

$$\int \sqrt{2x-x^2} dx = \int_0^2 x d\sqrt{2x-x^2}$$

$$21. = \int_x^a e^{-t^2} dt + \int_a^{x^2} e^{-t^2} dt$$

一. 填空

$$1. \int_0^2 \sqrt{2x-x^2} dx = -\frac{4}{3}\sqrt{2}.$$

$$= -\int_a^x e^{-t^2} dt + \int_a^{x^2} e^{-t^2} dt$$

$$2. (1) \text{ 若 } f(x) = \int_x^{x^2} e^{-t^2} dt, \text{ 则 } f'(x) = e^{-x^4} - e^{-x^2}.$$

$$= -e^{-x^2} + e^{-x^4}$$

$$(2) \text{ 若 } f(x) = \int_0^x x f(t) dt, \text{ 则 } f'(x) = x f(x) + \int_0^x f(t) dt$$

$$\int_0^x f(t) dt + x f(x)$$

$$3. \text{ 对连续函数 } f(x), \text{ 若 } f(x) = x + 3 \int_0^1 f(t) dt, \text{ 则函数 } f(x) = x - \frac{3}{4}.$$

$$4. \lim_{n \rightarrow \infty} \int_0^{\frac{1}{2}} \frac{x^n}{\sqrt{1+x}} dx = \frac{1}{4}.$$

$$\int_0^1 f(t) dt = a$$

$$a = \frac{1}{2} t^2 + 3at \Big|_0^1$$

$$a = \int_0^1 t + 3a dt$$

$$a = \frac{1}{2} + 3a$$

$$a = -\frac{1}{4}$$

二. 选择题

$$1. \text{ 设 } f(x) = \int_0^{\sin x} t^2 dt, g(x) = x^3 + x^4, \text{ 则当 } x \rightarrow 0 \text{ 时, } f(x) \text{ 是 } g(x) \text{ 的 ()}$$

A. 等价无穷小 B. 同阶但非等价的无穷小 C. 高阶无穷小 D. 低阶无穷小

三. 计算 (写出计算过程)

$$1. \text{ 设 } f(x) = \begin{cases} x^2, & x < 0, \\ e^{x+1}, & x \geq 0 \end{cases}, \text{ 求 } \int_0^2 f(x-1) dx.$$

$$2. \int_0^{\frac{\pi}{3}} x \sin x dx.$$

$$3. \text{ 求由参数方程 } \begin{cases} x = \int_0^t \sin u^2 du \\ y = \int_0^t \cos u^2 du \end{cases} \text{ 确定的函数 } y = y(x) \text{ 的导数 } \frac{dy}{dx}.$$

4. 求 $\lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt}{x^2}$

四. 证明题

1. 证明 对常数 $a > 0$, 有 $\int_0^a x^3 f(x^2) dx = \frac{1}{2} \int_0^{a^2} xf(x) dx$.