1 Final Stuff

2 Lagrangian Stuff

If youre trying to minimize a certain parameter, you can write it as

$$S = \int \mathcal{L}(q, q'; t) dt$$

And then you plug that into the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = 0$$

You can also 100% just use the equation $\mathcal{L} = T - U$ to get the equation of motion.

2.1 $\ddot{q}$

if your lagrangian depends on even more derivatives of q, then you get

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial \mathcal{L}}{\partial \ddot{q}} = 0$$

You can get this from the same method as the normal E-L equation plus integrating by parts twice

3 Rocket Equation

$$F = ma \rightarrow ma = -u\frac{dm}{dt} - F_{ext}$$

4 N2L

F = ma

5 Fourier Everything

Realistically this is the main thing that I need to know

6 Integral bs

oh shit I need to write down the fuck ass trig integrals

7 Rotation and Orbit bs

idk this'll probably be important to me

8 Harmonic Oscillators

8.1 Damped

8.2 Undamped

8.3 Resonance

9 DERIVATIONS DERIVATIONS DERIVATIONS

9.1 Euler-Lagrange Equation

I know the vibes of how to do but I've never actually done it from scratch

A function is minimized when the change is 0 if you nudge it in either direction (critical point).

$$\delta S = \int \mathcal{L}(q + \delta q, \dot{q} + \delta \dot{q}; t) - \int \mathcal{L}(q, \dot{q}; t)$$

implement a first order taylor series

$$\mathcal{L}(q+\delta q, \dot{q}+\delta\dot{q};t) = \mathcal{L}(q,\dot{q};t) + \delta q\frac{\partial}{\partial q}L(q,\dot{q};t) + \delta\dot{q}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t) + \dots$$

$$\int \mathcal{L}(q+\delta q, \dot{q}+\delta\dot{q};t) - \int \mathcal{L}(q,\dot{q};t) = \int dt(\delta q\frac{\partial}{\partial q}L(q,\dot{q};t) + \delta\dot{q}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t)$$

get rid of $\delta\dot{q}$ using integration by parts

$$\int dt\delta\dot{q}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t) \qquad u = \frac{\partial}{\partial \dot{q}}L(q,\dot{q};t) \qquad v' = \delta\dot{q} \to v = \delta q$$

$$\delta q\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t)\Big|_{t2}^{t1} - \int dt\delta q\frac{d}{dt}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t) \to -\int dt\delta q\frac{d}{dt}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t)$$

$$\int dt(\delta q\frac{\partial}{\partial q}L(q,\dot{q};t) + \delta\dot{q}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t)) =$$

$$\int dt(\delta q\frac{\partial}{\partial q}L(q,\dot{q};t) - \delta q\frac{d}{dt}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t)) \to$$

$$\int dt(\frac{\partial}{\partial q}L(q,\dot{q};t) - \frac{d}{dt}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t))\delta q$$

because $\delta q \to 0$ we get our EL equation

$$\frac{\partial}{\partial q}L(q,\dot{q};t) - \frac{d}{dt}\frac{\partial}{\partial \dot{q}}L(q,\dot{q};t) = 0$$

9.2 Fourier Shenanigans

We're given the fourier series and transform formulae, its just a matter of implementing it in a way that doesn't suck.

I'm just going to do the same midterm problem that I missed earlier.

Given the harmonic oscillator equation (damped)

$$m\ddot{x} + c\dot{x} + kx = 0$$

Find that same function, except in terms of frequency instead

$$f(\omega) = \int_{\infty}^{\infty} f(t)e^{i\omega t}dt$$

$$m\int_{\infty}^{\infty} \ddot{x}e^{i\omega t}dt + c\int_{\infty}^{\infty} \dot{x}e^{i\omega t}dt + k\int_{\infty}^{\infty} xe^{i\omega t}dt = 0$$

do some integration by parts

$$c\int_{\infty}^{\infty} \dot{x}e^{i\omega t}dt \qquad v' = \dot{x} \to v = x \qquad u = e^{i\omega t} \to du = i\omega e^{i\omega t}$$

$$c(xe^{i\omega t}\Big|_{-\infty}^{\infty} - i\omega\int_{\infty}^{\infty} xe^{i\omega t}dt)$$

$$m\int_{\infty}^{\infty} \ddot{x}e^{i\omega t}dt \qquad v' = \ddot{x} \to v = \dot{x} \qquad u = e^{i\omega t} \to du = i\omega e^{i\omega t}$$

$$\dot{x}e^{i\omega t}\Big|_{-\infty}^{\infty} - i\omega\int_{\infty}^{\infty} \dot{x}e^{i\omega t}dt \to$$

$$\dot{x}e^{i\omega t}\Big|_{-\infty}^{\infty} - i\omega xe^{i\omega t}\Big|_{-\infty}^{\infty} + \omega^2\int_{\infty}^{\infty} xe^{i\omega t}dt$$

$$m\int_{\infty}^{\infty} \ddot{x}e^{i\omega t}dt + c\int_{\infty}^{\infty} \dot{x}e^{i\omega t}dt + k\int_{\infty}^{\infty} xe^{i\omega t}dt = 0 =$$

$$\dot{x}e^{i\omega t}\Big|_{-\infty}^{\infty} - i\omega xe^{i\omega t}\Big|_{-\infty}^{\infty} + \omega^2\int_{\infty}^{\infty} xe^{i\omega t}dt + xe^{i\omega t}\Big|_{-\infty}^{\infty} - i\omega\int_{\infty}^{\infty} xe^{i\omega t}dt$$

$$+k\int_{\infty}^{\infty} xe^{i\omega t}dt = 0$$

9.3 Angular momentum/Orbit Stuff

angular momentum is defined at $l = r \times v$

if angular momentum is conserved in a central force, then its derivative is 0

$$\frac{d}{dt}(r \times mv) = \frac{dr}{dt} \times v + r \times m\frac{dv}{dt} = v \times v + r \times F = 0 + 0$$

because F is in the direction of r because that's the definition of a central force.

10 WHAT TO WRITE DOWN

WRITE DOWN THE HARMONIC OSCILLATOR EQUATIONS

WRITE DOWN SOME TRIG INTEGRALS

WRITE DOWN SOME VECTOR STUFF

realistically there's no way I don't understand physics