计算（写出计算过程）

1. $\int e^{\sqrt{1+x}}dx$

解 令 $t=\sqrt{1+x}$，则 $x=t^2-1, dx=2tdt$，代入所求积分并由分部积分法得

$$\int e^{\sqrt{1+x}}dx=2\int te^t dt=2\int tde^t=2(te^t-\int e^t dt)=2te^t-2e^t+C=2e^{\sqrt{1+x}}(\sqrt{1+x}-1)+C$$

2. $\int \frac{dx}{\sin x+\tan x}$

解 令 $t=\tan\frac{x}{2}$，则 $\sin x=\frac{2t}{1+t^2},\cos x=\frac{1-t^2}{1+t^2},dx=\frac{2dt}{1+t^2}$，代入所求积分转化为有理函数积分，得 $\int \frac{dx}{\sin x+\tan x}=\int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+\frac{2t}{1-t^2}}=\int \frac{1-t^2}{t(1-t^2+1+t^2)}dt$

$$=\frac{1}{2}(\int \frac{dt}{t}-\int tdt)=\frac{1}{2}(\ln|t|-\frac{1}{2}t^2)+C$$

$$=\frac{1}{2}\ln\left|\tan\frac{x}{2}\right|-\frac{1}{4}\tan^2\frac{x}{2}+C$$

3. $\int x\ln(1+x^2)dx$

解　由第一换元法（凑微法）和分部积分法得

$$\int x\ln(1+x^2)dx=\int \ln(1+x^2)d\frac{x^2}{2}=\frac{1}{2}\int \ln(1+x^2)d(1+x^2)\underset{=\!=\!=\!=\!=}{u=1+x^2}\frac{1}{2}\int \ln udu$$

$$=\frac{1}{2}(u\ln u-\int ud(\ln u))=\frac{1}{2}(u\ln u-\int 1du)=\frac{1}{2}[(1+x^2)\ln(1+x^2)-(1+x^2)]+C$$ 。

4. $\int \arctan(\sqrt{x})dx$

解　由第二换元法和分部积分法得

$$\int \arctan(\sqrt{x})dx\underset{=\!=\!=\!=\!=}{u=\sqrt{x}}2\int u\arctan udu==\int \arctan ud(u^2)$$

$$=u^2\arctan u-\int \frac{u^2}{1+u^2}du=u^2\arctan u-\int \frac{1+u^2-1}{1+u^2}du=u^2\arctan u-\int du+\int \frac{1}{1+u^2}du$$

$$=u^2\arctan u-u+\arctan u+C=x\arctan\sqrt{x}-\sqrt{x}+\arctan\sqrt{x}+C$$ 。

5. $\int \frac{dx}{(x+1)(x^2+1)}$。

解 根据有理函数积分法，设 $\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$，通分得

$1 = A(x^2+1) + (Bx+C)(x+1) = (A+B)x^2 + (B+C)x + (A+C)$，比较同次幂系数得

$\begin{cases} A+B=0 \\ B+C=0 \\ A+C=1 \end{cases}$，解得 $A = C = \frac{1}{2}, B = -\frac{1}{2}$，于是

$$\int \frac{dx}{(x+1)(x^2+1)} = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{x-1}{x^2+1}dx = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{2}(\int \frac{x}{x^2+1}dx - \int \frac{1}{x^2+1}dx)$$

$$= \frac{1}{2}\ln|x+1| - \frac{1}{4}\int \frac{d(1+x^2)}{1+x^2} + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln(1+x^2) + \frac{1}{2}\arctan x + C$$