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LA assignment questions

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Section : A

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Q1) Find the equations of the parabola $y = A + Bx + Cx^2$ that passes through 3 point $(1,1)$, $(2,-1)$ and $(3,1)$ using gaussian elimination.

Q2) Find the LU decomposition for the matrix

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

Q3) Let $T: R^3 \to R^3$ defined by
$$T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$$

(i) Find the matrix T relative to the standard basis of R^3.

(ii) Find the basis for 4 fundamental subspaces of T.

(iii) Find the eigen values and eigen vectors of T

(iv) Decompose $T = QR$

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Q4) Fit a best straight line $y = c + dx$ for the following data using least square principle.

x	-4	1	2	3
y	4	6	10	8

Q5) Find the projection matrices P and Q onto the plane $x_1 + x_2 + 3x_3 + 4x_5 = 0$ and its orthogonal complement respectively.

Q6) For which range of number 'a', the matrix A is positive definite?

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

Which 3x3 matrix (symmetric) B produces these function. $f = x^T A x$

Where $f = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3)$

Q7) Find the SVD of A, $U \Sigma V^T$ where

$$A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$$

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Ans 1) $y = A + Bx + Cx^2$

$1 = A + B + C \quad ; \quad (1,1)$

$-1 = A + 2B + 4C \quad ; \quad (2,-1)$

$1 = A + 3B + 9C \quad ; \quad (3,1)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{array}\right] \quad \begin{aligned} R_2 &= R_2 - R_1 \\ R_3 &= R_3 - R_1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 1 \\ 0 & \boxed{1} & 3 & -2 \\ 0 & 0 & \boxed{2} & 4 \end{array}\right] \quad R_3 = R_3 - 2R_2$$

$2C = 4 \quad ; \quad C = 2$

$B + 3C = -2 \quad ; \quad B = -8$

$A + B + C = 1 \quad ; \quad A = 7$

$\therefore \quad y = \underline{\underline{7 - 8x}} + 2x^2$

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Ans 2)

$$A = \begin{bmatrix} \boxed{2} & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & \boxed{2} & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix} \quad \begin{aligned} R_2 &= R_2 - 2R_1 \\ R_3 &= R_3 + 5R_1 \\ R_4 &= R_4 - 5R_1 \end{aligned}$$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & \boxed{3} & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix} \quad \begin{aligned} R_3 &= R_3 + 2R_2 \\ R_4 &= R_4 + 2R_2 \end{aligned}$$

$$U = \begin{bmatrix} \boxed{2} & 5 & 2 & -5 \\ 0 & \boxed{2} & -1 & -4 \\ 0 & 0 & \boxed{3} & 5 \\ 0 & 0 & 0 & \boxed{-4} \end{bmatrix} \quad R_4 = R_4 - 3R_3$$

$$U = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

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Ans 3) $T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$

(i) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

Standard basis of $R^3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$T(1,0,0) \Rightarrow (1,0,1)$
$T(0,1,0) \Rightarrow (2,1,1)$
$T(0,0,1) \Rightarrow (-1,1,-2)$

(ii) $\left[\begin{array}{ccc|c} \boxed{1} & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & 2 & b_3 \end{array}\right]$

$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & \boxed{1} & 1 & b_2 \\ 0 & -1 & -1 & b_3 - b_2 \end{array}\right]$ $R_3 = R_3 - R_1$

$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_3 - b_1 + b_2 \end{array}\right]$ $R_3 = R_3 + R_2$

$\boxed{\text{Rank} = \rho(T) = 2}$

Four fundamental spaces are,

$C(T) = \{(1,0,1), (2,1,1)\}$
$C(T') = \{(1,2,-1), (0,1,1)\}$
$N(T^T) = (-1,1,1) \longrightarrow b_3 - b_1 + b_2$
$N(T) = (3,-1,1) \longrightarrow y = b_2 - z\ ;\ x = b_1 - 2b_2 + 3z$

$z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$

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(iii) $|T - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix} = 0$$

$= (1-\lambda)\left[(1-\lambda)(-2-\lambda)-1\right] - 2\left[0-1\right) - 1(0-1+\lambda)$

$= (1-\lambda)^2(-2-\lambda) \cancel{-1} + \cancel{\lambda} + 2 \cancel{+1} - \cancel{\lambda}$

$= (\lambda^2+1-2\lambda)(-2-\lambda) + 2$

$= -\cancel{2\lambda^2} - \lambda^3 - \cancel{2} - \lambda + 4\lambda + \cancel{2\lambda^2} + \cancel{2}$

$= -\lambda^3 + 3\lambda = 0$

$\lambda = 0, \sqrt{3}, -\sqrt{3}$ $\Rightarrow$ Eigen Values

$|T - \lambda I|[x] = 0$

$\lambda = \sqrt{3}$

$$\begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 1 & 1 & -3.73 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(x, y, z) = (0, 0, 0)$

$\lambda = -\sqrt{3}$

$$\begin{bmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 1 & 1 & -0.2732 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(x, y, z) = (0, 0, 0)$

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$\lambda = \sqrt{3}$ General solution

$$\left[\begin{array}{ccc|c} -\sqrt{3}+1 & 2 & -1 & 0 \\ 0 & -\sqrt{3}+1 & 1 & 0 \\ 1 & 1 & -\sqrt{3}-2 & 0 \end{array}\right] \times \left(\frac{-\sqrt{3}-1}{2}\right)$$

$$= \left[\begin{array}{ccc|c} 1 & -\sqrt{3}-1 & (\sqrt{3}+1)/2 & 0 \\ 0 & -\sqrt{3}+1 & 1 & 0 \\ 1 & 1 & -\sqrt{3}-2 & 0 \end{array}\right] \quad R_1 \to \frac{R_1}{-\sqrt{3}+1}$$

$R_3 = R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & -\sqrt{3}-1 & (\sqrt{3}+1)/2 & 0 \\ 0 & -\sqrt{3}+1 & 1 & 0 \\ 0 & \sqrt{3}+2 & (-3\times\sqrt{3}-5)/2 & 0 \end{array}\right] \times \left(\frac{-\sqrt{3}-1}{2}\right)$$

$R_2 = R_2 / (-\sqrt{3}+1)$

$$\left[\begin{array}{ccc|c} 1 & -\sqrt{3}-1 & (\sqrt{3}+1)/2 & 0 \\ 0 & 1 & (-\sqrt{3}-1)/2 & 0 \\ 0 & \sqrt{3}+2 & (-3\times\sqrt{3}-5)/2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & (-\sqrt{3}-3)/2 & 0 \\ 0 & 1 & (-\sqrt{3}-1)/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1 = R_1 - (-\sqrt{3}-1) \times R_2$$

$$x_1 = \frac{\sqrt{3}+3}{2} x_3$$

$$x_2 = \frac{\sqrt{3}+1}{2} x_3$$

General solution $= x_3 \begin{bmatrix} (\sqrt{3}+3)/2 \\ (\sqrt{3}+1)/2 \\ 1 \end{bmatrix}$

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General Solution

$\lambda = -\sqrt{3}$

$$\left[\begin{array}{ccc|c} \sqrt{3}+1 & 2 & -1 & 0 \\ 0 & \sqrt{3}+1 & 1 & 0 \\ 1 & 1 & \sqrt{3}-2 & 0 \end{array}\right] \times \frac{\sqrt{3}-1}{2}$$

$R_1 = R_1 / (\sqrt{3}+1)$

$$\left[\begin{array}{ccc|c} 1 & \sqrt{3}-1 & (-\sqrt{3}+1)/2 & 0 \\ 0 & \sqrt{3}+1 & 1 & 0 \\ 1 & 1 & \sqrt{3}-2 & 0 \end{array}\right]$$

$R_3 = R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & \sqrt{3}-1 & (-\sqrt{3}+1)/2 & 0 \\ 0 & \sqrt{3}+1 & 1 & 0 \\ 0 & -\sqrt{3}+2 & \frac{3\times\sqrt{3}-5}{2} & 0 \end{array}\right] \times \frac{\sqrt{3}-1}{2}$$

$R_2 = \frac{R_2}{\sqrt{3}+1}$

$$\left[\begin{array}{ccc|c} 1 & \sqrt{3}-1 & (-\sqrt{3}+1)/2 & 0 \\ 0 & 1 & (\sqrt{3}-1)/2 & 0 \\ 0 & -\sqrt{3}+2 & (3\times\sqrt{3}-5)/2 & 0 \end{array}\right]$$

$R_3 = R_3 - (-\sqrt{3}+2) \times R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & (\sqrt{3}-3)/2 & 0 \\ 0 & 1 & (\sqrt{3}-1)/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \text{General Solution} \begin{bmatrix} \frac{-\sqrt{3}+3}{2} \\ \frac{-\sqrt{3}+1}{2} \\ 1 \end{bmatrix}$$

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$\lambda = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(x, y, z) = (3, -1, 1)$$

After gaussian elimi

$x_1 = 3x_3$

$x_2 = -x_3$

$x_3 = x_3$

$$x_3 \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

(iv) $T = QR$

$$a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$q_1 = \frac{a}{\|a\|} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$q_2 = \frac{b}{\|b\|}$$

$$b = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3/2 \times 1 \\ 0 \times 0 \\ 3/2 \times 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \end{bmatrix}$$

$$q_2 = \begin{bmatrix} 1/2 \,/\, \sqrt{3}/\sqrt{2} \\ 1 \,/\, \sqrt{3}/\sqrt{2} \\ -1/2 \,/\, \sqrt{3}/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} \\ \sqrt{2}/\sqrt{3} \\ -1/\sqrt{6} \end{bmatrix}$$

$$q_3 = \frac{c}{\|c\|} \Rightarrow c = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 3/2 \times 1 \\ 0 \\ 3/2 \times -1 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix}$$

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$T = QR$

$R = Q^T . T$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}$$

Ans 4)

x	-4	1	2	3
y	4	6	10	8

$y = c + dx$

$$\underset{A}{\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{\hat{x}}{\begin{bmatrix} c \\ d \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}}$$

$$\hat{x} = (A^T A)^{-1} A^T . B$$

$$A^T . A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}$$

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$$A^T \cdot B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 30/116 & -2/116 \\ -2/116 & 4/116 \end{bmatrix} \begin{bmatrix} 28 \\ 39 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 772/116 \\ 60/116 \end{bmatrix}$$

Ans 5) $x_1 + x_2 + 3x_3 + 4x_4 = 0$

$$\begin{bmatrix} \boxed{1} & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Rightarrow \underset{x_1}{a} = -\underset{x_2}{b} - 3\underset{x_3}{c} - 4\underset{x_4}{d}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = b\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Projection = $A \cdot (A^T \cdot A)^{-1} A^T$

$$A^T \cdot A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 10 & 12 \\ 4 & 12 & 17 \end{bmatrix}$$

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$$(A^T \cdot A)^{-1} = \begin{bmatrix} 26/27 & -1/4 & -4/27 \\ -1/9 & 2/3 & -4/9 \\ -4/27 & -4/9 & 11/27 \end{bmatrix}$$

$$A \cdot (A^T \cdot A)^{-1} = \begin{bmatrix} -1/27 & -1/9 & -4/27 \\ 26/27 & -1/9 & -4/27 \\ -1/9 & 2/3 & -4/9 \\ -4/27 & -4/9 & 11/27 \end{bmatrix}$$

P~~rojection~~

$$Proj_V = A(A^T \cdot A)^{-1} A^T = \begin{bmatrix} 26/27 & -1/27 & -1/9 & -4/27 \\ -1/27 & 26/27 & -1/9 & -4/27 \\ -1/9 & -3/27 & 6/9 & -12/27 \\ -4/27 & -4/27 & 4/9 & 11/27 \end{bmatrix}$$

$$I = Proj_V + Proj_{V\perp}$$

$$\text{Projection } V^{\perp} = \begin{bmatrix} 1/27 & 1/27 & 1/9 & 4/27 \\ 1/27 & 1/27 & 1/9 & 4/27 \\ 1/9 & 3/27 & 3/9 & 12/27 \\ 4/27 & 4/27 & 4/9 & 16/27 \end{bmatrix}$$

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Ans5) Considering the question had no mistake and it was x_5 then; the matrix is 1×5 (since its a 5D plane)

$$Q = A(A^T \cdot A)^{-1} \cdot A^T$$

$$A = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix} \left(\begin{bmatrix} 1 & 1 & 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 3 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/27 \\ 1/27 \\ 3/27 \\ 0 \\ 4/27 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 0 & 4 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/27 & 1/27 & 3/27 & 0 & 4/27 \\ 1/27 & 1/27 & 3/27 & 0 & 4/27 \\ 3/27 & 3/27 & 1/3 & 0 & 12/27 \\ 0 & 0 & 0 & 0 & 0 \\ 4/27 & 4/27 & 12/27 & 0 & 16/27 \end{bmatrix}$$

$$P = I - Q \Rightarrow P = \begin{bmatrix} 26/27 & -1/27 & -3/27 & 0 & -4/27 \\ -1/27 & 26/27 & -3/27 & 0 & -4/27 \\ -3/27 & -3/27 & 18/27 & 0 & -12/27 \\ 0 & 0 & 0 & 1 & 0 \\ -4/27 & -4/27 & -12/27 & 0 & 11/27 \end{bmatrix}$$

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Ans 6) (i) $\begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix} \approx \begin{bmatrix} a & 2 & 2 \\ 0 & (a^2-4)/a & (2a-4)/a \\ 0 & (2a-4)/a & (a^2-4)/a \end{bmatrix}$

$a > 0 \quad , \quad \frac{a^2-4}{a} > 0$

$a > 2$

$a(a^2-4) - 2(2a-4) + 2(4-2a)$

$a^3 - 12a + 16 > 0$

$(a+4)(a-2)(a-2) > 0$

$a > -4, \quad a > 2$

$-4 < a < \infty$

(ii) $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$= \begin{bmatrix} a_{11}x_1 + a_{21}x_2 + a_{31}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{32}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix}$

$a_{12} + a_{21} = -2$

$a_{31} + a_{13} = 0$

$a_{33} + a_{32} = -2$

Symmetric

$a_{12} = a_{21} = -1$

$a_{23} = a_{32} = -1$

$a_{31} = a_{13} = 0$

Required matrix $= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

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Ans 7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$A^T A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

Eigen Values $\Rightarrow |A^T A - \lambda I| = 0 \Rightarrow \lambda^2 - 90\lambda = 0$

$\Rightarrow (10-\lambda)((40-\lambda)^2 - 1600) + 20(-20 \times (40-\lambda) + 800) - 20(-800 + 20(40-\lambda)) = 0$

$\lambda = 0, \quad \lambda = 90$

Eigen Vectors $\Rightarrow x_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$v_1 = \dfrac{x_1}{\|x_1\|} = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$

Since x_1 and x_2 are orthogonal

$v_2 = \dfrac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$

$\therefore V = \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$ (check $V^T V = I$)

$\Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \\ 0 & 0 \end{bmatrix}$ where $\sqrt{\lambda_1} > \sqrt{\lambda_2}$

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~~[illegible]~~

$= \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ Where $\sigma_1 = \sqrt{\lambda_1}$, $\sigma_2 = \sqrt{\lambda_2}$

$\sigma_1 = \sqrt{90}$, $\sigma_2 = 0$

$U_1 = \dfrac{Av_1}{\sigma_1} = \begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix}$

U_2 can't be calculated by using above formulae since $\sigma_2 = 0$.

U_2 and V_3 are orthogonal vectors, w.r.t $\lambda = 0$

$(A \cdot A^T - 0 \cdot I)x = 0$

$$\begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$10x - 20y - 20z = 0$

$\Rightarrow x = 2y + 2z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Let $x_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $x_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

Clearly x_2 and x_3 are orthogonal to V_1

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~~Current Solution~~

Find U_2 and U_3 using Gram Schmidt process

$$U_2 = \frac{x_2}{\|x_2\|} \quad (x_2 \perp u_1)$$

$$\therefore \quad U_2 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$$

To find u_3,

$x_3 \perp$ to u_1 but not to u_2

$$C = x_3 - (u_1^T x_3)u_1 - (u_2^T x_3)u_2$$

$$= \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - (0)\begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix} - \begin{bmatrix} 2/\sqrt{5} & \frac{1}{\sqrt{5}} & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2/5 \\ -4/5 \\ 1 \end{bmatrix}$$

(where $C \perp u_1$, $C \perp U_2$)

$$\therefore \quad u_3 = \frac{C}{\|C\|} = \begin{bmatrix} 2/3\sqrt{5} \\ -4/3\sqrt{5} \\ \sqrt{5}/3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & \sqrt{5}/3 \end{bmatrix}$$

Hence, $\underline{U \Sigma V^T = A}$