PHYS 325

Aiden Sirotkine

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Contents

Chapter 1

PHYS325

I missed everything from the first 2 weeks because my laptop exploded whoopsies

Chapter 2

Equations of motion

derive the equations of motion to solve for a trajectory $\vec{r}(t)$ which is a position vector with respect to time

2.1 Newton's Second Law

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{r}(t)}{dt^2} \qquad \vec{F} = \frac{d\vec{p}(t)}{dt}$$

2.2 Strategy

1. choose a reference frame and coordinates
2. identify all the relevant forces (external forces)
   make a force diagram lol
3. integrate N2 for a given force $\vec{F}(\vec{r}, \dot{\vec{r}}, t)$ to find $\vec{r}(t)$

4. fix integration constants from inital or boundary conditions. (e.g. $\vec{v}_0 = \vec{v}(t = 0) = 0$)

5.

$$\vec{F} = 0$$
$$fromN2L0 = \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$
$$\rightarrow \vec{v} = const = \vec{v}_0$$
$$\vec{r}(t) = \int v_0 dt = v_0 t + r_0$$

6. if F is constant

$$\dot{\vec{v}} = \ddot{\vec{r}} = \vec{a} = \frac{\vec{F}_0}{m}$$
$$\int dv = \int \frac{F_0}{m} dt \rightarrow \vec{v}(t) = \frac{F_0}{m} t + \vec{v}_0$$
$$\vec{r}(t) = \int dr = \int \vec{v} dt =$$
$$\frac{1}{2}\frac{\vec{F}_0}{m} t^2 + \vec{v}_0 t + \vec{r}_0$$

only valid for constant force

2.2.1 Time Dependent Force

$$\frac{d\vec{v}}{dt} = \vec{a} = \frac{\vec{F}}{m}$$

separation of variables

$$d\vec{v} = \frac{\vec{F}(t)}{m}dt$$

$$\int d\vec{v} = \int \frac{\vec{F}(t)}{m}dt$$

$$\vec{v}(t) = \frac{\vec{F}}{m} + \vec{C} \qquad \vec{F} = \int F(t)dt$$

C is the integration constant

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{r} = \int \vec{v}dt$$

2.2.2 Example: Forced Harmonic Oscillator

A particle m moves along $-\infty < x < \infty$. It is subjected to a force $F = F_0 \cos(\alpha t)$. It starts at time $t = 0, x_0 = x(t = 0) = 0, v_0 = v(t = 0) = 0$

1. coordinate system is just 1D
2. force is $F = F_0 \cos(\alpha t)$

3. equation of motion from N2L is $\frac{dv}{dt} = a = \frac{F}{m}$

4. separation of variables

$$dv = \frac{F}{m}dt = \frac{F_0}{m}dt = \frac{F_0}{m}\cos\alpha t dt$$
$$v(t) = \int dv = \frac{F_0}{m}\int \cos(\alpha t)dt = \frac{F_0}{m}\frac{1}{2}\sin\alpha t + C_1$$
$$x(t) = intdx = \int vdt =$$
$$\int \frac{F_0}{\alpha m}\sin(\alpha t) + C_1 dt =$$
$$-\frac{F_0}{\alpha^2 m}\cos(\alpha t) + C_1(t) + C_2$$

5. find initial conditions

$$0 = v_0 = \frac{F_0}{\alpha m}\sin\alpha 0 + C_1 \rightarrow C_1 = 0$$
$$0 = x_0 = -\frac{F_0}{\alpha^2 m}\cos(\alpha 0) + 0 + C_2 \rightarrow$$
$$C_2 = \frac{F_0}{\alpha^2 m}$$
$$x(t) = \frac{F_0}{\alpha^2 m}(1 - \cos(\alpha t))$$
$$v(t) = \frac{F_0}{\alpha m}\sin(\alpha t)$$

2.3 Position Dependent Force

focusing on 1 dimension for simplicity

get the equation of motion from Newton's 2nd Law

$F(x) = ma = m\frac{dv}{dt}$

chain rule

$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$

$mv\frac{dv}{dx} = F(x)$

separation of variables

$mv\,dv = F(x)dx$

Use definite integrals. relabel v = v' and x = x' (not derivatives)

$m\int_{v_0}^{v} v'\,dv' = \int_{x_0}^{x} F(x')\,dx'$

$\frac{1}{2}m(v^2 - v_0^2) = \int_{x_0}^{x} F(x')\,dx'$

solve for v

It looks like change in kinetic energy and work

$\Delta T = \frac{1}{2}m(v^2 - v_0^2) \qquad W = \int F(x)\,dx$

if force is conservative, it is path independent,

and it can be written as a gradient of a potential $\vec{F} = -\nabla U$

$F = -\frac{dU}{dx}$

$T - T_0 = \int_{x_0}^{x} -\frac{dU}{dx}\,dx' = -\left(U(x) - U(x_0)\right)$

$E = T + U(x) = T_0 + U(x_0)$

Conservation of Mechanical Energy

$E = T + U(x) = \frac{1}{2}mv^2 + U(x)$

$v = \pm\sqrt{\frac{2}{m}(E - U(x))}$

use $v = \frac{dx}{dt}$ to find $x(t)$

2.4 Analyzing the Potential

infer velocity and position

given energy, what is motion

$E = E_0 = E(x_0) = T(x_0) + U(x_0)$

$U(x_0) = E_0 = \text{const} \to T(x_0) = 0 \to v(x_0) = C$

2.4.1 case 2: energy of particle is _

$E = E_1 = T(x) + U(x)$

particle can move between x_{1a} and x_{1b}

$x_0 : E_1 = E = T + U(x_0)$

potential energy is at minimum, kinetic at max

$v(x_0)$ is maximal

2.4.2 Case 3: $E = E_2 = U(x_2)$

$T(x_2) = 0$ so $v = 0$