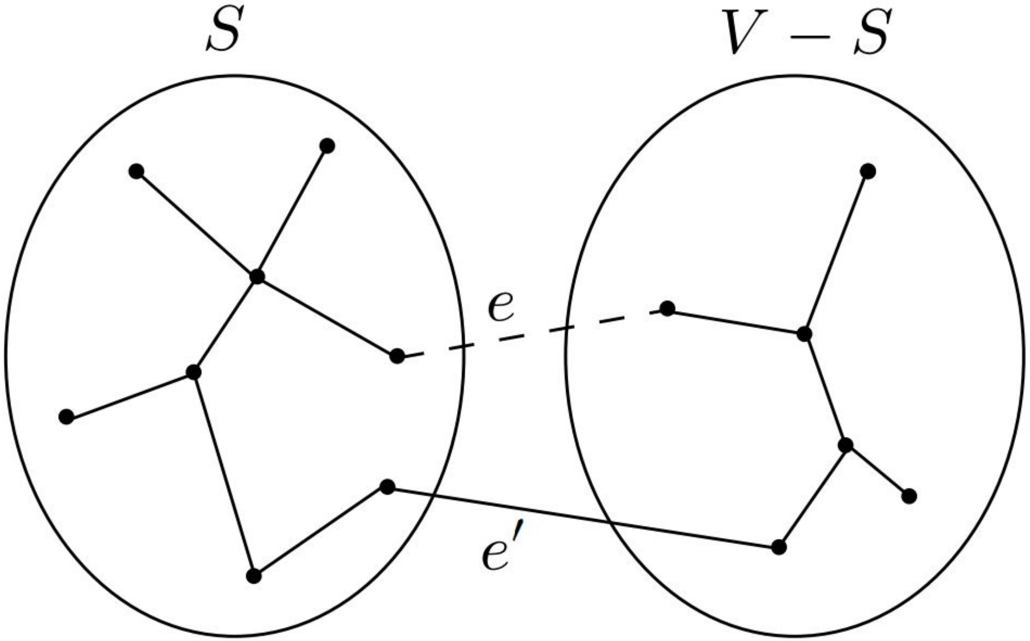
The Problem: Maxcut

- A Classically NP-Hard problem ($O(2^n)$).
- Given graph G = (V, E), connected vertices with differing values add 1 to the cut.
- We want a solution with the most amount of cuts.



[2] A. Jin and X. -Y. Liu, "A Fast Machine Learning Algorithm for the MaxCut Problem"

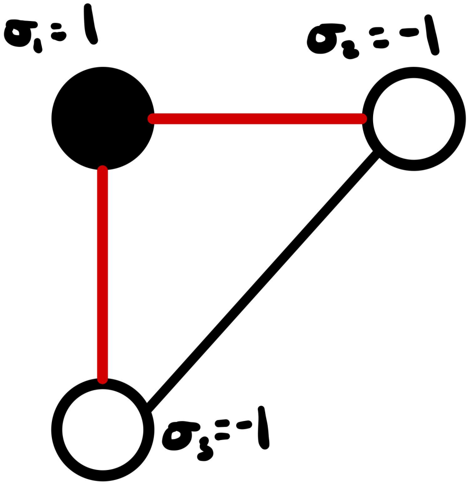
Another Perspective



We separate the vertices of a given graph into 2 sets, S and V and we want to maximize the # of edges between them.

Formulating as Ising model

- Binary sequence $\{0, 1\}^N$ becomes $\{-1, +1\}^N$ (Spin up and Spin down).



[2] A. Jin and X. -Y. Liu, "A Fast Machine Learning Algorithm for the MaxCut Problem"

Evaluating the Configuration

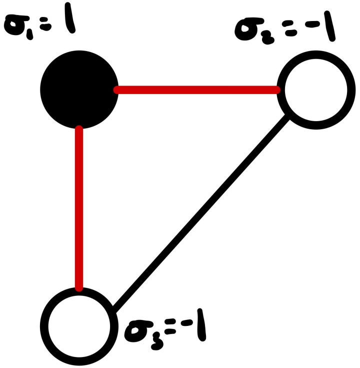
- Define a Hamiltonian
- Define Cut Value

$$H(\boldsymbol{\sigma}) = -\sum_{(i,j)\in E} \sigma_i \sigma_j$$

$$\begin{aligned} \mathrm{Cut}(G) &= \frac{1}{2}\sum_{(i,j)\in E}(1-\sigma_i\sigma_j) \\ &= \frac{1}{2}\sum_{(i,j)\in E} 1 - \frac{1}{2}\sum_{(i,j)\in E}\sigma_i\sigma_j \\ &= \frac{1}{2}|E| - \frac{1}{2}H(\boldsymbol{\sigma}) \end{aligned}$$

[2] A. Jin and X. -Y. Liu, "A Fast Machine Learning Algorithm for the MaxCut Problem"

Example



σ1σ2 = (1)(-1) = -1
σ2σ1 = (-1)(1) = -1
σ2σ3 = (-1)(-1) = 1
σ3σ2 = (-1)(-1) = 1
σ3σ1= (-1)(1) = -1
σ1σ3= (1)(-1) = -1
⇒ H(σ) = -(-4 + 2)
⇒ H(σ) = -(-2) = 2

$$H(\boldsymbol{\sigma}) = -\sum_{(i,j)\in E} \sigma_i \sigma_j$$

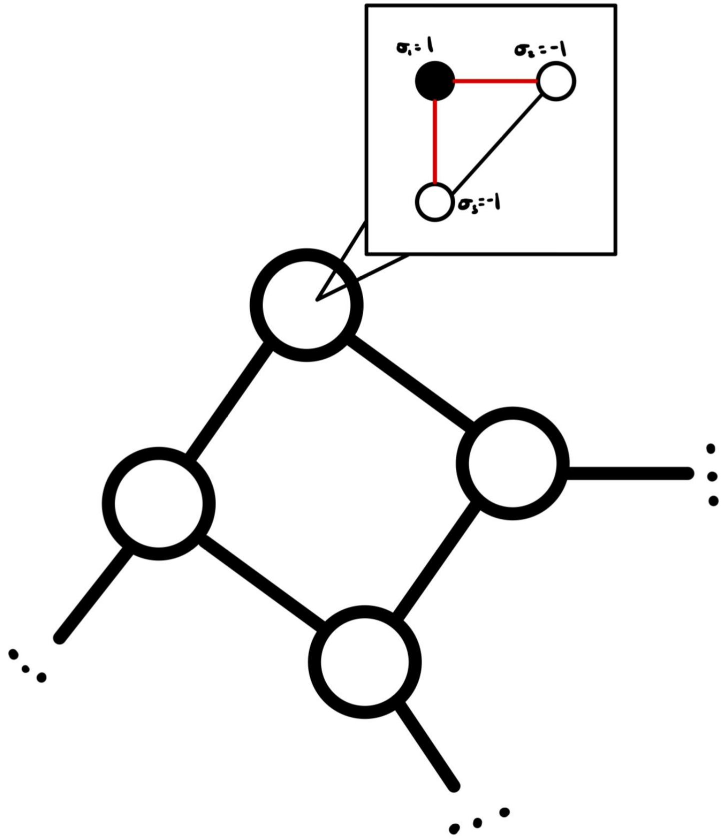
$$\mathrm{Cut}(G) = \frac{1}{2}|E| - \frac{1}{2}H(\boldsymbol{\sigma})$$

|E| = 6
H(σ) = 2
⇒ Cut(G) = ½ (6) - ½ (2)
⇒ Cut(G) = 3 - 1 = 2

What if...

We had a graph of all possible configurations

- How do we traverse it?
- How do we choose neighbors?
- Avoid local maximums?



Approaches

01

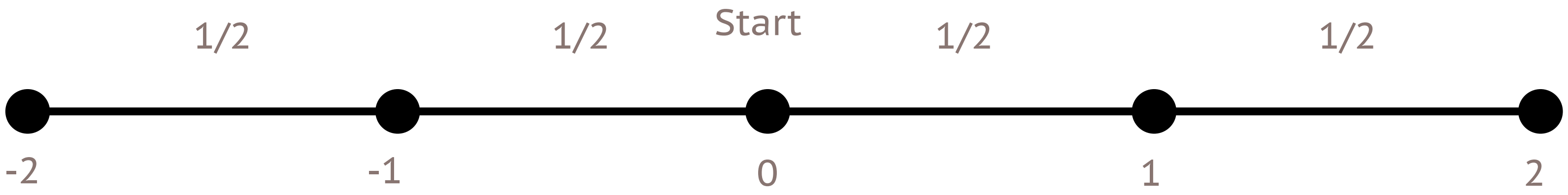
Classical Random Walk

02

Quantum Walk (QW)

Classical Random Walks

- Probabilistic node transitions

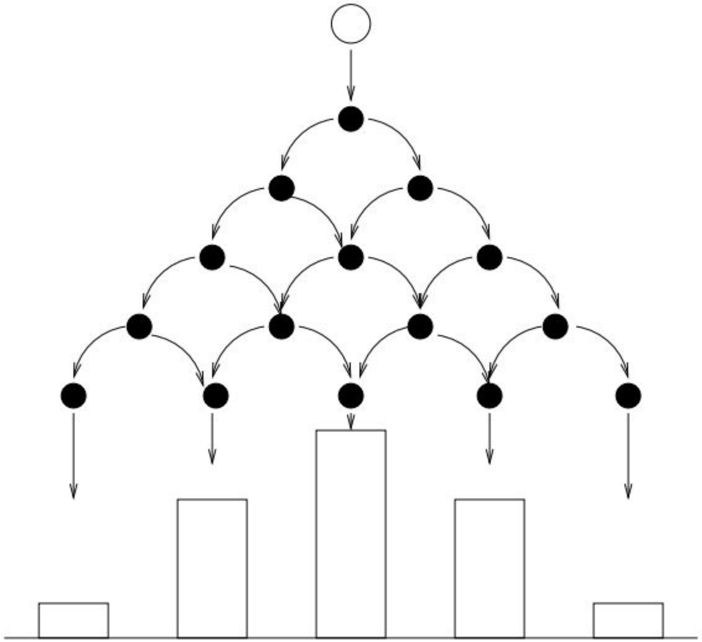


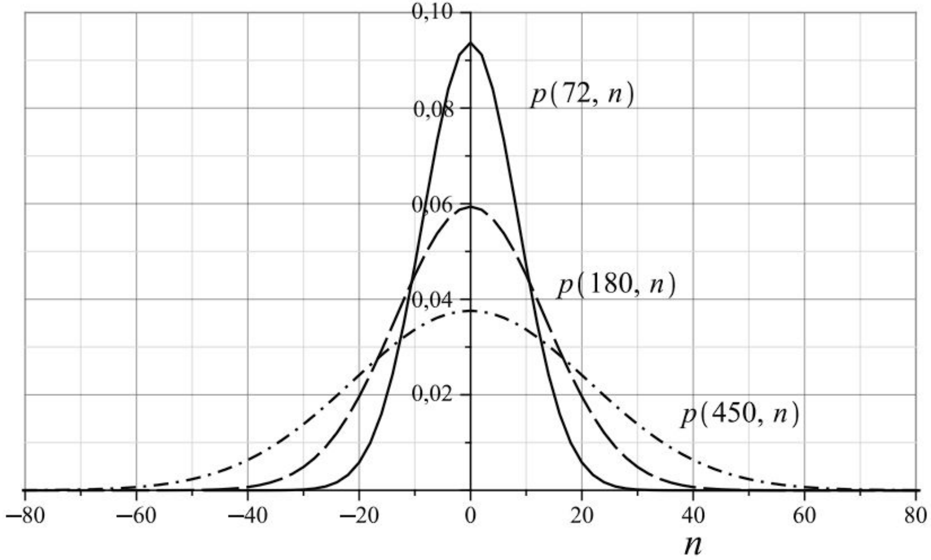
- Higher dimensionality

$$\mathbf{p}(t+1) = M\mathbf{p}(t)$$

[1] J. D. Hidary, "Quantum Walks" in Quantum Computing: An Applied Approach, Cham, Switzerland:Springer, 2019.

Gaussian Distribution

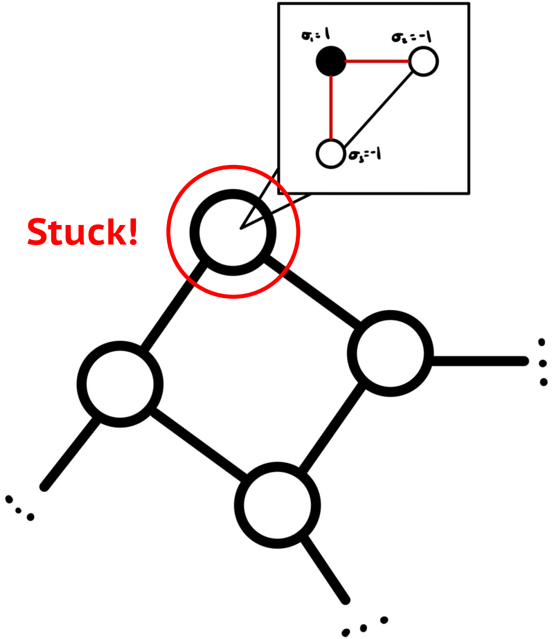




[9] Kempe, J. (2003). Quantum random walks - an introductory overview. ArXiv. https://doi.org/10.1080/00107151031000110776

Drawbacks

- No guarantee of finding the best solution.
- Some approaches get stuck at a local maximum.



Quantum Walks

- Can be used implemented physically without a computer.
- Useful for creating new quantum algorithms.
- Can be used to simulate complex physical systems.

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Continuous Time Quantum Walk

- Use the classical probability distribution to derive a variation of the schrodinger equation.

$$\mathbf{p}(t+1) = M\mathbf{p}(t)$$

$$\frac{d\mathbf{p}(t)}{dt} = -H\mathbf{p}(t)$$

Excludes $\hbar$ and i.

Bold **p**: is a vector

[1] J. D. Hidary, "Quantum Walks" in Quantum Computing: An Applied Approach, Cham, Switzerland:Springer, 2019.

Quantum Time Evolution

Solution to the differential equation:

$$\mathbf{p}(t) = e^{Ht}\mathbf{p}(0)$$

Unitary Time Evolution Operator:

$$U(t) = e^{\;iHt}$$

[1] J. D. Hidary, "Quantum Walks" in Quantum Computing: An Applied Approach, Cham, Switzerland:Springer, 2019.

Discrete-Time Quantum Walk

- Two operations are applied to a given position

$$|\psi(0)\rangle = |0\rangle|n = 0\rangle$$

- Coin

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- Shift

$$S|0\rangle|n\rangle = |0\rangle|n + 1\rangle,$$

$$S|1\rangle|n\rangle = |1\rangle|n - 1\rangle.$$

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Example

U^t applied once

$$|0\rangle \otimes |0\rangle \xrightarrow{H \otimes I} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle$$

$$\xrightarrow{S} \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |1\rangle + |1\rangle \otimes |-1\rangle\big)$$

Generalized

$$|\psi(t)\rangle = U^t|\psi(0)\rangle$$

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Example

Up to the 3rd case

$$
\begin{aligned}
|\psi(1)\rangle &= \frac{1}{\sqrt{2}}\big(|1\rangle|{-1}\rangle + |0\rangle|1\rangle\big),\\
|\psi(2)\rangle &= \frac{1}{2}\Big(- |1\rangle|{-2}\rangle + (|0\rangle + |1\rangle)|0\rangle + |0\rangle|2\rangle\Big),\\
|\psi(3)\rangle &= \frac{1}{2\sqrt{2}}\Big(|1\rangle|{-3}\rangle - |0\rangle|{-1}\rangle + (2|0\rangle + |1\rangle)|1\rangle + |0\rangle|3\rangle\Big)
\end{aligned}
$$

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Probability Distributions

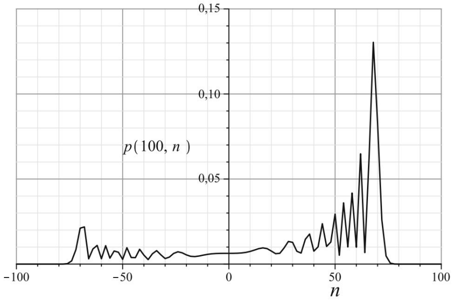


Fig. 3.4 Probability distribution after 100 steps of a quantum walk with the Hadamard coin starting from the initial condition $|\psi(0)\rangle = |0\rangle|n = 0\rangle$. The points where the probability is zero were excluded (n odd)

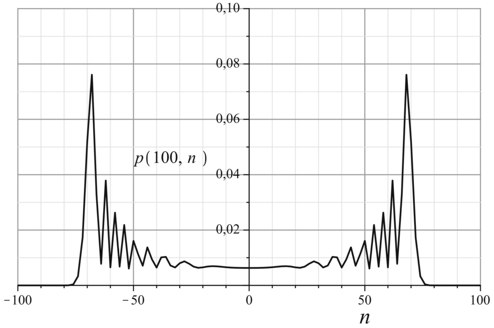


Fig. 3.5 Probability distribution after 100 steps of a Hadamard quantum walk starting from the initial condition (3.23)

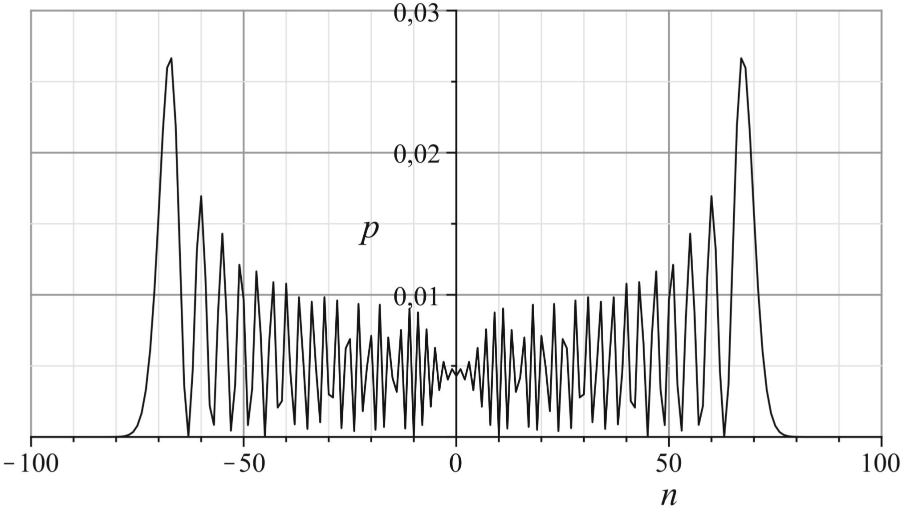
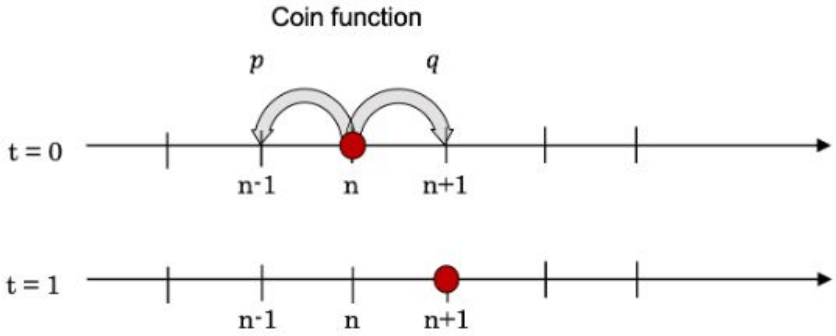


Fig. 3.7 Probability distribution at $t = 100$ with $\gamma = \left(2\sqrt{2}\right)^{-1}$ of a continuous-time quantum walk with initial condition $|\psi(0)\rangle = |0\rangle$

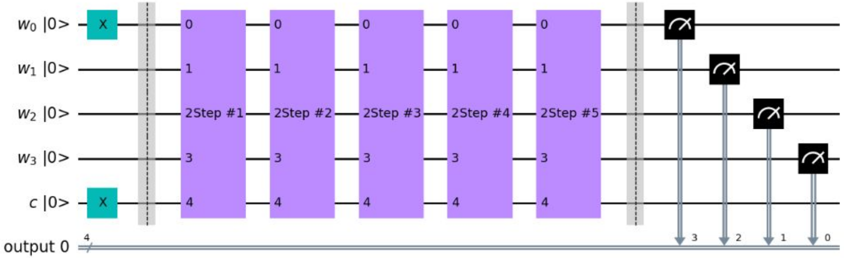
[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

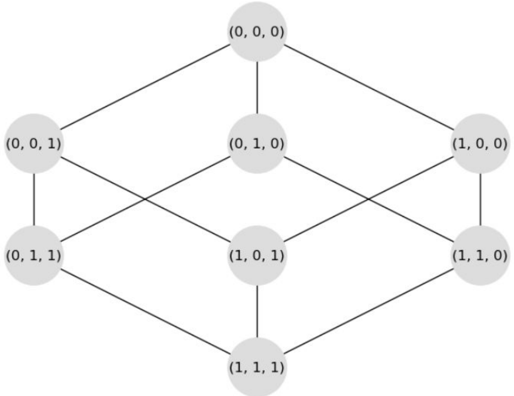
Implementation

Qiskit Community 1-D QW



QW through a hypercube





[4] TendTo. Quantum-random-walk-simulation. GitHub, https://github.com/TendTo/Quantum-random-walk simulation.
[5] Qiskit Community. Quantum walk. GitHub, https://github.com/qiskit-community/qiskit-communitytutorials/blob/master/terra/qis_adv/quantum_walk.ipynb.

Expected Results & Take-Home Message

- Expecting similar or high cut values then classical approach in [7]
- Evidence supporting quantum walk viability

References

- [1] J. D. Hidary, "Quantum Walks" in Quantum Computing: An Applied Approach, Cham, Switzerland:Springer, 2019.
- [2] A. Jin and X. -Y. Liu, "A Fast Machine Learning Algorithm for the MaxCut Problem," 2023 IEEE MIT Undergraduate Research Technology Conference (URTC), Cambridge, MA, USA, 2023, pp. 1-5, doi: 10.1109/URTC60662.2023.10534996.
- [3] Y. Aharonov, L. Davidovich, and N. Zagury. Quantum random walks. Physical Review A, 48(2):1687–1690, Aug 1993. https://journals.aps.org/pra/pdf/10.1103/PhysRevA.48.1687.
- [4] TendTo. Quantum-random-walk-simulation. GitHub, https://github.com/TendTo/Quantum-random-walk simulation.
- [5] Qiskit Community. Quantum walk. GitHub, https://github.com/qiskit-community/qiskit-communitytutorials/blob/master/terra/qis_adv/quantum_walk.ipynb.
- [6] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3
- [7] zhumingpassional, X. -Y. Liu, GitHub, https://github.com/zhumingpassional/Maxcut_CSCI
- [8] Lucas A (2014) Ising formulations of many NP problems. Front. Physics 2:5. doi: 10.3389/fphy.2014.00005.
- [9] Kempe, J. (2003). Quantum random walks - an introductory overview. ArXiv. https://doi.org/10.1080/00107151031000110776

Other References

- https://vixra.org/pdf/1909.0131v1.pdf
- https://lucaman99.github.io/blog/2019/08/03/Quantum-Random-Walks.html