Duration : 3 days (Friday 10:00 pm to Monday 10:00 pm)

1. Using only the 6 roman numerals (I, V, X, L, C, M) and the roman number system we can represent _

A. All numbers from 0 to infinity
B. All numbers from 1 to infinity
C. All numbers from 0 to 3999
D. All numbers from 1 to 3999

Ans. D
Explanation. Because of the rule that we cannot repeat a symbol more than twice, and the structure of the roman numeral system, we can only have finitely many values using the 6 symbols and 0 cannot be represented.

2. How many numbers are there whose representation in base 2 and base 3 are the same? For example the number "zero" is represented as "0" in base 2 and base 3.

Note: A number in base x is the representation of the number using the symbols 0, 1, … x-1 with the order 0 < 1 < … x-1.

A. Only 1 (i.e. the number "0")
B. Only 2
C. Some finite number
D. Infinite

Ans. B
Explanation. Only 0 and 1 satisfy this criteria, in all other cases, any representation in base 3 will be strictly greater than corresponding representation in base 2.

3. A number is represented as "38586" in base "x". Which of the following statements is true?

Note: A number in base x is the representation of the number using the symbols 0, 1, … x-1 with the order 0 < 1 < … x-1.

A. x can be 7 or 8
B. x can be 8 or 9
C. x can be 9 or 10
D. x can only be 10

Ans. C
Explanation. As long as the digits have value less than x, the number will be valid in base x. x must be at least 9. So C is the correct answer.

4. The number A in binary is “10101” and B in binary is “11010”. The value of A+B is

Note: A number in base x is the representation of the number using the symbols 0, 1, … x-1 with the order 0 < 1 < … x-1.

A. “101111” in binary, “1211” in base 3
B. “111111” in binary, “1202” in base 3
C. “101111” in binary, “1202” in base 3
D. “111111” in binary, “1211” in base 3

Ans. C
Explanation. A = 21, B = 26, A + B = 47
47 can be broken down as 32 + **15** = 32 + **8 + 4 + 2 + 1** = (101111)
47 in base 3 = 27 (3^3) + 20
= **1*** 27 (3^3) + **2** * 9 (3^2) + **0*** (3^1) + **2** *1 (3^0)
= (1202)

5. The decimal number 20 is same as

A. (10100) in base 2
B. (202) in base 3
C. (40) in base 5
D. All of the above

Ans. D
Explanation. 20 = 16 + 4 = 10100 in base 2
20 = **2** * 3^2 + **2** = 202 in base 3
20 = **4** * 5^1 + **0** = 40 in base 5

6. If a positive number has exactly “x” zeros at the end in its binary representation, then what is the largest power of two that is a factor of the number?

For example: the number “four” has two zeros at the end because it is written as “100” in binary.

A. x/2
B. x
C. x*2
D. x-2

Ans. B
Explanation. In the way we obtain the binary representation, we divide the number by 2 and note down the remainder. If we have remainder 0 x times, then the number is divisible by 2^x.

For example, the number six is written as “11**0**” in binary. It has one zero at the end, so x =1.

The largest power of 2 that is a factor of the number is **1** as $2^1 = 2$. (2^2 or higher is not a factor)

7. If there are exactly “x” zeros at the end of a number “A” in binary and “y” zeros at the end of a number “B” in binary, and A is divisible by B, then which of the following MUST be true?

A. x is greater than or equal to y
B. x is less than or equal to y
C. x is divisible by y
D. y is divisible by x

Ans. A
Explanation. Every multiple of B in base 2 will have at least the number of zeros of B in it. If A is a multiple of B then A must have an equal or more number of zeros than B in it.

8. Which of the following statements are true?
Note: A is a boolean variable (i.e. a variable which is either 0 or 1).

A. A XOR A = 0
B. A OR A = A
C. A AND A = A
D. All of the above

Ans. D
Explanation. We can verify them using truth tables.

9. The bitwise AND, XOR and OR operators are similar to their binary counterparts, except that they can be done on arbitrary integers.

The bitwise AND of two numbers A and B can be obtained by representing them in binary and calculating the AND of the corresponding bits and obtaining the result.
For example, 2 AND 3 = (10) AND (11) = (10) = 2 (because 1 AND 1 = 1 and 0 AND 1 = 0).
Similarly 2 OR 3 = (10) OR (11) = (11) = 3 (because 1 AND 1 = 0 and 0 AND 1 = 0)
Similarly 2 XOR 3 = (10) XOR (11) = (01) = 1 (because 1 XOR 1 = 0 and 0 XOR 1 = 1)

If the bitwise XOR of two numbers A and B is 9, then their bitwise AND can be

A. 2
B. 4
C. 6
D. All of the above

Ans. D
Explanation. 9 in binary is (1001). If XOR of two binary values is 1 then one of them is 0 and the other is 1 which implies that their AND is 0. So the A AND B must have 0 in the first and

fourth places. The remaining bits can be either 0 (when both the corresponding bits in A and B are 0) or 1 (when both the corresponding bits in A and B are 1).

So 2, 4, 6 are all possible options.
An example for 2 : A = (0010) = 2, B = (1011) = 11
An example for 4 : A = (0100) = 4, B = (1101) = 13
An example for 6 : A = (0110) = 6, B = (1111) = 15

10. Which of the following are equal to A + B ? (where OR, AND and XOR are the **bitwise operators** and A, B are non-negative integers)

A. (A XOR B) + 2 * (A AND B)
B. (A OR B) + (A AND B)
C. Both A and B
D. Neither A nor B

Ans. C
Explanation. A XOR B does the sum without carry. A AND B indicates the positions where carry should be propagated forward. So we can write A + B as (A XOR B) + 2 * (A AND B) where the multiplication by 2 results in shifting the positions to the next place and hence performing the carry.

A OR B is similar to A + B except in positions where both the corresponding bits are equal to 1. In this case a “1” should be carried forward. A AND B have “1” in such positions. In A XOR B, these positions will be 0 but in A OR B, they will be 1. So it is enough to add A AND B once and the result is same as the one above ((A XOR B) + 2 * (A AND B)).

You can also verify both by trial and error.