PHYS225

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Contents

Chapter 1

Relativity and Math Applications

Dr. Koptieva is Ukrainian, does gravity research
    Venn Diagram Course Plan:
    Very intense, might die.
    Special Relativity   Symmetries and other junk
    OFFICE HOURS
    Wednesday 12:30pm-3:30pm in Loomis 276
    or email TA

Course Goals

- time dialation, length contraction, Doppler shifts

- the postulates of relativity and how they imply the Lorentz transformations

- 4-vectors and the spacetime metric

- relativistic kinematics, using invariance an energy-momentum

- more stuff

1.1 TEXTBOOKS

REQUIRED
   Special Relativity for Enthusiastic Beginners by David Morin
   RECOMMENDED
   Basic Training in Mathematics: A Fitness Program for Science Students by R. Shankar

1.2 Homework

Opens Thursday August 24th
   Due Thursday August 31st

1.3 Midterm

October 10th, in-class

Chapter 2

Introduction to Special Relativity

First written down in 1905 by Albert Einstein. Passed every experimental test for 100 years.

    OMP (One Minute Paper: Write down one topic you are excited to learn about in PHYS225:

    I know that gravity is the curvature of spacetime, but I don't actually know anything about it, to learning about spacetime curvature in a more in-depth manner sounds fun.)

    The speed of light is constant within all reference frames which means everything else gets goofy.

    Let someone throw a ball on a moving train

Regular Relativity (Galilean Relativity):

Newton's Laws of Motions are the same in all reference frames.

    Time is universal and equal for all observers

$$v_{ball} = v_{train} + v_{throw}$$

2.0.1   Special Relavity:

EXTREMELY IMPORTANT EQUATION

$$v_{ball} = \frac{v_{train} + v_{throw}}{1 + v_{train}v_{throw}/c^2}$$

Not much changes with small values, however, for large numbers, the equation creates an effective upper bound. No addition of velocities can ever reach the speed of light.

Electromagnets

Maxwell's Equations Lorentz Forces

$$F = qv \times B$$

These are wholly related to Einstein's special relativity

2.0.2   Einstein's Theory of Relativity

- All laws of physics are the same in all inertial reference frames

- The speed of light in a vacuum is constant in all inertial frames

velocities vary depending on reference frames.

OMP (One Minute Paper: What does "special" mean in special relativity?)

Read Morin 1.1-1.2, Appendix F 6.6.1 - 6.6.3

Chapter 3

The Three Fundamental Effects

OMP List one or two examples of systems that you know from PHYS211 that can be used s clocks. In other words, their motion is cyclical with a definite period.

Answer: gravity based swinging pendulum and spring pendulum

- define the concepts of reference frame, event, simultaneity, time, and length

- Use these concepts to see the three fundamental effects of relativity:

  Rear Clock Ahead

  Time Dialation

  Lorentz (length) Contraction.

3.0.1   goofy paradox

You travel at the speed of light and someone shines a laser at you. How fast is it moving?

$$r_{laser} = \frac{c - c}{1 - (c^2/c^2)} = 0/0$$

That illiegal, which shows how the speed of light is not a valid inertial reference frame. If $v < c$, then you will see that the laser's speed is in fact c.

3.1   Reference Frame

What is a reference frame?
　　Imagine a network of synchronized clocks.
　　Everyone is the same reference frame sees the same clock/time.

3.1.1   Event

A thing that happens at a specific time and a specific place.
　　The reading on the clock in S at the location of the event.

3.1.2   Simultaneity

Two events that happen in different locations in space, but at the same time.

3.2 How do we measure a time elapsed in S

We look at the same clock at 2 different times and observe the difference of the readings.

3.2.1 How do we measure length in S

We measure two locations in S at different times

3.2.2 Length given constant velocity

Observe velocity over time.

fuck

Events that are simultaneous in one reference frame are NOT necessarily simultaneous in another because of constant speed of light.

Imagine a light in the middle of two trains with clocks on either side of the train. to measure Δt, you do

$$t_1 - t_2 = \frac{L(c+v)}{2c^2} - L(c-v)2c^2 = \frac{Lv}{c^2} = \text{ difference in clocks time}$$

3.3 Time Dialation

How to measure time elapsed in S' reference frame.

$$\Delta t = \gamma \Delta \tau$$

Where t is the time elapsed in S and

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \text{Gamma/Lorentz/boost factor}$$

γ is always bigger than 1, which means time for someone with a velocity appears slower than someone who is stationary. But velocity is relative to reference frame, so who has slower time?

They both are when they have to check the clock????? I don't get it. Acceleration fucks up special relativity.

3.4 Length Contraction

$$\Delta t' = \frac{L}{v}$$

$$\Delta t' = \frac{L}{\gamma v}$$

Planck's something

$$L_A = v \Delta t = \frac{L}{\gamma}$$

OMP
Which of the fundamental effects was most surprising?

Chapter 4

Vectors and Matrices

OMP: Give an example of a mathematical tool that is useful in physics

Answer: sine and cosine functions with regards to harmonic motion.

- Vector Manipulation

- Matrix Manipulation

- How Linear Transformations are used in physics

4.1 Vectors

Object with magnitude and direction.

Can be decomposed into unit vectors

$$\vec{v} = v_1\hat{x} + v_2\hat{y} + v_3\hat{z}, \qquad \vec{v} = \langle v_1, v_2, v_3 \rangle$$

Vectors have no preferred origin.

Vector Functions

- addition

- scalar multiplication

- dot product

- cross product

4.2 Matrix

- Rectangle of numbers.

- Matrices allow us to transform vectors.

- consists of m rows and n columns

$$A\vec{x} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} av_1 + bv_2 \\ cv_1 + dv_2 \end{bmatrix}$$

4.2.1 Einstein Summation Notation

Kronecker Delta δ^i_j

$$\delta^{ij} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

0 if $i \neq j$. 1 if $i = j$
    write v = Mx

$$M = \begin{bmatrix} M^1_1 & M^2_1 \\ M^1_2 & M^2_2 \end{bmatrix}$$

$$v_i = M_i^j x_j \qquad v_1 = M_1^1 x_1 + M_1^2 x_2 \qquad v_1 = M_2^1 x_1 + M_2^2 x_2$$

Matrix is considered an operator to turn a vector into another vector.

dot product

$$\vec{A} \cdot \vec{B} = \sum_{i,j=1}^{2} \delta^{ij} A_i B_j$$

Chapter 5

Deriving Lorentz Transformations

OMP: Name at least one way the Galilean Transformations of Newtonian Mechanics must be modified to be consistent with relativity.

5.1 Two Important Postulates of Relativity

1. All physical laws must be the same in all reference frames S and S'.

2. The speed of light is constant in all reference frames.

   $$x = ct, x' = ct', \rightarrow x - ct = 0, x' - ct' = 0 \rightarrow x' - ct' = (x - ct)$$

   Holy Fucking Shit I Fucking Give Up I have no idea what is happening

Chapter 6

Matrix Multiplication and Taylor Series

OMP:

We often make approximations for small quantities, but "small" is a dimensionless concept. How would we define a "small' (dimensionless) velocity in relativity?

- Learn how to multiply two matrices

- Relate matrix multiplication to linear transformations

- other stuff I didn't get in time

For a matrix M and a vector v

$$Mv = \begin{bmatrix} av_1 + bv_2 \\ cv_1 + dv_2 \end{bmatrix}$$

Multiplying a 2×2 matrix is treating the second matrix as a bunch of vectors.

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} N = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$MN = \begin{pmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{pmatrix} \quad NM \begin{pmatrix} ap + cq & bp + dq \\ ar + cs & br + ds \end{pmatrix}$$

Einstein notation is actually very useful here but I didnt fUCK-ING WRITE IT IN TIME

$$MN = M_j^i N_i^j$$

imagine person S" with velocity v_2 on train S' with velocity v_1 on ground S

$$(x''^0, x'''^1) \xrightarrow{\Delta_v 1} (x'^0, x''^1) \xrightarrow{\Delta_v 2} (x^0, x^1)$$

$$\begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \Delta_{v1} \Delta_{v2} \begin{pmatrix} x''^0 \\ x''^1 \end{pmatrix}$$

In HW you will show how multiplying these two matrices gets you the velocity addition formula

$$\frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}$$

$$\begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} = \Delta_{v1} \begin{pmatrix} x''^0 \\ x''^1 \end{pmatrix} = \begin{pmatrix} \gamma_1 & \gamma_1 \beta_1 \\ \gamma_1 \beta_1 & \gamma_1 \end{pmatrix} \begin{pmatrix} x''^0 \\ x''^1 \end{pmatrix}$$

$$\begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \Delta_{v2} \begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} = \begin{pmatrix} \gamma_2 & \gamma_2 \beta_2 \\ \gamma_2 \beta_2 & \gamma_2 \end{pmatrix} \begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix}$$

$$\begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \begin{pmatrix} \gamma_2 & \gamma_2\beta_2 \\ \gamma_2\beta_2 & \gamma_2 \end{pmatrix} \begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} =$$

$$\begin{pmatrix} \gamma_2(\gamma_1 x''^0 + \gamma_1\beta_1 x''^1) + \gamma_2\beta_2(\gamma_1\beta_1 x''^0) + \gamma_1 x''^1) \\ \gamma_2\beta_2(\gamma_1 x''^0 + \gamma_1\beta_1 x''^1 + \gamma_2(\gamma_1\beta_1 x''^0 + \gamma_1''^1)) \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} (\gamma_2\gamma_1 + \gamma_2\beta_2\gamma_1\beta_1)x''^0 + (\gamma_2\gamma_1\beta_1 + \gamma_2\beta_2\gamma_1)x''^1 \\ (\gamma_2\beta_2\gamma_1 + \gamma_2\gamma_1\beta_1)x''^0 + (\gamma_2\beta_2\gamma_1\beta_1 + \gamma_2\gamma_1)x''^1+ \end{pmatrix}$$

6.0.1   EINSTEIN NOTATION

$$x'^\mu = (\Delta_{v1})^\mu_\nu x''^\nu$$

$$x^\sigma = (\Delta_{v2})^\sigma_\mu ((\Delta_{v1})^\mu_\nu x''^\nu)$$

6.1   Taylor Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \cdot = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Example

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

What about things that arent defined everywhere such that $\ln 0$.

$$\ln 1 + x = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} = \cdots$$

Goofy approximation

$\sin x = x$ for small angles x (error proportional to x^3)

Can multiply series together

$$\sin^2 x = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \right)^2 = x^2 - \frac{x^4}{3} + \frac{2x^6}{45} + \cdots$$

Common variations

$$\frac{1}{1+x} = \frac{d}{dx}\ln(1+x) = \frac{d}{dx}\text{sum} = 1 - x + x^2 + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$(1+x)^\alpha = x + \alpha x + \cdots \quad \text{binomial expansion}$$

OMP:

Differentiate the first two terms of the Taylor series for $\sin x$ to obtain the first two terms of the Taylor series for $\cos x$

Chapter 7

Symmetrics and Group Theory

OMP:
 How owuld you define the concept of symmetry?

- Raise and lower indices with contractions

- Learn the concept of transpose, inverse, and determinant

- Define an orthogonal matrix as one which preserves the dot product

- Define a mathematical group

- Show that Lorentz boosts along a single axis form a group.

7.1   Types of Vectors

Column vector:
$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Row vector:

$$\vec{r} = \begin{pmatrix} r_1 & r_2 \end{pmatrix}$$

Convention/Notation:

$$M^i_j \ i \text{ is column, } j \text{ is rows}$$

Dot product is really just the product of a row vector and a column vector via matrix multiplication.

Matrix multiplication in the form of dot products

$$w \cdot v = w_i v^i \qquad Mv = M^i_j v^j$$

Fancy way to write dot products

$$w \cdot v = \delta * ij w_i v_j = w_i(v_j \delta^{ij})$$

Theres some shenanigans about northeast that I missed.

can also raise the index on w

$$w \cdot v = \delta * ij w_i v_j = v_i(w_j \delta^{ij}) = w^j v_j$$

Other stuff

Dot product is commutative so $v \cdot w = w \cdot v$. and Kronecker delta is symmetric so $\delta^{ij} = \delta^{ji}$

You can also have delta with lower index

$$w \cdot v = \delta_{ij} w^i v^j$$

Chapter 8

Invariant Interval and Spacetime Metric

OMP:

Draw a pair of coordinate axes to represent x and ct, and draw the path of a light ray on those axes.

Everything gets fucked up in different reference frames, gee would it be nice to have a quantity thats the same in every reference frame.

$$\Delta s^2 \equiv c^2(\Delta t^2) - (\Delta x^2) - (\Delta y^2) - (\Delta z^2)$$

Not actually square! Just a thing called an invariant interval.

Synchronize Coordinate Systems

$$\Delta s^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

For Lorentz boosts along the x-axis

Try to find numbers such that the Lorentz boosted stuff equals the non-boosted stuff.

$$ct' = \gamma ct - \gamma\beta x, \qquad x' = \gamma\beta ct - \gamma x$$

Do algebra to get

$$(ct')^2 - (x')^2 = (ct^2) - x^2$$

Works in all combinations of all directions but the equation for just x motion is convenient

Lorentz Transformations and rotations are similar and both have invariant thingies I think

$$\Delta s^2 = (x^0)^2 - \vec{x} \cdot \vec{x}$$

$$\Delta s^2 > 0 \text{ timelike separation}$$

You can do stuff and things to make locations the same and only vary in time. Pick $v = x/t$

$$\Delta \tau = t' = \sqrt{\Delta s^2}/c$$

If $c^2 t^2 < x^2$ then you cannot do a Lorentz transformation because $x/t > c$

What we do is make them happen at the same time, but separated in space

$$\Delta L = x' = \sqrt{-\Delta s^2}$$

Space like events can never effect each other, so you can always find a frame such that one happened before the other (both happened before each other??)

$\Delta s^2 = 0 =$ Light-like separation

8.1 Minkowski Spacetime Diagram

Chapter 9

MIDTERM SHENANIGANS

Taylor series will be on the text pretty extensively

probably something with Lorentz transformations forming a group (the lorentz transformation matrix)

Taylor Series for Lorentz transformations (because v/c is close to 0)

Office hours on Monday

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \qquad \beta << 1$$

Find γ for $\beta \approx 0$ Using a goofy Taylor Series.

Chapter 10

Relativistic Energy, Momentum, and Force

OMP:
    Define force and energy in classical mechanics in terms of momentum, mass, and/or time derivatives.

10.1 Momentum

Momentum is useful because it is conserved in the absence of external forces. We want to define momentum is special relativity so that it remains conserved at relativistic speeds.

$$u_{B,S} = \frac{u_0}{\gamma}$$

This means something surely. Define relativistic momentum as:

$$p = \gamma m v$$

I am so tired i genuinely could not get relativity at the moment if i wanted to

$$\Delta p_{y,B} = \frac{2mu_0}{\gamma}$$

Force is just the regular force equation but you have to use relativistic momentum

$$F = \frac{d\mathbf{p}}{dt}$$

$$\frac{d}{dt}(\gamma m v) = m(\dot{\gamma}v + \dot{v}\gamma)$$

Do some bullshit calculus

$$F = \gamma^3 ma$$

im gonna kms

10.2   Relativistic Energy

$$\Delta E = \int \mathbf{F} \cdot d\mathbf{l}$$

line integral

$$\int_0^{v_f} m\gamma^3 v \, dv$$

Kinetic Energy??

$$KE = (\gamma - 1)mc^2$$

that mc^2 looks awfully familiar

sdfg';sdf hjsiupkjf,chzgxndsfz;j,mnadsf;

$$M = 2\gamma_u m$$

10.3   Acceleration and Rapidity

$$v(r) + dv = \frac{v(r) + a\,d\tau}{1 + ad\tau v(r)/c^2}$$

Chapter 11

Vector Shenanigans

$$p = \gamma m v \qquad F = \frac{dp}{dt}$$

You can do the same thing for multiple directions

$$F = \langle F_x, F_y \rangle \qquad \vec{F} = m\langle \gamma^3 a_x, \gamma a_y \rangle$$

She derived it on the blackboard using a bunch of funky calculus
   Know gradient and curl which are both things you already know. Also use other coordinate systems. I am not terribly worried I'll be completely honest.

Chapter 12

4-Vector

OMP:

Remind yourself of the Dopper Effect in classical mechanics. If a fire truck is driving towards you, do you hear the siren at a higher or lower frequency?

Answer:

Higher frequency.

12.1 Time 4 vector

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} \iff \nu'^{\mu} \Lambda^{\mu}_{\nu}$$

Chapter 13

Relativistic Collisions

OMP:

Write down the energy-momentum 4-vector for a particle of mass m in its own rest frame, in natural units where $c = 1$

13.1 Energy Unit

Electron volt

$$E_k = qU \rightarrow 1eV = 1.6 * 10^{-19}C \times 1V = 1.6 * 10^{-19}J$$

$$E_0 = mc^2 \iff E_0 = m \qquad m_e = 511keV \qquad m_p = 938MeV$$

Different types of decay (pion, beta, alpha)

$$\pi^0 \rightarrow \gamma + \gamma \qquad n \rightarrow p + e^- + \bar{\nu}_e \qquad U \rightarrow Th + He$$

Smash particles together as well (matter anti-matter anhiilliation, Higgs boson)

$$e^+ + e^- = \gamma + \gamma \qquad p + \bar{p} = H$$

Velocity is ugly and unfun, use energy and momentum instead and something good might happen.

Use invariance of relativistic dot products. Compute the same quantity in 2 different frames, and set the result equal.

A charged pion of mass M decays most of the time into a muon and a neutrino. The neutrino is so light that we can approximate it as massless. What is the energy of the outgoing neutrino in the rest frame of the pion?

$$\pi^+ \to \mu^+ + \nu$$

Step 1 i sassign energy-momentum 4-vector to each of the particles

$$p_1^\mu = p_2^\mu + p_3^\mu$$

$$p_1 = (M, \vec{0}) \qquad p_2 = (E_2, \vec{p_2}) \qquad p_3 = (E_3, \vec{p_3})$$

We don't care about the direction of momentum in the 4-vectors, just energy.

Do some math and get a dot product

$$p_1 - p_3 = p_2$$

$$(p_1 - p_3) \cdot (p_1 - p_3) = p_2 \cdot p_2$$

$$p_1 \cdot p_1 + p_3 \cdot p_3 - 2p_1 \cdot p_3 = p_2 \cdot p_2$$

$$p_1 \cdot p_1 = M^2 \qquad p_2 \cdot p_2 = m^2 \qquad p_3 \cdot p_3 = 0 \text{ bc massless}$$

$$p_1 \cdot p_3 = ME_3 - 0 \cdot p_3 = ME_3$$

$$M^2 - 2ME_3 = m^2$$

$$E_3 = \frac{M^2 - m^2}{2M}$$

Makes sense, right dimensions, if the particles are massless then $E_3 = M/2$ and if $m > M$ then you get a negative answer which makes sense because you'd be turning a particle into a heavier particle which breaks the laws of physics.

Two fast enough protons can create a new particle with enough energy

$$p + p \rightarrow p + p + p - p$$

The antiproton has the same mass m_p as the proton. What is the minimum amount of energy required for the reaction to occur?

(Key strategy: use both the laboratory frame and the center-of-momentum frame (CM), where the total momentum is zero))

$$p_1^\mu + p_2^\mu \rightarrow k_1^\mu + k_2^\mu + k_3^\mu + k_4^\mu$$

$$p_1 = (E_{lab}, \vec{p}_{lab}) \qquad p_2 = (m_2, \vec{0})$$

$$p_1 \cdot p_1 = p_2 \cdot p_2 = m_p^2 \qquad p_1 \cdot p_2 = E_{lab}m - \vec{0}\vec{p} = E_{lab}m$$

We could find the boost to figure out the CM frame but we dont have to

$$\vec{p}_1 + \vec{p}_2 = 0 \qquad \vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4 = 0$$

Because of conservation of momentum and the direction of the original protons is opposite so they cancel out.

$$(p_1 + p_2)^2 = (k_1 + k_2 + k_3 + k_4)^2$$

Evaluate left side in lab frame and right side in CM frame

$$(p_1 + p_2) \cdot (p_1 + p_2) = p_1 \cdot p_1 + p_2 \cdot p_2 + 2p_1 \cdot p_2 = 2m_p^2 + 2E_{lab}m_p$$

$$4m_p \cdot 4m_p - \vec{0} \cdot \vec{0} = 16m_p^2$$

$$2m_p^2 + 2E_{lab}m_p = 16m_p^2 \longrightarrow E_{lab} = 7m_p$$

If you have to know the velocity of a particle

$$p^\mu = (E, \vec{p}) = (\gamma m, \gamma m \vec{v}) \implies v = p/E$$

If you need to find the velocity of the CM frame in the rest frame

$$p_{tot}^\mu = p_1^\mu + p_2^\mu + p_3^\mu + \ldots = (E_{tot}, \vec{p}_{tot}) \implies v_{CM} = \frac{\vec{p}_{tot}}{E_{tot}}$$

OMP:

In our first example of pion decay, how would you rearrange $p_1 = p_2 + p_3$ to find the energy of p_2 (the muon)?

Chapter 14

Vector Calculus and Maxwell's Equations

OMP:

Which of Maxwell's equations involve a surface integral? Which ones involve a line integral?

Thank the lord I've already done calc 3 this shit ez.

14.1 Line Integrals

Imagine how a vector field acts upon an object as it moves along a line

$$\lim_{\Delta t \to 0} \sum_{i=1}^{n} \vec{F}(\vec{r}(t_i)) \cdot \Delta \vec{r}_i = \int_C \vec{F} \cdot d\vec{r}$$

Use the fundamental theorem of calculus and some shenanigans with vector fields to get

$$\int_C \nabla f \cdot d\vec{r} \equiv f(b) - f(a) \text{ for a curve going from a to b}$$

Line integrals are path independent. The line integral for a closed line along any conservative vector field is 0.

14.2 Surface Integral

$$\lim_{\Delta u, v \to 0} \sum_i \vec{F}(u, v) \cdot \vec{n}(u_i, v_i)_i \Delta u \Delta v \equiv \int_S \vec{F} \cdot d\vec{S}$$

$$\int_S \vec{F} \cdot d\vec{S} = \iint \vec{F}(\vec{r}(u, v)) \cdot \left(\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right) du\, dv$$

Where the goofy cross product is a normal vector relative to the surface

$$\int_R df = \oint_{\partial R} f$$

$$\int_V (\boldsymbol{\nabla} \cdot \vec{F}) d^3 x = \oint_{\partial V} \vec{F} \cdot dS$$

14.3 Stoke's Theorem

$$\int_S (\boldsymbol{\nabla} \times \vec{F}) \cdot d\vec{S} = \oint_{\partial S} \vec{F} d\vec{r}$$

$$\int_S (\boldsymbol{\nabla} \times (\nabla f)) \cdot d\vec{S} = \oint_{\partial S} \nabla f \cdot d\vec{r} = 0$$

The the curl of a gradient is ALWAYS 0

14.4   More Index Notation

$$\epsilon^{ijk} = \begin{cases} 1, (ij) = \text{ even permutation of } (1, 2, 3) \\ -1, (ijk) = \text{ odd permutation of } (1, 2, 3) \\ 0 \text{ otherwise} \end{cases}$$

This is called the (totally antisymmetric) Levi-Civita symbol. Swapping two adjacent number sturns a permutation from even to odd and vice versa.

Curl in index notation:

$$\boldsymbol{\nabla} \times F \equiv \epsilon^{ijk} \partial_j \vec{F}_k$$

$$\int_V (\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times A)) d^3x = \oint_{\partial V} (\boldsymbol{\nabla} \times A) \cdot d\vec{S} = \oint_{\partial \partial V} A??? = 0$$

Divergence of a curl MUST be 0

Consider the vector field $F = \frac{1}{r^2}\hat{r}$ in spherical coordinates. compute divergence

$$\boldsymbol{\nabla} \cdot F = \frac{1}{r^2}\frac{\partial r^2 F_r}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}(1) = 0$$

but if we compute the divergence on a general ball B_R of radius R, we get not 0

$$\int_V \delta^{(3)}(x) d^3x = \begin{cases} 1, \text{ V contains the origin} \\ 0 \text{ otherwise} \end{cases}$$

14.5 Gauss's Law

$$\frac{Q_{enc}}{\epsilon_0} = \oint E \cdot dS = \int \boldsymbol{\nabla} \cdot E \, dV \rightarrow \boldsymbol{\nabla} \cdot E = \frac{\rho}{\epsilon_0}$$

$$0 = \oint B \cdot dS$$

14.6 Faraday's Law

$$-\frac{d}{dt} \int B \cdot dS = \oint E \cdot dr = \int (\boldsymbol{\nabla} \times E) \cdot dS = -\frac{\partial B}{\partial t} dS$$

OMP:

To apply Stoke's Theorem, do we want to integrate a curl as a line integral or a surface integral?

Chapter 15

Complex Numbers and Fourier Transforms

OMP: In what areas of physics or math have you encountered complex numbers before, or is this your first time being introduced to them?

I know what a complex number is.

$$z = x + iy$$

Makes sure that polynomials have the sufficient amount of answers

You can add complex numbers by just combining like terms kinda similarly to a vector.

The best thing about multiplying complex numbers is that $i^2 = -1$ so you always get just a regular complex number instead of some shenanigans.

To divide complex numbers, you have to multiply the top and bottom of the fraction by the conjugate of the denominator

to make is nice and good.

$$\text{conjugate} \equiv z* \equiv \overline{z}$$

multiplying a number by its conjugate gets you the modulus or the magnitude of z

$$|z| = z * \times z = \sqrt{x^2 + y^2}$$

if $|z| = 1$ then we get nice shenanigans

$$z = x + iy = \cos\theta + i\sin\theta$$

proof via taylor series nonsense on the blackboard
   This gets you
$$e^{i\pi} = -1$$

You can also use this to take fractional powers of i

$$z = re^{i\pi} \qquad z* = re^{-i\pi}$$

   And we can use this trig shenanigans to graph complex numbers in polar form
   We can derive more goofy trig identities in complex forms which is currently being done on the whiteboard.
   Can be used to solve PDE's???
   Given some PDE

$$\frac{\partial^2 f(t, x)}{\partial t^2} = v^2 \frac{\partial^2 f(t, x)}{\partial x^2}$$

Guess a solution
$$f = Ae^{iwt}e^{ikx}???$$

Do some stuff and things to get something.

Get an integral of sdjilan j

$$f(x, t) = \int_{-\infty}^{\infty} = dk \widetilde{f}(k) e^{ikx} e^{ivkt}$$

Also works in more dimensions????

$$(\nabla^2 + C)f = 0$$

Where that thing is the Lapacian operator.

OMP: write i in polar form

Chapter 16

Maxwell's Equations and Special Relativity

OMP: name 1 topic from each category that you learned about in this course for the first time

Maxwell's Equations

$$\boldsymbol{\nabla}{\cdot}E = \frac{p}{\epsilon_0} \qquad \boldsymbol{\nabla}{\cdot}B = 0 \qquad \boldsymbol{\nabla}{\times}E = -\frac{\partial B}{\partial t} \qquad \boldsymbol{\nabla}{\times}B = \mu_0 J + \mu_0\epsilon_0\frac{\partial E}{\partial t}$$

Take curl of both sides of third equation to get

$$\boldsymbol{\nabla} \times \left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\boldsymbol{\nabla} \times B) = -\mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}$$

Fourier transform w 4d wave vector

$$E(t, \vec{x}) = E_0 e^{-i\vec{k}_\mu\vec{x}^\mu}$$