PHYS 435 Midterm 1

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Chapter 1

PHYS435 Midterm 2

All magneto and electrostatics and maybe a little bit of electrodynamics with Ampere's Law.

REMEMBER YOU MAGNETIC DIPOLE EQUATIONS

and you magnetic field from a current equations

If you have both a current and a magnetization density, you can just put them together and try to solve.

All you do is solve both parts independently.

Solve the perma-magnetic with the magnetic pseudopotential and solve the moving current with the other math.

This midterm is open book open note so if I just write down absolutely every topic ever then I can look back at this notebook and be absolutely set

This midterm goes over basically all of electrostatics and magnetostatics

1.1 Calculus Junk

Know my Divergence and curls and junk

1.1.1 Divergence Theorem

Connects a volume to a surface

$$\iiint_V d^3V\, \nabla \cdot F = \iint_S d^2S\, \hat{n} \cdot F$$

1.1.2 Poisson's Equation and Green's Functions (Magnetic Pseudopotential)

We also do have a kind of Green's function approach with the magnetic pseudo-potential

$$\boldsymbol{\nabla} \cdot \vec{H} = -\boldsymbol{\nabla} \cdot \vec{M} \qquad \vec{H} = -\vec{\nabla}\phi_M(\vec{r})$$

$$\nabla^2\phi_m(\vec{r}) = -\rho_m(\vec{r}) = +\boldsymbol{\nabla} \cdot \vec{M}$$

1.2 Multipoles

They're like a just a heuristic thing to figure out roughly what you distance dependence is.

It's based off of a taylor series that I'm sure you can derive if you really want to.

monopole is 1/r, dipole is $1/r^2$, quadrupole is $1/r^3$

1.3 Dielectrics

I genuinely have no idea but there's some indentities and some polarizability stuff.

Dielectrics are not conductors, but are made of atoms that can conduct electricity.

Each atom has a symmetric charge distribution

Now apply an electric field so that the dielectric has a dipole moment.

$$\vec{p} = \alpha\vec{E}$$

Where α is the polarizability coefficient.

1.3.1 Spring Model of Atoms

The dipole moment is proportional to the displacement of the positive and negative charges relative to each other.

1.3.2 Polarization Density

Because dielectrics are like continuous

$$\vec{P} = \vec{p} \cdot \text{atomic density} = pn$$

$$\vec{P} = n\alpha\vec{E}$$

Consider an electronic suscebtilibility factor X

$$\vec{P} = \epsilon_0 X \vec{E} \qquad X = \frac{nq^2}{\epsilon_0 \kappa}$$

We find the polarizability constants of single atoms but I doubt that's super necessary

1.3.3 Bound Charge Density

I think this is finding the new charge density from a polarized dielectric

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r} - \vec{r'}|^3}$$

consider polarization density and $\vec{p} = \int d^3r \, \vec{P}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int\limits_V d^3r' \, \frac{\vec{P} \cdot \vec{r}}{|\vec{r} - \vec{r'}|^3}$$

we know that the derivative of $1/r$ is $-1/r^2 = -r/r^3$, so

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int\limits_V d^3r' \, \vec{P} \cdot -\vec{\nabla}_r \frac{1}{|\vec{r} - \vec{r'}|} = \frac{1}{4\pi\epsilon_0} \int\limits_V d^3r' \, \vec{P} \cdot \vec{\nabla}_{r'} \frac{1}{|\vec{r} - \vec{r'}|}$$

You do some math

$$\vec{\nabla}_{r'} \cdot \frac{\vec{P}}{|\vec{r} - \vec{r'}|} = \frac{\vec{\nabla} \cdot \vec{P}}{|\vec{r} - \vec{r'}|} + \vec{P} \cdot \vec{\nabla}_{r'} \cdot \frac{1}{|\vec{r} - \vec{r'}|}$$

and then get

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\limits_V d^3r'\,\vec{P}\cdot\vec{\nabla}_{r'}\frac{1}{|\vec{r}-\vec{r'}|} =$$

$$\frac{1}{4\pi\epsilon_0}\int\limits_V d^3r'\,\vec{\nabla}_{r'}\cdot\frac{\vec{P}}{|\vec{r}-\vec{r'}|} - \frac{1}{4\pi\epsilon_0}\int\limits_V d^3r'\,\frac{\vec{\nabla}\cdot\vec{P}}{|\vec{r}-\vec{r'}|}$$

Then do divergence theorem

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\limits_{\partial V} da\,\frac{\hat{n}\cdot\vec{P}}{|\vec{r}-\vec{r'}|} - \frac{1}{4\pi\epsilon_0}\int\limits_V d^3r'\,\frac{\vec{\nabla}\cdot\vec{P}}{|\vec{r}-\vec{r'}|}$$

With all of this shenanigans, we can say that

$$V = \frac{1}{4\pi\epsilon_0}\int\limits_V d^3r'\,\frac{p_b(\vec{r'})}{|\vec{r}-\vec{r'}|}$$

Where the bound charge density is given by

$$p_b(r) = -\vec{\nabla}\cdot\vec{P}(\vec{r})$$

and the surface bound charge density is given by

$$\sigma_b(\vec{r}) = \hat{n}\cdot\vec{p}(\vec{r})$$

So the total potential is given by integrating over the bulk bound charge and the surface bound charge.

1.3.4 $\vec{E}$ inside Dielectric

Consider a linear dielectric

$$\vec{p} = \alpha\vec{E} \qquad \vec{P} = n\alpha\vec{E} = \frac{\alpha}{a^3}\vec{E}$$

Define the susceptibility to polarization X

$$\vec{P} = \epsilon_0 X_E \vec{E} \qquad X_E = \frac{n\alpha}{\epsilon_0} = \frac{q^2 n}{\epsilon_0 \kappa}$$

A capacitor with a dielectric has both an external charge and a bound charge and a bound surface charge, the latter two are induced by both itself and the external charge.

$$E = \frac{\sigma_{total}}{\epsilon_0} \qquad \sigma_{tot} = \sigma_{Free} - p$$
$$E = \frac{1}{\epsilon_0}\left(\sigma_{free} - p\right) = \frac{1}{\epsilon_0}\left(\sigma_{free} - \epsilon_0 X\vec{E}\right)$$
$$\vec{E} = \frac{\sigma_{Free}}{\epsilon_0(1+X)} = \frac{\sigma_{Free}}{\epsilon_0\kappa}$$

There's some capacitance shenanigans but I don't really care at all

Consider a 3d bulk charge density

$$p_b(\vec{r}) = -\vec{\nabla} \cdot \vec{p}$$

This happens if X, the polarize susceptibility function, is position dependent.

The bound charges come from the dipole moment $\vec{p} = \epsilon_0 X\vec{E}$, which comes from the external electric field.
consider a total charge density

$$p_{tot} = p_{free} + p_{bound}$$

use Gauss's Law

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{1}{\epsilon_0}\left(p_{free}(\vec{r}) + p_{bound}(\vec{r})\right) = \frac{1}{\epsilon_0}\left(\vec{p}_{free}(\vec{r}) + \vec{\nabla} \cdot \vec{P}(\vec{r})\right)$$

$$\nabla \cdot \left(\vec{E}(\vec{r}) + \frac{1}{\epsilon_0}\vec{P}(\vec{r})\right) = \frac{p_f(\vec{r})}{\epsilon_0}$$

This equation is why we make the "Displacement field"

$$\vec{D}(\vec{r}) = \epsilon_0\vec{E} + \vec{P}(\vec{r})$$

I think that's the polarization density and not the dipole moment

$$\vec{\nabla} \cdot \vec{D}(\vec{r}) = \rho_{free}$$

In general nothing here should equal 0

1.3.5 Helmholtz Theorem

If you know the curl and divergence of a function, and the function does not diverge anywhere, then you know the function.

Consider a linear dielectric

$$\vec{p} = \epsilon_0 X \vec{E} \qquad \kappa = 1 + X$$
$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 X \vec{E} = \epsilon_0 \vec{E}(1 + X) = \epsilon_0 \vec{E} \kappa$$

$\vec{D}$ doesn't actually do anything, it just lets us do math easier.

If we have a parallel plate capacitor, and we consider the boundary between the bound charge density and the free charge density, we get

$$\hat{n} \cdot \vec{D}_1 - \hat{n} \cdot \vec{D}_2 = \sigma_{free}$$

If we have no external potential and thus $\sigma_{free} = 0$, then

$$\hat{n} \cdot \vec{D}_1 = \hat{n} \cdot \vec{D}_2 \rightarrow \kappa_1 \hat{n} \cdot \vec{E}_1 = \kappa_2 \hat{n} \cdot \vec{E}_2$$

This lets you determine the electric fields of dielectrics given both of their spring constants and some other stuff.

That is the end of lecture 16.

Chapter 2

NEW STUFF

2.1 Various Magnetic Fields

Everything can be solve with either Gauss's Law, superposition, or both.

- So you have your magnetic field $\vec{B}$
- You have your volume current density $\vec{J}$
- you have your surface current density $\vec{J}_b$
- you have your magnetization density $\vec{M}$
- You have your magnetic field density $\vec{A}$
- You have you helper field $\vec{H}$
- You have your magnetic pseudopotential ϕ_m
- You have a magnetic dipole susceptibility X_m

I think that covers all of it.

Then you use all the random equations that we learned to put them together

2.1.1 Coulomb Gauge

That just means that

$$\nabla^2 \vec{A} = -\mu \vec{J} \qquad \boldsymbol{\nabla} \cdot \vec{A} = 0$$

know that

$$\vec{B} = \boldsymbol{\nabla} \times \vec{A}$$

2.2 Magnetic Multipoles

I'm just gonna worry about the dipole moment for now

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}$$

dipole moment is calculated by

$$\vec{m} = \int d^3 r \, \frac{\vec{r} \times \vec{J}}{2}$$

and if we consider a current we get

$$\vec{m} = I * Area * \hat{n}$$

Consider a magnetization density vector field

$$\vec{M} = n(\vec{r})\vec{m}(\vec{r})$$

Where n is the local density

$$\vec{A} = \int d^3 \frac{\vec{M} \times \vec{r}}{r^3}$$

and we get the important

$$J_{bound} = \nabla \times \vec{M} \qquad K_{boundsurface} = \vec{M} \times \hat{n}$$

2.3 Boundary Value Problems

This one is connected to the homework 8 that I did and that I have the answers for

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_{free} + \vec{J}_{bound} \right)$$

That gets us the helper field

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \qquad \nabla \times \vec{H} = \mu_0 \vec{J}_{free}$$

That curl equation gives us some important stuff

$$\vec{H}_{1,//} + \vec{H}_{2,//} = \vec{K}_{free} \times \hat{n}$$

Where $\vec{K}_f$ is the free current density I think?

And, since $\boldsymbol{\nabla} \cdot \vec{B} = 0$, we get the other important

$$\hat{n}(\vec{B}_{1,\perp} + \vec{B}_{2,\perp}) = 0 \to \vec{B}_1 \cdot \hat{n} = \vec{B}_2 \cdot \hat{n}$$

For an ideal ferromagnet, there are no free currents and the bound currents are wholly determined by $\vec{M}$

There's a thingy

$$\vec{M} = X_m \hat{H}_{in} \qquad X_m = \frac{M}{H_{in}}$$

So for a linear paramagnet we get

$$\vec{B}_{in} = \mu_0(\vec{H}_{in} + \vec{M}) = \mu_0(1 + X_m)\vec{H}_{in}$$

okay I get it now so $\vec{J}$ is the volume current density which is amperes/m^2 and $\vec{K}$ is the surface current density which is just teslas / m.

2.4 Magnetic Pseudopotential

It's just a thing that connects to $\vec{H}$ that allows you to get something that looks like a Green's function.

2.5 UNITS

$\vec{B}$ is Volt-second / meters^2