MATH 257

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Chapter 1

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My laptop died and I skipped some lectures to go to a part time job fair but I know every basic thing about matrices and vectors so I should be fine

Chapter 2

Column Vectors and Basis Vectors

If you take the columns of a vector, then you get a couple vectors that span a space.

Solving a linear system is the same as finding the linear combinations that equal a certain result

2.1 Matrix Vector Multiplication

$$\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = ac_1 + bc_2 + cc_3$$

2.2 Transformations

You can multiply a vector by a matrix to transform it in a certain way

2.2.1 Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

2.3 Elementary Matrices

An elementary matrix is a matrix gotten by doing a single elementary row operation on the identity matrix.

To find the inverse of an elementary matrix, you just do the opposite of the row operation to an identity matrix.

2.4 Invertible Matrices

Suppose A and B are invertible. Then:

- A^{-1} is invertible then $(A^{-1})^{-1} = A$
- AB is invertible if $(AB)^{-1} = A^{-1}B^{-1}$
- A^T is invertible iff $(A^T)^{-1} = (A^{-1})^T$

2.5 LU Decomposition

idk what it is but it's probably important

It stands for lower upper decomposition.

You can find a upper and lower triangular matrices L and U such that $A = LU$

You know a matrix can be decomposed if you can put the matrix in echelon form with just row operations from a higher row to a lower row.

2.5.1 How To Steps

1. Row reduce
2. Find elementary matrices $E_1, E_2 \ldots$
3. L $= E_1^{-1}, E_2^{-1}, \ldots$
4. U $=$ echelon form of original matrix that you already calculated

2.5.2 Solving a thingy

to solve $Ax = b$, you can solve $Ux = c$ such that $Lc = b$.

2.5.3 Inner Product

$$v \cdot w = v^T w$$

2.5.4 Norm

$$||v|| = \sqrt{v \cdot v}$$

2.5.5 Distance

$$\text{dist}(v, w) = ||v - w||$$

2.6 Orthogonality

if twe vectors are orthogonal or perpendicular to each other, then

$$v \cdot w = 0$$

2.6.1 Pairwise Orthogonal

A set of vectors is pairwise orthogonal if they are all orthogonal to each other.

2.6.2 Orthonormal Set

A set of unit vectors that are all orthogonal to each other.

2.7 Subsets/ Subspaces

A non-subset H of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$ if it satisfies the two following:

- if $u, v \in H$, then $u + v \in H$
  (closed under addition)
- if $u \in H$ and c is scalar, then $cu \in H$
  (closed under scalar multiplication)

subspaces are pretty useful

2.7.1 Column Space

The space created by spanning the columns of a matrix

2.7.2 Null Space

The space created by all the solutions of the equation $Ax = 0$.