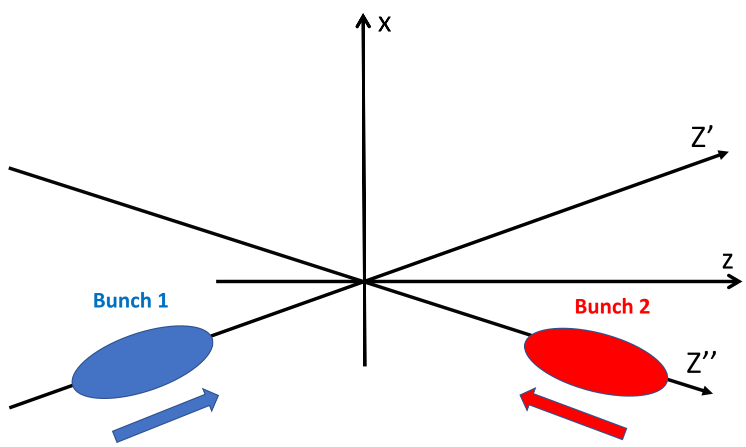
Conventions

- The bunch labelled 1 travels along the (positive) z' axis direction, while the bunch labelled 2 moves along the negative z'' direction, as depicted in the Figure. Both bunches have a positive velocity along the x axis, which points outwards of the ring.
- The time $t = 0$ corresponds to the time when the centre of both bunches is at the nominal interaction point ($x = z = 0$). Negative (positive) times correspond to what happens before (after) the collision. At a given time t, the bunch #1 is centered around $z' = ct$, while the bunch #2 is centered around $z'' = -ct$.
- α denotes the half crossing angle: $\alpha = +15$mrad.
- The axes x' and x'' (not shown) are in the (x, z) plane, orthogonal to z' and z'', respectively, and their direction is defined by $z' \wedge x' = z'' \wedge x'' = z \wedge x$
- The particles in the bunches are supposed to be distributed according to a 3-dimensional Gaussian distribution. For the bunch #1 for example, the bunch "sizes" σ_x, σ_y and σ_z denote the width of this Gaussian in the x', y and z' direction.

The 4-dimensional vertex distribution in (x, y, z, t) is obtained from the overlap of the two Gaussian distributions that describe the bunches.



Overlap of the two Gaussian distributions

$$
\begin{aligned}
f(X,Y,Z,t) \quad \propto \quad & \delta(x_1 = x_2 = X, y_1 = y_2 = Y, z_1 = z_2 = Z) \\
& \times \exp\left(-\frac{(z_1' - ct)^2}{2\sigma_z^2}\right) \exp\left(-\frac{{x_1'}^2}{2\sigma_x^2}\right) \exp\left(-\frac{y_1^2}{2\sigma_y^2}\right) \\
& \times \exp\left(-\frac{(z_2'' + ct)^2}{2\sigma_z^2}\right) \exp\left(-\frac{{x_2''}^2}{2\sigma_x^2}\right) \exp\left(-\frac{y_2^2}{2\sigma_y^2}\right)
\end{aligned}
$$

with

$$
\begin{aligned}
x' &= x\cos\alpha - z\sin\alpha, \quad x'' = x\cos\alpha + z\sin\alpha \\
z' &= z\cos\alpha + x\sin\alpha, \quad z'' = z\cos\alpha - x\sin\alpha
\end{aligned}
$$

One obtains:

$$
\begin{aligned}
f(X,Y,Z,t) \quad \propto \quad & \exp\left[-Z^2\left(\frac{\sin^2\alpha}{\sigma_x^2} + \frac{\cos^2\alpha}{\sigma_z^2}\right)\right] \\
& \times \exp\left[-X^2\left(\frac{\cos^2\alpha}{\sigma_x^2} + \frac{\sin^2\alpha}{\sigma_z^2}\right)\right] \\
& \times \exp\left(-t^2\frac{c^2}{\sigma_z^2}\right) \exp\left(Xt\frac{2c\sin\alpha}{\sigma_z^2}\right) \\
& \times \exp\left(-\frac{Y^2}{\sigma_y^2}\right)
\end{aligned}
$$

The distributions along the y and z directions are Gaussian, centered around the origin, of width

$$
\boxed{
\begin{aligned}
\sigma_{Y_{vtx}} &= \sigma_y/\sqrt{2} \\
\sigma_{Z_{vtx}} &= 1\Big/\sqrt{2\left(\frac{\cos^2\alpha}{\sigma_z^2} + \frac{\sin^2\alpha}{\sigma_x^2}\right)}
\end{aligned}
}
$$

The variables X and ct show a small correlation. Under a rotation of the variables (X, ct) by an angle θ, given by

$$
\tan 2\theta = 2\frac{\sin\alpha}{\cos^2\alpha}\frac{\sigma_x^2}{\sigma_z^2 - \sigma_x^2}
$$

the rotated variables are distributed according to the product of two Gaussian functions. With σ_z of a few mm, σ_x of a few and $\sin\alpha = 15\times10^{-3}$, this angle is tiny and the correlation between X and ct can be neglected. The distributions of the vertex along the x direction and along the time variable are then given by Gaussian functions, of width

$$\boxed{\begin{array}{rcl} \sigma_{X_{vtx}} & = & 1/\sqrt{2\left(\frac{\cos^2\alpha}{\sigma_x^2}+\frac{\sin^2\alpha}{\sigma_z^2}\right)} \simeq \sigma_x/\sqrt{2} \\ \sigma_t & = & \sigma_z/(c\sqrt{2}) \end{array}}$$